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India's Number 1 Education App

## MATHS

## BOOKS - SURA MATHS (TAMIL

 ENGLISH)
## DISCRETE MATHEMATICS

Exercise 121

1. Determine whether $*$ is a binary operation
on the sets given below.
(i) $a * b=a$. $|b|$ on R .
(ii) $a * b=\min (a, b)$ on $A=\{1,2,3,4,5\}$
(iii) $(a * b)=a \sqrt{b}$ is binary on R .

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$$
\begin{aligned}
& \text { 2. On } \quad \text { Z, define } \otimes \text { by } \\
& (m \otimes n)=m^{n}+n^{m}: \forall m, n \in Z .
\end{aligned}
$$

binary on $Z$ ?

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3. Let $*$ be defined on R by
$(a * b)=a+b+a b-7$. Is $*$ binary on R? If
so, find $3 *\left(-\frac{7}{15}\right)$.

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4. Let $A=\{a+\sqrt{5} b: a, b \in Z\}$. Check whether the usual multiplication is a binary operation on A .

## 5. Fill in the following table so that the binary

## operation $*$ on $A=\{a, b, c\}$ is commutative.



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# 6. Let <br> $$
A=\left(\begin{array}{llll} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{array}\right)
$$ <br> $B=\left(\begin{array}{llll}0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1\end{array}\right)$ <br> $C=\left(\begin{array}{llll}1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1\end{array}\right)$ <br> by any three boolean 

matrices of the same type. Find (i) $A \vee B$,
$A \wedge B,(\mathrm{iii})(A \vee A) \wedge C,(\mathrm{iv})(A \wedge B) \vee C$.

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7. (i) Let $M=\left\{\left(\begin{array}{ll}x & x \\ x & x\end{array}\right): x \in R-\{0\}\right\}$ and let $*$ be the matrix multiplication. Determine whether $M$ is closed under *. If so, examinie the existence of identity, existence of inverse properties for the operation $*$ on M .

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8. Let $A$ be $Q /\{1\}$. Define * on $A$ by $x^{*} y=x+y-x y$. Is

* binary on A? If so, examine the commutative and association properties satisfied by * on A.


## Exercise 122

1. Let p : Jupiter is a planet and q : India is an
island be any two simple statements. Give verbal sentence describing each of the following statements:
(i) $\sim p$
(ii) $p \wedge \sim q$
(iii) $\sim p \vee q$
(iv) $p \rightarrow \sim q$
(v) $p<\Rightarrow q$

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2. Write each of the following sentences in symbolic form using statement variables $p$ and
q.
(i) 19 is not a prime number and all the angles of a triangle are equal.
(ii) 19 is a prime number or all the angles of a triangle are not equal.
(iii) 19 is a prime number and all the angles of a triangle are equal.
(iv) 19 is not a prime number.

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3. Determine the truth value of each of the following statements.
(i) If $6+2=5$, then the milk is white.
(ii) China is an Europe or $\sqrt{3}$ is an integer.
(iii) It is not true that $5+5=9$ or Earth is a planet.
(iv) 11 is a prime number and all the sides of a rectangle are equal.

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4. Which one of the following sentences is a proposition?
(i) $4+7=12$
(ii) What are you doing?
(iii) $3^{n} \leq 81, n \in N$
(iv) Peacock is our national bird
(v) How tall this mountain is?

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5. Write the converse, inverse, and contrapositive of each of the following implication.
(i) If $x$ and $y$ are numbers such that $x=y$, then
$x^{2}=y^{2}$.
(ii) If a quadrilateral is a square then it is a rectangle.
6. Show that (i) $\sim(p \wedge q) \equiv \sim p \vee \sim q$
(ii) $\sim(p \rightarrow q) \equiv p \wedge \sim q$.

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7. Prove that $q \rightarrow p \equiv \sim p \rightarrow \sim q$

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8. Show that $p \rightarrow q$ and $q \rightarrow p$ are not equivalent.

## - Watch Video Solution

9. Show that $\sim(p<\Rightarrow q) \equiv p<\Rightarrow \sim q$

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10. Check whether the statement
$p \rightarrow(q \rightarrow p)$ is a tautology or a contradiction
without using the truth table.

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11. Using truth table check whether the statements $\sim(p \vee q) \vee(\sim p \wedge q)$ and $\sim p$ are logically equivalent.

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12. $\quad$ Prove $\quad p \rightarrow(q \rightarrow r) \equiv(p \wedge q) \rightarrow r$ without using truth table.

- Watch Video Solution

13. Prove that $p \rightarrow(\sim q \vee r) \equiv \sim p \vee(\sim q \vee r)$
using truth table.

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## Exercise 123

1. A binary operation on a set $S$ is a function
from
A. $S \rightarrow S$
B. $(S \times S) \rightarrow S$
C. $S \rightarrow(S \times S)$
D. $(S \times S) \rightarrow(S \times S)$

Answer: B

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2. Subtraction is not binary operation in
A. R
B. Z
C. N

## D. Q

## Answer: C

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3. Which one of the following is a binary operation on N ?

A. Subtraction

B. Multiplication

## C. Division

## D. All the above

Answer: B

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4. In the set R of real number $*$ is defined as
follows. Which one of the following is not a binary operation on $R$ ?
A. $a * b=\min (a, b)$
B. $a * b=\max (a, b)$
C. $a * b=a$
D. $a * b=a^{b}$

Answer: D

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5. The operation $*$ defined by $a * b=\frac{a b}{7}$ is not a binary operation on
A. $Q^{+}$
B. Z
C. R
D. C

## Answer: C

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6. In the set Q define $a \odot b=a+b+a b$. For what value of $y, 3 \odot(y \odot 5)=7$ ?

$$
\text { A. } y=\frac{2}{3}
$$

$$
\begin{aligned}
& \text { B. } y=-\frac{2}{3} \\
& \text { C. } y=-\frac{3}{2} \\
& \text { D. } y=4
\end{aligned}
$$

Answer: B

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7. If $a * b=\sqrt{a^{2}+b^{2}}$ on the real numbers
then $*$ is
A. commutative but not associative

## B. associative but not commutative

C. both commutative and associative
D. neither commutative nor associative

## Answer: C

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8. Which one of the following statements has the truth value $T$ ?
A. $\operatorname{six} x$ is an even function
B. Every square matrix is non-singular
C. The product of complex number and its
conjugate is purely imaginary
D. $\sqrt{5}$ is an irrational number

## Answer: D

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9. Which one of the following statements has truth value $F$ ?
A. Chennai is in India or $\sqrt{2}$ is an integer.
B. Chennai is in India or $\sqrt{2}$ is an irrational number
C. Chennai is in China or $\sqrt{2}$ is an integer
D. Chennai is in China or $\sqrt{2}$ is an irrational
number

Answer: C

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10. If a compound statement involves 3 simple
statements, then the number of rows in the
truth table is
A. 9
B. 8
C. 6
D. 5

Answer: B

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11. Which one is the inverse of the statement $(p \vee q) \rightarrow(p \wedge q) ?$
A. $(p \wedge q) \rightarrow(p \vee q)$
B. $\sim(p \vee q) \rightarrow(p \wedge q)$
C. $(\sim p \vee \sim q) \rightarrow(\sim p \wedge \sim q)$
D. $(\sim p \wedge \sim q) \rightarrow(\sim p \vee \sim q)$

Answer: D

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12. Which one is the contrapositive of the statement $(p \vee q) \rightarrow r$ ?

$$
\begin{aligned}
& \text { A. } \sim r \rightarrow(\sim p \wedge \sim q) \\
& \text { B. } \sim r \rightarrow(p \vee q) \\
& \text { C. } r \rightarrow(p \wedge q) \\
& \text { D. } p \rightarrow(q \vee r)
\end{aligned}
$$

Answer: A

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13. In the last column of the truth table for $\sim(p \vee \sim q)$ the number of final outcomes of the truth value ' $F$ ' are
A. 1
B. 2
C. 3
D. 4

Answer: C

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14. Which one of the following is incorrect?

For any two propostitions $p$ and $q$, we have

$$
\begin{aligned}
& \text { A. } \sim(p \vee q) \equiv \sim p \wedge \sim q \\
& \text { B. } \sim(p \wedge q) \equiv \sim p \vee \sim q \\
& \text { C. } \sim(p \vee q) \equiv \sim p \vee \sim q \\
& \text { D. } \sim(\sim p) \equiv p
\end{aligned}
$$

Answer: C

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15. The dual of $\sim(p \vee q) \vee[p \vee(p \wedge \sim r)]$ is

$$
\begin{aligned}
& \text { A. } \sim(p \wedge q) \wedge[p \vee(p \wedge \sim r)] \\
& \text { B. } \sim p(\wedge q) \vee[p \wedge(p \wedge \sim r)] \\
& \text { C. } \sim(p \wedge q) \wedge[p \wedge(p \wedge r)] \\
& \text { D. } \sim(p \wedge q) \wedge[p \wedge(p \vee \sim r)]
\end{aligned}
$$

Answer: D

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16. The proposition $p \wedge(\sim p \vee q)$ is
A. a tautology
B. a contradiction
C. logically equivalent to $p \wedge q$
D. logically equivalent to $p \vee q$

## Answer: C

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17. Determine the truth value of each of the following statements:
(a) $4+2=5$ and $6+3=9$
(b) $3+2=5$ and $6+1=7$
(c ) $4+5=9$ and $1+2=4$
(d) $3+2=5$ and $4+7=11$

A. (a)-F, (b)-T, (c)-F, (d)-T<br>B. (a)-T, (b)-F, ( c)-T, (d)-F<br>C. (a)-T, (b)-T, (c )-F, (d)-F<br>D. (a) $-\mathrm{F},(\mathrm{b})-\mathrm{F},(\mathrm{c})-\mathrm{T},(\mathrm{d})-\mathrm{T}$

Answer: A

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18. Which one of the following is not true?
A. Negation of a negation of a statement is
the statement itself.
B. If the last column of the truth table contains only T then it is a tautology.
C. If the last column of its truth table contains only F then it is a contradiction
D. If $p$ and $q$ are any two statements then
$p<\Rightarrow q$ is a tautology.

## Answer: D

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## Government Exam Questions

1. Let * be a binary operation on set $Q$ of
rational numbers defined as $a * b=\frac{a b}{8}$.
Write the identity for $*$, If any.
2. A fair coin is tossed a fixed number of times.

If the probability of getting seven heads is
equal to that of getting nine heads, find the probability of getting exactly two heads.

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## Additional Questions

1. The binary operation $*$ defined on a set $s$ is
said to be commutative if
A. $a * b \in S \forall a, b \in S$
B. $a * b=b * a \forall a, b \in S$
C. $(a * b) * c=a *(b * c) \forall a, b \in S$
D. $a * b=e \forall a, b \in S$

Answer: B

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2. If $*$ is defined by $a * b=a^{2}+b^{2}+a b+1$,
then $(2 * 3) * 2$ is
A. 20
B. 40
C. 400
D. 445

Answer: D

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3. The number of binary operations that can be defined on a set of 3 elements is
A. $3^{2}$
B. $3^{3}$
C. $3^{9}$
D. $3^{1}$

Answer: C

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4. The identify element of
$\left.\left\{\left(\begin{array}{ll}x & x \\ x & x\end{array}\right)\right\} x \in R, x \neq 0\right\} \quad$ under matrix
multiplication is

$$
\begin{aligned}
& \text { A. }\left(\begin{array}{ll}
1 & 0 \\
0 & 1
\end{array}\right) \\
& \text { B. }\left(\begin{array}{ll}
\frac{1}{4 x} & \frac{1}{4 x} \\
\frac{1}{4 x} & \frac{1}{4 x}
\end{array}\right) \\
& \text { C. }\left(\begin{array}{ll}
\frac{1}{2} & \frac{1}{2} \\
\frac{1}{2} & \frac{1}{2}
\end{array}\right) \\
& \text { D. }\left(\begin{array}{ll}
\frac{1}{2 x} & \frac{1}{2 x} \\
\frac{1}{2 x} & \frac{1}{2 x}
\end{array}\right)
\end{aligned}
$$

Answer: C

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5. Which one of the following is not a statement?
A. $2+3=5$
B. How beautiful is this flower?
C. Delhi is the capital of Tamil Nadu
D. A triangle has found angles

Answer: B

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6. Which of the following is a tautology?

$$
\begin{aligned}
& \text { A. } p \vee q \\
& \text { B. } p \wedge q \\
& \text { C. } q \vee \sim q \\
& \text { D. } q \wedge \sim q
\end{aligned}
$$

## Answer: C

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7. Which of the following is a contradiction?

## A. $p \vee q$

B. $p \wedge q$
C. $q \vee \sim q$
D. $q \wedge \sim q$

Answer: D

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8. The identify element in the group
$\{R-\{1\}, x\}$ where $a * b=a+b-a b$ is
A. 0
B. 1
C. $\frac{1}{a-1}$
D. $\frac{a}{a-1}$

Answer: A

## D Watch Video Solution

9. Define $*$ on $Z$ by
$a * b=a+b+1 \forall a, b \in Z$.
Then
the
identity element of $z$ is
A. 1
B. 0
C. 2
D. -1

Answer: D

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10. A binary operation $*$ is defined on the set
$a * b=\frac{a b}{4}$. Then $3 *\left(\frac{1}{5} * \frac{1}{2}\right)$ is

> A. $\frac{3}{160}$
> B. $\frac{5}{160}$
> C. $\frac{3}{10}$
> D. $\frac{3}{40}$

Answer: A
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Fill In The Blanks

1. If $a * b=a^{2} b^{2}-a b$ then $3 *(1 * 1)$
A. 0
B. 1
C. 2
D. 4

Answer: A

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2. The number whose multiplication inverse does not exist in C.
A. 0
B. 1
C. 0
D. 1

Answer: B

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3. Let p : Kamala is going to school
q : There are 20 students in the class. Then
Kamala is not going to school or there are 20
students in the class is represented by
A. $p \vee q$
B. $p \wedge q$
C. $\sim p$
D. $\sim p \vee q$

Answer: D
4. If $p$ is true and $q$ is unknown, then
A. $\sim p$ is true
B. $p \vee(\sim q)$ is false
C. $p \wedge(\sim p)$ is true
D. $p \vee q$ is true

Answer: D

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5. + is not a binary operation on
A. ~
B. z
C. Chennai is in China or $\sqrt{2}$ is an integer
D. $Q-\{0\}$

Answer: D

- Watch Video Solution

6.     - is a binary operation on
A. ~
B. $Q-\{0\}$
C. $\mathrm{R}-\{0\}$
D. Z

Answer: D

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## 7. Which of the following is a statement?

A. $7+2<10$
B. Wish you all success
C. All the best

D. How old are you?

Answer: A
8. In $(N, *), x * y=\max (\mathrm{x}, \mathrm{y}), x, y \in N$, then
$7 *(-7)$
A. 7
B. -7
C. 0
D. -49

Answer: A

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9. $\ln (S, *)$, is defined by $x * y=x$ where x , $y \in S$, then
A. associative
B. Commutative
C. associative and commutative
D. neither associative nor commutative

Answer: A

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10. The number of commutative binary operations which can be defined on a set containing n elements is

$$
\begin{aligned}
& \text { A. } n \frac{n(n+1)}{2} \\
& \text { B. } n^{n^{2}} \\
& \text { C. } n^{\frac{n}{2}} \\
& \text { D. } n^{2}
\end{aligned}
$$

Answer: D

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Choose The Incorrect Statement

1. Which of the following are not statements?
(i) $3+4=8$
(ii) Then sun is a planet
(iii) Switch on the light
(iv) Where are you going?
A. (i), (ii)
B. (ii), (iii)
C. (iii), (iv)
D. (iv) only

## Answer: C

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2. If $p$ is true and $q$ is false, then which of the following is not true?
A. $p \rightarrow q$ is F
B. $p \vee q$ is T
C. $p \wedge q$ is F
D. $p<\Rightarrow q$ is T

## Answer: D

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3. $p \vee q$ is false when

A. $p$ is $T, q$ is $F$
B. $p$ is $F, q$ is $T$
C. p is $\mathrm{T}, \mathrm{q}$ is T
D. $p$ is $F, q$ is $F$
4. Which of the following is not a binary operation on R?
A. $a * b=a b$
B. $a * b=a-b$
C. $a * b=\sqrt{a b}$
D. $a * b=\sqrt{a^{2}+b^{2}}$

Answer: C
5. Which of the following is a contradiction?
A. $p \vee q$
B. $p \wedge q$
C. $p \vee \sim q$
D. $p \wedge \sim p$

Answer: D
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Match The Following


The correct match is

> A. (i)-b, (ii)-c, (iii)-d, (iv)-a
B. (i)-c, (ii)-d, (iii)-b, (iv)-a
C. (i)-c, (ii)-d, (iii)-a, (iv)-b
D. (i)-b, (ii)-c, (iii)-d, (iv)-a

## Answer: D

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The correct match is
A. (i)-a, (ii)-b, (iii)-c, (iv)-d
B. (i)-b, (ii)-c, (iii)-d, (iv)-a
C. (i)-c, (ii)-b, (iii)-d, (iv)-a

> D. (i)-d, (ii)-c, (iii) -a, (iv)-b

## Answer: D

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3. Let $\mathrm{p}: 2$ is the only even prime $\mathrm{q}: \sin \mathrm{x}$ is an even function be simple statements.

| List-I |  | List - II |  |
| :---: | :--- | :--- | :--- | :--- |
| i. | $p \vee p$ | a) | 2 is not the only even <br> prime |
| ii. | $\sim p$ | b) | $\sim q$ |
| iii. | $\sin x$ is an odd <br> function | c) | 2 is the only even <br> prime or sin $x$ is an <br> even function |
| iv. | $p \leftrightarrow q$ | d) | 2 is the only even <br> prime if and only if sin <br> x is an even function. |

The Correct match is
A. (i)-c, (ii)-a, (iii)-d, (iv)-b
B. (i)-b, (ii)-a, (iii)-b, (iv)-d
C. (i)-b, (ii)-d, (iii)-c, (iv)-a
D. (i)-c, (ii)-a, (iii)-b, (iv)-d

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4. Let $\left.G\left\{\left(\begin{array}{ll}a & 0 \\ 0 & 0\end{array}\right)\right\} / a \in R-\{0\}\right\}$ and *
is the matrix multiplication.

|  | List - I | List - II |  |
| :---: | :---: | :---: | :---: |
| i. | $\begin{aligned} & \left(\begin{array}{cc} a b & 0 \\ 0 & 0 \end{array}\right) \in \mathrm{G} \forall a, \\ & b \in \mathbb{R}-\{0\} \end{aligned}$ | a) | Inverse element |
| ii. | $\left(\begin{array}{ll}1 & 0 \\ 0 & 1\end{array}\right) \in \mathrm{G}$ | b) | $\left(\mathrm{G},{ }^{*}\right)$ is closed |
| iii. | $\left(\begin{array}{cc}a b c & 0 \\ 0 & 0\end{array}\right) \in \mathrm{G}$ | c) | Identity element |
| iv. | $\left(\begin{array}{ll}\frac{1}{a} & 0 \\ 0 & 0\end{array}\right) \in \mathrm{G} \forall a \in \mathrm{G}$ | d) | (G, *) is associative |

The Correct match is
A. (i)-b, (ii)-c, (iii)-d, (iv)-a
B. (i)-a, (ii)-b, (iii)-c, (iv)-d
C. (i)-b, (ii)-c, (iii)-d, (iv)-a
D. (i)-d, (ii)-c, (iii)-b, (iv)-a

Answer: A

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## 2 Marks

1. Let $*$ be a binary operation on set $Q$ of rational numbers defined as $a * b=\frac{a b}{8}$. Write the identity for *, If any.

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2. Show that $p \vee(\sim p)$ is a tautology.

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3. Show that $p \vee(q \wedge r)$ is a contingency.

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4. In the set of integers under the operation * defined by $a * b=a+b-1$. Find the identify element.

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5. Let $S$ be the set of positive rational numbers and is defined by $a * b=\frac{a b}{2}$. Then find the identity element and the inverse of 2.
6. Let $\mathrm{G}=\left\{1, w, w^{2}\right\}$ where w is a complex cube root of unity. Then find the inverse of $w^{2}$. Under usual multiplication.

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## 3 Marks

1. Write the truth value for each of the following statements.
(1) $3+5=8$ and $\sqrt{2}$ is an irrational number.
(2) 5 is a positive integer or a square is a rectangle.
(3) Chennai is not a Tamilnadu.

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2. In $(z, *)$ where $*$ is defined by $a * b=a^{b}$, prove that $*$ is not a binary operation on $z$.

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3. Let $\mathrm{G}=\{1, \mathrm{i}, 1,-\mathrm{i}\}$ under the binary operation multiplication. Find the inverse of all the elements.

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4. In $(z, *)$ where $*$ is defined as
$a * b=a+b+2$. Verify the commutative and associative axiom.

## 5. Construct the truth table for $(\sim p) \vee(q \wedge r)$

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## 5 Marks

1. Let $S$ be a non-empty set and 0 be a binary operation on s defined by x $0 \mathrm{y}=\mathrm{x}, \mathrm{x}, y \in S$.

Determine whether 0 is commutative and association.
2. $\ln (N, *)$ where $*$ is defined by $x * y=\max$
$(x, y)$ check the closure axion and identify anion.

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