# ©゙ doubtnut 

India's Number 1 Education App

## MATHS

## BOOKS - MCGROW HILL EDUCATION MATHS (HINGLISH)

## APPLICATIONS OF DERIVATIVES

## Illustration

1. Find the rate of change of volume of a sphere with respect to its radius when $\mathrm{r}=4 \mathrm{~cm}$

## - Watch Video Solution

2. A particle moves along the curve $12 y=x^{3}$. . Which coordinate changes at faster rate at $x=10$ ?
3. A point move in a straight line so that its distance from the start in $t$ sec is equal to $s=\frac{1}{4} t^{4}-4 t^{3}+16 t^{2}$. What will be acceleration and at what times is its velocity equal to zero?

## - Watch Video Solution

4. A body whose mass is 3 kg performs rectilinear motion according to the formula $s=1+t+t^{2}$, where s is measured the kinetic energy $\frac{1}{2} m v^{2}$ and t in second.
Determine the kinetic energy $\frac{1}{2} m v^{2}$ of the body in 5 sec after its start.

## - Watch Video Solution

5. Find the increment and differential of the function, $f(x)=2 x^{2}-3 x+2$ when x changes to 1.99 from 2.
6. Derive the equation of tangent and normal at $\left(x_{0}, y_{0}\right)$ of the curve $y=\log x$

## - Watch Video Solution

7. Verify Rolles theorem for function $f(x)=4^{\sin x}$ on $[0, \pi]$

## - Watch Video Solution

8. The number of values of $k$ for which the equation $x^{3}-3 x+k=0$ has two distinct roots lying in the interval $(0,1)$ is three (b) two (c) infinitely many (d) zero

## - Watch Video Solution

9. The function $y=\sqrt{2 x-x^{2}}$ (A) increases in ( 0,2 ) (B) increases in ( 0 , 1) but decreases in (1,2) (C) Decreases in (0,2) (D) Increases in (1,2) but decreases in $(0,1)$

## - Watch Video Solution

10. Find the interval of monotonicity of $y=\frac{1-x+x^{2}}{1+x+x^{2}}$

## - Watch Video Solution

11. Find the extrema of $y=2 x^{3}-3 x^{2}$. Y is differentiable function and $y^{\prime}=6 x^{2}-6 x=6 x(x-1)$

## - Watch Video Solution

12. Find the greatest and least value of $y=x^{3}-3 x^{2}+6 x-2$ on
$[-1,1] \quad y \quad$ is differentiable function of $x$ and
$y^{\prime}(x)=3 x^{2}-6 x+6=3\left(x^{2}-2 x+2\right)=3(x-1)^{2}>0$.

## - Watch Video Solution

## Solved Examples Concept Based Single Correct Answer Type Questions

1. The approximate value of $\cos 31^{\circ}$ is (Take $1^{\circ}=0.0174$ )
A. 0.52
B. 0.851
C. 0.641
D. 0.681

## Answer: B

Watch Video Solution
2. The tangent at $A(2,4)$ on the curve $y=x^{3}-2 x^{2}+4$ cuts the x axis at $T$ then length of $A T$ is
A. $(2,0)$
B. $\left(\frac{7}{2}, 0\right)$
C. $\left(\frac{11}{9}, 0\right)$
D. $\left(\frac{14}{9}, 0\right)$

## Answer: D

## - Watch Video Solution

3. The slope of the tangent to the curve $x=t^{2}+3 t-8, y=2 t^{2}-2 t-5$ at the point $(2,-1)$, is
A. $2 / 3$
B. $6 / 7$
C. $4 / 5$
D. $3 / 2$

## Answer: B

## - Watch Video Solution

4. The interval in which $y=\frac{1}{4 x^{3}-9 x^{2}+6 x}$ is increasing is
A. $(-\infty, \infty)$
B. $(0,1 / 2)$
C. $(1 / 2,1)$
D. $(1, \infty)$

## Answer: C

5. $y=x-\log (1+x)$ increasing in
A. 1
B. 0
C. -1
D. $\frac{1}{2}$

## Answer: B

## - Watch Video Solution

6. A covered box of volume $72 \mathrm{~cm}^{3}$ and the base sides in a ratio of $1: 2$
is to be made. The length all sides so that the total surface area is the least possible is
A. $2,4,9$
B. 8,3,3
C. 6,6,2
D. 6,3,4

## Answer: D

## - Watch Video Solution

7. A point on the curve $y=x^{3}-3 x+5$ at which the tangent line is parallel to $y=-2 x$ is
A. $(1,3)$
B. $(0,5)$
C. $\left(\frac{1}{\sqrt{3}}, 5-\frac{8 \sqrt{3}}{9}\right)$
D. $\left(\frac{1}{\sqrt{2}}, 0\right)$

## Answer: C

## - Watch Video Solution

8. The difference between the greatest and the least values of the function $f(x)=\sin 2 x-x$ on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
A. $\frac{\pi}{2}$
B. 1
C. 2
D. $-\frac{\pi}{2}$

## Answer: A

## - Watch Video Solution

9. The point of inflection of $y=x^{3}-5 x^{2}+3 x-5$ is
A. $\frac{1}{2}$
B. $\frac{3}{4}$
C. $\frac{7}{4}$
D. $\frac{5}{3}$

Answer: D

## - Watch Video Solution

10. The rate of change of the function $f(x)=3 x^{5}-5 x^{3}+5 x-7$ is minimum when
A. $\frac{3}{4}$
B. $\frac{5}{4}$
C. $\frac{2}{3}$
D. $\frac{3}{2}$

## Answer: B

## - Watch Video Solution

Solved Examples Level 1 Single Correct Answer Type Questions

1. A spherical balloon is expanding. If the radius in increasing at the rate of 2 inches per minute the rate at which the volume increases (in cubic inches per minute) when the radius is 5 inches is
A. $100 \pi$
B. $1000 \pi$
C. $2000 \pi$
D. $500 \pi$

## Answer: C

## - Watch Video Solution

2. An object is moving in the clockwise direction around the unit circle $x^{2}+y^{2}=1$. As it passes through the point $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$, its $y$ coordinate is decreasing at the rate of 3 unit per second. The rate at which the $x$-coordinate changes at this point is (in unit per second)
A. 2
B. $3 \sqrt{3}$
C. $\sqrt{3}$
D. $2 \sqrt{3}$

## Answer: B

## - Watch Video Solution

3. An approximate value of $\cos 40^{\circ}$ is
A. 0.7688
B. 0.7071
C. 0.7117
D. 0.7

Answer: A
4. The value of $x$ for which the tangents to the curves $y=x \cos x, y=(\sin x) / x$ are parallel to the axis of x are roots of (respectively)
A. $\sin x=x, \tan x=x$
B. $\cot x=x, \sec x=x$
C. $\cot x=x, \tan x=x$
D. $\tan x=x, \cot x=x$

## Answer: C

## - Watch Video Solution

5. The length of the subtangent to the ellipse $x=a \cos t, y=b \sin t$ at $t=\pi / 4$ is
A. a
B. b
C. $b / \sqrt{2}$
D. $a / \sqrt{2}$

## Answer: D

## - Watch Video Solution

6. Find the angle of intersection of $y=a^{x} a n d y=b^{x}$
A. $\frac{\log a b}{1+\log a b}$
B. $\frac{\log a / b}{1+(\log a)(\log b)}$
C. $\frac{\log a b}{1+(\log a)(\log b)}$
D. none of these

## Answer: B

7. For the parabola $y^{2}=16 x$, the ratio of the length of the subtangent to the abscissa is
A. 2:1
B. 1:1
C. $x: y$
D. $x^{2}: y$

## Answer: A

## - Watch Video Solution

8. If the tangent to the curve $x^{3}-y^{2}=0$ at $\left(m^{2},-m^{2}\right)$ is parallel to
$y=-\frac{1}{m} x-2 m^{3}$, then the value of $m^{3}$ is
A. $(1 / 3)$
B. $1 / 6$
C. $2 / 3$
D. $-2 / 3$

## Answer: C

## (D) Watch Video Solution

9. The function $y=\frac{a x+b}{(x-1)(x-4)}$ has turning point at $P(2,1)$. Then find the value of $a$ and $b$.
A. $c=2, d=0$
B. $c=1, d=0$
C. $c=1, d=-1$
D. $c=1, d=1$

## Answer: B

10. The distance between the origin and the normal to the curve $y=e^{2 x}+x^{2}$ at $x=0$ is
A. $1 / \sqrt{5}$
B. $2 / \sqrt{5}$
C. $3 / \sqrt{5}$
D. $2 / \sqrt{3}$

## Answer: B

## - Watch Video Solution

11. The function $\mathrm{f}(\mathrm{x})=(\sin x)+[\cos x], 0<x \leq \pi / 2$
A. is continuous on $(0, \pi / 2)$
B. is strictly decreasing in $(0, \pi / 2)$
C. is stricitly increasing in $(0, \pi / 2)$
D. has global maximum value 2

## Answer: A

## - View Text Solution

12. if $f$ is an increasing function and $g$ is a decreasing function on an interval I such that fog exists then
A. $f o g$ is a decreasing function
B. $g$ of is an increasing function
C. fog is an increasing function
D. none of these

## Answer: A

13. If a le $0 f(x)=e^{\wedge}(a x)+e^{\wedge}(-a x)^{\wedge}$ and $S=\{x: f(x)$ is monotonically increasing then S equals
A. $S=\{x: x>0\}$
B. $S=\{x: x<0\}$
C. $S=\{x: x>1\}$
D. $S=\{x: x<1\}$

## Answer: B

## - Watch Video Solution

14. Equation of the horizonatl tangent to the curve $y=e^{x}+e^{-x}$ is
A. $y=-2$
B. $y=-1$
C. $y=2$
D. none

## Answer: C

## - Watch Video Solution

15. If $f(x) \operatorname{and} g(x)$ be two function which are defined and differentiable for all $x \geq x_{0}$. If $f\left(x_{0}\right)=g\left(x_{0}\right) \operatorname{and}^{\prime}(x)>g^{\prime}(x)$ for all $f>x_{0}$, then prove that $f(x)>g(x)$ for all $x>x_{0}$.
A. $f(x)<g(x)$ for some $x>x_{0}$
B. $\mathrm{f}(\mathrm{x})=\mathrm{g}(\mathrm{x})$ for some $x>x_{0}$
C. $f(x)>g(x)$ for all $x>x_{0}$
D. none of these

## Answer: C

16. If $f(x)=2 x \cot ^{-1} x+\log \left(\sqrt{1+x^{2}}-x\right.$ then $\mathrm{f}(\mathrm{x})$
A. decreases on $(-\infty, \infty)$
B. decreases on $[0, \infty)$
C. neither decreases nor increases on $[0, \infty]$
D. increases on ( $-\infty, \infty$ )

## Answer: D

## - Watch Video Solution

17. The equation $x^{4}-7 x+2=0$ has
A. exactly two real and distinct solutions
B. has four real roots
C. no real root
D. all the four roots lie between 0 and 2

Answer: A

## - Watch Video Solution

18. The maximum value of $x^{1 / x}$ is
A. $(1 / e)^{e}$
B. $e^{1 / e}$
C.e
D. $1 / e$

## Answer: B

## - Watch Video Solution

19. Let $P(x)=a_{0}+a_{1} x^{2}+a_{2} x^{4}++a_{n} x^{2 n}$ be a polynomial in a real
A. neither a maximum nor a minimum
B. only one maximum
C. only one minimum
D. none of these

## Answer: C

## - Watch Video Solution

20. $f(x)=\frac{x}{\sin x}$ and $g(x)=\frac{x}{\tan x}$, where $0<x \leq 1$ then in the interval
A. $f(x)$ and $g(x)$ are increasing functions
B. both $f(x)$ and $g(x)$ are decreasing functions
C. $f(x)$ is an increasing function
D. $g(x)$ is an increasing function

## Answer: C

## - Watch Video Solution

21. Examine the validity of Lagrange's mean value theorem for the function $f(x)=x^{2 / 3}$ in the interval $[-1,1]$.
A. $(0,0)$ is a point of maximum
B. $(0,0)$ is not a point of minimum
C. $(0,0)$ is a critical point
D. There is no crtical point

## Answer: C

## (D) Watch Video Solution

22. Let $f(x)=\frac{a x+b}{c x+d}(d a-c b \neq 0, c \neq 0)$ then $\mathrm{f}(\mathrm{x})$ has
A. a critical point
B. no point of inflection
C. a maximum
D. a minimum

## Answer: B

## (D) Watch Video Solution

23. If the only point of inflection of the function $f(x)-(x-a)^{m}(x-b)^{n}, m, n e N$ and $m \neq n$ is at $x=a$ then
A. $(a, 0),(b, 0)$ are the only critical points of $f$
B. there are $m+n$ critical points of $f$
C. there are exactly three critical points of $f$
D. none of these

## - Watch Video Solution

24. A ball is dropped from a platform 19.6 m high. Its position function is
A. $x=-4.9 t^{2}+19.6(0 \leq t \leq 1)$
B. $x=-4.9 t^{2}+19.6(0 \leq t \leq 2)$
C. $x=-9.8 t^{2}+19.6(0 \leq t \leq 2)$
D. $x=-4.9+19.6(0 \leq t \leq 2)$

## Answer: B

## - Watch Video Solution

25. Let $\mathrm{f}(\mathrm{n})=20 n-n^{2}(n=1,2,3 \ldots)$, then
A. $f(n) \rightarrow \infty$ as $n \rightarrow \infty$
B. $f(n)$ has no maximum
C. The maximum value of $f(n)$ is greater than 200
D. none of these

## Answer: D

## - View Text Solution

26. plot the curve $y=[x]^{2}$
A. $(1,1)$
B. $(2,4)$
C. $(2 / 3,4 / 9)$
D. $(4 / 3,16 / 9)$
27. The smallest value of $M$ such that $\left|x^{2}-3 x+2\right| \leq M$ for all $x$ in $\left[1, \frac{5}{2}\right]$
A. $1 / 4$
B. $3 / 4$
C. $5 / 4$
D. $5 / 16$

## Answer: B

## - Watch Video Solution

28. The no. of solutions of the equation $a^{f(x)}+g(x)=0$ where $\mathrm{a}>0$, and $g(x)$ has minimum value of $1 / 2$ is :-
A. one
B. two
C. infinitely many
D. zero

## Answer: D

- Watch Video Solution

29. The minimum value of $f(x)=|3-x|+|2+x|+|5-x|$ is
A. 0
B. 7
C. 8
D. 10

## Answer: B

30. $x(x-2)(x-4), 1 \leq x \leq 4$, will satisfy mean value theorem at
A. 1
B. 2
C. $5 / 2$
D. $7 / 2$

## Answer: A

## - Watch Video Solution

31. If $\sqrt{x}+\sqrt{y}=\sqrt{a}$ then $\frac{d y}{d x}=$ ?
A. 2 a
B. $a$
C. $a / 2$
D. $\sqrt{a}$

## Answer: B

## - Watch Video Solution

32. x and y be two variables such that $x>0$ and $x y=1$. Then the minimum value of $x+y$ is
A. 1
B. $1 / 2$
C. 2
D. $1 / 4$

## Answer: C

## (D) Watch Video Solution

33. if $f(x)=\left(\frac{\sin (x+\alpha)}{\sin (x+\beta), \alpha \neq \beta}\right.$ then $\mathrm{f}(\mathrm{x})$ has
A. $\beta-\alpha=k \pi$
B. $\beta-\alpha \neq k \pi$
C. $\beta-\alpha=2 k \pi$
D. none of the abve

## Answer: B

## - Watch Video Solution

34. The tangent to the curve $y=x^{3}-6 x^{2}+9 x+4,0 \leq x \leq 5$ has maximum slope at x which is equal to
A. 2
B. 3
C. 4
D. none of these

Answer: D

## - Watch Video Solution

35. The values of parameter $a$ for which the point of minimum of the function $f(x)=1+a^{2} x-x^{3} \quad$ satisfies the inequality
$\frac{x^{2}+x+2}{x^{2}+5 x+6}<0$ are
$(2 \sqrt{3}, 3 \sqrt{3})$
$-3 \sqrt{3},-2 \sqrt{3})$
$(-2 \sqrt{3}, 3 \sqrt{3})(\mathrm{d})(-2 \sqrt{2}, 2 \sqrt{3})$
A. an empty set
B. $(-3 \sqrt{3},-2 \sqrt{3})$
C. $(2 \sqrt{3}, 3 \sqrt{3})$
D. $(-3 \sqrt{3},-2 \sqrt{3}) \cup(2 \sqrt{3}, 3 \sqrt{3})$

## Answer: C

36. Find the point of intersection of the tangents drawn to the curve $x^{2} y=1-y$ at the points where it is intersected by the curve $x y=1-y$.
A. $(0,-1)$
B. $(1,1)$
C. $(0,1)$
D. none of these

## Answer: C

37. The equation of the tangent to the curve $y=(2 x-1) e^{2(1-x)}$ at the point of its maximum, is
A. $y=1$
B. $x=1$
C. $x+y=1$
D. $x-y=-1$

## Answer: A

## - Watch Video Solution

38. If the function $f(x)=x^{2}+\alpha / x$ has a local minimum at $\mathrm{x}=2$, then the value of $\alpha$ is

## - Watch Video Solution

39. Three normals are drawn to the parabola $y^{2}=4 x$ from the point $(c, 0)$. These normals are real and distinct when
A. $c=0$
B. $c=1$
C. $c=2$
D. $\mathrm{c}=3$

## Answer: D

## - Watch Video Solution

40. The function $f(x)=[\log (x-1)]^{2}(x-1)^{2}$ has :
A. loca extremum at $x=1$
B. point of inflection at $x=1$
C. local extremum at $\mathrm{x}=2$
D. point of inflection at $x=2$

## Answer: C

41. If $f(x)=\log x$ satisfies Lagrange's theorem on $[1, e]$ then value of $c \in(1, e)$ such that the tangent at $c$ is parallel to line joining $(1, f(1))$ and $(e, f(e))$ is
A. $e-\frac{3}{2}$
B. $\frac{1+e}{2}$
C. $e-1$
D. $e-\frac{1}{2}$

## Answer: C

## - Watch Video Solution

42. The value of $c$ for which the conclusion of Lagrange's theorem holds for the function $\mathrm{f}(\mathrm{x})=\sqrt{a^{2}-x^{2}}, a>1$ on the interval $[1, \mathrm{a}]$ is
A. $\frac{a(a+1)}{2}$
B. $\frac{1+a}{2}$
C. $\frac{\sqrt{a(a+1)}}{2}$
D. $\frac{a(a-1)}{2}$

## Answer: C

## - Watch Video Solution

43. Let $f(x)\left\{\begin{array}{ll}|x-2|+a, & \text { if } x \leq 2 \\ 4 x^{2}+3 x+1, & \text { if } x>2\end{array}\right.$. If $\mathrm{f}(\mathrm{x})$ has a local minimum at $x=2$, then
A. $a>21$
B. $a \leq 21$
C. $a>30$
D. $a>24$

## Answer: B

44. If $y=m x+2$ is parallel to a tangent to curve $e^{4 y}=1+16 x^{2}$ then
A. $|m|<1$
B. $|m|<1$
C. $|m|>1$
D. $|m| \geq 1$

## Answer: A

## (D) Watch Video Solution

45. Given the function $f(x)=x^{2} e^{-2 x}, x>0$. Then $\mathrm{f}(\mathrm{x})$ has the maximum value equal to
A. $e^{-2}$
B. $(2 e)^{-1}$
C. $e^{-1}$
D. none of these

## Answer: A

## - Watch Video Solution

46. if $f(x)=(x-4)(x-5)(x-6)(x-7)$ then.
A. $f^{\prime}(x)=0$ has four real roots
B. three roots of $\mathrm{f}^{\prime}(\mathrm{x})=0$ lie in $(4,5) \cup(5,6) \cup(6,7)$
C. the equation $f^{\prime}(x)=$ has only two roots
D. three roots of $\mathrm{f}^{\prime}(\mathrm{x})=0$ lie $(3,4) \cup(4,5) \cup(5,6)$

## Answer: B

47. If $f(x)=\frac{x^{2}-1}{x^{2}+1}$. For every real number $x$, then the minimum value of $f$. does not exist because $f$ is unbounded is not attained even through $f$ is bounded is equal to 1 is equal to -1
A. does not exist because f is unbounded
B. is not attained even though $f$ is bounded
C. is equal to 1
D. is equal to -1

## Answer: D

## - Watch Video Solution

48. For all $x \in(0,1)$
A. $e^{x}<1+x$
B. $\log _{e}(1+x)<x$
C. $\sin x>x$
D. $\log _{e} x>x$

## Answer: B

## - Watch Video Solution

49. Let $h(x)=f(x)-(f(x))^{2}+(f(x))^{3}$ for every real $x$. Then,
A. $h$ increases whenever $f$ decreases
B. $h$ decreases whenever $f$ increases
C. h increases or decreases accordingly as $f$ increases or decreases
D. nothing can be claimed in general

## Answer: C

## - Watch Video Solution

50. Let $f(x)=a x^{3}+b x^{2}+c x+d, b^{2}-3 a c>0, a>0, c<0$. Then $f(x)$ has
A. local maximum at some $x \in R^{+}$
B. a local maximum at some $x \in R^{-}$
C. a local minima at $\mathrm{x}=0$
D. local minima at some $\quad x \in R^{-} \quad$ Itbr.

$$
R^{+}=(0, \infty), R^{-}=(-\infty, 0)
$$

## Answer: B

## - View Text Solution

51. If $f(x)=\left\{\begin{array}{ll}3-x^{2}, & x \leq 2 \\ \sqrt{a+14}-|x-48|, & x>2\end{array}\right.$ and if $\mathrm{f}(\mathrm{x})$ has a local maxima at $x=2$, then greatest value of $a$ is
A. a cannot be determined
B. least value of a is 2011
C. greater value of a is 2011
D. `a ge 3010

## Answer: C

## - Watch Video Solution

52. The total number of local maxima and local minima of the function
$\mathrm{f}(\mathrm{x})=\left\{\begin{array}{ll}(2+x)^{3}, & -3<x \leq-1 \\ x^{2 / 3}, & -1<x<2\end{array}\right.$ is
A. 0
B. 1
C. 2
D. 3
53. If the function $g:(-\infty, \infty) \rightarrow\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ is given by $g(u)=2 \tan ^{-1}\left(e^{u}\right)-\frac{\pi}{2}$. Then, $g$ is
A. even and is strictly increasing in $(0, \infty)$
B. odd and is strictly decreasing in $(-\infty, \infty)$
C. odd and is strictly increasing in $(-\infty, \infty)$
D. neither even nor odd, but is strictly increasing in $(-\infty, \infty)$

## Answer: C

## - Watch Video Solution

54. If $f(x)=x^{a} \log x$ and $f(0)=0$ then the value of $\alpha$ for which Rolle's theorem can be applied in $[0,1]$ is
A. -1
B. $-1 / 2$
C. 0
D. $1 / 2$

## Answer: D

## - Watch Video Solution

55. Suppose the cubic $x^{3}-p x+q$ has three distinct real roots, where $p>0$ and $q>0$. Then which one of the following holds?
A. $\mathrm{f}(\mathrm{x})$ has minima at $\sqrt{\frac{p}{3}}$ and maxima at $-\sqrt{\frac{p}{3}}$
B. $\mathrm{f}(\mathrm{x})$ has minima at $-\sqrt{\frac{p}{3}}$ and maxima at $\sqrt{\frac{p}{3}}$.
C. $\mathrm{f}(\mathrm{x})$ has minima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$
D. $\mathrm{f}(\mathrm{x})$ has maxima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$
56. Given $\mathrm{P}(\mathrm{x})=x^{4}+a x^{3}+b x^{2}+c x+d$ such that $\mathrm{x}=0$ is the only real root of $\mathrm{P}^{\prime}(\mathrm{x})=0$. If $\mathrm{P}(-1)$ It $\mathrm{P}(1)$, then $\in$ the $\int$ ervall $[-1,1]$
A. $P(-1)$ is the minimum but $P(1)$ is not the maximum of $P$
B. neither $P(-1)$ is the minimum nor $P(1)$ is the maximum of $P$
C. $P(-1)$ is the minimum and $P(1)$ is the maximum of $P$
D. $P(-1)$ is not minimum but $P(1)$ is the maximum of $P$

## Answer: D

## - Watch Video Solution

57. 

Let $f: R \vec{R}$
be defined by $f(x)=\{k-2 x$, if $x \leq-12 x+3, f x \succ 1\}$. If f has a local minimum at $x=1$, then a possible value of k is (1) 0 (2) $-\frac{1}{2}$ (3) -1 (4) 1
A. $-1 / 2$
B. -1
C. 1
D. 0

## Answer: B

## - Watch Video Solution

58. The value of $K$ in order that $f(x)=\sin x-\cos x-K x+5$ decreases for all positive real value of $x$ is given by
A. $a \geq \sqrt{2}$
B. $a<\sqrt{2}$
C. $a \geq 1$
D. $a<1$

## - Watch Video Solution

59. The curve that passes through the point $(2,3)$ and has the property that the segment of any tangent to it lying between the coordinate axes is bisected by the point of contact, is given by
A. $2 y-3 x=0$
B. $y=\frac{6}{x}$
C. $x^{2}+y^{2}+13$
D. $\left(\frac{x}{2}\right)^{2}+\left(\frac{y}{3}\right)^{2}=2$

## Answer: B

## - Watch Video Solution

60. A spherical balloon is filled with 4500 p cubic meters of helium gas.

If a leak in the balloon causes the gas to escape at the rate of $72 \pi$ cubic meters per minute, then the rate (in meters per minute) at which the radius of the balloon decreases 49 minutes after the leakage began is (1) $\frac{9}{7}$ (2) $\frac{7}{9}$ (3) $\frac{2}{9}$ (4) $\frac{9}{2}$
A. $7 / 9$
B. $2 / 9$
C. $9 / 2$
D. $9 / 7$

## Answer: B

## D Watch Video Solution

## Solved Examples Level 2 Single Correct Answer Type Questions

1. The point $M(x, y)$ of the graph of the function $y=e^{-|x|}$ so that area bounded by the tangent at $M$ and the coordinate axes is greatest is
A. $\left(1, e^{-1}\right)$
B. $\left(2, e^{-2}\right)$
C. $\left(-2, e^{2}\right)$
D. $(0,1)$

## Answer: A

## - View Text Solution

2. The abscissa of the point on the curve $a y^{2}=x^{3}$, the normal at which cuts off equal intercepts from the coordinate axes is
A. 2
B. 4
C. -4
D. -2

## Answer: B

## - Watch Video Solution

3. v24
A. (0.4/3)
B. $(0,2 / 3)$
C. $(1,2 / 3)$
D. $(2,4 / 3)$

## Answer: A

4. If the tangent to the curve $2 y^{3}=a x^{2}+x^{3}$ at the point $(\mathrm{a}, \mathrm{a})$ cuts off intercept $\alpha$ and $\beta$ on the co-ordinate axes, (where $\alpha^{2}+\beta^{2}=61$ ) then $a^{2}$ equals $\qquad$
A. 16
B. 28
C. 30
D. 31

## Answer: C

## - Watch Video Solution

5. The co-ordinates of the points on the barabola $y^{2}=8 x$, which is at minium distance from the circle $x^{2}+(y+6)^{2}=1$ are
A. $(2,-4)$
B. $(18,-12)$
C. $(2,4)$
D. none of these

## Answer: A

## - Watch Video Solution

6. The equation $e^{x-8}+2 x-17=0$ has :-
A. two real roots
B. one real root
C. eight real roots
D. four real roots

## Answer: B

7. The maximum and minimum value of $f(x)$ $=a b \sin x+b \sqrt{1-a^{2}} \cos x+c$ lie in the interval (assuming $|a|<1, b>0)$
A. $[b-c, b+c]$
B. $(b-c, b+c)$
C. $[c-b, b+c]$
D. none of these

## Answer: C

## - Watch Video Solution

8. The maximum area of the rectangle whose sides pass through the vertices of a given rectangle of sides $a a n d b$ is $2(a b)$ (b) $\frac{1}{2}(a+b)^{2}$ $\frac{1}{2}\left(a^{2}+b^{2}\right)$ (d) noneofthese
A. $(1 / 2)(a b)^{2}$
B. $(1 / 2)(a+b)$
C. $(1 / 2)(a+b)^{2}$
D. none of these

## Answer: C

## - Watch Video Solution

9. Find the image of interval $[-1,3]$ under the mapping specified by the function $f(x)=4 x^{3}-12 x$.
A. $[-2,0]$
B. $[-8,72]$
C. $[-8,0]$
D. $[8,72]$

## Answer: B

10. The difference between the greatest and the least value of the function $f(x)=\cos x+\frac{1}{2} \cos 2 x-\frac{1}{3} \cos 3 x$
A. $3 / 8$
B. $2 / 3$
C. $8 / 7$
D. $9 / 4$

## Answer: D

## - Watch Video Solution

11. The maximum distance of the point $(a, 0)$ from the curve $2 x^{2}+y^{2}-2 x=0$ is -
A. $\sqrt{1-2 a+2 a^{2}}$
B. $\sqrt{1-2 a+a^{2}}$
C. $\sqrt{1+2 a+2 a^{2}}$
D. $\sqrt{1+a+a^{2}}$

## Answer: A

## - Watch Video Solution

12. The sides of the rectangle of the greatest area, that can be inscribed in the ellipse $x^{2}+2 y^{2}=8$, are given by
A. $4 \sqrt{2}, 4$
B. $4,2 \sqrt{2}$
C. $2, \sqrt{2}$
D. $2 \sqrt{2}, 2$

## Answer: B

13. The area of the region bounded by the curve $y=x^{3}$, its tangent at $(1,1)$ and $x$-axis is
A. $x^{2}+y^{2}+24 x-28 y+2=0$
B. $2\left(x^{2}+y^{2}\right)+12 x-8 y-8=0$
C. $3\left(x^{2}+y^{2}\right)-24 x+10 y+8=0$
D. none of these

## Answer: D

## - Watch Video Solution

14. Let $f(x)=6 x^{4 / 3}-3 x^{1 / 2}, x \in[-1,1]$. Then
A. The maximum value of $f(x)$ on $[-1,1]$ is 3
B. The maximum value of $f(x)$ on $[-1,1]$ is 9
C. The maximum value of $f(x)$ on $[-1,1]$ is 0
D. none of these

## Answer: B

## - View Text Solution

15. Let $g(x)=(\log (1+x))^{-1}-x^{-1}, x>0$ then
A. $1<g(x)<2$
B. $-1<g(x)<0$
C. $0<g(x)<1$
D. none of these

## Answer: C

16. Range of $\frac{x^{2}-x+1}{x^{2}+x+1}$ is
A. $1 / 2$
B. 1
C. 2
D. 3

## Answer: D

17. If the tangent at $(1,1)$ on $y^{2}=x(2-x)^{2}$ meets the curve again at $P$, then find coordinates of $P$.
A. $(4,4)$
B. $(-1,2)$
C. (9/4,3/8)
D. none of these

## - Watch Video Solution

18. If the curves $y^{2}=6 x, 9 x^{2}+b y^{2}=16$ intersect each other at right angles then the value of $b$ is: (1) 6 (2) $\frac{7}{2}$ (3) 4 (4) $\frac{9}{2}$
A. 2
B. 4
C. $9 / 2$
D. none of these

## Answer: C

## - Watch Video Solution

19. Find the distance of the point on $y=x^{4}+3 x^{2}+2 x$ which is nearest to the line $y=2 x-1$
A. $4 / \sqrt{5}$
B. $3 / \sqrt{5}$
C. $2 / \sqrt{5}$
D. $1 / \sqrt{5}$

## Answer: D

## - Watch Video Solution

20. A given right cone has volume $p$, and the largest right circular cylinder that can be inscribed in the cone has volume $q$. Then $p: q$ is $9: 4$
(b) 8:3 (c) 7:2 (d) none of these
A. $9: 4$
B. $8: 3$
C. 7:2
D. none of these

## Answer: A

## - Watch Video Solution

21. The set of all values of $a$ for which the function $f(x)=\left(a^{2}-3 a+2\right)\left(\cos ^{2} \frac{x}{4}-\sin ^{2} \frac{x}{4}\right)+(a-1) x+\sin 1$ does not possess critical points is (A) $[1, \infty)(B)(0,1) \cup(1,4)$ (C) $(-2,4)$
(D) $(1,3) \cup(3,5)$
A. $[1, \infty)$
B. $(-2,4)$
C. $(1,3) \cup(3,5)$
D. $(0,1) \cup(1,4)$

## - Watch Video Solution

22. Let $x, p \in R, x+1>0, p \neq 0,1$. Then
A. $(1+x)^{p}>1 p x$ for $p>0$
B. $(1+x)^{p}>1+p x$ for $p \in(-\infty, 0) \cup(1, \infty)$
C. $(1+x)^{p}>1+p x$ for $0<p<1$
D. $(1+x)^{p}<1+p x$ for $p<1$

## Answer: B

## - View Text Solution

23. If $f(x)=\frac{a \sin x+b \cos x}{c \sin x+d \cos x}$ is decreasing for all $x$, then

$$
\text { A. } a d-b c<0
$$

B. $a d-b c>0$
C. $a b-c d>0$
D. $a b-c d<0$

## Answer: A

## - Watch Video Solution

24. In the interval $[0,1]$, the function $x^{25}(1-x)^{75}$ takes its maximum value at the point O (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{3}$
A. 0
B. $1 / 3$
C. $1 / 2$
D. $1 / 4$

## Answer: D

25. The set of values of p for which the equation $p x^{2}=\ln x$ possess a single root is
A. $1 / 2$
B. $1 / 2 e$
C. $1 / e$
D. $2 e^{-1}$

## Answer: B

## - Watch Video Solution

## Solved Examples Numerical Answer Type Questions

1. If $f(x)=|x-7|+|x-10|+|x-12|$ has a minimum at $\mathrm{x}=\mathrm{k}$, then
the value of $k$ is
2. Let $f\left(x-=\tan ^{-1}\left(\frac{1-x}{1+x}\right)\right.$. Then difference of the greatest and least value of $f(x)$ on $[0,1]$ is:

## (D) Watch Video Solution

3. The absolute maximum value of $f(x)=\frac{5}{3 x^{4}+8 x^{3}-18 x^{2}+60}$

## - View Text Solution

4. Number of real roots of the equation $3 x^{5}+15 x-8=0$ is

## - Watch Video Solution

5. If the value of greater of $\sin x+\tan x$ and $2 x(0<x<\pi / 2)$ at $\pi / 4$ is $g(\pi / 4)$ then $g(\pi / 4)$ is equal to

## - Watch Video Solution

6. If the greatest value of $y=\frac{x}{\log x}$ on $\left[e, e^{3}\right]$ is $u$ then $u$ is equal to (given $\mathrm{e}=2.71$ )

## (D) Watch Video Solution

7. If ( $u, v$ ) are the coordinates of the point on the curve $x^{3}=y(x-4)^{2}$ where the ordinate is minimum then $u v$ is equal to

## - Watch Video Solution

8. If $A$ gt $0, B$ gt 0 and $A+B=\frac{\pi}{3}$,then the maximum value of $\tan A \tan B$, is

## - Watch Video Solution

9. If $f(\theta)=64 \sec \theta+27 \operatorname{cosec} \theta$ when $\theta$ lies in $(0, \pi / 4)$ then min $f(\theta)$ is equal to

## D View Text Solution

10. Show that the area of the triangle formed by the positive $x$-axis and the normal and tangent to the circle $x^{2}+y^{2}=4$ at $(1, \sqrt{3})$ is $2 \sqrt{3}$

## - Watch Video Solution

11. A curve is represented by the equations $x=\sec ^{2} \operatorname{tandy}=\cot t$, where $t$ is a parameter. If the tangent at the point $P$ on the curve
where $t=\frac{\pi}{4}$ meets the curve again at the point $Q$, then $|P Q|$ is equal to $\frac{5 \sqrt{3}}{2}$ (b) $\frac{5 \sqrt{5}}{2}$ (c) $\frac{2 \sqrt{5}}{3}$ (d) $\frac{3 \sqrt{5}}{2}$

## - Watch Video Solution

12. If the slope of line through the origin which is tangent to the curve $y=x^{3}+x+16$ is $m$, then the value of $m-4$ is $\qquad$ .

## - Watch Video Solution

13. If the point on $y=x \tan \alpha-\frac{a x^{2}}{24^{2} \cos ^{2} \alpha}(\alpha>0)$ where the tangent is parallel to $\mathrm{y}=\mathrm{x}$ has an ordinate $u^{2} / 4 a$, then $\cos ^{2} \alpha$ is equal to
14. $f: R \rightarrow R$ be defined as $f(x)=|x|+\left|x^{2}-1\right|$. The total number of points at which $f$ attains either local maximum or a level minimum is

## - View Text Solution

15. The number of non-zero integral solution of $K$ for which the equation $\frac{x^{3}}{3}-4 x=K$ has three distinct solution is

## - Watch Video Solution

16. The least integral value of $x$ where $f(x)=(\log )_{\frac{1}{2}}\left(x^{2}-2 x-3\right)$ is monotonically decreasing is $\qquad$
17. Let $P(x)$ be a polynomial of degree 5 having extremum at $x=-1,1$ and $\lim _{x \rightarrow 0}\left(\frac{P(x)}{x^{3}}-1\right)=7$. The value of $|P(7)|$ is

## - Watch Video Solution

18. Let $f(x)=\left\{\begin{array}{ll}\left|x^{2}-2 x\right|+a, & 0 \leq x<5 / 2 \\ -2 x+5, & x \geq 5 / 2\end{array}\right.$. If $\mathrm{f}(\mathrm{x})$ has a maximum at $x=5 / 2$, then the greatest value of $|a|$ is

## - Watch Video Solution

## Exercise Concept Based Single Correct Answer Type Questions

1. The length of the tangent to the curve $x=a \sin ^{3} t, y=a \cos ^{3} t(a>0)$ at an arbitrary is
A. $a \cos ^{2} t$
B. $a \sin ^{2} t$
C. $\frac{a \sin ^{2} t}{\cot t}$
D. $\frac{a \cos ^{2} t}{\sin t}$

## Answer: A

## - Watch Video Solution

2. Equation of normal to $x=2 e^{t}, y=e^{-t}$ at $t=0$ is
A. $x+y-4=0$
B. $x+2 y-4=0$
C. $2 x-y-3=0$
D. $x-2 y-3=0$

## Answer: C

3. A point moves according $s=\frac{2}{9} \sin \frac{\pi}{2} t+s_{0}$. The acceleration at the end of first second is
A. $-\frac{\pi}{18}$
B. $-\frac{\pi^{2}}{18}$
C. $\frac{\pi}{18}$
D. $\frac{\pi^{2}}{18}$

## Answer: B

## - Watch Video Solution

4. Let $\mathrm{f}(\mathrm{x})=\mathrm{x} \log \mathrm{x}+1$ then the set $\{\mathrm{x}: f(x)>0\}$ is equal to
A. $(1, \infty)$
B. $(1 / e, \infty)$
C. $[e, \infty)$
D. $(0,1) \cup(1, \infty)$

## Answer: D

## - Watch Video Solution

5. On the curve $y=x^{3}$, the point at which the tangent line is parallel to the chord through the point $(-1,-1)$ and $(2,8)$ is
A. $(1,1)$
B. $\left(\frac{1}{2}, \frac{1}{8}\right)$
C. $\left(\frac{1}{3}, \frac{1}{27}\right)$
D. $\left(\frac{1}{2},-\frac{1}{8}\right)$

## Answer: A

## - Watch Video Solution

6. Let $f(x)=2 x^{2}-\log x$, then
A. $f$ increases on $(0, \infty)$
B. f decrease on $\left(\frac{1}{2}, \infty\right)$
C. f increases on $\left(\frac{1}{2}, \infty\right)$
D. f decreases on $(0,1)$

## Answer:

## - Watch Video Solution

7. Let $f(x)=\frac{3}{4} x^{4}-x^{3}-9 x^{2}+7$, then the number of critical points in $[-1,4]$ is
A. 4
B. 3
C. 2
D. 1

## Answer: C

## - Watch Video Solution

8. On the curve $x^{3}=12 y$, find the interval at which the abscissa changes at a faster rate than the ordinate.
A. $(-2,2) \sim\{0\}$
B. $(-3,3) \sim\{0\}$
C. $(1,4)$
D. $(2,4)$

## Answer: A

## - Watch Video Solution

9. Find the value of $a$ if the curves $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{4}=1 a n d y^{3}=16 x$ cut orthogonally.
A. 1
B. $\frac{2 \sqrt{3}}{3}$
C. $3 \sqrt{3}$
D. $5 \sqrt{5})$

## Answer: B

10. The least value of $g(t)=8 t-t^{4}$ on $[-2,1]^{`}$ is
A. -16
B. -20
C. -32
D. 7

## Answer: C

## - Watch Video Solution

## Exercise Level 1 Single Correct Answer Type Questions

1. Let $f(x)=\tan ^{-1} x$ and $g(x)=\frac{x}{1+x^{2}}, x>0$ then
A. $f(x)<g(x), \quad$ on $\quad(0, \infty)$
B. $f(x) \leq g(x)$ on $[1, \infty)$
C. $g(x)<f(x)$ on $(0, \infty)$
D. none of these

## Answer: C

2. Let $f(x)=(x-2)(x-3)(x-4)(x-5)(x-6)$ then
A. $f^{\prime}(x)=0$ has five real roots
B. four roots of $f^{\prime}(x)=0$ lie in $(2,3) \cup(3,4) \cup(4,5) \cup(5,6)$
C. the equation $f^{\prime}(x)$ has only three roots
D. four roots of $f^{\prime}(x)=0$ lie in $(1,2) \cup(2,3) \cup(3,4) \cup(4,5)$

## Answer: B

## - Watch Video Solution

3. Let $f(x)=(x-3)^{5}(x+1)^{4}$ then
A. $x=-1$ is point of minima
B. $x=-1$ is point of maxima
C. $x=7 / 9$ is a point of maxima
D. $x=-1$ is neither a point of maxima and minima

## Answer: B

## - Watch Video Solution

4. 

The
normal to the curve
$x=a(\cos \theta-\theta \sin \theta), y=a(\sin \theta-\theta \cos \theta)$ at any point,$\theta$, is such that
A. makes a constant angle with the $x$-axis
B. is at a constant distance from the origin
C. does not touch a fixed circle
D. passes through the origin

## Answer: B

## - Watch Video Solution

5. The number of values of $k$ for which the equation $x^{3}-3 x+k=0$ has two distinct roots lying in the interval $(0,1)$ is three (b) two (c) infinitely many (d) zero
A. -1
B. 1
C. 3
D. none of these

## Answer: D

## - Watch Video Solution

6. If the sum of the squares of the intercepts on the axes cut off by tangent to the curve $x^{\frac{1}{3}}+y^{\frac{1}{3}}=a^{\frac{1}{3}}, a>0$ at $\left(\frac{a}{8}, \frac{a}{8}\right)$ is 2 , then $a=$ 1 (b) 2 (c) 4 (d) 8
A. 1
B. 2
C. 4
D. 8

## Answer: C

## - Watch Video Solution

7. If the area of the triangle included between the axes and any tangent to the curve $x^{n} y=a^{n}$ is constant, then find the value of $n$.
A. $1 / 2$
B. 1
C. $3 / 2$
D. 2

## Answer: B

8. If the tangent at any point on the curve $x^{4}+y^{4}=c^{4}$ cuts off intercepts a and b on the coordinate axes, the value of $a^{-\frac{4}{3}}+b^{-\frac{4}{3}}$ is
A. $a^{-4 / 3}$
B. $a^{-1 / 2}$
C. $a^{1 / 2}$
D. none of these

## Answer: A

## - Watch Video Solution

9. The interval of increase of the function
$y=x-2 \sin x$ if $0 \leq x \leq 2 \pi$, is
A. $(0, \pi)$
B. $\left(0, \frac{\pi}{2}\right)$
C. $(\pi / 2, \pi)$
D. $(\pi / 3,5 \pi / 3)$

## Answer: D

## - Watch Video Solution

10. All possible value of $f(x)=(x+1)^{\frac{1}{3}}-(x-1)^{\frac{1}{3}}$ on [0,1] is 1 (b) 2
(c) 3 (d) $\frac{1}{3}$
A. 1
B. 2
C. '3
D. $2^{1 / 3}$

## Answer: C

11. Let f be a function defined by $f(x)=2 x^{2}-\log |x|, x \neq 0$ then
A. $f$ increases on $[-1 / 2,0] \cup[1 / 2, \infty)$
B. f decrease on $(-\infty, 0)$
C. f increases on $(-\infty,-1 / 2)$
D. f decreases on $[1 / 2, \infty]$

## Answer:

## - Watch Video Solution

12. The shortest distance of $(0,0)$ from the curve $y=\frac{e^{x}+e^{-x}}{2}$ is
A. $1 / 2$
B. $1 / 3$
C. 2
D. none of these

## Answer: D

## - Watch Video Solution

13. The normal to the circle $x^{2}+y^{2}-2 x-2 y=0$ passing through
$(2,2)$ is
A. $x=y$
B. $2 x+y-6=0$
C. $x+2 y-6=0$
D. $x+y-4=0$

## Answer: A

## - Watch Video Solution

14. If $f(x)=x$ for $x \leq 0$
$=0$ for $x>0$ then $\mathrm{f}(\mathrm{x})$ at $\mathrm{x}=0$ is
A. decreases on $(0, \infty)$
B. increases on $(0, \infty)$
C. decreases on $(1, \infty)$
D. neither increases nor decreases on $(0, \infty)$

Answer: B

## - Watch Video Solution

15. The value of k so that the equation $x^{3}-12 x+k=0$ has distinct roots in $[0,2]$ is
A. 4
B. 2
C. -2
D. none of these

## Answer: D

## - Watch Video Solution

16. Let $f(x)=6 x^{4 / 3}-3 x^{1 / 3}$ defined on $[-1,1]$ then
A. maximum value of $f$ is 7
B. maximum value of $f$ is 5
C. maximum value of f is 9
D. minimum value of $f$ is $-3 / 2$

## Answer: C

17. An equation of tangent line at an inflection point of $f(x)=x^{4}-6 x^{3}+12 x^{2}-8 x+3$ is
A. $y=3 x+4$
B. $y=4$
C. $y=3 x+2$
D. none of these

## Answer: D

- Watch Video Solution

18. The number of real roots of the equation $2 x^{3}-3 x^{2}+6 x+6=0$ is
A. 1
B. 2
C. 3
D. none of these

Answer: A

## - Watch Video Solution

19. Let $\mathrm{f}(\mathrm{x})=(x-2)\left(x^{4}-4 x^{3}+6 x^{2}-4 x+1\right)$ then value of local minimum of $f$ is
A. $-2 / 3$
B. $-(4 / 5)^{4}$
C. $-4^{4} / 5^{5}$
D. $-(4 / 5)^{5}$

## Answer: C

## - View Text Solution

20. Let $f(x)=x^{2}-2|x|+2, x \in[-1 / 2,3 / 2]$ then
A. $\min f(x)=1 / 2 x \in[-1 / 2,3 / 2]$
B. $\min f(x)=1 x \in[-1 / 2,3 / 2]$
C. $\max f(x)=3 / 2 x \in[-1 / 2,3 / 2]$
D. none of these

## Answer:

## D Watch Video Solution

21. The function $f(x)=\frac{|x-1|}{x^{2}}$ is
A. -1
B. 3
C. 2
D. $1 / 2$

## Answer: C

## - Watch Video Solution

22. The function $f(x)=x^{x}$ decreases on the interval (a) ( $0, e$ ) (b)
$(0,1)(\mathrm{c})(0,1 / e)(\mathrm{d})(1 / e, e)$
A. $(0, \mathrm{e})$
B. $(0,1)$
C. (0, 1/e)
D. none of these

## Answer: C

## - Watch Video Solution

23. The interval of increase of the function
$f(x)=x-e^{x}+\tan (2 \pi / 7)$ is (a) $(0, \infty)$ (b) $(-\infty, 0)$ (c) $(1, \infty)$
(d) $(-\infty, 1)$
A. $(-\infty, 1)$
B. $(0, \infty)$
C. $(-\infty, 0)$
D. $(1, \infty)$

## Answer: C

## - Watch Video Solution

24. Let $f(x)=x^{2}+p x+q$. The value of $(\mathrm{p}, \mathrm{q})$ so that $\mathrm{f}(1)=3$ is an extreme value of $f$ on $[0,2]$ is
A. $(-2,2)$
B. $(1,4)$
C. $(-2,4)$
D. $(-2,3)$

## Answer: C

## - Watch Video Solution

25. The number of inflection points of a function given by a third degree polynomial is exactly
A. 2
B. 1
C. 3
D. 0

## Answer: B

26. Let $f(x)=2 \tan ^{-1} x+\sin ^{-1} \frac{2 x}{1+x^{2}}$ then
A. $\max \mathrm{f}(\mathrm{x})=\pi / 2$
B. $\min f(x)=\pi / 4$
C. $\max \mathrm{f}(\mathrm{x})=2 \pi$
D. none of these

## Answer: C

## - Watch Video Solution

27. If the normal to the curve $x^{3}=y^{2}$ at the point $\left(m^{2},-m^{3}\right)$ is $y=m x-2 m^{3}$, then the value of $m^{2}$ is
A. 1
B. $1 / 2$
C. $1 / 3$
D. $2 / 3$

## Answer: D

## - View Text Solution

28. Let $f(x)=2 \sin x+\cos 2 x(0 \leq x \leq 2 \pi)$ and $g(x)=x+\cos x$ then
A. g is a decreasing function
B. fincreases on $(0, \pi / 2)$
C. f increases on $(0, \pi / 6) \cup(\pi / 2,5 \pi / 6)$
D. f decreases on $(0, \pi / 2)$

## Answer: C

29. In the interval $(0 \pi / 2)$ the fucntion $f(x)=\tan ^{n} x \cot ^{n}$ attains
A. 1
B. 0
C. 2
D. $1 / 2$

## Answer: C

## - Watch Video Solution

30. Find the number of points of local extrema of $f(x)=3 x^{4}-4 x^{3}+6 x^{2}+a x+b$ where $a, b \in R$
A. 4
B. 3
C. 1
D. 2

## Answer: C

## - Watch Video Solution

31. The shortest distance between line $\mathrm{y}-\mathrm{x}=1$ and curve $x=y^{2}$ is
A. $3 / 8$
B. $3 \sqrt{2} / 4$
C. $3 / 4$
D. $3 \sqrt{2} / 8$

## Answer: D

## - Watch Video Solution

32. The set of values of $p$ for which the points of extremum of the function, $f(x)=x^{3}-3 p x^{2}+3\left(p^{2}-1\right) x+1$ lin in the interval $(-2,4)$ is
A. $(-1,0)$
B. $(-2,4)$
C. $(-1,5)$
D. $(-1,3)$

## Answer: D

## - Watch Video Solution

33. If A gt $\mathrm{O}, \mathrm{B}$ gt 0 and $\mathrm{A}+\mathrm{B}=\frac{\pi}{3}$, then the maximum value of $\tan \mathrm{A} \tan \mathrm{B}$, is
A. $1 / 3$
B. $1 / 2$
C. $1 / \sqrt{2}$
D. $\sqrt{3} / 2$

## Answer: A

## - Watch Video Solution

34. The maximum value of $|\mathrm{x} \log \mathrm{x}|$ for $0<x \leq 1$ is
A. 0
B. $1 / e$
C. $2 e^{-1}$
D. none of these

## Answer: B

35. The greatest value of the function $\log _{x} 1 / 9-\log _{3} x^{2}(x>1)$ is
A. 2
B. 0
C. -4
D. -2

## Answer: C

## - Watch Video Solution

36. Let $f$ be differentiable for all $x$, If $f(1)=-2 a n d f^{\prime}(x) \geq 2$ for all $x \in[1,6]$, then find the range of values of $f(6)$.
A. $f(6)<8$
B. $f(6) \geq 8$
C. $f(6) \geq 10$
D. $f(6) \geq 5$

## Answer: B

## - Watch Video Solution

37. An extremum value of the function
$f(x)=\left(\sin ^{-1} x\right)^{3}+\left(\cos ^{-1} x\right)^{3}(-1 \leq x \leq 1)$ is
A. $7 \pi^{3} / 8$
B. $\pi^{3} / 8$
C. $\pi^{3} / 32$
D. $\pi^{3} / 16$

## Answer: C

- Watch Video Solution

38. Let $f(x)=x \log x+3 x$. Then
A. f increases in $\left(e^{-4}, \infty\right)$
B. f increases in $(0, \infty)$
C. f decreases in $(0, \infty)$
D. f decreases in $\left(0, e^{-2}\right)$

## Answer: A

## - Watch Video Solution

39. Let $f(x)=x^{2} . e^{-x^{2}}$ then which one is incorrect? (A) $f(x)$ has local maxima at $x=-1$ and $x=1$ (B) $f(x)$ has local minima at $x=0$ (C) $f(x)$ is strictly decreasing on $x \in R(\mathrm{D})$ Range of $f(x)$ is $\left[0 . \frac{1}{e}\right]$,
A. $\max f(x)=e^{-1}$
B. $\max f(x)=4 e^{-2}$
C. $\min f(x)=e^{-1}$
D. $\min f(x)>0$

## Answer: B

## - Watch Video Solution

40. The minimum value of $f(x)=|3-x|+|2+x|+|5-x|$ is
A. 0
B. 7
C. 8
D. 10

## Answer: B

41. Let $f(x)=2+2 x-3 x^{2 / 3}$ on $[-1,10 / 3]$. Then f has
A. Absolute maximum at an end point
B. Absolute minimum at an interior point
C. Absolute minimum is $f(10 / 3)$
D. Absolute minimum is $f(-1)$

## Answer: D

## - Watch Video Solution

42. If $f$ and $g$ are defined on $[0, \infty)$ by $f(x)=\lim _{n \rightarrow \infty} \frac{x^{n}-1}{x^{n}+1}$ and $g(x)=\int_{0}^{x} f(t) d t$. Then
A. $g$ has local maximum at $x=1$
B. $g$ has local minimum at $x=1$
C. g is an increasing function on $(0, \infty)$
D. g is a decreasing function on $(0, \infty)$

## Answer: B

## - View Text Solution

43. Let the function $f(x)=\sin x+\cos x$, be defined in $[0,2 \pi]$, then $f(x)$
A. $x=17 \pi / 4$ is a point of minima
B. $x=13 \pi / 4$ is a point of maxima
C. $x=21 \pi / 4$ is a point of minima
D. $x=29 \pi / 4$ is a point of maxima

## Answer: C

## - Watch Video Solution

44. If $f(x)=x e^{x(1-x)}$, then $\mathrm{f}(\mathrm{x})$ is
A. increasing on $[-1 / 2,1]$
B. decreases on $R$
C. increasing on $R$
D. decreasing on $[-1 / 2,1]$

## Answer: A

## - Watch Video Solution

45. The tangent to the curve $y=e^{x}$ drawn at the point $\left(c, e^{c}\right)$ intersects the line joining $\left(c-1, e^{c-1}\right)$ and $\left(c+1, e^{c+1}\right)$ (a) on the left of $n=c(\mathrm{~b})$ on the right of $n=c(\mathrm{c})$ at no points (d) at all points
A. on the left of $x=c$
B. on the right of $x=c$
C. at no point
D. at all points

Answer: A

## - Watch Video Solution

Exercise Level 2 Single Correct Answer Type Questions

1. Find the critical points(s) and stationary points (s) of the function

$$
f(x)=(x-2)^{2 / 3}(2 x+1)
$$

A. -1 and 2
B. 1
C. 1 and $-1 / 2$
D. 1 and $1 / 2$

## Answer: B

2. The function $f(x)=\frac{x^{3}}{4}-\sin \pi x+3$ on $[-2,2]$ takes the value
A. 1
B. $16 / 3$
C. 6
D. 8

## Answer: A

## - Watch Video Solution

3. The greatest value of the function $f(x)=\tan ^{-1} x-\frac{1}{2} \log x$ in $\left[\frac{1}{\sqrt{3}}, \sqrt{3}\right]$ is
A. $\pi / 2+(1 / 2) \log 3$
B. $\pi / 6+(1 / 4) \log 3$
C. $\pi / 6+(1 / 2) \log 3$
D. $\pi / 4-(1 / 4) \log 3$

## Answer: B

## - Watch Video Solution

4. Equations of those tangents to $4 x^{2}-9 y^{2}=36$ which are prependicular to the straight line $2 y+5 x=10$, are
A. $5(y-3)=(x-\sqrt{117 / 4})$
B. $5(y-2)=2(x-\sqrt{18})$
C. $5(y+2)=2(x-\sqrt{18})$
D. none of these

## Answer: D

## - Watch Video Solution

5. if $\mathrm{a}, \mathrm{b}, \mathrm{c}$ are real then find the intervial in which $f(x)=\left|\begin{array}{lll}x+a^{2} & a b & a c \\ a b & x+b^{2} & b c \\ a c & b c & x+c^{2}\end{array}\right|$ is decreasing.
A. $\left(-(2 / 3)\left(a^{2}+b^{2}+c^{2}\right), 0\right)$
B. $\left(0,(2 / 3)\left(a^{2}+b^{2}+c^{2}\right)\right)$
C. $\left((1 / 3)\left(a^{2}+b^{2}+c^{2}\right), 0\right)$
D. none of these

## Answer: A

## - Watch Video Solution

6. A channel 27 m wide falls at a right angle into another channel 64 m wide. The greatest length of the log that can be floated along this system of channels is
A. 120
B. 125
C. 100
D. 110

## Answer: B

## - View Text Solution

7. For $a \in[\pi, 2 \pi]$ and $n \in Z$ the critical points of $g$
$f(x)=\frac{1}{3} \sin a \tan ^{3} x+(\sin a-1) \tan x+\frac{\sqrt{a-2}}{8-a}$ are
A. $x=n \pi(n \in I)$ as critical points
B. no critical points
C. $x=2 n \pi(n \in I)$ as critical points
D. $x=(2 n+1) \pi(n \in I)$ as critical points

## Answer: B

8. The value of $a$ for which the function $f(x)=(4 a-3)(x+\log 5)+2(a-7) \frac{\cot x}{2} \frac{\sin ^{2} x}{2}$ does not possess critical points is $\left(-\infty,-\frac{4}{3}\right)$ (b) $(-\infty,-1)[1, \infty)$ (d) $(2, \infty)$
A. $(-\infty,-4 / 3]$
B. $(-\infty,-1)$
C. $[1, \infty)$
D. $(0,00)$

## Answer: A

## - Watch Video Solution

9. The interval to which a may belong so that the function $f(x)=\left(1-\frac{\sqrt{21-4 a-a^{2}}}{a+1}\right) x^{3}+5 x+100 \quad$ is increasing for $x \in R$
A. $[-7,0]$
B. $[-6,0]$
C. $[1,4]$
D. $[2,3]$

## Answer: D

## - Watch Video Solution

10. The muinimum area of the triangle formed by the tangent to $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}=1$ and the coordinate axes is
A. $a b$
B. $\frac{a^{2}+b^{2}}{2}$
C. $(a+b)^{2} / 4$
D. $2 a b$

## Answer: A

- Watch Video Solution

11. The set of all x for which $\log (1+x) \leq x$ is equal to ...... .
A. $(1, \infty)$
B. $(0, \infty)$
C. $(-1, \infty)$
D. none of these

## Answer: C

12. The minimum value of $2^{x^{2}-3 \wedge}(3+27)$ is $2^{27}$ (b) 2 (c) 1 (d) none of these
A. $2^{27}$
B. 2
C. 1
D. none of these

## Answer: D

## - Watch Video Solution

13. If $f(x)= \begin{cases}|x|, & \text { for } \\ 1, & f \text { or }\end{cases}$ $0<|x| \leq 2$
$x=0$ . Then, at $\mathrm{x}=0, f$ has
A. a local maximum
B. no local maximum
C. a local minimum
D. no extremum

## Answer: A

## (D) Watch Video Solution

14. If $f(x)=x e^{x(1-x)}$, then $\mathrm{f}(\mathrm{x})$ is
A. increasing on $[-1 / 2,1]$
B. decreases on $R$
C. increasing on $R$
D. decreasing on $[-1 / 2,1]$

Answer: A

## - Watch Video Solution

15. If $f(x)=\left\{\begin{array}{ll}x^{\alpha} \log x & x>0 \\ 0 & x=0\end{array}\right.$ and Rolle's theorem is applicable to $f(x)$ for $x \in[0,1]$ then $\alpha$ may equal to (A) -2 (B) -1 (C) 0 (D) $\frac{1}{2}$
A. -1
B. $-1 / 2$
C. 0
D. $1 / 2$

## Answer: D

## - Watch Video Solution

16. A cone is made from a circular sheet of radius $\sqrt{3}$ by cutting out a sector and giving the cut edges of the remaining piece together. The maximum volume attainable for the cone is (A) $\frac{\pi}{3}$ (B) $\frac{\pi}{6}$ (C) $\frac{2 \pi}{3}$
$3 \sqrt{3} \pi$

$$
\text { A. } \pi / 3
$$

B. $\pi / 6$
C. $2 \pi / 3$
D. $3 \sqrt{3} \pi$

## Answer: C

## - Watch Video Solution

17. The dimension of the rectangle of maximum area that can be inscribed in the ellipse $(x / 4)^{2}+(y / 3)^{2}=1$ are
A. $\sqrt{8}, \sqrt{2}$
B. 4,3
C. $2 \sqrt{8}, 3 \sqrt{2}$
D. none of these

## Answer: C

18. Consider $f(x)=a x^{4}+c x^{2}+d x+e$ has no point o inflection Then which of the following is/are possible?
A. $b^{2}-4 a c>0$
B. $3 b^{2}-8 a c=0$
C. $3 b^{2}-8 a c>0$
D. $3 b^{2}-8 a c<0$

## Answer: C

## - Watch Video Solution

19. The smallest value of $M$ such that $\left|x^{2}-3 x+2\right| \leq M$ for all $x$ in $\left[1, \frac{5}{2}\right]$
A. $3 / 4$
B. $3 / 8$
C. $3 / 16$
D. $7 / 4$

## Answer: C

## - Watch Video Solution

20. The point in the interval $[0, \pi]$ for which the curve $\mathrm{y}=(1 / 2) x$ and $y=\sin x$ are farthest apart is
A. $\pi / 2$
B. $\pi / 4$
C. $\pi / 6$
D. $\pi$

## Answer: D

21. The points at which the tangents to the curve $a x^{2}+2 h x y+b y^{2}=1$ is parallel to $y$-axis is
A. $(0,0)$
B. where $h x+b y=0$ meets it
C. where $a x+h y$ meets it
D. none of these

## Answer: B

## - Watch Video Solution

22. If the point on $y=x \tan \alpha-\frac{a x^{2}}{32 \cos ^{2} \alpha},(\alpha>0)$ where the tangent is parallel to $y=x$ has an ordinate $\frac{4}{a}$ then $4 \sin ^{2} \alpha$ equals to:
A. $\pi / 2$
B. $\pi / 6$
C. $\pi / 3$
D. none of these

## Answer: D

## - Watch Video Solution

23. Let $f(x)=\left\{\begin{array}{ll}|x-1|+a, & x \leq 1 \\ 2 x+3, & x>1\end{array}\right.$. If $\mathrm{f}(\mathrm{x})$ has local minimum at $\mathrm{x}=1$ and $a \geq 5$ then the value of $a$ is
A. 5
B. 6
C. $11 / 2$
D. $15 / 2$

## Answer: A

24. Let $g(x)=\int_{0}^{x} f(t) d t$ and $f(x)$ satisfies the equation $f(x+y)=f(x)+f(y)+2 x y-1$ for all $x, y \in R$ and $\mathrm{f}^{\prime}(0)=2$ then
A. g increases on $(0, \infty)$ and decreases on $(-\infty, 0)$
B. g increases on $(0, \infty)$
C. g decreases on $(0, \infty)$ and increases $(-\infty, 0)$
D. g decreases on $(-\infty, \infty)$

## Answer: B

## - View Text Solution

25. The area of the triangle formed by the positive $x$-axis with the normal and the tangent to the circle $x^{2}+y^{2}=4$ at $(1, \sqrt{3})$ is

$$
\text { A. } 2 \sqrt{3}
$$

B. $\sqrt{3}$
C. $4 \sqrt{3}$
D. `3

## Answer: A

## - Watch Video Solution

26. The interval in which the function $y=f(x)=\frac{x-1}{x^{2}-3 x+3}$ transforms the real line is
A. $[1 / 3,2]$
B. $[-1 / 3,2]$
C. $[-1 / 3,1]$
D. none of these

## Answer: D

27. Angle at which the circle $x^{2}+y^{2}=16$ can be seen from $(8,0)$ is
A. $\pi / 6$
B. $\pi / 4$
C. $\pi / 2$
D. $\pi / 3$

## Answer: D

## (D) Watch Video Solution

28. The critical points of the function $f(x)=(x+2)^{2 / 3}(2 x-1)$ are
A. -1 and 2
B. 1
C. 1 and $-1 / 2$
D. -1 and -2

## Answer: D

## D Watch Video Solution

29. The function $f(x)=\frac{\log (\pi+x)}{\log (e+x)} \mathrm{s}$ is
A. increasing on $[0, \infty)$
B. decreasing on $[0, \infty)$
C. increasing on $[0, \pi / e)$ and decreasing on $[\pi / e, \infty)$
D. decreasing on $[0, \pi / e)$ and increasing on $[\pi / e, \infty)$

## Answer: B

## - Watch Video Solution

A. 12
B. 10
C. 8
D. 14

## Answer: C

## - Watch Video Solution

31. The greatest vaue of the function
$f(x)=\cot ^{-1} x+(1 / 2) \log x$ on $[1, \sqrt{3}]$ is
A. $(\pi / 6)+0.25 \log 3$
B. $(\pi / 3)-0.25 \log 3$
C. $\pi / 4$
D. $\tan ^{-1} e-1 / 2$

## - Watch Video Solution

32. A particle is moving along the parabola $y^{2}=4(x+2)$. As it passes through the point $(7,6)$ its $y$-coordinate is increasing at the rate of 3 units per second. The rate at which $x$-coordinate change at this instant is (in units/sec)
A. 4
B. 6
C. 8
D. 9

## Answer: D

33. The perimeter of a rectangle is fixed at 24 cm . If the length I of the rectangle is increasing at the rate of 1 cm per second, the value of $I$ for which the area of rectangle start to decrease is
A. 2 cm
B. 6 cm
C. 4 cm
D. 8 cm

## Answer: B

## - Watch Video Solution

34. The rate at which fluid level inside vertical cylindrical tank of radius $r$ drop if we pump fluid out at the rate of $3 \mathrm{~cm}^{3} / \mathrm{min}$ is
A. $-\frac{1}{\pi r^{2}}$
B. $\frac{3}{\pi r^{2}}$
C. $\frac{2}{\pi r^{2}}$
D. $\frac{4}{\pi r}$

## Answer: B

## - Watch Video Solution

35. The length $x$ of a rectangle is increasing at the rate of $3 \mathrm{~cm} / \mathrm{sec}$. and the width y is increasing at the rate of $2 \mathrm{~cm} / \mathrm{sec}$. If $\mathrm{x}=10 \mathrm{~cm}$ and $\mathrm{y}=6 \mathrm{~cm}$, then the rate of change of its area is
A. 14
B. 12
C. 8
D. 4

## Answer: A

36. if $f(x)$ be a twice differentiable function such that $f(x)=x^{2}$ for $x=1,2,3$, then
A. $f^{\prime \prime}(x)=2 \forall x \in(1,3)$
B. $\mathrm{f}^{\prime \prime}(\mathrm{x})=2$ for some $x \in(1,3)$
C. $f^{\prime \prime}(x)=3 \forall x \in(2,3)$
D. $f^{\prime \prime}(x)=f^{\prime}(x)$ for some x in $(2,3)$

## Answer: B

## - Watch Video Solution

37. A tangent drawn to the curve $y=f(x)$ at $P(x, y)$ cuts the x -axis and y -axis at $A$ and $B$ respectively such that $B P: A P=2: 1$. Given that $f(1)=1$. Answer the question: Equation of curve is (A) $y=\frac{1}{x}$ (B) $y=\frac{1}{x^{2}}$ (C) $y=\frac{1}{x^{3}}$ (D) none of these
A. equation of the curve is $x \frac{d y}{d x}-3 y=0$
B. normal at $(1,1)$ is $x+3 y=4$
C. curve passes through $(2,1 / 8)$
D. equation of the curve is $x \frac{d y}{d x}+4 y=0$

## Answer: C

## - Watch Video Solution

38. If $f(x)=x^{3}+b x^{2}+c x+d$ and $0<b^{2}<c$.then in $(-\infty, \infty)$
A. has no local minima
B. has no local maxima
C. is strictly increasing on $R$
D. is strictly decreasing on $R$

## Answer: C

## Exercise Numerical Answer Type Questions

1. If the tangent at $(16,64)$ on the curve $y^{2}=x^{3}$ meets the curve again at $Q(u, v)$ then $u v$ is equal to $\qquad$

## - Watch Video Solution

2. If $f(x)=\left\{\begin{array}{ll}3, & x=0 \\ -x^{2}+3 x+k, & 0<x<1 \\ a x+b, & 1 \leq x \leq 2\end{array}\right.$ satisfies the hypothesis of
the Lagrange's theorem then $(a+b) / k$ is equal to

## - View Text Solution

3. If the slope of a line that passes through the origin which is tangent to $y=x^{3}+x+54$ is m , then m is equal to
4. If $A$ is the area of triangle formed by positive $x$-axis and the normal and the tangents to $x^{2}+y^{2}=9$ at $(1, \sqrt{8})$ then A is equal to $(\sqrt{2}=1.41)$

## (D) Watch Video Solution

5. Let $f(x)=\left[\begin{array}{ll}x^{3 / 5,} & \text { if } x \leq 1 \\ -(x-2)^{3} & \text { if } x>1\end{array}\right.$, then the number of critical points on the graph of the function are. $\qquad$

## - Watch Video Solution

6. The minimum value of $\sqrt{e^{x^{2}}-1}$ is
7. Let $f(x)=\left\{\begin{array}{ll}|x-1|+a, & x \leq 1 \\ 2 x+3, & x>1\end{array}\right.$. If $\mathrm{f}(\mathrm{x})$ has local minimum $\mathrm{x}=1$ and $a \geq 5$ then a is equal to

## - Watch Video Solution

8. Let P be a variable point on the elipse $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}=1$ with foci $F_{1}$ and $F_{2}$. If A is the area of the triangle $P F_{1} F_{2}$, then maximum value of $A$ is

## - Watch Video Solution

9. The maximum value of $|\mathrm{x} \log \mathrm{x}|$ for $0<x \leq 1$ is (e=2.71)

## - View Text Solution

10. If $\mathrm{f}(\mathrm{x})=\log _{x} 1 / 9-\log _{3} x^{2}(x>1)$ then $|\max \mathrm{f}(\mathrm{x})|$ is equal to
11. Let $f(x)=\cos ^{2} x+\cos x+3$ then greatest value of $\mathrm{f}(\mathrm{x})+$ least value of $f(x)$ is equal to

## - Watch Video Solution

12. The greatest value of the function $y=\sin ^{2} x-20 \cos x+1$ is

## - Watch Video Solution

13. If $f(x)=a \log |x|+b x^{2}+x$ has its extremum values at $x=-1 a n d x=2, \quad$ then $\quad a=2, b=-1 \quad a=2, b=-1 / 2$ $a=-2, b=1 / 2(\mathrm{~d})$ none of these

## - Watch Video Solution

14. If $\mathrm{V}(\mathrm{x})$ is larger of $e^{x}-1$ and $(1+x) \log (1+x)$ for $x \in(0, \infty)$ then $\log (V(8)+1)$ is equal to

## - View Text Solution

15. A cylindrical vessel of volume $25 \frac{1}{7}$ cu metres, open at the top is to be manufactured from a sheet of metal. If $r$ and $h$ are the radius and height of the vessel so that amount of metal I sused in the least possible then rh is equal to

## - View Text Solution

16. The altitude of a cylinder of the greatest possible volume which can be inscribed in a sphere of radius $3 \sqrt{3}$ is
17. A straight line I with negative slope passes through $(8,2)$ and cuts the coordinate axes at P and Q . Find absolute minimum value of "OP+OQ where $O$ is the origin-

## - Watch Video Solution

## Question For Previous Year S Aieee Jee Main Paper

1. If $2 a+3 b+6 c+0(a, b, c \in R)$ then the quadractic equation $a x^{2}+b x+c=0$ has
A. at least one root in $[0,1]$
B. at least one root in $[2,3]$
C. at least one root
D. none of these
2. The maximum distance from origin of a point on the curve $x=a \sin t-b \sin \left(\frac{a t}{b}\right), y=a \cos t-b \cos \left(\frac{a t}{b}\right)$, borth $\mathrm{a}, \mathrm{b}>0$ is
A. $a-b$
B. $a+b$
C. $\sqrt{a^{2}+b^{2}}$
D. $\sqrt{a^{2}-b^{2}}$

## Answer: B

## - Watch Video Solution

3. If the function $f(x)=2 x^{3}-9 a x^{2}+12 x^{2} x+1$, wherea $>0$, attains its maximum and minimum at pandq, respectively, such that $p^{2}=q$, then $a$ equal to 1 (b) 2 (c) $\frac{1}{2}$ (d) 3
A. 1
B. 2
C. $1 / 2$
D. 3

## Answer: B

## (D) Watch Video Solution

4. If $u=\sqrt{a^{2} \cos ^{2} \theta+b^{2} \sin ^{2} \theta}+\sqrt{a^{2} \sin ^{2} \theta+b^{2} \cos ^{2} \theta}$, then the difference between maximum and minimum values of $u^{2}$ is
A. $(a+b)^{2}$
B. $2 \sqrt{a^{2}+b^{2}}$
C. $2\left(a^{2}+b^{2}\right)$
D. $(a-b)^{2}$

## - Watch Video Solution

5. A function $y=f(x)$ has a second order derivative $f(x)=6(x-1)$.

If its graph passes through the point $(2,1)$ and at that point the tangent to the graph is $y=3 x-5$ then the function is
A. $(x+1)^{3}$
B. $(x-1)^{3}$
C. $(x-1)^{2}$
D. $(x+1)^{2}$

## Answer: B

## - Watch Video Solution

6. The normal to the curve $x=a(1+\cos \theta), y=a \sin \theta$ at ' $\theta$ ' always passes through the fixed point
A. $(0,0)$
B. $(0, a)$
C. $(a, 0)$
D. $(a, a)$

## Answer: C

## - Watch Video Solution

7. A function is matched below against an interval, where it is supposed to be increasing. Which of the following pairs is incorrectly matched?

$$
\begin{array}{ll}
\text { A. } \begin{array}{ll}
\text { Interval } & \text { Function } \\
(-\infty, 1 / 3) & \left(3 x^{2}-2 x+1\right) \\
\text { B. Interval } & \text { Function } \\
(-\infty,-4) & \left(x^{3}-6 x^{2}+6\right)
\end{array}
\end{array}
$$

Interval Function
C. $(-\infty, \infty)\left(x^{3}-3 x^{2}\right)$

Interval Function
D. $(2, \infty) \quad\left(2 x^{3}-3 x^{2}-12 x+6\right)$

## Answer: A

## - Watch Video Solution

8. 

The
normal
to the
curve
$x=a(\cos \theta+\theta \sin \theta), y=a(\sin \theta-\theta \cos \theta)$ at any $\theta$ is such that
A. it passes through $\left(\frac{a \pi}{2},-a\right)$
B. it is at constant distance from origin
C. it passes through origin
D. it makes angle $\frac{\pi}{2}+\theta$ which the x -axis

## Answer: B

9. Let $f$ be differentiable for all $x$, If $f(1)=-2 a n d f^{\prime}(x) \geq 2$ for all $x \in[1,6]$, then find the range of values of $f(6)$.
A. $f(6)<5$
B. $f(6)=5$
C. $f(6) \geq 8$
D. $f(6)<8$

## Answer: C

## - Watch Video Solution

10. A spherical iron ball 10 cm in radius is coated with a layer of ice of uniform thickness that melts at a rate of $50 \mathrm{~cm}^{3} / \mathrm{min}$. When the thickness of ice is 5 cm , then the rate at which the thickness of ice decreases, is:
A. $1 / 54 \pi \mathrm{~cm} / \mathrm{min}$
B. $5 / 6 \pi \mathrm{~cm} / \mathrm{min}$
C. $1 / 36 \pi \mathrm{~cm} / \mathrm{min}$
D. $1 / 8 \pi \mathrm{~cm} / \mathrm{min}$

## Answer: D

## - Watch Video Solution

11. The function $f(x)=\frac{x}{2}+\frac{2}{x}$ has a local minimum at $x=2$ (b) $x=-2 x=0$ (d) $x=1$
A. $x=1$
B. $x=2$
C. $x=-2$
D. $x=0$

## Answer: B

12. A value of $c$ for which the conclusion of Mean value theorem holds for the function $f(x)=\log _{e} x$ on the interval $[1,3]$ is
A. $2 \log _{3} e$
B. $(1 / 2) \log 3$
C. $\log _{3} c$
D. $\log 3$

## Answer: A

## - Watch Video Solution

13. The function $f(x)=\tan ^{-1}(\sin x+\cos x)$ is an increasing function in (1) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ (2) $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$ (3) $\left(0, \frac{\pi}{2}\right)$ (4) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
A. $(\pi / 4, \pi / 2)$
B. $(-\pi / 2, \pi / 4)$
C. $(0, \pi / 2)$
D. $(-\pi / 2, \pi / 2)$

## Answer: B

## (D) Watch Video Solution

14. Suppose the cubic $x^{3}-p x+q$ has three distinct real roots, where $p>0$ and $q>0$. Then which one of the following holds?
A. The cubic has minima at $\sqrt{\frac{p}{3}}$ and maxima at $-\sqrt{\frac{p}{3}}$
B. The cubic has minima at $-\sqrt{\frac{p}{3}}$ and maxima at $\sqrt{\frac{p}{3}}$
C. The cubic has minima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$
D. The cubic has maxima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$
15. Given $\mathrm{P}(\mathrm{x})=x^{4}+a x^{3}+b x^{2}+c x+d$ such that $\mathrm{x}=0$ is the only real root of $\mathrm{P}^{\prime}(\mathrm{x})=0$. If $\mathrm{P}(-1)$ It $\mathrm{P}(1)$, then $\in$ the $\int$ erval $[-1,1]$
A. $P(-1)$ is the minimum but $P(1)$ is not the maximum of $P$
B. neither $P(-1)$ is the minimum nor $P(1)$ is the maximum of $P$
C. $P(-1)$ is the minimum and $P(1)$ is the maximum of $P$
D. $P(-1)$ is not minimum but $P(1)$ is the maximum of $P$

## Answer: D

## - Watch Video Solution

16. 

Let
$f: R \vec{R}$
be defined
by
$f(x)=\{k-2 x$, if $x \leq-12 x+3, f x \succ 1\}$. If f has a local minimum at $x=1$, then a possible value of k is (1) 0 (2) $-\frac{1}{2}$ (3) -1 (4) 1
A. $-1 / 2$
B. -1
C. 1
D. 0

## Answer: B

## - Watch Video Solution

17. The curve that passes through the point $(2,3)$ and has the property that the segment of any tangent to it lying between the coordinate axes is bisected by the point of contact, is given by
A. $2 y-3 x=0$
B. $y=6 / x$
C. $x^{2}+y^{2}+13$
D. $\left(\frac{x}{2}\right)^{2}+\left(\frac{y}{3}\right)^{2}=2$

## Answer: B

## - Watch Video Solution

18. Let be a function defined by $f(x)= \begin{cases}\frac{\tan x}{x}, & x \neq 0 \\ 1, & x=0\end{cases}$

Statement-1: $x=0$ is a point on minima of $f$
Statement-2: $f^{\prime}(0)=0$

## - Watch Video Solution

19. Let $a, b$ be such that the function $f$ given by $f(x)=\ln |x|+b x^{2}+a x, x \neq 0$ has extreme values at $x=1$ and $x=2$. Statement 1: f has local maximum at $x=1$ and at $x=2$. Statement 2: $a=\frac{1}{2}$ and $b=\frac{-1}{4}$ (1) Statement 1 is false, statement 2 is true (2) Statement 1 is true, statement 2 is true; statement 2 is a correct explanation for statement 1 (3) Statement 1 is true, statement 2
is true; statement 2 is not a correct explanation for statement 1 (4) Statement 1 is true, statement 2 is false

## - Watch Video Solution

20. A spherical balloon is filled with 4500p cubic meters of helium gas.

If a leak in the balloon causes the gas to escape at the rate of $72 \pi$ cubic meters per minute, then the rate (in meters per minute) at which the radius of the balloon decreases 49 minutes after the leakage began is (1) $\frac{9}{7}$ (2) $\frac{7}{9}$ (3) $\frac{2}{9}$ (4) $\frac{9}{2}$
A. $7 / 9$
B. $2 / 9$
C. $9 / 2$
D. $9 / 7$

## Answer: B

21. The cost of running a bus from A to B , is Rs. $\left(a v+\frac{b}{v}\right)$, where $v$ $\mathrm{km} / \mathrm{h}$ is the average speed of the bus. When the bus travels at $30 \mathrm{~km} / \mathrm{h}$, the cost comes out to be Rs. 75 while at $40 \mathrm{~km} / \mathrm{h}$, it is Rs. 65.Then the most economical speed (in $\mathrm{km} / \mathrm{h}$ ) of the bus is :
A. 45
B. 50
C. 60
D. 40

## Answer: C

## - Watch Video Solution

22. If the surface area of a sphere of radius $r$ is increasing uniformly at the rate $8 \frac{(c m)^{2}}{s}$, then the rate of change of its volume is:
A. constant
B. proportional to $\sqrt{r}$
C. proportional to $r^{2}$
D. proportional to r

## Answer: D

## - Watch Video Solution

23. The real number k for which the equation, $2 x^{3}+3 x+k=0$ has two distinct real roots in $[0,1]$ (1) lies between 2 and 3 (2) lies between -1 and 0 (3) does not exist (4) lies between 1 and 2
A. lies between 2 and 3
B. lies between 1 and 0
C. does not exist
D. lies between 1 and 2

## Answer: C

## - Watch Video Solution

24. The maximum area of a right angled triangle with hypotenuse $h$ is
A. $\frac{h^{2}}{2 \sqrt{2}}$
B. $\frac{h^{2}}{2}$
C. $\frac{h^{2}}{\sqrt{2}}$
D. $\frac{h^{2}}{4}$

## Answer: C

## - Watch Video Solution

25. Statement-1: The equation $\mathrm{x} \log \mathrm{x}=2-x$ is satisfied by at least one value of x lying between 1 and 2

Statement-2: The function $f(x)=x \log x$ is an increasing function in $[1,2]$ and $g(x)=2-x$ is a decreasing function in $[1,2]$ and the graphs represented by these functions intersect at a point in [1,2]

## - View Text Solution

26. Statement 1 : The function $x^{2}\left(e^{x}+e^{-x}\right)$ is increasing for all $x>0$ Statement 2: The functions $x^{2} e^{x}$ and $x^{2} e^{-x}$ are increasing for all $x>0$ and the sum of two infunctions in any interval $(\mathrm{a}, \mathrm{b})$ is an increasing function in $(a, b)$.

## - Watch Video Solution

27. If $f$ and $g$ are differentiable functions in $[0,1]$ satisfying $f(0)=2=g(1), g(0)=0$ and $f(1)=6$, then for some $c \in] 0,1[(1)$
$2 f^{\prime}(c)=g^{\prime}(c)$
(2) $2 f^{\prime}(c)=3 g^{\prime}(c)$
(3) $f^{\prime}(c)=g^{\prime}(c)$
$f^{\prime}(c)=2 g^{\prime}(c)$
A. $2 f^{\prime}(c)=g^{\prime}(c)$
B. $2 f^{\prime}(c)=3 g^{\prime}(c)$
C. $f^{\prime}(c)=g^{\prime}(c)$
D. $f^{\prime}(c)=2 g^{\prime}(c)$

## Answer: D

## - Watch Video Solution

28. If $\mathrm{x}=-1$ and $\mathrm{x}=2$ are extreme points of $\mathrm{f}(\mathrm{x})=\alpha \log |x|+\beta x^{2}+x$, then
A. $\alpha=-6, \beta=\frac{1}{2}$
B. $\alpha-6, \beta=-\frac{1}{2}$
C. $\alpha=2, \beta=-\frac{1}{2}$
D. $\alpha=2, \beta=\frac{1}{2}$

## Answer: C

## - Watch Video Solution

29. If the volume of a sphere increase at the rate of , $2 \pi \mathrm{~cm}^{3} / \mathrm{sec}$, then the rate of increase of its radius (in $\mathrm{cm} / \mathrm{sec}$ ), when the volume is $288 \pi \mathrm{~cm}^{3}$ is :
A. $\frac{1}{9}$
B. $\frac{1}{6}$
C. $\frac{1}{36}$
D. $\frac{1}{24}$

## Answer: C

- Watch Video Solution

30. If non-zero real number b and c are such that $\min f(x)>\max \mathrm{g}(\mathrm{x})$
where

$$
=x^{2}+2 b x+2 c^{2} \text { and } g(x)=-x^{2}-2 c x+b^{2}(x \in R) \text { then }\left|\frac{c}{b}\right|
$$

lies in the interval
A. $\left[\frac{1}{\sqrt{2}}, \sqrt{2}\right]$
B. $\left[0, \frac{1}{2}\right]$
C. $\left[\frac{1}{2}, \frac{1}{\sqrt{2}}\right]$
D. $[\sqrt{2}, \infty]$

## Answer: D

## - Watch Video Solution

31. Let $f^{\prime}(x)>0$ and $g^{\prime}(x)<0$ for all $x \in R$ Then
A. $g(f(x))>g(f(x-1))$
B. $f(g(x))>f(g(x+1))$
C. $f(g(x))>f(g(x-1))$
D. $g(f(x))<g(f(x+1))$

## Answer: B

## - Watch Video Solution

32. If Rolle's theorem holds for the function $f(x)=2 x^{3}+a x^{2}+b x$ in the interval $[-1,1]$ for the point $c=\frac{1}{2}$, then the value of $2 \mathrm{a}+\mathrm{b}$ is
A. 1
B. -1
C. 2
D. -2

## Answer: B

33. Let $f(x)$ be a polynomial of degree four having extreme values at $x=1$ and $x=2$. If $(\lim )_{x \rightarrow 0}\left[1+\frac{f(x)}{x^{2}}\right]=3$, then $\mathrm{f}(2)$ is equal to :
(1) $-8(2)-4(3) 0(4) 4$
A. -8
B. -4
C. 0
D. 4

## Answer: C

## D Watch Video Solution

34. The equation of a normal to the curve, $\sin y=x\left(\frac{\sin \pi}{3}+y\right)$ at $x=0$ is
A. $2 x+\sqrt{3} y=0$
B. $2 y-\sqrt{3} x=0$
C. $2 y+\sqrt{3}=0$
D. $2 x-\sqrt{3} y=0$

## Answer: A

## - Watch Video Solution

35. Let $k$ and $K$ be the minimum and the maximum values of the function $f(x)=\frac{(1+x)^{0.6}}{1+x^{0.6}}$, and $x \in[0,1]$ respectively,then the ordered pair $(k, K)$ is equal to
A. $\left(1,2^{0.6}\right)$
B. $\left(2^{-0.4}, 2^{0.6}\right)$
C. $\left(2^{-0.6}, 1\right)$
D. $\left(2^{-0.4}, 1\right)$

## Answer: D

## - Watch Video Solution

36. From the top of a 64 metres high tower, a stone is thrown upward vertically with the velocity of $48 \mathrm{~m} / \mathrm{s}$. The greatest height (in metres) attained by stone, assuming the value of the gravitational acceleration $g-32 m / s^{2}$, is
A. 100
B. 88
C. 128
D. 112

## Answer: A

37. If $x=2 \cos t+\cos 2 t, y=2 \sin t-\sin 2 t$, then at $t=\frac{\pi}{4}, \frac{d y}{d x}$
A. 4
B. $2 \sqrt{2}$
C. 2
D. $\sqrt{2}$

## Answer: C

## D Watch Video Solution

38. Tangents are drawn to $x^{2}+y^{2}=16$ from the point $P(0, h)$. These tangents meet the $x-a \xi s$ at $A a n d B$. If the area of triangle $P A B$ is minimum, then $h=12 \sqrt{2}$ (b) $h=6 \sqrt{2} h=8 \sqrt{2}$ (d) $h=4 \sqrt{2}$
A. $4 \sqrt{3}$
B. $3 \sqrt{3}$
C. $3 \sqrt{2}$
D. $4 \sqrt{2}$

## Answer: D

## - Watch Video Solution

39. Consider $f(x)=\tan ^{-1}\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right), x \in\left(0, \frac{\pi}{2}\right)$. A normal to $y=f(x)$ at $x=\frac{\pi}{6}$ also passes through the point: (1) (0,0) (2) $\left(0, \frac{2 \pi}{3}\right)$ (3) $\left(\frac{\pi}{6}, 0\right)$ (4) $\left(\frac{\pi}{4}, 0\right)$
A. $(0,0)$
B. $\left(0, \frac{2 \pi}{3}\right)$
C. $\left(\frac{\pi}{6}, 0\right)$
D. $\left(\frac{\pi}{4}, 0\right)$

## Answer: B

40. If $m$ and $M$ are the minimum and the maximum values of $4+\frac{1}{2} \sin ^{2} 2 x-2 \cos ^{4} x, x \in R$ then
A. $\frac{9}{4}$
B. $\frac{15}{4}$
C. $\frac{7}{4}$
D. $\frac{1}{4}$

## Answer: A

## - Watch Video Solution

41. If the tangent at a point $P$ with parameter $t$, on the curve $x=4 t^{2}+3, y=8 t^{3}-1 t \in R$ meets the curve again at a point Q, then the coordinates of $Q$ are

$$
\text { A. }\left(16 t^{2}+3,-64 t^{3}-1\right)
$$

B. $\left(4 t^{2}+3,-8 t^{3}-1\right)$
C. $\left(t^{2}+3, t^{3}-1\right)$
D. $\left(t^{2}+3,-t^{3}-1\right)$

## Answer: D

## - Watch Video Solution

42. Let $f(x)=\sin ^{4} x+\cos ^{4} x$. Then f is increasing function in the interval
A. $] \frac{5 \pi}{8}, \frac{3 \pi}{4}[$
B. $] \frac{\pi}{2}, \frac{5 \pi}{8}[$
C. $] \frac{\pi}{4}, \frac{\pi}{2}[$
D. $] 0, \frac{\pi}{4}[$

## Answer: C

43. Let C be a curve given by $y=1+\sqrt{4 x-3}, x>\frac{3}{4}$. If P is a point on C such that the tangent at P has slope $\frac{2}{3}$, then a point through which the normal at P passes, is
A. $(1,7)$
B. $(3,-4)$
C. $(4,-3)$
D. $(2,3)$

## Answer: A

## - Watch Video Solution

44. The normal to the curve $y(x-2)(x-3)=x+6$ at the point where the curve intersects the $y-a \xi s$, passes through the point :
$\left(\frac{1}{2},-\frac{1}{3}\right)(2)\left(\frac{1}{2}, \frac{1}{3}\right)$ (3) $\left(-\frac{1}{2},-\frac{1}{2}\right)(4)\left(\frac{\frac{1}{2,1}}{2}\right)$
A. $\left(\frac{1}{2}, \frac{1}{3}\right)$
B. $\left(-\frac{1}{2},-\frac{1}{2}\right)$
C. $\left(\frac{1}{2}, \frac{1}{2}\right)$
D. $\left(\frac{1}{2},-\frac{1}{3}\right)$

## Answer: C

## - Watch Video Solution

45. The tangent at the point $(2,-2)$ to the curve, $x^{2} y^{2}-2 x=4(1-y)$ does not pass through the point:
A. $(4,1 / 3)$
B. $(8,5)$
C. $(-4,-9)$
D. $(-2,-7)$

## Answer: D

## - Watch Video Solution

46. A tangent drawn to the curve $y=f(x)$ at $P(x, y)$ cuts the $x$ and $y$ axes at $A$ and $B$, respectively, such that $A P: P B=1: 3$. If $f(1)=1$ then the curve passes through $\left(k, \frac{1}{8}\right)$ where k is
A. $\left(\frac{1}{3}, 24\right)$
B. $\left(\frac{1}{2}, 4\right)$
C. $\left(2, \frac{1}{8}\right)$
D. $\left(3, \frac{1}{28}\right)$

## Answer: C

47. If a point P has co-ordinates $(0,-2)$ and $Q$ is any point on the circle $x^{2}+y^{2}-5 x-y+5=0$, then the maximum value of $(P Q)^{2}$ is
: (a) $\frac{25+\sqrt{6}}{2}$ (b) $14+5 \sqrt{3}$ (c) $\frac{47+10 \sqrt{6}}{2}$ (d) $8+5 \sqrt{3}$
A. $\frac{25+\sqrt{6}}{2}$
B. $14+5 \sqrt{3}$
C. $\frac{47+10 \sqrt{6}}{2}$
D. $8+5 \sqrt{3}$

## Answer: B

## - Watch Video Solution

48. The function $f$ defined by
$f(x)=x^{3}-6 x^{2}-36 x+7$ is increasing, if
A. increasing on $R$
B. decreasing on R
C. decreasing on $(0, \infty)$ and increasing on $(-\infty, 0)$
D. increasing on $(0, \infty)$ and decreasing on $(-\infty, 0)$

## Answer: A

## - Watch Video Solution

49. Twenty metres of wire is available for fencing off a flower-bed in the form of a circular sector. Then the maximum area (in sqm ) of the flower-bed is: 25 (2) 30 (3) 12.5 (4) 10
A. 30
B. 12.5
C. 10
D. 25
50. Let $\mathrm{f}(\mathrm{x})$ be a polynomial of degree four having extreme values at $\mathrm{x}=1$
and $\mathrm{x}=2$. If $\lim _{x \rightarrow 0}\left(1+\frac{f(x)}{x^{2}}\right)=3$, then $\mathrm{f}(2)$ is equal to
A. $5 / 2$
B. $9 / 2$
C. $1 / 2$
D. $3 / 2$

## Answer: B

## - Watch Video Solution

51. If the curves $y^{2}=6 x, 9 x^{2}+b y^{2}=16$ intersect each other at right angles then the value of $b$ is: (1) 6 (2) $\frac{7}{2}$ (3) 4 (4) $\frac{9}{2}$
A. 6
B. $7 / 2$
C. 4
D. $9 / 2$

## Answer: D

## - Watch Video Solution

52. If a right circular cone, having maximum volume is inscribed in a sphere of radius 3 cm , then the curved surface area (in $\mathrm{cm}^{2}$ ) of this cone is
A. $6 \sqrt{3 \pi}$
B. $6 \sqrt{2} \pi$
C. $8 \sqrt{2} \pi$
D. $8 \sqrt{3} \pi$

Answer: D

## - View Text Solution

53. Let $f(x)=x^{2}+\left(\frac{1}{x^{2}}\right)$ and $g(x)=x-\frac{1}{x} \xi n R-\{-1,0,1\}$. If
$h(x)=\left(\frac{f(x)}{g(x)}\right)$ then the local minimum value of $h(x)$ is: (1) 3 (2) -3
(3) $-2 \sqrt{2}$ (4) $2 \sqrt{2}$
A. -3
B. $-2 \sqrt{2}$
C. $2 \sqrt{2}$
D. 3

## Answer: C

54. if $\theta$ denotes the acute angle between the curves, $y=10-x^{2}$ and $y=2+x^{2}$ at a point of their intersection, then $|\tan \theta|$ is equal to
A. $4 / 9$
B. $7 / 17$
C. $8 / 17$
D. $8 / 15$

## Answer: D

## - Watch Video Solution

55. The tangent to the curve $y t=x e^{x^{2}}$ passing through the point $(1, \mathrm{e})$ also passes through the point
A. $(4 / 3,2 e)$
B. $(2,3 \mathrm{e})$
C. $(5 / 3,2 e)$
D. $(3,6 e)$

## Answer: A

## - Watch Video Solution

56. The tangent to the curve $y=x^{2}-5 x+5$. parallel to the line
$2 y=4 x+1$, also passes through the point :
A. $(1 / 4,7 / 2)$
B. $(7 / 2,1 / 4)$
C. $(-1 / 8), 7)$
D. $(1 / 8,-7)$

## Answer: D

57. The equation of a tangent to the parabola, $x^{2}=8 y$, which makes an angle $\theta$ with the positive direction of $x$-axis, is:
A. $x=y \cot \theta+2 \tan \theta$
B. $x=y \cot \theta-2 \tan \theta$
C. $y=x \tan \theta-2 \cot \theta$
D. $y=x \tan \theta+2 \cot \theta$

## Answer: A

## - Watch Video Solution

58. Let $f(x)=-\frac{x}{\sqrt{a^{2}+x^{2}}}-\frac{d-x}{\sqrt{b^{2}+(d-x)^{2}}}, x \in R$, where a, b and d are non-zero real constants. Then,
A. $f$ is decreasing function of $x$
B. $f$ is neither increasing nor decreasing function of $x$
C. $\mathrm{f}^{\prime}$ is not a continuous function of x
D. $f$ is an increasing function of $x$

## Answer: D

## - Watch Video Solution

59. Find the area of the largest rectangle with lower base on the $x$-axis and upper vertices on the curve $y=12-x^{2}$.
A. $20 \sqrt{2}$
B. $18 \sqrt{3}$
C. 32
D. 36

## Answer: C

60. The maximum values of $3 \cos \theta+5 \sin \left(\theta-\frac{\pi}{6}\right)$ for any real value of $\theta$ is:
A. $\sqrt{19}$
B. $\sqrt{\frac{79}{2}}$
C. $\sqrt{31}$
D. $\sqrt{34}$

## Answer: A

## - Watch Video Solution

61. Let $f(x)=x^{3}-3(a-2) x^{2}+3 a x+7$ and $f(x)$ is increasing in $(0,1]$ and decreasing is $[1,5)$, then roots of the equation $\frac{f(x)-14}{(x-1)^{2}}=0$ is (A) 1 (B) 3 (C) 7 (D) -2
A. 6
B. 5
C. 7
D. -7

## Answer: C

## - Watch Video Solution

62. The maximum value of the function
$f(x)=3 x^{3}-18 x^{2}+27 x-40 \quad$ on the set
$S=\left\{x \in R: x^{2}+30 \leq 11 x\right\}$ is:
A. 122
B. -222
C. -122
D. 222

## - Watch Video Solution

63. A helicopter flying along the path $y=7+x^{\frac{3}{2}}$, A soldier standint at point $\left(\frac{1}{2}, 7\right)$ wants to hit the helicopter when it is closest from him, then minimum distance is equal to (a) $\frac{1}{6} \frac{\sqrt{2}}{3}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3} \sqrt{\frac{2}{3}}$ (d) $\sqrt{\frac{5}{2}}$
A. $\frac{1}{2}$
B. $\frac{1}{3} \sqrt{\frac{7}{3}}$
C. $\frac{1}{6} \sqrt{\frac{7}{3}}$
D. $\frac{\sqrt{5}}{6}$

## Answer: C

64. The shortest distance between the point $\left(\frac{3}{2}, 0\right)$ and the curve $y=\sqrt{x},(x>0)$, is
A. $\frac{\sqrt{5}}{2}$
B. $\frac{5}{4}$
C. $\frac{3}{2}$
D. $\frac{\sqrt{3}}{2}$

## Answer: A

## - Watch Video Solution

65. The maximum volume (in cu.m) of the right circular cone having slant height 3 m is
A. $3 \sqrt{3} \pi$
B. $6 \pi$
C. $2 \sqrt{3} \pi$
D. $\frac{4}{3} \pi$

## Answer: C

## - Watch Video Solution

## Question For Previous Year S B Architecture Entrance Examination Papers

1. The slope of the normal to curve $y=x^{3}-4 x^{2}$ at $(2,-1)$ is
A. $\frac{1}{4}$
B. $\frac{1}{2}$
C. 4
D. -4

Answer: A
2. For the curve $x=t^{2}-1, y=t^{2}-t$, the tangent line is perpendicular to $x$-axis, then $t=$ (i) 0 (ii) $\infty$ (iii) $\frac{1}{\sqrt{3}}$ (iv) $-\frac{1}{\sqrt{3}}$
A. $t=0$
B. $t=1$
C. $t=\frac{1}{\sqrt{3}}$
D. $t=\frac{1}{2}$

## Answer: B

## - Watch Video Solution

3. If $f(x)=4^{\sin x}$ satisfies the Rolle's theorem on $[0, \pi]$, then the value of $c \in(0, \pi)$ for which $f^{\prime}(c)=0$ is
A. $c=\frac{\pi}{6}$
B. $c=\frac{\pi}{4}$
C. $c=\frac{\pi}{2}$
D. $c=\frac{\pi}{3}$

## Answer: C

## (D) Watch Video Solution

4. $f(x)=\sqrt{25-x^{2}}$ in $[1,5]$
A. $\sqrt{3}$
B. $\sqrt{5}$
C. $\sqrt{15}$
D. 2

## Answer: C

5. Let $f(x)=\left\{\begin{array}{ll}|x-1|+a & \text { if } x \leq 1 \\ 2 x+3 & \text { if } x>1\end{array}\right.$. If $\mathrm{f}(\mathrm{x})$ has a local minimum at $\mathrm{x}=1$, then
A. $a>5$
B. $0<a \leq 5$
C. $a \leq 5$
D. $a=5$

## Answer: C

## (D) Watch Video Solution

6. If $m$ be the slope of the tangent to the curve $e^{2 y}=1+4 x^{2}$, then
A. $|m| \leq 1$
B. $|m|>1$
C. $|m| \geq 1$
D. $|m|<1$

## Answer: A

## - Watch Video Solution

7. Let $f:(-\infty, \infty) \rightarrow(-\infty, \infty)$ be acontinuous and differentiable function and let $f^{\prime}($.$) denote the derivative of f($.$) .If f(0)=-2$ and $f^{\prime}(x)$ $\leq 3$ for each $x \in[0,2]$, then the largest possible value of $\mathrm{f}(2)$ is
A. 1
B. 2
C. 3
D. 4

## Answer: D

8. Let $f[1,2] \rightarrow(-\infty, \infty)$ be given by $f(x)=\frac{x^{4}+3 x^{2}+1}{x^{2}+1}$ then find value of in $\left[f_{\max }\right]$ in $[-1,2]$ where [.] is greatest integer function :
A. 1
B. $\frac{29}{5}$
C. $\frac{21}{5}$
D. $\frac{28}{5}$

## Answer: B

## (D) Watch Video Solution

9. Let $y=f(x)$ be a curve which passes through (3,1) and is such that normal at any point on it passes through (1,1). Then $y=f(x)$ describes
A. a circle of area $\pi$
B. an ellipse of area $2 \pi$
C. an ellipse of area $3 \pi$
D. a circle of area $4 \pi$

## Answer: D

## - Watch Video Solution

10. Let $f(x)=\left\{\begin{array}{ll}x \sin \frac{\pi}{x}, & 0<x \leq 1 \\ 0 & x=0\end{array}\right.$.Then $\mathrm{f}^{\prime}(\mathrm{x})=0$ for
A. exactly two value of $x$
B. no value of $x$
C. infinitely many values of $x$
D. exactly one value of $x$

## Answer: C

11. Let $f(x)=\left[1-x^{2}\right], x \in R$, where [] is the greatest integer function. Then
A. $f$ is increasing
B. $x=0$ is the point of maxima of $f$
C. $f$ is continuous at $x=0$
D. $f$ is decreasing

## Answer: D

## - View Text Solution

12. A particle is constrained to move along the curve $y=\sqrt{x}$ starting at the origin at time $\mathrm{t}=0$. The point on the curve where the abscissa and the ordinate are changing at the same rate is
A. $\left(\frac{1}{2}, \frac{1}{\sqrt{2}}\right)$
B. $\left(\frac{1}{8}, \frac{1}{2 \sqrt{2}}\right)$
C. $\left(\frac{1}{4}, \frac{1}{2}\right)$
D. $(1,1)$

## Answer: C

## - Watch Video Solution

13. If the tangent and the normal to $x^{2}-y^{2}=4$ at a point cut off intercepts $a_{1}, a_{2}$ on the $x$-axis respectively $\& b_{1}, b_{2}$ on the y -axis respectively. Then the value of $a_{1} a_{2}+b_{1} b_{2}$ is equal to:
A. -1
B. 0
C. 4
D. 1

## Answer: B

## - Watch Video Solution

14. Let $f$ be a differentiable function defined on $R$ such that $f(0)=-3$. If $f^{\prime}(x) \leq 5$ for all $x$ then
A. $f(2)>7$
B. $f(2) \leq 7$
C. $f(2)>8$
D. $f(2)=8$

## Answer: B

## - Watch Video Solution

15. Let f be a function defined on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ by $f(x)=$ $3 \cos ^{4} x-6 \cos ^{3} x-6 \cos ^{2} x-3$. Then the range of $f(x)$ is
A. $[-12,-3]$
B. $[-6,-3]$
C. $[-6,3]$
D. $(-12,3]$

## Answer: A

## - Watch Video Solution

16. The function $\mathrm{f}(\mathrm{x})=x e^{-x}$ has
A. neither a maximum nor a minimum at $x=1$
B. a minimum at $x=1$
C. a maximum at $x=1$
D. a maximum at $x=-1$

## Answer: C

## - Watch Video Solution

17. Each side of a square is increasing at the uniform rate of $1 \mathrm{~m} / \mathrm{sec}$. If after sometime the area of the square is increasing at the rate of $8 m^{2}$ $/ \mathrm{sec}$, then the area of square at that time in sq. meters is
A. 4
B. 9
C. 16
D. 25

## Answer: C

18. Find the rate of change of the volume of a sphere with respect to its surface area when the radius is 2 cm .
A. 4
B. 3
C. 2
D. 1

## Answer: D

## - Watch Video Solution

19. If $m$ is the slope of the tangent to the curve $e^{y}=1+x^{2}$, then
A. $[0,1]$
B. $(1, \infty)$
C. $(-\infty,-1)$
D. $[-1,1]$

## Answer: D

## - Watch Video Solution

20. $\mathrm{f}(\mathrm{x})=|x \log x|, x>0$ is monotonically decreasing in
A. $\left(0, \frac{1}{e}\right)$
B. $\left[\frac{1}{e}, 1\right]$
C. $(1, \mathrm{e})$
D. $(e, \infty)$

## Answer: B

## - View Text Solution

21. Let $f(x)=\left|x-x_{1}\right|+\left|x-x_{2}\right|$, where $x_{1}$ and $x_{2}$ are distinct real numbers. Then the number of points at which $f(x)$ is minimum
A. 1
B. 2
C. 3
D. more than 3

## Answer: B

## - Watch Video Solution

22. The maximum value of $\mathrm{f}(\mathrm{x})=2 \sin x+\sin 2 x$, in the interval $\left[0, \frac{3}{2} \pi\right]$ is
A. $\sqrt{2}+1$
B. $2 \sqrt{3}$
C. $\frac{3 \sqrt{3}}{2}$
D. $\sqrt{3}$

## Answer: C

## - Watch Video Solution

23. The abscissae of a point, tangent at which to the curve $y=e^{x} \sin x, x \in[0, \pi]$ has maximum slope is
A. $\frac{\pi}{4}$
B. $\frac{\pi}{2}$
C. $\pi$
D. 0

## Answer: B

24. Let $\mathrm{p}(\mathrm{x})$ be a real polynomial of degree 4 having extreme values $x=1$ and $x=2$. if $\lim _{x \rightarrow 0} \frac{p(x)}{x^{2}}=1$, then $p(4)$ is equal to
A. 16
B. 32
C. 64
D. 8

## Answer: A

## - Watch Video Solution

25. Water is running into an underground right circular conical reservoir, which is 10 m deep and radius of the base is 5 m . If the rate of change in the volume of water in the reservoir is $\frac{3}{\sqrt{2}} \pi m^{3} / / \mathrm{min}$, then the rate (in $\mathrm{m} / \mathrm{min}$ ) at which water rises in it, when the water level is $4 m$ is
A. $3 / 2$
B. $3 / 8$
C. $1 / 8$
D. $1 / 4$

## Answer: A

## - View Text Solution

26. If $f(x)=\left|x^{2}-16\right|$ for all $x \in R$, then the total number of points of R at which $\mathrm{f}: R \rightarrow R$ attains local extreme values of
A. 1
B. 2
C. 3
D. 4

## Answer: C

## - View Text Solution

27. A real valued function $f(x)=C \log |x|+D x^{3}+x, x \neq 0$ where C and D are constant, has critical points at $x=-1$ and $x=2$. Then the ordered pair (C,D) is
A. $\left(\frac{2}{3},-\frac{1}{9}\right)$
B. $\left(-\frac{1}{9}, \frac{2}{3}\right)$
C. $\left(-\frac{2}{3}, \frac{1}{9}\right)$
D. $\left(-\frac{1}{9}, \frac{2}{3}\right)$

## Answer: A

