MATHS

BOOKS - SURA MATHS (TAMIL ENGLISH)

MODAL QUESTION PAPER

Section I

1. A= $\begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$, B=adj A and C =3A, then $\frac{|adjB|}{|C|}$=

A. $\frac{1}{3}$

B. $\frac{1}{9}$

C. $\frac{1}{4}$

D. 1

Answer:

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2. If the inverse of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}^n$ is $\frac{1}{11}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then the ascending order of a, b, c, d is

A. a, b, c, d

B. d, b, c, a

C. c, a, b, d

D. b, a, c, d

Answer:

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3. The least value of n satisfying $\left[\frac{\sqrt{3}}{2} + \frac{i}{2}\right]^n = 1$ is

A. 30

B. 24

C. 12

D. 18

Answer:

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4. The principal argument of $\frac{3}{-1+i}$ is

A. $\frac{-5\pi}{6}$

B. $\frac{-2\pi}{3}$

C. $\frac{-3\pi}{4}$

D. $\frac{-\pi}{2}$

Answer:

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5. The polynomial $x^3 + 2x + 3$ has :

A. one negative and two real roots

B. one positive and two imaginary roots

C. three real roots

D. no solution

Answer:

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6. The domain of the function defined by $f(x) = \sin^{-1}\sqrt{x-1}$ is

A. [1, 2]

B. [-1, 1]

C. [0, 1]

D. [-1, 0]

Answer:

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7. If x + y = k is a normal to the parabola $y^2 = 12x$ then the value of k is

A. 3

B. -1

C. 1

D. 9

Answer:

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8. The circle passing through (1,-2) and touching the axis of x at (3,0) passing through the point

A. (-5, 2)

B. (2, -5)

C. (5, -2)

D. (-2, 5)

Answer:

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9. The volume of the parallelepiped with its edges represented by the vectors $\hat{i}+\hat{j}, \hat{i}+2\hat{j}, \hat{i}+\hat{j}+\pi\hat{k}$ is

A. $\frac{\pi}{2}$

B. $\frac{\pi}{3}$

C. π

D. $\frac{\pi}{4}$

Answer:

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10. If the line $\frac{x-2}{3}=\frac{y-1}{-5}=\frac{z+2}{2}$ lies in the plane $x+3y-az+\beta=0$ then $($ is

A. (-5, 5)

B. (-6, 7)

C. (5, -5)

D. (6, -7)

Answer:

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11. The function $\sin^4 x+\cos^4 x$ is increasing in the interval

A. $\left[\frac{5\pi}{8},\frac{3\pi}{4}\right]$

B. $\left[\frac{\pi}{2}, \frac{5\pi}{8}\right]$

C. $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

D. $\left[0, \frac{\pi}{4}\right]$

Answer:

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12. The curve $y = ax^4 + bx^2 \quad \text{with} \quad ab > 0$

A. has no horizontal tangent

B. is concave up

C. is concave down

D. has no points of inflection

Answer:

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13. If $u = (x - y)^2$, then $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}$ is

A. 1

B. -1

C. 0

D. 2

Answer:

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14. The value of $\int_0^{\pi} \frac{dx}{1 + 5^{\cos x}}$ is.....

A. $\frac{\pi}{2}$

B. π

C. $\frac{3\pi}{2}$

D. 2π

Answer:

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15. The volume of solid of revolution of the region bounded by $y^2 = x(a - x)$ about x-axis is

A. πa^3

B. $\frac{\pi a^3}{4}$

C. $\frac{\pi a^3}{5}$

D. $\frac{\pi a^3}{6}$

Answer:

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16. If m, n are the order and degree of the differential equation $\left[\frac{d^4y}{dx^4} + \frac{d^2y}{dx^2}\right]^{\frac{1}{2}} = a\frac{d^2y}{dx^2}$ respectively, then the value of $4m - n$ is

A. 15

B. 12

C. 14

D. 13

Answer:

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17. The solution of the differential equation $\frac{dy}{dx} = \frac{y}{x} + \frac{\phi\left(\frac{y}{x}\right)}{\phi'\left(\frac{y}{x}\right)}$ is

A. $x\phi\left(\frac{y}{x}\right) = k$

B. $\phi\left(\frac{y}{x}\right) = kx$

C. $y\phi\left(\frac{y}{x}\right) = k$

D. $\phi\left(\frac{y}{x}\right) = ky$

Answer:

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18. If P{X=0}=1-P{X=1}. If E{X}=3Var(X), then P{X=0} is

A. $\frac{2}{3}$

B. $\frac{2}{5}$

C. $\frac{1}{3}$

D. $\frac{1}{5}$

Answer:

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19. Which one is the contrapositive of the statement $(p \vee q) \rightarrow r$?

A. $\neg r \rightarrow (\neg p \wedge \neg q)$

B. $\neg r \rightarrow (p \vee q)$

C. $r \rightarrow (p \wedge q)$

D. $p \to (q \vee r)$

Answer:

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Section Ii

1. Solve the following system of linear equations by Cramer's rule 2x - y = 3, x + 2y = -1.

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2. If z_1, z_2 and z_3 are complex numbers such that $|z_1| = |z_2| = |z_3| = |z_1 + z_2 + z_3| = 1$, find the value of $\left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right|$.

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3. Find the value of $\sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right)$.

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4. Find the equation of the parabola with vertex (-1, -2), axis parallel to y-axis and passing through (3, 6).

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5. If $\hat{a}, \hat{b}, \hat{c}$ are three unit vectors such that $\hat{b}$ and $\hat{c}$ are non-parallel and $\hat{a} \times \left(\hat{b} \times \hat{c}\right) = \frac{1}{2}\hat{b}$, find the angle between $\vec{a}$ and $\vec{c}$.

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6. If the mass m(x) (in kilograms) of a thin rod of length x(in metres) is given by, $m(x) = \sqrt{3x}$ then what is the rate of change of mass with respect to the length when it is x = 27 meters.

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7. Evaluate : $\int_0^{\infty} e^{-ax} x^n dx$, where $a > 0$.

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8. Show that $y = ax + \frac{b}{x}, x \neq 0$ is a solution of the differential equation $x^2 y'' + xy' - y = 0$.

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9. Find the mean of a random variable X, whose probability density function is $f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{for } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$.

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10. Let $*$ be a binary operation on set Q of rational numbers defined as $a * b = \frac{ab}{8}$. Write the identity for $*$, If any.

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Section Iii

1. Find the inverse of $\begin{bmatrix} 2 & -1 \\ 5 & -2 \end{bmatrix}$ by Gauss Jorden method.

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2. If $\omega \neq 1$ is a cube root of unity, show that the roots of the equation $(z-1)^3 + 8 = 0$ are $-1, 1-2\omega, 1-2\omega^2$.

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3. Find all real numbers satisfying $4^x - 3\left(2^{x+2}\right) + 2^5 = 0$

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4. Find the centre, foci and eccentricity of the hyperbola $12x^2 - 4y^2 - 24x + 32y - 127 = 0$

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5. Find the image of the point whose position vector is $\hat{i} + 2\hat{j} + 3\hat{k}$ in the plane $\vec{r} \cdot \left(\hat{i} + 2\hat{j} + 4\hat{k}\right) = 38.$

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6. Evaluate : $\lim_{x \to 0^+} x \log x.$

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7. Find a linear approximation for the following functions at the indicated points.

$f(x) = x^3 - 5x + 12, x_0 = 2$

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8. By using the properties of definite integrals, evaluate $\int_0^3 |x - 1| dx$

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9. Solve : $\frac{dy}{dx} + 2y \cot x = 3x^2 \text{cosec}^2 x.$

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10. A fair coin is tossed a fixed number of times. If the probability of getting seven heads is equal to that of getting nine heads, find the probability of getting exactly two heads.

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Section Iv

1. If z = x + iy and $\arg\left(\frac{z-i}{z+2}\right)=\frac{\pi}{4}$. Show that $x^2+y^2+3x-3y+2=0$.

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2. Solve the equation :

$3x^4-16x^3+26x^2-16x+3=0$

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3. Solve : $\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$

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4. A rod of length 1.2 m moves with its ends always touching the coordinate axes. The locus of a point Pon the rod, which is 0.3 m from the end in contact with x-axis is an ellipse. Find the eccentricity.

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5. Find the non-parametric and Cartesian equations of the plane passing through the point (4, 2, 4) and is perpendicular to the planes 2x + 5y + 4z + 1 = 0 and 4x + 7y + 6z + 2 = 0.

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6. A steel plant is capable of producing x tonnes per day of a law-grade steel and y tonnes per day of a hight-grade steel, where $y = \frac{40 - 5x}{10 - x}$. If the fixed market price of low-grade steel is half that of high-grade steel, then what should be optimal productions in law-grade steel and high-grade steel in order to have maximum receipts.

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7. Let $z(x, y) = xe^y + ye^{-x}, x = e^{-t}, y = st^2, s, t \in \mathbb{R}$. Find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$.

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8. Find the area of the region bounded between the parabola $x^2 = y$ and the curve y=|x|.

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9. Water at temperature $100^\circ C$ cools in 10 minutes to $80^\circ C$ in a room temperature of $25^\circ C$.

Find

(i) The temperature of water after 20 minutes

(ii) The time when the temperature is $40^\circ C$

$\left[\log_e \frac{11}{15} = -0.3101, \log_e 5 = 1.6094\right]$

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10. Suppose a discrete random variable can only take the values 0, 1, and 2. The probability mass function is defined by

$$f(x) = \begin{cases} \frac{x^2+1}{k}, & \text{for x=0,1,2} \\ 0 & \text{otherwise} \end{cases}$$

Find (i) the value of k (ii) cumculative distribution function (iii) $P(X \geq 1)$.

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11. Using truth table check whether the statements $\sim(p \vee q) \vee (\sim p \wedge q)$ and $\sim p$ are logically equivalent.

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12. Prove by vector method that $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$.

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13. Find the equations of tangent and normal to the curve $y^2 - 4x - 2y + 5 = 0$ at the point where it cuts the x-axis.

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Part I

1. The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -3 & -4 \end{bmatrix}$ is

A. 2

B. 1

C. 3

D. 4

Answer:

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2. If $0 \leq \theta \leq \pi$ and the system of equations x+(sin $\theta)y - (\cos\theta)z = 0, (\cos\theta)x - y + z = 0, (\sin\theta)x + y - z = 0$ has a non -trivial solution then θ is

A. $\frac{5\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{2\pi}{3}$

D. $\frac{3\pi}{4}$

Answer:

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3. If $|z_1|$ = 1, $|z_2| = 2, |z_3| = 3$ and $|9z_1z_2 + 4z_1z_3 + z_2 z_3 = 12|$, then the value of $|z_1 + z_2 + z_3|$ is

A. 2

B. 1

C. 4

D. 3

Answer:

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4. Which one of the points $i, -2+i, 2$ and 3 is farthest from the origin?

A. 3

B. $-2+i$

C. i

D. 2

Answer:

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5. If α, β and γ are the zeros of $x^3 + px^2 + qx + r$, then $\Sigma\frac{1}{\alpha}$ is

A. $\frac{q}{r}$

B. $\frac{-p}{r}$

C. $\frac{-q}{r}$

D. $\frac{-q}{p}$

Answer:

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6. The range of $\sec^{-1} x$ is

A. $[-\pi, \pi] \backslash \left\{\frac{\pi}{2}\right\}$

B. $[0, \pi] \backslash \left\{\frac{\pi}{2}\right\}$

C. $[0, \pi] \backslash \left\{\frac{\pi}{2}\right\}$

D. $[-\pi, \pi] \backslash \left\{\frac{\pi}{2}\right\}$

Answer:

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7. If P (x,y) be any point on $16x^2 + 25y^2 = 400$ with foci $F_1(3, 0)$ and $F_2(-3, 0)$ then $PF_1 + PF_2$ is

A. 6

B. 8

C. 12

D. 10

Answer:

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8. If the length of the perpendicular from the origin to the plane $2x + 3y + \lambda z = 1, \lambda > 0$ is $\frac{1}{5}$ then the value of is λ is

A. 0

B. 1

C. $2\sqrt{3}$

D. $3\sqrt{2}$

Answer:

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9. The system of linear equations x+y+z =2, 2x+y-z=3, 3x+2y+kz= has a unique solution if

A. $k \neq 0$

B. $-1 < k < 1$

C. $-2 < k < 2$

D. k = 0

Answer:

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10. If A is an orthogonal matrix, then |A| is

A. 1

B. -1

C. ± 1

D. 0

Answer: C

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11. If z is a complex number such that Re (z) = Im(z), then

A. $Re\left(z^2\right) = 0$

B. $Im\left(z^2\right) = 0$

C. $Re\left(z^2\right) = Im\left(z^2\right)$

D. $Re(z^2) = -Im(z^2)$

Answer:

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12. If z is any complex number, then the points $z, iz, -z, -iz$

A. form a square

B. form a trapenzium

C. are collinear

D. lie on a circle $|z| = \sqrt{2}$ with centre (0,0) and radius $\sqrt{2}$

Answer:

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13. If $\sin\alpha$ and $\cos\alpha$ are the roots of $25x^2 + 5x - 12 = 0$ then the value of $\sin 2\alpha$ is

A. $\frac{12}{25}$

B. $\frac{-12}{25}$

C. $\frac{-24}{25}$

D. $\frac{4}{5}$

Answer:

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14. If a and b are odd integers then the roots of the equation $2ax^2 + (2a + b)x + b = 0 (a \neq 0)$ are

A. rational

B. irrational

C. non real

D. rational and equal

Answer:

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15. If $4\cos^{-1}x + \sin^{-1}x = \pi,$ then x is

A. $\frac{3}{2}$

B. $\frac{1}{\sqrt{2}}$

C. $\frac{\sqrt{3}}{2}$

D. $\frac{2}{\sqrt{3}}$

Answer:

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16. The domain of the function $\cos^{-1}(2x-1)$ is

A. [0, 1]

B. [-1, 1]

C. (-1, 1)

D. $(0, \pi)$

Answer:

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17. If 5x + 9 = 0 is the directrix of the hyperbola $16x^2 - 9y^2 = 144$ then its corresponding focus is

A. $\left(\frac{-5}{3}, 0\right)$

B. (5, 0)

C. (-5, 0)

D. $\left(\frac{5}{3}, 0\right)$

Answer:

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18. If $ax^2 + by^2 + (a + b - 4)xy - ax - by - 20 = 0$ represent the circle then its centre is

A. $\left(\frac{1}{2}, \frac{1}{2}\right)$

B. $\left(-\frac{1}{2}, -\frac{1}{2}\right)$

C. (1, 1)

D. (-1, -1)

Answer:

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19. The foot of the perpendicular from A (1, 0, 0) to the line $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$ is

A. (3, -4, -2)

B. (5, -8, -4)

C. (-3, 4, 2)

D. (2, -3, 4)

Answer:

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20. If A is a non-singular matrix such that $A^{-1} = \begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$, then $\left(A^T\right)^{-1}$=

A. $\begin{bmatrix} -5 & 3 \\ 2 & 1 \end{bmatrix}$

B. $\begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$

C. $\begin{bmatrix} -1 & -3 \\ 2 & 5 \end{bmatrix}$

D. $\begin{bmatrix} 5 & -2 \\ 3 & -1 \end{bmatrix}$

Answer:

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21. If adj A = $\begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$ and adj B = $\begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}$ then adj (AB) is

A. $\begin{bmatrix} -7 & -1 \\ 7 & -9 \end{bmatrix}$

B. $\begin{bmatrix} -6 & 5 \\ -2 & -10 \end{bmatrix}$

C. $\begin{bmatrix} -7 & 7 \\ -1 & -9 \end{bmatrix}$

D. $\begin{bmatrix} -6 & -2 \\ 5 & -10 \end{bmatrix}$

Answer:

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22. If $|z-2+i| \leq 2$, then the greatest value of $|z|$ is

A. $\sqrt{3}-2$

B. $\sqrt{3}+2$

C. $\sqrt{5}-2$

D. $\sqrt{5}+2$

Answer:

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23. If $x^2+y^2=1$, then the value of $\frac{1+x+iy}{1+x-iy}$.

A. x - iy

B. 2x

C. $-2iy$

D. x + iy

Answer:

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24. The product of all four values of $\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)^{\frac{3}{4}}$ is

A. -2

B. -1

C. 1

D. 2

Answer:

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25. The polynomial $x^3 + 2x + 3$ has :

A. One negative and two imaginary zeros

B. imaginary zeros

C. Three real zeros

D. No zeros

Answer:

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26. The value of $\sin^{-1}(\cos x), 0 \leq x \leq \pi$ is

A. $\pi - x$

B. $x - \frac{\pi}{2}$

C. $\frac{\pi}{2} - x$

D. $x - \pi$

Answer:

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27. An ellipse has OB, as semi minor axis, F and F' its foci and the angle FBF' is a right angle. Then the eccentricity of the ellipse is :

A. $\frac{1}{\sqrt{2}}$

B. $\frac{1}{2}$

C. $\frac{1}{4}$

D. $\frac{1}{\sqrt{3}}$

Answer:

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28. The vertex of the parabola $x^2 = 8y - 1$ is :

A. $\left(-\frac{1}{8}, 0\right)$

B. $\left(\frac{1}{8}, 0\right)$

C. $\left(0, \frac{1}{8}\right)$

D. $\left(0, -\frac{1}{8}\right)$

Answer:

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29. The value of $\left[\hat{i} + \hat{j}, \hat{j} + \hat{k}, \hat{k} + \hat{i}\right]$ is equal to :

A. 0

B. 1

C. 2

D. 4

Answer:

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30. The angle between the lines $\frac{x-2}{3}=\frac{y+1}{-2}, z=2$ and $\frac{x-1}{1}=\frac{2y+3}{3}, \frac{z+5}{2}$ is

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

Answer:

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31. A stone is thrown up vertically. The height it reaches at time t seconds is given by $x = 80t - 16t^2$. The stone reaches the maximum height in time t seconds is given by

A. 2

B. 2.5

C. 3

D. 3.5

Answer:

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32. The value of the limit $\lim_{x \to 0} \left(\cot x - \frac{1}{x} \right)$ is

A. 0

B. 1

C. 2

D. ∞

Answer:

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33. If $f(x, y) = e^{xy}$, then $\frac{\partial^2 f}{\partial x \, \partial y}$ is equal to

A. xye^{xy}

B. $(1 + xy)e^{xy}$

C. $(1 + y)e^{xy}$

D. $(1 + x)e^{xy}$

Answer:

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34. The value of $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left(\frac{2x^7 - 3x^5 + 7x^3 - x + 1}{\cos^2 x} \right) dx$ is

A. 4

B. 3

C. 2

D. 0

Answer:

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35. The value of $\int_0^{\frac{\pi}{6}} \cos^3 3x dx$

A. $\frac{2}{3}$

B. $\frac{2}{9}$

C. $\frac{1}{9}$

D. $\frac{1}{3}$

Answer:

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36. The value of $\int_0^{\pi} \frac{dx}{1+5^{\cos x}}$ is :

A. $\frac{\pi}{2}$

B. π

C. $\frac{3\pi}{2}$

D. 2π

Answer:

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37. The solution of the differential equation $\frac{dy}{dx} + \frac{1}{\sqrt{1-x^2}} = 0$ is

A. $y + \sin^{-1} x = c$

B. $x + \sin^{-1} y = 0$

C. $y^2 + 2\sin^{-1} x = c$

D. $x^2 + 2\sin^{-1} y = 0$

Answer:

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38. If $\sin x$ is the integrating factor of the linear differential equation $\frac{dy}{dx} + Py = Q$, then P is

A. $\log \sin x$

B. cosx

C. tanx

D. cotx

Answer:

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39. The order and degree of the differential equation $[x + y']^2 = (y')^2$

A. 1, 2

B. 1, 1

C. 2, 1

D. 2, 2

Answer:

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Part Ii

1. If adj A = $\begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, find A^{-1}.

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2. Simplify : $\left(\sin\frac{\pi}{6}+i\cos\frac{\pi}{6}\right)^{08}$.

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3. Form a polynomial equation with integer coefficients with $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ as a root.

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4. Find the value of $\tan^{-1}\left(\tan\frac{3\pi}{5}\right)$.

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5. Condition for y = mx + c to be a tangent to the circle $x^2+y^2=a^2$ is

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6. If $2\hat{i}-\hat{j}+3\hat{k}, 3\hat{i}+2\hat{j}+\hat{k}, \hat{i}+m\hat{j}+4\hat{k}$ are coplanar, find the value of m.

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7. Show that

$\left(2+i\sqrt{3}\right)^{10}-\left(2-i\sqrt{3}\right)^{10}$ is purely imaginary.

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8. Determine the number of positive and negative roots of the equation $x^9-5x^8-14x^7=0.$

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9. Find the value of

$$\cot^{-1}\left(\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{4}{5}\right)$$

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10. If the system of linear equation x + 2ay + az = 0, x + 3by + bz = 0, x + 4cy + cz = 0 has a non-trival solution then show that a, b, c are in H.P.

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11. Simplify : $i^{59} + \frac{1}{i^{59}}$.

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12. If α, β and γ are the roots of the cubic equation $x^3 + 2x^2 + 3x + 4 = 0$, for a cubic equation roots are

$2\alpha, 2\beta, 2\gamma$

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13. Find the principal value of

$\text{cosec}^{-1}(-1)$

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14. Find the equation of the parabola if the curve is opened upward, vertex is (-1, -2) and the length of the latus rectum is 4.

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15. If $2\hat{i} - \hat{j} + 3\hat{k}, 3\hat{i} + 2\hat{j} + \hat{k}, \hat{i} + m\hat{j} + 4\hat{k}$ are coplanar, find the value of m.

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16. Determine whether the pair of straight lines $\vec{r} = \left(2\hat{i} + 6\hat{j} + 3\hat{k}\right) + t\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right), \vec{r} = \left(2\hat{i} - 3\hat{k}\right) + s\left(\hat{i} + 2\hat{j} + 3\right.$

are parallel.

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17. Using Rolle's theorem find the point on the curve $y = x^2 + 1, \ -2 \leq x \leq 2$ where the tangent is parallel to x-axis.

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18. Find differential dy for $y = \frac{(1-2x)^3}{3-4x}$.

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19. Evaluate : $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} x \cos x dx.$

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20. Write the Maclaurin series expansion of e^{-x}.

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Part Iii

1. Find the inverse of the non-singular matrix A = $\begin{bmatrix} 0 & 5 \\ -1 & 6 \end{bmatrix}$, by Gauss Jordan method.

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2. In a competitive examination, one mark is awarded for every correct answer while $\frac{1}{4}$ mark is deducted for every wrong answer. A student answered 100 questions and got 80 marks. How many questions did he answer correctly ? (Use Cramer's rule to solve the problem).

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3. State triangle inequality.

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4. Find the condition that the roots of $ax^3 + bx^2 + cx + d = 0$ are in geometric progression. Assume $a, b, c, d \neq 0$.

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5. Find the domain of $\sin^{-1}(2 - 3x^2)$.

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6. Find the equation of the parabola with vertex (-1, -2), axis parallel to y-axis and passing through (3, 6).

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7. if the normal at the point t_1 on the parabola $y^2 = 4ax$ meets the parabola again in the point t_2 then prove that $t_2 = -\left(t_1 + \frac{2}{t_1}\right)$

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8. If $\hat{a}, \hat{b}, \hat{c}$ are three unit vectors such that $\hat{b}$ and $\hat{c}$ are non-parallel and $\hat{a} \times \left(\hat{b} \times \hat{c}\right) = \frac{1}{2}\hat{b}$, find the angle between $\vec{a}$ and $\vec{c}$.

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9. Determine whether the pair of straight lines

$\vec{r} = \left(2\hat{i} + 6\hat{j} + 3\hat{k}\right) + t\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right), \vec{r} = \left(2\hat{j} - 3\hat{k}\right) + s\left(\hat{i} + 2\hat{j} + 3\right.$

are parallel. Find the shortest distance between them.

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10. Prove that

$\tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right) + \tan\left(\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right) = \frac{2b}{a}.$

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11. If $F(\alpha) = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$

Show that = $F(\alpha)^{-1} = F(-\alpha)$

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12. Find the values of the real numbers x and y, if the complex numbers (3 - i) x - (2-i) y + 2i + 5 and 2x + (-1 + 2i)y + 3 + 2i are equal

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13. Find a polynomial equation of minimum degree with rational coefficients, having 2i+3 as a root.

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14. Solve : $\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$

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15. The parabolic communication antenna has a focus at 2 m distance from the vertex of the antenna. Find the width of the antenna 3 m from the vertex.

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16. Prove that $\left[\vec{a}\times\vec{b},\ \vec{b}\times\vec{c},\ \vec{c}\times\vec{a}\right]=\left[\vec{a},\ \vec{b},\ \vec{c}\right]^2$

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17. Evaluate $\lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{e^x-1}\right)$.

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18. Evaluate $\int_{-1}^{1} \log\left(\frac{3-x}{3+x}\right) dx.$

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19. From the differential equations by eliminating arbitrary constants given in bracket $Y = e^{3x} \quad (C\cos 2x + D\sin 2x)$, {C, D}.

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20. Draw the Geometrical diagram for the sum of two complex numbers, z_1 and z_2, any verify the result.

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Part Iv

1. Solve : $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$

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2. Find the value of k for which the equations kx-2y+z=1, x-2ky+z=-2,x-2y+kz=1 have

(i) no solution

(ii) unique solution

(iii) infinitely many solution

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3. Suppose z_1, z_2 and z_3 are the vertices of an equilateral triangle inscribed in the circle |z| = 2. If $z_1 = 1 + i\sqrt{3}$ then find z_2 and z_3.

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4. If z = x + iy and $\arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$. Show that $x^2 + y^2 + 3x - 3y + 2 = 0$.

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5. If $2+i$ and $3-\sqrt{2}$ are the roots of the equation $x^6-13x^5+62x^4-126x^3+65x^2+127x-140=0$ then find all the roots.

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6. Find the value of $\tan^{-1}(-1)+\cos^{-1}\left(\frac{1}{2}\right)+\sin^{-1}\left(\frac{-1}{2}\right)$

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7. If $\cot^{-1}\left(\frac{1}{7}\right)=\theta$, find the value of $\cos\theta$.

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8. For the ellipse $4x^2+y^2+24x-2y+21=0$, find the centre, vertices, and the foci. Also prove that the length of latus rectum is 2.

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9. Find the number of solution of the eqution $\tan^{-1}(x-1)+\tan^{-1}x+\tan^{-1}(x+1)=\tan^{-1}(3x)$

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10. Find the parametric form vector eqution and Cartesian equations of the plane passing through the points $(2,2,1),(1,-2,3)$ and parallel to the straight line passing through the points $(2,1,-3)$ and $(-1,5,-8)$.

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11. Prove by vector method that the perpendiculars (altitudes) from the vertices to the sides of a triangle are concurrent.

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12. If a tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ makes intercepts h and k on the co-ordinate axes then show that $\frac{a^2}{h^2}+\frac{b^2}{k^2}=1$.

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13. Solve the following systems of linear equation by Cramer's rule:

3x+3y-z=11, 2x-y+2z=9, 4x+3y+2z=25

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14. Test for consistency and if possible solve the following system of equations by rank method.

2x-y+z=2,6x-3y+3z=6,4x-2y+2z=4.

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15. If z = x + iy and arg $\left(\frac{z-1}{z+1}\right)=\frac{\pi}{2}$, show that $x^2+y^2-1=0$.

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16. Sketch the graph of sinx in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $\sin^{-1}$ in [-1, 1].

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17. Find the equation of the ellipse whose eccentricity is $\frac{1}{2}$, one of the foci is (2, 3) and a directrix is x = 7. Also find the length of the major and minor axes of the ellipse.

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18. A tunnel through a mountain for a four lane highway is to have a elliptical opening. The total width of the highway (not the opening) is to be 16m , and the height at the edge of the road must be sufficient for a truck 4m high to clear if the highest point of the opening is to be 5m approximately. How wide must the opening be ?

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19. By vector method, Prove that $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

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20. Find the Vector and Cartesian equation of the plane containing the line $\frac{x-2}{2} = \frac{y-2}{3} = \frac{z-1}{3}$ and parallel to the line $\frac{x+1}{3} = \frac{y-1}{2} = \frac{z+1}{1}$

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21. A conical water tank with vertex down of 12 meters height has a radius of 5 meters at the top. If water flows into the tank at a rate 10 cubic m/min, how fast is the depth of the water increases when the water is 8 metres deep?

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22. Prove that among all the rectangles of the given area, square has the least perimeter.

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23. If $u(x, y) = \frac{x^2 + y^2}{\sqrt{x + y}}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{3}{2}u$.

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24. Solve : $\frac{dy}{dx} + 2y \cot x = 3x^2 \text{cosec}^2 x$.

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25. The rate at which the population of a city increases at any time is propotional to the population at that time. If there were 1,30,000 people in the city in 1960 and 1,60,000 in 1990 what population may be anticipated in 2020.[loge(16/13)=.2070;e.42=1.52]

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26. Find the area of the region bounded by $y = \cos x$, $y = \sin x$, the lines $x = \frac{\pi}{4}$ and $x = \frac{5\pi}{4}$.

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