MATHS

BOOKS - SURA MATHS (TAMIL ENGLISH)

COMBINATORICS AND MATHEMATICAL INDUCTION

Exercise 4 1

1. A person went to a restaurant for dinner. In the menu card, the person saw 10 Indian and 7 Chinese food items. In how many ways the person can select either an Indian or a Chinese food ?

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2. There are 3 types of toy car and 2 types of toy train are available in a shop. Find the number of ways a baby can buy a toy car and a toy train ?

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3. How many two - digit numbers can be formed using 1,2,3,4,5 without repetition of digits ?

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4. There persons enter into a conference hall in which there are 10 seats. In how many ways they can take their seats ?

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5. In how many ways 5 persons can be seated in a row ?

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6. A mobile has a pass code of distinct digits. What is the maximum number of attempts one makes to retrieve the pass code ?

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7. Given four flags of different colours, how many different signals can be generated if each signal requires to use of 3 flags. One below the other ?

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8. Four children are running a race .

(i) In how many ways can the first two places be filled ?

(ii) In how many different ways could they finish the race ?

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9. Count the number of three - digit numbers which can be formed from the digits 2,4,6,8, if

(i) repetitions of digits is allowed ?

(ii) repetitions of digits is not allowed ?

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10. How many three - digit numbers are there with 3 in the unit place ?

(i) With repetition (ii) without repetition

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11. How many numbers are three between 100 and 500 with the digits 0,1,2,3,4,5, ? If

(i) repetition of digits allowed

(ii) the repetition of digits is not allowed ?

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12. How many three-digit odd numbers can be formed by using the digits 0,1,2,3,4,5 ? If

(i) the repetition of digits is not allowed

(ii) the repetition of digits is allowed

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13. Count the numbers between 999 and 10,000 subject to the condition that there are .

(i) no restriction .

(ii) no digit is repeated .

(iii) at least one of the digits is repeated .

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14. How many three- digit numbers, which are divisible by 5, can be formed using the digits 0,1,2,3,4, 5 if

(i) repetition of digits are not allowed ?

(ii) repetition of digits are allowed ?

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15. To travel from a place A to place B, there are two different bus routes B_1, B_2 , two different train routes T_1, T_2 and one air route A_1 . From place B to place C there is one bus route say B_1 two different train routes say T_1, T_2 and one air route A_1 . Find the number of routes of commuting from place A to place C via place B without using similar mode of transportation.

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16. How many numbers are there between 1 and 1000 (both inclusive) which are divisible neither by 2 nor by 5 ?

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17. How many strings can be formed using the letters of the word LOTUS if the word .

(i) either starts with L or ends with S.

(ii) neither starts with L nor ends with S ?

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18. (i) Count the total number of ways of answering 6 objective type questions , each question having 4 choices.

(ii) In how many ways 10 pigeons can be placed in 3 different pigeon holes ?

(iii) Find the number of ways of distributing 12 distance prizes to 10 students ?

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19. Find the value of

6!

(ii) 4 ! + 5 !

(iii) 3! - 2!

(iv) 3! $\times$ 4!

(v) $\frac{12!}{9! \times 3!}$ (vi) $\frac{(n+3)!}{(n+1)!}$

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20. Evaluate $\frac{n!}{r!(n-r)!}$ when

(i) n = 6, r= 2

(ii) n = 10 , r = 3

(iii) for any n with r = 2

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21. Find the value of n if

(i) $(n+1)! = 20(n-1)!$

(ii) $\frac{1}{8!} + \frac{1}{9!} = \frac{n}{10!}$

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Exercise 4 2

1. If $.^{n-1}P_3 : P_4 = 1, 10$ find n.

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2. IF $.^{10}P_{r-1} = 2 \times 6P_r$ find r

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3. (i) Suppose 8 people enter an event in a swimming meet. In how many ways could the gold, silver and bronze prizes be awarded ?

(ii) Three men have 4 coats , 5 waist coats and 6 caps. In how many ways can they wear them ?

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4. Determine the number of permutations of the letters of the word SIMPLE if all are taken at a time ?

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5. A test consists of 10 multiple choice questions . In how many ways can the test be answered if

(i) Each question has four choices ?

(ii) The first four questions have three choices and the remaining have five choices ?

(iii) Question number n has n + 1 choices ?

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6. A student appears in an objective test which contain 5 multiple choice questions. Each question has 4 choices out of which one correct answer.

(i) What is the maximum number of different answers can the students give ?

(ii) How will the answer change if each question may have more than one correct answers ?

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7. How many strings can be formed from the letters of the word ARTICLE, so that vowels occupy the even places ?

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8. 8 woman and 6 man are standing in a line.

(i) How many arrangements are possible if any individual can stand in any position ?

(ii) In how may arrangements will be 6 men be standing next to one another ?

(iii) In how many arrangements will no two men be standing next to one another ?

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9. Find the distinct permutations of the letters or the word MISSISSIPPI ?

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10. How many ways can the product $a^2b^3c^4$ be expressed without exponents ?

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11. In how many ways 4 mathematics books , 3 physics books, 2 chemistry books and 1 biology book can be arranged on a shelf so that all books of the same subjects are together ?

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12. In how many ways can the letters of the word SUCCESS be arranged so that all S's are together ?

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13. A coin is tossed 8 times.

(i) How many different sequences of heads and tails are possible ?

(ii) How many different sequences containing six heads and two tails are possible ?

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14. How many strings are there using the letters of the word INTERMEDIATE , if

(i) The vowels and consonants are alternative

(ii) All the vowels are together

(iii) Vowels are never together .

(iv) No two vowels are together .

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15. Each of the digits 1,1,2,3,3 and 4 is written on separate card. The six cards are then laid out in a row to form a 6-digit number.

(i) How many distinct 6-digit numbers are there ?

(ii) How many of these 6-digit numbers are even ?

(iii) How many of these 6-digit numbers are divisible by 4 ?

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16. If the letters of the word GARDEN are permuted in all possible ways and the strings thus formed are arranged in the dictionary order, then find the ranks of the words (i) GARDEN (ii) DANGER.

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17. Find the number of strings that can be made using all letters of the word THING. If these words are written as in a dictionary , what will be the 85^{th} string ?

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18. if the letters of the word FUNNY are permuted in all possible ways and the strings thus formed are arranged in the dictionary order, find the rank of the word FUNNY .

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19. Find the sum of all 4-digit numbers that can be formed using digits 1, 2, 3, 4, and 5 repetitions not allowed?

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20. Find the sum of all 4-digit numbers that can be formed using digits 0,2,5,7,8 without repetitions ?

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Exercise 4 3

1. If $.^{n}C_{12} = .^{n}C_{9}$ find $.^{21}C_{n}$

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2. If $.^{15}C_{2r-1} = .^{15}C_{2r+4}$ find r

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3. If $.^{n}P_{r} = 720.$ If $.^{n}C_{r} = 120$ find n, r

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4. Prove that $.^{15}C_{3} + 2 \times .^{15}C_{4} + .^{15}C_{5} = .^{17}C_{5}$

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5. Prove that $.^{35}C_{5} + \sum_{r=0}^{4^{(39-r)}} C_{4} = .^{40}C_{5}$

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6. If $.^{n+2}C_8 : .^{(n-2)}P_4 = 57:16$, find the value of n.

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7. Prove that $.^{2n}C_n = \frac{2^n \times 1 \times 3 \times \ldots (2n-1)}{n!}$

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8. Prove that if $1 \leq r \leq n$ then $n \times .^{(n-1)}C_{r-1} = (n-r+1).^{n}C_{r-1}$

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9. A Kabaddi coach has 14 players ready to play. How many different teams of 7 players could the coach put on the court ?

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10. There are 15 persons in a party and if each 2 of then shakes hands with each other , how many handshakes happen in the party ?

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11. How many chords can be drawn through 20 points on a circle ?

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12. In a parking lot one hundred one year old cars, are parked. Out of them five are to be chosen at random for to check its pollution devices. How many differenet set of five cars can be chosen ?

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13. How many ways can a team of 3 boys, 2 girls and 1 transgender be selected from 5 boys , 4 girls and 2 transgenders ?

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14. Find the total number of subsets of a set with .

(i) 4 elements (ii) 5 elements

(iii) n elements

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15. A trust has 25 members .

(i) How many ways 3 officers can be selected ?

(ii) In how many ways can a president, Vice president and a Secretary be selected ?

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16. How many ways a committee of six persons from 10 persons can be chosen along with a chair person and a secretary ?

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17. How many different selections of 5 books can be made from 12 different books if,

(i) Two particular books are always selected ?

(ii) Two particular books are never selected ?

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18. There are 5 teachers and 20 students . Out of them a committee of teachers and 3 students is to be formed. Find the number of ways in which this can be done. Further find in how many of these committees

(a) a particular teacher is included ?

(ii) a particular student is excluded ?

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19. In an examination a student has a answer 5 questions, out of 9 questions in which 2 are compulsory . In how many ways a student can answer the questions ?

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20. Determine the number of 5 card combinations out of a deck of 52 cards if there is exactly three aces in each combination .

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21. Find the number of ways of forming a committee of 5 members out of 7 Indians and 5 Americans so that always Indians will be majority in the committee.

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22. A committee of 7 peoples has to be formed from 9 men and 4 women . In how many can this be done when then committee consists of

(i) exactly 3 women ?

(ii) at least 3 woman ?

(iii) at most 3 women ?

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23. 7 relatives of a man comprises 4 ladies and 3 gentlemen, his wife also has 7 relatives , 3 of them are ladies and 4 gentlemen. In how many ways can they invite a dinner party of 3 ladies and 3 gentlemen , so that there are 3 of man's relative and 3 of the wife ,s relatives ?

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24. A box contains two white balls, three black balls and four balls. In how many ways can three balls be drawn from the box, if atleast one black ball is to be included in the draw ?

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25. Find the number of strings of 4 letters that can be formed with the letters of the word EXAMINATION.

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26. How many triangles can be formed by 15 points , in which 7 of them lie on one line and the remaining 8 on another parallel line ?

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27. There are 11 points in a plane. No three of these lies in the same straight line except 4 points, which are collinear. Find ,

(i) the number of straight lines that can be obtained from the pairs of these points ?

(ii) the number of triangles that can be formed for which the points are their vertices ?

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28. A polygon has 90 diagonals. Find the number of its sides ?

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Exercise 4 4

1. By the principle of mathematical induction, prove that, for $n \geq 1$

$$1^3 + 2^3 + 3^3 + \ldots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$$

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2. BY the principle of mathematical induction , prove that for n ≥ 1

$$1^2 + 3^2 + 5^2 + \ldots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$$

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3. Prove that the sun of first n' non-zero even numbers in $n^2 + n$

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4. By the principle of mathematical induction , prove that for $n \leq 1$

$$1.2 + 2.3 + 3.4 + \ldots + n(n+1) = \frac{n(n+1)(n+2)}{3}$$

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5. Using the mathematical induction, show that for any natural number

$n \leq 2$

$$\left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right)\left(1 - \frac{1}{4^2}\right)\ldots\left(1 - \frac{1}{n^2}\right) = \frac{n+1}{2n}$$

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6. Using the mathematical induction , show that for any natural number

$n \leq 2$

$$\frac{1}{1+2}+\frac{1}{1+2+3}+\frac{1}{1+2+3+4}+\ldots.+\frac{1}{1+2+3+\ldots.+n}=$$

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7. Using the mathematical induction , show that for any natural number n,

$$\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+..+\frac{1}{n(n+1)(n+2)}=\frac{n(n+3)}{4(n+1)(n+2)}$$

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8. Using the mathematical induction , show that for any natural number n,

$$\frac{1}{2.5}+\frac{1}{5.8}+\frac{1}{8.11}+\ldots+\frac{1}{(3n-1)(3n+2)}=\frac{n}{6n+4}$$

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9. Prove by mathematical induction that $1!+(2\times 2!)+(3\times 3!)+\ldots+(n\times n!)=(n+1)!-1$

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10. Using the mathematical induction show that for any natural number n, $x^{2n} - y^{2n}$ is divisible by (x +y)

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11. By the principle of mathematical induction , prove that for $n \leq 1, 1^2 + 2^2 + 3^2 + \ldots + n^2 > \frac{n^3}{3}$

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12. Use induction to prove that $n^3 - 7n + 3$ is divisible by 3, for all natural numbers n „.

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13. Use induction to prove that $5^{n+1} + 4 \times 6^n$ when divided by 20 leaves a remainder 9 for all natural numbers n .

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14. Use induction to prove that $10^n + 3 \times 4^{n+2} + 5$ is divisible by 9 for all natural numbers n .

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15. Prove that using the Mathematical induction

$$\sin(\alpha) + \sin\left(\alpha + \frac{\pi}{6}\right) + \sin\left(\alpha + \frac{2\pi}{6}\right) + .. + \sin\left(\alpha + \frac{(n-1)\pi}{6}\right) = \frac{\sin\left[\alpha + \frac{(n-1)\pi}{12}\right] \times \sin\left(\frac{n\pi}{12}\right)}{\sin\left(\frac{\pi}{12}\right)}$$

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Exercise 4 5

1. The sum of the digits at the 10th place of all numbers formed with the help of 2,4,5,7 taken all at a time is

A. 432

B. 108

C. 36

D. 18

Answer: B

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2. In an examination there are three multiple choice questions and each question has 5 choices Number of ways in which a student can fail to get all answer correct is

A. 125

B. 124

C. 64

D. 63

Answer: B

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3. The number of ways in which of following prize be given to a class of 30 boys first and second in mathematics, first and second in physics, first in chemistry and first in English is

A. $30^4 \times 29^2$

B. $30^3 \times 29^3$

C. $30^2 \times 29^4$

D. 30×29^5

Answer: A

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4. The number of 5 digit numbers all digits of which are odd is

A. 25

B. 5^5

C. 5^6

D. 625

Answer: B

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5. In 3 fingers , the number of ways four rings can be worn is .....ways

A. $4^3 - 1$

B. 3^4

C. 68

D. 64

Answer: B

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6. If $.^{n+5}P_{n+1} = \left(\frac{11(n-1)}{2}\right).^{n+3}P_n$ then the value of n are

A. 7 and 11

B. 6 and 7

C. 2 and 11

D. 2 and 6

Answer: B

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7. The product of r consecutive positive integers is divisible by

A. r!

B. (r-1)!

C. (r+1)!

D. r^r

Answer: A

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8. The number of five digit telephone numbers having at least one of their digits repeated is

A. 90000

B. 10000

C. 30240

D. 69760

Answer: D

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9. If $.^{a^{2-a}}C_2 = a^{2-a}C_4$ then the value of a is

A. 2

B. 3

C. 4

D. 5

Answer: B

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10. There are 10 points in a plane and 4 of them are collinear. The number of straight lines joining any two points is

A. 45

B. 40

C. 39

D. 38

Answer: B

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11. The number of ways in which a host lady invite 8 people for a party of 8 out of 12 people of whom two do not want to attend the party together is

A. $2 \times^{11} C_7 +^{10} C_8$

B. $.^{11} C_7 + .^{10} C_8$

C. $.^{12} C_8 -^{10} C_6$

D. $.^{10} C_6 + 2!$

Answer: C

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12. The number of parallelograms that can be formed from a set of four parallel lines intersecting another set of three parallel lines.

A. 6

B. 9

C. 12

D. 18

Answer: D

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13. Everybody in a room shakes hands with everybody else. The total number of shake hands is 66. The number of persons in the room is

A. 11

B. 12

C. 10

D. 6

Answer: B

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14. Number of sides of a polygon having 44 diagonals is

A. 4

B. 4!

C. 11

D. 22

Answer: C

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15. If 10 lines are drawn in a plane such that no two of them are parallel and no three are concurrent, then the total number of points of intersection are

A. 45

B. 40

C. 10!

D. 2^{10}

Answer: A

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16. In a plane are 10 points are there out of which 4 points are collinear, then the number of triangles formed is

A. 110

B. $.^{10}C_3$

C. 120

D. 116

Answer: B

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17. In $.^{2n}C_3 \therefore^{n} C_3 = 11:1$ then n is

A. 5

B. 6

C. 11

D. 7

Answer: B

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18. $.^{(n-1)}C_r + .^{(n-1)}C_{(r-1)}$ is

A. $.^{n+1}C_r$

B. $.^{(n-1)}C_r$

C. $.^{n}C_r$

D. $.^{n}C_{r-1}$

Answer: C

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19. The number of ways of choosing 5 cards out of a deck of 52 which include at least one king is

A. $.^{52}C_5$

B. $.^{48}C_5$

C. $.^{52}C_5 + .^{48}C_5$

D. $.^{52}C_5 - .^{48}C_5$

Answer: D

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20. The number of rectangles that a chessboard has

A. 81

B. 9^9

C. 1296

D. 6561

Answer: C

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21. The number of 10 digit number that can be written by using the digits 2 and 3 is

A. $.^{10}C_2 + .^{9}C_2$

B. 2^{10}

C. $2^{10} - 2$

D. 10!

Answer: B

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22. If P stands for P_r then the sum of the series $1 + P_1 + 2P_2 + 3P_3 + \ldots +$ nPn is

A. P_{n+1}

B. $P_{n+1} - 1$

C. $P_{n+1} + 1$

D. $.^{n+1}P_{n-1}$

Answer: B

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23. The product of first n odd natural numbers equals:

A. $.^{2n}C_n \times .^{n}P_n$

B. $\left(\frac{1}{2}\right)^n .^{2n}C_n \times .^{n}P_n$

C. $\left(\frac{1}{4}\right)^n . \times .^{2n}C_n \times .^{2n}P_n$

D. $.^{n}C_n \times .^{n}P_n$

Answer: B

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24. If $''C_4, ''C_5, ''C_6$ are in AP then value of n is

A. 14

B. 11

C. 9

D. 5

Answer: A

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25. 1 +3 + 5 + 7 + ... 17 is equal to

A. 101

B. 81

C. 71

D. 61

Answer: B

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Additional Problems Section A

1. Value of $\frac{7!}{2!}$ is

A. 2520

B. 2250

C. 2205

D. 2500

Answer: A

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2. The number of words that can be formed out of the letters of the word " COMMITTEE"

A. $\frac{9!}{(2!)^3}$

B. $\frac{9!}{(2!)^2}$

C. $\frac{9!}{2!}$

D. 9!

Answer: A

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3. Rank of the word " MOTHER " is

A. 310

B. 300

C. 308

D. 309

Answer: D

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4. If $\frac{1}{7!}+\frac{1}{8!}=\frac{A}{9!}$ then the value of A is

A. 7^2

B. 8^2

C. 9

D. 9^2

Answer: D

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5. The sum of the digits in the unit's place of all the 4-digit numbers formed by 3,4,5 and 6, without repetition , is

A. 432

B. 108

C. 36

D. 72

Answer: B

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6. Assertion (A) : Every body in a room shakes hands with everybody else. The total number of persons in the room is n . The number of hand shakes is $\frac{n(n-1)}{2}$

Reason (R) : The number of handshakes is $.^{n}C_{2}$

A. Both A and R are true and R is the correct explanation of A

B. Both A and R are true but R is not the correct explanation of A

C. A is true R is false

D. A is false R is true

Answer: A

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7. Mark the incorrect statement of the following

A. Factorial of a natural number n is the product of the first n natural numbers

B. The numbers ways of arranging n unlike objects is n!

C. Order matters for combination

D. The number of combinations of n different things taken r at time is $.^{n}C_{r}$

Answer: C

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Additional Problems Section B

1. If $nP_r = 11880$ and $nC_r = 495$ find n and r .

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2. Find n if (n + 1) ! = 12 $\times (n-1)!$

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3. A room has 6 dorrs . In how many ways can a man enter the room through one door and come out through a different door ?

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4. In how many ways can the letters of the word PENCIL be arranged so that N is always next of E.

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Additional Problems Section C

1. Count the number of positive integers greater than 7000 and less than 8000 which are divisible by 5, provided that no digits are repeated.

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2. If ${}^{(n+2)}C_7 : {}^{(n-1)}P_4 = 13:24$ find n.

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3. Find the rank of the word " SCHOOL"

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4. Show that $\frac{(2n)!}{n!} = 2^n\{1, 3, 5, \ldots(2n-1)\}$

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5. Prove that $n!(n+2) = n! + (n+1)!$

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6. If (n +2)!= 60 (n -1)! Find n .

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Additional Problems Section D

1. Find the sum of all 4-digit numbers that can be formed using the digits 1,2,4,6 and 8.

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2. Show that $\frac{(2n)!}{n!} = 2^n\{1, 3, 5, \ldots(2n-1)\}$

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3. If the letters of the word APPLE are permuted in all possible ways and the strings then formed are arranged in the dictionary order show that the rank of the word APPLE is 12.

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4. A van has 8 seats. It has two seats in the froot with two row of three seats behind. The van belongs to a family, consisting of seven members, $F, M, S_1, S_2, S_3, D_1, D_2$. How many ways can the family sit in the van if

i. Ther are no restriction?

ii. Either F or M drives the van ?

iii. D_1, D_2 sits next to a window and is driving ?

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5. Using Mathemtical induction, show that for any natural number n,

$$\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\ldots+\frac{1}{n(n+1)}=\frac{n}{n+1}$$

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