MATHS

BOOKS - MCGROW HILL EDUCATION MATHS (HINGLISH)

LIMITS AND CONTINUITY

Illustration

1. Evaluate $\lim_{x \to 2} \frac{x^2 + x + 2}{x^3 + 1}$

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2. Evaluate $\lim_{x \to 2} \frac{3 + \sin x}{\cos x}$

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3. Evaluate $\lim_{x \to 1} \frac{x^3 - 1}{x^2 - 1}$

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4. Evaluate $\lim_{x \to a} \frac{\sqrt{4a + 3x} - \sqrt{x + 6a}}{\sqrt{2a + 5x} - \sqrt{3a + 4x}}$

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5. Evaluate $\lim_{x \to 0} \frac{a^{\sin x} - 1}{x \cos x}$

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6. To find $\lim_{x \to 0} \left(1 + \left(x^2 + 3x\right)\right)^{1/\sin x}$

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7. What is wrong with the following application of L'Hopital's rule ?

$$\lim_{x \to 1} \frac{x^3 + 3x - 4}{2x^2 + x - 3}$$

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8. What is wrong with the following application of L'Hopital's rule ?

$$\lim_{x \to 0+} (\sin x)^x$$

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9. What is wrong with the following application of L'Hopital's rule ?

$$\lim_{x \to 0} (1 - 3x)^{4/x}$$

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10. What is wrong with the following application of L'Hopital's rule ?

$$\lim_{x \to 0} \frac{(1 + x)^{1/x} - e}{x}$$

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11. $\lim_{n \to \infty} \frac{1}{n} = 0, \lim_{n \to \infty} \frac{1}{n^2}$

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12. To show that $\lim_{x \to 1} \sin \frac{1}{x-1}$ does not exist.

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13. To show $\lim_{x \to 0} x \sin \frac{1}{x} = 0.$

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14. Find $\lim_{x \to 0+} \frac{|x|}{x}$

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15. Examine the continuity at origin $f(x) = \begin{cases} \frac{|x|}{x} & ,x \neq 0 \\ 1 & ,x = 0 \end{cases}$

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Solved Examples Single Correct Answer

1. Let $f(x) = \begin{cases} x-|x| & ,x \neq 0 \\ 1 & ,x = 0 \end{cases}$ then

A. $\lim_{x \to 0+} f(x) = 1$

B. $\lim_{x \to 0-} f(x) = 0$

C. $\lim_{x \to 0+} f(x) \neq \lim_{x \to 0-} f(x)$

D. $\lim_{x \to 0+} f(x)$ does not exist

Answer: C

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2. Let $f(x) = \left[\frac{\sin x}{x}\right], x \neq 0$, where [.] denotes the greatest integer function then $\lim_{x \to 0} f(x)$

A. does not exist

B. is equal to 1

C. is equal to 0

D. $\lim_{x \to 0+}$ f(x)=1

Answer: C

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3. $(\lim)_{x \to \infty} \frac{(x+1)^{10} + (x+2)^{10} + \ + (x+100)^{10}}{x^{10} + 10^{10}} isequa < o$ 0 (b) 1 (c) 10 (d) 100

A. 100

B. 0

C. 1

D. 10

Answer: A

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4. $\lim_{x \to 1} \left(\frac{1}{1-x} - \frac{3}{1-x^3} \right)$ is equal

A. 1

B. -1

C. 2

D. $-\frac{1}{2}$

Answer: B

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5. $\lim_{x \to \pi} \frac{\sin 3x}{\sin 2x}$ is equal to

A. $\frac{3}{2}$

B. 0

C. 1

D. $-\frac{3}{2}$

Answer: D

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6. If $f(x) = \begin{cases} mx^2 + x + n & ,x < 0 \\ nx + m & ,0 \leq x \leq 1 \\ 2nx^3 + x^2 - 2x + m & ,x > 1 \end{cases}$

and $\lim_{x \to 0} f(x)$ and $\lim_{x \to 1}$ f(x) exist then

A. m=2,n=1

B. m=0,n=1

C. m=1,n=2

D. m=1,n=1

Answer: D

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7. Let $f(x) = \begin{cases} -2\sin x & , \text{ if} x \leq -\frac{\pi}{2} \\ A\sin x + B & , \text{ if} -\frac{\pi}{2} < x < \frac{\pi}{2} \\ \cos x & , \text{ if} x \geq \frac{\pi}{2} \end{cases}$

Then

A. A=1,B=-1

B. A=-1,B=1

C. A=0,B=2

D. A=-2,B=2

Answer: B

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8. The function $f(x) = \frac{x^2 - 1}{x^3 - 1}$ is not defined for x=1. The value of f(1) so that the function extended by this value is continuous is

A. $\frac{2}{3}$

B. $\frac{1}{3}$

C. 1

D. 0

Answer: A

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9. Let $y = -\frac{2^{\frac{1}{x}} - 1}{2^{\frac{1}{x}} + 1}$, then

A. $\lim_{x \to 0} y = -1$

B. $\lim_{x \to 0} y = 1$

C. $\lim_{x \to 0+} y = -1$

D. $\lim_{x \to 0+} y = \lim_{x \to 0-} y$

Answer: C

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10. The value of $\lim_{x \to \infty} \frac{x^6}{6^x}$ is

A. 1

B. 0

C. -1

D. not a finite number

Answer: B

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Solved Examples Level 1 Single Correct Answer

1. $\lim_{x \to \infty} \left(\frac{x+1}{x+2} \right)^{2x+1}$ is

A. e

B. e^{-2}

C. e^{-1}

D. 1

Answer: B

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2. $\lim_{x \to 0} \frac{(\cos x)^{1/2} - (\cos x)^{1/3}}{\sin^2 x}$ is

A. $1/6$

B. $-1/12$

C. $2/3$

D. $1/3$

Answer: B

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3. $\lim_{x \to 1} \frac{1 + \log x - x}{1 - 2x - x^2}$ equals

A. 1

B. 0

C. -1

D. $-1/2$

Answer: B

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4. $\lim_{x \to 0} \frac{\tan x - x}{x^2 \tan x}$ equals

A. $1/3$

B. $2/3$

C. $1/2$

D. 1

Answer: A

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5. The value of $\lim_{n \to \infty} \left(\frac{1}{1-n^4} + \frac{8}{1-n^4} + \ldots + \frac{n^3}{1-n^4} \right)$ is

A. $1/4$

B. $1/8$

C. $1/2$

D. none of these

Answer: D

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6. $\lim_{x \to \pi/3} \frac{2\sin(x - \pi/3)}{1 - 2\cos x}$ is

A. $1/\sqrt{2}$

B. $2/\sqrt{3}$

C. $2/3$

D. $1/3$

Answer: B

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7. $\lim_{x \top i/4} \frac{1-\cot^3 x}{2-\cot x-\cot^3 x}$, is

A. $11/4$

B. $3/4$

C. $1/2$

D. none of these

Answer: B

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8. $\lim_{x \to 1} \left[\left(\frac{4}{x^2 - x^{-1}} - \frac{1 - 3x + x^2}{1 - x^3} \right)^{-1} + 3 \frac{x^4 - 1}{x^3 - x^{-1}} \right]$

A. 3

B. 2

C. 4

D. $28/3$

Answer: A

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9. $\lim_{x \to 3} \frac{\sqrt{1 - \cos 2(x - 3)}}{x - 3}$

A. $= \sqrt{2}$

B. does not exist

C. $= 1$

D. $= -\sqrt{2}$

Answer: B

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10. Let $f: R \to [0, \infty)$ be such that $\lim_{x \to 5} f(x)$ exists and $\lim_{x \to 5} \frac{[f(x)]^2 - 9}{\sqrt{|x-5|}} =$ is equal to:

A. 0

B. 1

C. 2

D. 3

Answer: D

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11. If $f(x) = \begin{cases} \frac{\sin(1+[x])}{[x]} \text{for} [x] \neq 0 \\ 0 \quad \text{for} \quad [x] - 0 \end{cases}$ where $[x]$ denotes the greatest integer not exceeding x, then $\lim_{x \to 0^-} f(x) =$

A. 1

B. 0

C. -1

D. none of these

Answer: B

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12. Let $f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right) + \sin\left(\frac{1}{x^2}\right) & ; x \neq 0 \\ 0 & ; x = 0 \end{cases}$, then $\lim_{x \to \infty} f(x)$ is equal to

A. 0

B. $-1/2$

C. 1

D. none of these

Answer: C

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13. which of the following limits equal to $\frac{1}{2}$: (A) $\lim_{n \to \infty} \left(\frac{1}{1.3} + \frac{1}{3.5} + \ldots + \frac{1}{(2n-1)(2n+1)} \right)$

A. $1/4$

B. $1/2$

C. 1

D. none of these

Answer: B

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14. If $\lim_{x \to 0} (1 + ax)^{b/x} = e^4$, where a and b are natural numbers then

A. a=4,b=2

B. a=8,b=4

C. a=16,b=8

D. none of these

Answer: D

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15. $\lim_{x \to 0} \frac{a^x - 1}{\sqrt{a + x} - \sqrt{a}}$ is

A. $2\sqrt{a} \log a$

B. $\sqrt{a}$ log a

C. log a

D. none of these

Answer: A

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16. $f(x) = 3x^{10} - 7x^8 + 5x^6 - 21x^3 + 3x^2 - 7$, then is the value of $\lim_{h \to 0} \frac{f(1-h) - f(1)}{h^3 + 3h}$ is

A. $50/3$

B. $22/3$

C. 13

D. none of these

Answer: D

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17. $\lim_{x \to 0} \frac{e^{x^2} - \cos x}{x^2} isequal \to$ " (a) 3/2 (b) 1/2, (c) 2/3, (d) none of these

A. $1/2$

B. $3/2$

C. $2/3$

D. 2

Answer: B

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18. Let f(x)= $< x >^*$, where $< x >^*$ is the distance from x to the integer nearest to x then $\lim_{x \to 2}$ f(x) is

A. 2

B. 1

C. 0

D. none of these

Answer: C

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19. $\lim_{x \to 0} \frac{(1 - \cos 2x)(3 + \cos x)}{x \tan 4x}$ is equal to

A. $1/2$

B. 1

C. 2

D. $-1/4$

Answer: C

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20. $\lim_{x \to 0} \frac{\sin\left(\pi \cos^2 x\right)}{x^2}$ is equal to

A. $-\pi$

B. π

C. $\pi / 2$

D. 1

Answer: B

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21. Let f(x)=sgn (sgn (sgn x)). Then $\lim_{x \to 0}$ f(x) is

A. 1

B. 2

C. 0

D. none of these

Answer: D

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22. $\lim (x \to a_-)\left\{\frac{|x|^3}{a} - \left[\frac{x}{a}\right]^3\right\}, (a < 0)$, where $[x]$ denotes the greatest integer less than or equal to x is equal to:

A. $a^2 - 3$

B. $a^2 - 1$

C. a^2

D. none of these

Answer: C

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23. $\lim_{x \to 0}\left\{\frac{1+\tan x}{1+\sin x}\right\}^{\text{cosec x}}$ is equal to

A. e

B. e^{-1}

C. 1

D. none of these

Answer: C

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24. $\lim_{n \to \infty} (6^n + 5^n)^{\frac{1}{n}}$ is equal to -

A. 6

B. 5

C. $5/6$

D. e

Answer: A

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25. If $f(x) = \left(\frac{x^2 + 5x + 3}{x^2 + x + 2} \right)^x$ then $\lim_{x \to \infty} f(x)$ is equal to

A. e^{-4}

B. e^{3}

C. e^{2}

D. e^{4}

Answer: D

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26. $\lim_{x \to \infty} \left[\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right]$ is equal to

A. 0

B. $1/2$

C. log 2

D. none of these

Answer: B

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27. If $\left(\lim\right)_{x \to -a} \frac{x^9 + a^9}{x + a} 9$, find the real value of a.

A. -7

B. -1

C. 7

D. none of these

Answer: B

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28. Let $f(x) = [x] + [-x]$, where $[x]$ denotes the greastest integer less than or equal to x . Then, for any integer m

A. f is continuous at x=m

B. $\lim_{x \to m+}$ f(x) exists and is equal to f(m)

C. $\lim_{x \to m}$ f(x) exists but is not equal to f(m)

D. f is differentiable at x=m

Answer: C

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29. Let $f(x)$ be a continuous function defined for $1 \leq x \leq 3$. If $f(x)$ takes rational values for all x and $f(2) = 10$ then the value of $f(1.5)$ is :

A. 8

B. $\frac{f(1) + f(3)}{3}$

C. 20

D. none of these

Answer: D

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30. Let f(x)= $\begin{cases} |x|\cos(1/x)+9x^2 & x \neq 0 \\ k & x=0 \end{cases}$

then f is continuous if the value k is

A. 9

B. 6

C. 0

D. none of these

Answer: C

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31. If f(x)=tan $(\pi/4-x)/\cot 2x$ for $x \neq \pi/4$. The value of $f(\pi/4)$ so that f is continuous at $x=\pi/4$ is

A. $1/3$

B. $1/2$

C. $1/4$

D. $1/6$

Answer: B

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32. Let a function f be defined by f(x)=$\frac{x-|x|}{x}$ for $x \neq 0$ and f(0)=2. Then f is

A. continuous nowhere

B. continuous everywhere

C. continuous for all x except x =1

D. continuous for all x except x=0

Answer: D

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33. If f(x) is a continuous function satisfying f(x)f(1/x) =f(x)+f(1/x) and f(1) $>$ 0 then $\lim_{x \to 1}$ f(x) is equal to

A. 2

B. 1

C. 3

D. none of these

Answer: A

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34. The function $f(x) = (x-1)^{\frac{1}{(2-x)}}$ is not defined at x = 2. The value of f(2) so that f is continuous at x = 2 is

A. 1

B. e

C. $1/e$

D. $1/e^2$

Answer: C

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35. Let $f(x) = \frac{x + x^2 + \ldots . + x^n - n}{x - 1}, x \neq 1,$ then value of $f(1)$ so that f is continuous is (A) n (B) $\frac{n(n-1)}{2}$ (C) $\frac{n(n+1)}{2}$ (D) $\frac{n+1}{2}$

A. n

B. $\frac{n+1}{2}$

C. $\frac{n(n+1)}{2}$

D. $\frac{n(n-1)}{2}$

Answer: C

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36. If f(u) $= \frac{1}{u^2+u-2}$, where $u = \frac{1}{x-1}$, then the points of discontinuity of f are x = ...

A. 1,2

B. 1,-2

C. 1,1/2,2

D. none of these

Answer: C

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37. Let $f(x)$ and $\phi(x)$ be defined by $f(x) = [x]$ and $\phi(x) = \{0, x \in I \text{ and } x^2, x \in R - I[.] =$ G.I.F

A. g is continuous at x=1

B. f is continuous at x=2

C. g o f is a continuous function

D. f o g is a continuous function

Answer: C

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38. If a, b are chosen from $\{1, 2, 3, 4, 5, 6, 7\}$ randomly with replacement. The probability that $\lim_{x \to 0} \left(\frac{a^x + b^x}{2} \right)^{2/x} = 7$ is

A. $1/7$

B. $2/49$

C. $4/7$

D. $4/49$

Answer: B

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39. The value of $\lim_{x \to \pi/2} \frac{\cot x - \cos x}{(\pi - 2x)^3}$ is

A. $\frac{1}{16}$

B. $\frac{1}{8}$

C. $\frac{1}{4}$

D. $\frac{\pi}{2}$

Answer: A

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40. If $\lim_{x \to 0} \frac{\{(a-n)nx - \tan x\}\sin nx}{x^2} = 0$, where n is non-zero real number, then a is equal to

A. 0

B. $\frac{n}{n+1}$

C. n

D. n+(1/n)

Answer: D

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41. Find derivative of $\tan 3x$ by first principle

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42. The value of $\lim_{x \to 0} \frac{e^{nx} - \left(1 + nx + \frac{n^2}{2}x^2\right)}{x^3} (n > 0)$ is

A. $\frac{n^2}{6}$

B. $\frac{n^3}{3}$

C. $\frac{n^3}{6}$

D. $1/6$

Answer: C

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43. Let $f(x) = \frac{(e^x - 1)^{2n}}{\sin^n(x/a)(\log(1 + (x/a)))^n}$ for $x \neq 0$. If $f(0) = 16^n$ and f is a continuous function, then the value of a is

A. 16

B. 2

C. 8

D. 4

Answer: D

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44. Let $f(x) = \begin{cases} x + a & ,x < 0 \\ |x - 1| & ,x \geq 0 \end{cases}$ and $g(x) = \begin{cases} x + 1 & , \text{if } x < 0 \\ (x - 1)^2 + b & ,x \geq 0 \end{cases}$ If gof is continuous $(a > 0)$ then

A. a=2,b=0

B. a=2,b=1

C. a=1,b=0

D. a=1,b=1

Answer: C

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45. If f:R $\rightarrow$ R is function defined by f(x) = $[x]^3 \cos\left(\frac{2x-1}{2}\right)\pi$, where [x] denotes the greatest integer function, then f is :

A. discontinuous only at x=0

B. discontinuous only at non-zero integral value of x

C. continuous only at x=0

D. continuous for every real x.

Answer: D

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46. If $\lim_{x \to 0} \left[1 + x \log\left(1 + b^2\right)\right]^{\frac{1}{x}} = 2b \sin^2 \theta, b > 0$ and $\theta \in (-\pi, \pi]$, then the value of θ is

A. $\pm \frac{\pi}{4}$

B. $\pm \frac{\pi}{3}$

C. $\pm \frac{\pi}{6}$

D. $\pm \frac{\pi}{2}$

Answer: D

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47. Let $f: R \to [0, \infty)$ be such that $\lim_{x \to 3}$ f(x) exists and $\lim_{x \to 3} \frac{(f(x))^2 - 4}{\sqrt{|x - 3|}} = 0$. Then $\lim_{x \to 3} f(x)$ equals

A. 0

B. 1

C. 2

D. 3

Answer: C

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48. The value of $\lim_{x \to 0} \left(\left[\frac{11x}{\sin x} \right] + \left[\frac{21 \sin x}{x} \right] \right)$, where [x] is the greatest integer less than or equal to x is

A. 32

B. 31

C. 11

D. 21

Answer: B

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49. If $\lim_{x \to 0} \frac{x^n - \sin^n x}{x - \sin^n x}$ is nonzero and finite , then n in equal to

A. 1

B. 4

C. 2

D. 3

Answer: A

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50. Value of $\lim_{n \to \infty} \sum_{r=1}^{n} \tan^{-1}\left(\frac{1}{2r^2}\right)$ is

A. $\pi/8$

B. $\pi/2$

C. $\pi/4$

D. $\pi/3$

Answer: C

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51. If f is a continuous function and $x^3 - (\sqrt{5}+1)x^2 + (\sqrt{5}-2+f(x))x + 2\sqrt{5} - \sqrt{5}f(x) = 0$ satisfies for $x \in R$ then $f(\sqrt{5})$ is equal to

A. $2 - \sqrt{5}$

B. $5 + \sqrt{5}$

C. $3 - \sqrt{5}$

D. can not be determined

Answer: D

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52. If x =u is a point of discontinuity of f(x)= $\lim_{n \to \infty} \cos^{2n} x$, then the value of cos u is

A. 0

B. $1/2$

C. $(-1)^n$

D. 1

Answer: C

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Solved Examples Level 2 Single Correct Answer

1. The value of $\lim_{x \to \beta} \frac{1 - \cos(ax^2 + bx + c)}{(x - \beta)^2}$ where $\alpha, \beta$are the distinct roots of $ax^2 + bx + c = 0$ is

A. $(a - b)^2$

B. $\frac{(\alpha-\beta)^2}{2}$

C. $\frac{1}{2}a^2(\alpha-\beta)^2$

D. none of these

Answer: C

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2. The integer $'n'$ for which $\left(\lim\right)_{x\to 0}\frac{(\cos x-1)(\cos x-e^x)}{x^n}$ is a finite non-zero number, is 1 b. 2 c. 3 d. 4

A. 1

B. 2

C. 3

D. 4

Answer: C

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3. $f(x) = \underset{x \to \infty}{\text{Lim}} \frac{x^{2n} - 1}{x^{2n} + 1}$

A. f(x)=1 for |x|=1

B. $f(x) = \begin{cases} 1 & \text{for} |x| > 1 \\ -1 & \text{for} |x| < 1 \end{cases}$

C. $f(x) = \begin{cases} 1 & \text{for} |x| > 1 \\ -1 & \text{for} |x| \geq 1 \end{cases}$

D. f is not defined for any value of x

Answer: B

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4. If $(\lim)_{x \to a} \frac{a^x - x^a}{x^x - a^a} = -1 and a > 0,$ then find the value of $a.$

A. 1

B. 0

C. e

D. none of these

Answer: A

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5. The value of $\lim_{x \to 0} \left(\frac{e^x + e^{-x} - 2}{x^2} \right)^{\frac{1}{x^2}}$ equals

A. $e^{1/2}$

B. $e^{1/4}$

C. $e^{1/3}$

D. $e^{1/12}$

Answer: D

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6. If $\lim_{x \to \infty} \left\{ \frac{x^2 + 1}{x + 1} - (ax + b) \right\} = 0,$ then find the values of a and b.

A. a=1,b=-2

B. a=1,b=-1

C. a=-1,b=1

D. a=2,b=-2

Answer: B

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7. Let $f(x) = g(x)\frac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}}$, where g is a continuous function then $\lim_{x \to 0}$ f(x) exist if

A. g(x)=x+2

B. $g(x) = x^2 + 4$

C. $g(x) = xh(x), h(x)$ is a polynomial

D. g(x) is a constant function

Answer: C

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8. The value of f(0) , so that the function

$$f(x) = \frac{\sqrt{a^2 - ax + x^3} - \sqrt{a^2 + ax + x^2}}{\sqrt{a + x} - \sqrt{a - x}}$$

become continuous for all x, is given by

A. $a\sqrt{a}$

B. $\sqrt{a}$

C. $-\sqrt{a}$

D. $-a\sqrt{a}$

Answer: C

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9. If $f(x) = \frac{2 - (256 - 7x)^{1/8}}{(5x + 32)^{1/5} - 2} (x \neq 0)$, then for f to be continuous on [-1,1] , f(0) is equal to

A. -1

B. 1

C. 2^6

D. none of these

Answer: D

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10. Let $f(x) = \frac{x(1 + a\cos x) - b\sin x}{x^3}, x \neq 0$ and $f(0) = 1$. The value of a and b so that $f(x)$ is a continuous function are.

A. $5/2, 3/2$

B. $5/2, -3/2$

C. $-5/2, -3/2$

D. none of these

Answer: C

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11. $f(x) = \begin{cases} (3/x^2)\sin 2x^2 & \text{if} x < 0 \\ \frac{x^2+2x+c}{1-3x^2} & \text{if} x \geq 0 \end{cases}$ then in order that f be continuous at x=0, the value of c is

A. 2

B. 4

C. 6

D. 8

Answer: C

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12. The function $f(x) = (\sin 2x)^{\tan^2 2x}$ is not defined at $x = \frac{\pi}{4}$. The value of $f(\pi/4)$, so that f is continuous at $x = \pi/4$, is

A. $\sqrt{e}$

B. 1

C. 2

D. none of these

Answer: D

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13. Let $f(x) = \frac{\log\left(1 + x + x^2\right) + \log\left(1 - x + x^2\right)}{\sec x - \cos x}, x \neq 0$ .Then the value of f(0) so that f is continuous at x=0 is

A. 1

B. 0

C. 2

D. none of these

Answer: A

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14. The value of $k(k > 0)$ for which the function $f(x) = \frac{(e^x - 1)^4}{\sin(x^2/k^2)\log\{1 + (x^2/2)\}}, x \neq 0$, f(0) =8 may be continuous function is

A. 1

B. 4

C. 2

D. 3

Answer: C

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15. The function $f(x) = \frac{\log(1 + ax) - \log(1 - bx)}{x}$ is not defined at x = 0. The value which should be assigned to f at x = 0 so that it is continuous at x = 0, is

A. a-b

B. a+b

C. log a + log b

D. none of these

Answer: B

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16. Let $f(x) = \frac{\log(1+x^2)}{x^4 - 26x^2 + 25}$Then 1)$f$ is continuous on ([6,10](2) f is continuous on [-2,2](3) f is continous on [-6,6]4) f is continuous on [1,])

A. f is continuous at on [6,10]

B. f is continuous on [-2,2]

C. f is continuous on [-6,6]

D. f is continuous on [1,7]

Answer: A

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17. Let $f(x) = \frac{\sin\left(\pi \cos^2 x\right)}{x^2}, x \neq 0$. The value of f(0) so that f is a continuous function is

A. $-\pi$

B. π

C. $\pi/2$

D. 1

Answer: B

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18. Let $f(x) = \begin{cases} \frac{e^{\alpha x} - e^x}{x^2} & x \neq 0 \\ 3/2 & x = 0 \end{cases}$ The value of α so that f is a continuous function is

A. 1

B. 0

C. 4

D. 2

Answer: D

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19. Let f be a continuous function on R such that $f(1/2^n) = (\sin e^n)e^{-n^2} + \frac{2n^2}{n^2+1}$ .Then the value of f(0) is

A. 1

B. $1/2$

C. 2

D. none of these

Answer: C

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20. Let $f(x) = \lim_{n \to \infty} \frac{\sin x}{1 + (2 \sin x)^{2n}}$ then f is discontinuous at

A. π

B. $\pi / 3$

C. $\pi / 4$

D. $\pi / 6$

Answer: D

Watch Video Solution

21. Let f be a non-zero continuous function satisfying f(x+y)=f(x)f(y) for all , x,y $\in$ R . If f(2)=9 then f(3) is

A. 1

B. 27

C. 9

D. none of these

Answer: B

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22. Let f be a function on [0,1] defined by $f(x) = (1/2)^n, (1/2)^{n+1} \leq x < (1/2)^n$, n=0,1,2,... Then

A. f is a continuous function

B. f is continuous except x=1/2

C. f is continuous except for finitely many points

D. The sets of points where f is not continuous is infinite.

Answer: D

View Text Solution

23. If $f(x)$ is continuous at $x=0$, where

$$f(x)=\begin{cases}\frac{\sin(a+1)x+\sin x}{x}, \text{ for } x<0\\ c, \text{ for } x=0\\ \frac{\sqrt{x+bx^2}-\sqrt{x}}{b\sqrt{x}}, \text{ for } x>0\end{cases}$$, then

A. a=-3/2, b=0, c=1/2

B. a=-3/2, b=1 , c=1/2

C. a=-3/2 , b ε R , c=-1/2

D. none of these

Answer: C

Watch Video Solution

24. Let $f(x)=\begin{cases}\frac{72^x-9^x-8^x+1}{\sqrt{2}-\sqrt{1+\cos x}}; x\neq 0 \text{ and } k\log 2\log 3; x=0\end{cases}$ .If f is continuous functioin at $x=0$, then k=

A. $\sqrt{2}$

B. 24

C. $18\sqrt{3}$

D. $24\sqrt{2}$

Answer: D

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25. Let f be a function defined on R by $f(x) = [x] + \sqrt{x - [x]}$ then

A. f is not continuous at every $x \in I$

B. f is not continuous at every $x \in R\sim I$

C. f is a continuous function

D. none of these

Answer: B

Watch Video Solution

26. The function $f: R\text{-}\{0\} \to R$ given by $f(x) = \dfrac{\tan x - x}{x - \sin x}$ can be made continuous at x=0 by defining f(0) as

A. 1

B. 0

C. 2

D. 4

Answer: C

View Text Solution

27. For $x \neq 1$, f is defined by $f(x) = \dfrac{1}{\log x} - \dfrac{1}{x-1}$ The value of f(1), so that f is a continous function is

A. 1

B. $\dfrac{1}{2}$

C. 0

D. 2

Answer: B

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28. The number of points where $f(x) = \begin{cases} [\cos \pi x] & ,0 \leq x \leq 1 \\ |2x-3|[x-2] & ,1 < x \leq 2 \end{cases}$ ([x] is the greatest integer less than or equal to x) is discontinuous is

A. 2

B. 4

C. 3

D. 1

Answer: B

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29. If $f(x) = \begin{cases} x^2 + Ax + 5 & x \in Q \\ 1 + x & x \in R\text{~}Q \end{cases}$

is continuous at exactly two points , then the possible values of A are in

A. $(1, \infty)$

B. $(-3, \infty)$

C. $(5, \infty) \cup (-\infty, -3)$

D. $(-\infty, \infty)$

Answer: C

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30. $\lim_{x \to 1} \frac{nx^{n+1} - nx^n + 1}{(e^x - e^2)\sin \pi x}$

A. $\frac{n(n+1)}{e^2 - e}$

B. $\frac{n}{(e^2 - e)\pi}$

C. $\frac{n}{(e - e^2)\pi}$

D. $\frac{n^2(n+1)}{(e^2-e)\pi}$

Answer:

Watch Video Solution

Solved Examples Numberical Answer

1. Let $f(x)=\frac{1-\cos x\sqrt{\cos 2x}}{x^2}, x\neq 0$ The value of f(0) so that f is a continuous function is

Watch Video Solution

2. Let $f(x)=\frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{\tan x}, x\neq 0$ Then $\lim_{x\to 0} f(x)$ is equal to

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3. If $\lim_{x \to 0} \frac{\sin 2x + a \sin x}{x^3} = b$ then the value of b-2a is equal to

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4. Let $f(x) = \frac{\sqrt{2} - (\cos x + \sin x)}{1 - \sin 2x}$, $x \neq \pi/4$. The value $f(\pi/4)$ so that f is continuous is $(\sqrt{2} = 1.41)$

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5. The number of discontinuities of the greatest integer function f(x)=[x-1], $x \in \left(-\frac{11}{2}, 105\right)$ is equal to

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6. If $\lim_{x \to 0} \frac{axe^x - b\log(1+x)}{x^2} = \frac{5}{2}$ then the value of 2a+b is

View Text Solution

7. $\lim_{x \to 1} \frac{nx^{n+1} - nx^n + 1}{(e^x - e^2)\sin \pi x}$

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8. If $\lim_{x \to 2^-} \frac{ae^{\frac{1}{|x+2|}} - 1}{2 - e^{\frac{1}{|x+2|}}} = \lim_{x \to 2^+} \sin\left(\frac{x^4 - 16}{x^5 + 32}\right)$, then a is

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9. Let f be a continuous function on R satisfying f(x+y) =f(x)f(y) for all x, y $\in$ R and f(1)=4 then f(3) is equal to

View Text Solution

10. $\lim_{x \to 0} \tan\left(\frac{\pi}{4} + x\right)^{\frac{1}{x}} =$

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11. Let $f(x) = \lim_{n \to \infty} n^2\left(x^{1/n^2} - 1\right), x > 0$. If f satisfies f(xy)=4kf(x)+f(y) for x , y $>$ 0, then k is equal to

View Text Solution

12. Let $0 \leq \beta_r \leq 1$ and $\sum_{r=1}^{k} \cos^{-1} \beta_r = \frac{k\pi}{2}$ for any $k \geq 1$ and $A = \sum_{r=1}^{k} (\beta_r)^r$, then $\lim_{x \to A} \frac{\left(1 + x^2\right)^{1/3} - (1 - 2x)^{1/4}}{x + x^2}$ is equal to

View Text Solution

13. The value $\lim_{x \to \tan^{-1} 3} \frac{\tan^6 x - 2\tan^5 x - 3\tan^4 x}{\tan^2 x - 4\tan x + 3}$

Watch Video Solution

14. The value of $e\left[\lim_{x \to 0}\left(\frac{\sin x}{x}\right)^{\frac{\sin x}{x - \sin x}} + \lim_{x \to 1} x^{\frac{1}{1-x}}\right]$

View Text Solution

15. If $\lim_{x \to 1} \left(1 + \alpha x + \beta^2 x\right)^{(\gamma/(x-1))} = e^3$ then the value of $2\beta\gamma + \alpha\gamma$ is equal to

View Text Solution

16. Let $f(x) = \dfrac{\sqrt{3x^2+2} + \sqrt[3]{x^3+3}}{\sqrt[4]{x^4+5} - \sqrt[5]{x^4+6}}$ then $\lim_{x \to \infty}$ f(x) is equal to

View Text Solution

Exercise Single Correct Answer

1. $\lim_{n \to \infty} \dfrac{1}{n^2}$ (1+2+...+n) is equal to

A. 0

B. 1

C. $\dfrac{1}{2}$

D. $\dfrac{1}{4}$

Answer: C

Watch Video Solution

2. $\lim_{n \to \infty} \left[\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots + \frac{1}{n(n+1)} \right] =$

A. 0

B. $\frac{1}{2}$

C. $\frac{1}{3}$

D. 1

Answer: D

Watch Video Solution

3. $\lim_{x \to 1} \frac{x^m - 1}{x^n - 1}$ is equal to

A. $\frac{m}{n}$

B. 1

C. 0

D. $\frac{m-1}{n-1}$

Answer: A

Watch Video Solution

4. $\lim_{\alpha \to 0} \frac{\sin(\alpha^n)}{(\sin \alpha)^m} (m, n \in I^+)$ is equal to

A. 1 if $m > n$

B. 0 if $n > m$

C. 1 if $> m$

D. 1 if m=n

Answer: B

View Text Solution

5. $\lim_{x \to \infty} \left(1 + \frac{k}{x}\right)^{mx}$ is equal to

A. m^k

B. e^m

C. e^{mk}

D. $e^{m/k}$

Answer: C

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6. $f(x) = \begin{cases} \frac{a|x^2 - x - 2|}{2 + x - x^2}, & x < 2 \text{ and } b, x = 2 \text{ and } \frac{x - [x]}{x - 2}, x > 2 \end{cases}$ is continuous at $x = 2$ and $[]$ is GLF. then valueof a and b are

A. a=1,b=1

B. a=1,b=2

C. a=2,b=1

D. a=2,b=2

Answer: A

Watch Video Solution

7. Test the following functions for continuity

(1) $\frac{2x^5 - 8x^2 + 11}{x^4 + 4x^2 + 8x^2 + 8x + 4}$

(2) $f(x) = \frac{3\sin^3 x + \cos^2 x + 1}{4\cos x - 2}$

A. R

B. R~{0}

C. R ~ {-1}

D. R~{-1,-2}

Answer: A

Watch Video Solution

8. Let $f(x) = \begin{cases} ax - b & ,x \leq 1 \\ 3x & ,1 < x < 2 \\ bx^2 - a & ,x \geq 2 \end{cases}$ If f is continuous function then (a,b) is equal to

A. (1,2)

B. (1,3)

C. (3,6)

D. (6,3)

Answer: D

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9. If $f(x) = \begin{cases} \cos^{-1}(\cot x) & ,x < \frac{\pi}{2} \\ a(x[x] - 1) & ,x \geq \frac{\pi}{2} \end{cases}$ The value of a for f to be continuous at $x = \frac{\pi}{2}$ is

A. $\frac{\pi}{2(\pi - 1)}$

B. $\pi - 1$

C. $\frac{\pi}{\pi+1}$

D. $\frac{\pi}{\pi-1}$

Answer: A

View Text Solution

10. Let $f(x) = \begin{cases} \frac{\sin 4x}{\log(1+3x)} & ,x \neq 0 \\ A+1 & ,x = 0 \end{cases}$ The value of A for f to be continuous at x=0 is

A. $\frac{2}{3}$

B. $\frac{1}{3}$

C. 1

D. $\frac{3}{4}$

Answer: B

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Exercise Level 1 Single Correct Answer

1. The value of $\lim_{x \to \infty} x\left[\tan^{-1}\left(\frac{x+1}{x+2}\right) - \tan^{-1}\left(\frac{x}{x+2}\right)\right]$ is

A. 1

B. 0

C. $1/2$

D. $1/4$

Answer: C

Watch Video Solution

2. Evaluate $\lim_{x \to 0} \left(1 + \tan^2 \sqrt{x}\right)^{\frac{1}{2x}}$

A. e

B. $e^{1/2}$

C. $e^{1/4}$

D. $e^{-1/2}$

Answer: B

Watch Video Solution

3. If $f(x)$ is continuous at $x = 0$, where $f(x) = \frac{\sin(a+x) - \sin(a-x)}{\tan(a+x) - \tan(a-x)}, x \neq 0$, then $f(0) =$

A. cos a

B. $\frac{1}{2}\cos a$

C. $\cos^2 a$

D. none of these

Answer: D

Watch Video Solution

4. $\lim_{x \to \pi/6} \frac{\sin(x - \pi/6)}{\sqrt{3}/2 - \cos x}$ is equal to

A. 2

B. 1

C. -1

D. $\sqrt{3}/2$

Answer: A

Watch Video Solution

5. $\lim_{x \to \pi/2} \frac{\cos x}{\sqrt[3]{(1 - \sin x)}}$ is equal to

A. 1

B. 2

C. $21/3$

D. none of these

Answer: D

View Text Solution

6. If $x^2 + 2x^3 \leq x + f(x) \leq x^3 - 2x^3$for value of x near 0 then $\lim_{x \to 0} \frac{f(x)}{x}$ is

A. -1

B. 0

C. 1

D. none of these

Answer: A

Watch Video Solution

7. If $f(x) = \frac{\sin^{-1}([x] + x)}{[x]}, [x] \neq 0$

=0, [x] = 0 where [x] denotes the greatest integer less than or equal to x,

then $\lim_{x \to 0} f(x)$ is

A. 1

B. -1

C. 0

D. none of these

Answer: D

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8. The function $f(x) = [x]\cos\left(\frac{2x-1}{2}\right)\pi$ where [ ] denotes the greatest integer function, is discontinuous

A. all x

B. all integer points

C. no x

D. x which is not integer

Answer: C

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9. Let $f(y) = \sin\frac{y-a}{2}\tan\frac{\pi y}{2a}, y \neq a$. The value of f(a) so that f is a continuous function is

A. π/a

B. $-a/\pi$

C. $\pi/2a$

D. none of these

Answer: B

View Text Solution

10. Let $f(x) = (1+\sin x)^{\text{cosec x}}$, the value of f(0) so that f is a continuous function is

A. e

B. $e^{1/2}$

C. e^2

D. 1

Answer: A

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11. Let $f(x) = \begin{cases} x+1 & ,x \leq 1 \\ 3-ax^2 & ,x > 1 \end{cases}$ The value of a so that f is continuous is

A. $1/2$

B. 1

C. 2

D. 3

Answer: B

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12. Let $f(x) = [x^2 + 1]$, ($[x]$ is the greatest integer less than or equal to x)`. Ther

A. on [1,3]

B. for all x in [1,3] except four points

C. for all x in [1,3] except seven points

D. for all x in [1,3] except eight points

Answer: D

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13. The function $y = \sqrt{\frac{1}{2} - \cos^2 x}$ is not continuous at

A. $x = \pi/4$

B. $x = 3\pi/4$

C. $x = 5\pi/4$

D. none of these

Answer: D

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14. The number of points at which the function $f(x) = \frac{1}{x - [x]}([.]$ denotes, the greatest integer function) is not continuous is

A. 1

B. 2

C. 3

D. none of these

Answer: D

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15. Let f(x)=x, x $\in$ Q , f(x)=1-x , x $\in$ R ~ Q then f is continuous only at x=

A. $1/2$

B. 1

C. 0

D. 2

Answer: A

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16. $\lim_{x \to 0} \frac{x \tan 2x - 2x \tan x}{(1 - \cos 2x)^2}$ equal

A. 2

B. -2

C. $1/2$

D. $-1/2$

Answer: C

Watch Video Solution

17. Let $f(x) = \frac{\sqrt{1-\cos(x-2)}}{x-2}, x \neq 2$. The $\lim_{x \to 2}$ f(x)

A. exists and is equal to $\sqrt{2}$

B. does not exist because $\lim_{x \to 2+} f(x)$ doesn't exist

C. equal to 1

D. doesn't exist because $\lim_{x \to 2+} f(x) \neq \lim_{x \to 2-} f(x)$

Answer: D

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18. The value of f(0) so that the function

$$f(x) = \frac{\sqrt{1+x} - (1+x)^{\frac{1}{3}}}{x}$$

becomes continuous is equal to

A. $1/6$

B. $1/4$

C. $1/3$

D. 3

Answer: A

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19. The set of all points of continuous of fofof, where $f(x) = sgn(x)$ is

A. R~{0}

B. R ~ {1,0,1}

C. R ~ {-1,1}

D. none of these

Answer: A

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20. The set of all points of discontinuity of f(x)=$\frac{x-1}{x^3+6x^2+11x+6}$

A. ϕ

B. {-1}

C. {-1,-2,-3}

D. {1,2,3}

Answer: C

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21. The number of continuous functions on R which satisfy $(f(x))^2 = x^2$ for $x \in R$ is

A. 2

B. 4

C. 1

D. infinitely many

Answer: B

View Text Solution

22. The value of $\lim_{x \to 0} \frac{\sqrt{x^2+1}-1}{\sqrt{x^2+16}-4}$ is

A. 3

B. 4

C. 1

D. 2

Answer: B

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23. $\lim_{x \to a^-} \frac{\sqrt{x-b}-\sqrt{a-b}}{(x^2-a^2)}, (a > b)$ is

A. $\frac{1}{4a}$

B. $\frac{1}{a\sqrt{a-b}}$

C. $\frac{1}{2a\sqrt{a-b}}$

D. $\frac{1}{4a\sqrt{a-b}}$

Answer: D

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24. The value of $\lim_{x \to \infty} \left(\sqrt{x^2+1} - \sqrt{x^2-1}\right)$ is

A. -1

B. 1

C. 0

D. none of these

Answer: C

Watch Video Solution

25. $\lim_{x \to \infty} x^{\frac{3}{2}} \left(\sqrt{x^3 + 1} - \sqrt{x^3 - 1} \right)$

A. 1

B. -1

C. 0

D. none of these

Answer: A

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26. The value of f(1) so that f(x)=$(e^x - e) / (x - 1)$ is continuous at x =1 is

A. e^{-1}

B. e^2

C. $e^{1/2}$

D. e

Answer: D

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27. The function $f(x) = a[x+1] + b[x-1]$, $(a \neq 0, b \neq 0)$ where [x] is the greatest integer function, then the condition for which $f(x)$ is continuous at x=1 is

A. a=2b

B. a=b

C. a+b=0

D. a+2b=0

Answer: C

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28. Let $f(x) = \left[2x^3 - 6\right]$, where [x] is the greatest integer less than or equal to x. Then the number of points in (1,2) where f is discontinuous is

A. 5

B. 7

C. 13

D. 12

Answer: C

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29. Discuss the continuity of f(x), where $f(x) = \lim_{n \to \infty} \left(\sin\frac{\pi x}{2}\right)^{2n}$

A. f is continuous on [0,2]

B. f is continuous on R~ $\{2n, n \in I\}$

C. f is continuous on R ~ $\{(2n+1), n \in I\}$

D. $\lim_{x \to 0+} f(x) = 1$

Answer: C

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30. If $f(x) = \frac{1}{2}x - 1$, then on the interval $[0, \pi]$

A. tan f(x) and $\frac{1}{f(x)}$ are both continuous

B. tan f(x) and $\frac{1}{f(x)}$ are both discontinuous

C. tan f(x) and $f^{-1}(x)$ are both continuous

D. tan (f(x)) is continuous but $\frac{1}{f(x)}$ is not

Answer: D

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31. If $\lim_{x \to 0} [\cot(\pi/4 + x)]^{1/x} = Ae^2$ then the value of A is

A. e^2

B. e^{-4}

C. e

D. e^{3}

Answer: B

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32. Let $f(x) = \frac{[1/2 + x] - [1/2]}{x}$, $-1 \leq x \leq 2$ and f(0) =0([x] is greatest integer less than or equal to x) then

A. f is not continuous at x=0

B. f is a continuous function on [-1/2,1/2]

C. f is continuous on [-1,0]

D. f is continuous on [0,2]

Answer: B

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33. Let f and g functions such that f+g is a continuous function then

A. f and g are continuous function

B. f or g is a continuous function

C. f may be discontinuous but g is continuous

D. both f and g may be discontinuous

Answer: D

View Text Solution

34. $\lim_{x \to 0} \frac{\left(1+x^2\right)^{1/3} - (1-2x)^{1/4}}{x + x^2}$ is

A. 2

B. $1/2$

C. $3/2$

D. cannot be determined

Answer: B

View Text Solution

35. The set of all points for which $f(x) = |x||x-1| + \frac{1}{[\ x+1\ ]}$ ([x] is the greatest integer function) is continuous is

A. R

B. $R\sim I$

C. $R\sim(I \cup [-1, 0))$

D. none of these

Answer: C

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36. Let $f(x) = \begin{cases} \left(\left(e^{[x]} - e^{\{x\}}\right)e^{-x} + A\right) & ,x < 0 \\ \frac{2\sin\{x\}}{\tan\{x\}} & ,x > 0 \\ 2 & ,x = 0 \end{cases}$ The value of A so that f is continous at x=0 is ([x] is greatest integer function and {x} is the fractional part of x) is

A. e^{-1}

B. $3 - e^{-1}$

C. $2 - e^{-1}$

D. 2

Answer: B

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37. If $\lim_{x \to 0}\left(\frac{\sin 2x}{x^3} + a + \frac{b}{x^2}\right) = 0$ then then value of 3a +b is

A. 2

B. -2

C. -1

D. 0

Answer: A

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38. For continuous functions f and g on R, let f(a)=4, f'(a)=6, g(a)=2, g'(a)=1 .

Then the value of $\lim_{x \to a} \frac{\sqrt{f(x)g(a)} - \sqrt{g(x)f(a)}}{(x-a)\left(\sqrt{f(x)g(a)} + \sqrt{g(x)f(a)}\right)}$

A. 0

B. 3

C. $1/4$

D. $2/3$

Answer: C

View Text Solution

39. Let $f(x) = \begin{cases} \frac{\sqrt{1+ax}-\sqrt{1-ax}}{x} & ,-1 \leq x < 0 \\ \frac{2x+1}{x-2} & ,0 \leq x \leq 1 \end{cases}$ The value of a so f is continuous on [-1,1] is

A. $1/2$

B. $-1/2$

C. 1

D. -1

Answer: B

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Exercise Level 2 Single Correct Answer

1. If α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$, then evaluate $\lim_{x \to \frac{1}{\alpha}} \sqrt{\frac{1 - \cos(cx^2 + bx + a)}{2(1 - \alpha x)^2}}$

A. c(a-b)

B. $\frac{c}{2\alpha}\left(\frac{1}{\alpha}-\frac{1}{\beta}\right)$

C. $\frac{1}{\beta}\left(\frac{1}{\alpha}-\frac{1}{\beta}\right)$

D. $\left|\left(\frac{1}{\alpha}-\frac{1}{\beta}\right)\frac{c}{2\alpha}\right|$

Answer: D

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2. Let f be a continuous function satisfying f(x) f(y)=f(x)+f(y)+f(xy)-2 for all x,y $\in$ R and f(2)=5 then $\lim_{x\to 4} f(x)$ is

View Text Solution

3. The value of $\lim_{x\to\infty}\frac{e^{1/x}-e^{-1/x}}{e^{1/x}+e^{-1/x}}$ tan (1/x) is

A. 1

B. 0

C. -1

D. none of these

Answer: B

View Text Solution

4. Let f be a continuous function satisfying f(x+y)=f(x)+f(y) for all x,y $\in$ R and f(1)=5 then $\lim_{x \to 4}$ f(x) is equal to

A. 4

B. 80

C. 0

D. none of these

Answer: D

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5. Let $f(x) = e^x sgn(x + [x])$, where sgn is the signum function and [x] is the greatest integer function . Then

A. $\lim_{x \to 0+} f(x) = 0$

B. $\lim_{x \to 0+} f(x) = -1$

C. $\lim_{x \to 0+} f(x) = 1$

D. $\lim_{x \to 0-} f(x) = 1$

Answer: C

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6. The value of $\lim_{x \to -4} \frac{\tan \pi x}{x + 4} + \lim_{x \to \infty} \left(1 + \frac{1}{x^2}\right)^x$ is

A. greater than 3

B. less than 2

C. equal to 0

D. equal to 1

Answer: A

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7. The value of $\lim_{x \to 0} \left(\frac{\sin x}{x} \right)^{\frac{\sin x}{x - \sin x}}$ is

A. e

B. 1

C. e^2

D. 1/e

Answer: D

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8. The value of $\lim_{x \to 0} \frac{(\sin x - \tan x)^2 - (1 - \cos 2x)^4 + x^5}{7(\tan^{-1} x)^7 + (\sin^{-1} x)^6 + 3\sin^5 x}$ equal to :

A. 0

B. 1

C. $1/3$

D. 2

Answer: C

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9. Let $f(x) = x - 1$ and $g(x) = \frac{1}{x}$. Then the set of points where $gofog$ is continuous is

A. R~{0}

B. R~{1}

C. $(-\infty, \infty)$

D. $(-\infty, \infty)$~$\{0, 1\}$

Answer: D

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10. Let $f(x) = \lim_{n \to \infty} \frac{\left[x^2\right] + \left[(2x)^2\right] + \ldots + \left[(nx)^2\right]}{n^3}$ then the set of all points of continuity of f([x] denotes the greatest integer function)

A. $(-\infty, \infty) \sim \{0\}$

B. $(-\infty, \infty) \sim I$

C. $(-\infty, \infty)$

D. $(-\infty, \infty) \sim \{0, 1\}$

Answer: C

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11. If : $f(x) \begin{matrix} = 1, & \ldots \text{x is rational} \\ = 0, & \ldots \text{x is irrational} \end{matrix}$ then : $\lim_{x \to 0} f(x) =$

A. f is discontinuous for every real x

B. f is continuous on R

C. f is continuous at the points where x is rational

D. f is continuous at the points where x is irrational .

Answer: A

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12. Let f(x) be given that $f(x) = \begin{cases} x & \text{if x is rational} \\ 1-x & \text{if x is irrational} \end{cases}$

The number of points at which f(x) is continuous, is

A. discontinuous at x=1/2

B. continuous at x=1/2

C. continuous everywhere

D. discontinuous everywhere

Answer: B

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13. Let $f: R \to R$ be any function. Defining $g: R \to R$ by $g(x) = |f(x)|$ for $x \to R$. Then g, is

A. one-one if f is one-one

B. discontinuous if f is discontinuous

C. continuous if f is continuous

D. onto if f is onto

Answer: C

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14. Let f be a function defined on R by $f(x) = \lim_{n \to \infty} \frac{\log(3+x) - x^{2n} \sin x}{1 + x^{2n}}$ then

A. f is continuous on R

B. f is continuous on R ~ {-1,1}

C. f is continuous on R ~{0}

D. none of these

Answer: B

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15. If $f(x) = \left(\frac{x^2 + 5x + 3}{x^2 + x + 2}\right)^x$ then $\lim_{x \to \infty} f(x)$ is equal to

A. e^4

B. e^3

C. e^2

D. 2^4

Answer: A

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16. Let $f(x)$ be defined for all $> x0$ and be continuous. Let f(x) satisfies $f\left(\frac{x}{y}\right) = f(x) - f(y)$ for all $x, yandf(e) = 1.$ Then $f(x)$ is bounded (b) $f\left(\frac{1}{x}\right)\overrightarrow{0}\,as x\,\overrightarrow{0}\ f(x)$ is bounded (d) $f(x) = (\log)_e x$

A. f is bounded

B. f(1/x) $\rightarrow$ 0 as x $\rightarrow$ 0

C. xf (x) $\rightarrow$ 1 as x $\rightarrow$ 0

D. f(x)=log x

Answer: D

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17. Given the function $f(x) = \frac{1}{1-x}$, the numbers of discontinuities of $f^{3n} = fof\ldots of\ (3\text{n times})$ is

A. 1

B. 2

C. 3

D. infinite

Answer: B

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18. If $f(x) = \frac{1 + \sin x - \cos x}{1 - \sin x - \cos x}, x \neq 0$. The value of f(0) so that f is a continuous function is

A. 1

B. -2

C. -1

D. 2

Answer: C

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19. If $f(x) = \begin{cases} ax+1, & x \leq \frac{\pi}{2} \\ \sin x + b, & x > \frac{\pi}{2} \end{cases}$ is continuous, then

A. a=1,b=0

B. $a = b\frac{\pi}{2} + 1$

C. $b = a\frac{\pi}{2}$

D. a=b=pi//2`

Answer: C

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20. If $f(x) = \begin{cases} \frac{x^3+x^2-16x+20}{(x-2)^2}, & x \neq 2 \\ k, & x = 2 \end{cases}$ is continuous at x = 2, then the value of k is

A. 5

B. 7

C. 6

D. 4

Answer: B

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21. $f(x) = \begin{cases} \frac{2^{x+2}-16}{4^x-16} & \text{if } x \neq 2 \\ k & \text{if } x = 2 \end{cases}$,$x = 2$.

A. $1/2$

B. 1

C. 2

D. e

Answer: A

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22. If the function $f(x) = \dfrac{\cos^2 x - \sin^2 x - 1}{\sqrt{x^2+1}-1}, x \neq 0$, is continuous at x = 0, then f(0) is equal to

A. -2

B. -1

C. 0

D. -4

Answer: D

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23. If a and b are positive integers then

A. $\lim_{x \to 0+} \dfrac{x}{a}\left[\dfrac{b}{x}\right] = \dfrac{a}{b}$

B. $\lim_{x \to 0+} \dfrac{x}{a}\left[\dfrac{b}{x}\right] = ab$

C. $\lim_{x \to 0+} \dfrac{a}{x}\left[\dfrac{x}{b}\right] = \dfrac{b}{a}$

D. $\lim_{x \to 0+} \frac{x}{a}\left[\frac{b}{x}\right] = \frac{b}{a}$

Answer: D

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24. Let $f(x) = \frac{\tan\left[e^2\right]x^2 - \tan\left[-e^2\right]x^2}{\sin^2 x}, x \neq 0$ then the value of $f(0)$ so that f is a continuous function is

A. 15

B. 0

C. 7

D. 8

Answer: A

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25. The value of f(0) so that the function f(x)= $\frac{\cos ax - \cos bx}{x^2}, x \neq 0$ is continuous is given by

A. a-b

B. $a^2 - b^2$

C. $\frac{b^2 - a^2}{2}$

D. $\frac{a^2 + b^2}{2}$

Answer: C

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Exercise Numerical Answer

1. Find the value of $\lim_{x \to 1} (1 + \sin \pi x)^{\cos \pi x}$

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2. $(\lim)_{x \overrightarrow{\infty}} \left(\frac{x^2 + 2x - 1}{2x^2 - 3x - 2} \right)^{\frac{2x+1}{2x-1}} isequa < o\ 0$ (b) ∞ (c) $\frac{1}{2}$ (d) none of these

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3. Find the value of $\lim_{x \to \infty} \left(\frac{\pi}{4} - \tan^{-1} \frac{x+1}{x+2} \right)$.

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4. Find the value of $\left| \frac{\lim_{x \to 1} \frac{x^x - 1}{x \log x}}{\lim_{x \to 0} \frac{\log(1 - 3x)}{x}} \right|$

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5. If $\lim_{x \to 2} \frac{A \sin(x-2) + B \cos(x-2) + 5}{x^2 - 4}$=1 , then |A-B| is equal to

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6. If $\frac{5x-2}{x} < f(x) < \frac{5x^2-4x}{x^2}$ and $\frac{\sin x^2}{x} < g(x) < \frac{\log(1+x^2)}{x}$ then $\left| \lim_{x \to \infty} f(x) - \lim_{x \to \infty} g(x) \right|$ is equal to

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7. If $\lim_{x \to 0} \left[1 + x + \frac{f(x)}{x}\right]^{1/x} = e^3$, then the value of $\ln\left(\lim_{x \to 0} \left[1 + \frac{f(x)}{x}\right]^{1/x} \right)$ is _____.

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8. If $\lim_{x \to 2} \left(\frac{-ax + \sin(x-2) + 2a}{x + \sin(x-2) - 2} \right)^{\frac{2-x}{\sqrt{2}-\sqrt{x}}} = \frac{1}{4^{\sqrt{2}}}$ then least $1 + a^2$ is

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9. Let m and n be two integers greater than 1. if $\lim_{\alpha \to 0} \left(\frac{e^{\cos(\alpha^n)} - e}{\alpha^m} \right) = -\left(\frac{e}{2}\right)$ then the value of $\frac{n}{m}$ is

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10. Let $f(x) = \lim_{n \to \infty} \frac{x^{2n-1} + ax^2 + bx}{x^{2n} + 1}$. If f is continuous for x $\in$ R, then the value of a+8b is

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11. Find the value of f(0) so that the function $f(x) = \frac{1}{24} \frac{(4^x - 1)^3}{\sin\left(\frac{x}{4}\right) \log\left(1 + \frac{x^2}{3}\right) (\log 2)^3}$, $x \neq 0$ is continuous in R is .

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12. Find the value of f(0) so that the function $f(x) = \frac{[\log(1 + x/12) - \log(1 - x/8)]}{x}$, $x \neq 0$ is continuous on [0,8].

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13. Find the value of f(0) so that the function $f(x) = \frac{1}{8}\frac{1-\cos^2 x+\sin^2 x}{\sqrt{x^2+1}-1}, x \neq 0$ is continuous .

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14. Find the value of f(1) so that the function $f(x) = \frac{\left(\sqrt[3]{x}^2 - \left(2x^{1/3}-1\right)\right)}{4(x-1)^2}, x \neq 1$ is continuous at x=1.

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15. Let $f(x) = x^2$ if x is rational and $f(x) = 1 - x^2$ if x is irrational then the number of points of continuity of f is

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16. Let $f(x) = \frac{\cos x - \sin x}{\cos 2x} + \sin^2 x, x \neq \frac{\pi}{4}$. The value of $f(\pi/4)$ so that f is continuous on $(0, \pi/2)$ is $\left(\sqrt{2} = 1.41\right)$

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Questions For Previous Years Aieee Jee Main Papers

1. f is defined on [-5,5] as f(x)=$\begin{cases} x \text{ if } x \text{ is rational} \\ -x \text{ if } x \text{ is irrational} \end{cases}$

A. f(x) is continuous at every x, except x =0

B. f(x) is discontinuous at every x, except x=0

C. f(x) is continuous everywhere

D. f(x) is discontinuous everywhere

Answer: B

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2. $\lim_{x \to 0} \frac{\sqrt{1-\cos 2x}}{\sqrt{2}x}$ is

A. 1

B. -1

C. 0

D. does not exist

Answer: D

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3. $\lim_{x \to \infty} \left(\frac{x^2 + 5x + 3}{x^2 + x + 3} \right)^x$

A. e^4

B. e^2

C. e^3

D. e

Answer: A

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4. If $\lim_{x \to 0} \frac{\log(3+x) - \log(3-x)}{x} = k$, the value of k is

A. $-\frac{1}{3}$

B. $\frac{2}{3}$

C. $-\frac{2}{3}$

D. 0

Answer: B

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5. $\lim_{x \to \pi/2} \frac{\left(1 - \tan\frac{x}{2}\right)(1 - \sin x)}{\left(1 + \tan\frac{x}{2}\right)(\pi - 2x)^3}$ का मान है :

A. 0

B. $\frac{1}{32}$

C. ∞

D. $\frac{1}{8}$

Answer: B

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6. If $\lim_{x \to \infty} \left(1 + \frac{a}{x} + \frac{b}{x^2}\right)^{2x} = e^2$, then the values of a and b are

A. $a \in R, b = 2$

B. $a = 1, b \in R$

C. $a \in R, b = \ \in R$

D. a=1 and b=2

Answer: B

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7. Let $f(x) = \frac{1 - \tan x}{4x - \pi}, x \neq \frac{\pi}{4}, x \in \left[0, \frac{\pi}{2}\right]$. If f(x) is continuous in $\left[0, \frac{\pi}{2}\right]$, then $f\left(\frac{\pi}{4}\right)$ is

A. $-\frac{1}{2}$

B. $\frac{1}{2}$

C. 1

D. -1

Answer: A

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8. Let α and β be the distinct root of $ax^2 + bx + c = 0$ then $\lim_{x \to 0} \frac{1 - \cos\left(ax^2 + bx + c\right)}{(x - \alpha)^2}$ is equal to

A. $-\frac{a^2}{2}(\alpha - \beta)^2$

B. $\frac{1}{2}(\alpha - \beta)^2$

C. $\frac{a^2}{2}(\alpha - \beta)^2$

D. 0

Answer: C

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9. The function $f: R - \{0\} \to R$ given by $f(x) = \frac{1}{x} - \frac{2}{e^2 x - 1}$ can be made continuous at x=0 by defining f(0) as

A. 2

B. -1

C. 0

D. 1

Answer: D

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10. Let $f: R \to R$ be a positive increasing function with $\lim_{x \to \infty} \frac{f(3x)}{f(x)} = 1$.Then $\lim_{x \to \infty} \frac{f(2x)}{f(x)}$ is

A. $3/2$

B. 3

C. 1

D. $2/3$

Answer: C

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11. The value of p and q for which the function

$$f(x) = \begin{cases} \frac{\sin(p+1)x + \sin x}{x} & , \quad x < 0 \\ q & , \quad x = 0 \\ \frac{\sqrt{x+x^2} - \sqrt{x}}{x^{1/2}} & , \quad x > 0 \end{cases}$$

is continuous for all x in R, are

A. $p = \frac{1}{2}, q = \frac{3}{2}$

B. $p = \frac{1}{2}, q = -\frac{3}{2}$

C. $p = \frac{5}{2}, q = \frac{1}{2}$

D. $p = -\frac{3}{2}, q = \frac{1}{2}$

Answer: D

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12. Let $f: R \to [0, \infty)$ be such that $\lim_{x \to 5} f(x)$ exists and $\lim_{x \to 5} \frac{[f(x)]^2 - 9}{\sqrt{|x-5|}} =$ is equal to:

A. 0

B. 1

C. 2

D. 3

Answer: D

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13. Deine $F(x)$ as the product of two real functions $f_1(x) = x, x \in R,$ and $f_2(x) = \left\{\sin\left(\frac{1}{x}\right), \quad \text{if} \quad x \neq 0, 0 \quad \text{if} \quad x = 0 \right.$ follows : $F(x) = \{f_1(x).\ f_2(x)$ if $x \neq 0, 0,$ if x = 0. Statement-1 : F(x) is continuous on R. Statement-2 : $f_1(x)$ and $f_2(x)$ are continuous on R.

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14. The value of $\lim_{x \to 2} \frac{\sqrt{1 - \cos 2(x-2)}}{x-2}$, is

A. equals $\frac{1}{\sqrt{2}}$

B. does not exist

C. equals $\sqrt{2}$

D. equals $-\sqrt{2}$

Answer: B

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15. If $f: \overrightarrow{RR}$ is a function defined by $f(x) = [x]\cos\left(\frac{2x-1}{2}\right)\pi$ where [x] denotes the greatest integer function, then f is (1) continuous for every real x (2) discontinuous only at $x = 0$ (3) discontinuous only at non-zero integral values of x (4) continuous only at $x = 0$

A. discontinuous only at x=0

B. discontinuous only at non-zero integral value of x

C. continuous only at x=0

D. continuous for every real x.

Answer: D

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16. $\lim_{x \to 0} \frac{(1-\cos 2x)(3+\cos x)}{x \tan 4x}$ is equal to

A. $1/2$

B. 1

C. 2

D. $-1/4$

Answer: C

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17. If the function $f(x) = \begin{cases} \frac{\sqrt{2+\cos x}-1}{(\pi-x)^2} & ; x \neq \pi \\ k & ; x = \pi \end{cases}$ is continuous at $x = 1$, then k equals:

A. 2

B. $1/4$

C. $1/2$

D. 0

Answer: B

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18. $\lim_{x \to 0} \frac{\sin\left(\pi \cos^2 x\right)}{x^2}$ is equal to

A. $\pi / 2$

B. 1

C. $-\pi$

D. π

Answer: D

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19. If f(x) is continuous and $f\left(\frac{9}{2}\right) = \frac{2}{9}$, then : $\lim_{x \to 0} f\left(\frac{1 - \cos 3x}{x^2}\right)$=

A. $\frac{9}{2}$

B. $\frac{2}{9}$

C. 0

D. $\frac{8}{9}$

Answer: B

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20. If $Lt_{x \to 2} \frac{\tan(x-2)\left[x^2+(k-2)x-2k\right]}{x^2-4x+4} = 5$. then K is equal to

A. 0

B. 1

C. 2

D. 3

Answer: D

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21. if $\lim_{x \to \infty} \frac{e^{x^2}-\cos x}{\sin^2 x}$ is

A. 3

B. $\frac{3}{2}$

C. $\frac{5}{4}$

D. 2

Answer: B

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22. $\lim_{x \to 0} \frac{(1-\cos 2x)(3+\cos x)}{x \tan 4x}$ is equal to

A. 4

B. 3

C. 2

D. $\frac{1}{2}$

Answer: C

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23. Let k be a non-zero real number . If $f(x) = \begin{cases} \frac{(e^x - 1)^2}{\sin\left(x\frac{\pi}{k}\right)\log\left(1+\frac{x}{4}\right)} & ,x \neq 0 \\ 12 & ,x = 0 \end{cases}$ is a continuous function, then the value of k is

A. 1 pi

B. 2 pi

C. 3 pi

D. 4 pi

Answer: C

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24. Let $p = \lim_{x \to 0^+} \left(1 + \tan^2 \sqrt{x}\right)^{\frac{1}{2x}}$ then log p is equal to`

A. 2

B. 1

C. $\frac{1}{2}$

D. $\frac{1}{4}$

Answer: C

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25. If $\lim_{x \to \infty} \left(1 + \frac{a}{x} - \frac{4}{x^2}\right)^{2x} = e^3$, the a is equal to

A. 2

B. $\frac{3}{2}$

C. $\frac{1}{2}$

D. $\frac{2}{3}$

Answer: A

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26. Let $a, b \in R, (a \in 0)$. If the funtion f defined as

$$f(x) = \begin{cases} \frac{2x^2}{a} & 0 \leq x < 1 \\ a & 1 \leq x < \sqrt{2} \\ \frac{2b^2 - 4b}{x^3} & \sqrt{2} < x < \infty \end{cases}$$ is a continous in $[0, \infty)$. Then, (a,b)=

A. $\left(-\sqrt{2}, 1 - \sqrt{3} \right)$

B. $\left(\sqrt{2}, \ -1 + \sqrt{3} \right)$

C. $\left(\sqrt{2}, 1 + \sqrt{3} \right)$

D. $\left(-\sqrt{2}, 1 + \sqrt{3} \right)$

Answer: C

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27. $\lim_{x \to 0} \frac{(1 - \cos 2x)^2}{2x \tan x - x \tan 2x}$ is

A. 2

B. $-\frac{1}{2}$

C. -2

D. $\frac{1}{2}$

Answer: A

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28. Let $p = \lim_{x \to 0^+} \left(1 + \tan^2 \sqrt{x}\right)^{\frac{1}{2x}}$ then log p is equal to`

A. 2

B. 1

C. $\frac{1}{2}$

D. $\frac{1}{4}$

Answer: C

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29. If $\lim_{x \to \infty} \left(1 + \frac{a}{x} - \frac{4}{x^2}\right)^{2x} = e^3$, the a is equal to

A. 2

B. $\frac{3}{2}$

C. $\frac{1}{2}$

D. $\frac{2}{3}$

Answer: B

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30. $\lim_{x \to 3} \frac{\sqrt{3x} - 3}{\sqrt{2x - 4} - \sqrt{2}}$ is equal to

A. $\sqrt{3}$

B. $1/\sqrt{2}$

C. $\frac{\sqrt{3}}{2}$

D. $\frac{1}{2\sqrt{2}}$

Answer: B

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31. Find a for which $\lim_{n\to\infty} \frac{1^a+2^a+3^a+\ldots+n^a}{(n+1)^{a-1}[(na+1)+(na+2)+\ldots+(na+n)]} = \frac{1}{60}$

A. 7

B. 8

C. $\frac{15}{2}$

D. $\frac{17}{2}$

Answer: A

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32. The value of $\lim_{x\to\pi/2} \frac{\cot x - \cos x}{(\pi-2x)^3}$ is

A. $1/4$

B. $1/24$

C. $1/16$

D. $1/8$

Answer: C

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33. For each $t \in R$ let $[t]$ be the greatest integer less than or equal to t then $\lim_{x \to 0^+} x\left(\left[\frac{1}{t}\right] + \left[\frac{2}{t}\right] + \ldots + \left[\frac{15}{t}\right]\right)$ (1) is equal to 0 (2) is equal to 15 (3) is equal to 120 (4) does not exist (in R)

A. is equal to 0

B. is equal to 15

C. is equal to 120

D. does not exist (in R)

Answer: B

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34. $\lim_{x \to 0} \frac{x \tan 2x - 2x \tan x}{(1 - \cos 2x)^2}$ equal

A. $-1/2$

B. $1/4$

C. $1/2$

D. 1

Answer: C

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35. $\lim_{x -> 0} \frac{(27 + x)^{\frac{1}{3}} - 3}{9 - (27 + x)^{\frac{2}{3}}}$

A. $-\frac{1}{3}$

B. $\frac{1}{6}$

C. $-\frac{1}{6}$

D. $\frac{1}{3}$

Answer: C

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36. If the function f defined as $f(x)=\frac{1}{x}-\frac{k-1}{e^{2x}-1}, x \neq 0,$ is continuous at $x=0,$ then the ordered pair $(k, f(0))$ is equal to :

A. (2,1)

B. (3,1)

C. (3,2)

D. (1/3,2)

Answer: B

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37. $\lim_{y \to \infty} \frac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4} =$ (a) $\frac{1}{4\sqrt{2}}$ (b) $\frac{1}{2\sqrt{2}}$ (c) $\frac{1}{2\sqrt{2}\left(1+\sqrt{2}\right)}$ (d) does not exist

A. exists and equals $\frac{1}{4\sqrt{2}}$

B. does not exist

C. exists and equals $\frac{1}{2\sqrt{2}}$

D. exists and equals $\frac{1}{2\sqrt{2}\left(\sqrt{2}+1\right)}$

Answer: A

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38. For each $x \in R$, let [x] be the greatest integer less than or equal to x,

Then,

$\lim_{x \to 0^-} \frac{x([x]+|x|)\sin[x]}{[x]}$ is equal to

A. $-\sin 1$

B. 0

C. 1

D. sin 1

Answer: A

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39. For each $t \in \mathbb{R}$, let $[t]$ be the greatest integer less than or equal to t.

Then, $\lim_{x \to 1+} \frac{(1-|x|+\sin|1-x|)\sin\left(\frac{\pi}{2}[1-x]\right)}{|1-x|[1-x]}$

A. equals -1

B. equals 1

C. does not exist

D. equals 0

Answer: D

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40. Let [x] denote the greatest integer less than or equal to Then:

$$\lim_{x \to 0} \frac{\tan\left(\pi \sin^2 x\right) + \left(|x| - \sin(x[x])\right)^2}{x^2} :$$

A. equals π

B. equals 0

C. equals $\pi + 1$

D. does not exist

Answer: C

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41. $\lim_{x \to 0} \frac{x \cot(4x)}{\sin^2 x \cot^2(2x)}$ is equal to

A. 2

B. 0

C. 4

D. 1

Answer: D

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42. $\lim_{x \to \frac{\pi}{4}} \frac{\cot^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)}$ is

A. 4

B. $8\sqrt{2}$

C. 8

D. $4\sqrt{2}$

Answer: D

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43. $\lim_{x \to 1-} \frac{\sqrt{\pi} - \sqrt{2\sin^{-1} x}}{\sqrt{1-x}}$ is equal to

A. $\frac{1}{\sqrt{2\pi}}$

B. $\sqrt{\frac{\pi}{2}}$

C. $\sqrt{\frac{2}{\pi}}$

D. $\sqrt{\pi}$

Answer: C

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44. Let f:R $\rightarrow$ R be a function defined as $f(x) = \begin{cases} 5 & \text{if} x \leq 1 \\ a + bx & \text{if} 1 < x < 3 \\ b + 5x & \text{if} 3 \leq x < 5 \\ 30 & \text{if} x \geq 5 \end{cases}$

then f is

A. continuous at a=5 and b=5

B. contiuous at a=-5 and b=10

C. continuous at a=0 and b=5

D. not continuous for any values of a and b

Answer: A

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Previous Years B Architecture Entrance Examination Paper

1. If $\lim_{x \to a} \left(\frac{f(x)}{g(x)} \right)$ exists, then

A. $\lim_{x \to c}$ f(x) exist

B. neither $\lim_{x \to c}$ f(x) nor $\lim_{x \to c}$ g(x) may exist

C. $\lim_{x \to c}$ g(x) exist

D. both $\lim_{x \to c}$ f(x) and $\lim_{x \to c}$ g(x) must exist

Answer: B

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2. Let $f:(-\infty,\infty)\to(-\infty,\infty)$ be such that f(.) is continuous at 0 and let $f\left(\frac{x+y}{2}\right)=\frac{f(x)+f(y)}{2}$ for x , y $\in(-\infty,\infty)$

Statement-1: f(.) is continuous at every point on $(-\infty,\infty)$

Statement-2: $f(x+h)+f(0)=2f(x)+f(2h)$ for $x,h\in(-\infty,\infty)$.

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3. Statement-1: $\lim_{x\to1}\sin\frac{\pi}{4}\left(\frac{x|x|-1}{|x|-1}\right)$ exists

Statement-2: $\lim_{x\to1}\tan\frac{\pi}{4}\left(\frac{x|x|-1}{|x|-1}\right)$ exists

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4. Let $f:R\to\left(\frac{-1}{2},\frac{1}{2}\right)$ be an odd function such that $\lim_{x\to0}$ f(x) exists. Then, $\lim_{x\to0}\frac{1}{2f(x)-1}$ equals

A. 0

B. $\frac{1}{2}$

C. 2

D. -1

Answer: D

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5. The values of α and β such that $\lim_{x \to \infty} \left[\frac{x^2+1}{x-1} - \alpha x - 2\beta\right] = \frac{3}{2}$ are

A. $\alpha = -1, \beta = \frac{3}{4}$

B. $\alpha = 1, \beta = -\frac{1}{4}$

C. $\alpha = -1, \beta = \frac{5}{4}$

D. $\alpha = 1, \beta = \frac{-3}{4}$

Answer: B

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6. Let f(x)=|x|+[x-1], where [.] is greatest integer function , then f(x) is

A. continuous at x=0 as well as at x=1

B. continuous at x=0 but not at x=1

C. continuous at x=1 but not at x=0

D. neither continuous at x=0 or nor at x=1

Answer: D

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7. If for some real number a, $Lt_{x \to 0} \frac{\sin 2x + a \sin x}{x^3}$ exists, then the limits is equal to

A. -2

B. -1

C. 1

D. 2

Answer: B

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8. If $f(x) = \begin{vmatrix} \sin x & \cos x & \tan x \\ x^3 & x^2 & x \\ 2x & 1 & 1 \end{vmatrix}$ then $\lim_{x \to 0} \frac{f(x)}{x^2}$ is

A. 1

B. -1

C. 0

D. 2

Answer: A

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9. Let f(x)=[x] where [x] be the greatest integer less than or equal to x.

$g(x) = \begin{cases} 0 & ,x \in Z \\ x^2 & ,x \in R - Z \end{cases}$ z is the set of integers , $\phi(x) = fog(x)$ and $\Psi(x) = gof(x)$. Then on the set R-Z .

A. both ϕ and Φ are continuous

B. neither ϕ nor Φ is continuous

C. ϕ is continuous and Φ is not continuous

D. Φ is continuous and ϕ is not continuous

Answer: A

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10. $\lim_{x \to 1} (1 - x + [x - 1] + [1 - x]) =$ where [.] denotes the greatest integer function

A. is equal to 0

B. is equal to 1

C. does not exist

D. is equal to -1

Answer: C

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11. $\lim_{x \to 0} \frac{\log(\sin 7x + \cos 7x)}{\sin 3x}$ equals.

A. $\frac{1}{3}\log 7$

B. $\frac{7}{3}$

C. $\frac{14}{3}$

D. $\frac{1}{3}$

Answer: B

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12. Let f(x)=$x\left[\frac{1}{x}\right]$ for all $x(\neq 0) \in R$, where for each $t \in R$, $[t]$ denotes the greatest integer less than or equal to t. Then

A. $\lim_{x \to 1/3+} f(x) = 1$

B. $\lim_{x \to 1/2-} f(x) = 1$

C. $\lim_{x \to 2-} f(x) = 1$

D. $\lim_{x \to 0+} f(x) = 0$

Answer: B

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