MATHS

BOOKS - MCGROW HILL EDUCATION MATHS (HINGLISH)

MATHEMATICAL INDUCTION AND BINOMIAL THEOREM

Illustration

1. Find the middle term of $\left(\frac{3}{2}x - \frac{2}{5}y\right)^{10}$

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2. Find the middle terms of $\left(\frac{1}{2}x + \frac{2}{3}\sqrt{x}\right)^{11}$

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3. Find numerically greatest term in the expansion of $(5-3x)^7$ when x=2/3

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Solved Examples Concept Based Single Correct Answer Type Questions

1. Suppose $a \in R$. If the coefficient of x^5 in the expansion of $\left(ax + \frac{1}{x^3}\right)^{17}$ is 680, then a is equal to

A. ± 2

B. ± 1

C. $\pm 1/2$

D. $\pm 1/3$

Answer: B

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2. Term independent of x in the expansion of $\left(3x^2 - \frac{1}{3x}\right)^9$.

A. $\frac{1}{9}$

B. $\frac{2}{17}$

C. $\frac{7}{18}$

D. $\frac{11}{18}$

Answer: C

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3. Let a_r denote the coefficient of y^{r-1} in the expansion of $(1+2y)^{10}$. If $\frac{a_{r+2}}{a_r} = 4$, then r is equal to

A. 2

B. 4

C. 3

D. 5

Answer: C

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4. Suppose $k \in R$. Let a be the coefficient of the middle term in the expansion of $\left(\frac{k}{x} + \frac{x}{k}\right)^{10}$ and b be the term independent of x in the expansion of $\left(\frac{k^2}{x} + \frac{x}{k}\right)^{10}$. If $\frac{a}{b} = 1$, then k is equal to

A. 1

B. 2

C. -3

D. any non-zero number

Answer: A

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5. The remainder when $6^n - 5n$ is divided by 25, is

A. 1

B. 24

C. 0

D. n

Answer: A

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6. If coefficients of r th and (r + 1)th term in the expansion of $(3 + 2x)^{74}$ are equal, then r is equal to:

A. 28

B. 29

C. 30

D. 31

Answer: C

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7. If the coefficients of three consecutive terms in the expansion of $(1+x)^n$ are in the ratio 1:7:42, then find the value of n.

A. 45

B. 55

C. 65

D. 75

Answer: B

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8. The coefficient of x^9 in the expansion of $E=(1+x)^9+(1+x)^{10}+\ldots+(1+x)^{100}$ is

A. $^{101}C_9$

B. $^{101}C_{10}$

C. $^{100}C_9$

D. $^{100}C_{10}$

Answer: B

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9. If the 7th tern in the binomial expansion of $\left(\frac{3}{(84)^{\frac{1}{3}}}+\sqrt{3}\ln x\right)^9$, $x>0$ is equal to 729 then x can be (A) e^2 (B) e (C) $\frac{e}{2}$ (D) $2e$

A. e, 1/e

B. 2e,1/2e

C. e^2, e^{-2}

D. 1/e

Answer: A

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10. Sum of the series

$$S = \sum_{j=0}^{8} \frac{1}{(j+1)(j+2)} \left({}^{8}C_{j}\right) \text{ is}$$

A. $\frac{1003}{90}$

B. $\frac{1013}{90}$

C. $\frac{1023}{90}$

D. $\frac{1033}{90}$

Answer: B

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Solved Examples Level 1 Single Correct Answer Type Questions

1. If sum of the coefficients in the expansion of $(x+y)^n$ is 2048, then the greatest coefficient in the expansion is:

A. ${}^{10}C_6$

B. ${}^{11}C_6$

C. ${}^{10}C_7$

D. ${}^{12}C_6$

Answer: B

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2. Prove that : Find the 3^{rd} term from the end in the expansion of $\left(x^{-2/3}-\frac{3}{x^2}\right)^8$.

A. $25x^2$

B. -70

C. 70

D. $-25x^2$

Answer: C

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3. If $P(x)=\frac{1}{\sqrt{3x+1}}\left[\left(\frac{1+\sqrt{3x+1}}{5}\right)^n-\left(\frac{1-\sqrt{3x+1}}{5}\right)^n\right]$ is a 5th degree polynomial, then value of n is

A. 9

B. 11

C. 23

D. 21

Answer: B

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4. The two consecutive terms whose coefficients in the expansion of $(3+2x)^{94}$ are equal, are

A. 37th, 38th

B. 38th, 39th

C. 47th, 48th

D. 48th, 49th

Answer: B

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5. Coefficient of the constant term in the expansion of $E=\left(x^{2/3}+4x^{1/3}+4\right)^5\left(\frac{1}{x^{1/3}-1}+\frac{1}{x^{2/3}+x^{1/3}+1}\right)^{-9}$ is

A. 74

B. 98

C. 148

D. 168

Answer: D

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6. The coefficient of x^4 in the expansion of $\left(1+x+x^2\right)^6$ is

A. 240

B. 735

C. 820

D. 936

Answer: B

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7. Coefficient of x^{17} in the polynomial $P(x)=\prod_{r=0}^{17}\left(x+{}^{35}C_r\right)$ is

A. 2^{34}

B. ${}^{36}C_{17}$

C. $2^{35} - {}^{36}C_{17}$

D. 0

Answer: A

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8. The coefficient of x^{20} in the expansion of $\left(1 + \frac{1}{1!}x + \frac{1}{2!}x^2 + \ldots + \frac{1}{20!}x^{20}\right)^3$ is:

A. 0

B. $\frac{3}{20!}$

C. $\frac{3^{20}}{20!}$

D. ${}^{60}C_{20}$

Answer: C

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9. The coefficient of x^7 in the expansion of $\left(1+\frac{1}{1!}x+\frac{1}{2!}x^2+\frac{1}{3!}x^3+\frac{1}{4!}x^4+\frac{1}{5!}x^5\right)^2$ is:

A. $\frac{2}{7!}\left({}^8C_3\right)$

B. $\frac{2^{10}}{5!}$

C. $\frac{4}{15}$

D. $\frac{7}{20!}$

Answer: A

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10. Sum of the series $\sum_{k=1}^{\infty}\sum_{r=0}^{k}\frac{2^{2r}}{7^k}\left({}^kC_r\right)$ is:

A. $1/7$

B. $4/7$

C. 2.5

D. 5.2

Answer: C

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11. If the 6th term from the beginning is equal to the 6th term from the end in the expansion of $\left(2^{1/5}+\frac{1}{3^{1/5}}\right)^{n}$, then n is equal to:

A. 7

B. 9

C. 10

D. 12

Answer: C

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12. If the coefficients of x^7 and x^8 in the expansion of $\left[2+\frac{x}{3}\right]^n$ are equal, then the value of n is : (A) 15 (B) 45 (C) 55 (D) 56

A. 56

B. 55

C. 47

D. 19

Answer: B

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13. In the expansion of $\left(x^3-\frac{1}{x^2}\right)^{15}$, the constant term,is

A. ${}^{15}C_9$

B. 0

C. $-\left({}^{15}C_9\right)$

D. ${}^{15}C_{11}$

Answer: C

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14. If the 4 th term in the expansion of $\left(ax + \frac{1}{x}\right)^n$ is (5)/(2), f or $allx$in` R then the values of a and n are respectively

A. $5, \frac{1}{2}$

B. $6, -1/2$

C. $3, 1/3$

D. $6, 1/2$

Answer: D

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15. If the 6th term in the expansion of $\left[\frac{1}{x^{\frac{8}{3}}}+x^2\log_{10}x\right]^8$ is 5600, then x =

A. 2

B. $\sqrt{5}$

C. $\sqrt{10}$

D. 10

Answer: D

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16. If the (r + 1)th term in the expansion of $\left(\frac{a^{1/3}}{b^{1/6}}+\frac{b^{1/2}}{a^{1/6}}\right)^{21}$ has equal exponents of both a and b, then value of r is

A. 8

B. 9

C. 10

D. 11

Answer: B

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17. If A and B are the coefficients of x^n in the expansion $(1+x)^{2n}$ and $(1+x)^{2n-1}$ respectively, then

A. A=B

B. A=2B

C. 2A = B

D. none of these

Answer: B

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18. If x^{2k} occurs in the expansion of $\left(x+\frac{1}{x^2}\right)^{n-3}$, then

A. n - 2k is a multiple of 2

B. n- 2k is a multtiple of 3

C. k=0

D. none of these

Answer: B

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19. The coefficient of the term independent of x in the exampansion of $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}$ is 210 b. 105 c. 70 d. 112

A. 210

B. 105

C. 70

D. 35

Answer: A

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20. If $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \ldots + C_n x^n$, prove that

$C_1 + 2C_2 + 3C_3 + \ldots + nC_n = n \cdot 2^{n-1}$

A. 2^{n-1}

B. $n\left(2^{n-1}\right)$

C. $n\left(2^{n-1}\right) + 2^n$

D. $(n+1)2^n$

Answer: C

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21. $(1+x)^n = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + \ldots + C_n x^n$, prove that $C_0 - 2C_1 + 3C_2 - 4C_3 + \ldots + (-1)^n (n+1) C_n = 0$

A. -1

B. 0

C. 1

D. 2

Answer: B

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22. If $n \in N, n > 1,$ then value of $E = a - {}^nC_1(a-1) + {}^nC_2(a-2) + ... + (-1)^n(a-n)({}^nC_n)$ is

A. a

B. 0

C. a^2

D. 2^n

Answer: B

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23. Suppose ABC is a triangle and n is a natural number, then sum of the series

$S = \sum_{r=0}^{n} {}^{n}C_r, a^{n-r}b' \cos[nA - (n-r)B]$ is:

A. $(a+b)^n$

B. $(a-b)^n$

C. c^n

D. c^{2n}

Answer: C

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24. Find the positive integer just greater than $(1 + 0.0001)^{10000}$.

A. 2

B. 3

C. 4

D. 5

Answer: B

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25. The coefficient of the middle term in the binomial expansion in powers of x of$(1+\alpha x)^4$ and of$(1-\alpha x)^6$ is the same, if α equals $-\frac{5}{3}$ b. $\frac{10}{3}$ c. $-\frac{3}{10}$ d. $\frac{3}{5}$

A. $-\frac{3}{10}$

B. $\frac{10}{3}$

C. $-\frac{5}{3}$

D. $\frac{3}{5}$

Answer: A

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26. For natural numbers m, n if $(1-y)^m(1+y)^n = 1 + a_1y + a_2y^2 + \ldots\ldots$ and $a_1 = a_2 = 10$, then (m, n) is :

A. (35, 45)

B. (20, 45)

C. (35, 20)

D. (45, 35)

Answer: A

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27. The sum of the series

$.^{20}C_0 - .^{20}C_1 + .^{20}C_2 - .^{20}C_3 + \ldots - . + .^{20}C_{10}$ is -

A. $-\left({}^{20}C_{10}\right)$

B. $\frac{1}{2}\left({}^{20}C_{10}\right)$

C. 0

D. ${}^{20}C_{10}$

Answer: B

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28. The sum of the coefficients of all odd degree terms in the expansion of $\left(x+\sqrt{x^3-1}\right)^5+\left(x-\sqrt{x^3-1}\right)^5, (x>1)$ is :

A. 5

B. 6

C. 7

D. 8

Answer: C

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29. The coefficient of $x^r[0 \leq r \leq (n-1)]$ in lthe expansion of $(x+3)^{n-1} + (x+3)^{n-2}(x+2) + (x+3)^{n-3}(x+2)^2 + \ + (x+2)^{n-1}$ is ^ $nC_r(3^r - 2^n)$ b. ^ $nC_r\left(3^{n-r} - 2^{n-r}\right)$ c. ^ $nC_r\left(3^r + 2^{n-r}\right)$ d. none of these

A. $3^{n-r} - 2^{n-r}$

B. ${}^nC_r(3^r - 2^r)$

C. ${}^nC_r\left(2^{n-r} - 2^{n-r}\right)$

D. none of these

Answer: C

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30. Number of zeroes at the end of $99^{1001} + 1$?

A. 2

B. 4

C. 1002

D. 1004

Answer: A

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31. The co-efficient of $x^k(0 \leq k \leq n)$ in the expansion of $E = 1 + (1+x) + (1+x)^2 + \ldots . + (1+x)^n$ is

A. ${}^nC_k$

B. ${}^{n+1}C_k$

C. ${}^{n+1}C_{k+1}$

D. none of these

Answer: C

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32. Find the number of irrational terms in the expansion of $\left(5^{1/6}+2^{1/8}\right)^{100}$.

A. 96

B. 97

C. 98

D. 99

Answer: B

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33. The sum of the rational terms in the expansion of $\left(2^{1/5}+\sqrt{3}\right)^{20}$, is

A. 71

B. 85

C. 97

D. none of these

Answer: D

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34. In the expansion of $\left(x^3 - \frac{1}{x^2}\right)^n$, $n \in N$ if sum of the coefficients of x^5 and x^{10} is 0 then n is

A. 5

B. 10

C. 15

D. 20

Answer: C

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35. If in the expansion of $(1+ax)^n$, $n \in$ N, the coefficient of x and x^2 are 8 and 24 respectively, then

A. n = 3, p = 2

B. n= 4, p = 2

C. n = 4, p = 3

D. n = 5, p = 3

Answer: B

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36. The greatest value of the term independent of x, as α varies over R, in the expansion of $\left(x\cos\alpha+\frac{\sin\alpha}{x}\right)^{20}$ is

A. ${}^{20}C_{10}$

B. ${}^{20}C_{15}$

C. ${}^{20}C_{19}$

D. $\frac{{}^{20}C_{10}}{2^{10}}$

Answer: D

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37. Sum of the last 20 coefficients in the expansion of $(1+x)^{39}$, when expanded in ascending powers of x, is

A. 2^{19}

B. 2^{18}

C. ${}^{40}C_{20} - 2^{19}$

D. 2^{38}

Answer: D

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38. If $(1+x)^{15} = C_0 + C_1 x + C_2 x^2 + \ + C_{15} x^{15}$, then find the su of $C_1 + 2C_3 + 3C_4 + \ + 14C_{15}$.

A. $13\left(2^{14}\right) + 1$

B. $13\left(2^{15}\right)$

C. $13\left(2^{14}\right)$

D. none of these

Answer: A

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39. Let $R = \left(2+\sqrt{3}\right)^{2n}$ and $f = R - [R]$ where [] denotes the greatest integer function, then $R(1-f)$ is equal to

A. 1

B. 2^{2n}

C. 2^{2n-1}

D. ${}^{2n}C_n$

Answer: A

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40. If [ ] denotes the greatest integer function, then $\left[\left(\sqrt{2}+1\right)^6\right)$ is equal

A. 199

B. 198

C. 197

D. 196

Answer: C

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41. Sum of the coefficients of the terms of degree m in the expansion of $(1+x)^n(1+y)^n(1+z)^n$ is

A. $({}^nC_m)^3$

B. $3({}^nC_m)$

C. ${}^nC_{3m}$

D. ${}^{3n}C_m$

Answer: D

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42. The remainder when 2^{2000} is divided by 17 is

A. 1

B. 2

C. 8

D. 12

Answer: A

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43. If $\left(1+x+x^2\right)^{48}=a_0+a_1x+a_2x^2+\ldots+a_{96}x^{96}$, then value of $a_0-a_2+a_4-a_6+\ldots+a_{96}$ is

A. -1

B. 0

C. 1

D. 48

Answer: C

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44. Sum of the series S = $3^{-1}\left({}^{10}C_0\right)-{}^{10}C_1+(3)\left({}^{10}C_2\right)-3^2\left({}^{10}C_3\right)+\ldots+3^9\left({}^{10}C_{10}\right)$ is

A. 2^9

B. $2^{10}-1$

C. $\frac{1}{3}(2^{11}-2)$

D. $\frac{1}{3}(2^{10})$

Answer: D

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45. The coefficient of x^{53} in the expansion $\sum_{m=0}^{100}$ ˆ $(100)C_m(x-3)^{100-m}2^m$ is ˆ $100C_{47}$ b. ˆ $100C_{53}$ c. $-^{100}C_{53}$ d. none of these

A. $^{100}C_{47}$

B. $^{100}C_{53}$

C. $-^{100}C_{53}$

D. 2^{53}

Answer: C

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46. Let $E = 1^{2017} + 2^{2017} + 3^{2017} + \ldots + 2016^{2017}$, then E is divisible by

A. 2016

B. 2017

C. 4033

D. 2016^2

Answer: B

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47. Let $\left(1 + x + x^2\right)^{100} = \sum_{r=0}^{200} a_r x^r$ and $a = \sum_{r=0}^{200} a_r$, then value of $\sum_{r=1}^{200} \frac{r a_r}{25a}$ is

A. 5

B. 4

C. 3

D. 2

Answer: B

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48. If $\left(1+x+x^2\right)^8 = a_0 + a_1x + a_2x^2 + \ldots a_{16}x^{16}$ for all values of x, then $\frac{a_5}{100}$ is equal to

A. 502

B. 504

C. 506

D. 508

Answer: B

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49. Sum of the last 12 coefficients in the binomial expansion of $(1+x)^{23}$ is:

A. 2^{22}

B. 2^{23}

C. $2^{23} - {}^{23}C_{11}$

D. $2^{23} - 2^{11}$

Answer: A

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50. The coefficients of x^3 and x^4 in the expansion of $\left(1+ax+bx^2\right)(1-2x)^{18}$

A. $\left(14, \frac{251}{3}\right)$

B. $\left(14, \frac{272}{3}\right)$

C. $\left(16, \frac{272}{3}\right)$

D. $\left(16, \frac{251}{3}\right)$

Answer: C

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51. If a, b and c are three consecutive coefficients terms in the expansion of $(1+x)^n$, then find n.

A. n

B. 1

C. 0

D. 2n

Answer: C

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52. Suppose $a_0 = 2017, a_1 a_2, \ldots, a_{n-1}, 2023 = a_n$, are in A.P. Let

$$S = \frac{1}{2^{n+1}} \sum_{r=0}^{n} ({}^{n}C_r) a_r - 1000$$

Then S is equal to

A. 10

B. 40

C. 1013

D. 2021

Answer: A

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53. The interval in which x must lie so that the

greatst term in the expansion of $(1+x)^{2n}$ has the greatest

coefficient,is

A. $\left(\frac{n-1}{n}, n(n-1)\right)$

B. $\left(\frac{n}{n+1}, \frac{n+1}{n}\right)$

C. $\left(\frac{n}{n+2}, \frac{n+2}{n}\right)$

D. none of these

Answer: B

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54. If the coefficients of the rth, (r +1)th and (r + 2)th terms in the binomial expansion of (1 + y)^(m)` are in A.P., then m and r satisfy the equation

A. $m^2 - m(4r+1) + 4r^2 - 2 = 0$

B. $m^2 - m(4r-1) + 4r^2 + 2 = 0$

C. $m^2 - m(4r-1) + 4r^2 - 2 = 0$

D. $m^2 - m(4r+1) + 4r^2 + 2 = 0$

Answer: A

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55. Let x > - 1, then statement $P(n) : (1+x)^n > 1+nx$

is true for

A. all $n \in N$

B. all $n > 1$

C. all $n > 1$ provided $x \neq 0$

D. none of these

Answer: C

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56. For $n \in N$, $2^{3n} + 1$ is divisisble by _____

A. 8

B. 16

C. 32

D. none of these

Answer: D

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57. $x^{2n-1} + y^{2n-1}$ is divisible by $x + y$

A. x+y

B. $(x + y)^2$

C. $x^3 + y^3$

D. none of these

Answer: A

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58. For each $n \in N$, $n(n+1)(2n+1)$ is divisible by

A. 6

B. 8

C. 15

D. 9

Answer: A

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59. If $a_n = \sqrt{7+\sqrt{7+\sqrt{7}+.....}}$ having n radical signs then by methods of mathematical induction which is true

A. $a_n > 7Vn \geq 1$

B. $a_n > 3Vn \geq 1$

C. $a_n < 4V \leq 1$

D. $a_n < 3Vn \geq 1$

Answer: C

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60. If $a_n = 2^{2^n} + 1$, then for n > 1, last digit of a_n is

A. 5

B. 7

C. 3

D. 4

Answer: B

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61. Let $S(k) = 1 + 3 + 5 + \ldots + (2k - 1) = 3 + k^2$. Then which of the following is true ? (A) S(1) is correct (B) S(k)=S(k+1) (C) $S(k) \neq S(k + 1)$ (D) Principal of mathematical induction can be used to prove the formula

A. $S(K) \neq S(K + 1)$

B. $S(K) \rightarrow S(K + 1)$

C. S(1) is correct

D. principle of mathematical induction can be used to prove the formula.

Answer: B

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62. Let $(1 + x)^n = \sum_{r=0}^{n} C_r x^r$ and ,

$$\frac{C_1}{C_0} + 2\frac{C_2}{C_1} + \frac{C_3}{C_2} + \ldots + n\frac{C_n}{C_{n-1}} = \frac{1}{k}n(n + 1),$$

then the value of k, is

A. 2

B. 3

C. 6

D. 12

Answer: A

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63. The last term in the binomial expansion of $\left(2^{\frac{1}{3}} - \frac{1}{\sqrt{2}}\right)^n$ is $\left(\frac{1}{3.9^{\frac{1}{3}}}\right)^{\log_3 8}$ then the 5th term from the beginning is

A. ${}^{10}C_6$

B. $2\left({}^{10}C_4\right)$

C. $\frac{1}{2}\left({}^{10}C_4\right)$

D. $-1{}^{10}C_6$

Answer: B

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64. Degree of the polynomial $\left(x^2-\sqrt{1-x^3}\right)^4+\left(x^2+\sqrt{1-x^3}\right)^4$ is

A. 7

B. 8

C. 9

D. 12

Answer: B

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65. Coefficient of $\frac{1}{x}$ in the expansion of $(1+x)^n(1+1/x)^n$ is

A. ${}^{2n}C_{n-1}$

B. ${}^{2n}C_n$

C. 1

D. 0

Answer: A

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66. Find the coefficient of x^4 in the expansion of $\left(1+x+x^2+x^3\right)^{11}$

A. 99

B. 330

C. 990

D. 1050

Answer: C

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67. Find the 6th term of the expansion $\left(y^{1/2}+x^{1/3}\right)^{n}$, if the binomial coefficient of 3rd term from the end is 45.

A. $252x^{5/3}y^{5/2}$

B. 45

C. $45x^{5/3}y^{5/2}$

D. none of these

Answer: A

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68. If a> 0 and coefficient of x^2, x^3, x^4 in the expansion of $(1+x/a)^6$ are in A.P., then a equals

A. $\left(4+\sqrt{7}\right)/3$

B. $\left(4+\sqrt{3}\right)/3$

C. $2-\sqrt{3}$

D. $2+\sqrt{3}$

Answer: A

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69. The sum of the coefficients of middle terms in the expansion of $(1+x)^{2n-1}$

A. ${}^{2n-1}C_{n-1}$

B. ${}^{2n-1}C_{n}$

C. ${}^{2n}C_{n}$

D. ${}^{2n}C_{n+1}$

Answer: C

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70. If a is a real number and if the middle term of $\left(\frac{a}{3}+3\right)^8$ is 1120, then value of a is

A. ± 2

B. ± 1

C. $\pm \sqrt{3}$

D. $\pm \sqrt{2}$

Answer: A

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71. Suppose F is the fractional part of $M = \left(\sqrt{13}+\sqrt{11}\right)^6$, then value of M (1- F) is

A. 128

B. 64

C. 32

D. 16

Answer: B

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72. The remainder left out when $8^{2n}(62)^{2n+1}$ is divided by 9 is (1) 0 (2) 2 (3) 7 (4) 8

A. 7

B. 8

C. 0

D. 2

Answer: B

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73. The coefficient of x^7 in the expansion of $\left(1-x-x^2+x^3\right)^6$ is :

A. 132

B. 144

C. -132

D. -144

Answer: D

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74. If n is a positive integer, then $\left(\sqrt{3}+1\right)^{2n}-\left(\sqrt{3}-1\right)^{2n}$ is (1) an irrational number (2) an odd positive integer (3) an even positive integer (4) a rational number other than positive integers

A. an odd positive integer

B. an even positive integer

C. a rational number but not a natural number

D. an irrational number

Answer: D

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75. The coefficient of x^n in the expansion of

$$\left(1+\frac{1}{1!}x+\frac{1}{2!}x^2\ldots+\frac{1}{n!}x^n\right)^2$$

A. $\frac{2^n}{n!}$

B. $\frac{2^n}{n}$

C. $n!$

D. $\frac{1}{n!}$

Answer: A

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76. The coefficient of x^n in the expansion of $(1+x)(1-x)^n$ is

A. $(-1)^n n$

B. $(-1)^n(1-n)$

C. $(-1)^{n-1}(n-1)$

D. $(n-1)$

Answer: B

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77. Sum of the coefficients of x^3 and x^6 in the expansion of $\left(x^2 - \frac{1}{x}\right)^9$ is

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78. For $n \geq 2$, let

$a_n = \sum_{r=0}^{n} \frac{1}{C_r^2}$, then value of $b_n = \sum_{r=1}^{n} \frac{1}{r^2 C_r^2}$ equals

A. $\frac{1}{n^2} a_n$

B. $\frac{1}{n^2} a_{n-1}$

C. a_n

D. a_n^2

Answer: B

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79. If in the expansion of $\left(x^3 - \frac{1}{x^2}\right)^n$ the sum of the coefficients of x^5 and x^{10} is 0, then the coefficient of x^{20} is:

A. ${}^{20}C_6$

B. $-{}^{20}C_6$

C. ${}^{15}C_5$

D. $-{}^{15}C_5$

Answer: D

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80. If

$(1+x)\left(1+x+x^2\right)\left(1+x+x^2+x^3\right)\ldots\left(1+x+x^2+\ldots+x^n\right)=a_0+$

, then value of a_1 is

A. m+1

B. n+1

C. n

D. m

Answer: C

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Solved Examples Level 2 Single Correct Answer Type Questions

1. If

$x^n = a_0 + a_1(1+x) + a_2(1+x)^2 + \ldots + a_n(1+x)^n = b_0 + b_1(1-x) +$

then for n = 201, (a_{101}, b_{101}) is equal to:

A. $\left(-{}^{201}C_{101}, -{}^{201}C_{101}\right)$

B. $\left({}^{201}C_{101}, -{}^{201}C_{101}\right)$

C. $\left(-{}^{201}C_{101}, {}^{201}C_{101}\right)$

D. $\left({}^{201}C_{101}, {}^{201}C_{101}\right)$

Answer: B

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2. The number of rational terms in the expansion of $\left(1+\sqrt{2}+\sqrt[3]{3}\right)^6$ is

A. 7

B. 11

C. 12

D. 12

Answer: A

Watch Video Solution

3. The sum to $(n+1)$ terms of the series $\frac{C_0}{2} - \frac{C_1}{3} + \frac{C_2}{4} - \frac{C_3}{5} + \ldots\ldots =$

A. $\frac{1}{n+1}$

B. $\frac{1}{n+2}$

C. $\frac{1}{n(n+1)}$

D. $\frac{1}{(n+1)(n+2)}$

Answer: D

Watch Video Solution

4. $E = \left(3^{\log_3 \sqrt{9^{|x-2|}}} + 7^{\left(\frac{1}{5}\right)\log_7(4).3^{|x-2|}} - 9\right)^7$

A. 3, 4

B. 3,1

C. 2,7

D. none of these

Answer: B

Watch Video Solution

5. Let a_n denote the term independent of x in the expansion of $\left[x + \frac{\sin(1/n)}{x^2}\right]^{3n}$, then $\lim_{x \to \infty} \frac{(a_n)n!}{{}^{3n}P_n}$ equals

A. 0

B. 1

C. e

D. $e/\sqrt{3}$

Answer: A

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6. Find the largest term in the expansion of $(3+2x)^{50}$, $where x = 1/5$.

A. ${}^{50}C_6 3^{44}\left(\frac{2}{5}\right)^6$

B. ${}^{50}C_7 3^{43}\left(\frac{2}{5}\right)^7$

C. ${}^{50}C_{43} 3^{7}\left(\frac{2}{5}\right)^7$

D. ${}^{50}C_{44} 3^{6}\left(\frac{2}{5}\right)^{44}$

Answer: A

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7. If n is odd, then sum of the series $C_0^2 - C_1^2 + C_2^2 - C_2^3 + \ldots . + (- 1)^n C_n^2$ is

A. 0

B. 1

C. ${}^{2n}C_n$

D. $- \left({}^{2n}C_n \right)$

Answer: A

Watch Video Solution

8. If $(1 + x)^n = C_0 + C_1 x + C_2 x^2 + \ldots + C_n x^n$, prove that

$C_0^2 - C_1^2 + C_2^2 - \ldots + (- 1)^n \cdot C_n^2 = 0$ or

$(- 1)^{n/2} \cdot \frac{n!}{(n/2)!(n/2)!}$, according as n is odd or even

Also , evaluate $C_0^2 + C_1^2 + C_2^2 - \ldots + (- 1)^n \cdot C_n^2$ for n

` = 10 and n= 11 .

A. ${}^nC_{\frac{n}{2}}(-1)^{n/2}$

B. ${}^n\frac{C}{\frac{n}{2}}$

C. ${}^nC_{n-1}$

D. $(-1)^{n/2}{}^{n}C_2$

Answer: A

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9. If $(1+x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$, then value of $C_0^2 + 2C_1^2 + 3C_2^2 + ... + (n+1)C^2n$ is

A. $(2n+1)\left({}^{2n}C_n\right)$

B. $(2n-1)\left({}^{2n}C_n\right)$

C. $\left(\frac{n}{2}+1\right)\left({}^{2n}C_n\right)$

D. $\left(1+\frac{n}{2}\right)\left({}^{2n-1}C_n\right)$

Answer: C

Watch Video Solution

10. If C_r stands for ${}^nC_r$, then the sum of the series $\frac{2\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!}{n!}\left[C_0^2 - 2C_1^2 + 3C_2^2 - \ldots + (-1)^n(n+1)C_n^2\right]$, where n is an even positive integers, is:

A. $(-1)^{n/2}(n+2)$

B. $(-1)^n(n+1)$

C. $(-1)^{n/2}(n+1)$

D. none of these

Answer: A

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11. The sum of coefficients of integral powers of x in the binomial expansion of $\left(1 - 2\sqrt{x}\right)^{50}$ is: (1) $\frac{1}{2}\left(3^{50}+1\right)$ (2) $\frac{1}{2}\left(3^{50}\right)$ (3) $\frac{1}{2}\left(3^{50}-1\right)$

(4) $\frac{1}{2}(2^{50}+1)$

A. $\frac{1}{2}(3^{50}+1)$

B. $\frac{1}{2}(3^{50})$

C. $\frac{1}{2}(2^{50}-2)$

D. $\frac{1}{2}(2^{50}+2)$

Answer: A

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12. Given that the 4th term in the expansion of $[2+(3/8x)]^{10}$ has the maximum numerical value. Then find the range of value of x.

A. $\left(-\frac{64}{21},-2\right)\cup\left(2,\frac{64}{21}\right)$

B. $\left(-\frac{60}{23},-2\right)\cup\left(2,\frac{64}{21}\right)$

C. $\left(-\frac{64}{21},-2\right)$

D. none of these

Answer: A

Watch Video Solution

13. Assuming x to be so small that x^3 and higher powers of x can be neglected, then value of $E = \left(1 - \frac{3}{2}x\right)^5 (2 + 3x)^6$, is

A. 64 + 96x + 720x^2

B. 65 - 97x + 721x^2

C. 64- 96x + 720x^2

D. 64 + 96x - 720x^2

Answer: D

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14. If $\left(1+x+x^2\right)^8 = a_0 + a_1x + a_2x^2 + \ldots a_{16}x^{16}$ for all values of x, then $\frac{a_5}{100}$ is equal to

A. 504

B. 506

C. 508

D. 502

Answer: A

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15. Coefficient of x^{11} in the expansion of $\left(1+x^2\right)\left(1+x^3\right)^7\left(1+x^4\right)^{12}$ is

1051 b. 1106 c. 1113 d. 1120

A. 1051

B. 1106

C. 1113

D. 1120

Answer: C

Watch Video Solution

16. The coefficients of three consecutive terms of $(1+x)^{n+5}$ are in the ratio $5:10:14$. Then, n is equal to :

A. 5

B. 6

C. 7

D. 8

Answer: B

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17. Sum of the series

$$\sum_{k=0}^{n} {}^{n}C_k(-1)^k \frac{1}{a_k}$$

where $a_k = \sum_{i=0}^{k} {}^{k}C_i b_i$

where $b_i = \sum_{j=0}^{i} {}^{i}C_j \left(\frac{-2}{3}\right)^j$

is

A. $\frac{1}{2^n}$

B. $\frac{1}{3^n}$

C. $\frac{1}{4^n}$

D. $\left(\frac{3}{4}\right)^n$

Answer: C

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18. Suppose m and n are positive integers and let

$S = \sum_{k=0}^{n} (-1)^k \frac{1}{k+m+1}(nC_k)$ and $T = \sum_{k=0}^{m} (-1)^k 1(k+n+1)(mC_k$

then $S - T$ is equal to

A. 0

B. $n^m - m^n$

C. $(n+1)^m - (m+1)^n$

D. $(1-n)^m - (1-m)^n$

Answer: A

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19. Coefficient of x^m in the expansion of

$S = (1+x)^{2m} + x(1+\cdot x)^{2m-1} + x^2(1+x)^{2m-2} + \ldots. + x^{2m}$ is

A. ${}^{2m}C_m + {}^{2m+1}C_{m-1}$

B. ${}^{2m+1}C_m$

C. 0

D. $2m^2 + 2m + 1$

Answer: B

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20. Sum of the first 20 terms of the series

$$\frac{1}{(2)(4)}+\frac{(1)(3)}{(2)(4)(6)}+\frac{(1)(3)(5)}{(2)(4)(6)(8)}+\ldots.$$ is

A. $\frac{1}{2}-\frac{1}{2^{40}}\left({}^{40}C_{20}\right)$

B. $\frac{1}{2}-\frac{1}{2^{41}}\left({}^{42}C_{21}\right)$

C. $\frac{1}{2}-\frac{1}{2^{42}}\left({}^{42}C_{21}\right)$

D. $\frac{1}{2}-\frac{1}{2^{43}}\left({}^{40}C_{20}\right)$

Answer: C

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21. The term independent of x in the expansion of

$$\left(1-\frac{1}{2x}+3x^{7}\right)\left(x^{2}+\frac{1}{3x}\right)^{10}$$ is

A. $10/3^8$

B. $-2/3$

C. $10/3^9$

D. $11/6$

Answer: A

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22. Let P(x) be a polynomial with real coefficients such that $\int_0^1 x^m P(1-x)dx = 0 \forall \min N \cup \{0\}$, then

A. $P(x) = x^n(1-x)^n$ for some $n \in N$

B. P(x) = $(1-x)^{2n}$ for some $n \in N$

C. P(x) = $1 - x^m(1-x)^n$ for some m, $n \in N$

D. P(x)=0

Answer: D

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23. Suppose [x] denote the greatest integer $\leq x$ and $n \in N$, then

$$\lim_{n \to \infty} \frac{\left[{}^{n}C_0 x^2\right] + \left[{}^{n}C_1 x^2\right] + \ldots. + \left[{}^{n}C_n x^2\right]}{2^n - 2}$$

is equal to

A. $\frac{1}{2}x^2$

B. x^2

C. $2x^2$

D. $4x^2$

Answer: B

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Solved Examples Numerical Answer Type Questions

1. The sum of the coefficients in the expansion of $\left(x^2-\frac{1}{3}\right)^{199}\left(x^3+\frac{1}{2}\right)^{200}$ is _____

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2. $\sum_{k=1}^{\infty}\sum_{r=1}^{k}\frac{1}{4^k}\left({}^kC_r\right)$ is equal to=_____

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3. If ab $\neq$ 0 and the sum of the coefficients of x^7 and x^4 in the expansion of $\left(\frac{x^2}{a}-\frac{b}{x}\right)^{11}$ is zero, then ab=____

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4. If t_r denotes the rth term in the expansion $\left(x+\frac{1}{x}\right)^{23}$, and $t_{12}=4t_{13}$, then $|x|$=_____

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5. The numbet of values of r for which the coefficients of rth and (r + 1)th terms in the expansion of $(1+x)^{3n}$ are in the ratio 1 :2, is __ _

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6. Let t, denote the rth term in the binomial expansion of $(1+a)^{50}$. If $t_{25}+t_{27}=\frac{125}{52}t_{26}$ then the sum of all possible values of a is ___

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7. If coefficient of x^{21} in the expansion of $(1+x)^{21}+(1+x)^{22}+\ldots+(1+x)^{30}$ is ${}^{31}C_r$, where $r \geq 20$, then r= ___

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8. The number of irrational terms in the binomial expansion of $\left(3^{1/5}+7^{1/3}\right)^{100}$ is ___

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9. If the expansion of $\left(\frac{3}{7}\sqrt{x}-\frac{5}{2}\frac{1}{x\sqrt{x}}\right)^{13n}$ $x>0$ contains a term independent of x, then n should be a multiple of _____

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10. The coefficient of the term independent of x in the expansion of $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}$

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11. The sum of the coefficients of all odd degree terms in the expansion of $\left(x+\sqrt{x^3-1}\right)^5+\left(x-\sqrt{x^3-1}\right)^5, (x>1)$ is :

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12. Let $[x]$ denote the greatest integer less than or equal to x. If $x = \left(\sqrt{3}+1\right)^5$, then $[x]$ is equal to

Watch Video Solution

13. Coefficient of the term independent of x in the expansion of $\left(\frac{1}{2}x^{1/3} + x^{-1/5}\right)^8$ is _____

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14. If the last tem in the binomial expansion of $\left(2^{\frac{1}{3}} - \frac{1}{\sqrt{2}}\right)^n$ $is\left(\frac{1}{3^{\frac{5}{3}}}\right)^{\log_3 8}$, then 5th term from the beginning is 210 b. 420 c. 105 d. none of these

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15. If $\sum_{r=0}(n)(-1)^r \frac{{}^nC_r}{{}^{r+3}C_r} = \frac{3}{a+3}$ then $\frac{3a}{4n} =$ _____ is equal to

Watch Video Solution

16. If n $\in$ N, then

$$\lim_{n\to\infty}\left[\sum_{k=0}^{n}\frac{1}{k+1}({}^nC_k)\left(k\sum_{l=1}^{k}({}^iC_k)\right)+\frac{1}{n+l}\right]^{1/n}$$ is equal to _____.

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17. If $\sum_{r=0}^{2n} a_r(x-2)^r = \sum_{r=0}^{2n} b_r(x-3)^r and a_k = 1$ for all $k \geq n$, then show that $b_n = {}^{2n+1}C_{n+1}$.

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18. Let S be the sum of the last 24 coefficients in the expansion of $(1+x)^{47}$ when expanded in ascending powers of x, then $\frac{S-2^{44}}{2^{44}}$=____

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19. Let $(1+x)^{10}=\sum_{r=0}^{10} c_r x^r$ and $(1+x)^7=\sum_{r=0}^{7} d_r, x^r$. If $P=\sum_{r=0}^{5} c_{2r}$ and $Q=\sum_{r=0}^{3} d_{2r+1}$, then $\frac{P}{Q}$ is equal to _____

Watch Video Solution

20. If $n \geq 2$, and $\left(1+x+x^2\right)^n=a_0+a_1x+a_2x^2+\ldots+a_{2n}x^{2n}$, then $a_0-2a_1+3a_2+\ldots+(2n+1)a_n$ = _____

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Exercise Concept Based Single Correct Answer Type Questions

1. If the coefficient of x in $\left(x^2+\frac{k}{x}\right)^5$ is 270, then k=

A. 3

B. -3

C. 9

D. -9

Answer: A

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2. If the fourth term in the expansion of $\left(px+\frac{1}{x}\right)^n$ is $\frac{5}{2}$, then $(n, p) =$

A. (5, 1/2)

B. (6, 1/2)

C. (8, 1/2)

D. (10, 1/2)

Answer: B

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3. If the coefficient of (3r + 4)th term and (2r - 2)th term in the expansion of $(1+x)^{20}$ are equal then r is equal to,

A. 4

B. 5

C. 7

D. 8

Answer: A

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4. If the sum of the coefficients in the expansion of $(1+2x)^n$ is 6561 , then the greatest coefficients in the expansion, is

A. 442

B. 446

C. 448

D. 472

Answer: C

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5. The coefficient of x20 in the expansion of $(1+x^2)^{40} \cdot \left(x^2+2+\frac{1}{x^2}\right)^{-5}$ is :

A. ${}^{10}C_5$

B. ${}^{15}C_5$

C. 0

D. 2

Answer: C

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6. If $(1+ax)^n = 1+10x+40x^2+\ldots$ then value of $\frac{a+n}{a-n}$ is

A. $5/7$

B. $7/3$

C. $-5/7$

D. $-7/3$

Answer: D

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7. Prove that the ratio of the coefficient of x^{10} in $\left(1-x^2\right)^{10}$ & the term independent of x in $\left(x-\frac{2}{x}\right)^{10}$ is $1:32$

A. $32:1$

B. $1:32$

C. $1:64$

D. $64:1$

Answer: B

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8. If coefficient of 9th, 10th and 11th terms of $(1+x)^n$ are in A.P., then the value of n can be

A. 15

B. 32

C. 23

D. 17

Answer: C

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9. (p + 2)th term from the end in the binomial expansion of $\left(x^2 - \frac{2}{x^2}\right)^{2n+1}$ is

A. ${}^{2n+1}C_{2n-p}(-2)^{2n-p}x^{2p+1-2n}$

B. ${}^{2n+1}C_{2n-p}(-2)^{2n-p}x^{2n-2p}$

C. ${}^{2n+1}C_{2n-p}(-2)^{2n-p}x^{2n-2p+1}$

D. ${}^{2n+1}C_{2n-p}(-2)^{2n-p}x^{2p-1+2n}$

Answer: A

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10. Let $C_r = {}^{15}C_r, 0 \leq r \leq 15$. Sum of the series $S = \sum_{r=1}^{15} r\frac{C_r}{C_{r-1}}$ is

A. 40

B. 60

C. 100

D. 120

Answer: D

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Exercise Level 1 Single Correct Answer Type Questions

1. If the middle term in the expansion of $\left(x^2+1/x\right)^n$ is $924x^6$, then find the value of n.

A. 8

B. 10

C. 12

D. 20

Answer: C

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2. The greatest term in the expansion of $\sqrt{5}\left(1+\frac{1}{\sqrt{5}}\right)^{20}$ is:

A. ${}^{20}C_5\left(\frac{1}{25}\right)$

B. ${}^{20}C_5\left(\frac{1}{5}\right)$

C. ${}^{20}C_5$

D. ${}^{20}C_5(5)$

Answer: A

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3. Let $S(\theta)$ denote the sum of coefficients in the expansion of $\left(\sqrt{2}-x\sin\theta+x^2\cos\theta\right)^{2n}$. Maximum value of $S(\theta)$ is

A. 4^n

B. 8^n

C. 2^n

D. 1

Answer: B

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4. Suppose

$(x+a)^n = T_0 + T_1 + T_2 + \ldots + T_n$

Then $(T_0 - T_2 + T_4 - \ldots)^2 + (T_1 - T_3 + T_5 - \ldots.)^2$ is equal to

A. $\left(x^2 + a^2\right)^n$

B. $\left(x^2 - a^2\right)^n$

C. $(x+a)^{2n}$

D. $(x-a)^{2n}$

Answer: A

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5. For $0 \leq r < 2n$, $\left({}^{2n+r}C_n\right)\left({}^{2n-r}C_n\right)$ cannot exceed

A. ${}^{4n}C_{n-}$

B. ${}^{4n}C_{2n}$

C. ${}^{6n}C_{3n}$

D. none of these

Answer: B

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6. If sum of the coefficients in the expansion $\left(2x^2 - 3cx + c^2\right)^{17}$ is zero , then c is equal to

A. 2,3

B. 1,2

C. 1,3

D. 2,-1

Answer: B

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7. Coefficient of constant term in the expansion of $\left(x^3+5(3)^{-\log\sqrt{3}\sqrt{x^3}}\right)^4$ is

A. 125

B. $\log_{\sqrt{3}}5$

C. $\log_3 5$

D. 150

Answer: D

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8. If the number of terms in the expansion of $\left(1+5x+10x^2+10x^3+5x^4+x^5\right)^{20}$ is m, then unit's place of 2^m is

A. 2

B. 8

C. 6

D. 4

Answer: A

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9. If P_n denotes the product of all the coefficients in the expansion of $(1+x)^n, n \in N$, show that, $\frac{P_{n+1}}{P_n} = \frac{(n+1)^n}{n!}$.

A. 21

B. 20

C. 19

D. 18

Answer: B

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10. Coefficient of the term independent of x in the expansion of $(1+x)^{2n}\left(\frac{x}{1-x}\right)^{-2n}$ is:

A. ${}^{2n}C_n$

B. $(-1)^n {}^{2n}C_n$

C. ${}^{n^2-n}C_n$

D. 0

Answer: B

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11. Find the coefficient of x^4 in the expansion of $\left(x/2-3/x^2\right)^{10}$.

A. $\frac{504}{256}$

B. $\frac{405}{256}$

C. $\frac{459}{512}$

D. $-\frac{135}{256}$

Answer: B

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12. If A and B are coefficients of x^r and x^{n-r} respectively in the expansion of $(1+x)^n$, then

A. $\alpha = \beta$

B. $\alpha + \beta = n$

C. $\alpha = k\beta$ for some constant k

D. $\alpha + \beta = 2^n$

Answer: A

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13. If the coefficients of $(2r+4)th$ and $(r-2)th$ terms in the expansion of $(1+x)^{18}$ are equal.Find r ?

A. 5

B. 6

C. 7

D. 9

Answer: B

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14. The total number of terms in the expansion of $(x+a)^{200}+(x-a)^{200}$ after simplification is

A. 101

B. 102

C. 201

D. 202

Answer: A

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15. Sum of all the digits of the coefficient of x^5 in the expansion of $\left(1+x^2\right)^5(1+x)^4$ is

A. 30

B. 40

C. 50

D. 60

Answer: D

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16. Given positive integers $r > 1, n > 2$ and that the coefficient of $(3rd)thand(r+2)th$ terms in the binomial expansion of $(1+x)^{2n}$ are equal. Then $n = 2r$ b. $n = 2r+1$ c. $n = 3r$ d. none of these

A. n = 2r

B. n=3r

C. n=2r+1

D. n=2r+2

Answer: A

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17. If $(1+px)^n = 1 + 24x + 264x^2 + \ldots$ then

A. p = 2, n = 6

B. p = 2, n = 12

C. p = 3, n = 6

D. none of these

Answer: B

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18. find the sum of the series $\sum_{r=0}^{n}(-1)^r \cdot {}^nC_r$

$\left[\frac{1}{2^r}+\frac{3^r}{2^{2r}}+\frac{7^r}{2^{3r}}+\frac{15^r}{2^{4r}}\ldots\text{up to m terms}\right]$

A. $\frac{2^{10}+1}{2^{20}}$

B. $\frac{2^{10}-1}{2^{20}}$

C. $\frac{2^{20}-1}{2^{20}}$

D. none of these

Answer: A

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19. The value of

$.^{n}C_{0}.^{n}C_{n}.^{n}C_{1}.^{n}C_{n-1}+\ldots+.^{n}C_{n}.^{n}C_{0}$ is

A. $^{m+n}C_{k}$

B. $^{m+n}C_{m+k}$

C. $^{m+n}C_{n+k}$

D. none of these

Answer: A

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20. Express: $\left(\left(x+\sqrt{x^{2}+1}\right.^{6}+\left(x-\sqrt{x^{2}+1}\right)^{6}\right.$ as a polynomial in x

A. 6

B. 7

C. 9

D. 10

Answer: C

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21. The number of distinct terms in the expansion of $\left(1+3x+3x^2+x^3\right)^7$ is

A. 18

B. 19

C. 28

D. 22

Answer: D

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22. If x > 0, then the number of positive real terms in the expansion of $(1+ix)^{4n}$ is

A. n+1

B. n-1

C. n=2r+1

D. 2n

Answer: A

Watch Video Solution

23. The total number of terms which depand on the value of x in the expansion of $\left(x^2-2+\frac{1}{x^2}\right)^n$ is

A. 2n

B. 2n+1

C. 2n-1

D. 0

Answer: A

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24. If the third term in the expansion of $\left(\frac{1}{x}+{}_{\mathrm{x}}(\log)_{10x}\right)^5$ is $1000,$ then find $x\cdot$

A. $\sqrt{10}$

B. 10

C. 100

D. $10\sqrt{10}$

Answer: C

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25. Let R $=\left(\sqrt{2}+1\right)^{2n+1}, n \in N$ and f= R- [R], where [] denote the greatest integer function, Rf is equal to

A. 1

B. 2^{2n+1}

C. $2^{2n}-1$

D. none of these

Answer: A

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26. The coefficient of x^{60} in $(1+x)^{51}\left(1-x+x^2\right)^{50}$ is

A. ${}^{50}C_{20}$

B. $-\left({}^{50}C_{20}\right)$

C. ${}^{51}C_{20}$

D. $-\left({}^{51}C_{20}\right)$

Answer: A

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27. Which is larger : $\left(99^{50}+100^{50}\right)$ or $(101)^{50}$.

A. $a<b$

B. $a=b$

C. $a>b$

D. $a-b=100^{49}$

Answer: A

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28. $\left(1+x-2x^2\right)^6=\sum_{r=0}^{12}a_r x^r$ then $a_2+a_4+\ldots..+a_{12}=$

A. 30

B. 31

C. 32

D. none of these

Answer: B

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29. The sum of the coefficients in the expansion of $\left(1+x-3x^2\right)^{4321}$ is

A. 0

B. 1

C. -1

D. 2^{4320}

Answer: C

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30. The greatest integer contained in $\left(\sqrt{3}+1\right)^6$ is

A. 208

B. 416

C. 415

D. 207

Answer: C

Watch Video Solution

31. The number of terms in the expansion of $\left(x^2+6x+9\right)^{30}$ is

A. 31

B. 61

C. 91

D. none of these

Answer: B

Watch Video Solution

32. The expansion of $(x+y+z)^n$ is given by

A. $\sum \frac{n!}{p!q!r!}x^p q^q z^r$ where p,q,r $\geq 0, p+q+r=n$

B. $\sum \frac{n!}{p!q!r!}x^p q^q z^r$ where p+q+r=n

C. $\sum \frac{(n+1)!}{p!q!r!}x^p q^q z^r$ where p+q+r=n

D. none of these

Answer: A

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33. The number of terms in the expansion of $(x+y+x)^{10}$, is

A. $\frac{1}{2}(n+1)(n+2)$

B. $\frac{1}{2}n(n+1)$

C. $\frac{1}{2}(n+1)(n+3)$

D. none of these

Answer: A

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34. Statement-1 The number of terms in the expansion of

$\left(x+\frac{1}{x}+1\right)^n$ is $(2n+1)$

Statement-2 The number of terms in the expansion of

$(x_1+x_2+x_3+\ldots+x_m)^n$is ${}^{n+m-1}C_{m-1}$.

A. ${}^{n+1}C_r$

B. ${}^{n+r-1}C_{r-1}$

C. ${}^{n+r+1}C_{r+1}$

D. ${}^{n+r-1}C_r$

Answer: B

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35. Find the coefficient of x^7 in the expansion of $\left(ax^2 + \frac{1}{bx}\right)^{11}$. (ii) the coefficient of x^{-7} in the expansion of $\left(ax + \frac{1}{bx^2}\right)^{11}$. Also , find the relation between a and b , so that these coefficients are equal .

A. ab=1

B. ab=11

C. ab=5

D. ab=6

Answer: A

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36. If x+y= 1, then value of $\sum_{r=0}^{n}(r)({}^{n}C_{r})x^{n-r}y^{r}$ is

A. 1

B. 0

C. nx

D. ny

Answer: D

Watch Video Solution

37. If sum of the coefficients in the expansion of $\left(x+\frac{1}{x}\right)^{n}$ is 128, then coefficient of x in the expansion of $\left(x+\frac{1}{x}\right)^{n}$ is

A. 51

B. 48

C. 35

D. 28

Answer: C

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38. In the binomial expansion $(a-b)^n, n \geq 5$ the sum of 5th and 6th terms is zero. Then find $\frac{a}{b}$

A. $\frac{n-5}{6}$

B. $\frac{6}{n-4}$

C. $\frac{n-4}{5}$

D. $\frac{5}{n-4}$

Answer: C

Watch Video Solution

39. $2C_0 + \frac{2^2}{2}C_1 + \frac{2^3}{3}C_2 + \ldots\ldots\ldots\ldots + \frac{2^{11}}{11}C_{10} = ?$

A. $\frac{1}{11}(2^{11} - 1)$

B. $\frac{1}{11}(3^{11} - 1)$

C. $\frac{1}{11}(11^3 - 1)$

D. $\frac{1}{11}(11^2 - 1)$

Answer: B

Watch Video Solution

40. $\frac{C_1}{C_0} + 2\frac{C_2}{C_1} + 3\frac{C_3}{C_2} + \ldots\ldots\ldots\ldots + n\frac{C_n}{C_{n-1}} = \frac{n(n+1)}{2}$

A. n(n -1)

B. n(n+1)

C. $n^2 - 1$

D. $n^2 + 1$

Answer: B

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41. If $(1+x)^n = \sum_{r=0}^{n} C_r x^r$, $\left(1+\frac{C_1}{C_0}\right)\left(1+\frac{C_2}{C_1}\right)\ldots\left(1+\frac{C_n}{C_{n-1}}\right)$ is equal to

A. $\frac{(n+1)!}{2^n}$

B. $\frac{(n+1)^n}{n!}$

C. $\frac{2^{n-1}}{n!}$

D. $\frac{2^n-1}{n!}$

Answer: B

Watch Video Solution

42. If $C_r = {}^nC_r$, then $C_0 - \frac{1}{3}C_1 + \frac{1}{5}C_2$..... upto (n+1) terms equal

A. $\int_0^1 x(1-x)^n dx$

B. $\int_0^1 \left(1-x^2\right)^n dx$

C. 0

D. $1+\int_0^1 x(1+x)^n dx$

Answer: B

Watch Video Solution

43. Let $\left(1+x^2\right)^2 \cdot (1+x)^n = \sum_{k=0}^{n+4} a_k \cdot x^k$ If a_1, a_2 and a_3 are iun AP, find n.

A. 6

B. 5

C. 7

D. 2,3,4

Answer: D

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44. Find the sum $\sum_{j=0}^{n}\left(\hat{}\ (4n+1)C_j + {}^{4n+1}C_{2n-j}\right)$.

A. $2^{4n} + {}^{4n+1}C_{2n}$

B. 2^{4n+1}

C. 2^{4n}

D. $2^{4n+1} + {}^{4n+1}C_n$

Answer: A

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45. The coefficient of X^{24} in the expansion of $(1+X^2)^{12}(1+X^{12})(1+X^{24})$

A. ${}^{12}C_6 + 2$

B. ${}^{12}C_6 + 1$

C. ${}^{12}C_6$

D. ${}^{12}C_6 + 3$

Answer: A

Watch Video Solution

46. Third term in expression of $\left(x + x^{\log_{10} x}\right)^5$ is 10^6 than possible value of x are

A. $1, 10$

B. $10, 10^{-5/2}$

C. $10, 10^{-3/2}$

D. $10^2, 10^{-3/2}$

Answer: B

Watch Video Solution

47. The numerically greatest term in the expansion of $(1+x)^{10}$ when $x = 2/3$, is

A. $210\left(\frac{3}{2}\right)^6$

B. $210(2/3)^4$

C. $210(3/2)^4$

D. none of these

Answer: B

Watch Video Solution

48. The greatest term in the expansion of $(3+5x)^{15}$, when x=1/5, is

A. ${}^{15}C_3\left(3^{13}\right)$

B. ${}^{15}C_4\left(3^{12}\right)$

C. ${}^{15}C_4\left(3^{10}\right)$

D. none of these

Answer: D

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49. If sum of the coefficients of x^7 and x^4 in the expansion of $\left(\frac{x^2}{a} - \frac{b}{x}\right)^{11}$ is zero, then

A. ab= 1

B. a= b

C. ab=-1

D. a+ b = 0

Answer: A

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50. The value of

$$\frac{19^3 + 6^3 + (3)(19)(6)(25)}{3^6 + 6(243)(2) + (15)(81)(4) + (20)(27)(8) + (15)(9)(16) + (6)(3)(32) +}$$

is

A. 19

B. 6

C. 25

D. 1

Answer: D

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51. In the expansion of $(x+a)^n$ if the sum of odd terms is P and the sum of even terms is Q, tehn $P^2-Q^2=\left(x^2-a^2\right)^n$ $4PQ=(x+a)^{2n}-(x-a)^{2n}$ $2\left(P^2+Q^2\right)=(x+a)^{2n}+(x-a)^{2n}$ none of these

A. AB

B. $2AB$

C. $4AB$

D. $A^2 - B^2$

Answer: D

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52. Coefficient of the term independent of x in the expansion of $\left(x + \frac{1}{x}\right)^4 \left(x - \frac{1}{x}\right)^{12}$ is

A. 192

B. 194

C. 196

D. 198

Answer: D

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53. For each $n \in N$, 3. $\left(5^{2n+1}\right) + 2^{3n+1}$ is divisible by

A. 19

B. 17

C. 23

D. 25

Answer: B

Watch Video Solution

54. The sum to 100 terms of the series

$1.2.3. + 2.3.4. + 3.4.5. + \ldots + n(n+1)(n+2) + \ldots.$

is integral multiple of

A. $\frac{1}{4}(n+1)(n+2)(n+3)$

B. $\frac{1}{4}(n+2)(n-2)(n-3)(n+3)$

C. $\frac{1}{4}n(n+1)(n+2)(n+3)$

D. none of these

Answer: C

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55. If $a, b, c, \in_N , a^n + b^n$ is divisible by c when n is odd but not when n is even, then value of c is

A. a+ b

B. a-b

C. $a^3 + b^3$

D. $a^3 + b^3$

Answer: A

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Exercise Level 2 Single Correct Answer Type Questions

1. Let S denote the set of real numbers x such that $\left(x+\sqrt{x^2-1}\right)^3+\left(x-\sqrt{x^2-1}\right)^3=2\cos\left(3\cos^{-1}x\right)$ Then S contains

A. one element

B. two elements

C. infinte elements

D. no element

Answer: D

Watch Video Solution

2. Value of

$\left({}^{40}C_0\right)\left({}^{40}C_{15}\right)-\left({}^{40}C_1\right)\left({}^{40}C_{16}\right)+\left({}^{40}C_2\right)\left({}^{40}C_{12}\right).....-\left({}^{40}C_{25}\right)\left({}^{40}C_{40}\right)$

equals

A. 0

B. $^{40}C_{25}$

C. $^{40}C_{20}$

D. -1

Answer: A

View Text Solution

3. The hundred's digit of 3^{100} is

A. 0

B. 1

C. 2

D. 7

Answer: A

Watch Video Solution

4. If $a, b, p \in N$ and p is prime, then $(a+b)^p - a^P - b^p$ is divisible by

A. p

B. p^2

C. $\frac{1}{2}p(p-1)$

D. $\frac{1}{2}p(p+1)$

Answer: A

Watch Video Solution

5. $C_0^2 + \frac{1}{2}C_1^2 + \frac{1}{3}C_2^2 + \ldots + \frac{1}{n+1}C_n^2$ equals

A. $\frac{1}{n}\left({}^{2n}C_n\right)$

B. $\frac{1}{n+1}\left({}^{2n+1}C_n\right)$

C. $\frac{1}{n}\left({}^{2n}C_{n+1}\right)$

D. 2^n

Answer: B

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6. Sum of the series $\sum_{k=0}^{n} \frac{(2n)!}{(k!)^2(n-k!)^2}$ equals

A. $n^n + {}^{2n}C_n$

B. $\left({}^{2n}C_n\right)^2$

C. ${}^{2n}C_n$

D. n^n

Answer: B

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7. Sum of the series

$S = n^2\left(\frac{C_0}{C_1}\right) + (n-1)^2\left(\frac{C_1}{C_2}\right) + \ldots + 1^2\left(\frac{C_{n-1}}{C_n}\right)$ equals

A. $\frac{1}{6}n(n+1)(2n+1)$

B. $\frac{1}{3}n(n+1)(n+2)$

C. $\frac{1}{6}n(n+1)(n+2)$

D. $\frac{1}{6}n^2(n+1)(n+2)$

Answer: C

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8. If $\sum_{r=0}^{2n} a_r(x-2)^r = \sum_{r=0}^{2n} b_r(x-3)^r and a_k = 1$ for all $k \geq n$, then show that $b_n = {}^{2n+1}C_{n+1}$.

A. n-1

B. n

C. $2n-1$

D. $n+1$

Answer: D

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9. Coefficient of x^{50} in the expansion of $(1+x)^{41}\left(1-x+x^2\right)^{40}$ is

A. 0

B. 1

C. ${}^{40}C_{19}$

D. ${}^{40}C_{29}$

Answer: A

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10. If $n \in N$, then $\sum_{i \leq j} ({}^{n}C_{i})\left({}^{n+i}C_{j}\right)$ equals

A. 2^{2n}

B. 2^{2n-1}

C. $2^{n+1}C_n$

D. $2n+1$

Answer: A

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Exercise Numerical Answer Type Questions

1. If the coefficient of x^2 and x^3 in the expansion of $(3+ax)^{11}$ are equal then a=_____

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2. If $\sum_{r=0}^{n}(3^r)({}^nC_r) = 4096$, then n=_____-

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3. Coefficient of x^7 in the expansion of $\left(1+x+x^2\right)^4$ is _____

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4. The value of $\frac{18^3+7^3+3.187.25}{3^6+62432+1581.4+2027.8+159.16+6.3.32+64}$, is

Watch Video Solution

5. If the sixth term in the expansion of

$$\left[3\log_{3\sqrt{9^{x-1}+7}}+\frac{1}{3^{\log 3\left(3^{x-1}+1\right)}}\right]^7$$

is 84, ttien sum of the possible values of x is _____.

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6. In the expansion of $\left(2-3x^3\right)^{20}$, if the ratio of 10^{th} term to 11^{th} term is $\frac{45}{22}$ then $x=$

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7. The expression

$\left[x+\left(x^3-1\right)^{\frac{1}{2}}\right]^5+\left[x-\left(x^3-1\right)^{\frac{1}{2}}\right]^5$ is a polynomial of degree

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8. Find the number of nonzero terms in the expansion of $\left(1+3\sqrt{2}x\right)^9+\left(1-3\sqrt{2}x\right)^9$.

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9. The sum of the coefficients of the first three terms in the expansion of $\left(x-\frac{3}{x^2}\right)^m, x\neq 0$, m being a natural number, is 559. Find the term of the expansion containing x^3.

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10. The value of $\left(\frac{{}^{50}C_0}{1}+\frac{{}^{50}C_2}{3}+\frac{{}^{50}C_4}{5}+\ldots.+\frac{{}^{50}C_{50}}{51}\right)$ is :

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11. If $n>2$, then prove that $C_1(a-1)-C_2\times(a-2)++(-1)^{n-1}C_n(a-n)=a, where C_r={}^n C_r$

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12. Suppose the sum of the coefficients in the expansion of $\left(1-5x+12x^3\right)^n$ is a and the sum of the coefficients in the expansion of $\left(1-2x+3x^2\right)^n$ is b. If $a=b^m$, then m = ___

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13. Let $C_r={}^{15}C_r, (0\le r\le 15)$, and

$$m=\frac{C_1}{C_0}+\frac{2C_3}{C_2}+\frac{3C_3}{C_2}+\ldots.+\frac{{}^{15}C_{15}}{C_{14}}$$

then m = _____

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14. Suppose the coefficient of the middle term in the expansion of $(1+x)^{2n}$ is A and the coefficients of two middle terms in the expansion of $(1+x)^{2n-1}$ are B and C, then $\frac{A}{B+C} =$ _____

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15. If n is an even natural number , then $\sum_{r=0}^{n} \frac{(-1)^r}{{}^nC_r}$ equals

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16. If a > 0 and the coefficient of x^5 in the expansion of $(1+ax)^2(1-x)^7$ is -21 then a=_____

Watch Video Solution

17. Coefficient of x^{11} in the expaJ}sion of $\left(1+3x+2x^2\right)^6$ is _____

View Text Solution

18. Find the term independent of x in the expansion of $\left(1+x+2x^3\right)\left[\left(3x^2/2\right)-(1/3)\right]^9$

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19. If a:b = 3:5, and sum of the coefficients of 5^{th} and 6^{th} terms in the expansion of $(a-b)^n$ is zero, then n=_____

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20. For n = 6, let

$$N = ({}^nC_0)^2 + ({}^nC_1)^2 + \ldots + ({}^nC_n)^2$$

If N- 204 = m!, then m = _____

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Questions From Previous Years Aieee Jee Main Papers

1. If the sum of the coefficients in the expansion of $(a+b)^n$ is 4096, then the greatest coefficient in the expansion is 924 b. 792 c. 1594 d. none of these

A. 792

B. 924

C. 1594

D. 2990

Answer: B

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2. Let a_r denote the coefficient of x^r in the expansion of $(1+x)^{P+q}$, then

A. $a_p = a_q$

B. $a_p = -a_q$

C. $a_p a_q = 1$

D. none of these

Answer: A

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3. Find the positive integer just greater than $(1+0.0001)^{10000}$.

A. 2

B. 3

C. 4

D. 5

Answer: B

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4. If $r, n \in Nr > 1, n > 2$ and the coefficient of (r +2)th term and (3r)th term in the expansion of $(1 + x)^{7n}$ are equal, then n equals

A. 3r

B. 3r+1

C. 2r

D. 2r+1

Answer: C

View Text Solution

5. The number of integral terms in the expansion of $\left(3^{1/2} + 5^{1/8}\right)^{256}$ is

A. 33

B. 34

C. 35

D. 32

Answer: A

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6. Let $S(k) = 1 + 3 + 5 + ... + (2k - 1) = 3 + k^2$. Then which of the following is true ? (A) S(1) is correct (B) S(k)=S(k+1) (C) $S(k) \neq S(k+1)$ (D) Principal of mathematical induction can be used to prove the formula

A. S(K) $\neq$ S(K + 1)

B. $S(K) \rightarrow S(K+1)$

C. S(1) is correct

D. Principle of mathematical induction can be used to prove the formula.

Answer: B

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7. The coefficient of the middle term in the binomial expansion in powers of x of$(1+\alpha x)^4$ and of$(1-\alpha x)^6$ is the same, if α equals $-\frac{5}{3}$ b. $\frac{10}{3}$ c. $-\frac{3}{10}$ d. $\frac{3}{5}$

A. $-\frac{3}{10}$

B. $\frac{10}{3}$

C. $-\frac{5}{3}$

D. $\frac{3}{5}$

Answer: A

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8. The coefficient of x^n in the expansion of $(1+x)(1-x)^n$ is

A. $(-1)^{n-1}(n-1)^2$

B. $(-1)^n(1-n)$

C. n-1

D. $(-1)^{n-1}n$

Answer: B

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9. In the coefficients of rth, $(r+1)th, and (r+2)th$ terms in the binomial expansion of $(1+y)^m$ are in A.P., then prove that $m^2 - m(4r+1) + 4r^2 - 2 = 0.$

A. $m^2 - m(4r+1) + 4r^2 - 2 = 0$

B. $m^2 - m(4r-1) + 4r^2 + 2 = 0$

C. $m^2 - m(4r-1) + 4r^2 - 2 = 0$

D. $m^2 - m(4r+1) + 4r^2 + 2 = 0$

Answer: A

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10. Find the coefficient of x^7 in the expansion of $\left(ax^2 + \frac{1}{bx}\right)^{11}$. (ii) the coefficient of x^{-7} in the expansion of $\left(ax + \frac{1}{bx^2}\right)^{11}$. Also , find the relation between a and b , so that these coefficients are equal .

A. $\frac{a}{b} = 1$

B. $ab = 1$

C. $a - b = 1$

D. $a + b = 1$

Answer: B

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11. For natural numbers m, n if $(1-y)^m(1+y)^n = 1 + a_1y + a_2y^2 + \ldots\ldots$ and $a_1 = a_2 = 10$, then (m, n) is :

A. (35,45)

B. $(20, 45)$

C. (35, 20)

D. (45, 35)

Answer: A

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12. The sum of the series

$.^{20}C_0 - .^{20}C_1 + .^{20}C_2 - .^{20}C_3 + \ldots - . + .^{20}C_{10}$ is -

A. $-\left(^{20}C_{10}\right)$

B. $\frac{1}{2}\left(^{20}C_{10}\right)$

C. 0

D. $^{20}C_{10}$

Answer: B

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13. In the binomial expansion of $(a-b)^n$, $n \geq 5$, the sum of the 5^{th} and 6^{th} terms is zero. Then, a/b equals

A. $\frac{5}{n-4}$

B. $\frac{6}{n-5}$

C. $\frac{n-5}{6}$

D. $\frac{n-4}{5}$

Answer: D

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14. The remainder left out when $8^{2n}(62)^{2n+1}$ is divided by 9 is (1) 0 (2) 2 (3) 7 (4) 8

A. 7

B. 8

C. 0

D. 2

Answer: D

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15. The coefficient of x^7 in the expansion of $\left(1-x-x^2+x^3\right)^6$ is :

A. 132

B. 144

C. -132

D. -144

Answer: D

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16. If n is a positive integer, then $\left(\sqrt{3}+1\right)^{2n}-\left(\sqrt{3}-1\right)^{2n}$ is (1) an irrational number (2) an odd positive integer (3) an even positive integer (4) a rational number other than positive integers

A. an odd positive integer

B. an even positive integer

C. a rational number other than a positive integer

D. an irrational number

Answer: D

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17. The coefficient of the term independent of x in the expansion of $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}$

A. 120

B. 210

C. 310

D. 4

Answer: B

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18. If the 7th tern in the binomial expansion of $\left(\frac{3}{(84)^{\frac{1}{3}}}+\sqrt{3}\ln x\right)^{9}, x>0$ is equal to 729 then x can be (A) e^2 (B) e (C) $\frac{e}{2}$ (D) $2e$

A. e^2

B. e

C. $\frac{1}{2}e$

D. $2e$

Answer: B

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19. Given positive integers $r > 1, n > 2, n$ being even and the coefficient of $(3r)th$ term and $(r+2)th$ term in the expansion of $(1+x)^{2n}$ are equal; find r

A. 2r

B. 2r-1

C. 3r

D. r+1

Answer: A

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20. The ratio of the coefficient of x^{15} to the term independent of x in the expansion of $\left(X^2+\frac{2}{x}\right)^{15}$ is

A. $7:16$

B. $7:64$

C. $1:4$

D. $1:32$

Answer: D

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21. The sum of the rational terms in the expansion of $\left(\sqrt{2}+\sqrt[5]{3}\right)^{10}$ is

A. 25

B. 32

C. 9

D. 41

Answer: D

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22. The coefficients of x^3 and x^4 in the expansion of $\left(1+ax+bx^2\right)(1-2x)^{18}$

A. $\left(14, \frac{251}{3}\right)$

B. $\left(14, \frac{272}{3}\right)$

C. $\left(16, \frac{272}{3}\right)$

D. $\left(16, \frac{251}{3}\right)$

Answer: C

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23. The number of terms in the expansion of $(1+x)^{101}\left(1+x^2-x\right)^{100}$ in powers of x is

A. 302

B. 301

C. 202

D. 101

Answer: C

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24. Find the coefficients of x^{50} in the expression $(1+x)^{1000}+2x(1+x)^{999}+3x^2(1+x)^{998}++1001x^{1000}$.

A. $\frac{(1000)!}{(50)!(950)!}$

B. $\frac{(1000)!}{(49)!(951)!}$

C. $\frac{(1001)!}{(51)!(950)!}$

D. $\frac{(1001)!}{(50)!(951)!}$

Answer: D

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25. If $\left(2+\frac{x}{3}\right)^{55}$ is expanded in the ascending powers of x and the coefficients of powers of x in two consecutive terms of the expansion are equal then these terms are (A) 7,8 (B) 8,9 (C) 27,28 (D) 28,29

A. 7^{th} and 8^{th}

B. 8^{th} and 9^{th}

C. 28^{th} and 29^{th}

D. 27^{th} and 28^{th}

Answer: B

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26. If $1 + x^4 + x^5 = \sum_{i=0}^{5} a_i(1+x)^i$, for all x in R, then a_2 is :

A. -4

B. 6

C. -8

D. 10

Answer: A

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27. The coefficient of x^{1012} in the expansion of $\left(1 + x^n + x^{253} \hat{} (10)\right.$, where $n \leq 22$ is a positive integer), is

A. 1

B. ${}^{10}C_4$

C. $4n$

D. $^{253}C_4$

Answer: B

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28. The sum of coefficients of integral powers of x in the binomial expansion of $\left(1-2\sqrt{x}\right)^{50}$ is: (1) $\frac{1}{2}\left(3^{50}+1\right)$ (2) $\frac{1}{2}\left(3^{50}\right)$ (3) $\frac{1}{2}\left(3^{50}-1\right)$ (4) $\frac{1}{2}\left(2^{50}+1\right)$

A. $\frac{1}{2}\left(3^{50}+1\right)$

B. $\frac{1}{2}\left(3^{50}\right)$

C. $\frac{1}{2}\left(3^{50}-1\right)$

D. $\frac{1}{2}\left(2^{50}+1\right)$

Answer: C

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29. If the coefficients of the three successive terms in the binomial expansion of $(1+x)^n$ are in the ratio $1:7:42$ then the first of these terms in the expansion is

A. 6^{th}

B. 7^{th}

C. 8^{th}

D. 9^{th}

Answer: B

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30. The term independent of x in the binomial expansion of $\left(1-\frac{1}{x}+3x^5\right)\left(2x^2-\frac{1}{x}\right)^8$ is:

A. 400

B. 496

C. -400

D. -496

Answer: A

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31. If the number of terms in the expansion of $\left(1-\frac{2}{x}+\frac{4}{x^2}\right)^n, x \neq 0,$ is 28, then the sum of the coefficients of all the terms in this expansion, is : (1) 64 (2) 2187 (3) 243 (4) 729

A. 64

B. 2187

C. 243

D. 729

Answer: B

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32. FOr $\xi nr, x \neq -1$, If $(1+x)^{2016} + x(1+x)^{215} + x^2(1+x)^{2014} + + x^{2016} = \sum_{i=0}^{2016} a_i x^i$, then a_{17} is equal to -

A. $\frac{2017!}{17!2000!}$

B. $\frac{2016!}{17!1999!}$

C. $\frac{2016!}{16!}$

D. $\frac{2017!}{2000!}$

Answer: A

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33. If the coefficients of x^{-2} and x^{-4} the expansion of $\left(x^{\frac{1}{3}} + \frac{1}{2x^{\frac{1}{3}}}\right)^{18}$, are m and n respectively, then $\frac{m}{n}$ is equal to

A. 27

B. 182

C. $\frac{5}{4}$

D. $\frac{4}{5}$

Answer: B

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34. The value of $\left(.^{21}C_1 - .^{10}C_1\right) + \left(.^{21}C_2 - .^{10}C_2\right) + \left(.^{21}C_3 - .^{10}C_3\right) + \left(.^{21}C_4 - .^{10}C_4\right.$ is

A. $2^{21} - 2^{11}$

B. $2^{21} - 2^{10}$

C. $2^{20} - 2^9$

D. $2^{20} - 2^{10}$

Answer: D

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35. If $(27)^{999}$ is divided by 7, then the remainder is .

A. 1

B. 2

C. 3

D. 6

Answer: D

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36. The coefficient of x^{-5} in the binomial expansion of $\left(\frac{x+1}{x^{\frac{2}{3}}-x^{\frac{1}{3}}+1}-\frac{x-1}{x-x^{\frac{1}{2}}}\right)^{10}$, where $x \neq 0, 1$, is

A. 1

B. 4

C. -4

D. -1

Answer: A

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37. The sum of the coefficients of all odd degree terms in the expansion of $\left(x+\sqrt{x^3-1}\right)^5+\left(x-\sqrt{x^3-1}\right)^5, (x>1)$ is :

A. 0

B. 1

C. 2

D. -1

Answer: C

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38. If n is the degree of the polynomial, $\left[\frac{2}{\sqrt{5x^3+1}-\sqrt{5x^3-1}}\right]^8+\left[\frac{2}{\sqrt{5x^3+1}+\sqrt{5x^3-1}}\right]^8$ and m is the coefficient of x^{12} is

A. $(8, 5(10^4))$

B. $\left(12, 8(10)^4\right)\Big)$

C. $(12, (20^4))$

D. $(24, (10^8))$

Answer: C

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39. The coefficient of x^{10} in the expansion of $(1+x)^2(1+x^2)^3(1+x^3)^4$ is equal to

A. 50

B. 52

C. 44

D. 56

Answer: B

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40. The coefficient of x^2 in the expansion of the product $\left(2-x^2\right)\left(1+2x+3x^2\right)^6+\left(1-4x^2\right)^6\Big)$ is:

A. 107

B. 106

C. 108

D. 155

Answer: B

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41. Ratio of the 5^{th} term from the beginning to the 5^{th} term from the end in the binomial expansion of $\left(2^{1/3}+\frac{1}{2(3)^{1/3}}\right)^{10}$ is

A. $1:4(16)^{1/3}$

B. $1:2(6)^{1/3}$

C. $2(36)^{1/3}:1$

D. $4(36)^{1/3}:1$

Answer: D

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42. The total number of irrational terms in the binomial expansion of $\left(7^{1/5}-3^{1/10}\right)^{60}$ is :

A. 55

B. 49

C. 48

D. 54

Answer: D

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43. If the fractional part of the number $\frac{2^{403}}{15}$ is $\frac{k}{15}$ then k is equal to

A. 6

B. 8

C. 4

D. 14

Answer: B

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44. The coefficient of t^4 in the expansion of $\left(\frac{1-t^6}{1-t}\right)^3$ is 3k. The value of k is _____.

A. 14

B. 15

C. 10

D. 12

Answer: B

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45. If the third term in the binomial expansion of $\left(1+x^{\log_2 x}\right)^5$ equals 2560, then a possible value of x is:

A. $1/4$

B. $4\sqrt{2}$

C. $1/8$

D. $2\sqrt{2}$

Answer: A

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46. The positive value of λ for which the co-efficient of x^2 in the expression $x^2\left(\sqrt{x}+\frac{\lambda}{x^2}\right)^{10}$ is 720, is

A. 4

B. $2\sqrt{2}$

C. $\sqrt{5}$

D. 3

Answer: A

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47. The sum of the real values of x for which the middle term in the binomial expansion of $\left(\frac{x^3}{3}+\frac{3}{x}\right)^8$ equals 5670 is :

A. 6

B. 8

C. 0

D. 4

Answer: C

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48. Let $(x+10)^{50}+(x-10)^{50}=a_0+a_1x+a_2x^2+\ldots+a_{50}x^{50}$ for all $x \in R$, then $\frac{a_2}{a_0}$ is equal to

A. 12.5

B. 12

C. 12.75

D. 12.25

Answer: D

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49. The sum of the series $2.\ ^{20}C_0 + 5.\ ^{20}C_1 + 8.\ ^{20}C_2 + 11.\ ^{20}C_{20}$ is equal

A. 2^{23}

B. 2^{24}

C. 2^{26}

D. 2^{25}

Answer: D

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50. The sum of the coefficients of all even degree terms is x in the expansion of : $\left(x+\sqrt{x^3-1}\right)^6+\left(x-\sqrt{x^3-1}\right)^6, (x>1)$ is equal to

A. 32

B. 24

C. 29

D. 26

Answer: B

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51. If the fourth term in the binomial expansion of $\left(\sqrt{\frac{1}{x^{1+\log_{10}x}}}+x^{\frac{1}{12}}\right)^6$ is equal to 200 , and $x>1$, then the value of x is :

A. 100

B. 10

C. 10^3

D. 10^4

Answer:

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52. If the fourth term in the binomial expansin of $\left(\frac{2}{x} + x^{\log_6 x}\right)^6 (x > 0)$ is 20×8^7, then the value of x is

A. 8^2

B. 8^3

C. 8

D. 8^{-2}

Answer: A

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53. If some three consecutive coefficeints in the binomial expanison of $(x+1)^n$ in powers of x are in the ratio 2 : 15 : 70, then the average of these three coefficients is

A. 232

B. 964

C. 625

D. 227

Answer: A

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54. If the coefficients of x^2 and x^3 are both zero, in the expansion of the expression $\left(1+ax+bx^2\right)(1-3x)$ in powers of x, then the ordered pair (a,b) is equal to

A. (28, 861)

B. -28315

C. (-21, 714)

D. (-54, 315)

Answer: B

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55. The smallest natural number n, such that the coefficient of x in the expansion of $\left(x^2+\frac{1}{x^3}\right)^n$ is $.^nC_{23}$, is

A. 23

B. 58

C. 38

D. 35

Answer: C

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56. The cofficient of x^{18} in the product $(1+x)(1-x)^1\left(1+x+x^2\right)^9$ is

A. 126

B. -84

C. -126

D. 84

Answer: D

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57. The term independent of x in the expansion of $\left(\frac{1}{60}-\frac{x^8}{81}\right).\left(2x^2-\frac{3}{x^2}\right)^6$ is equal to

A. -36

B. -108

C. 36

D. -72

Answer: A

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58. If $.^{20}C_1 + (2^2).^{20}C_2 + (3^2).^{20}C_3 + ... + (20^2).^{20}C_{20} = A\left(2^{\beta}\right)$, then the ordered pair (A, β) is equal to

A. (420, 19)

B. (420, 18)

C. (380, 18)

D. (380, 19)

Answer: B

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59. Consider the statement : $P(n): n^2 - n + 41$ is prime." Then, which one of the following is true?

A. Both P(3) and P(5) are true

B. P(3) is false but P(5) is true

C. Both P(3) and P(5) are false

D. P(5) is false but P(3) is true

Answer: A

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Questions From Previous Years B Architecture Entrance Examination Papers

1. In the expansion of $\left(x^3 - \frac{1}{x^2}\right)^n$, $n \in N$ if sum of the coefficients of x^5 and x^{10} is 0 then n is

A. 455

B. 105

C. 605

D. 120

Answer: D

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2. If $(1+x)\left(1+x+x^2\right)\left(1+x+x^2+x^3\right)....\left(1+x+x^2+...+x^n\right)=a_0$, then the value of a_1 is

A. m+1

B. (n+1)

C. n

D. m

Answer: C

View Text Solution

3. If the 7th terms from the beginning and end in the expansion of $\left(\sqrt[3]{2}+\frac{1}{\sqrt[3]{2}}\right)^{n}$ are equal, find the value of n.

A. 9

B. 12

C. 6

D. 3

Answer: A

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4. The remainder when 7^{128} is divided by 10 is

A. 1

B. 3

C. 7

D. 9

Answer: A

Watch Video Solution

5. The value of the sum

$$\sum_{j=0}^{8} \frac{1}{(j+1)(j+2)} \binom{8}{j}$$ is

A. $\frac{1003}{90}$

B. $\frac{1013}{90}$

C. $\frac{1023}{90}$

D. $\frac{1033}{90}$

Answer: B

View Text Solution

6. If

$x^n = a_0 + a_1(1+x) + a_2(1+x)^2 + \ldots. + a_n(1+x)^n = b_0 + b_1(1-x)$

then for n=101, (a_{50}, b_{50}) equals:

A. $\left(-{}^{101}C_{50}, {}^{101}C_{50}\right)$

B. $\left({}^{101}C_{50}, -{}^{101}C_{50}\right)$

C. $\left(-{}^{101}C_{50}, -{}^{101}C_{50}\right)$

D. $\left({}^{101}C_{50}, {}^{101}C_{50}\right)$

Answer: A

View Text Solution

7. If the third term in the expansion of $\left(\frac{1}{x} + {}_{\mathrm{x}}(\log)_{10x}\right)^5$ is 1000, then find x.

A. 10

B. 1

C. $\frac{1}{\sqrt{10}}$

D. 100

Answer: D

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8. If the sum of the coefficients in the expansion of $(x+y)^n$ is 2048, then the greatest coefficient in the expansion is:

A. ${}^{10}C_6$

B. ${}^{11}C_6$

C. ${}^{11}C_7$

D. ${}^{12}C_6$

Answer: B

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9. If $\left(1+x+x^2\right)^8 = a_0 + a_1x + a_2x^2 + \ldots a_{16}x^{16}$ for all values of x, then $\frac{a_5}{100}$ is equal to

A. 502

B. 504

C. 506

D. 508

Answer: B

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10. Coefficient of t^{24} in $\left(1+t^2\right)^{12}\left(1+t^{12}\right)\left(1+t^{24}\right)$ is :

A. ${}^{12}C_6 + 13$

B. ${}^{12}C_6 + 2$

C. ${}^{12}C_6 + 1$

D. ${}^{12}C_6$

Answer: B

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11. Sum of the last 30 coefficients of powers of x in the binomial expansion of $(1+x)^{59}$ is:

A. 2^{29}

B. 2^{28}

C. $2^{59}-2^{29}$

D. 2^{58}

Answer: D

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12. If in the expansion of $(1+x)^m(1-x)^n$, the coefficients of x and x^2 are 3 and -6 respectively, then m is:

A. $10:7$

B. $8:11$

C. $10:13$

D. $7:10$

Answer: D

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13. For a positive integer n, if the mean of the binomial coefficients in the expansion of $(a+b)^{2n-3}$ is 16, then n is equal to:

A. 5

B. 7

C. 9

D. 4

Answer: A

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14. If the digits at ten's and hundred's places in $(11)^{2016}$ are x and y respectively, then the order pair (x, y) is equal to:

A. (1, 8)

B. (1, 6)

C. (6, 1)

D. (8, 1)

Answer: C

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15. Let t_r denote the rth term in the binomial expansion of $(a+1)^{50}$. If $t_{25}+t_{27}=\frac{125}{52}t_{26}$, then the sum of 52 all the values of a is:

A. $1/2$

B. $3/2$

C. 2

D. $5/2$

Answer: D

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16. Let $(1+x)^n = C_0 + C_1x + C_2x^2 + \ + \ldots. + C_nx^n$ where $C_r = {}^nC_r$ and

$(C_0 + C_1)(C_1 + C_2)\ldots..(C_{n-1} + C_n) = AC_1C_2\ldots..C_n$

Then for n = 5, A is equal to

A. $\frac{3125}{24}$

B. $\frac{625}{24}$

C. $\frac{324}{5}$

D. $\frac{128}{3}$

Answer: C

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17. For $\beta \neq 0$, if the coefficient of x^3 in the binomial expansion of $(1+\beta x)^6$ and the coefficient of x^4 in the binomial expansion of $(1-\beta x)^8$ are equal, then the value of β is

A. $2/7$

B. $-2/7$

C. $-1/7$

D. $1/7$

Answer: A

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18. What is the sum of all the coefficients in the expansion of $(1+x)^n$?

A. 512

B. 252

C. 256

D. 352

Answer: B

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19. If r is the remainder obtained on dividing 98^5 by 12, then the coefficient . of x^3 in the binomial expansion of $\left(1+\frac{x}{2}\right)^{2r}$ is:

A. $\frac{55}{2}$

B. $\frac{91}{2}$

C. 102

D. 70

Answer: D

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20. The coefficient of x^5 in the expansion of $(1-x)\left(\frac{x^3-6}{2x^2}\right)^{10}$ is:

A. 405

B. $\frac{405}{256}$

C. $-\frac{1405}{256}$

D. $-\frac{405}{256}$

Answer: D

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