



MATHS

BOOKS - NCERT MATHS (ENGLISH)

CONTINUITY AND DIFFERENTIABILITY

Short Answer

1. Examine continuity of the function $f(x) = x^3 + 2x^2 - 1$ at $x = 1$.

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2. $f(x) = \begin{cases} 3x + 5 & \text{if } x \geq 2 \\ x^3 & \text{if } x \leq 2 \end{cases}$ at $x = 2$

A. $f(x)$ is dicontinuous at $x = 2$.

B. $f(x)$ is continuous at $x = 2$.

C. Can not be determined

D. None of these

Answer: A

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3. Check the continuity of $f(x) = \begin{cases} \frac{1 - \cos 2x}{x^2} & \text{if } x \neq 0 \\ 5 & \text{if } x = 0 \end{cases}$ at $x = 0$.

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4. Check the continuity of $f(x) = \begin{cases} \frac{2x^2 - 3x - 2}{x - 2} & \text{if } x \neq 2 \\ 5 & \text{if } x = 2 \end{cases}$ at $x = 2$.

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5. Check the continuity of $f(x) = \begin{cases} \frac{|x - 4|}{2(x - 4)} & \text{if } x \neq 4 \\ 0 & \text{if } x = 4 \end{cases}$ at $x = 4$.

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$$6. f(x) = \begin{cases} |x| \cos \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases} \text{ at } x = 0.$$



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7. Find that the function is continuous or discontinuous at the indicated

$$\text{point } f(x) = \begin{cases} |x - a| \sin \frac{1}{x-a} & \text{if } x \neq a \\ 0 & \text{if } x = a \end{cases} \text{ at } x = a$$



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$$8. \text{ Check the continuity of } f(x) = \begin{cases} \frac{e^{1/x}}{1 + e^{1/x}} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases} \text{ at } x = 0$$



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9. Check the continuity of $f(x) = \begin{cases} \frac{x^2}{2} & \text{if } 0 \leq x \leq 1 \\ 2x^2 - 3x + \frac{3}{2} & \text{if } 1 < x \leq 2 \end{cases}$ at $x = 1$

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10. $f(x) = |x| + |x - 1|$ at $x = 1$.

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11. Find the value of k for which the function $f(x) = \begin{cases} 3x - 8 & \text{if } x \leq 5 \\ 2k & \text{if } x > 5 \end{cases}$ is continuous at $x = 5$

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12. If $f(x) = \begin{cases} \frac{2^{x+2} - 16}{4^x - 16} & \text{if } x \neq 2 \\ k & \text{if } x = 2 \end{cases}$ is 'continuous' at $x = 2$. then find 'k'.



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$$13. f(x) = \begin{cases} \frac{\sqrt{1+kx} - \sqrt{1-kx}}{x} & \text{if } -1 \leq x < 0 \\ \frac{2x+1}{x-1} & \text{if } 0 \leq x \leq 1 \end{cases} \text{ at } x = 0.$$



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$$14. f(x) = \begin{cases} \frac{1 - \cos kx}{x \sin x} & \text{if } x \neq 0 \\ \frac{1}{2} & \text{if } x = 0 \end{cases} \text{ at } x = 0$$



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$$15. \text{ Prove that the function } f \text{ defined by } f(x) = \begin{cases} \frac{x}{|x| + 2x^2} & \text{if } x \neq 0 \\ k & \text{if } x = 0 \end{cases}$$

remains discontinuous at $x = 0$, regarding the choice of k .



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16. Find the values of a and b such that the function f defined by

$$f(x) = \begin{cases} \frac{x-4}{|x-4|} + a & \text{if } x < 4 \\ a + b & \text{if } x = 4 \\ \frac{x-4}{|x-4|} + b & \text{if } x > 4 \end{cases}$$

is a continuous function at $x = 4$.



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17. Given the function $f(x) = \frac{1}{x+2}$. Find the points of discontinuity of the function $f(f(x))$



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18. Find all point of discontinuity of the function $f(t) = \frac{1}{t^2 + t - 2}$, where $t = \frac{1}{x-1}$



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19. Show that the function $f(x) = |\sin x + \cos x|$ is continuous at $x = \pi$.

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20. Examine the differentiability of f , where f is defined by

$$f(x) = \begin{cases} x[x] & \text{if } 0 \leq x < 2 \\ (x-1)x & \text{if } 2 \leq x < 3 \end{cases} \text{ at } x = 2$$

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21. If $f(x) = x^2 \sin' \frac{1}{x}$, where $x \neq 0$, then the value of the function f at $x = 0$, so that the function is continuous at $x = 0$ is

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22. Examine the differentiability of f , where f is defined by

$$f(x) = \begin{cases} 1+x & \text{if } x \leq 2 \\ 5-x & \text{if } x > 2 \end{cases} \text{ at } x = 2.$$

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23. Show that $f(x) = |x - 3|$ is continuous but not differentiable at $x = 3$.

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24. A function $f: \mathbb{R} \rightarrow \mathbb{R}$ satisfies that equation $f(x + y) = f(x)f(y)$ for all $x, y \in \mathbb{R}$, $f(x) \neq 0$. Suppose that the function $f(x)$ is differentiable at $x = 0$ and $f'(0) = 2$. Prove that $f'(x) = 2f(x)$.

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25. Differentiate $2^{\cos^2 x}$

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26. Differentiate $\frac{8^x}{x^8}$

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27. Differentiate $\log(x + \sqrt{a^2 + x^2})$ with respect to x :

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28. Differentiate $\log[\log(\log x^5)]$

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29. Differentiate $\sin \sqrt{x} + \cos^2 \sqrt{x}$

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30. Differentiate $\sin^n(ax^2 + bx + c)$



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31. Differentiate $\cos(\tan \sqrt{x+1})$



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32. Differentiate $\sin x^2 + \sin^2 x + \sin^2(x^2)$



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33. Differentiate $\sin^{-1} \frac{1}{\sqrt{x+1}}$



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34. $(\sin x)^{\cos x}$



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35. Differentiate $\sin^m x \cdot \cos^n x$



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36. Differentiate $(x + 1)^2(x + 2)^3(x + 3)^4$



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37. Simplify: $\cos^{-1}((\sin x + \cos x)/(\sqrt{2}))$



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38. Differentiate $\tan^{-1} \left\{ \sqrt{\frac{1 - \cos x}{1 + \cos x}} \right\}$ with respect to x :



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39. Differentiate : $\tan^{-1}(\sec x + \tan x)$



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40. Differentiate the following function with respect to x :

$$\tan^{-1}\left(\frac{a+x}{1-ax}\right)$$



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41. Differentiate $\cos^{-1}(4x^3 - 3x)$ with respect to x , if $x \in \left(\frac{1}{2}, 1\right)$



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42. Differentiate $\tan^{-1}\left(\frac{3a^2x - x^3}{a^3 - 3ax^2}\right)$,



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43. $y = \tan^{-1}\left(\frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}}\right)$



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44. If $x = a\left(t + \frac{1}{t}\right)$ and $y = a\left(t - \frac{1}{t}\right)$, prove that $\frac{dy}{dx} = \frac{x}{y}$

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45. Find $\frac{dy}{dx}$, when $x = e^{\theta}\left(\theta + \frac{1}{\theta}\right)$ and $y = e^{-\theta}\left(\theta - \frac{1}{\theta}\right)$

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46. If $x = 3 \cos \theta - \cos^3 \theta$ and $y = 3 \sin \theta - \sin^3 \theta$ find $\frac{dy}{dx}$

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47. If $\sin x = \frac{2t}{1+t^2}$, $\tan y = \frac{2t}{1-t^2}$, find $\frac{dy}{dx}$.

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48. If $x = \frac{1 + \log t}{t^2}$, $y = \frac{3 + 2 \log t}{t}$, find $\frac{dy}{dx}$.

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49. If $x = e^{\cos 2t}$ and $y = e^{\sin 2t}$, prove that $\frac{dy}{dx} = -\frac{y \log x}{x \log y}$

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50. If $x = a \sin 2t(1 + \cos 2t)$ and $y = b \cos 2t(1 - \cos 2t)$, show that at $t = \frac{\pi}{4}$, $\frac{dy}{dx} = \frac{b}{a}$.

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51. If $x = 3 \sin t - \sin 3t$, $y = 3 \cos t - \cos 3t$, find $\frac{dy}{dx}$ at $t = \frac{\pi}{3}$.

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52. Differentiate $\frac{x}{\sin x}$ w.r.t. $\sin x$.

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53. Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ w. r. t $\tan^{-1} x$, where $x \neq 0$.

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54. If $\sin(xy) + \frac{y}{x} = x^2 - y^2$, find $\frac{dy}{dx}$.

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55. Find $\frac{dy}{dx}$ if $\sec(x+y) = xy$

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56. Find $\frac{dy}{dx}$ if, $\tan^{-1}(x^2 + y^2) = a$

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57. If $(x^2 + y^2)^2 = xy$, find $\frac{dy}{dx}$

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58. If $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, find $\frac{dy}{dx}$ and $\frac{dx}{dy}$. Also, show that $\frac{dy}{dx} \frac{dx}{dy} = 1$.

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59. If $x = e^{x/y}$, prove that $\frac{dy}{dx} = \frac{x - y}{x \log x}$

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60. If $y^x = e^{y-x}$, prove that $\frac{dy}{dx} = \frac{(1 + \log y)^2}{\log y}$

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61. If

$y = (\cos x)^{\cos x} \wedge (\cos x) \wedge (((((\infty))))))$, prove that $\frac{dy}{dx} = -\frac{y^2 \tan x}{(1 - y \log \cos x)}$

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62. If $x \sin(a + y) + \sin a \cdot \cos(a + y) = 0$. Prove that :

$$\frac{dy}{dx} = \left(\frac{\sin^2(a + y)}{\sin a} \right)$$

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63. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$

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64. If $y = \tan^{-1} x$, find $\frac{d^2y}{dx^2}$ in terms of y alone.

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65. Verify Rolles theorem for function $f(x) = x(x - 1)^2$ on $[0, 1]$

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66. Verify Rolles theorem for function $f(x) = \sin^4 x + \cos^4 x$ on $\left[0, \frac{\pi}{2}\right]$

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67. Verify Rolles theorem for function $f(x) = \log(x^2 + 2) - \log 3$ on $[-1, 1]$

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68. Verify Rolles theorem for the function: $f(x) = x(x + 3)e^{-x/2}$ on $[-3, 0]$.

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69. Verify Rolles theorem for the function $f(x) = \sqrt{4 - x^2}$ on $[-2, 2]$.

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70. Discuss the applicability of Rolles theorem on the function $f(x) = x^2 + 1$ when $0 < x < 1$ and $f(x) = 3 - x$ when $x > 1$.

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71. Using Rolle's theorem find the point in $(0, 2\pi)$ on the curve $y = \cos x - 1$, where tangent is parallel to x axis.

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72. Using Rolle's theorem, find the point on the curve $y = x(x - 4)$, $x \in [0, 4]$, where the tangent is parallel to X-axis.

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73. Verify mean value theorem for the function $f(x) = \frac{1}{4x - 1}$ in $[1, 4]$

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74. Verify mean value theorem for the function $f(x) = x^3 - 2x^2 - x + 3$ in $[0, 1]$

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75. Verify Rolles theorem for function $f(x) = \sin x - \sin 2x$ on $[0, \pi]$

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76. Verify mean value theorem for the function $f(x) = \sqrt{25 - x^2}$ in $[1, 5]$

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77. Find the point on the parabola $y = (x - 3)^2$, where the tangent is parallel to the line joining $(3,0)$ and $(4,1)$

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78. Using mean value theorem, prove that there is a point on the curve $y = 2x^2 - 5x + 3$ between the points $A(1,0)$ and $B(2,1)$, where tangent is parallel to the chord AB . Also, find that point.

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79. Find the values of a and b so that the function $f(x) = \begin{cases} x^2 + 3x + a, & \text{if } x \leq 1 \\ bx + 2, & \text{if } x > 1 \end{cases}$ is differentiable at

each $x \in R$.

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80. If $x^m y^n = (x + y)^{m+n}$, prove that $\frac{dy}{dx} = \frac{y}{x}$.

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81. If $x = \sin t, y = \sin pt$, prove that

$$(1 - x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + p^2 y = 0.$$

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82. If $y = x^{\tan x} + \sqrt{\frac{x^2 + 1}{2}}$, find $\frac{dy}{dx}$

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83. Examine continuity of the function $f(x) = x^3 + 2x^2 - 1$ at $x = 1$.

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84. $f(x) = \begin{cases} 3x + 5 & \text{if } x \geq 2 \\ x^3 & \text{if } x \leq 2 \end{cases}$ at $x = 2$

A. $f(x)$ is discontinuous at $x = 2$.

B. $f(x)$ is continuous at $x = 2$.

C. Can not be determined

D. None of these

Answer: A

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85. $f(x) = \begin{cases} \frac{1 - \cos 2x}{x^2} & \text{if } x \neq 0 \\ 5 & \text{if } x = 0 \end{cases}$ at $x = 0$.

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$$86. f(x) = \begin{cases} \frac{2x^2 - 3x - 2}{x - 2} & \text{if } x \neq 2 \\ 5 & \text{if } x = 2 \end{cases} \text{ at } x = 2.$$

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$$87. f(x) = \begin{cases} \frac{|x-4|}{2(x-4)} & \text{if } x \neq 4 \\ 0 & \text{if } x = 4 \end{cases} \text{ at } x = 4.$$

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$$88. f(x) = \begin{cases} |x| \cos \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases} \text{ at } x = 0.$$

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$$89. f(x) = \begin{cases} |x| \sin \frac{1}{x-a} & \text{if } x \neq a \\ 0 & \text{if } x = a \end{cases} \text{ at } x = a$$

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$$90. f(x) = \begin{cases} \frac{e^{1/x}}{1+e^{1/x}} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases} \text{ at } x = 0$$

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$$91. \begin{cases} \frac{x^2}{2} & \text{if } 0 \leq x \leq 1 \\ 2x^2 - 3x + \frac{3}{2} & \text{if } 1 < x \leq 2 \end{cases} \text{ at } x = 1$$

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$$92. f(x) = |x| + |x - 1| \text{ at } x = 1. \text{ discuss the continuity}$$

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$$93. f(x) = \begin{cases} 3x - 8 & \text{if } x \leq 5 \\ 2k & \text{if } x > 5 \end{cases} \text{ at } x = 5$$

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94.

If

$$f(x) = \begin{cases} \frac{2^{x+2} - 16}{4^x - 16}, & \text{if } x \neq 2k, \\ \end{cases} \quad \text{if } f(x) \text{ is continuous at } x = 2, f \in \mathbb{R}$$


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$$95. f(x) = \begin{cases} \frac{\sqrt{1+kx} - \sqrt{1-kx}}{x} & \text{if } -1 \leq x < 0 \\ \frac{2x+1}{x-1} & \text{if } 0 \leq x \leq 1 \end{cases} \quad \text{at } x = 0.$$


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$$96. f(x) = \begin{cases} \frac{1 - \cos kx}{x \sin x} & \text{if } x \neq 0 \\ \frac{1}{2} & \text{if } x = 0 \end{cases} \quad \text{at } x = 0$$


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$$97. \text{ Prove that the function } f \text{ defined by } f(x) = \begin{cases} \frac{x}{|x| + 2x^2} & \text{if } x \neq 0 \\ k & \text{if } x = 0 \end{cases}$$

remains discontinuous at $x = 0$, regardless of the choice of k .


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98. Find the values of a and b such that the function f defined by

$$f(x) = \begin{cases} \frac{x-4}{|x-4|} + a & \text{if } x < 4 \\ a + b & \text{if } x = 4 \\ \frac{x-4}{|x-4|} + b & \text{if } x > 4 \end{cases}$$

is a continuous function at $x = 4$.

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99. If the function $f(x) = \frac{1}{x+2}$, then find the points of discontinuity of the composite function $y = f\{f(x)\}$.

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100. Find all points of discontinuity of the function $f(t) = \frac{1}{t^2 + t - 2}$, where $t = \frac{1}{x-1}$.

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101. Show that the function $f(x) = |\sin x + \cos x|$ is continuous at $x = \pi$.

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102. Examine the differentiability of f , where f is defined by

$$f(x) = \begin{cases} x[x] & \text{if } 0 \leq x < 2 \\ (x-1)x & \text{if } 2 \leq x < 3 \end{cases} \text{ at } x = 2$$

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103. $f(x) = \begin{cases} x^2 \sin' \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases} \text{ at } x = 0.$

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104. $f(x) = \begin{cases} 1+x & \text{if } x \leq 2 \\ 5-x & \text{if } x > 2 \end{cases} \text{ at } x = 2.$ Examine the differentiability

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105. Show that $f(x) = |x - 5|$ is continuous but not differentiable at $x = 5$.

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106. A function $f: \mathbb{R} \rightarrow \mathbb{R}$ satisfies the equation $f(x + y) = f(x) \cdot f(y)$ for all $x, y \in \mathbb{R}$, $f(x) \neq 0$. Suppose that the function is differentiable at $x = 0$ and $f'(0) = 2$, then prove that $f' = 2f(x)$.

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107. Differentiate the function with respect to x : $2^{\cos^2 x}$

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108. Differentiate $\frac{8^x}{x^8}$

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109. Differentiate $\log(x + \sqrt{x^2 + a})$

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110. Differentiate: $\log[\log(\log x^5)]$

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111. Differentiate $\sin \sqrt{x} + \cos^2 \sqrt{x}$

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112. $\sin^n(ax^2 + bx + c)$

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113. Differentiate $\cos(\tan \sqrt{x+1})$

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114. Differentiate $\sin x^2 + \sin^2 x + \sin^2(x^2)$

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115. Differentiate $\sin^{-1} \frac{1}{\sqrt{x+1}}$

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116. Differentiate $(\sin x)^{\cos x}$

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117. $\sin^m x \cdot \cos^n x$



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118. $(x + 1)^2(x + 2)^3(x + 3)^4$



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119. The derivative of $\cos^{-1}\left(\frac{\sin x + \cos x}{\sqrt{2}}\right)$, $-\frac{\pi}{4} < x < \frac{\pi}{4}$, w.r.t. x is



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120. Differentiate $\tan^{-1}\left\{\sqrt{\frac{1 - \cos x}{1 + \cos x}}\right\}$, $-\pi < x < \pi$ with respect to

x :



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121. Differentiate $\tan^{-1}(\sec x + \tan x)$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$



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122. Differentiate the following functions with respect to x :

$$\tan^{-1}\left(\frac{a+x}{1-ax}\right) \quad \text{(ii) } \tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right), \quad -\frac{\pi}{2} < x < \frac{\pi}{2}$$

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$$123. y = \sec^{-1} \frac{1}{2x^2 - 1}, \quad 0 < x < \frac{1}{\sqrt{2}}$$

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124. Differentiate $\tan^{-1}\left(\frac{3a^2x - x^3}{a^3 - 3ax^2}\right), \quad -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$

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$$125. y = \tan^{-1}\left(\frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}}\right), \quad \text{where } -1 < x < 1, x \neq 0$$

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126. If $x = t + \frac{1}{t}$, $y = t - \frac{1}{t}$, then dy/dx is equal to

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127. Find $\frac{dy}{dx}$, when $x = e^\theta \left(\theta + \frac{1}{\theta} \right)$ and $y = e^{-\theta} \left(\theta - \frac{1}{\theta} \right)$

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128. Find $\frac{dy}{dx}$ if $x = 3 \cos \theta - 2 \cos^3 \theta$, $y = 3 \sin \theta - 2 \sin^3 \theta$.

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129. If $\sin x = \frac{2t}{1+t^2}$, $\tan y = \frac{2t}{1-t^2}$, find $\frac{dy}{dx}$.

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$$130. x = \frac{1 + \log t}{t^2}, y = \frac{3 + 2 \log t}{t}$$

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$$131. \text{ If } x = e^{\cos 2t} \text{ and } y = e^{\sin 2t}, \text{ then prove that } \frac{dy}{dx} = -\frac{y \log x}{x \log y}.$$

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$$132. \text{ If } x = a \sin 2t(1 + \cos 2t) \text{ and } y = b \cos 2t(1 - \cos 2t), \text{ then show that } \left(\frac{dy}{dx} \right)_{t=\pi/4} = \frac{b}{a}.$$

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$$133. \text{ If } x = 3 \sin t - \sin 3t, y = 3 \cos t - \cos 3t, \text{ find } \frac{dy}{dx} \text{ at } t = \frac{\pi}{3}.$$

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134. Differentiate $\frac{x}{\sin x}$ w.r.t. $\sin x$.

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135. Differentiate $\tan^{-1} \left(\frac{\sqrt{1+x^2}-1}{x} \right)$ w.r.t. $\tan^{-1} x$, when $x \neq 0$.

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136. Differentiate $\sin(xy) + \frac{x}{y} = x^2 - y$

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137. Differentiate $\sec(x+y) = xy$

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138. $\tan^{-1}(x^2 + y^2) = a$



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139. $(x^2 + y^2)^2 = xy$



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140. If $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, then show that $\frac{dy}{dx} \cdot \frac{dx}{dy} = 1$.



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141. If $x = e^{x/y}$, then prove that $\frac{dy}{dx} = \frac{x - y}{x \log x}$.



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142. If $y^x = e^{y-x}$, then prove that $\frac{dy}{dx} = \frac{(1 + \log y)^2}{\log y}$



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143. If $y = (\cos x)^{(\cos x)^{(\cos x)^{\dots\infty}}}$, then show that $\frac{dy}{dx} = \frac{y^2 \tan x}{y \log \cos x - 1}$

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144. If $x \sin(a + y) + \sin a \cdot \cos(a + y) = 0$, then prove that

$$\frac{dy}{dx} = \frac{\sin^2(a + y)}{\sin a}$$

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145. If $\sqrt{1 - x^2} + \sqrt{1 - y^2} = a(x - y)$, then prove that

$$\frac{dy}{dx} = \sqrt{\frac{1 - y^2}{1 - x^2}}$$

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146. If $y = \tan^{-1} x$, then find $\frac{d^2y}{dx^2}$ in term of y alone.



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147. $f(x) = x(x - 1)^2$ in $[0, 1]$



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148. $f(x) = \sin^4 x + \cos^4 x$ in $\left[0, \frac{\pi}{2}\right]$



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149. $f(x) = \log(x^2 + 2) - \log 3$ in $[-1, 1]$



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150. verify Rolle's theorem for the function $f(x) = x(x + 3)e^{-\frac{x}{2}}$ in $[-3, 0]$



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151. Verify Rolles theorem for the function $f(x) = \sqrt{4 - x^2}$ on $[-2, 2]$.

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152. Discuss the applicability of Rolle's theorem on the function given by

$$f(x) = \begin{cases} x^2 + 1 & \text{if } 0 \leq x \leq 1 \\ 3 - x & \text{if } 1 \leq x \leq 2 \end{cases}$$

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153. Find the points on the curve $y = (\cos x - 1)$ in $[0, 2\pi]$, where the tangent is parallel to $X -$ axis.

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154. Using Rolle's theroem, find the point on the curve $y = x(x - 4)$, $x \in [0, 4]$, where the tangent is parallel to X-axis.

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155. $f(x) = \frac{1}{4x - 1}$ in $[1, 4]$

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156. $f(x) = x^3 - 2x^2 - x + 3$ in $[0, 1]$

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157. $f(x) = \sin x - \sin 2x$ in $[0, \pi]$

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158. $f(x) = \sqrt{25 - x^2}$ in $[1, 5]$

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159. Find a point on the curve $y = (x - 3)^2$, where the tangent is parallel to the chord joining the points $(3, 0)$ and $(4, 1)$.

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160. Using mean value theorem, prove that there is a point on the curve $y = 2x^2 - 5x + 3$ between the points $A(1, 0)$ and $B, (2, 1)$, where tangent is parallel to the chord AB . Also, find that point.

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161. Find the values of p and q , so that

$$f(x) = \begin{cases} x^2 + 3x + p & \text{if } x \leq 1 \\ qx + 2 & \text{if } x > 1 \end{cases} \text{ is differentiable at } x = 1$$

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162. If $x^m y^n = (x + y)^{m+n}$, Prove that $\frac{dy}{dx} = \frac{y}{x}$.

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163. If $x = \sin t$ and $y = \sin pt$, prove that

$$(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + p^2y = 0.$$



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164. Find the values of $\frac{dy}{dx}$, if $y = x^{\tan x} + \sqrt{\frac{x^2 + 1}{2}}$.



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Objective Type

1. If $f(x) = 2x$ and $g(x) = \frac{x^2}{2} + 1$, then which of the following can be a discontinuous functions?

A. $f(x) + g(x)$

B. $f(x) - g(x)$

C. $f(x) \cdot g(x)$

D. $\frac{g(x)}{f(x)}$

Answer: D



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2. The function $f(x) = \frac{4 - x^2}{4x - x^3}$ is discontinuous at

- A. discontinuous at only one point
- B. discontinuous at exactly two points
- C. discontinuous at exactly three points
- D. None of the above

Answer: C



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3. The set of points where the function f given by $f(x) = |2x - 1| \sin x$ is differentiable is

A. R

B. $R - \left\{ \frac{1}{2} \right\}$

C. $(0, \infty)$

D. None of these

Answer: B



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4. The function $f(x) = \cot x$ is discontinuous on set

A. $\{x = n\pi : n \in Z\}$

B. $\{x = 2n\pi : n \in Z\}$

C. $\left\{ x = (2n + 1)\frac{\pi}{2}, n \in Z \right\}$

D. $\left\{ x = \frac{n\pi}{2}, n \in Z \right\}$

Answer: A

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5. The function $f(x) = e^{|x|}$ is

- A. continuous everywhere but not differentiable at $x = 0$
- B. continuous and differentiable everywhere
- C. not continuous at $x = 0$
- D. None of the above

Answer: A

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6. if $f(x) = x^2 \sin\left(\frac{1}{x}\right)$, $x \neq 0$ then the value of the function f at $x = 0$ so that the function is continuous at $x = 0$

A. 0

B. -1

C. 1

D. None of the above

Answer: A



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7. If $f(x) = \begin{cases} mx + 1 & \text{if } x \leq \frac{\pi}{2} \\ \sin x + n & \text{if } x > \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$, then find

the relation between m and n .

A. $m = 1, n = 0$

B. $m = \frac{n\pi}{2} + 1$

C. $n = \frac{m\pi}{2}$

D. $m = n = \frac{\pi}{2}$

Answer: C



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8. If $f(x) = |\sin x|$, then

A. f is everywhere differentiable

B. f is everywhere continuous but not differentiable at $x = n\pi, n \in \mathbb{Z}$

C. f is everywhere continuous but not differentiable at

$$x = (2n + 1)\frac{\pi}{2}, n \in \mathbb{Z}$$

D. None of the above

Answer: B



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9. If $y = \log\left(\frac{1 - x^2}{1 + x^2}\right)$, then $\frac{dy}{dx}$ is equal to

A. $\frac{4x^3}{1 - x^4}$

B. $\frac{-4x}{1-x^4}$

C. $\frac{1}{4-x^4}$

D. $\frac{-4x^3}{1-x^4}$

Answer: B



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10. If $y = \sqrt{\sin x + y}$, then $\frac{dy}{dx}$ is equal to

A. $\frac{\cos x}{2y-1}$

B. $\frac{\cos x}{1-2y}$

C. $\frac{\sin x}{1-2y}$

D. $\frac{\sin x}{2y-1}$

Answer: A



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11. The derivative of $\cos^{-1}(2x^2 - 1)$ w.r.t. $\cos^{-1} x$ is

A. 2

B. $\frac{-1}{2\sqrt{1-x^2}}$

C. $\frac{2}{x}$

D. $1 - x^2$

Answer: A



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12. If $x = t^2$ and $y = t^3$, then $\frac{d^2y}{dx^2}$ is equal to

A. $\frac{3}{2}$

B. $\frac{3}{4t}$

C. $\frac{3}{2t}$

D. $\frac{3}{2t}$

Answer: B

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13. The value of c in Rolle's theorem for the function $f(x) = x^3 - 3x$ in the interval $[0, \sqrt{3}]$ is

A. 1

B. -1

C. $\frac{3}{2}$

D. $\frac{1}{3}$

Answer: A

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14. For the function $f(x) = x + \frac{1}{x}$, $x \in [1, 3]$, the value of c for mean value theorem is

A. 1

B. $\sqrt{3}$

C. 2

D. None of these

Answer: B



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15. If $f(x) = 2x$ and $g(x) = \frac{x^2}{2} + 1$, then which of the following can be a discontinuous functions?

A. $f(x) + g(x)$

B. $f(x) - g(x)$

C. $f(x) \cdot g(x)$

D. $\frac{g(x)}{f(x)}$

Answer: D



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16. The function $f(x) = \frac{4 - x^2}{4x - x^3}$ is discontinuous at

- A. discontinuous at only one point
- B. discontinuous at exactly two points
- C. discontinuous at exactly three points
- D. None of the above

Answer: C



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17. The set of points where the function f given by $f(x) = |2x - 1| \sin x$ is differentiable is

A. \mathbb{R}

B. $\mathbb{R} - \left\{ \frac{1}{2} \right\}$

C. $(0, \infty)$

D. None of these

Answer: B



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18. The function $f(x) = \cot x$ is discontinuous on set

A. $\{x = n\pi : n \in \mathbb{Z}\}$

B. $\{x = 2n\pi : n \in \mathbb{Z}\}$

C. $\left\{x = (2n + 1)\frac{\pi}{2}, n \in \mathbb{Z}\right\}$

D. $\left\{x = \frac{n\pi}{2}, n \in \mathbb{Z}\right\}$

Answer: A



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19. The function $f(x) = e^{|x|}$ is

- A. continuous everywhere but not differentiable at $x = 0$
- B. continuous and differentiable everywhere
- C. not continuous at $x = 0$
- D. None of the above

Answer: A



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20. If $f(x) = x^2 \sin \frac{1}{x}$, where $x \neq 0$, then the value of the function f at $x = 0$, so that the function is continuous at $x = 0$ is

- A. 0
- B. -1
- C. 1
- D. None of the above

Answer: A

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21. If $f(x) = \begin{cases} mx + 1 & \text{if } x \leq \frac{\pi}{2} \\ \sin x + n & \text{if } x > \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$, then

A. $m = 1, n = 0$

B. $m = \frac{n\pi}{2} + 1$

C. $n = \frac{m\pi}{2}$

D. $m = n = \frac{\pi}{2}$

Answer: C

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22. If $f(x) = |\sin x|$, then

A. f is everywhere differentiable

B. f is everywhere continuous but not differentiable at $x = n\pi, n \in \mathbb{Z}$

C. f is everywhere continuous but not differentiable at

$$x = (2n + 1)\frac{\pi}{2}, n \in \mathbb{Z}$$

D. None of the above

Answer: B

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23. If $y = \log\left(\frac{1 - x^2}{1 + x^2}\right)$, then $\frac{dy}{dx}$ is equal to

A. $\frac{4x^3}{1 - x^4}$

B. $\frac{-4x}{1 - x^4}$

C. $\frac{1}{4 - x^4}$

D. $\frac{-4x^3}{1 - x^4}$

Answer: B

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24. If $y = \sqrt{\sin x + y}$, then $\frac{dy}{dx}$ is equal to

A. $\frac{\cos x}{2y - 1}$

B. $\frac{\cos x}{1 - 2y}$

C. $\frac{\sin x}{1 - 2y}$

D. $\frac{\sin x}{2y - 1}$

Answer: A



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25. The derivative of $\cos^{-1}(2x^2 - 1)$ w.r.t. \cos^{-1} is

A. 2

B. $\frac{-1}{2\sqrt{1-x^2}}$

C. $\frac{2}{x}$

D. $1 - x^2$

Answer: A



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26. If $x = t^2$ and $y = t^3$, then $\frac{d^2y}{dx^2}$ is equal to

A. $\frac{3}{2}$

B. $\frac{3}{4t}$

C. $\frac{3}{2t}$

D. $\frac{3}{2t}$

Answer: B



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27. The value of c in Rolle's theorem for the function $f(x) = x^3 - 3x$ in the interval $[0, \sqrt{3}]$ is

A. 1

B. -1

C. $\frac{3}{2}$

D. $\frac{1}{3}$

Answer: A



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28. For the function $f(x) = x + \frac{1}{x}$, $x \in [1, 3]$, the value of c for mean value theorem is

A. 1

B. $\sqrt{3}$

C. 2

D. None of these

Answer: B

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Fillers

1. An example of a function which is continuous every where but fails to be differentiable exactly at two point is $\hat{a}\hat{e}'\hat{a}\hat{e}'$.

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2. Derivative of x^2 w.r.t. x^3 is

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3. If $f(x) = |\cos x|$, then $f' \left(\frac{\pi}{4} \right)$ is equal to $\hat{a}\hat{e}'\hat{a}\hat{e}'$.



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4. For the curve $\sqrt{x} + \sqrt{y} = 1$, $\frac{dy}{dx}$ at $(1/4, 1/4)$ is 1/2 (b) 1 (c) -1 (d) 2



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5. Rolle's theorem is applicable for the function $f(x) = |x - 1|$ in $[0, 2]$.



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6. An example of a function which is continuous every where but fails to be differentiable exactly at two point is $\hat{a}\hat{e}\hat{a}\hat{e}$.



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7. Derivative of x^2 w.r.t. x^3 is



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8. If $f(x) = |\cos x|$, then $f'\left(\frac{\pi}{4}\right)$ is equal to

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9. For the curve $\sqrt{x} + \sqrt{y} = 1$, $\frac{dy}{dx}$ at $\left(\frac{1}{4}, \frac{1}{4}\right)$ is

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10. Rolle's theorem is applicable for the function $f(x) = |x - 1|$ in $[0, 2]$.

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True False

1. If f is continuous on its domain D ; then $|f|$ is also continuous on D

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2. If f is continuous on its domain D ; then $|f|$ is also continuous on D

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3. The composition of two continuous function is a continuous function.

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4. Trigonometric and inverse trigonometric functions are differentiable in their respective domain.

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5. If f, g is continuous at $x = 0$, then f and g are separately continuous at $x = 0$.

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6. If f is continuous on its domain D ; then $|f|$ is also continuous on D

 [Watch Video Solution](#)

7. If f is continuous on its domain D , then $|f|$ is also continuous on D .

 [Watch Video Solution](#)

8. The composition of two continuous function is a continuous function.

 [Watch Video Solution](#)

9. Trigonometric and inverse trigonometric functions are differentiable in their respective domain.

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10. If $f \cdot g$ is continuous at $x = 0$, then f and g are separately continuous at $x = 0$.



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