



MATHS

BOOKS - NCERT MATHS (ENGLISH)

APPLICATION OF DERIVATIVES

Short Answer Types Questions

1. A spherical ball of salt is dissolving in water in such a manner that the rate of decrease of volume at any instant is proportional to the surface. Prove that the radius is decreasing at

a constant rate.



2. If the area of circle increases at a uniform rate, then prove that the perimeter varies inversely as the radius.



3. A kite is moving horizontally at a height of 151.5m. If the speed of the kite is $10\frac{m}{s}$, how fast is the string being let out, when the kite is 250 m away from the boy who is flying the kite? The height of the boy is 1.5 m. (A) 8 m/s (B) 12 m/s (C) 16 m/s (D) 19 m/s

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4. Two men A and B start with velocities v at the same time from the junction of two roads

inclined at $45\,^\circ$ to each other.If they travel by different roads,find the rate at which they are being separated.



5. Find angle θ , 0 < θ < $\frac{\pi}{2}$, which increase

twice as fast as sine

6. Using differentials, find the approximate value of $(1.999)^5$

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7. Find the approximate volume of metal in a hollow spherical shell whose internal and external radii are 3 cm and 3.0005 cm, respectively.

8. A man 2m tall, walks at the rate of $1\frac{2}{3}m/sec$ towards a street light which is $5\frac{1}{3}$ m above the ground. At what rate is tip of his shadow moving? At what rate is the length of the shadow changing when he is $3\frac{1}{13}m$ from the base of the light?

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9. A swimming pool is to be drained by cleaning. If L represents the number of litres of water in the pool t seconds after the pool

has been plugged off to drain and $L = 2000(10 - t)^2$. How fast is the water ruining out at the end of 5 seconds? What is the average rate at which the water flows out during the first 5 seconds? What is 2 Watch Video Solution

10. The volume of a cube is increasing at a constant rate. Prove that the increase in surface area varies inversely as the length of the edge of the cube.

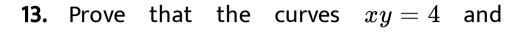


11. x and y are the sides of two squares such that $y = x - x^2$. Find the rate of the change of the area of the second square with respect to the first square.

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12. Prove that the curve $y=x^2$ and xy=k

intersect orthogonally if $8k^2 = 1$.



 $x^2 + y^2 = 8$ touch each other.

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14. Find the required point be $P(x_1, y_1)$. The

tangent to the curve $\sqrt{x}+\sqrt{y}=4$ at which

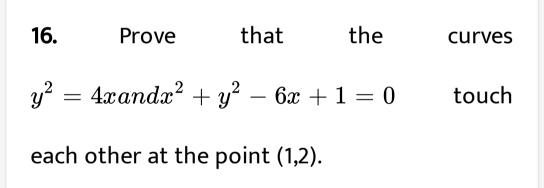
tangent is equally inclined to the axes.



15. Find the angle of intersection of the curves

$$\mathsf{y} = 4 - x^2$$
 and $y = x^2$

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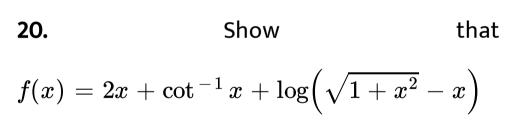
17. Find the equation(s) of normal(s) to the curve $3x^2 - y^2 = 8$ which is (are) parallel to the line x + 3y = 4.

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18. At what points on the curve $x^2+y^2-2x-4y+1=0$, the tangents are

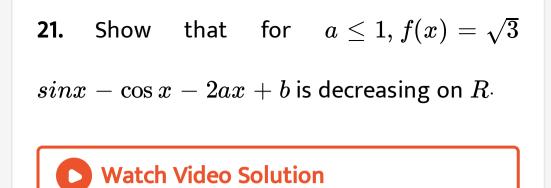
parallel to the $y - a\xi s$?

19. Show that the line $\frac{d}{a} + \frac{y}{b} = 1$ touches the curve $y = be^{-\frac{x}{a}}$ at the point where it crosses the y-axis.



is increasing in R

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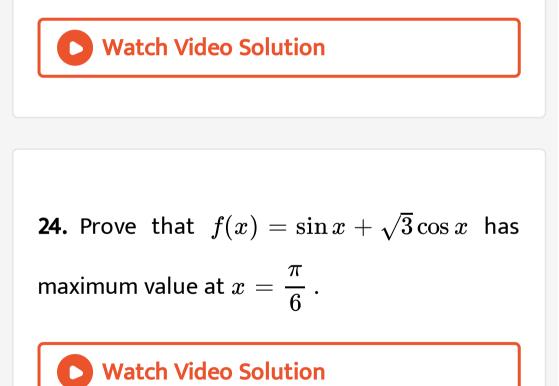


22. Show that $f(x) = an^{-1}(\sin x + \cos x)$ is

an increasing function on the interval $(0, \ \pi/4)$.



23. At what points, the slope of the curve $y = -x^3 + 3x^2 + 9x - 27$ at point (x, y) is given by maximum slope.



Long Answer Types Questions

1. If the sum of lengths of hypotenuse and a side of a right angled triangle is given, show that area of triangle is maximum, when the angle between them is $\frac{\pi}{3}$.

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2. Find the points of local maxima, local minima and the points of inflection of the function $f(x) = x^5 - 5x^4 + 5x^3 - 1$. Also,

find the corresponding local maximum and

local minimum values

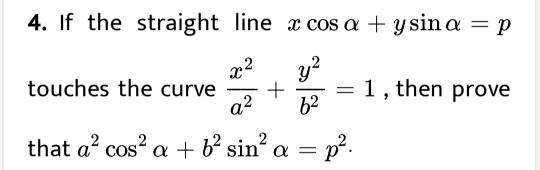


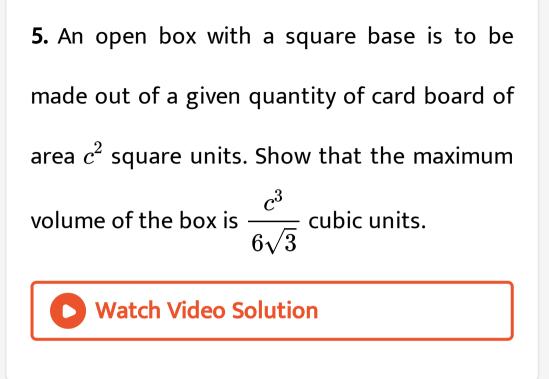
3. A telephone company in a town has 500 subscribers on its list and collects fixed charges of Rs 300 per subscriber. The company proposes to increase the annual subscription and it is believed that every increase of Rs 1 one subscriber will discontinue the service.



revenue?

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6. Find the dimensions of the rectangle of perimeter 36cm which will sweep out a volume as large as possible when revolved about one of its sides.



7. The sum of the surface areas of a sphere and a cube is given. Show that when the sum of their volumes is least, the diameter of the sphere is equal to the edge of the cube.



8. AB is a diameter of a circle and C is any point on the circle. Show that the area of ABC is maximum, when it is isosceles.

9. A metal box with a square base and vertical sides is to contain $1024cm^3$ of water, the material for the top and bottom costs Rs. $5percm^2$ and the material for the costs Rs. 2. $50percm^2$. Find the least cost of the box.



10. The sum of the surface areas of a cuboid with sides x, 2x and $\frac{x}{3}$ and a sphere is given to be constant. Prove that the sum of their volumes is minimum, if x is equal to three times the radius of sphere. Also find the minimum value of the sum of their volumes.

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Objective Types Questions

1. The sides of an equilateral triangle are increasing at the rate of 2 cm/sec. How far is the area increasing when the side is 10 cms?

A.
$$10cm^{2}/s$$

B. $\sqrt{3}cm^{2}/s$
C. $10\sqrt{3}cm^{2}/s$
D. $\frac{10}{3}cm^{2}/s$

Answer: C

2. A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground,away from the wall at the rate of 10 cm/s. How fast is the angle between the ladder and the ground decreasing when the foot of the ladder is 2 m away from the wall?

A.
$$\frac{1}{10}$$
 rad/s
B. $\frac{1}{20}$ rad/s

C. 20 rad/s

D. 10 rad/s

Answer: B



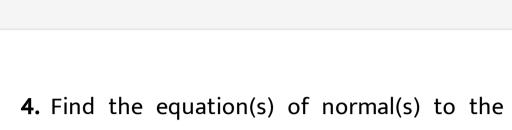
3. The curve y
$$=x^{rac{1}{5}}$$
 has at (0,0)

A. a vertical tangent (parallel to Y-axis)

B. a horizontal tangent (parallel to X-axis)

- C. an oblique tangent
- D. no tangent

Answer: A



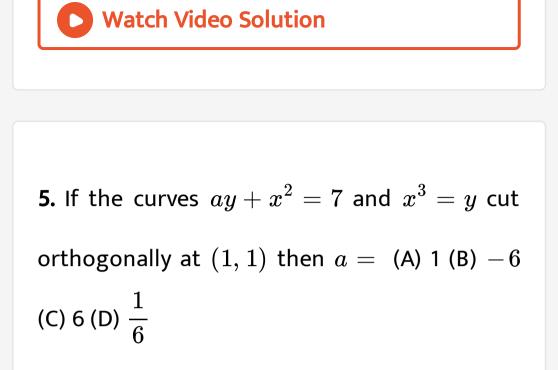
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curve $3x^2 - y^2 = 8$ which is (are) parallel to the line x + 3y = 4.

A.
$$3X - Y = 8$$

- B. 3X + Y + 8 = 0
- C. $X + 3Y \pm 8 = 0$
- $\mathsf{D}.\,X+3Y=0$

Answer: C



A. 1

B. 0

C.-6

D. 6

Answer: D



6. If $y = x^4 - 12$ and if x changes from 2 to

1.99. what is the appoinmate change in y.

A. 0.32

 $B.\,0.032$

C. 5.68

D. 5.968

Answer: A



7. Find the equation of the tangent to the curve $(1+x^2)y = 2-x$, where it crosses the x-axis.

A.
$$x+5y=2$$

B.
$$x - 5y = 2$$

C.
$$5x - y = 2$$

D.
$$5x + y = 2$$

Answer: A



8. The points at which the tangents to the curve $y = x^3 - 12x + 18$ are parallel to the X-axis are

A.
$$(2-2), (-2, -34)$$

B. $(2, 34), (-2, 0)$
C. $(0, 34), (-2, 0)$
D. $(2, 2), (-2, 34)$

Answer: D

9. The tangent to the curve $y = e^{kx}$ at a point (0,1) meets the x-axis at (a,0), where $a \in [-2, -1]$. Then $k \in$ (a) $\left[-\frac{1}{2}, 0\right]$ (b) $\left[-1, -\frac{1}{2}\right] [0, 1]$ (d) $\left[\frac{1}{2}, 1\right]$

A.
$$(0, 1)$$

B. $\left(-\frac{1}{2}, 0\right)$
C. $(2, 0)$
D. $(0, 2)$

Answer: B



10. The slope of the tangent to the curve $x = t^2 + 3t - 8$, $y = 2t^2 - 2t - 5$ at the point (2, -1) is (a)22/7 (b) 6/7 (c) 7/6 (d) -6/7

A.
$$\frac{22}{7}$$

B. $\frac{6}{7}$
C. $-\frac{6}{7}$
D. -6

Answer: B

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11. What is the angle between these two curves $x^3 - 3xy^2 + 2 = 0$ and $3x^2y - y^3 - 2 = 0$

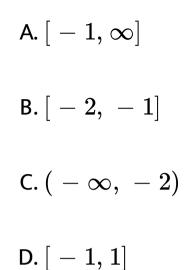
A.
$$\frac{\pi}{4}$$

B. $\frac{\pi}{3}$
C. $\frac{\pi}{2}$
D. $\frac{\pi}{6}$

Answer: C

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12. 24. Find the intervals in which the following function is (a) increasing and (b) decreasing $f(x) = 2x^3 + 9x^2 + 12x - 1$



Answer: B

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13. The function $f\!:\!R o R$ be defined by $f(x)=2x+\cos x$ then f



B. has a maximum at x=0

C. is a decreasing function

D. is in increasing function

Answer: D

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14. If $y = x(x-3)^2$ decreases for the values of x given by

A.
$$1 < x < 3$$

B.
$$x < 0$$

C. $x > 0$
D. $0 < x < rac{3}{2}$

Answer: A

15. The function
$$f(x) = 4 \sin^3 x - 6 \sin^2 x + 12 \sin x + 100$$
 is strictly

A. increasing in π , $\frac{3\pi}{2}$ B. decreasing in $\left(\frac{\pi}{2}, \pi\right)$ C. decreasing in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ D. decreasing in $\left[0, \frac{\pi}{2}\right]$

Answer: B



16. Which of the following functions are decreasing on $(0, \pi/2)$? (i) $\cos x$ (ii) $\cos 2x$ (iii) $\tan x$ (iv) $\cos 3x$

A. sin2x

B. tanx

C. cosx

D. cos3x

Answer: C

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17. The function $f(x) = \tan x - x$

A. always increases

B. always decreases

C. never increases

D. sometimes increases and sometimes

decreases

Answer: A

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18. If x is real, then the minimum value of the expression $x^2 - 8x + 17$ is

A. - 1

B. 0

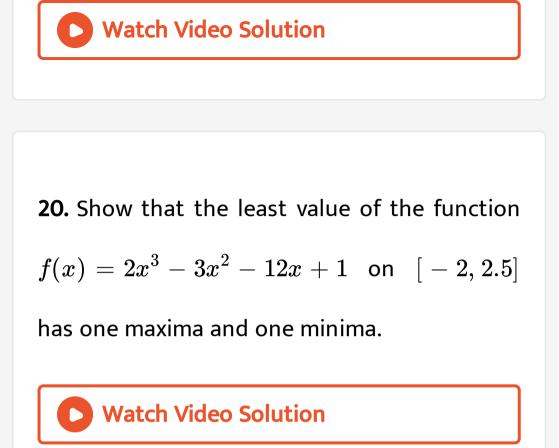
C. 1

D. 2

Answer: C

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19. Find the least value of the function $f(x) = x^3 - 18x^2 + 96x$ in the interval [0,9] is ?



21. Show that The maximum value of sinx. cosx in R is $\frac{1}{2}$



22. At $x = \frac{5\pi}{6}$, $f(x) = 2\sin 3x + 3\cos 3x$ is (a) 0 (b) maximum (c) minimum (d) none of these

A. maximum

B. minimum

C. zero

D. neither maximum nor minimum

Answer: D

23.	The	maximum	slope	of	curve	у
=	$-x^3$ +	$-3x^2+9x$ -	– 27 is			
	A. O					
E	3. 12					
(C. 16					
[). 32					
Answer: B						

24. The function $f(x) = x^x$ has a stationary point at

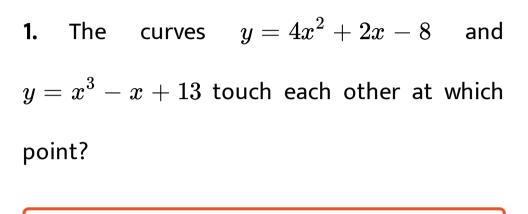
A. x=e
B. x
$$=$$
 $\frac{1}{e}$

D.
$$x=\sqrt{E}$$

Answer: B

25. Show that the maximum value of
$$\left(\frac{1}{x}\right)^x$$
 is
 $e^{1/e}$.
A. e
B. e^e
C. $e^{1/e}$
D. $\left(\frac{1}{e}\right)^{1/e}$
Answer: C





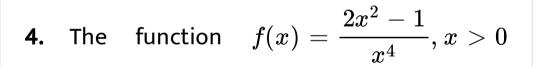
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2. The equation of normal to the curve y=tanx

at (0,0) is

3. Find the values of 'a' for which the function $f(x) = \sin x - ax + 4$ is increasing function on R.





decreases in the interval

5. The least value of the function
$$f(x) = ax + rac{b}{x}(x > 0, a > 0, b > 0)$$