

MATHS

BOOKS - VIKAS GUPTA MATHS (HINGLISH)

BINOMIAL THEOREM

Exercise 1 : Single problems

1. Let

$$N = 2^{1224} - 1, \alpha = 2^{153} + 2^{77} + 1 \text{ and } \beta = 2^{408} - 2^{204} + 1.$$

Then which of the following statement is correct ?

- A. α divides N but β does not
- B. β divides N but α does not
- C. α and β both divide N
- D. neither α nor β divides N

Answer: C



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2. If $(1 + x + x^2)^n = \sum_{r=0}^{2n} a_r x^r$, then $a_r - {}^n C_1 \cdot a_{r-1} + {}^n C_2 a_{r-2} - {}^n C_3 a_{r-3} + \dots + (-1)^r {}^n C_r a_0$ is equal to : (r is not multiple of 3)

A. 0

B. ${}^n C_r$

C. a_r

D. 1

Answer: A



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3. The coefficient of the middle term in the binomial expansion in powers of x of $(1 + \alpha x)^4$ and of $(1 - \alpha x)^6$ is the same, if α equals $-\frac{5}{3}$ b. $\frac{10}{3}$ c. $-\frac{3}{10}$ d. $\frac{3}{5}$

A. $-\frac{5}{3}$

B. $\frac{3}{5}$

C. $-\frac{3}{10}$

D. $\frac{10}{3}$

Answer: C

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4. If $(1 + x)^{2010} = C_0 + C_1x + C_2x^2 + \dots + C_{2010}x^{2010}$

then the sum of series $C_2 + C_5 + C_8 + \dots + C_{2009}$ equals to

:

A. $\frac{1}{2}(2^{2010} - 1)$

B. $\frac{1}{3}(2^{2010} - 1)$

C. $\frac{1}{2}(2^{2009} - 1)$

D. $\frac{1}{3}(2^{2009} - 1)$

Answer: B



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5. Let $\alpha_n = (2 + \sqrt{3})^n$. Find $\lim_{n \rightarrow \infty} (\alpha_n - [\alpha_n])$ ($[.]$ denotes greatest integer function)

A. 1

B. $\frac{1}{2}$

C. $\frac{1}{3}$

D. $\frac{2}{3}$

Answer: A



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6. The number $N = {}^{20}C_7 - {}^{20}C_8 + {}^{20}C_9 - {}^{20}C_{10} + \dots - {}^{20}C_{20}$ is not divisible by :

A. 3

B. 7

C. 11

D. 19

Answer: C

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7. The value of the expression $\log_2 \left(1 + \frac{1}{2} \sum_{k=1}^{11} {}^{12}C_k \right)$:

A. 11

B. 12

C. 13

D. 14

Answer: A

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8. The constant term in the expansion of $\left(x + \frac{1}{x^3}\right)^{12}$ is :

- A. 26
- B. 169
- C. 260
- D. 220

Answer: D

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9. If $\frac{3}{4!} + \frac{4}{5!} + \frac{5}{6!} + \dots + 50\text{term} = \frac{1}{3!} - \frac{1}{(k-3)!}$, then

sum of coefficients in the expansion

$(1 + 2x_1 + 3x_2 + \dots + 100x_{100})^k$ is:

- A. $(5050)^{49}$
- B. $(5050)^{51}$
- C. $(5050)^{52}$

D. $(5050)^{50}$

Answer: D

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10. Statement-1: The remainder when $(128)^{(128)^{128}}$ is divided by 7 is 3. because Statement-2: $(128)^{128}$ when divided by 3 leaves the remainder 1.

- A. Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
- B. Statement-1 is true, statement-2 is true and statement-2 is not the correct explanation for statement-1.
- C. Statement-1 is true, statement-2 is false.
- D. Statement-1 is false, statement-2 is true.

Answer: D

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11. If $n > 3$, then $xyz^n C_0 - (x-1)(y-1)(z-1)^n C_1 + (x-2)(y-2)(z-2)^n C_2 - (x-3)(y-3)(z-3)^n C_3 + \dots + (-1)^n (x-n)(y-n)(z-n)^n C_n$ equals :

- A. xyz
- B. $x + y + z$
- C. $xy + yz + zx$
- D. 0

Answer: D

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12. If $\alpha_1, \alpha_2, \dots, \alpha_n$ are the n, n^{th} roots of unity, $\alpha_r = e^{\frac{i2(r-1)\pi}{n}}, r = 1, 2, \dots, n$ then ${}^n C_1 \alpha_1 + {}^n C_2 \alpha_2 + \dots + {}^n C_n \alpha_n$ is equal to :

- A. $\left(1 + \frac{\alpha_2}{\alpha_1}\right)^n - 1$

B. $\frac{\alpha_1}{2} [(1 + \alpha_1)^n - 1]$

C. $\frac{\alpha_1 + \alpha_{n-1} - 1}{2}$

D. $(\alpha_1 + \alpha_{n-1})^n - 1$

Answer: A



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13. The remainder when $2^{30} \cdot 3^{20}$ is divided by 7 is :

A. 1

B. 2

C. 4

D. 6

Answer: B



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14. ${}^{26}C_0 + {}^{26}C_1 + {}^{26}C_2 + \dots + {}^{26}C_{13}$ is equal to :

A. $2^{25} - \frac{1}{2} \cdot {}^{26}C_{13}$

B. $2^{25} + \frac{1}{2} \cdot {}^{26}C_{13}$

C. 2^{13}

D. $2^{26} + \frac{1}{2} \cdot {}^{26}C_{13}$

Answer: B

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15. If a_r is the coefficient of x^r in the expansion of $(1 + x + x^2)^n$ ($n \in N$). Then the value of

$(a_1 + 4a_4 + 7a_7 + 10a_{10} + \dots)$ is equal to :

A. 3^{n-1}

B. 2^n

C. $\frac{1}{3} \cdot 2^n$

D. $n \cdot 3^{n-1}$

Answer: D

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16. Let $\binom{n}{k}$ represents the combination of ' n ' things taken ' k ' at a time, then the value of the sum $\binom{99}{97} + \binom{98}{96} + \binom{97}{95} + \dots + \binom{3}{1} + \binom{2}{0}$ equals-

- A. $\binom{99}{97}$
- B. $\binom{100}{98}$
- C. $\binom{99}{98}$
- D. $\binom{100}{97}$

Answer: D

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17. The last digit of $9! + 3^{9966}$ is :

A. 1

B. 3

C. 7

D. 9

Answer: D



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18. Let x be the 7^{th} term from the beginning and y be the 7^{th} term from the end in the expansion of $\left(3^{1/3} + \frac{1}{4^{1/3}}\right)^n$. If $y = 12x$ then the value of n is :

A. 9

B. 8

C. 10

D. 11

Answer: A



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19. ${}^{10}(C_0)^2 - {}^{10}(C_1)^2 + {}^{10}(C_2)^2 - \dots - ({}^{10}C_9)^2 + ({}^{10}C_{10})^2 =$

A. $10!$

B. $({}^{10}C_5)^2$

C. $-{}^{10}C_5$

D. ${}^{10}C_5$

Answer: C



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20. Find the ratio of the coefficient of x^{15} to them independent of x in the expansion of $\left(x^2 + \frac{2}{x}\right)^{15}$.

A. $1:4$

B. 1 : 32

C. 7 : 64

D. 7 : 16

Answer: B



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21. In the expansion of $(1 + x)^2(1 + y)^3(1 + z)^4(1 + w)^5$, the sum of the coefficient of the terms of degree 12 is :

A. 61

B. 71

C. 81

D. 91

Answer: D



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$$22. \text{ If } \sum_{r=0}^n \left(\frac{r^3 + 2r^2 + 3r + 2}{r + 1} \right)^n C_r = \frac{2^4 + 2^3 + 2^2 - 2}{3}$$

A. 2

B. 2^2

C. 2^3

D. 2^4

Answer: A

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Exercise-2 : One or More than One Answer is/are Correct

1. The number

$N = {}^{20}C_7 - {}^{20}C_8 + {}^{20}C_9 - {}^{20}C_{10} + \dots - {}^{20}C_{20}$ is divisible by

:

A. 3

B. 4

C. 7

D. 19

Answer: A::B::C::D



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2.

If

$$(1 + x + x^2 + x^3)^{100} = a_0 + a_1 + a_2x^2 + \dots + a_{300}x^{300},$$

then

A. $a_1 = 100$

B. $a_0 + a_1 + a_2 + \dots + a_{300}$ is divisible by 1024

C. coefficients equidistant from beginning and end are equal

D.

$$a_0 + a_2 + a_4 + \dots + a_{300} = a_1 + a_3 + a_5 + \dots + a_{299}$$

Answer: A::B::C::D

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3. $\sum_{r=0}^4 (-1)^r {}^{16}C_r$ is divisible by :

A. 5

B. 7

C. 11

D. 13

Answer: A::B::D

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4. Arrange the expansion of $\left(x^{1/2} + \frac{1}{2x^{1/4}}\right)$ in decreasing powers of x . Suppose the coefficient of the first three terms form an arithmetic progression. Then the number of terms in the expression having integer powers of x is -

A. 0

B. 2

C. 4

D. 8

Answer: A::C::D



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5. Let $(1 + x^2)^2 \cdot (1 + x)^n = \sum_{k=0}^{n+4} a_k \cdot x^k$ If a_1, a_2 and a_3 are in

AP , find n .

A. 6

B. 4

C. 3

D. 2

Answer: B::C::D



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$$6. \sum_{i=0}^n \sum_{j=0}^n \sum_{k=0}^n \binom{n}{i} \binom{n}{j} \binom{n}{k}, \binom{n}{r} = {}^n C_r:$$

- A. is less than 500 if $n = 3$
- B. is greater than 600 if $n = 3$
- C. is less than 5000 if $n = 4$
- D. is greater than 4000 if $n = 4$

Answer: C::D

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$$7. \text{ If } {}^{100}C_6 + 4. \quad {}^{100}C_7 + 6. \quad {}^{100}C_8 + 4. \quad {}^{100}C_9 + {}^{100}C_{10}$$

has the value equal to ${}^x C_y$, then the possible value (s) of $x + y$ can be :

- A. 112

B. 114

C. 196

D. 198

Answer: B::D



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8. If the co-efficient of x^{2r} is greater than half of the co-efficient of x^{2r+1} in the expansion of $(1 + x)^{15}$, then the possible value of 'r' equal to :

A. 5

B. 6

C. 7

D. 8

Answer: A::B::C



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9. Let $f(x) = 1 + x^{111} + x^{222} + x^{333} \dots \dots \dots + x^{999}$ then

$f(x)$ is divisible by

A. $x + 1$

B. x

C. $x - 1$

D. $1 + x^{222} + x^{444} + x^{666} + x^{888}$

Answer: A::D

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Exercise-4 : Subjective Type Problems

1. The sum of series

$$3 \cdot {}^{2007}C_0 - 8 \cdot {}^{2007}C_1 + 13 \cdot {}^{2007}C_2 - 18 \cdot {}^{2007}C_3 + \dots \dots \dots \text{upto}$$

2008 terms is K, then K is :

Answer: 0



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2. In the polynomial function

$$f(x) = (x - 1)(x^2 - 2)(x^3 - 3) \dots \dots (x^{11} - 11) \quad \text{the}$$

coefficient of x^{60} is :

Answer: 1



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3. If

$$\sum_{r=0}^{3n} a_r(x - 4)^r = \sum_{r=0}^{3n} A_r(x - 5)^r \quad \text{and} \quad a_k = 1 \quad \forall K \geq 2n \quad \text{and} \quad \sum_{r=0}^{3n}$$

$$\sum_{r=0}^{3n} B_r(x - 9)^r \quad \text{and} \quad \sum_{r=0}^{3n} d_r(x - 12)^r = \sum_{r=0}^{3n} D_r(x - 13)^r \quad \text{and} \quad d_K$$

. Then find the value of $\frac{A_{2n} + D_{2n}}{B_{2n}}$.

Answer: 2



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4. If $3^{101} - 2^{100}$ is divided by 11, the remainder is

Answer: 2

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5. Find the hundred's digit in the co-efficient of x^{17} in the expansion of $(1 + x^5 + x^7)^{20}$.

Answer: 4

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6. Let $n \in N, S_n = \sum_{r=0}^{3n} \binom{3n}{r}$ and $T_n = \sum_{r=0}^n \binom{3n}{3r}$,

then $|S_n - 3T_n|$ equals

Answer: 2

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7. Find the sum of possible real values of x for which the sixth term of

$$\left(3^{\log_3 \sqrt{9^{|x-2|}}} + 7^{\frac{1}{5} \log_7 (3^{|x-2|-9})} \right)^7 \text{ equals } 567.$$

Answer: 4

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8. Let q be a positive with $q \leq 50$.

If ${}^{98}C_{30} + 2 \cdot {}^{97}C_{30} + 3 \cdot {}^{96}C_{30} + \dots + 68 \cdot {}^{31}C_{30} + 69$ the sum

Find the sum of the digits of q .

Answer: 5

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9. The remainder when $\left(\sum_{k=1}^5 {}^{20}C_{2k-1} \right)^6$ is divided by 11, is :

Answer: 3

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10. Let $a = 3^{\frac{1}{223}} + 1$ and for all $n \geq 3$ let $f(n) = C(n, 0)a^{n-1} - C(n, 1) \cdot a^{n-2} - C(n, 2)a^{n-3} - \dots + (-1)^n$. If the value of $f(2007) + f(2008) = 3^k$ where $k \in \mathbb{N}$ then the value of k is

Answer: 9

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11. Let the sum of all divisors of the form $2^p \cdot 3^q$ (with p, q positive integers) of the number $19^{88} - 1$ be λ . Find the unit digit of λ .

Answer: (4)

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12. Find the sum of possible real values of x for which the sixth term of $\left(3^{\log_3 \sqrt{9^{|x-2|}}} + 7^{\frac{1}{5} \log_7 (3^{|x-2|-9})} \right)^7$ equals 567.

Answer: (4)

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13. Let $1 + \sum_{r=1}^{10} (3^r C(10, r) + rC(10, r)) = 2^{10}(\alpha 4^5 + \beta)$ where $\alpha, \beta \in N$ and $f(x) = x^2 - 2x - k^2 + 1$ If α, β lies betweenm the roots of $f(x) = 0$ then find the smalles positive integral value of k

Answer: 5



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14. if $S_n = C_0C_1 + C_1C_2 + \dots + C_{n-1}C_n$ and $\frac{S_{n+1}}{S_n} = \frac{15}{4}$

then n is

Answer: 6



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