MATHS

BOOKS - SURA MATHS (TAMIL ENGLISH)

DIFFERENTIAL CALCUS - DIFFERENTIABILITY AND METHODS OF DIFFERENTIATION

Exercise 10 1

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1. Find the derivatives of the following functions using first principle.

$f(x)=6$

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2. Find the derivatives of the following functions using first principle.

$f(x)=-4x+7$

3. Find the derivatives of the following functions using first principle.

$$f(x) = -x^2 + 2$$

4. Find the derivatives from the left and from the right at x=1 (if they exist) of the following functions. Are the functions differentiable at x=1?

$$f(x) = |x - 1|$$

5. Find the derivatives from the left and from the right at x=1 (if they exist) of the following functions. Are the functions differentiable at x=1?

$$f(x) = \sqrt{1 - x^2}$$

6. Find the derivatives from the left and from the right at x=1 (if they exist) of the following functions. Are the functions differentiable at x=1?

$$f(x) = \begin{cases} x, & x \leq 1 \\ x^2, & x > 1 \end{cases}$$

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7. Determine whether the following function is differentiable at the indicated values.

$f(x) = x|x|$ at x=0

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8. Determine whether the following function is differentiable at the indicated values.

$f(x) = |x^2 - 1|$ at x=1

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9. Determine whether the following function is differentiable at the indicated values.

$f(x) = |x| + |x - 1|$ at x=0, 1

10. Determine whether the following function is differentiable at the indicated values.

$f(x) = \sin|x|$ at x=0

11. Show that the following functions are not differentiable at the indicated value of x.

$f(x) = \begin{cases} -x + 2, & x \leq 2 \\ 2x - 4, & x > 2 \end{cases}, x = 2$

12. Show that the following functions are not differentiable at the indicated value of x.

$$f(x) = \begin{cases} 3x, & x < 0 \\ -4x, & x \geq 0 \end{cases}, x = 0$$

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13. If $f(x) = |x + 100| + x^2$, test whether $f'(100)$ exists.

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14. Examine the differentiability of functions in R by drawing the diagrams.

$|\sin x|$

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15. Examine the differentiability of functions in R by drawing the diagrams.

$|\cos x|$

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1. Find the derivatives of the following functions with respect to corresponding independent variables.

$$f(x) = x - 3\sin x$$

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2. Find the derivatives of the following functions with respect to corresponding independent variables.

$$y = \sin x + \cos x$$

3. Find the derivatives of the following functions with respect to corresponding independent variables.

$f(x) = x \sin x$

4. Find the derivatives of the following functions with respect to corresponding independent variables.

$y = \cos x - 2 \tan x$

5. Find the derivatives of the following functions with respect to corresponding independent variables.

$g(t) = t^3 \cos t$

6. Find the derivatives of the following functions with respect to corresponding independent variables.

$$g(t) = 4 \sec t + \tan t$$

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7. Find the derivatives of the following functions with respect to corresponding independent variables.

$$y = e^x \sin x$$

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8. Find the derivatives of the following functions with respect to corresponding independent variables.

$$y = \frac{\tan x}{x}$$

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9. Find the derivatives of the following functions with respect to corresponding independent variables.

$$y = \frac{\sin x}{1 + \cos x}$$

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10. Find the derivatives of the following functions with respect to corresponding independent variables.

$$y = \frac{x}{\sin x + \cos x}$$

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11. Find the derivatives of the following functions with respect to corresponding independent variables.

$$y = \frac{\tan x - 1}{\sec x}$$

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12. Find the derivatives of the following functions with respect to corresponding independent variables.

$$y = \frac{\sin x}{x^2}$$

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13. Find the derivatives of the following functions with respect to corresponding independent variables.

$$y = \tan\theta(\sin\theta + \cos\theta)$$

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14. Find the derivatives of the following functions with respect to corresponding independent variables.

$$y = \cos ecx. \cot x$$

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15. Find the derivatives of the following functions with respect to corresponding independent variables.

$y = x \sin x \cos x$

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16. Find the derivatives of the following functions with respect to corresponding independent variables.

$y = e^{-x} \log x$

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17. Find the derivatives of the following functions with respect to corresponding independent variables.

$y = (x^2 + 5) . \log(1 + x)e^{-3x}$

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18. Find the derivatives of the following functions with respect to corresponding independent variables.

$$y = \sin x°$$

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19. Find the derivatives of the following functions with respect to corresponding independent variables.
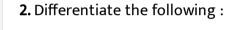
$$y = \log_{10} x$$

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20. Draw the function f'(x) if $f(x) = 2x^2 - 5x + 3$

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Exercise 10 3

1. Differentiate the following :

$$y = \left(x^2 + 4x + 6\right)^5$$

2. Differentiate the following :

$$y = \tan 3x$$

3. Differentiate the following :

$$y = \cos(\tan x)$$

4. Differentiate the following :

$$y = \sqrt[3]{1 + x^3}$$

5. Differentiate the following :

$$y = e^{\sqrt{x}}$$

6. Differentiate the following :

$$y = \sin(e^x)$$

7. Differentiate the following :

$$F(x) = \left(x^3 + 4x\right)^7$$

8. Differentiate the following :

$$h(t) = \left(t - \frac{1}{t}\right)^{\frac{3}{2}}$$

9. Differentiate the following :

$$f(t) = \sqrt[3]{1 + \tan t}$$

10. Differentiate the following :

$$y = \cos\left(a^3 + x^3\right)$$

11. Differentiate the following :

$$y = e^{-mx}$$

12. Differentiate the following :

$$y = 4 \sec 5x$$

13. Differentiate the following :

$$y = (2x - 5)^4 (8x^2 - 5)^{-3}$$

14. Differentiate the following :

$$y = (x^2 + 1) \sqrt[3]{x^2 + 2}$$

15. Differentiate the following :

$$y = xe^{-x^2}$$

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16. Differentiate the following :

$$s(t) = \sqrt[4]{\frac{t^3 + 1}{t^3 - 1}}$$

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17. Differentiate the following :

$$f(x) = \frac{x}{\sqrt{7 - 3x}}$$

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18. Differentiate the following :

$$y = \tan(\cos x)$$

19. Differentiate the following :

$$y = \dfrac{\sin^2 x}{\cos x}$$

20. Differentiate the following :

$$y = 5^{\frac{-1}{x}}$$

21. Differentiate the following :

$$y = \sqrt{1 + 2\tan x}$$

22. Differentiate the following :

$$y = \sin^3 x + \cos^3 x$$

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23. Differentiate the following :

$$y = \sin^2(\cos kx)$$

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24. Differentiate the following :

$$y = \left(1 + \cos^2 x\right)^6$$

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25. Differentiate the following :

$$y = \frac{e^{3x}}{1 + e^x}$$

26. Differentiate the following :

$$y = \sqrt{x + \sqrt{x}}$$

27. Differentiate the following :

$$y = e^{x \cos x}$$

28. Differentiate the following :

$$y = \sqrt{x + \sqrt{x + \sqrt{x}}}$$

29. Differentiate the following :

$$y = \sin\big(\tan(\sqrt{\sin x})\big)$$

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30. Differentiate the following :

$$y = \sin^{-1}\left(\frac{1 - x^2}{1 + x^2}\right)$$

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Exercise 10 4

1. Find the derivatives of the following :

$$y = x^{\cos x}$$

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2. Find the derivatives of the following :

$$y = x^{\log x} + (\log x)^x$$

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3. Find the derivatives of the following :

$$\sqrt{xy} = e^{(x-y)}$$

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4. Find the derivatives of the following :

$$x^y = y^x$$

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5. Find the derivatives of the following :

$$(\cos x)^{\log x}$$

6. Find the derivatives of the following :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

7. Find the derivatives of the following :

$$\sqrt{x^2 + y^2} = \tan^{-1}\left(\frac{y}{x}\right)$$

8. Find the derivatives of the following :

$$\tan(x + y) + \tan(x - y) = x$$

9. if cos(xy)=x, show that $\dfrac{dy}{dx} = \dfrac{-(1 + y\sin(xy))}{x\sin xy}$.

10. Find the derivatives of the following :

$\tan^{-1}\sqrt{\dfrac{1 - \cos x}{1 + \cos x}}$

11. Find the derivatives of the following :

$\tan^{-1}\left(\dfrac{6x}{1 - 9x^2}\right)$

12. Find the derivatives of the following :

$\cos\left(2\tan^{-1}\sqrt{\dfrac{1 - x}{1 + x}}\right)$

13. Find the derivatives of the following :

$$x = a\cos^3 t, y = a\sin^3 t$$

14. Find the derivatives of the following :

$$x = a(\cos t + t\sin t), y = a(\sin t - t\cos t)$$

15. Find the derivatives of the following :

$$x = \frac{1 - t^2}{1 + t^2}, y = \frac{2t}{1 + t^2}$$

16. Find the derivatives of the following :

$$\cos^{-1}\left(\frac{1 - x^2}{1 + x^2}\right)$$

17. Find the derivatives of the following :

$$\sin^{-1}\left(3x - 4x^3\right)$$

18. Find the derivatives of the following :

$$\tan^{-1}\left(\frac{\cos x + \sin x}{\cos x - \sin x}\right)$$

19. Find the derivative of $\sin x^2$ with respect to x^2.

20. Find the derivative of $\sin^{-1}\left(\dfrac{2x}{1+x^2}\right)$ with respect to $\tan^{-1} x$.

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21. If $u = \tan^{-1}\dfrac{\sqrt{1+x^2}-1}{x}$ and $v = \tan^{-1} x$, find $\dfrac{du}{dv}$.

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22. Find the derivative with $\tan^{-1}\left(\dfrac{\sin x}{1+\cos x}\right)$ with respect to $\tan^{-1}\left(\dfrac{\cos x}{1+\sin x}\right)$.

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23. If $y = \sin^{-1} x$ then find y''.

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24. If $y = e^{\tan^{-1}x}$, show that $(1 + x^2)y'' + (2x - 1)y' = 0$.

25. If $y = \dfrac{\sin^{-1}x}{\sqrt{1 - x^2}}$ show that $(1 - x^2)y_2 - 3xy_1 - y = 0$.

26. If $x = a(\theta + \sin\theta), y = a(1 - \cos\theta)$ then prove that at $\theta = \dfrac{\pi}{2}, y'' = \dfrac{1}{a}$.

27. If $\sin y = x \sin(a + y)$, then prove that $\dfrac{dy}{dx} = \dfrac{\sin^2(a + y)}{\sin a}, a \neq n\pi$.

28. If $y = \left(\cos^{-1} x\right)^2$ prove that $\left(1 - x^2\right)\dfrac{d^2 y}{dx^2} - x\dfrac{dy}{dx} - 2 = 0$. Hence find y_2 when x=0

Exercise 10 5

1. $\dfrac{d}{dx}\left(\dfrac{2}{\pi}\sin x^\circ\right)$ is

A. $\dfrac{\pi}{180}\cos x^\circ$

B. $\dfrac{1}{90}\cos x^\circ$

C. $\dfrac{\pi}{90}\cos x^\circ$

D. $\dfrac{2}{\pi}\cos x^\circ$

Answer: B

2. If $y = f(x^2 + 2)$ and $f'(3) = 5$, then $\dfrac{dy}{dx}$ at x=1 is

A. 5

B. 25

C. 15

D. 10

Answer: D

3. If $y = \dfrac{1}{4}u^4$ and $u = \dfrac{2}{3}x^3 + 5$, then $\dfrac{dy}{dx}$ is

A. $\dfrac{1}{27}x^2(2x^3 + 15)^3$

B. $\dfrac{2}{27}x(2x^3 + 5)^3$

C. $\dfrac{2}{27}x^2(2x^3 + 15)^3$

D. $\dfrac{2}{27}x(2x^3 + 5)^3$

Answer: C

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4. If $f(x) = x^2 - 3x$, then the points at which $f(x) = f'(x)$ are

    A. both positive integers

    B. both negative integers

    C. both irrational

    D. one rational and another irrational

Answer: C

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5. If $y = \dfrac{1}{a-z}$, then $\dfrac{dz}{dy}$ is

    A. $(a-z)^2$

B. $-(z-a)^2$

C. $(z+a)^2$

D. $-(z+a)^2$

Answer: A

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6. If $y = \cos\left(\sin x^2\right)$, then $\dfrac{dy}{dx}$ at $x = \sqrt{\dfrac{\pi}{2}}$ is

A. -2

B. 2

C. $-2\sqrt{\dfrac{\pi}{2}}$

D. 0

Answer: D

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7. If $y = mx + c$ and $f(0) = f'(0) = 1$, then $f(2)$ is

A. 1

B. 2

C. 3

D. -3

Answer: C

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8. If $f(x) = x \tan^{-1} x$, then $f'(1)$ is

A. $1 + \dfrac{\pi}{4}$

B. $\dfrac{1}{2} + \dfrac{\pi}{4}$

C. $\dfrac{1}{2} - \dfrac{\pi}{4}$

D. 2

Answer: B

9. d/dx(`e^(x+5logx))

A. $e^x . x^4 (x + 5)$

B. $e^x . x(x + 5)$

C. $e^x + \dfrac{5}{x}$

D. $e^x - \dfrac{5}{x}$

Answer: A

10. If the derivative of $(ax - 5)e^{3x}$ at x=0 is -13, then the value of a is

A. 8

B. -2

C. 5

D. 2

Answer: D

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11. $x = \dfrac{1 - t^2}{1 + t^2}, y = \dfrac{2t}{1 + t^2}$ then $\dfrac{dy}{dx}$ is

A. $-\dfrac{y}{x}$

B. $\dfrac{y}{x}$

C. $-\dfrac{x}{y}$

D. $\dfrac{x}{y}$

Answer: C

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12. If $x = a \sin \theta$ and $y = b \cos \theta$, then $\dfrac{d^2 y}{dx^2}$ is

A. $\dfrac{a}{b^2} \sec^2 \theta$

B. $- \dfrac{b}{a} \sec^2 \theta$

C. $- \dfrac{b}{a^2} \sec^3 \theta$

D. $\dfrac{b^2}{a^2} \sec^3 \theta$

Answer: C

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13. The differential coefficient of $\log_{10} x$ with respect to $\log_x 10$ is

A. 1

B. $- (\log_{10} x)^2$

C. $(\log_x 10)^2$

D. $\dfrac{x^2}{100}$

Answer: B

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14. If $f(x) = x + 2$, then $f'(f(x))$ at x=4 is

A. 8

B. 1

C. 4

D. 5

Answer: B

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15. If $y = \dfrac{(1-x)^2}{x^2}$, then $\dfrac{dy}{dx} =$ is

A. $\dfrac{2}{x^2} + \dfrac{2}{x^3}$

B. $-\dfrac{2}{x^2} + \dfrac{2}{x^3}$

C. $-\dfrac{2}{x^2} - \dfrac{2}{x^3}$

D. $-\dfrac{2}{x^3} + \dfrac{2}{x^2}$

Answer: D

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16. If pv = 81, then $\dfrac{dp}{dv}$ at v=9 is

A. 1

B. -1

C. 2

D. -2

Answer: B

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17. If $f(x) = \begin{cases} x - 5 & \text{if } x \leq 1 \\ 4x^2 - 9 & \text{if } 1 < x < 2 \\ 3x + 4 & \text{if } x \geq 2 \end{cases}$, then the right hand derivative

of $f(x)$ at $x = 2$ is............. .

A. 0

B. 2

C. 3

D. 4

Answer: C

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18. It is given that f'(a) exists,then $\lim\limits_{x \to a} \dfrac{x f(a) - a f(x)}{x - a}$ is:

A. $f(a) - a f'(a)$

B. $f'(a)$

C. $- f'(a)$

D. $f(a) + af'(a)$

Answer: A

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19. If $f(x) = \begin{cases} x + 1 & \text{when} \quad x < 2 \\ 2x - 1 & \text{when} \quad x \geq 2 \end{cases}$, then $f'(2)$ is.............. .

A. 0

B. 1

C. 2

D. does not exist

Answer: D

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20.

If $g(x) = (x^2 + 2x + 3)$, $f(x)$ and $f(0) = 5$ and $\lim\limits_{x \to 0} \dfrac{f(x) - 5}{x} = 4$ the

A. 20

B. 14

C. 18

D. 12

Answer: B

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21. If $f(x) = \begin{cases} x + 2 & -1 < x < 3 \\ 5 & x = 3 \\ 8 - x & x > 3 \end{cases}$, then at x=3, $f'(x)$ is

A. 1

B. -1

C. 0

D. does not exist

Answer: D

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22. The derivative of $f(x) = x|x|$ at x=-3 is

A. 6

B. -6

C. does not exist

D. 0

Answer: A

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23. If $f(x) = \begin{cases} 2a - x & \text{for} \quad -a < x < a \\ 3x - 2a & \text{for} \quad x \geq a \end{cases}$, then which one of the following is true?

A. f(x) is not differentiable at x=a

B. f(x) is discontinuous at x=a

C. f(x) is continuous for all x in R

D. f(x) is differentiable for all $x \geq a$

Answer: A

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24. If $f(x) = \begin{cases} ax^2 - b, & -1 < x < 1 \\ \frac{1}{|x|}, & \text{elsewhere} \end{cases}$ is differentiable at x=1, then

A. $a = \dfrac{1}{2}, b = \dfrac{-3}{2}$

B. $a = \dfrac{-1}{2}, b = \dfrac{3}{2}$

C. $a = -\dfrac{1}{2}, b = -\dfrac{3}{2}$

D. $a = \dfrac{1}{2}, b = \dfrac{3}{2}$

Answer: C

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25. The number of points in R in which the function $f(x) = |x - 1| + |x + 3| + \sin x$ is not differentiable, is

A. 3

B. 2

C. 1

D. 4

Answer: B

Additional Problems Section A 1 Mark

1. If $y = \sin^{-1} x + \cos^{-1} x$ then $\dfrac{dy}{dx}$ is

A. 1

B. π

C. $\dfrac{\pi}{2}$

D. 0

Answer: D

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2. If $x = a(\theta + \sin\theta), y = a(1 + \cos\theta)$ then $\dfrac{dy}{dx}$ is

A. $\tan\dfrac{\theta}{2}$

B. $-\tan\dfrac{\theta}{2}$

C. $\cot\dfrac{\theta}{2}$

D. $-\cot\dfrac{\theta}{2}$

Answer: B

3. If $y = a + bx^2$, where a, b are arbitrary constants, then

A. $\dfrac{d^2y}{dx^2} = 2xy$

B. $x\dfrac{d^2y}{dx^2} = y_1$

C. $x\dfrac{d^2y}{dx^2} - \dfrac{dy}{dx} + y = 0$

D. $x\dfrac{d^2y}{dx^2} = 2xy$

Answer: B

4. Match list

List I   List II

i. $\begin{bmatrix} a & b \\ b & a \end{bmatrix}$ (a) identity

ii. $\begin{bmatrix} 0 & b \\ -b & 0 \end{bmatrix}$ (b) singular matrix

iii. $\begin{bmatrix} a & a \\ b & b \end{bmatrix}$ (c) Skew-Symmetric

iv. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (d) Symmetric

The correct match is

(i) (ii) (iii) (iv)

   A. d c b a

   B. c d b a

   C. b a d c

   D. b d a c

Answer: B

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5. Choose the correct statement

A. Derivative of odd function is odd

B. Derivative of even function is even

C. Inverse of odd function is even

D. Inverse function of sinx is $\sin^{-1} x$

Answer: D

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6. Assertion (A) : $f(x) = \begin{cases} x + 1, & x < 2 \\ 2x - 1, & x \geq 2 \end{cases}$ then $f'(2)$ does not exist.

Reason (R) : f(x) is not continuous at 2.

A. Both A and R are true and R is the correct explanation of A

B. Both A and R are true but R is not the correct explanation of A

C. A is true R is false

D. A is false R is true

Answer: C

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1. Differentiate : 2^x.

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2. Differentiate $\dfrac{\sin(ax + b)}{\cos(cx + d)}$.

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3. Differentiate $x^2(x + 1)^3(x + 2)^4$ with respect to 'x'.

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4. If $y = 500e^{7x} + 600e^{-7x}$ show that $\dfrac{d^2y}{dx^2} = 49y$.

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1. If $f(x) = 2x^2 + 3x - 5$, then prove that $f'(0) + 3f'(-1) = 0$

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2. Show that $f(x) = x^2$ is differentiable at x=1 and find $f'(1)$.

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3. Differentiate $f(x) = e^{2x}$ from first principles.

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4. If $y = \sqrt{x+1} + \sqrt{x-1}$, prove that $\sqrt{x^2-1}\dfrac{dy}{dx} = \dfrac{1}{2}y$

5. If $x = a\sec^3\theta$ and $y = a\tan^3\theta$ find $\dfrac{dy}{dx}$ at $\theta = \dfrac{\pi}{3}$.

6. Differentiate $\log(1+x^2)$ with respect to $\tan^{-1}x$.

Additional Problems Section D 5 Marks

1. If $y = \sqrt{\dfrac{1+e^x}{1-e^x}}$, show that $\dfrac{dy}{dx} = \dfrac{e^x}{(1-e^x)\sqrt{1-e^{2x}}}$.

2. Differentiate $\tan^{-1}(\sec x + \tan x)$, $-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$ with respect to 'x'.

3. Differentiate x^x with respect to x log x.

4. If $x = 4z^2 + 5$, $y = 6z^2 + 7z + 3$, find $\dfrac{d^2y}{dx^2}$.