# びdoubtnut 

## MATHS

## BOOKS - JEEVITH PUBLICATIONS MATHS (KANNADA ENGLISH)

## APPLICATION OF DERIVATIVES

Five Marks Questions With Answers

1. Find the rate of change of the area of a circle with respect to its radius $r$ when
(a) $r=3 \mathrm{~cm}$.

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2. Find the rate of change of the area of a circle with respect to its radius $r$ when
(b) $r=4 \mathrm{~cm}$.

## D Watch Video Solution

3. The volume of a cube is increasing at the rate of $8 \mathrm{~cm}^{3} / \mathrm{s}$. How fast is the surface area
increasing when the length of an edge is 12 cm ?

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4. The radius of a circle is increasing uniformly at the ratio $3 \mathrm{~cm} / \mathrm{s}$. Find the rate at which the area of the circle is increasing when the radius is 10 cm .
5. An edge of a variable cube is increasing at the rate of $3 \mathrm{~cm} / \mathrm{s}$. How fast is the volume of the increasing when the edge is 10 cm long?

## D Watch Video Solution

6. A stone is dropped into a quiet lake and waves in circles at the speed of $5 \mathrm{~cm} / \mathrm{s}$. At the instant when the radius of the circular wave is 8 cm , how fast is the enclosed area increasing?
7. The radius of a circle is increasing at the rate of $0.7 \mathrm{~cm} / \mathrm{s}$. What is the rate of increase of its circumference?

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8. The length $x$ of rectangle is decreasing at the rate of $5 \mathrm{~cm} /$ minute and width y is increasing at the rate of $4 \mathrm{~cm} /$ minute. When $x=8 \mathrm{~cm}$ and $y=6 \mathrm{~cm}$, find the rate of change of
(i) the perimeter and (ii) the Area of the rectangle.

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9. The length $x$ of rectangle is decreasing at
the rate of $5 \mathrm{~cm} /$ minute and width y is increasing at the rate of $4 \mathrm{~cm} /$ minute. When $x=8 \mathrm{~cm}$ and $\mathrm{y}=6 \mathrm{~cm}$, find the rate of change of
(i) the perimeter and (ii) the Area of the rectangle.

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10. A balloon which always remain spherical on inflation, is being inflated by pumping in 900 cubic cm of gas per second. Find the rate at which the radius of the balloon increases when the radius is 15 cm .

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11. A balloon which remains spherical has a
vertical radius. Find the rate at which its
volume is increasing with the radius when the radius is 10 cm .

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12. A ladder 5 m long is leaning against a well.

The bottom of the ladder is pulled along the ground, away from the well, at the rate of 2 $\mathrm{m} / \mathrm{s}$. How fat is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall?
13. A particle move along the curve $6 y=x^{3}+2$.Find the points on the curve at which $y$-coordinate is changing 8 times as fast as the $x$-coordinates.

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14. The radius of an air bubble is increasing at the rate of $\frac{1}{2} \mathrm{~cm} / \mathrm{s}$. At what rate is the volume of the bubble increasing when the radius is 1 cm ?
15. A balloon, which always spherical, has a variable diameter $3 / 2(2 x+1)$. Find the rate of change of its volume with respect to $x$.

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16. Sand is pouring from a pipe at the rate of
$12 \mathrm{~cm}^{3} / \mathrm{s}$. The falling sand forms a cone on
the ground in such a way that the height of
the cone is always one-sixth of the base. How fast height of the sand cone increasing when the height is 4 cm ?

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17. The total cost $C(x)$ in rupees associated with the production of $x$ units of an item is given by
$C(x)=0.007 x^{3}-0.003 x^{2}+15 x+4000$.
Find the marginal cost when $x=17$ units are produced.

## Watch Video Solution

18. The total revenue in rupees received from
the sale of $x$ units of a product is given by
$R(x)=13 x^{2}+26 x+15$. Find the marginal revenue when $x=7$.

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19. A man of height 2 metres walks at a uniform speed of $5 \mathrm{~km} / \mathrm{h}$ away from a lamp
post which is 6 metres high. Find the rate at which the length of his shadow increases.

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20. A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lowermost. Its semi-vertical angle is $\tan ^{-1}(0.5)$. Water is poured into it at a constant rate of 5 cubic metre per hour. Find the rate at which the level of the water is
rising at the instant when the depth of water in the tank is 4 m .

## D Watch Video Solution

21. A car starts from a point $P$ at time $t=0$ seconds and stops at point $Q$. The distance $x$, in metres, covered by it, in $t$ seconds is given
by $x=t^{2}\left(2-\frac{t}{3}\right)$. Find the time taken by it to reach Q and also find the distance between $P$ and Q .
22. A circular disc of radius 3 cm is being heated. Due to expansion, its radius increased at the rate of $0.05 \mathrm{~cm} / \mathrm{s}$. Find the rate at which its area is increasing when radius is 3.2 cm.

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Three Marks Questions With Answers A Decreasing And Increasing Functions

1. Show that the function given by $f(x)=3 x+17$ is strictly increasing on R.

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2. Show that the function given by $f(x)=e^{2 x}$ is strictly increasing on R .

## D Watch Video Solution

3. Find the intervals in which the function $f$ given by $f(x)=x^{2}-4 x+6$ is (a) strictly increasing (b) strictly decreasing.

## - Watch Video Solution

4. Find the intervals in which the function $f$
given by $f(x)=2 x^{2}-3 x$ is
(a) strictly increasing (b) strictly decreasing.

## - Watch Video Solution

5. Find the intervals in which the function $f$
given by $f(x)=2 x^{3}-3 x^{2}-36 x+7$ is
(a) strictly increasing (b) strictly decreasing?

- Watch Video Solution

6. Find the intervals in which the functions $f$
given by $f(x)=4 x^{3}-6 x^{2}-72 x+30$ is
(a) strictly increasing (b) strictly decreasing.
7. Show that the function given by
$f(x)=\sin x$ is
(a) strictly increasing in $\left(0, \frac{\pi}{2}\right)$.

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8. Show that the function given by
$f(x)=\sin x$ is
(b) strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$.

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9. Show that the function given by
$f(x)=\sin x$ is
(c) neither increasing nor decreasing in $(0, \pi)$.

## D Watch Video Solution

10. Prove that the function given by
$f(x)=\cos x$ is
(a) strictly decreasing in $(0, \pi)$.

## D Watch Video Solution

11. Prove that the function given by
$f(x)=\cos x$ is
(b) strictly increasing in $(\pi, 2 \pi)$.

- Watch Video Solution

12. Prove that the function given by
$f(x)=\cos x$ is
(c) neither increasing nor decreasing in
$(0,2 \pi)$.
13. Find the intervals in the function $f$ is given
by $\quad f(x)=\sin x+\cos x, 0 \leq x \leq 2 \pi \quad$ is
strictly increasing or strictly decreasing.

## - Watch Video Solution

14. Find the interval in which the following functions are strictly increasing or decreasing.
(a) $x^{2}+2 x=5$
15. Find the interval in which the following functions are strictly increasing or decreasing.
(b) $10-6 x-2 x^{2}$

- Watch Video Solution

16. Find the interval in which the following
functions are strictly increasing or decreasing.
$-2 x^{3}-9 x^{2}-12 x+1$

- Watch Video Solution

17. Find the interval in which the following functions are strictly increasing or decreasing.
(d) $6-9 x-x^{2}$

## D Watch Video Solution

18. Find the interval in which the following
functions are strictly increasing or decreasing.
(e) $(x+1)^{3}(x-3)^{3}$.

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## 19.

Show
that
$y=\log (1+x)-\frac{2 x}{2+2 x}, x>-1$, is an
increasing function of $x$ throughout its domain.

## D Watch Video Solution

20. Find the values of x for which $\mathrm{y}=[x(x-2)]^{2}$
is an increasing function.

## D Watch Video Solution

21. Prove that $y=\frac{4 \sin \theta}{(2+\cos \theta)}-\theta$ is an
increasing on $\theta$ in $\left[0, \frac{\pi}{2}\right]$.

- Watch Video Solution

22. Prove that the logarithmic function is strictly increasing on $(0, \infty)$.
23. Prove that the function $f$ given by
$f(x)=x^{2}-x+1$ is neither increasing nor decreasing strictly on (-1,1).

## D Watch Video Solution

Three Marks Questions With Answers B Tangents And Normals

1. Find the slope of the tangent to the curve

$$
y=3 x^{4}-4 x \text { at } x=4 .
$$

2. Find the slope of the tangent to the curve
$y=\frac{x-1}{x-2}, x \neq 2$ at $x=10$.

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3. Find the slope of the tangent to curve $y=x^{3}-x+1$ at the point whose $x$ coordinate is 2.

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4. Find the slope of the tangent to the curve $y=x^{3}-3 x+2$ at the point whose $x$ coordinate is 3 .

## D Watch Video Solution

5. Find the slope of the normal to the curve $x=a \cos ^{3} \theta, y=a \sin ^{3} \theta$ at $\theta=\frac{\pi}{3}$.

## D Watch Video Solution

6. Find the slope of the normal to the curve
$x=1-a \sin \theta, y=b \cos ^{2} \theta$ at $\theta=\frac{\pi}{2}$.

## D Watch Video Solution

7. Find points at which the tangent to the
curve $y=x^{3}-3 x^{2}-9 x+7$ is parallel to
the $x$-axis.

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8. Find the point on the curve $y=(x-2)^{2}$ at which the tangent is parallel to the chord joining the points $(2,0)$ and $(4,4)$.

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9. Find the equations of the tangent and normal to the given curve at the given points.
(i) $y=x^{4}-6 x^{3}-10 x+5$ at $(0,5)$

## - Watch Video Solution

10. Find the equations of the tangent and normal to the given curve at the given points.
(ii) $y=x^{4}-6 x^{3}+13 x^{2}-10 x+5$ at $(1,3)$

- Watch Video Solution

11. Find the equations of the tangent and normal to the given curve at the given points.
(iii) $y=x^{3}$ at $(1,1)$

- Watch Video Solution

12. Find the equations of the tangent and normal to the given curve at the given points.
(iv) $y=x^{2}$ at $(0,0)$.

## D Watch Video Solution

13. Find the equations of the tangent and normal to the given curve at the given points.
(v) $x=\cos t, y=\sin t$ at $t=\frac{\pi}{4}$

## D Watch Video Solution

14. Find the slope of the tangent to the curve $y=x^{3}-x$ at $x=2$.

- Watch Video Solution

15. Find the point at which the tangent to the
curve $y=\sqrt{4 x-3}-1$ has its slope $\frac{2}{3}$.

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16. Find the point on the curve $y=x^{3}-11 x+5$ at which the tangent is $y=x-11$.

## D Watch Video Solution

17. Find the equation of all lines having slope -1
that are tangent to the curve
$y=\frac{1}{x+1}, x \neq-1$.
18. Find the equation of all lines having slope
-2 that are tangent to the curve
$y=\frac{1}{x-3}, x \neq 3$.

- Watch Video Solution

19. Find the equation of all lines having slope 0
which are tangents to the curve
$y=\frac{1}{x^{2}-2 x+3}$.

- Watch Video Solution

20. Find points on the curve $\frac{x^{2}}{9}+\frac{y^{2}}{16}=1$ at which the tangents are
(a) parallel X-axis.

- Watch Video Solution

21. Find points on the curve $\frac{x^{2}}{9}+\frac{y^{2}}{16}=1$ at which the tangents are
(b) parallel to Y -axis.
22. Find the equation of the tangent line to
the curve $y=x^{2}-2 x+7$ which is
(a) parallel to the line $2 x-y+9=0$

## - Watch Video Solution

23. Find the equation of the tangent line to
the curve $y=x^{2}-2 x+7$ which is
(b) perpendicular to the line $5 y-15 x=13$.

## D Watch Video Solution

24. Show that the tangents to the curve $y=7 x^{3}+11$ at the points where $\mathrm{x}=2$ and $\mathrm{x}=$
-2 are parallel.

## D Watch Video Solution

25. Find the points on the curve $y=x^{3}$ at which the slope of the tangent is equal to the
$y$-coordinate of the point.
26. For the curve $y=4 x^{3}-2 x^{5}$, find all the points at which the tangent passes through the origin.

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Three Marks Questions With Answers C Approximations

1. Using differentials, find the approximate
value of each of the following upto 3 place of

## decimal.

(i) $\sqrt{25.3}$

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2. Using differentials, find the approximate
value of each of the following upto 3 place of decimal.
(ii) $\sqrt{49.5}$

- Watch Video Solution

3. Using differentials, find the approximate value of each of the following upto 3 place of decimal.
(iii) $\sqrt{0.6}$

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4. Using differentials, find the approximate
value of each of the following upto 3 place of decimal.
(iv) $(0.009)^{1 / 3}$
5. Find the approximate value of $f(2.01)$, where $f(x)=4 x^{2}+5 x+2$.

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6. Find the approximate value of $f(5.001)$, where $f(x) j=x^{3}-7 x^{2}+15$.

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7. Find the approximate change in the volume
$V$ of a cube of side $x$ metre caused by increasing the side by $1 \%$.

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8. Find the approximate change in the surface area of a cube of side $x$ metre caused by decreasing the side by $1 \%$.

## D Watch Video Solution

9. If the radius of a sphere is measured as 7 m
with an error of 0.02 m , then find the approximate error in calculating its volume.

## D Watch Video Solution

## Exercise

1. Find the local maxima and local minima. If
any of the following function. Also, find the
local maximum and the local minimum values,
as the case may be as follows:
(i) $f\left(x=x^{2}\right)$

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2. Find the local maxima and local minima. If any of the following function. Also, find the local maximum and the local minimum values, as the case may be as follows:
(ii) $g(x)=x^{3}-3 x$
3. Find the local maxima and local minima. If any of the following function. Also, find the local maximum and the local minimum values, as the case may be as follows:
(iii) $h(x)=\sin x+\cos x, 0<x<\frac{\pi}{2}$

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4. Find the local maxima and local minima. If any of the following function. Also, find the local maximum and the local minimum values,
as the case may be as follows:
(iv) $f(x)=\sin x-\cos x, 0<x<2 \pi$

## D Watch Video Solution

5. Find the local maxima and local minima. If any of the following function. Also, find the local maximum and the local minimum values, as the case may be as follows:
(v) $f(x)=x^{3}-6 x^{2}+9 x+15$
6. Find the local maxima and local minima. If any of the following function. Also, find the local maximum and the local minimum values, as the case may be as follows:
(vi) $g(x)=\frac{x}{2}+\frac{2}{x}, x>0$

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7. Prove that the following functions do not have maxima or minima:
(a) $f(x)=e^{x}$
8. Prove that the following functions do not have maxima or minima:
(b) $g(x)=\log x$

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9. Prove that the following functions do not have maxima or minima:
(c) $h(x)=x^{3}+x^{2}+x+1$

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10. Find the absolute maximum value and the absolute minimum value of the following functions in the given interval.
(a) $f(x)=x^{3}, x \in[-2,2]$

## D Watch Video Solution

11. Find the absolute maximum value and the absolute minimum value of the following functions in the given interval .
(b) $f(x)=\sin x+\cos x, x \in[0, \pi]$
12. Find the absolute maximum value and the absolute minimum value of the following functions in the given interval.
(c) $f(x)=4 x-\frac{1}{2} x^{2}, x \in\left[-2, \frac{9}{2}\right]$

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13. Find the absolute maximum value and the absolute minimum value of the following
functions in the given interval .
(d) $f(x)=(x-1)^{2}+3 x, x \in[-3,1]$

## D Watch Video Solution

14. Find both the maximum value and the minimum value $3 x^{4}-8 x^{3}+12 x^{2}-48 x+25$ on the interval $[0,3]$.

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# 15. Find the maximum value of <br> $2 x^{3}-24 x+107$ in the interval $[1,3]$. Find the 

 maximum value of the same function in $[-3,-1]$.
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16. It is given that at $x=1$, the function
$x^{4}-62 x^{2}+a x+9$ attains its maximum
value, on the interval [0,2]. Find the value of a.

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17. Find two number whose sum is 24 and whose product is larger as possible.

## D Watch Video Solution

18. Find two positive numbers $x$ and $y$ such
that $x+y=60$ and $x y^{3}$ is maximum.

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19. Find two positive numbers $x$ and $y$ such that their sum is 35 and the product is $x^{3} y^{5}$ is maximum.

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20. Find two positive numbers whose sum is 16 and the sum of whose cubes is minimum.

## D Watch Video Solution

21. A square piece of tin of side 18 cm is to be made into a box without top, by cutting off square from each corner and foling up the flaps of the box. What should be the side of the square to be cut off so that the volume of the box is maximum possible.

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22. A rectangular sheet of tin 45 cm by 24 cm
is to made into a box without top, by cutting-
off square from each other corner and folding
up the flaps. What should be the side of the
square to be cut-off so that the volume of the box is maximum?

## D Watch Video Solution

## Try Yourself

1. Find the rate of change of the area of a circle with respect to its radius $r$ when $r=5 \mathrm{~cm}$.
2. The volume of a cube is increasing at the rate of 9 cubic $\mathrm{cm} / \mathrm{sec}$. How fast is the surface area increasing when the length of an edge is 10 cm ?

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3. The volume of a cube is increasing at a rate of 9 cubic centimetres per second. How fast is
the surface area increasing when the length of an edge is 10 centimetres per second. How fast
is the surface area increasing when the length of an edge is 10 centimetres?

## D Watch Video Solution

4. A stone is dropped into a quiet lake and waves in circles at a speed of 4 cm per second.

At the instant, when the radius of the circular
wave is 10 cm , how fast is the enclosed area increasing?
5. The length $x$ of a rectangle is decreasing at the rate of $3 \mathrm{~cm} / \mathrm{min}$ and the width y is increasing at the rate of $2 \mathrm{~cm} / \mathrm{min}$. When $x=10 \mathrm{~cm}$ and $\mathrm{y}=6 \mathrm{~cm}$, find the ration of change (i) the perimeter and (ii) the area of the reactangle.

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6. The length $x$ of a rectangle is decreasing at the rate of $3 \mathrm{~cm} / \mathrm{min}$ and the width y is increasing at the rate of $2 \mathrm{~cm} / \mathrm{min}$. When
$x=10 \mathrm{~cm}$ and $y=6 \mathrm{~cm}$, find the ration of change (i)
the perimeter and (ii) the area of the reactangle.

## - Watch Video Solution

7. Find the intervals in the function $f$ is given
by $\quad f(x)=\sin x+\cos x, 0 \leq x \leq 2 \pi \quad$ is
strictly increasing or strictly decreasing.

## D Watch Video Solution

8. Find the slope of the tangent to the curve $y=x^{3}-x$ at $x=2$.

## D Watch Video Solution

9. Find the point at which the tangent to the
curve $y=\sqrt{4 x-3}-1$ has its slope $\frac{2}{3}$.

## D Watch Video Solution

10. Use differential to approximate $\sqrt{36.6}$

## - Watch Video Solution

11. Find the approximate value of $f(3.02)$, where $f(x)=3 x^{2}+5 x+3$.

## - Watch Video Solution

12. If the length of three sides of a trapezium other than base are equal to 10 cm then find the area of the trapezium when it is maximum.
13. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.
