MATHS

BOOKS - NTA MOCK TESTS

NTA JEE MOCK TEST 53

Mathematics

1. Let $A = [a_{ij}]$ be a square matrix of order 3 and $B = [b_{ij}]$ be a matrix such that $b_{ij} = 2^{i-j} a_{ij}$ for $1 \le i, j \le 3$, $\forall i, j \in N$.

If the determinant of A is same as its order, then the value of $\left|\left(B^T\right)^{-1}\right|$ is

A. $\frac{1}{3}$

B. 3

C. 9

D. $\frac{1}{27}$

Answer: A

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2. A person predicts the outcome of 20 cricket matches of his home team. Each match can result either in a win, loss or tie for the home team. The total number of ways in which he can make the predictions, so that exactly 10 predictions are correct, is equal to

A. $.^{20}C_{10}.2^{10}$

B. $.^{20}C_{10}.3^{20}$

C. $.^{20}C_{10}.3^{01}$

D. $.^{20}C_{10}.5^{20}$

Answer: A

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3. The quadratic equation whose roots are the arithmetic mean and the harmonic mean of the roots of the equation $x^2+7x-1=0$ is

A. $14x^2+14x-45=0$

B. $45x^2-14x+14=0$

C. $14x^2+45x-14=0$

D. $45x^2+14x-45=0$

Answer: C

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4. If $a+b+c=3$ and $a>0, b>0, c>0$ then the greatest value of $a^2b^2c^2$ is

A. $\frac{3^{10}2^4}{7^7}$

B. $\frac{3^9 2^4}{7^7}$

C. $\frac{3^8 2^4}{7^7}$

D. $\frac{3^9 2^3}{7^6}$

Answer: A

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5. The point at which the line segment joining A(1, 1) and B (5, 5) subtends an obtuse angle is

A. (7, 7)

B. (0, 5)

C. (2, 4)

D. (1, 5)

Answer: C

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6. The average weight of students in a class of 35 students is 40 kg. If the weight of the teacher is included, then the average raises by $\frac{1}{3}$ kg. The weight of the teacher is

A. 40.5 kg

B. 50 kg

C. 41 kg

D. 52 kg

Answer: D

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7. Two straight roads OA and OB intersect at O. A tower is situated within the angle formed by them and subtends angles of 45° and 30° at the points A and B where the roads are nearest to it. If

OA = 400 meters and

OB = 300 meters, than the height of the tower is

A. $250\sqrt{2}$ meters

B. 500 meters

C. $50\sqrt{14}$ meters

D. $100\sqrt{7}$ meters

Answer: C

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8. The value of $\lim_{x \to -\infty} \frac{x^2 \tan\left(\frac{1}{x}\right)}{\sqrt{4x^2 - x + 1}}$ is equal to

A. 1

B. $\frac{1}{2}$

C. -1

D. $-\frac{1}{2}$

Answer: D

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9. The range of the function $f(x) = \sin^{-1}\left[x^2 - \frac{1}{3}\right] - \cos^{-1}\left[x^2 + \frac{2}{3}\right]$ is (where, [x] represents the greatest integer value of x)

A. $[-\pi, 0]$

B. $\{-\pi, 0\}$

C. $\{0, \pi\}$

D. $\{0, \pi, -\pi\}$

Answer: B

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10. The point on the curve $6y = 4x^3 - 3x^2$, the tangent at which makes an equal angle with the coordinate axes is

A. $\left(1, -\frac{1}{6}\right)$

B. $\left(-1, -\frac{7}{6}\right)$

C. $\left(-\frac{1}{2}, -\frac{5}{24}\right)$

D. $\left(\frac{1}{2}, -\frac{1}{24}\right)$

Answer: C

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11. Let $\int \frac{dx}{\sqrt{x^2+1}-x} = f(x) + C$ such that $f(0) = 0$ and C is the constant of integration, then the value of $f(1)$ is

A. $\frac{1}{\sqrt{2}} + \frac{1}{2}\ln\left(1+\sqrt{2}\right)$

B. $\frac{1}{2} + \frac{1}{\sqrt{2}}\ln\left(1+\sqrt{2}\right)$

C. $\frac{1}{2} + \frac{1}{2}\ln\sqrt{2}+1$

D. $\frac{1}{\sqrt{2}} + \frac{1}{2}\left(1+\ln\left(1+\sqrt{2}\right)\right)$

Answer: D

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12. The solution of the differential equation $x\frac{dy}{dx} = y\ln\left(\frac{y^2}{x^2}\right)$ is (where, c is an arbitrary constant)

A. $y = x.\,e^{cx+1}$

B. $y = x.\,e^{cx-1}$

C. $y = x^2.\,e^{cx+1}$

D. $y = x.\,e^{cx^2+\frac{1}{2}}$

Answer: D

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13. Let P be the image of the point (3, 1, 7) with respect to the plane $x - y + z = 3$. The equation of the plane passing through P and parallel to $x - 2y + 3z = 7$ is

A. $x - 2y + 3z = 2$

B. $2x - 4y + 6z = 7$

C. $x + 2y - 6z + 2 = 0$

D. $x - 2y + 3z + 2 = 0$

Answer: D

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14. A person goes to the office either by a car, or scooter, or bus, the probability of which being $\frac{2}{7}, \frac{3}{7}, \frac{2}{7}$ respectively. The probability that he reaches office late if he takes a car, or scooter, or bus is $\frac{2}{9}, \frac{1}{9}, \frac{4}{9}$ respectively. If he reaches office in time, the probability that he travelled by car is k, then the value of $24k + 7$

is equal to

A. 7

B. 14

C. $\frac{21}{2}$

D. $\frac{31}{2}$

Answer: B

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15. The solution of the system of equations $x+(\cos\alpha)y=1$ and $(\cos\alpha)x+4y=2$ satisfy $x\geq\frac{4}{5}$ and $y\leq\frac{1}{2}$, then the value of α can lie in the inverval

A. $\alpha\in\left[\frac{\pi}{4},\frac{\pi}{3}\right]$

B. $\alpha\in\left[0,\frac{\pi}{6}\right]$

C. $\alpha\in\left[\frac{\pi}{6},\frac{\pi}{3}\right]$

D. $\alpha\in\left[\frac{\pi}{3},\frac{\pi}{2}\right]$

Answer: D

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16. $\sum_{r=0}^{n}\left(\frac{r+2}{r+1}\right)\cdot {}^{n}C_{r}$ is equal to :

A. $\frac{2^{n}(n+2)-1}{n+1}$

B. $\frac{2^{n}(n+1)-1}{n+1}$

C. $\frac{2^{n}(n+4)-1}{n+1}$

D. $\frac{2^{n}(n+3)-1}{n+1}$

Answer: D

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17. If the complex numbers $\sin x + i\cos 2x$ and $\cos x - i\sin 2x$ are conjugate of each other, then the number of values of x in the inverval $[0, 2\pi)$ is equal to (where, $i^2 = -1$)

A. 0

B. 1

C. 2

D. 3

Answer: A

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18. A variable line through the point $\left(\frac{6}{5}, \frac{6}{5}\right)$ cuts the coordinate axes at the points A and B respectively. If the point P divides AB internally in the ratio $2:1$, then the equation of the locus of P is

A. $5xy = (2x + y)$

B. $5xy = 2(2x + y)$

C. $5xy = (x + 2y)$

D. $5xy = 2(x + 2y)$

Answer: B

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19. The value of $\tan 63^\circ - \cot 63^\circ$ is equal to

A. $\dfrac{\sqrt{5}-1}{\sqrt{5}+1}\sqrt{10+2\sqrt{5}}$

B. $\dfrac{2}{\sqrt{5}+1}\sqrt{10+2\sqrt{5}}$

C. $\dfrac{\sqrt{5}-1}{4}\sqrt{10-2\sqrt{5}}$

D. $\dfrac{\sqrt{5}-1}{4}\sqrt{10+2\sqrt{5}}$

Answer: A

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20. The locus of a point which moves such that the difference of its distances from the points (5, 0) and $(\,-5, 0)$ is 6 units is a conic, whose length of the latus rectum (in units) is equal to

A. 4

B. $\frac{16}{3}$

C. 8

D. $\frac{32}{3}$

Answer: D

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21. If the lengths of the sides of a right- angled triangle ABC, right angled at C, are in arithmetic progression, then the value of $5(\sin A + \sin B)$ is

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22. If $f(0) = 0$, $f(3) = 3$ and $f'(3) = 4$, then the value of $\int_0^1 x f''(3x) dx$ is equal to

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23. The minimum value of x which satisfies the inequality $\sin^{-1} x \geq \cos^{-1} x$ is λ, then the value of 2λ is (use $\sqrt{2} = 1.41$)

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24. Let $\overrightarrow{a}$ be a unit vector coplanar with $\hat{i}-\hat{j}+2\hat{k}$ and $2\hat{i}-\hat{j}+\hat{k}$ such that $\overrightarrow{a}$ is perpendicular to $\hat{i}-2\hat{j}+\hat{k}$. If the projecton of $\overrightarrow{a}$ along $\hat{i}-\hat{j}+2\hat{k}$ is λ, then the value of $\frac{1}{\lambda^2}$ is equal to

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25. A tangent and a normal are drawn at the point P(8, 8) on the parabola $y^2=8x$ which cuts the axis of the parabola at the points A and B respectively. If the centre of the circle through P, A and B is C, then the sum f $\sin(\angle PCB)$ and $\cot(\angle PCB)$ is equal to

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