MATHS

BOOKS - MCGROW HILL EDUCATION MATHS (HINGLISH)

COMPLEX NUMBERS

Solved Examples

1. If $a+ib=\sum_{k=1}^{101} i^{k}$, then (a, b) equals

A. (0, 1)

B. (0, 0)

C. (0, -1)

D. (1, 1)

Answer: A

Watch Video Solution

2. If $\left(\frac{1+i}{1-i}\right)^n = -1, n \in N$, then least value of n is

A. 1

B. 2

C. 3

D. 4

Answer: B

Watch Video Solution

3. The conjugate of a complex number z is $\frac{2}{1-i}$. Then Re(z) equals

A. -1

B. 0

C. 1

D. 2

Answer: C

Watch Video Solution

4. The number of complex numbers z such that $|z-i|=|z+i|=|z+1|$ is

A. 0

B. 1

C. 2

D. infinite

Answer: B

Watch Video Solution

5. If $z+2|z|=\pi+4i$, then Im (z) equals

A. π

B. 4

C. $\sqrt{\pi^2+16}$

D. None of these

Answer: B

Watch Video Solution

6. If $|z|=z+3-2i$, then z equals

A. $7/6+i$

B. $-7/6+2i$

C. $-5/6+2i$

D. $5/6+i$

Answer: C

Watch Video Solution

7. If $\omega(\neq 1)$ is a cube root of unity and $\left(1+\omega^2\right)^{11}=a+b\omega+c\omega^2$, then (a, b, c) equals

A. (1, 1, 0)

B. (0, 1, 1)

C. (1, 0, 1)

D. (1, 1, 1)

Answer: A

Watch Video Solution

8. If $x^2+y^2=1$ and $x\neq -1$ then $\dfrac{1+y+ix}{1+y-ix}$ equals

A. 1

B. x + iy

C. 2

D. y + ix

Answer: D

Watch Video Solution

9. Write the value of $arg(z) + arg(z)$.

A. 0

B. π

C. 2π

D. None of these

Answer: A

Watch Video Solution

10. If $z \in C$ and $2z = |z| + i$, then z equals

A. $\dfrac{\sqrt{3}}{6} + \dfrac{1}{2}i$

B. $\dfrac{\sqrt{3}}{6} + \dfrac{1}{3}i$

C. $\dfrac{\sqrt{3}}{6} + \dfrac{1}{4}i$

D. $\dfrac{\sqrt{3}}{6} + \dfrac{1}{6}i$

Answer: A

Watch Video Solution

11. If $z = \left(\dfrac{1}{\sqrt{3}} + \dfrac{1}{2}i\right)^7 + \left(\dfrac{1}{\sqrt{3}} - \dfrac{1}{2}i\right)^7$, then

A. Re (z) = 0

B. Im (z) = 0

C. Re (z) $>$ 0, Im (z) $<$ 0

D. Re (z) $<$ 0, Im (z) $>$ 0

Answer: B

Watch Video Solution

12. If $\omega(\neq 1)$ is a complex cube root of unity and $\left(1+\omega^4\right)^n=\left(1+\omega^8\right)^n$, then the least positive integral value of n is

A. 2

B. 3

C. 6

D. 12

Answer: B

Watch Video Solution

13. If $z = \dfrac{1 + \cos\theta + i\sin\theta}{\sin\theta + i(1 + \cos\theta)}(0 < \theta < \pi/2)$ then |z| equals

A. $2|\sin\theta|$

B. $2|\cos\theta|$

C. 1

D. $|\cot(\theta/2)|$

Answer: C

Watch Video Solution

14. All the roots of $(z+1)^4 = z^4$ lie on

A. a straight line parallel to x-axis

B. a straight line parallel to y-axis

C. a circle with centre at -1 + 0i

D. a circle with centre at 1 + i

Answer: B

Watch Video Solution

15. Given $\alpha, \beta,$ respectively, the fifth and the fourth non-real roots of units, then find the value of $(1+\alpha)(1+\beta)\left(1+\alpha^2\right)\left(1+\beta^2\right)\left(1+\alpha^4\right)\left(1+\beta^4\right)$

A. α

B. β

C. $\alpha\beta$

D. 0

Answer: D

Watch Video Solution

16. Suppose z_1, z_2, z_3 are vertices of an equilateral triangle whose circumcentre -3 + 4i, then $|z_1 + z_2 + z_3|$ is equal to

A. 5

B. $10\sqrt{3}$

C. 15

D. $15\sqrt{3}$

Answer: C

Watch Video Solution

17. If $z \neq 0$ lies on the circle $|z - 1| = 1$ and $\omega = 5/z$, then ω lies on

A. a circle

B. an ellipse

C. a straight line

D. a parabola

Answer: C

Watch Video Solution

18. If $\bar{z} = 3i + \frac{25}{z + 3i}$, then |z| cannot exceed

A. 3

B. 8

C. 16

D. 18

Answer: B

View Text Solution

19. If |z - 1| = |z + 1| = |z - 2i|, then value of |z| is

A. 1

B. 2

C. $5/4$

D. $3/4$

Answer: D

Watch Video Solution

20. Find number of all complex numbers z satisfying $\bar{z} = iz^2$

A. 1

B. 2

C. 3

D. 4

Answer: D

Watch Video Solution

Solved Examples Level 1

1. If $z \in C, z \notin R, \text{ and } a = z^2 + 3z + 5$, then a cannot take value

A. $-2/5$

B. $5/2$

C. $\frac{11}{4}$

D. $-\frac{11}{5}$

Answer: C

Watch Video Solution

2. Suppose a, b, c $\in C, \text{ and } |a| = |b| = |c| = 1 \text{ and } abc = a + b + c,$ then |bc + ca + ab| is equal to

A. 0

B. -1

C. 1

D. None of these

Answer: C

Watch Video Solution

3. The number of complex numbers z which satisfy $z^2 + 2|z|^2 = 2$ is

A. 0

B. 2

C. 3

D. 4

Answer: D

Watch Video Solution

4. Suppose $a \in R$ and the equation $z + a|z| + 2i = 0$ has no solution in C, then a satisfies the relation.

A. $|a| > 1$

B. $|a| \geq 1$

C. $|a| > \sqrt{2}$

D. $|a| \geq \sqrt{2}$

Answer: B

Watch Video Solution

5. Suppose A is a complex number and $n \in N$, such that $A^n = (A+1)^n = 1$, then the least value of n is 3 b. 6 c. 9 d. 12

A. 3

B. 6

C. 9

D. 18

Answer: B

Watch Video Solution

6. Let $z \neq i$ be any complex number such that $\frac{z-i}{z+i}$ is a purely imaginary number. Then $z+\frac{1}{z}$ is

A. a non-zero real number other than 1

B. a purely imaginary number

C. a non-zero real number

D. 0

Answer: C

Watch Video Solution

7. The point z_1, z_2, z_3, z_4 in the complex plane are the vertices of a parallogram taken in order, if and only if.

(1) $z_1 + z_4 = z_2 + z_3$ (2) $z_1 + z_3 = z_2 + z_4$

(3) $z_1 + z_2 = z_3 + z_4$ (4) $z_1 + z_3 \neq z_2 + z_4$

A. $z_1 + z_4 = z_2 + z_3$

B. $z_1 + z_3 = z_2 + z_4$

C. $z_1 + z_2 = z_3 + z_4$

D. None of these

Answer: B

Watch Video Solution

8. if the complex no z_1, z_2 and z_3 represents the vertices of an equilateral triangle such that $|z_1| = |z_2| = |z_3|$ then relation among z_1, z_2 and z_3

A. $z_1 + z_2 + z_3 = 0$

B. $z_1 + z_2 - z_3 = 0$

C. $z_1 - z_2 + z_3 = 0$

D. $z_1 + z_2 + z_3 \neq 0$

Answer: A

Watch Video Solution

9. $\sum_{k=1}^{6}\left(\sin, \frac{2\pi k}{7} - i \cos, \frac{2\pi k}{7}\right) = ?$

A. -1

B. 0

C. $-i$

D. i

Answer: D

Watch Video Solution

Watch Video Solution

10. The complex number $\sin(x) + i\cos(2x)$ and $\cos(x) - i\sin(2x)$ are conjugate to each other for

A. $x = n\pi, n \in I$

B. $x = \left(n + \frac{1}{2}\right)\pi, n \in I$

C. x = 0

D. no value of x

Answer: D

Watch Video Solution

11. If z_1 and z_2 are two complex number and a, b, are two real number then $|az_1 - bz_2|^2 + |bz_1 + az_2|^2$ equals

A. $(a^2 + b^2)|z_1 z_2|$

B. $(a^2 + b^2)(z_1^2 + z_2^2)$

C. $(a^2+b^2)|z_1|^2+|z_2|^2$

D. $2ab|z_1z_2|$

Answer: C

Watch Video Solution

12. a and b are real numbers between 0 and 1 such that the points $z_1=a+i, z_2=1+bi, z3=0$ form an equilateral triangle, then a and b are equal to

A. $a=b=2-\sqrt{3}$

B. $a=2-\sqrt{3}, b=\sqrt{3}-1$

C. $a=\sqrt{3}-1, b=2-\sqrt{3}$

D. None of these

Answer: A

Watch Video Solution

13. If $z \neq 0$ be a complex number and $\arg(z) = -\pi/4$, then

A. $Re(z^2) = 0$

B. $Im(z^2) = 0$

C. $Re(z^2) = Im(z^2)$

D. None of these

Answer: A

Watch Video Solution

14. Let $zandw$ be two non-zero complex number such that $|z| = |w|$ and $arg(z) + arg(w) = \pi$, then z equals. w (b) $-w$ (c) w (d) $-w$

A. w

B. $-w$

C. $\overline{w}$

D. $-\overline{w}$

Answer: D

Watch Video Solution

15. If $|z| = 1$ and $\omega = \dfrac{z-1}{z+1}$ (where $z \in -1$), then $\text{Re}(\omega)$ is

A. 0

B. $-\dfrac{1}{|z+1|^2}$

C. $\left|\dfrac{z}{z+1}\right|\dfrac{1}{|z+1|^2}$

D. $\dfrac{\sqrt{2}}{|z+|^2}$

Answer: A

Watch Video Solution

16. Let z and w be two complex numbers such that $|Z| \leq 1, |w| \leq 1$ and $|z - iw| = |z - i\overline{w}| = 2$ and z equals

A. 1 or i

B. i or -i

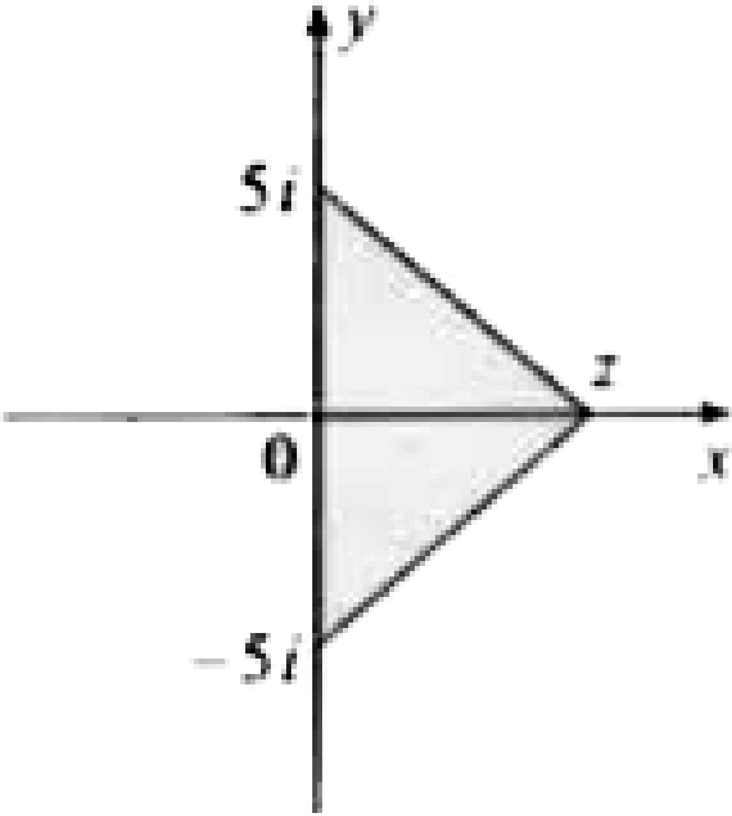
C. 1 or -1

D. i or -1

Answer: C

Watch Video Solution

17. The complex numbers z = x + iy which satisfy the equation $\left|\frac{z - 5i}{z + 5i}\right| = 1$, lie on



A. the x - axis

B. the straight line y = 5

C. a circle passing through origin

D. None of these

Answer: A

Watch Video Solution

18. The inequality $|z-4| < |z-2|$ represents

A. $Re(z) \geq 0$

B. $Re(z) < 3$

C. $Re(z) \leq 0$

D. $Rez > 3$

Answer: D

Watch Video Solution

19. If z_1 and z_2 are two complex numbers such that $\left|\frac{z_1 - z_2}{z_1 + z_2}\right| = 1,$ then

A. $z_2 = kz_1, k \in R$

B. $z_2 = ikz_1, k \in R$

C. $z_1 = z_2$

D. None of these

Answer: B

Watch Video Solution

20. For any complex number z, find the minimum value of $|z| + |z - 2i|$.

A. 0

B. 1

C. 2

D. None of these

Answer: C

Watch Video Solution

21. If $x = 2 + 5i$ then the value of $x^3 - 5x^2 + 33x - 19$ is

A. -5

B. -7

C. 7

D. 10

Answer: D

Watch Video Solution

22. If $z = x + iy$ and $w = \frac{1 - iz}{z - i}$, then $|w| = 1$ implies that in the complex plane (A)z lies on imaginary axis (B) z lies on real axis (C)z lies on unit circle (D) None of these

A. z lies on the imaginary axis

B. z lies on the real axis

C. z lies on the unit circle

D. None of these

Answer: B

Watch Video Solution

23. The real part of $z = \dfrac{1}{1 - \cos\theta + i\sin\theta}$ is

A. $\dfrac{1}{1 - \cos\theta}$

B. $\dfrac{1}{2}$

C. $\dfrac{1}{2}\tan\theta$

D. 2

Answer: B

Watch Video Solution

24. If the imaginary part of $\dfrac{2z + 1}{iz + 1}$ is -4, then the locus of the point representing z in the complex plane is

A. a straight line

B. a parabola

C. a circle

D. an ellipse

Answer: C

Watch Video Solution

25. Show that the area of the triagle on the argand plane formed by the complex numbers Z, iz and $z+iz$is $\frac{1}{2}|z|^2, \quad \text{where} \quad i=\sqrt{-1}.$

A. $\frac{1}{4}|z|^2$

B. $\frac{1}{8}|z|^2$

C. $\frac{1}{2}|z|^2$

D. $\frac{1}{2}|z|$

Answer: C

Watch Video Solution

26. If ω is a complex cube root of unity, then a root of the equation

$$\begin{vmatrix} x+1 & \omega & \omega^2 \\ \omega & x+\omega^2 & 1 \\ \omega^2 & 1 & x+\omega \end{vmatrix} = 0, \text{ is}$$

A. x = 1

B. $x = \omega$

C. $x = \omega^2$

D. x = 0

Answer: D

Watch Video Solution

27. Let z_1 and z_2 be two non - zero complex numbers such that $\frac{z_1}{z_2} + \frac{z_2}{z_1} = 1$ then the origin and points represented by z_1 and z_2

A. lie on a straight line

B. form a right triangle

C. form an equilateral triangle

D. None of these

Answer: C

Watch Video Solution

28. If $(1 + x + x^2)^n = a_0 + a_1x + a_2x^2 + \quad + a_{2n}x_{2n},$ find the value of $a_0 + a_6 + \quad + , n \in N.$

A. 1

B. 2^n

C. 2^{n-1}

D. 3^{n-1}

Answer: D

Watch Video Solution

29. Let $z = \begin{vmatrix} 1 & 1-2i & 3+5i \\ 1+2i & -5 & 10i \\ 3-5i & -10i & 11 \end{vmatrix}$, then

A. z is purely imaginary

B. z is purely real

C. z = 0

D. None of these

Answer: B

Watch Video Solution

30. if $(x+iy)^{\frac{1}{3}} = a+ib$ then $\left(\frac{x}{a}\right) + \left(\frac{y}{b}\right)$ equals to

A. $4\left(a^2 - b^2\right)$

B. $2\left(a^2 - b^2\right)$

C. $2(a^2 + b^2)$

D. None of these

Answer: A

Watch Video Solution

31. If $z \varepsilon C$, the minimum value of $|z| + |z - i|$ is attained at

A. exactly one point

B. exactly two points

C. infinite number of points

D. None of these

Answer: C

Watch Video Solution

32. For all complex numbers z_1, z_2 satisfying $|z_1| = 12 \text{ and } |z_2 - 3 - 4i| = 5$, the minimum value of $|z_1 - z_2|$ is

A. 0

B. 2

C. 7

D. 17

Answer: B

Watch Video Solution

33. If z lies on the circle $|z - 1| = 1$, then $\dfrac{z-2}{z}$ is

A. 0

B. 2

C. -1

D. None of these

Answer: D

Watch Video Solution

34. If $1, \omega, \ldots\ldots, \omega^{n-1}$ are the n^{th} roots of unity,then value of $\frac{1}{2-\omega}+\frac{1}{2-\omega^2}\ldots\ldots.+\frac{1}{2-\omega^{n-1}}$ equals

A. $\frac{1}{2^n-1}$

B. $\frac{n(2^n-1)}{2^n+1}$

C. $\frac{(n-2)2^{n-1}}{2^n-1}$

D. None of these

Answer: D

Watch Video Solution

35. If $\omega=\cos\frac{\pi}{n}+i\sin\frac{\pi}{n}$, then value of $1+\omega+\omega^2+\ldots+\omega^{n-1}$ is

A. $1 + i\cot\left(\frac{\pi}{2\pi}\right)$

B. $1 + i\tan\left(\frac{\pi}{n}\right)$

C. 1 + i

D. None of these

Answer: A

Watch Video Solution

36. If $|z| = 1 and z \neq \pm 1,$ then all the values of $\frac{z}{1 - z^2}$ lie on a line not passing through the origin $|z| = \sqrt{2}$ the x-axis (d) the y-axis

A. a line not passing through the origin.

B. |z| = 2

C. the x-axis

D. the y-axis

Answer: D

Watch Video Solution

37. The locus of the center of a circle which touches the circles $|z - z_1| = a, |z - z_2 = b|$ externally will be

A. an ellipse

B. a hyperbola

C. a circle

D. a pair of straight lines.

Answer: B

Watch Video Solution

38. If $\left|z^2 - 1\right| = |z|^2 + 1$, then z lies on

(a) a circle

(b) the imaginary axis

(c) the real axis

(d) an ellipse

A. a circle

B. the imaginary axis

C. the real axis

D. an ellipse

Answer: B

Watch Video Solution

39. If $z^2 + z + 1 = 0$ where z is a complex number, then the value of $\left(z + \frac{1}{z}\right)^2 + \left(z^2 + \frac{1}{z^2}\right)^2 + ... + \left(z^6 + \frac{1}{z^6}\right)^2$ is

A. 12

B. 18

C. 54

D. 6

Answer: A

Watch Video Solution

40. If $|z+4| \leq 3$, then the maximum value of $|z+1|$ is (1) 4 (B) 10 (3) 6 (4) 0

A. 4

B. 10

C. 6

D. 0

Answer: C

Watch Video Solution

41. Let z, w be complex numbers such that $\bar{z} + i\overline{w} = 0$ and $arg zw = \pi$

Then $arg z$ equals

A. $\frac{3\pi}{4}$

B. $\frac{\pi}{2}$

C. $\frac{\pi}{4}$

D. $\frac{5\pi}{4}$

Answer: A

Watch Video Solution

42. If $z_1 + z_2 + z_3 = 0$ and $|z_1| = |z_2| = |z_3| = 1,$ then value of $z_1^2 + z_2^2 + z_3^2$ equals

A. -1

B. 0

C. 1

D. 3

Answer: B

Watch Video Solution

43. If z satisfies the relation $|z - i|z|| = |z + i|z||$ then

A. Im(z) = 0

B. |z| = 1

C. Re(z) = 0

D. None of these

Answer: A

Watch Video Solution

44. If $\alpha$and $\beta$are different complex numbers with $|\beta| = 1$, then find $\left|\frac{\beta - \alpha}{1 - \overline{\alpha}\beta}\right|$.

A. 1

B. $|\alpha|$

C. 2

D. None of these

Answer: A

Watch Video Solution

45. Let z be not a real number such that $(1 + z + z^2) / (1 - z + z^2) \in R$, then prove tha $|z| = 1$.

A. 1

B. 2

C. $\sqrt{3}$

D. $2\sqrt{3}$

Answer: A

Watch Video Solution

46. If $\left|z-\frac{4}{z}\right|=2$, then the maximum value of $|z|$

A. 1

B. $2+\sqrt{2}$

C. $\sqrt{3}+1$

D. $\sqrt{5}+1$

Answer: D

Watch Video Solution

47. If $|\omega| = 2$, then the set of points $z = \omega - \frac{1}{\omega}$ is contained in or equal to the set of points z satisfying

A. circle

B. ellipse

C. parabola

D. hyperbola

Answer: B

Watch Video Solution

48. If $|z| = 1, z \neq 1$, then value of arg $\left(\frac{1}{1-z}\right)$ cannot exceed

A. $\pi/2$

B. π

C. $3\pi/2$

D. 2π

Answer: A

Watch Video Solution

49. If $z \neq 1$ and $\frac{z^2}{z-1}$ is real, the point represented by the complex numbers z lies

A. on circle with centre at the origin.

B. either on the real axis or on a circle not passing through the origin.

C. on the imaginary axis.

D. either on the real axis or on a circle passing through the origin

Answer: B

Watch Video Solution

50. If $3^{49}(x+iy)=\left(\frac{3}{2}+\frac{\sqrt{3}}{2}I\right)^{100}$ and $x=ky$ then k is: $-1/3$ b. $\sqrt{3}$

c. $-\sqrt{3}$ d.$-\frac{1}{\sqrt{3}}$

A. $\pm 1/3$

B. $\pm 2\sqrt{2}$

C. $\pm 1/\sqrt{3}$

D. $\pm 1/\sqrt{3}$

Answer: B

Watch Video Solution

51. If $(4+i)(z+\bar{z})-(3+i)(z-\bar{z})+26i=0$, then the value of $|z|^2$ is

A. 13

B. 17

C. 19

D. 11

Answer: B

Watch Video Solution

52. Let $z = a\left(\cos\frac{\pi}{5} + \mathrm{i}\sin\frac{\pi}{5}\right), a \in R, |a| < 1,$ then $S = z^{2015} + z^{2016} + z^{2017} + \ldots$ equals

A. $\dfrac{a^{2015}}{z-1}$

B. $\dfrac{a^{2015}}{1-z}$

C. $\dfrac{a^{2015}}{1-a}$

D. $\dfrac{a^{2015}}{a-1}$

Answer: A

Watch Video Solution

53. If $z=\sqrt{20i-21}+\sqrt{20i+21}$, than one of the possible value of |arg(z)| equals

A. $\pi/4$

B. $\pi/2$

C. $3\pi/8$

D. π

Answer: A

Watch Video Solution

54. If $(a+bi)^{11}=x+iy$, where a, b, x, y $\in$ R, then $(b+ai)^{11}$ equals

A. y + ix

B. $-y-ix$

C. $-x-iy$

D. x + iy

Answer: B

View Text Solution

55. If $a, b, x, y \in R, \omega \neq 1,$ is a cube root of unity and $(a + b\omega)^7 = x + y\omega$, then $(b + a\omega)^7$ equals

A. $y + x\omega$

B. $-y - x\omega$

C. $x + y\omega$

D. $-x - y\omega$

Answer: A

View Text Solution

Solved Examples Level 2

1. Let $z = \cos\theta + i\sin\theta$. Then the value of $\sum_{m \to 1-15} Img\left(z^{2m-1}\right)$ at $\theta = 2^\circ$ is:

A. $\frac{1}{\sin 2^\circ}$

B. $\frac{1}{3\sin 2^\circ}$

C. $\frac{1}{\sin 2^\circ}$

D. $\frac{1}{4\sin 2^\circ}$

Answer: D

Watch Video Solution

Solved Examples Level 3

1. Let $z = x + iy$ be a complex number where $xandy$ are integers. Then, the area of the rectangle whose vertices are the roots of the equation $zz^3 + zz^3 = 350$ is 48 (b) 32 (c) 40 (d) 80

A. 48

B. 32

C. 40

D. 80

Answer: A

Watch Video Solution

Solved Examples Level 4

1. Let z_1 and z_2 be two distinct complex numbers and $z = (1-t)z_1 + iz_2$, for some real number t with $0 < t < 1$ and $i = \sqrt{-1}$. If arg(w) denotes the principal argument of a non-zero compolex number w, then

A. $|z - z_1| + |z - z_2| = |z_1 - z_2|$

B. Arg $(z - z_1) = \text{Arg}(z - z_2)$

C. $\begin{vmatrix} z - z_1 & \bar{z} - \bar{z}_1 \\ z_2 - z_1 & \bar{z}_2 - \bar{z}_1 \end{vmatrix} = 0$

D. $Arg(z - z_1) = Arg(z_2 - z_1)$

Answer: B

Watch Video Solution

Solved Examples Level 5

1. For complex numbers $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$, (where $i = \sqrt{-1}$) we write $z_1 \cap z_2$ of $x_1 \leq x_2$ and $y_1 \leq y_2$, then for all complex number z with $1 \cap z$, we have $\frac{1-z}{1+z} \cap ....$ is

A. $\frac{1-z}{1+z} \cap -i$

B. $1 \cap \frac{1-z}{1+z}$

C. $\frac{1-z}{1+z} \cap 0$

D. $\frac{1+z}{1-z} \cap 0$

Answer: C

Watch Video Solution

Solved Examples Level 6

1. Let the complex numbers z_1, z_2 and z_3 be the vertices of an equailateral triangle. If z_0 is the circumcentre of the triangle , then prove that $z_1^2 + z_2^2 + z_3^2 = 3z_0^2$.

A. z_0^2

B. $3z_0^2$

C. z_0^3

D. $3z_0^3$

Answer: B

Watch Video Solution

Solved Examples Level 7

1. If ω is a complex cube root of unity, then the value of the expression

$1(2-\omega)\left(2-\omega^2\right)+2(3-\omega)\left(3-\omega^2\right)+\ldots+(n-1)(n-\omega)\left(n-\omega^2\right)(n \geq$

is equal to (A) $\frac{n^2(n+1)^2}{4}-n$ (B) $\frac{n^2(n+1)^2}{4}+n$ (C) $\frac{n^2(n+1)}{4}-n$

(D) $\frac{n(n+1)^2}{4}-n$

A. $\frac{1}{4}n^2(n+1)^2-n$

B. $\frac{1}{4}n^2(n+1)^2+n$

C. $\frac{1}{4}n^2(n+1)-n$

D. $\frac{1}{4}n(n+1)^2-n$

Answer: A

Watch Video Solution

Solved Examples Level 8

1. Prove that there exists no complex number z such that $|z| < \frac{1}{3}$ and $\sum_{n=1}^{n} a_r z^r = 1$, where $|a_r| < 2$.

A. 0

B. 1

C. 4

D. infinite

Answer: A

Watch Video Solution

Solved Examples Level 9

1. a, b, c are integers, not all simultaneously equal, and ω is cube root of unity $(\omega \neq 1)$, then minimum value of $|a + b\omega + c\omega^2|$ is 0 b. 1 c. $\frac{\sqrt{3}}{2}$ d. $\frac{1}{2}$

A. $\sqrt{3}$

B. 1

C. 2

D. 3

Answer: B

Watch Video Solution

Solved Examples Level 10

1. The region of Argand diagram defined by $|z-1| + |z+1| \le 4$ is

A. interior of an ellipse

B. exterior of a circle

C. interior and boundary of an ellipse

D. None of these

Answer: C

Watch Video Solution

Solved Examples Level 11

1. If z_1 and z_2, are two non-zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$ then $arg(z_1) - arg(z_2)$ is equal to (1) 0 (2) $-\frac{\pi}{2}$ (3) $\frac{\pi}{2}$ (4) $-\pi$

A. $-\pi$

B. $-\pi/2$

C. $\pi/2$

D. 0

Answer: D

Watch Video Solution

Solved Examples Level 12

1. If $|z-25i| \leq 15.$ then $|\text{maximum } arg(z) - \text{minimum } arg(z)|$ equals

A. $2\cos^{-1}(3/5)$

B. $2\cos^{-1}(4/5)$

C. $\pi/2 + \cos^{-1}(3/5)$

D. $\sin^{-1}(3/5) - \cos^{-1}(3/5)$

Answer: B

Watch Video Solution

Solved Examples Level 13

1. If |z| = 3, the area of the triangle whose sides are $z, \omega z$ and $z + \omega z$ (where ω is a complex cube root of unity) is

A. $9\sqrt{3}/4$

B. $3\sqrt{3}/2$

C. $5/2$

D. $8\sqrt{3}/3$

Answer: A

Watch Video Solution

Solved Examples Level 14

1. Find the greatest and the least value of $|z_1 + z_2|$ if $z_1 = 24 + 7iand|z_2| = 6.$

A. 31, 19

B. 25, 19

C. 31, 25

D. None of these

Answer: A

Watch Video Solution

Solved Examples Level 15

1. If α, β are the roots of $x^2 + px + q = 0, \text{ and } \omega$ is a cube root of unity, then value of $(\omega\alpha + \omega^2\beta)(\omega^2\alpha + \omega\beta)$ is

A. p^2

B. 3q

C. $p^2 - 2q$

D. $p^2 - 3q$

Answer: D

Watch Video Solution

Solved Examples Level 16

1. Maximum distance from the origin of the points z satisfying the relation $|z+1/z|=1$ is

A. $(\sqrt{5}+1)/2$

B. $(\sqrt{5}-1)/2$

C. $3-\sqrt{5}$

D. $(3+\sqrt{5})/2$

Answer: A

Watch Video Solution

Solved Examples Level 17

1. If $|z_1|+|z_2|=1 and z_1+z_2+z_3=0$ then the area of the triangle whose vertices are z_1, z_2, z_3 is $3\sqrt{3}/4$ b. $\sqrt{3}/4$ c. 1 d. 2

A. $3\sqrt{3}/4$

B. $\sqrt{3}/4$

C. 1

D. 2

Answer: A

Watch Video Solution

Solved Examples Level 18

1. An equation of straight line joining the complex numbers a and ib (where a, b ε R and a, b $\neq$ 0) is

A. $z\left(\frac{1}{a}-\frac{i}{b}\right)+\bar{z}\left(\frac{1}{a}+\frac{i}{b}\right)=2$

B. $z\left(\frac{1}{a}-\frac{i}{b}\right)+\bar{z}\left(\frac{1}{a}+\frac{i}{b}\right)=2$

C. $z(a+ib)+\bar{z}(a-ib)=2ab$

D. None of these

Answer: A

Watch Video Solution

Solved Examples Level 19

1. Two different non-parallel lines cut the circle $|z| = r$ at points a, b, c and d, respectively. Prove that these lines meet at the point given by $\dfrac{a^{-1}+b^{-1}-c^{-1}-d^{-1}}{a^{-1}b^{-1}-c^{-1}d^{-1}}$

A. $\dfrac{a^{-1}+b^{-1}+c^{-1}+d^{-1}}{a^{-1}b^{-1}+c^{-1}d^{-1}}$

B. $\dfrac{ab+cd}{a+b+c+d}$

C. $\dfrac{a^{-1}+b^{-1}-c^{-1}-d^{-1}}{a^{-1}b^{-1}-c^{-1}d^{-1}}$

D. None of these

Answer: C

Watch Video Solution

Solved Examples Numerical Answer Type Questions

1. If $1, x_1, x_2, x_3$ are the roots of $x^4 - 1 = 0$ and ω is a complex cube root of unity, find the value of $\frac{(\omega^2 - x_1)(\omega^2 - x_2)(\omega^2 - x_3)}{(\omega - x_1)(\omega - x_2)(\omega - x_3)}$

Watch Video Solution

2. Radius of the circle $z\bar{z} + (2 + 5.5i)z + (2 - 5.5i)\bar{z} + 4 = 0$ is __________

Watch Video Solution

3. Radius of the circle $\left|\frac{z-1}{z-3i}\right| = \sqrt{2}$

Watch Video Solution

4. Suppose z_1, z_2, z_3 are vertices of an equilateral triangle with incentre at 1.1 + 0i, then $z_1^2 + z_2^2 + z_3^3$ is equal to ________

View Text Solution

5. Let m = Slope of the line $|z+3|^2 - |z-3i|^2 = 24$, then m + 1.73 is equal to

Watch Video Solution

6. If $\omega \neq 1$ is a cube root of unity, then $\frac{1}{\pi}\sin^{-1}\left[\left(\omega^{73}+\omega^{83}\right)+\tan\frac{5\pi}{4}\right]$ is equal to ________

Watch Video Solution

7. $\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{181}+\left(\frac{1-\sqrt{3}i}{1+\sqrt{3}i}\right)^{181}$ is equal to ____________

Watch Video Solution

Watch Video Solution

8. Let z_1, z_2 be two complex numbers satisfying the equations $\left|\frac{z-4}{z-8}\right| = 1$ and $\left|\frac{z-8i}{z-12}\right| = \frac{3}{5}$, then $\sqrt{|z_1 - z_2|}$ is equal to ________

Watch Video Solution

9. If z is a complex number, then the minimum value of $|z-2.8| + |z-1.42|$ is __________

Watch Video Solution

10. If $\frac{3z_1}{5z_2}$ is purely imaginary, then $\left|\frac{2z_1 - z_2}{2z_1 + z_2}\right|$ is equal to _______

Watch Video Solution

11. If $\omega \neq 1$ is a complex cube root of unity, then $5.23 + \omega + \omega^{\left(\frac{1}{2}+\frac{3}{8}+\frac{9}{32}+\frac{27}{128}+\dots\right)}$ is equal to ________

Watch Video Solution

12. If conjugate of a complex number z is $\frac{2+5i}{4-3i}$, then |Re(z) + Im(z)| is equal to __________

Watch Video Solution

13. Let z be a complex number such that $Im(z) \neq 0$. If $\mathrm{a} = z^2 + 5z + 7$, then a cannot take value __________

Watch Video Solution

14. Let $z_k = \cos\left(\frac{2k\pi}{7}\right) + i\sin\left(\frac{2k\pi}{7}\right)$, for $\mathrm{k} = 1, 2, \ldots, 6,$ then $\log_7|1 - z_1| + \log_7|1 - z_2| + \ldots. + \log_7|1 - z_6|$ is equal to _______

Watch Video Solution

15. Let $S = \{z \in C : |z-2| = |z+2i| = |z-2i|\}$ then $\sum_{z \in S} |z+1.5|$ is equal to ________

Watch Video Solution

16. Suppose z satisfies the equation $z^2 + z + 1 = 0$. Let $\omega = \left(z + \frac{1}{z}\right)^2 + \left(z^2 + \frac{1}{z^2}\right)^2 + \left(z^3 + \frac{1}{z^3}\right) + \ldots +$ then $\left|\omega + \sqrt{301i}\right|$ is equal to __________

Watch Video Solution

17. Suppose $\omega \neq 1$ is cube root of unity. If $1(2-\omega)\left(2-\omega^2\right) + 2(3-\omega)\left(3-\omega^2\right) + \ldots. + (n-1)(n-\omega)\left(n-\omega^2\right) =$, then n is equal to __________

Watch Video Solution

18. If z_1 and z_2 are two nonzero complex numbers and θ is a real number, then $\frac{1}{|z_1|^2 + |z_2|^2}\left[|(\cos\theta)z_1 - (\sin\theta)z_2|^2 + |(\sin\theta)z_1 + (\cos\theta)z_2|^2\right]$ is equal to _______

Watch Video Solution

19. Eccentricity of the ellipse $|z-4| + |z-4i| = 10\sqrt{2}$ is ________

Watch Video Solution

20. Suppose a and b are two different complete numbers such that $\left|a + \sqrt{a^2 - b^2}\right| + \left|a - \sqrt{a^2 - b^2}\right| = |a+b| + 4$ then |a-b| is equal to ________

Watch Video Solution

21. Suppose z_1, z_2 and z_3 are three distinct complex numbers such that $|z_1| = |z_2| = |z_3| = \sqrt{3}$. If $z_1 + z_2 + z_3 = 0$, and Δ = area of triangle with vertices $A(z_1), B(z_2)$ and $C(z_3)$, then $\frac{32}{9}\Delta^2$ is equal to ________.

View Text Solution

22. Let P be a point on the circle $|z + 2 - 5i| = 6$ and A be the point (4 - 3i). Let D = max(AP), then $\frac{1}{32}$D is equal to ________.

Watch Video Solution

Exercise

1. The number of complex numbers satisfying (1 + i)z = i|z|

A. 0

B. 1

C. 2

D. infinite

Answer: B

Watch Video Solution

2. Suppose $a, b, c \in R$ and $C < 0$. Let z = a + $(b + ic)^{2015} + (b - ic)^{2015}$, then

A. Re (z) = 0

B. Im (z) = 0

C. $Re(z) > 0, Im(z) < 0$

D. $Re(z) < 0, Im(z) > 0$

Answer: B

Watch Video Solution

3. The number of solutions of $z^2 + |z| = 0$ is

A. 1

B. 2

C. 3

D. infinite

Answer: C

Watch Video Solution

4. The equation $\left|\frac{(1+i)z-2}{(1+i)z+4}\right| = k$ does not represent a circle when k is

A. 2

B. π

C. e

D. 1

Answer: D

Watch Video Solution

5. If $|z| \geq 5$, then least value of $\left|z - \frac{1}{z}\right|$ is

A. 5

B. $24/5$

C. 8

D. $8/3$

Answer: B

Watch Video Solution

6. Principal argument of $z = \dfrac{i-1}{i\left(1 - \cos\frac{2\pi}{7}\right) + \sin\frac{2\pi}{7}}$ is

A. $\dfrac{\pi}{28}$

B. $\frac{3\pi}{28}$

C. $\frac{17\pi}{28}$

D. $\frac{19\pi}{28}$

Answer: C

Watch Video Solution

7. If $(x + iy) = \sqrt{\frac{a + ib}{c + id}}$ then prove that $(x^2 + y^2)^2 = \frac{a^2 + b^2}{c^2 + d^2}$

A. $a^2 + b^2$

B. $\sqrt{a^2 + b^2}$

C. $\frac{a^2 + b^2}{c^2 + d^2}$

D. $\sqrt{\frac{a^2 + b^2}{c^2 + d^2}}$

Answer: A

Watch Video Solution

8. For any three complex numbers z_1, z_2, z_3, if $\Delta = \begin{vmatrix} 1 & z_1 & \overline{z_1} \\ 1 & z_2 & \overline{z_2} \\ 1 & z_3 & \overline{z_3} \end{vmatrix}$, then

A. $Re(\Delta) = 0$

B. $Im(\Delta) = 0$

C. $Re(\Delta) \geq 0$

D. $Im(\Delta) \leq 0$

Answer: B

Watch Video Solution

9. If $x, y, a, b \in R, a \neq 0$ and $(a + ib)(x + iy) = (a^2 + b^2)i$, then (x, y) equals

A. (a, b)

B. (a, 0)

C. (0, b)

D. (b, a)

Answer: D

Watch Video Solution

10. If $\omega(\neq 1)$ is a cube root of unity, then the value of $\tan\left[\left(\omega^{2017}+\omega^{2225}\right)\pi-\pi/3\right]$

A. $-\frac{1}{\sqrt{3}}$

B. $\frac{1}{\sqrt{3}}$

C. $-\sqrt{3}$

D. $\sqrt{3}$

Answer: C

Watch Video Solution

11. If z is purely imaginary and $Im(z) < 0$, then $arg(i\bar{z}) + arg(z)$ is equal to

A. π

B. 0

C. $\pi/2$

D. $-\pi/2$

Answer: C

Watch Video Solution

12. The inequality $a + ib > c + id$ is true when

A. $a > c, b > d > 0$

B. $a > c, b = d = 0$

C. $a > c, b = d > 0$

D. None of these

Answer: B

Watch Video Solution

13. Let $z \in C$ be such that $Re(z^2) = 0$, then

A. $|Re(z)| + Im(z) = 0$

B. $|Re(z)| = |Im(z)|$

C. $Re(z) + |Im(z)| = 0$

D. $Re(z) = 0 \text{ or } Im(z) = 0$

Answer: B

Watch Video Solution

14. If z_1, z_2 and z_3, z_4 are two pairs of conjugate complex numbers then $\arg\left(\frac{z_1}{z_4}\right) + arg\left(\frac{z_2}{z_3}\right)$ equals

A. $0, \pi$

B. $\pi, \ -\pi$

C. $\frac{\pi}{2}, \frac{3\pi}{2}$

D. $0, 2\pi$

Answer: D

Watch Video Solution

15. If $z = x + iy$ and $0 \leq \sin^{-1}\left(\frac{z-4}{2i}\right) \leq \frac{\pi}{2}$ then

A. $x = 4, 0 \leq y \leq 2$

B. $0 \leq x \leq 4, 0 \leq y \leq 2$

C. $x = 0, 0 \leq y \leq 2$

D. None of these

Answer: A

Watch Video Solution

16. If $a > 0$ and $z|z| + az + 3i = 0$, then z is

A. 0

B. purely imaginary

C. a positive real number

D. a negative real number

Answer: B

Watch Video Solution

17. If z is a complex numbers such that $z \neq 0$ and $\text{Re}(z) = 0$, then

A. $Re(z^2) = 0$

B. $Im(z^2) = 0$

C. $Re(z^2) = Im(z^2)$

D. $Im(z^2) < 0$

Answer: B

Watch Video Solution

18. If $z_k = \cos\left(\frac{k\pi}{10}\right) + i\sin\left(\frac{k\pi}{10}\right)$, then $z_1z_2z_3z_4$ is equal to

A. -1

B. 2

C. -2

D. 1

Answer: A

Watch Video Solution

19. If $|z_1| = |z_2| = 1, z_1z_2 \neq -1$ and $z = \frac{z_1 + z_2}{1 + z_1z_2}$ then

A. z is a purely real number

B. z is a purely imaginary number

C. |z| = 1

D. None of these

Answer: A

Watch Video Solution

20. If $z \in C$, then $Re\left(\bar{z}^2\right) = k^2, k > 0$, represents

A. an ellipse

B. a parabola

C. a circle

D. a hyperbola

Answer: D

Watch Video Solution

Exercise Level 1

1. If $\omega \neq 1$ is a cube root of unity, then $1, \omega, \omega^2$

A. are vertices of an equilateral triangle

B. lie on a straight line

C. lie on a circle of radius $\sqrt{3/2}$

D. None of these

Answer: A

Watch Video Solution

2. If α, β, γ are the cube roots of p then for any x, y and z $\dfrac{x\alpha + y\beta + z\gamma}{x\beta + y\gamma + z\alpha}$ is

A. $\omega, 1$

B. ω, ω^2

C. $\omega^2, 1$

D. $1, \omega, \omega^2$

Answer: B

Watch Video Solution

3. ABCD is a rhombus. Its diagonals AC and BD intersect at the point M and satisfy $BD = 2AC$.If he point D and M represent the complex numbers $1 + i$ and $2 - i$ respectively, then A represents the complex number......or......

A. $3 - \frac{1}{2}i, 1 + \frac{3}{2}i$

B. $3 + \frac{1}{2}i, 1 + \frac{3}{2}i$

C. $3 - \frac{1}{2}i, 1 - \frac{3}{2}i$

D. $3 + \frac{1}{2}i, 1 - \frac{3}{2}i$

Answer: C

Watch Video Solution

4. Let a, be the roots of the equation $x^2 + x + 1 = 0$. The equation whose roots are α^{19} and β^7 are:

A. $x^2 - x - 1 = 0$

B. $x^2 - x + 1 = 0$

C. $x^2 + x - 1 = 0$

D. $x^1 + x + 1 = 0$

Answer: D

Watch Video Solution

5. If ω is a cube root of unity then find the value of $\sin\left(\left(\omega^{10} + \omega^{23}\right) - \frac{\pi}{4}\right)$

A. $-\frac{\sqrt{3}}{2}$

B. $-\frac{1}{\sqrt{2}}$

C. $\frac{1}{\sqrt{2}}$

D. $\frac{\sqrt{3}}{2}$

Answer: C

Watch Video Solution

6. If $\omega(\neq 1)$ is a cube root of unity and $(1+\omega)^{2017} = A + B\omega$. Then A and B are respectively the numbers

A. 0, 1

B. 1, 1

C. 1, 0

D. $-1, 1$

Answer: B

Watch Video Solution

7. If ω is the complex cube root of unity then $\begin{vmatrix} 1 & 1+i+\omega^2 & \omega^2 \\ 1-i & -1 & \omega^2-1 \\ -i & -i+\omega-1 & -1 \end{vmatrix} =$

A. 0

B. 1

C. i

D. ω

Answer: A

Watch Video Solution

8. If ω is an imaginary cube root of unity, then $\left(1-\omega-\omega^2\right)^7$ equals

A. 128 ω

B. -128ω

C. 128 ω^2

D. $-128\omega^2$

Answer: D

Watch Video Solution

9. If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & 1 \\ 20 & 3 & i \end{vmatrix} = x + iy, i = \sqrt{-1}$ then

A. x = 3, y = 1

B. x = 1, y = 3

C. x = 0, y = 3

D. x = 0, y = 0

Answer: D

Watch Video Solution

10. If $arg(z) < 0,$ then find $arg(-z) - arg(z).$

A. π

B. $-\pi$

C. $-\frac{\pi}{2}$

D. $\frac{\pi}{2}$

Answer: A

Watch Video Solution

11. If z_1, z_2, z_3 are complex numbers such that $|z_1| = |z_2| = |z_3| = \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = 1$ then $|z_1 + z_2 + z_3|$ is equal to

A. equal to 1

B. less than 1

C. greater than 3

D. equal to 3

Answer: A

Watch Video Solution

12. Let z_1 and z_2 be nth roots of unity which subtend a right angle at the origin. Then n must be of the form (1) $4k+1$ (2) $4k+2$ (3) $4k+3$ (4) $4k$

A. 4k + 1

B. 4k + 2

C. 4k + 3

D. 4k

Answer: D

Watch Video Solution

13. The complex number z_1, z_2 and z_3 satisfying $\frac{z_1 - z_3}{z_2 - z_3} = \frac{1 - i\sqrt{3}}{2}$ are the vertices of a triangle which is :

A. of area zero

B. right-angled isosceles

C. equilateral

D. obtuse-angle isosceles

Answer: C

Watch Video Solution

14. Let $\omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$, then the value of the determinant $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1-\omega^2 & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{vmatrix}$, is

A. 3ω

B. $3\omega(\omega - 1)$

C. $3\omega^2$

D. $3\omega(1-\omega)$

Answer: B

Watch Video Solution

15. The inequality $|z-i| < |z+i|$ represents the region

A. $Re(z) > 0$

B. $Re(z) < 0$

C. $Im(z) > 0$

D. $Im(z) < 0$

Answer: C

Watch Video Solution

16. Show that if $iz^3 + z^2 - z + i = 0$, then $|z| = 1$

A. Re z = 0

B. Im z = 0

C. |z| = 1

D. None of these

Answer: C

Watch Video Solution

17. If $x + iy = \frac{1}{1 - \cos\theta + 2i\sin\theta}, \theta \neq 2n\pi, n \in I,$ then maximum value of x is

A. 1

B. 2

C. $\frac{1}{2}$

D. $\frac{1}{3}$

Answer: C

Watch Video Solution

18. The equation $z^3 = \bar{z}$ has

A. no solution

B. two solutions

C. five solutions

D. infinite number of solutions

Answer: C

Watch Video Solution

19. If $z = 5 + t + i\sqrt{25 - t^2}, (-5 \le t \le 5)$, then locus of z is a curve which passes through

A. 5 + 0i

B. $-2+3i$

C. 2 + 4i

D. $-2-3i$

Answer: C

Watch Video Solution

20. If ω is complex cube root of that $\frac{1}{a+\omega}+\frac{1}{b+\omega}+\frac{1}{c+\omega}=2\omega^2$ and $\frac{1}{a+\omega^2}+\frac{1}{b+\omega^2}+\frac{1}{c+\omega^2}=2\omega$ then the value of $\frac{1}{a+1}+\frac{1}{b+1}+\frac{1}{c+1}=$

A. 2

B. -2

C. $-1+\omega^2$

D. None of these

Answer: A

Watch Video Solution

21. if $|z - iRe(z)| = |z - Im(z)|$ where $i = \sqrt{-1}$ then z lies on

A. Re (z) = 2

B. Im (z) = 2

C. Re (z) + Im (z) = 2

D. None of these

Answer: D

Watch Video Solution

22. If ω is a complex cube root of unity, then value of expression $\cos\left[\{(1-\omega)(1-\omega^2)+\ldots+(12-\omega)(12-\omega^2)\}\frac{\pi}{370}\right]$

A. -1

B. 0

C. 1

D. $\sqrt{3}/2$

Answer: C

Watch Video Solution

23. If roots of the equation $z^2 + az + b = 0$ are purely imaginary then

A. $(b - \bar{b})^2 + (a + \bar{a})(\bar{a}b + a\bar{b}) = 0$

B. $(b - \bar{b})^2 + (a - \bar{a})^2 = 0$

C. $(b + \bar{b})^2 - (a - \bar{a})^2 = 0$

D. None of these

Answer: A

Watch Video Solution

24. The system of equations $|z+1-i| = \sqrt{2}$ and $|z| = 3$ has

A. no solution

B. one solution

C. two solutions

D. infinite number of solutions

Answer: A

Watch Video Solution

25. If $8\iota z^3$+$12z^2$-$18z$+27ι=0 then: a. $|z| = \frac{3}{2}$ b. $|z| = \frac{2}{3}$ c. $|z| = 1$

A. $|z| = \frac{3}{2}$

B. $|z| = \frac{2}{3}$

C. |z| = 1

D. $|z| = \frac{3}{4}$

Answer: A

Watch Video Solution

26. If a complex number z lies in the interior or on the boundary of a circle or radius 3 and center at $(-4, 0)$, then the greatest and least values of $|z+1|$ are

A. 4

B. 5

C. 10

D. 9

Answer: C

Watch Video Solution

27. If $x+iy=\dfrac{3}{2+\cos\theta+i\sin\theta}$, then show that $x^2+y^2=4x-3$

A. 2

B. 3

C. 4

D. 5

Answer: B

Watch Video Solution

28. Suppose z_1, z_2, z_3 represent the vertices A, B and C respectively of a ΔABC with centroid at G. If the mid point of AG is the origin, then

A. $z_1+z_2+z_3=0$

B. $2z_1+z_2+z_3=0$

C. $z_1+z_2+4z_3=0$

D. $4z_1+z_2+z_3=0$

Answer: D

Watch Video Solution

29. Suppose that three points z_1, z_2, z_3 are connected by the relation $az_1 + bz_2 + cz_3 = 0$, where a + b + c = 0, then the points are

A. vertices of a right triangle

B. vertices of an isosceles triangle

C. vertices of an equilateral triangle

D. collinear

Answer: D

Watch Video Solution

30. If the number $\dfrac{z-1}{z+1}$ is purely imaginary, then

A. |z| = 1

B. $|z| < 1$

C. $|z| > 1$

D. $|z| \geq 2$

Answer: A

Watch Video Solution

31. If z s a complex number such that $-\frac{\pi}{2} \leq argz \leq \frac{\pi}{2}$, then which of the following inequality is true

A. $|z - \bar{z}| \leq |z||arg(z) - arg(\bar{z})|$

B. $|z - \bar{z}| \leq |arg(z) - arg(\bar{z})|$

C. $|z - \bar{z}| > |z||arg(z) - arg(\bar{z})|$

D. None of these

Answer: A

Watch Video Solution

32. If $|\omega| = 1$, then the set of points $z = \omega + \frac{1}{\omega}$ is contained in or equal to the set of points z satisfying.

A. $|Re(z)| \leq 2$

B. $|z| \leq 1$

C. |z| = 1

D. $|Im(z)| \geq 2$

Answer: A

Watch Video Solution

33. The number of complex numbers z such that $|z| = 1 and \mid z/z + z/z = 1$ is $(arg(z) \in [0, 2\pi))$ 4 b. 6 c. 8 d. m or $ethan8$

A. 0

B. 2

C. 4

D. 8

Answer: D

Watch Video Solution

34. If $|z_1| = |z_2| = |z_3| = 1$ are twu complex numbers such that $|z_1| = |z_2| = \sqrt{2}$ and $|z_1 + z_2| = \sqrt{3}$, then $|z_1 - z_2|$ equation

A. $2\sqrt{2}$

B. $\sqrt{5}$

C. 3

D. $2 - \sqrt{2}$

Answer: B

Watch Video Solution

35. Let z_1, z_2, z_3 be three non-zero complex numbers such that $z_1\bar{z}_2 = z_2\bar{z}_3 = z_3\bar{z}_1$, then z_1, z_2, z_3

A. are vertices of an equilateral triangle

B. are vertices of an isosceles triangle

C. lie on a straight line

D. None of these

Answer: A

Watch Video Solution

36. If $|z_1| = |z_2| = |z_3| = 1$ then value of $|z_1 - z_3|^2 + |z_3 - z_1|^2 + |z_1 - z_2|^2$ cannot exceed

A. 3

B. 6

C. 9

D. 12

Answer: C

Watch Video Solution

37. Let $z_1 z_2, z_3,$ be three complex number such that $z_1 + z_2 + z_3 = 0$ and $|z_1| = |z_2| = |z_3| = 1$ then Let $|z_1^2 + 2z_2^2 + z_3^2|$ equals

A. 1

B. 2

C. 3

D. 4

Answer: A

Watch Video Solution

38. Let z_1, z_2, z_3 be three complex numbers such that $|z_1| = |z_2| = |z_3| = 1$ and $z = (z_1 + z_2 + z_3)\left(\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right)$, then $|z|$ cannot exceed

A. 1

B. 3

C. 6

D. 9

Answer: D

Watch Video Solution

39. Suppose z is a complex number such that $z \neq -1, |z| = 1,$ and $arg(z) = \theta$. Let $\omega = \frac{z(1-\bar{z})}{\bar{z}(1+z)}$, then Re (ω) is equal to

A. $1+\cos(\theta/2)$

B. $1-\sin(\theta/2)$

C. $-2\sin^2(\theta/2)$

D. $2\cos^2(\theta/2)$

Answer: C

Watch Video Solution

40. Let $a=Im\left(\frac{1+z^2}{2iz}\right)$, where z is any non-zero complex number. Then the set $A=\{a:|z|=1 \text{ and } z\neq \pm 1\}$ is equal to

A. (-1, 1)

B. [-1, 1]

C. [0, 1)

D. (-1, 0]

Answer: A

Watch Video Solution

41. Number of complex numbers such that |z| = 1 and $z = 1 - 2\bar{z}$ is

A. 0

B. 1

C. 2

D. infinite

Answer: A

Watch Video Solution

42. Let z_1, z_2 be two complex numbers such that $z_1 \neq 0$ and z_2 / z_1 is purely real, then $\left|\frac{2iz_1 + 5z_2}{2iz_1 - 5z_2}\right|$ is equal to

A. 3

B. 2

C. 1

D. 0

Answer: C

Watch Video Solution

43. If $z = i(1 + \sqrt{3})$, then $z^4 + 2z^3 + 4z^2 + 5$ is equal to

A. 5

B. -5

C. $2\sqrt{3}i$

D. $-2\sqrt{3}i$

Answer: A

View Text Solution

44. If the fourth roots of unity are z_1, z_2, z_3, z_4 and $z_1^2 + z_2^2 + z_3^2 + z_4^2$ is equal to :

A. $-a^2$

B. |a| - a

C. a + |a|

D. a^2

Answer: C

Watch Video Solution

45. Suppose $arg(z) = -5\pi/13$, then $arg\left(\frac{z+\bar{z}}{1+z\bar{z}}\right)$ is

A. $-5\pi/13$

B. $5\pi/13$

C. π

D. 0

Answer: C

Watch Video Solution

46. The number of values of $\theta \in (0, \pi]$, such that

$(\cos\theta + i\sin\theta)(\cos 3\theta + i\sin 3\theta)(\cos 5\theta + i\sin 5\theta)(\cos 7\theta + i\sin 7\theta)(\cos 9\theta$

is

A. 11

B. 13

C. 14

D. 16

Answer: B

Watch Video Solution

47. If $z \in C - \{0, -2\}$ is such that $\log_{(1/7)}|z-2| > \log_{(1/7)}|z|$ then

A. $Re(z) > 1$

B. $Re(z) < 1$

C. $Im(z) > 1$

D. $Im(z) < 1$

Answer: A

Watch Video Solution

48. Im $\left(\frac{2z+1}{iz+1}\right) = 5$ represents

A. a circle

B. a straight line

C. a parabola

D. an ellipse

Answer: A

Watch Video Solution

49. If z_1, z_2 are two complex numbers such that $Im(z_1 + z_2) = 0, Im(z_1 z_2) = 0$, then:

A. $z_1 = -\bar{z}_2$

B. $z_1 = z_2$

C. $z_1 = \bar{z}_2$

D. None of these

Answer: D

Watch Video Solution

50. The number $\dfrac{(1+i)^n}{(1-i)^{n-2}}$ is equal to

A. i^{n+1}

B. $-2i^{n+1}$

C. i^{n+2}

D. $-2i^{n+2}$

Answer: B

Watch Video Solution

51. Let $\omega \neq 1$, be a cube root of unity, and $f: I \rightarrow C$ be defined by $f(n) = 1 + \omega^n + \omega^{2n}$, then range of f is

A. {0}

B. {0, 3}

C. {0, 1, 3}

D. {0, 1}

Answer: B

Watch Video Solution

52. If $z+\frac{1}{z}=2\cos\theta, z \in \mathrm{C}$ then $\mathrm{z}^{2n}-2z^{n}\cos(n\theta)$ is equal to

A. 1

B. 0

C. -1

D. $-n$

Answer: C

Watch Video Solution

53. If $\omega \ne 1$ is a cube root of unity, then $z=\sum_{k=1}^{60}\omega^{k}-\prod_{k=1}^{30}\omega^{k}$ is equal to

A. 0

B. ω

C. ω^2

D. -1

Answer: D

Watch Video Solution

54. Let g(x) and h(x) be two polynomials with real coefficients. If $p(x) = g(x^3) + xh(x^3)$ is divisible by $x^2 + x + 1$, then

A. g(1) = 0, h(1) = 1

B. g(1) = 1, h(1) = 0

C. g(1) = 0, h(1) = 0

D. g(1) = 1, h(1) = 0

Answer: C

Watch Video Solution

55. If $x^2 - x + 1$ divides the polynomial $x^{n+1} - x^n + 1$, then n must be of the form

A. 3k + 1

B. 6k + 1

C. 6k - 1

D. 3k - 1

Answer: B

View Text Solution

Exercise Level 2

1. The complex number z_1, z_2, z_3 are the vertices A, B, C of a parallelogram ABCD, then the fourth vertex D is:

A. $\frac{1}{2}(z_1 + z_2)$

B. $\frac{1}{4}(z_1 + z_2 - z_3 - z_4)$

C. $\frac{1}{3}(z_1 + z_2 + z_3)$

D. $z_1 + z_3 - z_2$

Answer: D

Watch Video Solution

Exercise Level 3

1. If a, b and c are three integers such that at least two of them are unequal and $\omega(\neq 1)$ is a cube root of unit, then the least value of the expression $\left|a + b\omega + c\omega^2\right|$ is

A. 0

B. 1

C. $\frac{\sqrt{3}}{2}$

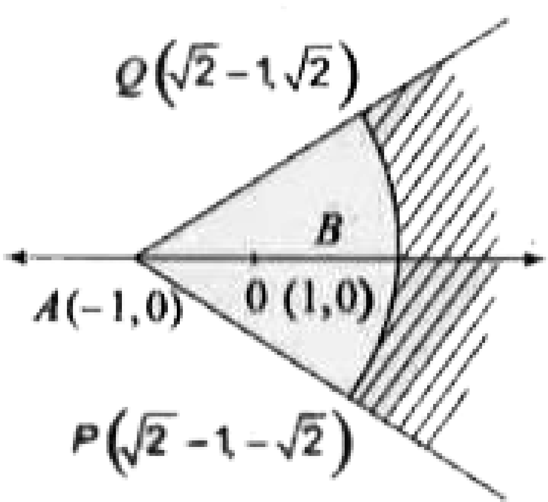
D. $\frac{1}{2}$

Answer: B

Watch Video Solution

Exercise Level 4

1. The shaded region in Figure is given by



A. $\left\{z : |z-1| < 2, |arg(z+1)| < \frac{\pi}{2}\right\}$

B. $\left\{z : |z+1| < 2, |arg(z+1)| < \frac{\pi}{2}\right\}$

C. $\left\{z : |z-1| > 2, |arg(z-1)| < \frac{\pi}{4}\right\}$

D. $\left\{z : |z+1| > 2, |arg(z+1)| < \frac{\pi}{4}\right\}$

Answer: D

Watch Video Solution

Exercise Level 5

1. If $w = \alpha + i\beta$ where $B\eta 0$ and $z \neq 1$ satisfies the condition that $\left(\frac{w - \overline{w}z}{1 - z}\right)$ is purely real then the set of values of z is

A. {z : |z| = 1}

B. $\{z : \bar{z} = z\}$

C. $\{z : |z| \neq 1\}$

D. $\{z : |z| = 1, z \neq 1\}$

Answer: D

Watch Video Solution

Exercise Level 6

1. Let z and w be non - zero complex numbers such that $zw = |z^2|$ and $|z - \bar{z}| + |w + \overline{w}| = 4$. If w varies, then the perimeter of the locus of z is

A. rectangle

B. square

C. rhombus

D. trapezium

Answer: B

Watch Video Solution

Exercise Level 7

1. Let $z = 1 - t + i\sqrt{t^2 + t + 2}$, where t is a real parameter.the locus of the z in argand plane is

A. a parabola

B. an ellipse

C. a hyperbola

D. a pair of straight lines.

Answer: C

Watch Video Solution

Exercise Level 8

1. $Z_1 \neq Z_2$ are two points in an Argand plane. If $a|Z_1| = b|Z_2|$, then prove that $\frac{aZ_1 - bZ_2}{aZ_1 + bZ_2}$ is purely imaginary.

A. in the 1st quadrant

B. in the 3rd quadrant

C. on the real axis

D. on the imaginary axis

Answer: D

Watch Video Solution

Exercise Level 9

1. If $z \in C$, then least value of the expression $|z| + |1 - z| + |z - 2|$ is

A. 4

B. $3/2$

C. 2

D. cannot be determined

Answer: C

Watch Video Solution

Exercise Level 10

1. If $k > 0, k \neq 1, \text{ and } z_1, z_2 \in C$, then $\left|\frac{z - z_1}{z - z_2}\right|$ = k represents

A. a circle

B. an ellipse

C. a parabola

D. a hyperbola

Answer: A

Watch Video Solution

Exercise Level 11

1. If $|z_1| = |z_2| = |z_3|$ and $z_1 + z_2 + z_3 = 0$, then z_1, z_2, z_3 are vertices of

A. are vertices of a right triangle

B. an equilateral triangle

C. an obtuse angled triangle

D. None of these

Answer: B

Watch Video Solution

Exercise Level 12

1. If $w = \cos\frac{\pi}{n} + i\sin\frac{\pi}{n}$ then value of $1 + w + w^2 + \ldots\ldots + w^{n-1}$ is :

A. 1 + i

B. $1 + i\tan(\pi/2n)$

C. $1 + i\cot(\pi / 2n)$

D. None of these

Answer: C

Watch Video Solution

Exercise Level 13

1. Let z_1, z_2 be two non-zero complex numbers such that $|z_1 + z_2| = |z_1 - z_2|$, then $\frac{z_1}{\bar{z}_1} + \frac{z_2}{\bar{z}_2}$ equals

A. 0

B. 1

C. -1

D. None of these

Answer: A

Watch Video Solution

Exercise Level 14

1. If $|z_1| = 2 \text{ and } (1-i)z_2 + (1+i)\bar{z}_2 = 8\sqrt{2}$, then

A. minimum value of $|z_1 - z_2|$ is 1

B. minimum value of $|z_1 - z_2|$ is 2

C. maximum value of $|z_1 - z_2|$ is 8

D. maximum value of $|z_1 - z_2|$ is 4

Answer: B

Watch Video Solution

Exercise Level 15

1. If z_1 lies on |z| = r, then equation of tangent at z_1 is

A. $\frac{z}{\bar{z}_1}+\frac{\bar{z}}{\bar{z}_1}=2$

B. $\frac{z}{\bar{z}_1}+\frac{\bar{z}}{\bar{z}_1}=r$

C. $\frac{z}{\bar{z}_1}+\frac{\bar{z}}{z_1}=2$

D. $\frac{z}{\bar{z}_1}+\frac{\bar{z}}{z_1}=r$

Answer: A

Watch Video Solution

Exercise Level 16

1. If $z \in C$, then minimum value of $|z-2+3i|+|z-1+i|$ is

A. $\sqrt{5}$

B. $2\sqrt{5}$

C. $\sqrt{13}-\sqrt{2}$

D. 0

Answer: A

Watch Video Solution

Exercise Level 17

1. If $a > 0$ and the equation $\left|z - a^2\right| + |z - 2a| = 3$, represents an ellipse, then 'a' belongs to the interval

A. (0, 3)

B. $(0, \infty)$

C. (1, 3)

D. $(3, \infty)$

Answer: A

Watch Video Solution

Exercise Level 18

1. If the points A(z), B(-z) and C(1-z) are the vertices of an equilateral triangle, then value of z is

A. $1 \pm \frac{i\sqrt{3}}{2}$

B. $\frac{1}{2}(1 \pm i)$

C. $\frac{1}{4}(1 \pm \sqrt{3}i)$

D. $\frac{1}{3}(1 \pm \sqrt{3}i)$

Answer: C

Watch Video Solution

Exercise Level 19

1. If $|z+1| + |z-3| \leq 10$, then the range of values of |z - 7| is

A. [0, 10]

B. [3, 13]

C. [2, 12]

D. [7, 9]

Answer: B

Watch Video Solution

Exercise Level 20

1. If $\omega \neq 1$ is a cube root of unity and $|z-1|^2 + 2|z-\omega|^2 = 3|z-\omega^2|^2$ then z lies on

A. a straight line

B. a parabola

C. an ellipse

D. a rectangular hyperbola

Answer: A

Watch Video Solution

Exercise Level 21

1. The least positive integer n for which $\left(\frac{1+i}{1-i}\right)^n = \frac{2}{\pi}\sin^{-1}\left(\frac{1+x^2}{2x}\right)$, where $x > 0$ and $i = \sqrt{-1}$ is

A. 2

B. 4

C. 8

D. 32

Answer: B

Watch Video Solution

Exercise Numerical Answer Type Questions

1. Suppose $a \in R$ and $z \in C$. If $|z| = 2$ and $\frac{z-\alpha}{z+\alpha}$ is purely imaginary, then 1.25 $|\alpha|$ is equal to __________.

Watch Video Solution

2. If $\alpha and \beta$ are roots of the equation $x^2 - 2x + 2 = 0$, then the least calue of n for which $\left(\left(\frac{\alpha}{\beta}\right)\right)^n = 1$ is

Watch Video Solution

3. Let $S = \left\{\frac{2\alpha + 3i}{2\alpha - 3i} : \alpha \in R\right\}$. All the points of S lie on a circle of radius __________.

Watch Video Solution

4. Let $\omega \neq 1$ be cube root of unity. Suppose $(1+\omega)^{2023} = A + B\omega$ where A, B $\in$ R, then $|A + Bi|^2$ is equal to ____________.

Watch Video Solution

5. If $(7+i)(z+\bar{z}) - (4+i)(z-\bar{z}) + 116i = 0$ then $z\bar{z}$ is equal to ____________.

Watch Video Solution

6. Let C_1 be the curve represented by $\frac{2z+i}{z-2}$ is purely imaginary, and C_2 be the curve represented by $arg\left(\frac{z+i}{z+1}\right) = \frac{\pi}{2}$. Let m = slope of the common chord of C_1 and C_2, then |m| is equal to _________.

Watch Video Solution

7. Let $a = 3 + 4i$, z_1 and z_2 be two complex numbers such that $|z_1| = 3$ and $|z_2 - a| = 2$, then maximum possible value of $|z_1 - z_2|$ is __________.

Watch Video Solution

8. Let α be the real and β, γ be the complex roots of $x^3 + 3x^2 + 3x + 28 = 0$, then $|\alpha + 2\beta + 2\gamma|$ = ________________.

Watch Video Solution

9. Let $z = \left(\frac{2}{i + \sqrt{3}}\right)^{200} + \left(\frac{2}{i - \sqrt{3}}\right)^{200}$, then $\left|z + (1.7)^2\right|$ = ________________.

Watch Video Solution

10. Let z be a non-zero complex number such that area of the triangle with vertices $P(z), Q\left(ze^{\pi i/6}\right)$ and $R\left(z\left(1+e^{\pi i/6}\right)\right)$ is 0.64, then z lies on a circle of diameter ________.

Watch Video Solution

11. Suppose $x, y \in R$. If x^2+y+4i is conjugate of $-3+x^2yi$, then maximum possible value of $(|x|+|y|)^2$ is equal to __________.

Watch Video Solution

12. All the roots of the equation $x^{11}-x^6-x^5+1=0$ lie on a circle of radius ________.

Watch Video Solution

13. If $13e^{i\tan^{-1}(5/12)}=a+ib$, then |a| + |b| is equal to _________.

Watch Video Solution

14. Suppose, $a, b, c \geq 0, c \neq 1, a^2 + b^2 + c^2 = c$. If $\left|\frac{a + ib}{2 - c}\right| = \frac{1}{2}$, then c is equal to __________.

Watch Video Solution

15. Let z_1, z_2, z_3 be the roots of $iz^3 + 5z^2 - z + 5i = 0$, then $|z_1| + |z_2| + |z_3|$ = ____________.

Watch Video Solution

16. Let $S = \left\{z \in C : z^2 = 4(i\bar{z})^2\right\}$, then $\sum_{z \in S} \left|z + \frac{1}{2}i\right|$ = __________.

Watch Video Solution

17. Let $z = \frac{1}{2}\left(\sqrt{3} + i\right)$, then $\left|\left(z^{101} + i^{103}\right)^{105}\right|$ = _______.

Watch Video Solution

18. If $\alpha = \cos\left(\frac{8\pi}{11}\right) + i\sin\left(\frac{8\pi}{11}\right)$ then $Re(\alpha + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5)$ is

Watch Video Solution

19. Eccentricity of conic $|z - 5i| + |z + 5i| = 25$ is __________.

Watch Video Solution

20. If α is a non-real root of $x^6 = 1$ then $\frac{\alpha^5 + \alpha^3 + \alpha + 1}{\alpha^2 + 1} =$

Watch Video Solution

Questions From Previous Years Aieee Jee Main Papers

1. Let $zandw$ be two non-zero complex number such that $|z| = |w|$ and $arg(z) + arg(w) = \pi$, then z equals. w (b) $-w$ (c) w (d) $-w$

A. $\overline{w}$

B. $-\overline{w}$

C. w

D. $-w$

Answer: B

Watch Video Solution

2. If $|z - 4| < |z - 2|$, then

A. $Re(z) > 0$

B. $Re(z) < 0$

C. $Re(z) > 3$

D. $Re(z) > 2$

Answer: C

Watch Video Solution

3. The locus of the center of a circle which touches the circles $|z - z_1| = a, |z - z_2 = b|$ externally will be

A. an ellipse

B. a hyperbola

C. a circle

D. a pair of straight lines.

Answer: B

Watch Video Solution

4. If $\left(\frac{1+i}{1-i}\right)^x = 1$, then

n $\in$ N.

A. x = 2n, where n is any positive integer

B. x = 4n + 1, where n is any positive integer

C. x = 2n + 1, where n is any positive integer

D. x = 4n, where n is any positive integer

Answer: D

Watch Video Solution

5. If z and w are two complex number such that $|zw| = 1$ and $arg(z) - arg(w) = \frac{\pi}{2}$, then show that $\bar{z}w = -i$.

A. -1

B. i

C. $-i$

D. 1

Answer: C

Watch Video Solution

6. Let z_1 and z_2 be the roots of $z^2 + az + b = 0$. If the origin, z_1 and z_2 from an equilateral triangle, then

A. $a^2 = 2b$

B. $a^2 = 3b$

C. $a^2 = 4b$

D. $a^2 = b$

Answer: B

Watch Video Solution

7. Let z, w be complex numbers such that $\bar{z} + i\overline{w} = 0$ and $arg zw = \pi$

Then $arg z$ equals

A. $\dfrac{3\pi}{4}$

B. $\frac{\pi}{2}$

C. $\frac{\pi}{4}$

D. $\frac{5\pi}{4}$

Answer: A

Watch Video Solution

8. If $z = x - iy$ and $z'^{\frac{1}{3}} = p + iq$, then $\frac{1}{p^2 + q^2}\left(\frac{x}{p} + \frac{y}{q}\right)$ is equal to

A. 2

B. -1

C. 1

D. -2

Answer: D

Watch Video Solution

9. If $\left|z^2 - 1\right| = |z|^2 + 1$, then z lies on (a) a circle (b) the imaginary axis (c) the real axis (d) an ellipse

A. a circle

B. the imaginary axis

C. the real axis

D. an ellipse

Answer: B

Watch Video Solution

10. If the cube roots of unity are $1, \omega, \omega^2$, then the roots of the equation $(x-1)^3 + 8 = 0$, arte

A. $-1, 1-2\omega, 1-2\omega^2$

B. $-1, 1+2\omega, 1+2\omega^2$

C. $-1, \ -1+2\omega, \ -1-2\omega^2$

D. $-1, \ -1, \ -1$

Answer: A

Watch Video Solution

11. If $w = \dfrac{z}{z - \frac{1}{3}i}$ and |w| =1, then z lies on

A. straight line

B. a parabola

C. an ellipse

D. a circle

Answer: A

Watch Video Solution

12. If z_1 and z_2, are two non-zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$ then $arg(z_1) - arg(z_2)$ is equal to (1) 0 (2) $-\frac{\pi}{2}$ (3) $\frac{\pi}{2}$ (4) $-\pi$

A. 0

B. $-\frac{\pi}{2}$

C. $\frac{\pi}{2}$

D. π

Answer: A

Watch Video Solution

13. $\sum_{k=1}^{10}\left(\frac{\sin(2k\pi)}{11} + i\frac{\cos(2k\pi)}{11}\right)$

A. $-i$

B. i

C. 1

D. -1

Answer: A

Watch Video Solution

14. If $z^2+z+1=0$ where z is a complex number, then the value of $\left(z+\frac{1}{z}\right)^2+\left(z^2+\frac{1}{z^2}\right)^2+\ldots.+\left(z^6+\frac{1}{z^6}\right)^2$ is

A. 12

B. 18

C. 54

D. 6

Answer: A

Watch Video Solution

15. If $|z+4| \leq 3$, then the maximum value of $|z+1|$ is (1) 4 (B) 10 (3) 6 (4) 0

A. 4

B. 10

C. 6

D. 0

Answer: C

Watch Video Solution

16. The conjugate of a complex number is $\frac{1}{i-1}$. Then the complex number is (1) $\frac{-1}{i-1}$ (2) $\frac{1}{i+1}$ (3) $\frac{-1}{i+1}$ (4) $\frac{1}{i-1}$

A. $\frac{-1}{i-1}$

B. $\frac{1}{i+1}$

C. $\frac{-1}{i+1}$

D. $\dfrac{1}{i-1}$

Answer: C

Watch Video Solution

17. If $\left|z-\dfrac{4}{z}\right|=2$, then the maximum value of $|z|$

A. 1

B. $2+\sqrt{2}$

C. $\sqrt{3}+1$

D. $\sqrt{5}+1$

Answer: D

Watch Video Solution

18. The number of complex numbers z such that $|z-1| = |z+1| = |z-i|$ is

A. 2

B. ∞

C. 0

D. 1

Answer: D

Watch Video Solution

19. If $\omega(\neq 1)$ be a cube root of unity and $(1+\omega)^7 = A + B\omega$, then A and B are respectively the numbers.

A. (-1, 1)

B. (0, 1)

C. (1, 1)

D. (1, 0)

Answer: C

Watch Video Solution

20. Let α, β be real and z be a complex number. If $z^2 + \alpha z + \beta = 0$ has two distinct roots on the line Re $z = 1$, then it is necessary that : (1) $b \in (0, 1)$ (2) $b \in (-1, 0)$ (3) $|b| = 1$ (4) $b \in (1, \infty)$

A. $\beta \in (1, \infty)$

B. $\beta \in (0, 1)$

C. $\beta(-1, 0)$

D. $|\beta| = 1$

Answer: A

Watch Video Solution

21. If $z \neq 1$ and $\frac{z^2}{z-1}$ is real, the point represented by the complex numbers z lies

A. on a circle with centre at the origin.

B. either on the real axis or on a circle not passing through the origin.

C. on the imaginary axis.

D. either on the real axis or on a circle passing through the origin

Answer: D

Watch Video Solution

22. The area of the triangle on the Arand plane formed by the complex numbers z, iz and z+iz is?

A. $2|z|^2$

B. $\frac{1}{2}|z|^2$

C. $4|z|^2$

D. $|z|^2$

Answer: B

Watch Video Solution

23. If z is a complex number of unit modulus of modulus and argument θ, then $arg\left(\frac{1+z}{1+\bar{z}}\right)$ equals to

A. $\frac{\pi}{2}-\theta$

B. θ

C. $\pi-\theta$

D. $-\theta$

Answer: B

Watch Video Solution

24. if $\frac{5z_2}{7z_1}$ is purely imaginary number then $\left|\frac{2z_1+3z_2}{2z_1-3z_2}\right|$ is equal to

A. 2

B. 5

C. 3

D. 1

Answer: D

Watch Video Solution

25. Let $a = Im\left(\frac{1+z^2}{2iz}\right)$, where z is any non-zero complex number. Then the set $A = \{a : |z| = 1 \text{ and } z \neq \pm 1\}$ is equal to

A. (-1, 1)

B. [-1, 1]

C. [0, 1)

D. (-1, 0]

Answer: A

Watch Video Solution

26. If a complex number z satisfies $z+\sqrt{2}|z+1|+i=0$, then |z| is equal to :

A. 2

B. $\sqrt{3}$

C. $\sqrt{5}$

D. 1

Answer: C

Watch Video Solution

27. If z is a complex number such that $|z| \geq 2$ then the minimum value of $\left|z + \frac{1}{2}\right|$ is

A. is strictly greater than 5/2

B. is strictly greater that 3/2 but less than 5/2

C. is equal to 5/2

D. lies in the interval (1, 2)

Answer: D

Watch Video Solution

28. Let $w(Imw \neq 0)$ be a complex number. Then the set of all complex numbers z satisfying the equal $w - \overline{w}z = k(1 - z)$, for some real number k, is :

A. $\{z : |z| = 1\}$

B. $\{z : z = \bar{z}\}$

C. $\{z : z \neq 1\}$

D. $\{z : |z| = 1, z \neq 1\}$

Answer: D

Watch Video Solution

29. If z_1, z_2 and z_3, z_4 are two pairs of conjugate complex numbers then $\arg\left(\frac{z_1}{z_4}\right) + arg\left(\frac{z_2}{z_3}\right)$ equals

A. 0

B. $\frac{\pi}{2}$

C. $\frac{3\pi}{2}$

D. π

Answer: A

Watch Video Solution

30. Let $z \neq i$ be any complex number such that $\frac{z-i}{z+i}$ is a purely imaginary number. Then $z+\frac{1}{z}$ is

A. 0

B. any non-zero real number other than 1

C. any non-zero real number

D. a purely imaginary number

Answer:

Watch Video Solution

31. For all complex numbers z of the form $1+i\alpha, \alpha \in \mathrm{R}$ if $\mathrm{z}^2 = x+iy$, then

A. $y^2 - 4x + 2 = 0$

B. $y^2 + 4x - 4 = 0$

C. $y^2 - 4x + 4 = 0$

D. $y^2 + 4x + 2 = 0$

Answer: B

Watch Video Solution

32. A complex number z is said to be uni-modular if $|z| = 1$. Suppose z_1 and z_2 are complex numbers such that $\frac{z_1 - 2z_2}{2 - z_1 z_2}$ is uni-modular and z_2 is not uni-modular. Then the point z_1 lies on a:

A. straight line parallel to the x-axis.

B. straight line parallel to the y-axis.

C. circle of radius 2.

D. circle of radius $\sqrt{2}$

Answer: C

Watch Video Solution

33. The largest value of r for which the region represented by the set $\{\omega \in \mathbb{C} / |\omega - 4 - i| \leq r\}$ is contained in the region represented by the set $\{z \in \mathbb{C} / |z - 1| \leq |z + i|\}$, is equal to

A. $\sqrt{17}$

B. $\sqrt{2}$

C. $\frac{3}{2}\sqrt{2}$

D. $\frac{5}{2}\sqrt{2}$

Answer: D

Watch Video Solution

34. If Z is a non-real complex number, then find the minimum value of $\frac{Imz^5}{Im^5 z}$

A. -1

B. -2

C. -4

D. -5

Answer: C

Watch Video Solution

35. A value of for which $\frac{2+3i\sin\theta}{1-2i\sin\theta}$ purely imaginary, is : (1) $\frac{\pi}{3}$ (2) $\frac{\pi}{6}$ (3) $\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$ (4) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

A. $\frac{\pi}{3}$

B. $\frac{\pi}{6}$

C. $\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

D. $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Answer: D

Watch Video Solution

36. The point represented by $2+i$ in the Argand plane moves 1 unit eastwards, then 2-units northwards and finally from there $2\sqrt{2}$ units in the south-westwards direction. Then its new position in the Argand plane is at the point represented by

A. 1 + i

B. 2 + 2i

C. $-2-2i$

D. $-1-i$

Answer: A

Watch Video Solution

37. Let $z=1+ai$ be a complex number, $a>0$,such that z^3 is a real number. Then the sum $1+z+z^2+\ldots+z^{11}$ is equal to:

A. 1365 $\sqrt{3}i$

B. $-1365\sqrt{3}i$

C. $-1250\sqrt{3}i$

D. 1250 $\sqrt{3}i$

Answer: B

Watch Video Solution

38. Let $z \in C$, the set of complex numbers. Thenthe equation, $2|z+3i|-|z-i|=0$ represents :

A. a circle with radius 8/3

B. a circle with diameter 10/3

C. an ellipse with length of major axis 16/3

D. an ellipse with length of minor axis 16/9

Answer: A

Watch Video Solution

39. The equation $Im\left(\frac{iz-2}{z-i}\right)+1=0, z\&\varepsilon; C, z\neq i$ represents a part of a circle having radius equalto.4

A. 2

B. 1

C. $3/4$

D. $1/2$

Answer: C

Watch Video Solution

40. If $\alpha, \beta \in C$ are the distinct roots of the equation $x^2-x+1=0$, then $\alpha^{101}+\beta^{107}$ is equal to

A. 0

B. 1

C. 2

D. -1

Answer: B

Watch Video Solution

41. The set of all $\alpha \in R$ for which $w = \dfrac{1 + (1 - 8\alpha)z}{1 - z}$ is a purely imaginary number, for all $z \in C$, satisfying |z| = 1 and $Re(z) \neq 1$, is

A. an empty set

B. {0}

C. $\left\{0, \dfrac{1}{4}, -\dfrac{1}{4}\right\}$

D. equal to R

Answer: B

Watch Video Solution

42. If $|z-3+2i|\leq 4$, (where $i=\sqrt{-1}$) then the difference of greatest and least values of $|z|$ is

A. $2\sqrt{13}$

B. 8

C. $4+\sqrt{13}$

D. $\sqrt{13}$

Answer: C

Watch Video Solution

43. The least positive integer n for which $\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^n=1$, is

A. 2

B. 3

C. 5

D. 6

Answer: B

Watch Video Solution

44. Let $A = \left\{\theta \in \left(-\frac{\pi}{2}, \pi\right) : \frac{3 + 2i\sin\theta}{1 - 2\sin\theta}\right.$ is purely imaginary }

Then the sum of the elements in A is

A. $\frac{5\pi}{6}$

B. π

C. $\frac{3\pi}{4}$

D. $\frac{2\pi}{3}$

Answer: A

Watch Video Solution

45. Let Z_0 is the root of equation $x^2+x+1=0$ and $Z=3+6i(Z_0)^{81}-3i(Z_0)^{93}$ Then arg (Z) is equal to (a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) π (d) $\frac{\pi}{6}$

A. $\frac{\pi}{4}$

B. $\frac{\pi}{6}$

C. $\frac{\pi}{3}$

D. 0

Answer: A

Watch Video Solution

46. Let z_1 and z_2 be any two non-zero complex numbers such that $3|z_1|=2|z_2|$. If $z=\frac{3z_1}{2z_2}+\frac{2z_2}{3z_1}$, then

A. Re (z) = 0

B. $|z|=\sqrt{\frac{5}{2}}$

C. $|z| = \frac{1}{2}\sqrt{\frac{17}{2}}$

D. Im (z) = 0

Answer:

Watch Video Solution

47. If $z = \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5$, then

A. I(z) = 0

B. $R(z) > 0 \text{ and } I(z) > 0$

C. $Re(z) < 0 \text{ and } I(z) > 0$

D. Re(z) = -3

Answer: A

Watch Video Solution

48. If x, y are real and $\left(-2-\frac{i}{3}\right)^3 = \frac{x+iy}{27}$ then value of $y-x$ is equal to (A) 91 (B) -91 (C) 85 (D) -85

A. -85

B. 85

C. -91

D. 91

Answer: D

Watch Video Solution

49. Let $\frac{z-\alpha}{z+\alpha}$ is purely imaginary and $|z| = 2, \alpha \varepsilon R$ then α is equal to (A) 2 (B) 1 (C) $\sqrt{2}$ (D) $\sqrt{3}$

A. 1

B. 2

C. $\sqrt{2}$

D. $\frac{1}{2}$

Answer: B

Watch Video Solution

50. Let Z_1 and Z_2 be two complex numbers satisfying $|Z_1| = 9$ and $|Z_2 - 3 - 4i| = 4$. Then the minimum value of $|Z_1 - Z_2|$ is

A. 0

B. 1

C. $\sqrt{2}$

D. 2

Answer: A

Watch Video Solution

51. If α and β be the roots of the equation $x^2 - 2x + 2 = 0$, then the least value of n for which $\left(\frac{\alpha}{\beta}\right)^n = 1$ is:

A. 2

B. 5

C. 4

D. 3

Answer: C

Watch Video Solution

52. if $z = \frac{\sqrt{3}}{2} + \frac{i}{2}\left(i = \sqrt{-1}\right)$, then $\left(1 + iz + z^5 + iz^8\right)^9$ is equal to:

A. 0

B. 1

C. $(-1 + 2i)^9$

D. -1

Answer: D

Watch Video Solution

53. All the points in the set $S=\left\{\frac{\alpha+i}{\alpha-i}:\alpha\in R\right\}(i=\sqrt{-1})lie$ on a

A. straight line whose slope is 1

B. circle whose radius is 1

C. circle whose radius is $\sqrt{2}$

D. straight line whose slope is -1

Answer: B

Watch Video Solution

54. Let $z\in C$ be such that $|z|<1$.If $\omega=\frac{5+3z}{5(1-z)}$ then

A. 5 Re (w) $>$ 4

B. 4 Im (w) $>$ 5

C. 5 Re (w) $>$ 1

D. 5 Im (w) $<$ 1

Answer: C

Watch Video Solution

55. If $a > 0$ and $z = \frac{(1+i)^2}{a-1}$ has magnitude $\sqrt{\frac{2}{5}}$ the $\bar{z} is$ equal to

A. $-\frac{1}{5} + \frac{3}{5}i$

B. $\frac{1}{5} - \frac{3}{5}i$

C. $-\frac{1}{5} - \frac{3}{5}i$

D. $-\frac{1}{5} - \frac{3}{5}i$

Answer: C

Watch Video Solution

56. If $z and w$ are two complex number such that $|zw| = 1 and arg(z) - arg(w) = \frac{\pi}{2}$, then show that $zw = -i$.

A. $\bar{z}w = i$

B. $z\overline{w} = \frac{1}{\sqrt{2}}(-1+i)$

C. $z\overline{w} = \frac{1}{\sqrt{2}}(1-i)$

D. $\bar{z}w = -i$

Answer: D

Watch Video Solution

57. The equation |z - i| = |z - 1| represents :

A. a circle of radius 1

B. a circle of radius 1/2

C. the line through the origin with slope -1

D. the line through the origin with slope 1.

Answer: D

Watch Video Solution

58. if $\frac{2z-n}{2z+n} = 2i - 1, n \in N \in N$ and $IM(z) = 10$, then

A. n = 20 and Re(z) = - 10

B. n = 40 and Re(z) = - 10

C. n = 40 and Re(z) = 10

D. n = 20 and Re(z) = 10

Answer: B

Watch Video Solution

Questions From Previous Years B Architecture Entrance Examination Paper

1. $\dfrac{5 + i\sin\theta}{5 - 3i\sin\theta}$ is a real number when

A. $\theta = \pi / 4$

B. $\theta = -\pi$

C. $\theta = -\pi / 2$

D. $\theta = \pi / 2$

Answer: B

Watch Video Solution

2. Two points P and Q in the Argand diagram represent z and $2z + 3 + i$. If P moves on a circle with centre at the origin and radius 4, then the locus of Q is a circle with centre

A. $-2 - i, 6$

B. $2 - i, 3$

C. $2 + i, 6$

D. $2+i, 3$

Answer: C

Watch Video Solution

3. Let z be a complex number such that |z| = 2, then maximum possible value of $\left|z+\frac{2}{z}\right|$ is

A. 1

B. 2

C. 3

D. 4

Answer: C

Watch Video Solution

4. If $i = \sqrt{-1}$, then $4 + 3\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^{127} + 5\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^{124}$ is equal to

A. $4\sqrt{3}i$

B. $2\sqrt{3}i$

C. $1 - \sqrt{3}i$

D. $1 + \sqrt{3}i$

Answer: A

Watch Video Solution

5. The real part of a complex number z having minimum principal argument and satisfying $|z - 5i| \leq 1$ is

A. $\frac{2}{5}\sqrt{6}$

B. 0

C. $\frac{2}{\sqrt{5}}$

D. $-\frac{1}{5}\sqrt{6}$

Answer: A

Watch Video Solution

6. Show that the area of the triangle on the Argand diagram formed by the complex number $z, izandz+iz$ is $\frac{1}{2}|z|^2$

A. 0

B. $\frac{1}{2}|z|^2$

C. $|z|^2$

D. $2|z|^2$

Answer: B

Watch Video Solution

7. Two circles in the complex plane are

$C_1 : |z - i| = 2$
$C_2 : |z - 1 - 2i| = 4$ then

A. C_1 and C_2 touch each other

B. C_1 and C_2 intersect at two distinct points

C. C_1 lies within C_2

D. C_2 lies within C_1

Answer: C

Watch Video Solution

8. If $z = i(i + \sqrt{2})$, then value of $z^4 + 4z^3 + 6z^2 + 4z$ is

A. -5

B. 3

C. 6

D. -9

Answer: B

Watch Video Solution

9. Suppose z is a complex number such that $z \neq -1, |z| = 1$ and $arg(z) = \theta$. Let $w = \frac{z(1-\bar{z})}{\bar{z}(1+z)}$, then Re(w) is equal to

A. $1 + \cos\left(\frac{\theta}{2}\right)$

B. $1 - \sin\left(\frac{\theta}{2}\right)$

C. $-2\sin^2\left(\frac{\theta}{2}\right)$

D. $2\cos^2\left(\frac{\theta}{2}\right)$

Answer: C

Watch Video Solution

10. If $|z_1| = |z_2| = |z_3| = 1$ and $z_1 + z_2 + z_3 = \sqrt{2} + i$, then the number $z_1\bar{z}_2 + z_2\bar{z}_3 + z_3\bar{z}_1$ is :

A. a positive real number

B. a negative real number

C. always zero

D. a purely imaginary number

Answer: D

Watch Video Solution

11. Let $S = \{z \in C : z(iz_1 - 1) = z_1 + 1, |z_1| < 1\}$. Then, for all $z \in S$, which one of the following is always true ?

A. $Rez + Imz < 0$

B. $Rez < 0$

C. $Rez - Imz > 1$

D. $Rez - Imz < 0$

Answer:

Watch Video Solution

12. If $(4+3i)^2 = 7+24i$, then a value of $(7+\sqrt{-576})^{1/2} - (7-\sqrt{-576})^{1/2}$ is :

A. $-6i$

B. $-3i$

C. 2i

D. 6

Answer: A

Watch Video Solution

13. Let $A = \{z \in \mathbb{C} : |z| = 25)$ and $B = \{z \in \mathbb{C} : |z + 5 + 12i| = 4\}$.

Then the minimum value of $|z - \omega|$, for $z \in A$ and $\omega \in B$, is

A. 7

B. 8

C. 9

D. 6

Answer: B

Watch Video Solution

14. If z_1, z_2 and z_3 are three distinct complex numbers such that

$|z_1| = 1, |z_2| = 2, |z_3| = 4, arg(z_2) = arg(z_1) - \pi, arg(z_3) = arg(z_1) + \pi$

, then $z_2 z_3$ is equal to

A. $-8iz_1^2$

B. $8iz_1^2$

C. $-\dfrac{8i}{z_i^2}$

D. $\dfrac{8i}{z_i^2}$

Answer: A

Watch Video Solution

15. The locus of the point $w = Re(z) + \dfrac{1}{z}$, where $|z| = 3$, in complex plane is :

A. parabola

B. a circle

C. an ellipse

D. a hyperbola

Answer: C

Watch Video Solution

16. Let $z(\neq -1)$ be any complex number such that |z| = 1. Then the imaginary part of $\frac{\bar{z}(1-z)}{z(1+\bar{z})}$ is : (Here θ = Arg(z))

A. $-\tan\left(\frac{\theta}{2}\right)\sin\theta$

B. $\tan\left(\frac{\theta}{2}\right)\cos\theta$

C. $-\tan\left(\frac{\theta}{2}\right)\cos\theta$

D. $\tan\left(\frac{\theta}{2}\right)\sin\theta$

Answer: C

Watch Video Solution

17. Let $u = \frac{1}{2}(-1+\sqrt{3}i)$ and $z = u - u^2 - 2$. Then the value of $z^4 + 3z^3 + 2z^2 - 11z - 6$ is :

A. 1

B. -1

C. 2

D. -2

Answer: A

View Text Solution