

MATHS

BOOKS - KUMAR PRAKASHAN KENDRA MATHS (GUJRATI ENGLISH)

PRINCIPLE OF MATHEMATICAL INDUCTION

Exercise 4 1

1. Prove the following by using the principle of mathematical induction

for all $n \in N$

 $1 + 3 + 3^2 + \ldots + 3^{n-1} = \frac{3^n - 1}{2}$

2. Prove the following by using the principle of mathematical induction for all $n \in N$ $1^3+2^3+3^3+\ldots +n^3=\left(rac{n(n+1)}{2}
ight)^2$ O Watch Video Solution

3. Prove the following by using the principle of mathematical induction

for all
$$n \in N$$

 $1 + \frac{1}{(1+2)} + \frac{1}{(1+2+3)} + \dots + \frac{1}{1+2+3+\dots+n} = \frac{2n}{n+1}$
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4. Prove the following by using the principle of mathematical induction

1.2.3.
$$+2.3.4+\ldots+n(n+1)(n+2)=rac{n(n+1)(n+2)(n+3)}{4}$$

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for all m C M

5. Prove the following by using the principle of mathematical induction for all $n \in N$ $1.3 + 2.3^2 + 3.3^3 + \ldots + n.3^n = \frac{(2n-1)3^{n+1} + 3}{4}$ Watch Video Solution

6. Prove the following by using the principle of mathematical induction

for all
$$n \in N$$
 $1.2 + 2.3 + 3.4 + \ldots + n. \ (n+1) = \left[rac{n(n+1)(n+2)}{3}
ight]$

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. .

...

7. Prove the following by using the principle of mathematical induction

for all
$$n \in N$$

 $1.3 + 3.5 + 5.7 + \ldots + (2n-1)(2n+1) = rac{n(4n^2 + 6n - 1)}{3}$

for all $n \in N$

$$1.2 + 2.2^2 + 3.2^3 + \ldots + n.2^n = (n-1)2^{n+1} + 2$$



9. Prove the following by using the principle of mathematical induction

for all
$$n \in N$$

 $rac{1}{2} + rac{1}{4} + rac{1}{8} + \dots + rac{1}{2^n} = 1 - rac{1}{2^n}$

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10. Prove the following by using the principle of mathematical induction

for all
$$n \in N$$

 $\frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + \dots + \frac{1}{(3n-1)(3n+2)} = \frac{n}{(6n+4)}$

for all
$$n \in N$$

$$\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \dots + \frac{1}{n(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$$

$$\textcircled{Watch Video Solution}$$

12. Prove the following by using the principle of mathematical induction for all $n \in N$ $a + ar + ar^2 + \dots + ar^{n-1} = \frac{a(r^n - 1)}{r - 1}$ Watch Video Solution

13. Prove the following by using the principle of mathematical induction

for all
$$n \in N$$

 $\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right) \times \ldots \times \left(1+\frac{(2n+1)}{n^2}\right) = (n+1)^2$

for all
$$n \in N$$

 $\left(1 + \frac{1}{1}\right)\left(1 + \frac{1}{2}\right)\left(1 + \frac{1}{3}\right)\dots\left(1 + \frac{1}{n}\right) = (n+1)$
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15. Prove the following by using the principle of mathematical induction
for all $n \in N$
 $1^2 + 3^2 + 5^2 + \dots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$

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16. Prove the following by using the principle of mathematical induction for all $n \in N$

$$rac{1}{1.4} + rac{1}{4.7} + rac{1}{7.10} + \dots + rac{1}{(3n-2)(3n+1)} = rac{n}{3n+1}$$

for all
$$n \in N$$

 $\frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \dots + \frac{1}{(2n+1)(2n+3)} = \frac{n}{3(2n+3)}$
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18. Prove the following by using the principle of mathematical induction

 $1+2+3+\ldots\ldots+n < rac{1}{8}(2n+1)^2$

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19. Prove the following by using the principle of mathematical induction

for all $n \in N$

for all $n \in N$

n(n+1)(n+5) is a multiple of 3

20. Prove the following by using the principle of mathematical induction for all $n\in N$ $10^{2n-1}+1$ is divisible by 11.

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21. Prove the following by using the principle of mathematical induction for all $n \in N$

 $x^{2n}-y^{2n}$ is divisible by x + y .

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22. Prove the following by using the principle of mathematical induction

for all $n \in N$

 $3^{2n+2}-8n-9$ is divisible by 8 .

for all $n \in N$

 $41^n - 14^n$ is a multiple of 27.

24. Prove the following by using the principle of mathematical induction

for all $n \in N$

 $\left(2n+7\right) < \left(n+3\right)^2$

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Practice Work

1. Prove the following by using the principle of mathematical induction

for all $n \in N$

$$\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \dots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$$

for all $n \in N$

 $1 + 2 + 2^2 + \ldots + 2^n = 2^{n+1} - 1$

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3. Prove the following by using the principle of mathematical induction

for all $n \in N$

$$igg(1-rac{1}{2^2}igg)igg(1-rac{1}{3^2}igg)igg(1-rac{1}{4^2}igg).\ldots\ldotsigg(1-rac{1}{n^2}igg)=rac{n+1}{2n}\qquad n\geq 2$$

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4. Prove the following by using the principle of mathematical induction for all $n \in N$ $3 \times 6 + 6 \times 9 + 9 \times 12 + \ldots + (3n)(3n+3) = 3n(n+1)(n+2)$

for all $n \in N$

 $a + (a + d) + (a + 2d) + \ldots + (a + (n - 1)d) = rac{n}{2}[2a + (n - 1)d]$

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6. Prove the following by using the principle of mathematical induction

for all $n \in N$

 $4 + 8 + 12 + \dots + 4n = 2n(n+1)$

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7. Prove the following by using the principle of mathematical induction

for all $n \in N$

 $7^n - 3^n$ is divisible by 4.

for all $n \in N$

 $2^{3n} - 1$ is divisible by 7.

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9. Prove the following by using the principle of mathematical induction

for all $n \in N$

 3^{2n} when divided by 8, the ramained is always 1.

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10. Prove the following by using the principle of mathematical induction

for all $n \in N$

 $10^{n} + 3.4^{n+2} + 5$ is divisible by 9.

for all $n \in N$

The sum of the cubes of three consecutive natural numbers is divisible by 9.

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12. Prove the following by using the principle of mathematical induction

for all $n \in N$

 $2.7^n + 3.5^n - 5$ is divisible by 24.

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13. Prove the following by using the principle of mathematical induction

for all $n \in N$

 $11^{n+2} + 12^{2n+1}$ is divisible by 133.

for all $n \in N$

n(n+1)(2n+1) is divisble by 6.

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15. Prove the following by using the principle of mathematical induction

for all $n \in N$

 $(2n+1)<2^n,n\geq 3$

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16. Prove the following by using the principle of mathematical induction

for all $n \in N$

 $3^n > 2^n$

for all $n \in N$

$$\left(1+x
ight)^n \leq \left(1+nx
ight)$$

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18. Prove the following by using the principle of mathematical induction

for all
$$n \in N$$
 $rac{1}{n+1} + rac{1}{n+2} + \dots rac{.1}{2n} > rac{13}{24}, n > 1$

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19. Prove the following by using the principle of mathematical induction

$$\sqrt{n} \leq rac{1}{\sqrt{1}} + rac{1}{\sqrt{2}} + \ldots + rac{1}{\sqrt{n}}$$

for all $n \in N$

for all
$$n \in N$$
 $1 + rac{1}{4} + rac{1}{9} + rac{1}{16} + \ldots + rac{1}{n^2} < 2 - rac{1}{n}, n \geq 2$

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Textbook Based Mcqs

1. For
$$P(n)\!:\!2^n < n\!!$$
...... Is true

A. P(1)

B. P(2)

C. Any P(n) , $n\in N$

D. P(4)

Answer: D

2. For
$$P(n): 2^n = 0$$
is true .

A. P(1)

B. P(3)

C. P(10)

D.
$$P(k) \Rightarrow P(k+1), k \in N$$

Answer: D

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$$\mathbf{3.1}+2+3+\ldots\ldots+(n+1)=rac{(n+1)(n+2)}{2}, n\in N.$$

A. For P(1) , L.H.S = 7 = R.H.S.

B. For P(1) , L.H.S = 3 = R.H.S.

C. $P(k) \Rightarrow P(k+1), k \in N$ is not true

D. By the principle of mathematical induction P(n) is true for all

 $n\in N$. Which is not true .

Answer: B

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4. If....is true and P(k) is true \Rightarrow P(k+1) is true, $k \ge -1$, then for all $n \in N \cup \{0, -1\}, P(n)$ is true.

A. P(-1)

B. P(0)

C. P(1)

D. P(2)

Answer: A

5. P(n) : $2^{2^n} + 1$ is a prime number . For n =, it is not true .

A. 1

B. 2

C. 0

D. 5

Answer: D

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6. $P(n): 2^n - 1$, for n =it is a prime number.

A. 1

B. 3

C. 4

D. 8

Answer: B



7. P(n) : $n^2 - n + 41$, for n =, it is not prime number .

A. 1 B. 2

C. 3

D. 41

Answer: D

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8. P(n): 2n + 1, for n =it is not a prime number.

C. 3

D. 4

Answer: D

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9. P(n) : 4n + 1 , for n =it is not a prime number

A. 1

B. 3

C. 7

D. 11

Answer: D

10. P(n) : $2^n > n^2$, for n =it is true.

A. 2

B. 3

C. 4

D. 5

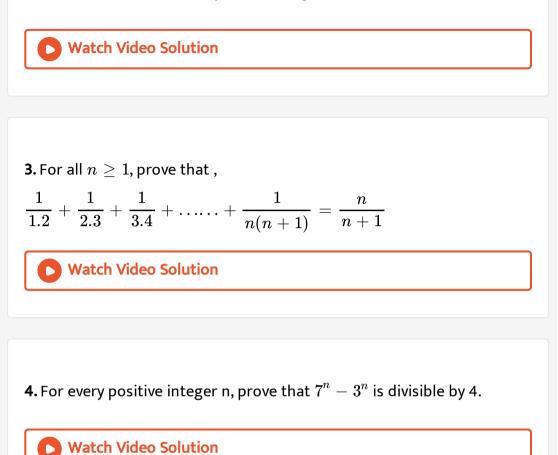
Answer: D

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Textbook Illustrations For Pratice Work

1. For all
$$n\geq 1$$
, prove that, $1^2+2^2+3^2+4^2+\ldots\ldots+n^2=rac{n(n+1)(2n+1)}{6}$

2. Prove that $2^n > n$ for all positive integers n.



5. Prove that
$$\left(1+x
ight)^n \geq (1+nx), f ext{ or } all ext{ natural number n, where}$$
 $x > -1$

for all $n \in N$

 $2.7^n + 3.5^n - 5$ is divisible by 24.

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7. Prove that,
$$1^2+2^2+\ldots +n^2>rac{n^3}{3}, n\in N$$

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8. Prove the rule of exponents $(ab)^n = a^n b^n$ by using principle of mathematical induction for every natural number.



Ncert Exemplar Problems Short Answer Type Questions

1. Give an example of a statement P(n) which is for all $n \ge 4$ but P(1) ,P(2)

and P(3) are not true, justify your answer.

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2. Give an example of a statement P(n) which is true for all n , justify your answer.	
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3. Prove the statement by the principle of mathematical induction :	
4^n-1 is divisible by 3, for each natural number n .	

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4. Prove the statement by the principle of mathematical induction :

 $2^{3n}-1$ is divisible by 7, for all natural numbers n .

5. Prove the statement by the principle of mathematical induction :

 n^3-7n+3 is divisible by 3, for all natural number n .

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6. Prove the statement by the principle of mathematical induction :

 $3^{2n}-1$ is divisible by 8, for all natural number n.

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7. Prove the statement by the principle of mathematical induction :

For any natural numbers $n, 7^n - 2^n$ is divisible by 5.

8. Prove the statement by the principle of mathematical induction :

For any natural numbers n, x^n-y^n is divisible by x - y , where x and y any integers with x
eq y



9. Prove each of the statements by the principle of mathematical induction :

 n^3-n is divisible by 6, for each natural number $n\geq 2. \,\, orall n\geq 2$

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10. Prove each of the statements by the principle of mathematical induction :

 $nig(n^2+5ig)$ is divisible by 6, for each natural number n.

11. Prove each of the statements by the principle of mathematical induction :

 $n^2 < 2^n$, for all natural numbers $n \geq 5$



12. Prove each of the statements by the principle of mathematical induction :

2n < (n+2)! for all natural number n .

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13. Prove each of the statements by the principle of mathematical induction :

$$\sqrt{n} < rac{1}{\sqrt{1}} + rac{1}{\sqrt{2}} + rac{1}{\sqrt{3}} + \ldots + rac{1}{\sqrt{n}},$$
 for all natural numbers $n > 2$

14. Prove each of the statements by the principle of mathematical induction :

 $2+4+6+\ldots+2n=n^2+n$ for all natural numbers n .



15. Prove each of the statements by the principle of mathematical induction :

 $1+2+2^n+\ldots$. $+2^n=2^{n+1}-1$ for all natural numbers n .

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16. Prove each of the statements by the principle of mathematical induction :

 $1+5+9+\ldots\ldots+(4n-3)=n(2n-1)$ for all natural numbers n .

1. Use the principle of mathematical induction :

A sequence a_1, a_2, a_3, \ldots is defined by letting $a_1 = 3$ and $a_k = 7a_{k-1}$, for all natural numbers k > 2. Show that $a_n = 3.7^{n-1}$, for all natural numbers.



2. Use the principle of mathematical induction :

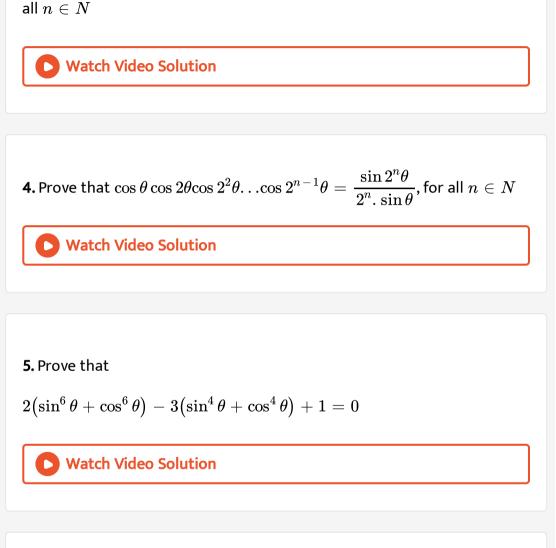
A sequence b_0 , b_1 , b_2 ,.... Is defined by letting $b_0 = 5$ and $b_k = 4 + b_{k-1}$, for all natural numbers k. show that $b_n = 5 + 4n$, for all natural number n using mathematical induction.

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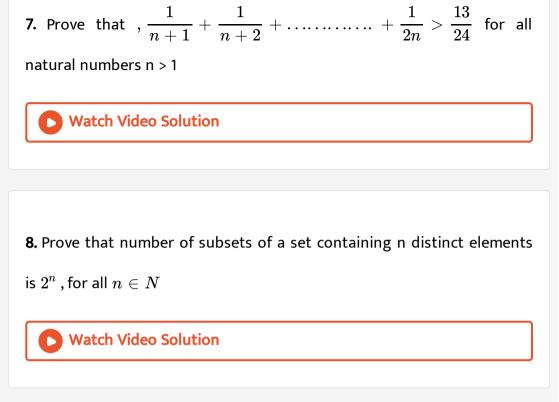
3. Use the principle of mathematical induction :

A sequence d_1, d_2, d_3, \ldots is defined by letting $d_1 = 2$ and d_k =

 $(d_k - 1)/(k)$, f or $all natural \nu mbers$, k gt= 2. $Showt^d_n = (2)/(n!)$, for



6. Show that
$$rac{n^5}{5}+rac{n^3}{3}+rac{7n}{15}$$
 is a natural number for all $n\in N$



Ncert Exemplar Problems Objective Type Questions

1. If $10^n + 3.4^{n+2} + k$ is divisible by 9 , for all $n \in N$, then the least positive integral value of k is

A. 5

B. 3

C. 7

Answer: A



2. For all
$$n \in N, 3.5^{2n+1}+2^{3n+1}$$
 is divisible by

A. 19

B. 17

C. 23

D. 25

Answer: B::C

3. if x^n-1 is divisible by x - k , then the least positive integral value of k is

A. 1 B. 2 C. 3 D. 4

Answer: A

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Ncert Exemplar Problems Fillers

1. If P(n) $: 2n < n!, n \in N$, then P(n) is true for all $n \geq \ldots \ldots \ldots$

1. Let P(n) be statement and let P(k) $\Rightarrow P(k+1)$,for some natural number k, then P(n) is true for all $n \in N$

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Ncert Exemplar Problems Question Of Module

1. By using the principle of mathematical induction , prove the following :

 $P(n) + 1 + 3 + 5 + \ldots + (2n-1) = n^2, n \in N$

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2. By using the principle of mathematical induction , prove the following :

$$P(n)\!:\!1^2+2^2+3^2+\ldots\ldots+n^2=rac{n}{6}(n+1)(2n+1), n\in N$$

3. By using the principle of mathematical induction , prove the following :

$$P(n): \frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \dots + \frac{1}{n(n+1)} = \frac{n}{n+1}, n \in N$$
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4. By using the principle of mathematical induction , prove the follwing :

$$P(n)\!:\!\left(1+x
ight)^n\geq 1+nx, x>(-1), n\in N$$

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5. Prove the following by using the principle of mathematical induction

for all $n \in N$

 $2.7^n + 3.5^n - 5$ is divisible by 24.

6. By using the principle of mathematical induction , prove the follwing :

$$igg(1+rac{1}{1}igg)igg(1+rac{1}{2}igg)igg(1+rac{1}{3}igg)\dots\dots\dotsigg(1+rac{1}{n}igg)=n+1,n\in N$$

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7. By using the principle of mathematical induction , prove the follwing :

 $P(n)\!:\!2+4+6+\ldots\,.+2n=n(n+1), n\in N$

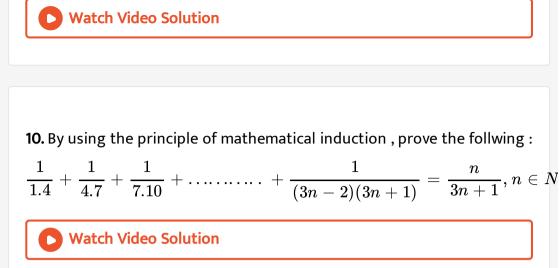
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8. By using the principle of mathematical induction , prove the follwing :

$$1 + rac{1}{1+2} + rac{1}{1+2+3} + \ldots + rac{1}{1+2+\ldots+n} = rac{2n}{n+1}, n \in N$$

9. By using the principle of mathematical induction , prove the follwing :

$$P(n)\!:\!rac{1}{2}+rac{1}{4}+rac{1}{8}+\ldots\ldots\,+rac{1}{2^n}=1-rac{1}{2^n},n\in N$$



11. By using the principle of mathematical induction , prove the follwing :

$$P(n)\!:\!\left(2n+7
ight)<\left(n+3
ight)^{2},n\in N$$