

MATHS

BOOKS - VIKAS GUPTA MATHS (HINGLISH)

AREA UNDER CURVES

EXERCISE (SINGLE CHOICE PROBLEMS)

1. The area enclosed by the curve

$$[x + 3y] = [x - 2] \text{ where } x \in [3, 4] \text{ is :}$$

(where $[.]$ denotes greatest integer function)

A. $\frac{2}{3}$

B. $\frac{1}{3}$

C. $\frac{1}{4}$

D. 1

Answer: B



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2. The area of the region enclosed by $y = x^2$ and $y = \sqrt{|x|}$ is

A. $\frac{1}{3}$

B. $\frac{2}{3}$

C. $\frac{4}{3}$

D. $\frac{16}{3}$

Answer: B



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3. Find the area enclosed by the figure described by the equation

$$x^4 + 1 = 2x^2 + y^2.$$

A. 2

B. $\frac{16}{3}$

C. $\frac{8}{3}$

D. $\frac{4}{3}$

Answer: C



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4. The area defined by $|y| \leq e^{|x|} - \frac{1}{2}$ in cartesian co-ordinate system, is :

A. $(2 - 2 \ln 2)$

B. $(4 - \ln 2)$

C. $(2 - \ln 2)$

D. $(2 - 2 \ln 2)$

Answer: D



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5. For each positive integer $n > a$, A_n represents the area of the region restricted to the following two inequalities :

$$\frac{x^2}{n^2} + y^2 \text{ and } x^2 + \frac{y^2}{n^2} < 1. \text{ Find } \lim_{n \rightarrow \infty} A_n.$$

A. 4

B. 1

C. 2

D. 3

Answer: A



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6. Find the ratio in which the area bounded by the curves $y^2 = 12x$ and $x^2 = 12y$ is divided by the line $x = 3$.

A. 7:15

B. 15:49

C. 1:3

D. 17:49

Answer: B

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7. The value of positive real parameter 'a' such that area of region bounded by parabolas $y = x - ax^2$, $ay = x^2$ attains its maximum value is equal to :

A. $\frac{1}{2}$

B. 2

C. $\frac{1}{3}$

D. 1

Answer: D

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8. For $0 < r < 1$, let n_r denotes the line that is normal to the curve $y = x^{\textcircled{r}}$ at the point $(1, 1)$ Let S_r denotes the region in the first quadrant bounded by the curve $y = x^r$, the x-axis and the line n_r '

Then the value of r the minimizes the area of S_r is :

- A. $\frac{1}{\sqrt{2}}$
- B. $\sqrt{2} - 1$
- C. $\frac{\sqrt{2} - 1}{2}$
- D. $\sqrt{2} - \frac{1}{2}$

Answer: B

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9. The area bounded by $|x| = 1 - y^2$ and $|x| + |y| = 1$ is:

- A. $\frac{1}{3}$
- B. $\frac{1}{2}$
- C. $\frac{2}{3}$

D. 1

Answer: C

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10. Point A lies on the curve $y = e^{x^2}$ and has the coordinate (x, e^{-x^2}) where $x > 0$. Point B has the coordinates $(x, 0)$. If 'O' is the origin, then the maximum area of the $\triangle AOB$ is

A. $\frac{1}{\sqrt{8}e}$

B. $\frac{1}{\sqrt{4}e}$

C. $\frac{1}{\sqrt{2}e}$

D. $\frac{1}{\sqrt{e}}$

Answer: A

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11. If the area enclosed between the curves $y = ax^2$ and $x = ay^2$ ($a > 0$) is 1 square unit, then find the value of a .

- A. $\frac{1}{\sqrt{3}}$
- B. $\frac{1}{2}$
- C. 1
- D. $\frac{1}{3}$

Answer: D



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12. Let $f(x) = x^3 - 3x^2 + 3x + 1$ and g be the inverse of it, then area bounded by the curve $y = g(x)$ with x-axis between $x = 1$ to $x = 2$ is (in square units):

- A. $\frac{1}{2}$
- B. $\frac{1}{4}$
- C. $\frac{3}{4}$

D. 1

Answer: B

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13. Area bounded by $x^2y^2 + y^4 - x^2 - 5y^2 + 4 = 0$ is equal to :

A. $\frac{4\pi}{2} + \sqrt{2}$

B. $\frac{4\pi}{3} - \sqrt{2}$

C. $\frac{4\pi}{3} - \sqrt{2}$

D. none of these

Answer: C

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14. Let $f: \mathbb{R} \rightarrow \mathbb{R}^+$ is an invertible function such that $f'(x) > 0$ and $f''(x) > 0 \forall x \in [1, 5]$. If $f(1) = 1$ and $f(5) = 5$ and

area under the curve $y = f(x)$ on x-axis from $x = 1 \rightarrow x = 5$ is 8 sq. units, then area bounded by $y = f^{-1}(x)$ on x-axis from $x = 1 \rightarrow x = 5$ is

a. 8 b. 12 c. 16 d. 20

A. 12

B. 16

C. 18

D. 20

Answer: B

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15. A circle centered at origin and having radius π units is divided by the curve $y = \sin x$ in two parts. Then area of the upper part equals to

:

A. $\frac{\pi^2}{2}$

B. $\frac{\pi^3}{4}$

C. $\frac{\pi^3}{2}$

D. $\frac{\pi^3}{8}$

Answer: C

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16. The area of the loop formed by $y^2 = x(1 - x^3)$ dx is:

A. $\int_0^1 \sqrt{x - x^4} dx$

B. $2 \int_0^1 \sqrt{x - x^4} dx$

C. $\int_{-1}^1 \sqrt{x - x^4} dx$

D. $4 \int_0^{1/2} \sqrt{x - x^4} dx$

Answer: B

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17. If $f(x) = \min \left[x^2, \sin \frac{x}{2}, (x - 2\pi)^2 \right]$, the area bounded by the curve $y = f(x)$, x-axis, $x = 0$ and $x = 2\pi$ is given by

Note: x_1 is the point of intersection of the curves x^2 and $\sin \frac{x}{2}$, x_2 is the point of intersection of the curves $\sin \frac{x}{2}$ and $(x - 2\pi)^2$

A.

$$\int_0^{x_1} \left(\sin \frac{x}{2} \right) dx + \int_{x_1}^{\pi} x^2 dx + \int_{\pi}^{x_2} (x - 2\pi)^2 dx + \int_{x_2}^{2\pi} \left(\sin \frac{x}{2} \right) dx$$

B. $\int_0^{x_1} x^2 dx + \int_{x_1}^{x_3} \left(\sin \frac{x}{2} \right) dx + \int_{x_2}^{2\pi} (x - 2\pi)^2 dx$, where $x_1 \in \left(0, \frac{\pi}{3} \right)$ and $x_2 \in \left(\frac{5\pi}{3}, 2\pi \right)$

C. $\int_0^{x_1} x^2 dx + \int_{x_1}^{x_2} \sin \left(\frac{x}{2} \right) dx + \int_{x_2}^{2\pi} (x - 2\pi)^2 dx$, where $x_1 \in \left(\frac{\pi}{3}, \frac{\pi}{2} \right)$ and $x_2 \in \left(\frac{3\pi}{2}, 2\pi \right)$

D. $\int_0^{x_1} x^2 dx + \int_{x_1}^{x_2} \sin \left(\frac{x}{2} \right) dx + \int_{x_2}^{2\pi} (x - 2\pi)^2 dx$, where $x_1 \in \left(\frac{\pi}{2}, \frac{2\pi}{3} \right)$ and $x_2 \in (\pi, 2\pi)$

Answer: B

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18. The area enclosed between the curves

$$|x| + |y| \geq 2 \text{ and } y^2 = 4\left(1 - \frac{x^2}{9}\right) \text{ is :}$$

- A. $(6\pi - 4)$ sq. units
- B. $(6\pi - 8)$ se. units
- C. $(3\pi - 4)$ se. units
- D. $(3\pi - 2)$ sq. units

Answer: B

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EXERCISE (ONE OR MORE THAN ONE ANSWER IS/ARE CORRECT)

1. $T_n = \sum_{r=2n}^{3n-1} \frac{r}{r^2 + n^2}, S_n = \sum_{r=2n+1}^{3n} \frac{r}{r^2 + n^2},$ then

$\forall n \in \{1, 2, 3, \dots\}$:

- A. $T_n > \frac{1}{2} \ln 2$
- B. $S_n < \frac{1}{2} \ln 2$

C. $T_n < \frac{1}{2} \ln 2$

D. $S_n > \frac{1}{2} \ln 2$

Answer: A::D

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2. If a curve $y = a\sqrt{x} + bx$ passes through point $(1, 2)$ and the area bounded by curve, line $x = 4$ and x-axis is 6, then :

A. $a = \frac{15}{4}$

B. $b = 3$

C. $a = -1$

D. $b = -\frac{7}{4}$

Answer: A::D

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3. Area enclosed by the curves $y = x^{2+1}$ and a normal drawn to it with gradient -1, is equal to:

A. $\frac{2}{3}$

B. $\frac{1}{3}$

C. $\frac{3}{4}$

D. $\frac{4}{3}$

Answer: D

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AXERCISE (COMPREHENSION TYPE PROBLEMS)

1. Let $f: A \rightarrow B$ $f(x) = \frac{x + a}{bx^2 + cx + 2}$, where A represent domain set and B represent range set of function $f(x)$ $a, b, c \in R$, $f(-1) = 0$ and $y = 1$ is an asymptote of $y = f(x)$ and $y = g(x)$ is the inverse of $f(x)$.
g(0) is equal to :

A. -1

B. -3

C. $-\frac{5}{2}$

D. $-\frac{3}{2}$

Answer: A



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2. Let $f: A \rightarrow B$ $f(x) = \frac{x + a}{bx^2 + cx + 2}$, where A represent domain set and B represent range set of function $f(x)$ $a, b, c \in R$, $f(-1) = 0$ and $y = 1$ is an asymptote of $y = f(x)$ and $y = g(x)$ is the inverse of $f(x)$.

Area bounded between the curves $y = f(x)$ and $y = g(x)$ is:

A. $2\sqrt{5} + \ln\left(\frac{3 - \sqrt{5}}{5 + \sqrt{5}}\right)$

B. $2\sqrt{5} + 2 \ln\left(\frac{3 + \sqrt{5}}{3 - \sqrt{5}}\right)$

C. $3\sqrt{5} + 4 \ln\left(\frac{3 - \sqrt{5}}{3 + \sqrt{5}}\right)$

D. $3\sqrt{5} + 2 \ln \left(\frac{3 - \sqrt{5}}{3 + \sqrt{5}} \right)$

Answer: D

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3. Let $f: A \rightarrow B$ $f(x) = \frac{x + a}{bx^2 + cx + 2}$, where A represent domain set and B represent range set of function $f(x)$ $a, b, c \in R$, $f(-1) = 0$ and $y = 1$ is an asymptote of $y = f(x)$ and $y = g(x)$ is the inverse of $f(x)$.

Area of region enclosed by asymptotes of curves $y = f(x)$ and $y = g(x)$ is:

- A. 4
- B. 9
- C. 12
- D. 25

Answer: B



4. For $j = 0, 1, 2, \dots, n$ let S_j be the area of region bounded by the x-axis and the curve $ye^x = \sin x$ for $j\pi \leq x \leq (j+1)\pi$

The value of S_0 is :

A. $\frac{1}{2}(1 + e^x)$

B. $\frac{1}{2}(1 + e^{-\pi})$

C. $\frac{1}{2}(1 - e^{-\pi})$

D. $\frac{1}{2}(e^\pi - 1)$

Answer: B

5. For $j = 0, 1, 2, \dots, n$ let S_j be the area of region bounded by the x-axis and the curve $ye^x = \sin x$ for $j\pi \leq x \leq (j+1)\pi$

The ratio $\frac{S_{2009}}{S_{2010}}$ equals :

A. e^{-x}

B. e^π

C. $\frac{1}{2}e^x$

D. $2e^\pi$

Answer: B

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6. For $j = 0, 1, 2, \dots, n$ let S_j be the area of region bounded by the x-axis and the curve $ye^x = \sin x$ for $j\pi \leq x \leq (j+1)\pi$

The value of $\sum_{j=0}^{\infty} S_j$ equals to :

A. $\frac{e^x(1 + e^x)}{2(e^\pi - 1)}$

B. $\frac{1 + e^\pi}{2(e^\pi - 1)}$

C. $\frac{1 + e^\pi}{e^\pi - 1}$

D. $\frac{e^\pi(1 + e^\pi)}{(e^\pi - 1)}$

Answer: B

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EXERCISE (SUBJECTIVE TYPE PROBLEMS)

1. Let f be a differentiable function satisfying the condition $f\left(\frac{x}{y}\right) = \frac{f(x)}{f(y)}$ ($y \neq 0, f(y) \neq 0$) $\forall x, y \in R$ and $f'(1) = 2$. If the smaller area enclosed by $y = f(x), x^2 + y^2 = 2$ is A , then find $[A]$, where $[.]$ represents the greatest integer function.

Answer: 1

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2. Let $f(x)$ be a function which satisfy the equation $f(xy) = f(x) + f(y)$ for all $x > 0, y > 0$ such that $f'(1) = 2$. Let A be the area of the region bounded by the curves $y = f(x), y = |x^3 - 6x^2 + 11x - 6|$ and $x = 0$, then find value

of $\frac{28}{17}A$.

Answer: 7

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3. If the area bounded by circle $x^2 + y^2 = 4$, the parabola $y = x^2 + x + 1$ and the curve $y = \left[\sin^2 \frac{x}{4} + \cos \frac{x}{4} \right]$, (where $[\]$ denotes the greatest integer function) and x-axis is $\left(\sqrt{3} + \frac{2\pi}{3} - \frac{1}{k} \right)$, then the numerical quantity is should be :

Answer: 6

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4. Let the function $f: [-4, 4] \rightarrow [-1, 1]$ be defined implicitly by the equation $x + 5y - y^5 = 0$. If the area of triangle formed by tangent and normal to $f(x)$ at $x = 0$ and the line $y = 5$ is A , find $\frac{A}{13}$.

Answer: 5

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5. Area of the region bounded by $[x]^2 = [y]^2$, if $x \in [1, 5]$, where $[]$ denotes the greatest integer function is:

Answer: 8

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6. Consider $y = x^2$ and $f(x)$ where $f(x)$, is a differentiable function satisfying

$$f(x+1) + f(z-1) = f(x+z) \forall x, z \in R \text{ and } f(0) = 0, f'(0) = 1$$

If area bounded by curve $y = x^2$ and $y = f(x)$ is Δ , find the value of $\left(\frac{3}{16}\Delta\right)$.

Answer: 2

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7. The least integer which is greater than or equal to the area of region in $x - y$ plane satisfying $x^6 - x^2 + y^2 \leq 0$ is:

Answer: 2



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8. The set of points (x,y) in the plane satisfying $x^{2/5} + |y| = 1$ form a curve enclosing a region of area $\frac{p}{q}$ square units, when p and q are relatively prime positive integers. Find $p - q$.

Answer: 1



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