MATHS

BOOKS - KUMAR PRAKASHAN KENDRA MATHS (GUJRATI ENGLISH)

PRINCIPLE OF MATHEMATICAL INDUCTION

Exercise 4 1

1. Prove the following by using the principle of mathematical induction for all $n \in N$

$$1 + 3 + 3^2 + \ldots\ldots\ldots\ldots + 3^{n-1} = \frac{3^n - 1}{2}$$

2. Prove the following by using the principle of mathematical induction for all $n \in N$

$$1^3 + 2^3 + 3^3 + \ldots\ldots\ldots\ldots\ldots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$$

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3. Prove the following by using the principle of mathematical induction for all $n \in N$

$$1 + \frac{1}{(1+2)} + \frac{1}{(1+2+3)} + \ldots\ldots + \frac{1}{1+2+3+\ldots\ldots + n} = \frac{2n}{n+1}$$

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4. Prove the following by using the principle of mathematical induction for all $n \in N$

$$1.2.3. + 2.3.4 + \ldots. + n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}$$

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5. Prove the following by using the principle of mathematical induction for all $n \in N$

$$1.3 + 2.3^2 + 3.3^3 + \ldots\ldots\ldots + n.3^n = \frac{(2n-1)3^{n+1} + 3}{4}$$

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6. Prove the following by using the principle of mathematical induction for all $n \in N$

$$1.2 + 2.3 + 3.4 + \ldots\ldots\ldots + n.(n+1) = \left[\frac{n(n+1)(n+2)}{3}\right]$$

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7. Prove the following by using the principle of mathematical induction for all $n \in N$

$$1.3 + 3.5 + 5.7 + \ldots\ldots + (2n-1)(2n+1) = \frac{n(4n^2+6n-1)}{3}$$

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8. Prove the following by using the principle of mathematical induction for all $n \in N$

$$1.2 + 2.2^2 + 3.2^3 + \dots\dots\dots + n.2^n = (n-1)2^{n+1} + 2$$

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9. Prove the following by using the principle of mathematical induction for all $n \in N$

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots\dots\dots + \frac{1}{2^n} = 1 - \frac{1}{2^n}$$

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10. Prove the following by using the principle of mathematical induction for all $n \in N$

$$\frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + \dots\dots + \frac{1}{(3n-1)(3n+2)} = \frac{n}{(6n+4)}$$

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11. Prove the following by using the principle of mathematical induction

for all $n \in N$

$$\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \ldots\ldots + \frac{1}{n(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$$

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12. Prove the following by using the principle of mathematical induction

for all $n \in N$

$$a + ar + ar^2 + \ldots\ldots. + ar^{n-1} = \frac{a(r^n - 1)}{r - 1}$$

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13. Prove the following by using the principle of mathematical induction

for all $n \in N$

$$\left(1 + \frac{3}{1}\right)\left(1 + \frac{5}{4}\right)\left(1 + \frac{7}{9}\right) \times \ldots\ldots \times \left(1 + \frac{(2n+1)}{n^2}\right) = (n+1)^2$$

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14. Prove the following by using the principle of mathematical induction

for all $n \in N$

$$\left(1+\frac{1}{1}\right)\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\ldots\ldots\left(1+\frac{1}{n}\right)=(n+1)$$

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15. Prove the following by using the principle of mathematical induction

for all $n \in N$

$$1^2+3^2+5^2+\ldots\ldots\ldots\ldots+(2n-1)^2=\frac{n(2n-1)(2n+1)}{3}$$

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16. Prove the following by using the principle of mathematical induction

for all $n \in N$

$$\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+\ldots\ldots+\frac{1}{(3n-2)(3n+1)}=\frac{n}{3n+1}$$

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17. Prove the following by using the principle of mathematical induction

for all $n \in N$

$$\frac{1}{3.5}+\frac{1}{5.7}+\frac{1}{7.9}+\ldots\ldots+\frac{1}{(2n+1)(2n+3)}=\frac{n}{3(2n+3)}$$

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18. Prove the following by using the principle of mathematical induction

for all $n \in N$

$$1+2+3+\ldots\ldots\ldots+n<\frac{1}{8}(2n+1)^2$$

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19. Prove the following by using the principle of mathematical induction

for all $n \in N$

n(n+1)(n+5) is a multiple of 3

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20. Prove the following by using the principle of mathematical induction

for all $n \in N$

$10^{2n-1} + 1$ is divisible by 11.

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21. Prove the following by using the principle of mathematical induction

for all $n \in N$

$x^{2n} - y^{2n}$ is divisible by x + y .

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22. Prove the following by using the principle of mathematical induction

for all $n \in N$

$3^{2n+2} - 8n - 9$ is divisible by 8 .

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23. Prove the following by using the principle of mathematical induction for all $n \in N$

$41^n - 14^n$ is a multiple of 27.

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24. Prove the following by using the principle of mathematical induction for all $n \in N$

$(2n + 7) < (n + 3)^2$

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Practice Work

1. Prove the following by using the principle of mathematical induction for all $n \in N$

$$\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \dots\dots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$$

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2. Prove the following by using the principle of mathematical induction for all $n \in N$

$1 + 2 + 2^2 + \ldots.. + 2^n = 2^{n+1} - 1$

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3. Prove the following by using the principle of mathematical induction for all $n \in N$

$$\left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right)\left(1 - \frac{1}{4^2}\right)\ldots\ldots\left(1 - \frac{1}{n^2}\right) = \frac{n+1}{2n} \qquad n \geq 2$$

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4. Prove the following by using the principle of mathematical induction for all $n \in N$

$3 \times 6 + 6 \times 9 + 9 \times 12 + \ldots.. + (3n)(3n+3) = 3n(n+1)(n+2)$

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5. Prove the following by using the principle of mathematical induction

for all $n \in N$

$a+(a+d)+(a+2d)+\ldots\ldots+(a+(n-1)d)=\frac{n}{2}[2a+(n-1)d]$

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6. Prove the following by using the principle of mathematical induction

for all $n \in N$

$4+8+12+\ldots\ldots.\ +4n=2n(n+1)$

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7. Prove the following by using the principle of mathematical induction

for all $n \in N$

7^n-3^n is divisible by 4.

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8. Prove the following by using the principle of mathematical induction

for all $n \in N$

$2^{3n} - 1$ is divisible by 7.

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9. Prove the following by using the principle of mathematical induction

for all $n \in N$

3^{2n} when divided by 8, the ramained is always 1.

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10. Prove the following by using the principle of mathematical induction

for all $n \in N$

$10^n + 3.4^{n+2} + 5$is divisible by 9.

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11. Prove the following by using the principle of mathematical induction for all $n \in N$

The sum of the cubes of three consecutive natural numbers is divisible by 9.

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12. Prove the following by using the principle of mathematical induction for all $n \in N$

$2.7^n + 3.5^n - 5$ is divisible by 24.

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13. Prove the following by using the principle of mathematical induction for all $n \in N$

$11^{n+2} + 12^{2n+1}$ is divisible by 133.

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14. Prove the following by using the principle of mathematical induction

for all $n \in N$

$n(n+1)(2n+1)$ is divisible by 6.

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15. Prove the following by using the principle of mathematical induction

for all $n \in N$

$(2n+1) < 2^n, n \geq 3$

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16. Prove the following by using the principle of mathematical induction

for all $n \in N$

$3^n > 2^n$

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17. Prove the following by using the principle of mathematical induction

for all $n \in N$

$(1+x)^n \leq (1+nx)$

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18. Prove the following by using the principle of mathematical induction

for all $n \in N$

$$\frac{1}{n+1} + \frac{1}{n+2} + \ldots . \frac{.1}{2n} > \frac{13}{24}, n > 1$$

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19. Prove the following by using the principle of mathematical induction

for all $n \in N$

$$\sqrt{n} \leq \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \ldots \ldots + \frac{1}{\sqrt{n}}$$

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20. Prove the following by using the principle of mathematical induction for all $n \in N$

$$1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\ldots\ldots+\frac{1}{n^2}<2-\frac{1}{n}, n \geq 2$$

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Textbook Based Mcqs

1. For $P(n): 2^n < n!$.......... Is true

A. P(1)

B. P(2)

C. Any P(n) , $n \in N$

D. P(4)

Answer: D

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2. For $P(n): 2^n = 0$ ....is true .

A. P(1)

B. P(3)

C. P(10)

D. $P(k) \Rightarrow P(k+1), k \in N$

Answer: D

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3. $1 + 2 + 3 + \dots\dots + (n+1) = \frac{(n+1)(n+2)}{2}, n \in N.$

A. For P(1) , L.H.S = 7 = R.H.S.

B. For P(1) , L.H.S = 3 = R.H.S.

C. $P(k) \Rightarrow P(k+1), k \in N$ is not true

D. By the principle of mathematical induction P(n) is true for all $n \in N$. Which is not true .

Answer: B

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4. If.....is true and P(k) is true $\Rightarrow$ P(k+1) is true, $k \geq -1$, then for all $n \in N \cup \{0, -1\}, P(n)$ is true.

A. P(-1)

B. P(0)

C. P(1)

D. P(2)

Answer: A

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5. $P(n) : 2^{2^n} + 1$ is a prime number . For n = ..., it is not true .

A. 1

B. 2

C. 0

D. 5

Answer: D

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6. $P(n) : 2^n - 1$, for n = ...it is a prime number.

A. 1

B. 3

C. 4

D. 8

Answer: B

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7. $P(n): n^2 - n + 41$, for n = ..., it is not prime number .

A. 1

B. 2

C. 3

D. 41

Answer: D

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8. $P(n): 2n + 1$, for n = ...it is not a prime number.

A. 1

B. 2

C. 3

D. 4

Answer: D

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9. $P(n): 4n+1$, for n = ...it is not a prime number

A. 1

B. 3

C. 7

D. 11

Answer: D

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10. $P(n): 2^n > n^2$, for n = ...it is true.

A. 2

B. 3

C. 4

D. 5

Answer: D

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Textbook Illustrations For Pratice Work

1. For all $n \geq 1$, prove that, $1^2 + 2^2 + 3^2 + 4^2 + \ldots\ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$

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2. Prove that $2^n > n$ for all positive integers n.

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3. For all $n \geq 1$, prove that ,

$$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots\ldots + \frac{1}{n(n+1)} = \frac{n}{n+1}$$

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4. For every positive integer n, prove that $7^n - 3^n$ is divisible by 4.

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5. Prove that $(1+x)^n \geq (1+nx), f \text{ or } all$ natural number n, where $x > -1$

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6. Prove the following by using the principle of mathematical induction for all $n \in N$

$2.7^{n} + 3.5^{n} - 5$ is divisible by 24.

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7. Prove that, $1^{2} + 2^{2} + \ldots\ldots + n^{2} > \frac{n^{3}}{3}, n \in N$

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8. Prove the rule of exponents $(ab)^{n} = a^{n}b^{n}$ by using principle of mathematical induction for every natural number.

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Ncert Exemplar Problems Short Answer Type Questions

1. Give an example of a statement P(n) which is for all $n \geq 4$ but P(1) ,P(2) and P(3) are not true, justify your answer.

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2. Give an example of a statement P(n) which is true for all n , justify your answer.

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3. Prove the statement by the principle of mathematical induction :

$4^n - 1$ is divisible by 3, for each natural number n .

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4. Prove the statement by the principle of mathematical induction :

$2^{3n} - 1$ is divisible by 7, for all natural numbers n .

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5. Prove the statement by the principle of mathematical induction :

$n^3 - 7n + 3$ is divisible by 3, for all natural number n .

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6. Prove the statement by the principle of mathematical induction :

$3^{2n} - 1$ is divisible by 8, for all natural number n.

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7. Prove the statement by the principle of mathematical induction :

For any natural numbers n, $7^n - 2^n$ is divisible by 5.

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8. Prove the statement by the principle of mathematical induction :

For any natural numbers n, $x^n - y^n$ is divisible by x - y , where x and y any integers with $x \neq y$

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9. Prove each of the statements by the principle of mathematical induction :

$n^3 - n$ is divisible by 6, for each natural number $n \geq 2. \ \forall n \geq 2$

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10. Prove each of the statements by the principle of mathematical induction :

$n(n^2 + 5)$ is divisible by 6, for each natural number n.

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11. Prove each of the statements by the principle of mathematical induction :

$n^2 < 2^n$, for all natural numbers $n \geq 5$

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12. Prove each of the statements by the principle of mathematical induction :

$2n < (n+2)!$ for all natural number n .

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13. Prove each of the statements by the principle of mathematical induction :

$\sqrt{n} < \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \ldots\ldots + \frac{1}{\sqrt{n}}$, for all natural numbers $n \geq 2$

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14. Prove each of the statements by the principle of mathematical induction :

$2 + 4 + 6 + \ldots + 2n = n^2 + n$ for all natural numbers n .

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15. Prove each of the statements by the principle of mathematical induction :

$1 + 2 + 2^n + \ldots.. + 2^n = 2^{n+1} - 1$ for all natural numbers n .

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16. Prove each of the statements by the principle of mathematical induction :

$1 + 5 + 9 + \ldots\ldots + (4n - 3) = n(2n - 1)$ for all natural numbers n .

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Ncert Exemplar Problems Long Answer Type Questions

1. Use the principle of mathematical induction :

A sequence $a_1, a_2, a_3, \ldots\ldots$ is defined by letting $a_1 = 3$ and $a_k = 7a_{k-1}$, for all natural numbers k > 2 . Show that $a_n = 3.7^{n-1}$, for all natural numbers .

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2. Use the principle of mathematical induction :

A sequence b_0, b_1, b_2,..... Is defined by letting $b_0 = 5$ and $b_k = 4 + b_{k-1}$, for all natural numbers k. show that b_n = 5 + 4n`, for all natural number n using mathematical induction.

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3. Use the principle of mathematical induction :

A sequence $d_1, d_2, d_3, \ldots\ldots\ldots$ is defined by letting $d_1 = 2$ and d_k =

(d_k - 1)/(k), f or $allnaturalvmbers$, k gt= 2. $Showt$^d_n = (2)/(n!) , for all $n \in N$

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4. Prove that $\cos\theta \cos 2\theta \cos 2^2\theta ... \cos 2^{n-1}\theta = \frac{\sin 2^n\theta}{2^n . \sin\theta}$, for all $n \in N$

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5. Prove that

$2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1 = 0$

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6. Show that $\frac{n^5}{5} + \frac{n^3}{3} + \frac{7n}{15}$ is a natural number for all $n \in N$

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7. Prove that , $\frac{1}{n+1}+\frac{1}{n+2}+\ldots\ldots\ldots\ldots+\frac{1}{2n}>\frac{13}{24}$ for all natural numbers n > 1

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8. Prove that number of subsets of a set containing n distinct elements is 2^n , for all $n \in N$

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Ncert Exemplar Problems Objective Type Questions

1. If $10^n+3.4^{n+2}+k$ is divisible by 9 , for all $n \in N$, then the least positive integral value of k is

A. 5

B. 3

C. 7

D. 1

Answer: A

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2. For all $n \in N, 3.5^{2n+1} + 2^{3n+1}$ is divisible by

A. 19

B. 17

C. 23

D. 25

Answer: B::C

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3. if $x^n - 1$ is divisible by x - k , then the least positive integral value of k is

A. 1

B. 2

C. 3

D. 4

Answer: A

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Ncert Exemplar Problems Fillers

1. If P(n) $: 2n < n!, n \in N$, then P(n) is true for all $n \geq \ldots\ldots\ldots\ldots\ldots$

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Ncert Exemplar Problems True False

1. Let P(n) be statement and let P(k) $\Rightarrow P(k+1)$,for some natural number k, then P(n) is true for all $n \in N$

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Ncert Exemplar Problems Question Of Module

1. By using the principle of mathematical induction , prove the follwing :

$P(n)+1+3+5+\ldots\ldots\ldots+(2n-1)=n^2, n \in N$

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2. By using the principle of mathematical induction , prove the follwing :

$P(n): 1^2+2^2+3^2+\ldots\ldots\ldots\ldots+n^2=\frac{n}{6}(n+1)(2n+1), n \in N$

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3. By using the principle of mathematical induction , prove the follwing :

$$P(n): \frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots\ldots + \frac{1}{n(n+1)} = \frac{n}{n+1}, n \in N$$

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4. By using the principle of mathematical induction , prove the follwing :

$$P(n): (1+x)^n \geq 1 + nx, x > (-1), n \in N$$

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5. Prove the following by using the principle of mathematical induction for all $n \in N$

$2.7^n + 3.5^n - 5$ is divisible by 24.

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6. By using the principle of mathematical induction , prove the follwing :

$$\left(1+\frac{1}{1}\right)\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\ldots\ldots\ldots\ldots\ldots\left(1+\frac{1}{n}\right)=n+1, n \in N$$

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7. By using the principle of mathematical induction , prove the follwing :

$$P(n): 2+4+6+\ldots\ldots+2n=n(n+1), n \in N$$

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8. By using the principle of mathematical induction , prove the follwing :

$$1+\frac{1}{1+2}+\frac{1}{1+2+3}+\ldots\ldots+\frac{1}{1+2+\ldots\ldots+n}=\frac{2n}{n+1}, n \in N$$

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9. By using the principle of mathematical induction , prove the follwing :

$$P(n): \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots\ldots\ldots+\frac{1}{2^n}=1-\frac{1}{2^n}, n \in N$$

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10. By using the principle of mathematical induction , prove the follwing :

$$\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+\ldots\ldots\ldots\ldots+\frac{1}{(3n-2)(3n+1)}=\frac{n}{3n+1}, n \in N$$

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11. By using the principle of mathematical induction , prove the follwing :

$$P(n):(2n+7)<(n+3)^2, n \in N$$

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