MATHS

BOOKS - SURA MATHS (TAMIL ENGLISH)

BINOMIAL THEOREM, SEQUENCES AND SERIES

Exercise 5 1

1. Expand

$$\left(2x^2 - \frac{3}{x}\right)^3$$

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2. Expand

$$\left(2x^2 - 3\sqrt{1-x^2}\right)^4 + \left(2x^2 + 3\sqrt{1-x^2}\right)^4$$

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3. Compute

102^4

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4. Compute

99^4

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5. Find the co-efficient of x^{15} in $\left(x^2+\frac{1}{x^3}\right)^{10}$

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6. Find the Co-efficient of x^6 and the co-efficient of x^2 in $\left(x^2-\frac{1}{x^3}\right)^6$

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7. Find the Co-efficient of x^4 in the expansion of $(1+x^3)^{50}\left(x^2+\frac{1}{x}\right)^5$.

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8. Find the constant term of $\left(2x^3-\frac{1}{3x^2}\right)^5$.

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9. Find the last two digits of the number 3^{600}.

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10. If n is a positive integer, show that, $9^{n+1} - 8n - 9$ is always divisible by 64.

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11. If n is an odd positive integer, prove that the coefficients of the middle terms in the expansion of $(x + y)^n$ are equal.

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12. In the binomial expansion of $(a+b)^n$, the coefficients of the 4^{th} and 13^{th} terms are equal to each other, find n.

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Exercise 5 2

1. Write the first 6 terms of the sequences whose n^{th} terms are given below and classify them as arithmetic progression, geometric progression, arithmetico-geometric progression, harmonic

progression and none of them.

$$\frac{1}{2^{n+1}}$$

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2. Write the first 6 terms of the sequences whose n^{th} terms are given below and classify them as arithmetic progression, geometric progression, arithmetico-geometric progression, harmonic progression and none of them.

$$\frac{(n+1)(n+2)}{n+3(n+4)}$$

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3. Write the first 6 terms of the sequences whose n^{th} terms are given below and classify them as arithmetic progression, geometric progression, arithmetico-geometric progression, harmonic progression and none of them.

$4\left(\frac{1}{2}\right)^n$

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4. Write the first 6 terms of the sequences whose n^{th} terms are given below and classify them as arithmetic progression, geometric progression, arithmetico-geometric progression, harmonic

progression and none of them.

$$\frac{(-1)^n}{n}$$

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5. Write the first 6 terms of the sequences whose n^{th} terms are given below and classify them as arithmetic progression, geometric progression, arithmetico-geometric progression, harmonic progression and none of them.

$$\frac{2n+3}{3n+4}$$

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6. Write the first 6 terms of the sequences whose n^{th} terms are given below and classify them as arithmetic progression, geometric progression, arithmetico-geometric progression, harmonic progression and none of them.

$$\frac{3n-2}{3^{n-1}}$$

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7. Write the first 6 terms of the sequences whose n^{th} term a_n is given below

$$a_n = \begin{cases} \text{n+1} & \text{if n is odd} \\ \text{n} & \text{if n is even} \end{cases}$$

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8. Write the first 6 terms of the sequences whose n^{th} term a_n is given below

$$a_n = \begin{cases} 1 & \text{if } n = 1 \\ 2 & \text{if } n = 2 \\ a_{n-1} + a_{n-2} & \text{if } n > 2 \end{cases}$$

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9. Write the first 6 terms of the sequences whose n^{th} term a_n is given below

$$a_n = \begin{cases} \text{n} & \text{if n is 1, 2, or 3} \\ a_{n-1} + a_{n-2} + a_{n-3} & \text{if } n > 3 \end{cases}$$

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10. Write the n^{th} term of the following sequences.

2, 2, 4, 4, 6, 6, ___

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11. Write the n^{th} term of the following sequences.

$$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \cdots$$

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12. Write the n^{th} term of the following sequences.

$$\frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \frac{9}{10}, \cdots$$

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13. Write the n^{th} term of the following sequences.

6, 10, 4, 12, 2, 14, 0, 16, -2, ____

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14. The product of three increasing numbers in GP is 5832. If we add 6 to the second number and 9 to the third number, then resulting numbers form an AP. Find the number in GP.

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15. Write the n^{th} term of the sequence $\frac{3}{1^2 2^2}, \frac{5}{2^2 3^2}, \frac{7}{3^2 4^2}, \ldots$ as a difference of two terms.

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16. If t_k is the k^{th} term of a G.P, then show that t_{n-k}, t_n, t_{n+k} also form a GP for any positive integer k.

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17. If a, b, c are in geometric progression, and if $a^{\frac{1}{x}} = b^{\frac{1}{y}} = c^{\frac{1}{z}}$, then prove that x, y, z are in arithmetic progression.

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18. The AM of two numbers exceeds their GM by 10 and HM by 16. Find the numbers.

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19. If the roots of the equation $(q-r)x^2+(r-p)x+p-q=0$ are equal, then show that p, q and r are in A.P.

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20. If a, b, c are respectively the $p^{th}q^{th}$ and r^{th} terms of a GP. Show that $(q-r)\log a+(r-p)\log b+(p-q)\log c=0.$

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Exercise 5 3

1. Find the sum of first 20 terms of the arithmetic progression having the sum of first 10 terms as 52 and the sum of the first 15 terms as 77.

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2. Compute the sum of first n terms of the following series :

8 + 88 + 888 + 8888 ____

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3. Compute the sum of first n terms of the following series :

6 + 66 + 666 + 6666 ...

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4. Compute the sum of first n terms of

$1 + (1 + 4) + (1 + 4 + 4^2) + (1 + 4 + 4^2 + 4^3) + ...$

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5. Find the general term and sum to n terms of the sequanece $1, \frac{4}{3}, \frac{7}{9}, \frac{10}{27}, \ldots$

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6. Find the value of n, if the sum to n terms of the series $\sqrt{3} + \sqrt{75} + \sqrt{243} + \ldots 435\sqrt{3}$. Is

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7. Show that the sum of $(m+n)^{th}$ and $(m-n)^{th}$ term of an A.P is equal to twice the m^{th} term.

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8. A man repays an amount of Rs.3250 by paying Rs.20 in the first month and then increases the payment by Rs.15 per month. How long will it take him to clear the amount?

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9. In a race, 20 balls are placed in a line at intervals of 4 meters, with the first ball 24 meters away from the starting point. A contestant is required to bring the balls back to the starting place one at a time.

How far would the contestant run to bring back all balls?

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10. The number of bacteria in a certain culture doubles every hour. If there were 30 bacteria present in the culture originally, how many bacteria will be present at the end of 2^{nd} hour, 4^{th} hour and n^{th} hour?

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11. What will Rs. 500 amounts to in 10 years after its deposit in a bank which pays annual interest rate of 10% compounded annually?

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12. In a certain town, a viral disease caused severe health hazards upon its people disturbing their normal life. It was found that on each day, the virus which caused the disease spread in Geometric Progression. The amount of infectious virus particle gets doubled each day, being 5 particles on the first

day. Find the day when the infections virus particles just grow over 1,50,000 units?

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Exercise 5 4

1. Expand the following in ascending powers of x and find the condition on x for which the binomial expansion is valid.

$$\frac{1}{5+x}$$

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2. Expand the following in ascending powers of x and find the condition on x for which the binomial expansion is valid.

$$\frac{2}{(3+4x)^2}$$

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3. Expand the following in ascending powers of x and find the condition on x for which the binomial expansion is valid.

$$(5+x^2)^{\frac{2}{3}}$$

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4. Expand the following in ascending powers of x and find the condition on x for which the binomial expansion is valid.

$(x+2)^{\frac{-2}{3}}$

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5. Find $\sqrt[3]{1001}$ approximately. (two decimal places).

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6. Prove that $\sqrt[3]{x^3+6}-\sqrt[3]{x^3+3}$ is approximately equal to $\frac{1}{x^2}$ when x is sufficiently large.

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7. Prove that $\sqrt{\frac{1-x}{1+x}}$ is approximately equal to $1-x+\frac{x^2}{2}$ when x is very small.

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8. Write the first 6 terms of the exponential series

e^{5x}

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9. Write the first 6 terms of the exponential series

e^{-2x}

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10. Write the first 6 terms of the exponential series

$e^{\frac{1}{2}x}$

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11. Write the first 4 terms of the logarithmic series

$\log(1 + 4x)$

Find the intervals on which the expansions are valid.

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12. Write the first 4 terms of the logarithmic series

$\log(1 - 2x)$

Find the intervals on which the expansions are valid.

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13. Write the first 4 terms of the logarithmic series

$$\log\left(\frac{1+3x}{1-3x}\right)$$

Find the intervals on which the expansions are valid.

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14. Write the first 4 terms of the logarithmic series

$$\log\left(\frac{1-2x}{1+2x}\right)$$

Find the intervals on which the expansions are valid.

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15. If $y = x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \ldots$ then show that $x = y - \frac{y^2}{2!} + \frac{y^3}{3!} - \frac{y^4}{4!} + \ldots$

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16. If p - q is small compared to either p or q, then show that $\sqrt{\frac{p}{q}} = \frac{(n+1)p + (n-1)q}{(n-1)p + (n+1)q}$

Hence find $\sqrt{\frac{15}{16}}$.

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17. Find the co-efficient of x^4 in the expansion of $\frac{3-4x+x^2}{e^{2x}}$.

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Exercise 5 5

1. The value of 2 + 4 + 6 + ...+ 2n is

A. $\frac{n(n-1)}{2}$

B. $\frac{n(n+1)}{2}$

C. $\frac{2n(2n+1)}{2}$

D. $n(n+1)$

Answer:

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2. The coefficient of x^6 in $(2+2x)^{10}$ is

A. ${}^{10}C_6$

B. 2^6

C. ${}^{10}C_6 2^6$

D. ${}^{10}C_6 2^{10}$

Answer:

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3. The coefficient of $x^8 y^{12}$ in the expansion of $(2x + 3y)^{20}$ is

A. 0

B. $2^8 3^{12}$

C. $2^8 3^{12} + 2^{12} 3^8$

D. ${}^{20}C_8 2^8 3^{12}$

Answer:

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4. If ${}^nC_{10} > {}^nC_r$ for all possible r, then a value of n is

A. 10

B. 21

C. 19

D. 20

Answer:

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5. If a is the arithmetic mean and g is the geometric mean of two numbers, then

A. $a \leq g$

B. $a \geq g$

C. a = g

D. $a > g$

Answer:

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6. If a, 8, b are in AP, a, 4, b are in GP, and if a, x, b are in HP then x is

A. 2

B. 1

C. 4

D. 16

Answer:

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7. The sequence $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}+\sqrt{2}}, \frac{1}{\sqrt{3}+2\sqrt{2}}, \ldots$ form an

A. AP

B. GP

C. HP

D. AGP

Answer:

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8. The HM of two positive numbers whose AM and GM are 16, 8 respectively is

A. 10

B. 6

C. 5

D. 4

Answer:

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9. If S_n denotes the sum of n terms of an AP whose common difference is d, the value of $S_n - 2S_{n-1} + S_{n-2}$ is

A. 0

B. 2d

C. 4d

D. d^2

Answer:

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10. The remainder when 38^{15} is divided by 13 is

A. 12

B. 1

C. 11

D. 5

Answer:

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11. The n^{th} term of the sequence 1, 2, 4, 7, 11, ... is

A. $n^3 + 3n^2 + 2n$

B. $n^3 - 3n^2 + 3n$

C. $\frac{n(n+1)(n+2)}{3}$

D. $\frac{n^2 - n + 2}{2}$

Answer:

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12. The sum up to n terms of the series $\frac{1}{\sqrt{1}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{7}} + \ldots$ is

A. $\sqrt{2n+1}$

B. $\frac{\sqrt{2n+1}}{2}$

C. $\sqrt{2n+1}-1$

D. $\frac{\sqrt{2n+1}-1}{2}$

Answer:

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13. The n^{th} term of the sequence $\frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \frac{15}{16}, \ldots$ is

A. $2^n - n - 1$

B. $1 - 2^{-n}$

C. $2^{-n} + n - 1$

D. 2^{n-1}

Answer:

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14. Sum of first 'n' terms of the series $\sqrt{2} + \sqrt{8} + \sqrt{18} + \ldots$ is ___.

A. $\frac{n(n+1)}{2}$

B. $2n(n+1)$

C. $\frac{n(n+1)}{2}$

D. 1

Answer:

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15. The value of the series $\frac{1}{2}+\frac{7}{4}+\frac{13}{8}+\frac{19}{16}+\ldots$ is

A. 14

B. 7

C. 4

D. 6

Answer:

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16. The sum of an infinite GP is 18. If the first term is 6, the common ratio is

A. $\frac{1}{3}$

B. $\frac{2}{3}$

C. $\frac{1}{6}$

D. $\frac{3}{4}$

Answer:

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17. The coefficient of x^5 in the series e^{-2x} is

A. $\frac{2}{3}$

B. $\frac{3}{2}$

C. $\frac{-4}{15}$

D. $\frac{4}{15}$

Answer:

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18. The value of $\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+\ldots$ is

A. $\frac{e^2+1}{2e}$

B. $\frac{(e+1)^2}{2e}$

C. $\frac{(e-1)^2}{2e}$

D. $\frac{e^2-1}{2e}$

Answer:

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19. The value of $1-\frac{1}{2}\left(\frac{2}{3}\right)+\frac{1}{3}\left(\frac{2}{3}\right)^2-\frac{1}{4}\left(\frac{2}{3}\right)^3+\ldots$ is

A. $\log\left(\frac{5}{3}\right)$

B. $\frac{3}{2}\log\left(\frac{5}{3}\right)$

C. $\frac{5}{3}\log\left(\frac{5}{3}\right)$

D. $\frac{2}{3}\log\left(\frac{2}{3}\right)$

Answer:

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Additional Problems

1. If the co-efficient of x in $\left(x^2 + \frac{\lambda}{x}\right)^5$ is 270, then λ =

A. 3

B. 4

C. 5

D. 6

Answer:

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2. The remainder when 52^{40} is divided by 17 is

A. 1

B. 3

C. 5

D. 6

Answer:

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3. The first and last term of an A.P. are 1 and 11. If the sum of its terms is 36, then the number of

terms will be

A. 5

B. 6

C. 7

D. 8

Answer:

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4. Choose the incorrect pair :

A. $\frac{d}{dx}(\sin x)$ $\cos x$

B. $\frac{d}{dx}(\tan x)$ $\sec^2 x$

C. $\frac{d}{dx}(\cos x)$ $\sin x$

D. $\frac{d}{dx}\log x$ $\frac{1}{x}$

Answer:

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5. Expand $(1+x)^{\frac{2}{3}}$ up to four terms for $|x| < 1$

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6. If a,b,c are in A.P., show that $(a-c)^2 = 4(b^2 - ac)$.

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7. Find all the values of $x \neq 0$ in $(-\pi, \pi)$ satisfying the equation $8^{1+\cos x+\cos^2 x+\ldots} = 4^3$

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8. For what value of n, the n^{th} term of the series "3 + 10 + 17 + .." and "63 + 65 + 67 + ..." are equal

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9. The $2^{nd}, 3^{rd}$ and 4^{th} terms in the binomial expansion of $(x+a)^n$ are 240, 720 and 1080 for a suitable value of x. Find x, a and n.

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10. Prove that $\sqrt[3]{x^3+7} - \sqrt[3]{x^3+4}$ is approximately equal to $\frac{1}{x^2}$ when x is large.

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