MATHS

BOOKS - SURA MATHS (TAMIL ENGLISH)

APPLICATIONS OF DIFFERENTIAL CALCULUS

Exercise 7 1

1. A point moves along a stright line in such a way that after t seconds its distance from the origin is $s = 2t^2 + 3t$ meters.

Find the average velocity of the points between t = 3 and t = 6 seconds.

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2. A point moves along a stright line in such a way that after t seconds its distance from the origin is $s = 2t^2 + 3t$ meters.

Find the instantaneous velocities at t = 3 and t = 6 seconds.

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3. A camera is accidentally knocked off an edge of a cliff 400 ft high. The camera falls a distance of $s = 16t^2$ in t seconds.

How long does the camera fall before it hits the ground?

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4. A camera is accidentally knocked off an edge of a cliff 400 ft high. The camera falls a distance of $s = 16t^2$ in t seconds.

What is the average velocity with which the camera falls during the last 2 seconds?

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5. A camera is accidentally knocked off an edge of a cliff 400 ft high. The camera falls a distance of $s = 16t^2$ in t seconds.

What is the instantaneous velocity of the camera when it hits the ground?

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6. A particle moves along a line according to the law $s(t) = 2t^3 - 9t^2 + 12t - 4$, where $t \geq 0$.

At what times the particle changes direction?

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7. A particle moves along a line according to the law $s(t) = 2t^3 - 9t^2 + 12t - 4$, where $t \geq 0$.

Find the total distance travelled by the particle in the first 4 seconds.

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8. A particle moves along a line according to the law $s(t) = 2t^3 - 9t^2 + 12t - 4$, where $t \geq 0$.

Find the particle's acceleration each time the velocity is zero.

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9. If the volume of a cube of side length x is $v = x^3$. Find the rate of change of the volume with respect to x when x = 5 units.

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10. If the mass m(x) (in kilograms) of a thin rod of length x (in meters) is given by, $m(x) = \sqrt{3}$ x then what is the rate of change of mass with respect to the length when it is x = 3 and x = 27 meters.

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11. A stone is dropped into a pond causing ripples in the form of concentric circles. The radius r of the outer ripple is increasing at a constant rate at 2 cm per second. When the radius is 5 cm find the rate of changing of the total area of the disturbed water?

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12. A beacon makes one revolution every 10 seconds. It is located on a ship which is anchored 5 km from a straight shore line. How fast is the beam moving along the shore line when it makes an angle of 45° with the shore?

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13. A conical water tank with vertex down of 12 meters height has a radius of 5 meters at the top. If water flows into the tank at a rate 10 cubic m/min, how fast is the depth of the water increases when the water is 8 metres deep?

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14. A ladder 17 metre long is leaning against the wall. The base of the ladder is pulled away from the wall at a rate of 5m/s. When the base of the ladder is 8 metres from the wall.

How fast is the top of the ladder moving down the wall?

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15. A ladder 17 metre long is leaning against the wall. The base of the ladder is pulled away from the wall at a rate of 5m/s. When the base of the ladder is 8 metres from the wall.

At what rate, the area of the triangle formed by the ladder, wall, and the floor, is changing?

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16. A police jeep, approaching an orthogonal intersection from the northern direction, is chasing a speeding car that has turned and moving straight east. When the jeep is 0.6 km north of the intersection and the car is 0.8 km to the east. The police deteremine with a radar that the distance between them and the cae is

increasing at 20 km/hr. If the jeep is moving at 60 km/hr at the instant of measurement, what is the speed of the car?

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17. A police jeep, approaching an orthogonal intersection from the northern direction, is chasing a speeding car that has turned and moving straight east. When the jeep is 0.6 km north of the intersection and the car is 0.8 km to the east. The police deteremine with a radar that the distance between them and the cae is increasing at 20 km/hr. If the jeep is moving at 60 km/hr at the instant of measurement, what is the speed of the car?

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Exercise 7 2

1. Find the slope of the tangent to the curves at the respective given points.

$y = x^4 + 2x^2 - x$ at x = 1

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2. Find the slope of the tangent to the curves at the respective given points.

$x = a\cos^3 t, y = b\sin^3 t \text{ at } t = \frac{\pi}{2}$

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3. Find the point on the curve $y = x^2 - 5x + 4$ at which the tangent is parallel to the line 3x + y = 7.

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4. Find the point on the curve $y = x^3 - 6x^2 + x + 3$ where the normal is parallel to the line x + y = 1729.

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5. Find the points on the curve $y^2 - 4xy = x^2 + 5$ for which the tangent is horizontal.

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6. Find the tangent and normal to the following curves at the givne points on the curve.

$y = x^2 - x^4$ at (1, 0)

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7. Find the tangent and normal to the following curves at the givne points on the curve.

$y = x^4 + 2e^x$ at (0, 2)

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8. Find the tangent and normal to the following curves at the givne points on the curve.

$y = x \sin x \quad \text{at} \quad \left(\frac{\pi}{2}, \frac{\pi}{2}\right)$

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9. Find the tangent and normal to the following curves at the givne points on the curve.

$x = \cos t, y = 2 \sin^2 t$ at $t = \frac{\pi}{3}$

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10. Find the equations of the tangents to the curve $y = 1 + x^3$ for which the tangent is orthogonal with the line x + 12y = 12.

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11. Find the equations of the tangents to the curve $y = \frac{x+1}{x-1}$ which are parallel to the line $x + 2y = 6$.

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12. Find the equation of tangent and normal to the curve given by $x = 7\cos t$ and $y = 2\sin t, t \in R$ at any point on the curve.

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13. Find the angle between the rectangular hyperboloa xy = 2 and the parabola $x^2 + 4y = 0$.

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14. Show that the two curves $x^2 - y^2 = r^2$ and $xy = c^2$ where c,r are constants, cut orthogonally.

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Exercise 7 3

1. Explain why Rolle's theorem is not applicable to the following functions in the respective intervals.

$$f(x) = \left|\frac{1}{x}\right|, x \in [-1, 1]$$

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2. Explain why Rolle's theorem is not applicable to the following functions in the respective intervals.

$f(x) = \tan x, x \in [0, \pi]$

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3. Explain why Rolle's theorem is not applicable to the following functions in the respective intervals.

$f(x) = x - 2\log x, x \in [2, 7]$

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4. Using the Rolle's theorem, determine the values of x at which the tangent is parallel to the x-axis for the following functions :

$f(x) = x^2 - x, x \in [0, 1]$

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5. Using the Rolle's theorem, determine the values of x at which the tangent is parallel to the x-axis for the following functions :

$$f(x) = \frac{x^2 - 2x}{x + 2}, x \in [-1, 6]$$

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6. Using the Rolle's theorem, determine the values of x at which the tangent is parallel to the x-axis for the following functions :

$f(x) = \sqrt{x} - \frac{x}{3}, x \in [0, 1]$

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7. Explain why Lagrange's mean value theorem is not applicable to the following functions in the respective intervals :

$$f(x) = \frac{x+1}{x}, x \in [-1, 2]$$

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8. Explain why Lagrange's mean value theorem is not applicable to the following functions in the respective intervals :

$$f(x) = |3x+1|, x \in [-1, 3]$$

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9. Using the Lagrange's mean value theorem determine the values of x at which the tangent is parallel to the secant line at the end points of the given interval:

$f(x) = x^3 - 3x + 2, x \in [-2, 2]$

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10. Using the Lagrange's mean value theorem determine the values of x at which the tangent is parallel to the secant line at the end points of the given interval:

$$f(x) = (x - 2)(x - 7), x \in [3, 11]$$

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11. Show that the value in the conclusion of the mean value theorem for

$f(x) = \frac{1}{x}$ on a closed interval of positive numbers [a, b] is $\sqrt{ab}$.

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12. Show that the value in the conclusion of the mean value theorem for

$f(x) = Ax^2 + Bx + C$ on any interval [a, b] is $\frac{a+b}{2}$

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13. A race car driver is racing at 20^{th} km. If his speed never exceeds 150 km/hr, what is the maximum distance he can cover in the next two hours.

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14. Suppose that for a function $f(x), f'(x) \leq 1$ for all $1 \leq x \leq 4$. Show that $f(4) - f(1) \leq 3$.

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15. Does there exist a differentiable function f(x) such that f(0) = -1, f(2) = 4 and $f'(x) \leq 2$ for all x. Justify your answer.

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16. Show that there lies a point on the curve $f(x) = x(x+3)e^{\frac{\pi}{2}}, \; -3 \le x \le 0$ where tangent drawn is parallel to the x-axis.

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17. Using mean value theorem prove that for, $a > 0, b > 0, \left|e^{-a} - e^{-b}\right| < |a - b|$.

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Exercise 7 4

1. Write the Maclaurin series expansion of the following functions :

e^x

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2. Write the Maclaurin series expansion of the following functions :

$\sin x$

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3. Write the Maclaurin series expansion of the following functions :

$\cos x$

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4. Write the Maclaurin series expansion of the following functions :

$\log(1-x), \ -1 \leq x \leq 1$

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5. Write the Maclaurin series expansion of the following functions :

$\tan^{-1}(x), \ -1 \le x \le 1$

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6. Write the Maclaurin series expansion of the following functions :

$\cos^2 x$

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7. Write down the Taylor series expansion, of the function $\log x$ about x = 1 upto three non zero terms for $x > 0$.

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8. Expand $\sin x$ in ascending powers $x - \frac{\pi}{4}$ upto three non-zero terms.

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9. Expand the polynomial $f(x) = x^2 - 3x + 2$ in powers of x - 1.

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Exercise 7 5

1. Evaluate the following limits, if necessary use l' Hopital Rule :

$$\lim_{x \to 0} \frac{1 - \cos x}{x^2}$$

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2. Evaluate the following limits, if necessary use l' Hopital Rule :

$$\lim_{x \to \infty} \frac{2x^2 - 3}{x^2 - 5x + 3}$$

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3. Evaluate the following limits, if necessary use l' Hopital Rule :

$$\lim_{x \to \infty} \frac{x}{\log x}$$

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4. Evaluate the following limits, if necessary use l' Hopital Rule :

$$\lim_{x \to \frac{\pi}{2}^-} \frac{\sec x}{\tan x}$$

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5. Evaluate the following limits, if necessary use l' Hopital Rule :

$$\lim_{x \to \infty} e^{-x}\sqrt{x}$$

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6. Evaluate the following limits, if necessary use l' Hopital Rule :

$$\lim_{x \to 0} \left(\frac{1}{\sin x} - \frac{1}{x}\right)$$

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7. Evaluate the following limits, if necessary use l' Hopital Rule :

$$\lim_{x \to 1^+} \left(\frac{2}{x^2 - 1} - \frac{x}{x - 1}\right)$$

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8. Evaluate the following limits, if necessary use l' Hopital Rule :

$$\lim_{x \to o^{+}} x^{x}$$

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9. Evaluate the following limits, if necessary use l' Hopital Rule :

$$\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^{x}$$

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10. Evaluate the following limits, if necessary use l' Hopital Rule :

$$\lim_{x \to \frac{\pi}{2}} (\sin x)^{\tan x}$$

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11. Evaluate the following limits, if necessary use l' Hopital Rule :

$$\lim_{x \to 0^+} (\cos x)^{\frac{1}{x^2}}$$

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12. If an initial amount A_0 of money is invested at an interest rate r compounded n times a year, the value of the investment after t years is $A = A_0\left(1 + \frac{1}{n}\right)^{nt}$. If the interest is compounded continuously, (that is as $n \to \infty$), show that the amount after t years is $A = A_0 e^{rt}$.

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Exercise 7 6

1. Find the absolute extrema of the following functions on the given closed interval.

$f(x) = x^3 - 12x + 10, [1, 2]$

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2. Find the absolute extrema of the following functions on the given closed interval.

$f(x) = 3x^4 - 4x^3, [-1, 2]$

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3. Find the absolute extrema of the following functions on the given closed interval.

$f(x) = 6x^{\frac{4}{3}} - 3x^{\frac{1}{3}}, [-1, 1]$

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4. Find the absolute extrema of the following functions on the given closed interval.

$f(x) = 2\cos x + \sin 2x, \left[0, \frac{\pi}{2}\right]$

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5. Find the intervals of monotonicities and hence find the local extremum for the following functions:

$$f(x) = 2x^3 + 3x^2 - 12x$$

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6. Find the intervals of monotonicities and hence find the local extremum for the following functions:

$$f(x) = \frac{x}{x-5}$$

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7. Find the intervals of monotonicities and hence find the local extremum for the following functions:

$$f(x) = \frac{e^x}{1 - e^x}$$

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8. Find the intervals of monotonicities and hence find the local extremum for the following functions:

$$f(x) = \frac{x^3}{3} - \log x$$

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9. Find the intervals of monotonicities and hence find the local extremum for the following functions:

$$f(x) = \sin x \cos x + 5, x \in (0, 2\pi)$$

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Exercise 7 7

1. Find intervals of concavity and points of inflexion for the following functions:

$f(x) = x(x - 4)^3$

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2. Find intervals of concavity and points of inflexion for the following functions:

$f(x) = \sin x + \cos x, 0 < x < 2\pi$

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3. Find intervals of concavity and points of inflexion for the following functions:

$$f(x) = \frac{1}{2}\left(e^{x} - e^{-x}\right)$$

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4. Find the local extrema for the following functions using second derivative test :

$$f(x) = -3x^{5} + 5x^{3}$$

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5. Find the local extrema for the following functions using second derivative test :

$f(x) = x \log x$

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6. Find the local extrema for the following functions using second derivative test :

$f(x) = x^2 e^{-2x}$

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7. For the function $f(x) = 4x^3 + 3x^2 - 6x + 1$ find the intervals of monotonicity, local extrema, intervals of concavity and points of inflection.

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Exercise 7 8

1. Find two positive numbers whose sum is 12 and their product is maximum.

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2. Find two positive numbers whose product is 20 and their sum is minimum.

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3. Find the smallest possible value of $x^2 + y^2$ given that x + y = 10.

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4. A garden is to be laid out in a rectangular area and protected by wire fence. What is the largest possible area of the fenced garden with 40 metres of wire.

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5. A rectangular page is to contain $24cm^2$ of print. The margins at the top and bottom of the page are 1.5 cm and the margins at other sides of the page is 1 cm. What should be the dimensions of the page so that the area of the paper used is minimum.

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6. A farmer plans to fence a rectangular pasture adjacent to a river. The pasture must contain 1,80,000 sq. mtrs in order to provide enough grass for herds. No fencing is needed along the river. What is the length of the minimum needed fencing material?

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7. Find the dimensions of the rectangle with maximum area that can be inscribed in a circle of radius 10 cm.

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8. Prove that among all the rectangles of the given perimeter, the square has the maximum area.

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9. Find the dimensions of the largest rectangle that can be inscribed in a semi circle of radius r cm.

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10. A manufacturer wants to design an open box having a square base and a surface area of 108 sq. cm. Determine the dimensions of the box for the maximum volume.

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11. The volume of a cylinder is given by the formula $V = \pi r^2 h.$ Find the greatest and least values of V if r + h = 6.

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Exercise 7 10

1. The volume of a sphere is increasing in volume at the rate of $3\pi cm^3 / \sec.$ The rate of change of its radius when radius is $\frac{1}{2}$ cm

A. 3 cm/s

B. 2 cm/s

C. 1 cm/s

D. $\frac{1}{2}$ cm/s

Answer:

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2. A balloon rises straight up at 10m/s. An observer is 40 m away from the spot where the balloon left the ground. Find the rate of change of the balloon's angle of elevation in radian per second when the balloon is 30 metres above the ground.

A. $\frac{3}{25}$ radians/sec

B. $\frac{4}{25}$ radians/sec

C. $\frac{1}{5}$ radians/sec

D. $\frac{1}{3}$ radians/sec

Answer:

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3. The position of a particle moving along a horizontal line of any time t is given by $s(t) = 3t^2 - 2t - 8$. The time at which the particle is at rest is

A. t = 0

B. $t = \frac{1}{3}$

C. t = 1

D. t = 3

Answer:

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4. A stone is thrown up vertically. The height it reaches at time t seconds is given by $x = 80t - 16t^2$. The stone reaches the maximum height in time t seconds is given by

A. 2

B. 2.5

C. 3

D. 3.5

Answer:

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5. The point on the curve $6y = x^3 + 2$ at which y- co ordinate is changing 8 times as fast as $x -$ co -ordinate is

A. (4, 11)

B. (4, -11)

C. (-4, 11)

D. (-4, -11)

Answer:

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6. The abscissa of the point on the curve $f(x) = \sqrt{8 - 2x}$ at which the slope of the tangent is -0.25 ?

A. -8

B. -4

C. -2

D. 0

Answer:

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7. The slope of the line normal to the curve $f(x) = 2\cos 4x$ at $x = \frac{\pi}{12}$ is

A. $-4\sqrt{3}$

B. -4

C. $\frac{\sqrt{3}}{12}$

D. $4\sqrt{3}$

Answer:

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8. Angle between $y^2 = x$ and $x^2 = y$ at the origin is

A. $\tan^{-1}\frac{3}{4}$

B. $\tan^{-1}\left(\frac{4}{3}\right)$

C. $\frac{\pi}{2}$

D. $\frac{\pi}{4}$

Answer:

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9. The value of the limit $\lim_{x \to 0}\left(\cot x - \frac{1}{x}\right)$ is

A. 0

B. 1

C. 2

D. ∞

Answer:

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10. The function $\sin^4 x + \cos^4 x$ is increasing in the interval

A. $\left[\frac{5\pi}{8}, \frac{3\pi}{4}\right]$

B. $\left[\frac{\pi}{2}, \frac{5\pi}{8}\right]$

C. $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

D. $\left[0, \frac{\pi}{4}\right]$

Answer:

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11. The number given by the Rolle's theorem for the function $x^3 - 3x^2, x \in [0, 3]$ is

A. 1

B. $\sqrt{2}$

C. $\frac{3}{2}$

D. 2

Answer:

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12. The number given by the Mean value theorem for the function $\frac{1}{x}, x \in [1, 9]$ is

A. 2

B. 2.5

C. 3

D. 3.5

Answer:

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13. The minimum value of the function $|3-x|+9$ is

A. 0

B. 3

C. 6

D. 9

Answer:

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14. The maximum slope of the tangent to the curve $y = e^x \sin x$, $x \in [0, 2\pi]$ is at

A. $x = \frac{\pi}{4}$

B. $x = \frac{\pi}{2}$

C. $x = \pi$

D. $x = \frac{3\pi}{2}$

Answer:

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15. The maximum value of the function $x^2e^{-2x}, x > 0$ is

A. $\frac{1}{e}$

B. $\frac{1}{2e}$

C. $\frac{1}{e^2}$

D. $\frac{4}{e^4}$

Answer:

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16. One of the closest points on the curve $x^2 - y^2 = 4$ to the point (6, 0) is

A. (2, 0)

B. $(\sqrt{5}, 1)$

C. $(3, \sqrt{5})$

D. $(\sqrt{13}, -\sqrt{3})$

Answer:

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17. The maximum value of the product of two positive numbers, when their sum of the squares is 200, is

A. 100

B. $25\sqrt{7}$

C. 28

D. $24\sqrt{14}$

Answer:

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18. The curve $y = ax^4 + bx^2 \text{ with } ab > 0$

A. has no horizontal tangent

B. is concave up

C. is concave down

D. has no points of inflection

Answer:

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19. The point of inflection of the curve $y = (x - 1)^3$ is

A. (0, 0)

B. (0, 1)

C. (1, 0)

D. (1, 1)

Answer:

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1. Using Rolle's theorem find the point on the curve $y = x^2 + 1, \ -2 \leq x \leq 2$ where the tangent is parallel to x-axis.

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2. Write the Maclaurin series expansion of e^{-x}

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3. Evaluate : $\lim_{x \to 0^+} x \log x.$

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4. Evaluate $\lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{e^x - 1}\right).$

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5. A steel plant is capable of producing x tonnes per day of a law-grade steel and y tonnes per day of a hight-grade steel, where

$y = \dfrac{40 - 5x}{10 - x}$. If the fixed market price of low-grade steel is half that of high-grade steel, then what should be optimal productions in law-grade steel and high-grade steel in order to have maximum receipts.

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6. Find the equations of tangent and normal to the curve $y^2 - 4x - 2y + 5 = 0$ at the point where it cuts the x-axis.

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7. Prove that among all the rectangles of the given area, square has the least perimeter.

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Additional Questions

1. If a particle moves is a straight line according to $s = t^3 - 6t^2 - 15t$, the time interval during which the velocity is negative and acceleration is positive is

A. $2 < t < 5$

B. $2 \leq t \leq 5$

C. $t \geq 2$

D. $t \leq 2$

Answer:

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2. The law of linear motion of a particle is given by $s = \frac{1}{3}t^3 - 16t$, the acceleration at the time when the velocity vanishes is

A. 4

B. 6

C. 2

D. 8

Answer:

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3. If the rate of increase of $s = x^3 - 5x^2 + 5x + 8$ is twice the rate of increase of x, then one value of x is

A. $\frac{3}{5}$

B. $\frac{10}{3}$

C. $\frac{3}{10}$

D. $\frac{1}{3}$

Answer:

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4. The point on the curve $y = x^2$ is the tangent parallel to X-axis is

A. (1, 1)

B. (2, 2)

C. (4, 4)

D. (0, 0)

Answer:

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5. The equation of the tangent to the curve $y = x^2 - 4x + 2$ at (4, 2) is

A. x + 4y + 12 = 0

B. 4x + y + 12 = 0

C. 4x - y - 14 = 0

D. x + 4y - 12 = 0

Answer:

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6. Equation of the normal to the curve $y = 2x^2 + 3\sin x$ at x = 0 is

A. x + y = 0

B. 3y = 0

C. x + 3y = 7

D. x + 3y = 0

Answer:

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7. The least value of a when $f(x) = x^2 + ax + 1$ is increasing on (1, 2) is

A. -2

B. 2

C. 1

D. -1

Answer:

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8. The value of $\lim_{x \to \infty} e^{-x}$ is

A. 0

B. ∞

C. e

D. $\frac{1}{e}$

Answer:

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9. The critical points of the function $f(x) = (x-2)^{\frac{2}{3}}(2x+1)$ are

A. $-1, 2$

B. $1, \ -\frac{1}{2}$

C. 1, 2

D. none

Answer:

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10. The equation of the tangent to the curve $x = t\cos t, y = t\sin t$ at the origin is ________

A. x = 0

B. y = 0

C. x + y = 0

D. x + y = 7

Answer:

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11. In LMV theorem, we have $f'(x_1) = \frac{f(b) - f(a)}{b - a}$ then $a < x_1$ ________

A. $< b$

B. $\leq b$

C. $= b$

D. $\neq b$

Answer:

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12. If the slope of the curve $2y^2 = ax^2 + b$ at (1, -1) is -1, then the values of a, b is _______

A. 2, 0

B. 0, 2

C. 0, 0

D. 2, 2

Answer:

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13. If the curves $y = 2e^x$ and $y = ae^{-x}$ intersect orthogonally, then a = ________

A. $\frac{1}{2}$

B. $-\frac{1}{2}$

C. 2

D. $2e^2$

Answer:

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14. The function -3x + 12 is ________ function on R.

A. decreasing

B. strictly decreasing

C. increasing

D. strictly increasing

Answer:

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15. The function $f(x) = x^9 + 3x^7 + 64$ is increasing on ________

A. R

B. $(-\infty, 0)$

C. $(0, \infty)$

D. None of these

Answer:

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16. If x + y = 8, then the maximum value of xy is

____

A. 8

B. 16

C. 30

D. 24

Answer:

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17. The curve $y = e^x$ is ________

A. convex

B. concave

C. convex upwards

D. concave upwards

Answer:

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18. $\lim_{x \to 0} \frac{x}{\tan x}$ is ________

A. 1

B. -1

C. 0

D. ∞

Answer:

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19. The statement "If f has a local extremum at c and if f'(c) exists then f'(c) = 0" is ________

A. the extreme value theorem

B. Fermats' theorem

C. Law of mean

D. Rolle's theorem

Answer:

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20. Identify the incorrect statement.

A. Every constant function is an increasing function.

B. Every constant function is a decreasing function.

C. Every identify function is an increasing function.

D. Every polynomial function is continuous

Answer:

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21. Identify the false statement

A. All the stationary numbers are critical numbers.

B. At the stationary point, the first derivative is zero.

C. At critical numbers, the first derivative does not exist.

D. All the critical numbers are stationary numbers.

Answer:

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22. A particle moves in a line so that so that $x = \sqrt{t}$. Show that the acceleration is negative and proportional to the cube of the velocity.

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23. A man 2 m high walks at a uniform speed of 5km/hr away from a lamp post 6 m high.

Find the rate at which the length of his shadow increases?

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24. At what point on the curve $y = x^2$ on [-2, 2] is the tangent parallel to X-axis?

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25. Find the maximum and minimum values of $f(x) = |x + 3| \forall x \in \mathbb{R}$.

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26. Find the intervals of increasing and decreasing function for $f(x) = x^3 + 2x^2 - 1$.

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27. Prove that $\frac{x}{1+x} < \log(1+x) \text{ for } x > 0$.

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28. Find the equation of normal to the cure $y = \sin^2 x$ at $\left(\frac{\pi}{3}, \frac{3}{4}\right)$.

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29. Verify LMV theorem for $f(x) = x^3 - 2x^2 - x + 3$ in [0, 1].

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30. The ends of a rod AB which is 5 m long moves along two grooves OX, OY which at the right angles. If A moves at a constant speed of $\frac{1}{2}m/\sec$, what is the speed of B, when it is 4m from O?

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31. A ball is thrown vertically upwards, moves according to the law $s = 13.8t - 4.9t^2$ where s is in metres and t is in seconds.

(i) Find the acceleration at t = 1

(ii) Find velocity at t = 1

(iii) Find the maximum height reached by the ball?

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32. If Rolle's theorem holds for $f(x) = x^3 + bx^2 + ax + 5$ on [1, 3] with $c = \left(2 + \frac{1}{\sqrt{3}}\right)$ find the values of a and b.

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33. Find the angle of intersection of the curves $2y^2 = x^3$ and $y^2 = 32x$.

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34. Prove that the semi-vertical angle of a cone of maximum volume and of given slant height is $\tan^{-1}(\sqrt{2})$.

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35. Show that the curves $4x = y^2$ and $4xy = k$ cut at right angles if $k^2 = 512$.

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