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# MATHS

# BOOKS - NCERT MATHS (ENGLISH)

# APPLICATION OF DERIVATIVES

## Short Answer Types Questions

**1.** A spherical ball of salt is dissolving in water in such a manner that the rate of decrease of volume at any instant is proportional to the surface. Prove that the radius is decreasing at a constant rate.

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**2.** If the area of circle increases at a uniform rate, then prove that the perimeter varies inversely as the radius.

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**3.** A kite is moving horizontally at a height of $151.5m$. If the speed of the kite is $10\frac{m}{s}$, how fast is the string being let out, when the kite is 250 m away from the boy who is flying the kite? The height of the boy is 1.5 m. (A) $8$ m/s (B) $12$ m/s (C) $16$ m/s (D) $19$ m/s

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**4.** Two men A and B start with velocities v at the same time from the junction of two roads inclined at $45^\circ$ to each other.If they travel by different roads,find the rate at which they are being separated.

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**5.** Find angle $\theta$, $0 < \theta < \frac{\pi}{2}$, which increase twice as fast as sine

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**6.** Using differentials, find the approximate value of $(1.999)^5$

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**7.** Find the approximate volume of metal in a hollow spherical shell whose internal and external radii are 3 cm and 3.0005 cm, respectively.

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**8.** A man 2m tall, walks at the rate of $1\frac{2}{3}m/sec$ towards a street light which is $5\frac{1}{3}$ m above the ground. At what rate is tip of his shadow moving? At what rate is the length of the shadow changing when he is $3\frac{1}{13}m$ from the base of the light?

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**9.** A swimming pool is to be drained by cleaning. If L represents the number of litres of water in the pool $t$ seconds after the pool has been plugged off to drain and $L = 2000(10 - t)^2$. How fast is the water ruining out at the end of 5 seconds? What is the average rate at which the water flows out during the first 5 seconds?

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**10.** The volume of a cube is increasing at a constant rate. Prove that the increase in surface area varies inversely as the length of the edge of the cube.

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**11.** x and y are the sides of two squares such that $y = x - x^2$. Find the rate of the change of the area of the second square with respect to the first square.

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**12.** Prove that the curve $y = x^2$ and $xy = k$ intersect orthogonally if $8k^2 = 1$.

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13. Prove that the curves $xy = 4$ and $x^2 + y^2 = 8$ touch each other.

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14. Find the required point be $P(x_1, y_1)$. The tangent to the curve $\sqrt{x} + \sqrt{y} = 4$ at which tangent is equally inclined to the axes.

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**15.** Find the angle of intersection of the curves

y $= 4 - x^2$ and $y = x^2$

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**16.** Prove that the curves

$y^2 = 4x and x^2 + y^2 - 6x + 1 = 0$ touch

each other at the point (1,2).

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17. Find the equation(s) of normal(s) to the curve $3x^2 - y^2 = 8$ which is (are) parallel to the line $x + 3y = 4$.

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18. At what points on the curve $x^2 + y^2 - 2x - 4y + 1 = 0$, the tangents are parallel to the $y - a\xi s$?

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**19.** Show that the line $\frac{d}{a}+\frac{y}{b}=1$ touches the curve $y=be^{-\frac{x}{a}}$ at the point where it crosses the y-axis.

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**20.** Show that $f(x)=2x+\cot^{-1}x+\log\left(\sqrt{1+x^2}-x\right)$ is increasing in $R$

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**21.** Show that for $a \leq 1, f(x) = \sqrt{3}$ $sinx - \cos x - 2ax + b$ is decreasing on $R$.

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**22.** Show that $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function on the interval $(0, \pi/4)$.

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23. At what points, the slope of the curve $y = -x^3 + 3x^2 + 9x - 27$ at point $(x, y)$ is given by maximum slope.

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24. Prove that $f(x) = \sin x + \sqrt{3} \cos x$ has maximum value at $x = \frac{\pi}{6}$.

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## Long Answer Types Questions

**1.** If the sum of lengths of hypotenuse and a side of a right angled triangle is given, show that area of triangle is maximum, when the angle between them is $\frac{\pi}{3}$.

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**2.** Find the points of local maxima, local minima and the points of inflection of the function $f(x) = x^5 - 5x^4 + 5x^3 - 1$. Also, find the corresponding local maximum and local minimum values

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**3.** A telephone company in a town has 500 subscribers on its list and collects fixed charges of Rs 300 per subscriber. The company proposes to increase the annual subscription and it is believed that every increase of Rs 1 one subscriber will discontinue the service.

Find what increase will bring maximum revenue?

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4. If the straight line $x \cos \alpha + y \sin \alpha = p$ touches the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then prove that $a^2 \cos^2 \alpha + b^2 \sin^2 \alpha = p^2$.

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**5.** An open box with a square base is to be made out of a given quantity of card board of area $c^2$ square units. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cubic units.

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**6.** Find the dimensions of the rectangle of perimeter 36cm which will sweep out a volume as large as possible when revolved about one of its sides.

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7. The sum of the surface areas of a sphere and a cube is given. Show that when the sum of their volumes is least, the diameter of the sphere is equal to the edge of the cube.

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8. $AB$ is a diameter of a circle and $C$ is any point on the circle. Show that the area of $ABC$ is maximum, when it is isosceles.

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**9.** A metal box with a square base and vertical sides is to contain $1024cm^3$ of water, the material for the top and bottom costs Rs. $5 per cm^2$ and the material for the costs Rs. $2.50 per cm^2$ . Find the least cost of the box.

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**10.** The sum of the surface areas of a cuboid with sides $x, 2x$ and $\frac{x}{3}$ and a sphere is given to be constant. Prove that the sum of their volumes is minimum, if $x$ is equal to three times the radius of sphere. Also find the minimum value of the sum of their volumes.

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## Objective Types Questions

**1.** The sides of an equilateral triangle are increasing at the rate of 2 cm/sec. How far is the area increasing when the side is 10 cms?

A. $10cm^2 / s$

B. $\sqrt{3}cm^2 / s$

C. $10\sqrt{3}cm^2 / s$

D. $\frac{10}{3}cm^2 / s$

**Answer: C**

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**2.** A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground,away from the wall at the rate of 10 cm/s. How fast is the angle between the ladder and the ground decreasing when the foot of the ladder is 2 m away from the wall?

A. $\frac{1}{10}$ rad/s

B. $\frac{1}{20}$ rad/s

C. 20 rad/s

D. 10 rad/s

**Answer: B**

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**3.** The curve y $= x^{\frac{1}{5}}$ has at (0,0)

A. a vertical tangent (parallel to Y-axis)

B. a horizontal tangent (parallel to X-axis)

C. an oblique tangent

D. no tangent

**Answer: A**

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**4.** Find the equation(s) of normal(s) to the curve $3x^2 - y^2 = 8$ which is (are) parallel to the line $x + 3y = 4$.

A. $3X - Y = 8$

B. $3X + Y + 8 = 0$

C. $X + 3Y \pm 8 = 0$

D. $X + 3Y = 0$

**Answer: C**

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**5.** If the curves $ay + x^2 = 7$ and $x^3 = y$ cut orthogonally at $(1, 1)$ then $a =$ (A) 1 (B) $-6$ (C) 6 (D) $\frac{1}{6}$

A. 1

B. 0

C. $-6$

D. 6

**Answer: D**

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**6.** If $y = x^4 - 12$ and if $x$ changes from 2 to 1.99. what is the appoinmate change in $y$.

A. $0.32$

B. $0.032$

C. $5.68$

D. $5.968$

**Answer: A**

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**7.** Find the equation of the tangent to the curve $(1+x^2)y = 2 - x$, where it crosses the x-axis.

A. $x + 5y = 2$

B. $x - 5y = 2$

C. $5x - y = 2$

D. $5x + y = 2$

**Answer: A**

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**8.** The points at which the tangents to the curve $y = x^3 - 12x + 18$ are parallel to the X-axis are

A. $(2 - 2), (-2, -34)$

B. $(2, 34), (-2, 0)$

C. $(0, 34), (-2, 0)$

D. $(2, 2), (-2, 34)$

**Answer: D**

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**9.** The tangent to the curve $y=e^{kx}$ at a point (0,1) meets the x-axis at (a,0), where $a\in[-2,-1]$ . Then $k\in$ (a) $\left[-\frac{1}{2},0\right]$ (b) $\left[-1,-\frac{1}{2}\right]$ $[0,1]$ (d) $\left[\frac{1}{2},1\right]$

A. $(0,1)$

B. $\left(-\frac{1}{2},0\right)$

C. $(2,0)$

D. $(0,2)$

**Answer: B**

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**10.** The slope of the tangent to the curve $x = t^2 + 3t - 8, \ y = 2t^2 - 2t - 5$ at the point $(2, \ -1)$ is

(a)$22/7$

(b) $6/7$

(c) $7/6$

(d) $-6/7$

A. $\frac{22}{7}$

B. $\frac{6}{7}$

C. $-\frac{6}{7}$

D. $-6$

**Answer: B**

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11. What is the angle between these two curves $x^3 - 3xy^2 + 2 = 0$ and $3x^2y - y^3 - 2 = 0$

A. $\frac{\pi}{4}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{2}$

D. $\frac{\pi}{6}$

**Answer: C**

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**12.** 24. Find the intervals in which the following function is (a) increasing and (b) decreasing

$f(x) = 2x^3 + 9x^2 + 12x - 1$

A. $[-1, \infty]$

B. $[-2, -1]$

C. $(-\infty, -2)$

D. $[-1, 1]$

**Answer: B**

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**13.** The function $f: R \rightarrow R$ be defined by $f(x) = 2x + \cos x$ then $f$

A. has minimum at $x=\pi$

B. has a maximum at x=0

C. is a decreasing function

D. is in increasing function

**Answer: D**

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**14.** If $y = x(x-3)^2$ decreases for the values of $x$ given by

A. $1 < x < 3$

B. $x < 0$

C. $x > 0$

D. $0 < x < \frac{3}{2}$

**Answer: A**

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**15.** The function $f(x) = 4\sin^3 x - 6\sin^2 x + 12\sin x + 100$ is strictly

A. increasing in $\pi, \frac{3\pi}{2}$

B. decreasing in $\left(\frac{\pi}{2}, \pi\right)$

C. decreasing in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

D. decreasing in $\left[0, \frac{\pi}{2}\right]$

**Answer: B**

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**16.** Which of the following functions are decreasing on $(0, \pi/2)$ ? (i) $\cos x$ (ii) $\cos 2x$ (iii) $\tan x$ (iv) $\cos 3x$

A. sin2x

B. tanx

C. cosx

D. cos3x

**Answer: C**

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**17.** The function $f(x) = \tan x - x$

A. always increases

B. always decreases

C. never increases

D. sometimes increases and sometimes decreases

**Answer: A**

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**18.** If $x$ is real, then the minimum value of the expression $x^2 - 8x + 17$ is

A. $-1$

B. 0

C. 1

D. 2

**Answer: C**

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**19.** Find the least value of the function $f(x) = x^3 - 18x^2 + 96x$ in the interval $[0, 9]$ is ?

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**20.** Show that the least value of the function $f(x) = 2x^3 - 3x^2 - 12x + 1$ on $[-2, 2.5]$ has one maxima and one minima.

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**21.** Show that The maximum value of sinx. cosx in R is $\frac{1}{2}$

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**22.** At $x = \frac{5\pi}{6}$, $f(x) = 2\sin 3x + 3\cos 3x$ is

(a) 0 (b) maximum (c) minimum (d) none of these

A. maximum

B. minimum

C. zero

D. neither maximum nor minimum

**Answer: D**

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**23.** The maximum slope of curve y $= -x^3 + 3x^2 + 9x - 27$ is

A. 0

B. 12

C. 16

D. 32

**Answer: B**

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**24.** The function $f(x) = x^x$ has a stationary point at

A. x=e

B. x $= \frac{1}{e}$

C. x=1

D. $x = \sqrt{E}$

**Answer: B**

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**25.** Show that the maximum value of $\left(\frac{1}{x}\right)^{x}$ is $e^{1/e}$.

A. e

B. $e^{e}$

C. $e^{1/e}$

D. $\left(\frac{1}{e}\right)^{1/e}$

**Answer: C**

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## Fillers

1. The curves $y = 4x^2 + 2x - 8$ and $y = x^3 - x + 13$ touch each other at which point?

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2. The equation of normal to the curve y=tanx at (0,0) is ....................

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**3.** Find the values of '$a$' for which the function $f(x) = \sin x - ax + 4$ is increasing function on $R$.

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**4.** The function $f(x) = \frac{2x^2 - 1}{x^4}, x > 0$ decreases in the interval

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**5.** The least value of the function

$$f(x) = ax + \frac{b}{x}(x > 0, a > 0, b > 0)$$

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