MATHS

BOOKS - KUMAR PRAKASHAN KENDRA MATHS (GUJRATI ENGLISH)

APPLICATION OF DERIVATIVES

Exercise 6 1

1. Find the rate of change of the area of a circle with respect to its radius r when

(a) r=3 cm (b) r=4 cm

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2. The volume of a cube is increasing at the rate of $8cm^3/s$. How fast is the surface area increasing when the length of an edge is 12 cm ?

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3. The radius of a circle is increasing uniformly at the rate of 3 cm/s. Find the rate at which the area of the circle is increasing when the radius is 10 cm.

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4. An edge of a variable cube is increasing at the rate of 3 cm/s. How fast is the volume of the cube increases when the edge is 10 cm long ?

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5. A stone is dropped into a quiet lake and waves move in circles at the speed of 5 cm/s. At the instant when the radius of the circular wave is 8 cm, how fast is the enclosed area increasing ?

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6. The radius of a circle is increasing at the rate of 0.7 cm/s. What is the rate of increase of its circumference ?

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7. The length x of a rectangle is decreasing at the rate of 5 cm/minute and the width y is increasing at the rate of 4 cm/minute. When x = 8 cm and y = 6 cm, find the rates of change of (a) the [erimeter, and (b) the area of the rectangle.

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8. A balloon, which always remains spherical on inflation, is being inflated by pumping in 900 cubic centimetres of gas per second. Find the rate at which the radius of the balloon increases when the radius is 15 cm.

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9. A balloon, which always remains spherical has a variable radius. Find the rate at which its volume is increasing with the radius when the later is 10 cm.

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10. A ladder 5 cm long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2cm/s. How fast is its height on the wall decreasing when the foot of the ladder is 4 cm away from the wall ?

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11. A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which the y-coordinate is changing 8 times as fast as the x-coordinate.

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12. The radius of an air bubble is increasing at the rate of $\frac{1}{2}$ cm/s. At what rate is the volume of the bubble increasing when the radius is 1 cm ?

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13. A balloon, which always remains spherical, has a variable diameter $\frac{3}{2}(2x+1)$. Find he rate of change of its volume with respect to x.

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14. Sand is pouring from a pipe at the rate of 12 cm^3/s. The falling sand forms a cone on the ground in such a way that the height of the cone is always one - sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm ?

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15. The total cost C(x) in Rupees associated with the production of x units of an item is given by

$C(x) = 0.007x^3 - 0.003x^2 + 15x + 4000.$

Find the marginal cost when 17 units are produced.

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16. The total revenue in Rupees received from the sale of x units of a product is give by $R(x) = 13x^2 + 26x + 15$. Find the marginal revenue when x = 7.

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17. The rate of change of the area of a circle with respect to its radius r at r = 6 cm is ....

A. 10π

B. 12π

C. 8π

D. 11π

Answer: B

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18. The total revenue in Rupees received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5.$ The marginal revenue, when x = 15 is ....

A. 116

B. 96

C. 90

D. 126

Answer: D

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Exercise 6 2

1. Show that the function given by $f(x) = 3x + 17$ is increasing on R.

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2. Show that the function given by $f(x) = e^{2x}$ is increasing on R.

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3. Show that the function given by f(x) = sin x is

(a) increasing in $\left(0, \frac{\pi}{2}\right)$

(b) decreasing in $\left(\frac{\pi}{2}, \pi\right)$

(c) neither increasing nor decreasing in $(0, \pi)$.

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4. Find the intervals in which the function f given by $f(x) = 2x^2 - 3x$ is

(a) incresing

(b) decreasing

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5. Find the intervals in which function f given by $f(x) = 2x^3 - 3x^2 - 36x + 7$ is

(a) increasing

(b) decreasing

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6. Find the intervals in which the following functions are strictly increasing or decreasing

:

$x^2 + 2x - 5$

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7. Find the intervals in which the following functions are strictly increasing or decreasing

:

$10 - 6x - 2x^2$

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8. Find the intervals in which the following functions are strictly increasing or decreasing :

$f(x) = 2x^3 - 9x^2 - 12x + 15$

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9. Find the intervals in which the functions are strictly increasing or decreasing:

$6 - 9x - x^2$

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10. Find the intervals in which the functions are strictly increasing or decreasing:

$(x+1)^3(x-3)^3$

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11. Show that $y = \log(1+x) - \frac{2x}{2+x}, x > -1,$ is an increasing function of x throughout its domain.

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12. Find the values of x for which $y = [x(x-2)]^2$ is an increasing function.

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13. Prove that $y = \frac{4\sin\theta}{(2+\cos\theta)} - \theta$ is an increasing function of θ in $\left[0, \frac{\pi}{2}\right]$.

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14. Prove that the logarithmic function is increasing on $(0, \infty)$

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15. Prove that the function f given by f(x) = $x^2 - x + 1$ is neither strictly increasing nor decreasing on (– 1, 1).

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16. Which of the following functions is decreasing on $\left(0, \frac{\pi}{2}\right)$.

A. cos x

B. cos 2x

C. cos 3x

D. tan x

Answer: A::B

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17. On which of the following intervals is the function f given by f(x) = x^{100} + sin x −1 decreasing ?

A. (0, 1)

B. $\left(\frac{\pi}{2}, \pi\right)$

C. $\left(0, \frac{\pi}{2}\right)$

D. None of these

Answer: D

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18. Find the least value of a such that the function given by $f(x) = x^2 + ax + 1$ is strictly increasing on (1, 2).

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19. Let I be any interval disjoint from $[-1, 1]$. Prove that the function f given by $f(x) = x + \frac{1}{x}$ is increasing on I.

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20. Prove that the function f given by f(x) = log sin x is increasing on $\left(0, \frac{\pi}{2}\right)$ and decreasing on $\left(\frac{\pi}{2}, \pi\right)$.

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21. Prove that the function f given by f (x) = log |cos x| is decreasing on $\left(0, \frac{\pi}{2}\right)$ and increasing on $\left(\frac{3\pi}{2}, 2\pi\right)$.

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22. Prove that the function given by $f(x) = x^3 - 3x^2 + 3x - 100$ is increasing in R.

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23. The interval in which $y = x^2 . e^{-x}$ is increasing is -----

A. $(-\infty, \infty)$

B. $(-2, 0)$

C. $(2, \infty)$

D. $(0, 2)$

Answer: D

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Exercise 6 3

1. Find the slope of the tangent to the curve $y = 3x^4 - 4x$ at x = 4.

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2. Find the slope of the tangent to the curve $y=\frac{x-1}{x-2}, x\neq 2$ at x = 10.

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3. Find the slope of the tangent to curve $y=x^3-x+1$ at the point whose x-coordinate is 2.

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4. Find the slope of the tangent to the curve $y = x^3 - 3x + 2$ at the point whose x - coordinate is 3.

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5. Find the slope of the normal to the curve $x = a\cos^3\theta, y = a\sin^3\theta$ at $\theta = \dfrac{\pi}{4}$.

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6. Find the slope of the normal to the curve $x = 1 - a\sin\theta, y = b\cos^2\theta$ at $\theta = \frac{\pi}{2}$.

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7. Find points at which the tangent to the curve $y = x^3 - 3x^2 - 9x + 7$ is parallel to the x-axis.

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8. Find a point on the curve y $=(x-2)^2$ at which the tangent is parallel to the chord joining the points (2, 0) and (4, 4).

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9. Find the point on the curve $y = x^3 - 11x + 5$ at which the tangent is $y = x - 11$.

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10. Find the equation of all lines having slope -1 that are tangents to the curve $y = \frac{1}{x-1}, x \neq 1.$

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11. Find the equation of all lines having slope 2 which are tangents to the curve $y = \frac{1}{x-3}, x \neq 3.$

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12. Find the equations of all lines having slope 0 which are tangent to the curve $y = \frac{1}{x^2 - 2x + 3}$.

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13. Find points on the curve $\frac{x^2}{9} + \frac{y^2}{16} = 1$ at which the tangents are

(i) parallel to X - axis

(ii) parallel to Y-axis.

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14. Find the equations of the tangent and normal to the given curves at the indicated points :

$y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at (0, 5)

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15. Find the equations of the tangent and normal to the given curves at the indicated points :

$y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at (1,3)

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16. Find the equations of the tangent and normal to the given curves at the indicated points :

$y = x^3$ at (1, 1)

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17. Find the equations of the tangent and normal to the given curves at the indicated points :

$y = x^2$ at (0, 0)

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18. Find the equations of the tangent and normal to the given curves at the indicated points :

$x = \cos t, y = \sin t$ at $t = \dfrac{\pi}{4}$

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19. Find the equation of the tangent line to the curve y = x^2 – 2x +7 which is

(a) parallel to the line 2x – y + 9 = 0

(b) perpendicular to the line 5y – 15x = 13.

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20. Show that the tangents to the curve $y = 7x^3 + 11$ at the point where x = 2 and $x = -2$ are parallel.

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21. Find the points on the curve $y = x^3$ at which the slope of the tangent is equal to the y - coordinate of the point.

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22. For the curve $y = 4x^3 - 2x^5$, find all the points at which the tangent passes through the origin.

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23. Find the points on the curve $x^2 + y^2 - 2x - 3 = 0$ at which the tangents are parallel to the X-axis.

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24. Find the equation of the normal at the point (am^2, am^3) for the curve $ay^2 = x^3$.

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25. Find the equation of the normals to the curve $y = x^3 + 2x + 6$ which are parallel to the line $x + 14y + 4 = 0$.

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26. Find the equations of the tangent and normal to the parabola $y^2 = 4ax$ at the point $(at^2, 2at)$.

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27. Prove that the curves $x = y^2$ and xy = k cuts at right angles if $8k^2 = 1$.

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28. Find the equations of the tangent and normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point (x_0, y_0).

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29. Find the equation of the tangent to the curve $y = \sqrt{3x - 2}$ which is parallel to the line $4x - 2y + 5 = 0.$

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30. The slope of the normal to the curve $y = 2x^2 + 3\sin x$ at x = 0 is ....

A. 3

B. $\frac{1}{3}$

C. -3

D. $-\frac{1}{3}$

Answer: D

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31. The line y = x + 1 is a tangent to the curve y^2 = 4x at the poin

A. (1, 2)

B. (2, 1)

C. (1, -2)

D. (-1, 2)

Answer: A

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Exercise 6 4

1. Using differentials, find the approzimate value of each of the following up to 3 places of decimal :

$\sqrt{25.3}$

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2. Using differentials, find the approximate value of each of the up to 3 places of decimal.

$\sqrt{49.5}$

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3. Using differentials, find the approximate value of each of the up to 3 places of decimal.

$\sqrt{0.6}$

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4. Using differentials, find the approximate value of each of the up to 3 places of decimal.

$(0.009)^{\frac{1}{3}}$

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5. Using differentials, find the approzimate value of each of the following up to 3 places of decimal :

$(0.999)^{\frac{1}{10}}$

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6. Using differentials, find the approximate value of each of the up to 3 places of decimal.

$(15)^{\frac{1}{4}}$

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7. Using differentials, find the approzimate value of each of the following up to 3 places of decimal :

$(26)^{\frac{1}{3}}$

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8. Using differentials, find the approximate value of each of the up to 3 places of decimal.

$(255)^{\frac{1}{4}}$

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9. Using differentials, find the approzimate value of each of the following up to 3 places of decimal :

$(82)^{\frac{1}{4}}$

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10. Using differentials, find the approzimate value of each of the following up to 3 places of decimal :

$(401)^{\frac{1}{2}}$

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11. Using differentials, find the approximate value of each of the up to 3 places of decimal.

$(0.0037)^{\frac{1}{2}}$

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12. Using differentials, find the approximate value of each of the up to 3 places of decimal.

$(26.57)^{\frac{1}{3}}$

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13. Using differentials, find the approximate value of each of the up to 3 places of decimal.

$(81.5)^{\frac{1}{4}}$

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14. Using differentials, find the approximate value of each of the up to 3 places of decimal.

$(3.968)^{\frac{3}{2}}$

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15. Using differentials, find the approximate value of each of the up to 3 places of decimal.

$(32.15)^{\frac{1}{5}}$

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16. Find the approximate value of $f(2.01)$, where $f(x) = 4x^2 + 5x + 2$.

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17. Find the approximate value of $f(5.001)$, where $f(x) = x^3 - 7x^2 + 15$.

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18. Find the approximate change in the volume V of a cube of side x meters caused by increasing the side by 1%.

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19. Find the approximate change in the surface area of a cube of side x metres caused by decreasing the side by 1%.

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20. If the radius of a sphere is measured as 7m with an error of 0.02 m, then find the approximate error in calculating its volume.

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21. If the radius of a sphere is measured as 9m with an error of 0.03m, then find the approximte error in calculating its surface area.

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22. If $f(x) = 3x^2 + 15x + 5,$ then the approximate value of f(3.02) is ....

A. 47.66

B. 57.66

C. 67.66

D. 77.66

Answer: D

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23. The approximate change in the volume of a cube of side x metres caused by increasing the side by 3% is ....

A. $0.06x^3(\text{met})^3$

B. $0.6x^3(\text{met})^3$

C. $0.09x^3(\text{met})^3$

D. $0.9x^3(\text{met})^3$

Answer: C

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Exercise 6 5

1. Find the maximum and minimum values, if any, of the functions given by

f (x) = $(2x-1)^2+3$

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2. Find the maximum and minimum values, if any, of the functions given by

f(x) = $9x^2+12x+2$

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3. Find the maximum and minimum values, if any, of the functions given by

$f(x) = -(x - 1)^2 + 10$

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4. Find the maximum and minimum values, if any, of the functions given by

g(x) =x^3 + 1

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5. Find the maximum and minimum values, if any, of the functions given by

$f(x) = |x + 2| - 1$

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6. Find the maximum and minimum values, if any, of the functions given by

$g(x) = -|x + 1| + 3$

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7. Find the maximum and minimum values, if any, of the functions given by

h(x) = sin(2x) + 5

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8. Find the maximum and minimum values, if any of the following functions given by :

$f(x) = |\sin(4x) + 3|$

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9. Find the maximum and minimum values, if any, of the functions given by

h(x) = x + 1, x $\in$ (– 1, 1)

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10. Find the local maxima and local minima, if any, of the functions. Find also the local maximum and the local minimum values, as the case may be:

f(x) =x^2

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11. Find the local maxima and local minima, if any, of the functions. Find also the local maximum and the local minimum values, as the case may be:

g(x) = $x^3 - 3x$

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12. Find the local maxima and local minima, if any, of the functions. Find also the local maximum and the local minimum values, as the case may be:

h(x) = sin x+ cos x , 0 $< x < \frac{\pi}{2}$

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13. Find the local maxima and local minima, if any, of the functions. Find also the local maximum and the local minimum values, as the case may be:

f(x) = sin x - cos x , 0 $< x < 2\pi$

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14. Find the local maxima and local minima, if any, of the functions. Find also the local maximum and the local minimum values, as the case may be:

f(x) $= x^3 - 6x^2 + 9x + 15$

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15. Find the local maxima and local minima, if any, of the functions. Find also the local maximum and the local minimum values, as the case may be:

g(x) $= \frac{x}{2} + \frac{2}{x} x > 0$

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16. Find the local maxima and local minima, if any, of the functions. Find also the local maximum and the local minimum values, as the case may be:

g(x) $= \frac{1}{x^2 + 2}$

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17. Find the local maxima and local minima, if any, of the functions. Find also the local maximum and the local minimum values, as the case may be:

f(x) = $x\sqrt{1-x}, 0 < \quad < 1$

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18. Prove that the following functions do not have maxima or minima :

$f(x) = e^x$

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19. Prove that the functions do not have maxima or minima:

g(x) = log x

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20. Prove that the functions do not have maxima or minima:

h(x) = $x^3 + x^2 + x + 1$

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21. Find the absolute maximum value and the absolute minimum value of the functions in the given intervals:

f(x) $= x^3, x \in [-2, 2]$

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22. Find the absolute maximum value and the absolute minimum value of the functions in the given intervals:

f (x) = sin x + cos x , $x \in [0, \pi]$

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23. Find the absolute maximum value and the absolute minimum value of the functions in the given intervals:

f(x) = 4x $-\frac{1}{2}x^2, x \in \left[-2, \frac{9}{2}\right]$

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24. Find the absolute maximum value and the absolute minimum value of the functions in the given intervals:

f(x) =$(x-1)^1+3, x \in$ [-3 ,1)

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25. Find the maximum profit that a company can make, if the profit function is given by $p(x)=41-72x-18x^2$

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26. Find both the maximum value and the minimum value of $f(x) = 3x^4 - 8x^3 + 12x^2 - 48x + 25$ on the interval [0, 3].

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27. At what points in the interval $[0, 2\pi]$, does the function $\sin(2x)$ attain its maximum value ?

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28. What is the maximum value of the function $\sin x + \cos x$?

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29. Find the maximum value of $2x^3 - 24x + 107$ in the interval [1, 3]. Find the maximum value of the same function in $[-3, -1]$.

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30. It is given that at x = 1, the function $x^4 - 62x^2 + ax + 9$ attains its maximum value, on the interval [0, 2]. Find the value of a.

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31. Find the maximum and minimum values of $x + \sin(2x)$ on $[0, 2\pi]$.

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32. Find two numbers whose sum is 24 and whose product is as large as possible.

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33. Find two positive numbers x and y such that $x + y = 60$ and xy^3 is maximum.

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34. Find two positive numbers x and y such that their sum is 35 and the product x^2y^5 is a maximum.

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35. Find two positive numbers whose sum is 16 and the sum of whose cubes is minimum.

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36. A square piece of tin of side 18 cm is to be made into a box without top, by cutting a square from each corner and folding up the flaps to form the box. What should be the side of the square to be cut off so that the volume of the box is the maximum possible.

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37. A rectangular sheet of tin 45 cm by 24 cm is to be made into a box without top, by cutting off square from each corner and folding up the flaps. What should be the side of the square to be cut off so that the volume of the box is maximum ?

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38. Show that of all rectangles inscribed in a given fixed circle, the square has the maximum area.

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39. Show that the right circular cylinder of given surface and maximum volume is such that its height is equal to the diameter of the base.

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40. Of all the closed cylindrical cans (right circular), of a given volume of 100 cubic centimetres, find the dimensions of the can which has the minimum surface area ?

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41. A wire of length 28 m, is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the length of the two pieces so that the combined area of the square and the circle is minimum ?

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42. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

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43. Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ time the radius of the base.

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44. Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$.

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45. Show that semi - vertical angle of right circular cone of given surface area and maximum volume is $\sin^{-1}\left(\frac{1}{3}\right)$.

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46. The point on the curve $x^2 = 2y$ which is nearest to the point (0, 5) is ....

A. $(2\sqrt{2}, 4)$

B. $(2\sqrt{2}, 0)$

C. (0, 0)

D. (2, 2)

Answer: A

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47. For all real values of x, the minimum value of $f(x) = \frac{1 - x + x^2}{1 + x + x^2}, \forall x \in R$ is ....

A. 0

B. 1

C. 3

D. $\frac{1}{3}$

Answer: D

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Miscelaneous Exercise 6

1. Using differentials, find the approximate value of each of the following:

$(a)\left(\frac{17}{81}\right)^{\frac{1}{4}}$ (b) $(33)^{-\frac{1}{5}}$

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2. Using differentials, find the approximate value of each of the following:

$(a)\left(\frac{17}{81}\right)^{\frac{1}{4}}$ (b) $(33)^{-\frac{1}{5}}$

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3. Show that the function given by $f(x) = \frac{\log x}{x}$ has maximum at x = e.

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4. The two equal sides of an isosceles triangle with fixed base b are decreasing at the rate of 3 cm. per second. How fast is the area decreasing when the two equal sides are equal to the base ?

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5. Find the equation of the normal to the curve $x^2 = 4y$ which passes through the point (1, 2).

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6. Show that the normal at any point θ to the curve

$x = a\cos\theta + a\theta\sin\theta, y = a\sin\theta - a\theta\cos\theta$

is at the constant distance from origin.

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7. Find the intervals in which the function f given by $f(x) = \dfrac{4\sin x - 2x - x\cos x}{2 + \cos x}$ is (i) increasing (ii) decreasing.

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8. Find the intervals in which the function f given $f(x) = x^3 + \frac{1}{x^3}, x \neq 0$ is

(i) increasing (ii) decreasing.

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9. Find the maximum area of an isosceles triangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with its vertex at one end of the major axis.

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10. A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2m and volume is $8m^3$. If building of tank costs Rs. 70 per sq. metres for the base and Rs. 45 per square metre for sides. What is the cost of least expensive tank ?

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11. The sum of the perimeter of a circle and square is k, where k is some constant. Prove that the sum of their areas is least when the side of square is double the radius of the circle.

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12. A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light through the whole opening.

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13. A point on the hypotenuse of a triangle is at distance a and b from the side of the triangle. Show that the minimum length of the hypotenuse is $\left(a^{\frac{2}{3}} + b^{\frac{2}{3}}\right)^{\frac{3}{2}}$.

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14. Find the points at which the function f given by $f(x) = (x-2)^4(x+1)^3$ has local maxima,

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15. Find the points at which the function f given by $f(x) = (x-2)^4(x+1)^3$ has local minima,

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16. Find the points at which the function f given by $f(x) = (x-2)^4(x+1)^3$ has point of inflexion.

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17. Find the absolute maximum and minimum values of the function given by

$f(x) = \cos^2 x + \sin x, x \in [0, \pi]$

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18. Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$.

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19. Let f be a function defined on [a,b] such that $f'(x) > 0$, for all $x \in (a, b)$. Then prove that f is an increasing function on (a, b).

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20. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume.

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21. Show that height of the cylinder of greatest volume which can be inscribed in a right circular cone of height h and semi vertical angle α is one - third that of the cone and the greatest volume of cylinder is $\frac{4\pi}{27}h^3 \tan^2 \alpha$.

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22. A cylindrical tank of radius 10 m is being filled with wheat at the rate of 314 cubic metre per hour. Then the depth of the wheat is increasing at the rate of ....

A. 1 m/h

B. 0.1 m/h

C. 1.1 m/h

D. 0.5 m/h

Answer: A

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23. The slope of the tangent to the curve $x = t^2 + 3t - 8, y = 2t^2 - 2t - 5$ at the point $(2, -1)$ is ....

A. $\frac{22}{7}$

B. $\frac{6}{7}$

C. $\frac{7}{6}$

D. $\frac{-6}{7}$

Answer: B

24. The line $y = mx + 1$ is a tangent to the curve $y^2 = 4x$ if the value of m is _____

A. 1

B. 2

C. 3

D. $\frac{1}{2}$

Answer: A

25. The normal at the point (1, 1) on the curve $2y + x^2 = 3$ is ....

A. $x + y = 0$

B. $x - y = 0$

C. $x + y + 1 = 0$

D. $x - y = 1$

Answer: B

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26. The normal to $x^2 = 4y$ passing through (1, 2) has equation ....

A. $x + y = 3$

B. $x - y = 3$

C. $x + y = 1$

D. $x - y = 1$

Answer: A

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27. The points on the curve $9y^2 = x^3$, where the normal to the curve makes equal intercepts with the axes are ....

A. $\left(4, \pm\frac{8}{3}\right)$

B. $4, \frac{-8}{3}$

C. $\left(4, \pm\frac{3}{8}\right)$

D. $\left(\pm 4, \frac{3}{8}\right)$

Answer: A

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Practice Work

1. Water comes out from a conical funnel at the rate of $5cm^3 / s$. When the slant height of a water cone is 4 cm, find the rate of decrease of slant height of a water cone. The semi vertical angle of a conical funnel is $\frac{\pi}{3}$.

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2. A ladder 20 feet long is leaning against a wall. The bottom of the ladder is 16 feet away from the wall. If the upper end of the ladder slide along wall is λ times the lower end of the ladder pull along the ground. Find the value of λ.

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3. If the height of the cone is constant then find the rate of change of its curved surface area with respect to its radius.

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4. A man 2m tall walks at a uniform speed of 4m/min away from a lamp post 6m high. Find the rate at which the length of his shadow decreases.

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5. According to Boil's law PV = C, where V = 600 cm^3, P = 150 SI/cm^2 and $\frac{dP}{dt} = 20SI/cm^2$. Find $\frac{dV}{dt}$.

Watch Video Solution

6. The measure of two sides of a triangle is 10 m and 15 m. The angle between them is increasing at the rate of 0.01 radi./sec. when the angle between them is $\frac{\pi}{3}$, find the rate of change of increase in third side.

Watch Video Solution

7. A cylinder is heated in such a way that its radius always remains twice of its height when the radius is 3 cm, find the rate of increase of its volume. The radius is increased at the rate of 2 cm/s. Also find the rate of increase of its total surface area.

Watch Video Solution

8. A kite is flying at a height 151.5 m from horiozontal. The speed of the kite is 10 m/s. The distance of the kite from a boy who flies the kite is 250 m. Find the rate of change of thread of the kite.

Watch Video Solution

9. Two trains start from the same place. A person travel in a train whose speed is 50 km/h and it go to south direction. Another person travel in another train whose speed is 60 km/h and it go to west direction. After two hours, find the rate of distance between them.

Watch Video Solution

10. The area of a triangle is increasing at the rate of $4cm^2/s$. Its altitude is increasing at the rate of 2 cm/s. When the length of its altitude is 20 cm and its area is $30cm^2$, find the rate of change of its base.

Watch Video Solution

11. Show that $f(x) = x^3 - 6x^2 + 12x - 18$ is an increasing function on R.

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12. Find the intervals in which $f(x) = \frac{4x^2 + 1}{x}$ is increasing or decreasing.

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13. Find the least value of a such that the function given by $f(x) = x^2 + ax + 1$ is strictly increasing on (1, 2).

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14. Prove that $f(x) = x^2 - x\sin x$ is increasing on $\left(0, \frac{\pi}{2}\right)$.

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15. Find the value of a for which the function $f(x) = ax^3 - 3(a+2)x^2 + 9(a+2)x - 1$ is decreasing.

Watch Video Solution

16. Find the in intervals in which $f(x) = \sin^4 x + \cos^4 x$ is increasing or decreasing $x \in \left(0, \frac{\pi}{2}\right)$.

Watch Video Solution

17. $f : (0, \pi) \rightarrow R, f(x) = 2x + \cot x$. Find the intervals in which f(x) is strictly increasing or strictly decreasing.

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18. Show that $f(x) = x^3 - 3x^2 + 4x, x \in R$ is strictly increasing on R.

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19. Without using differentiation, prove that $f(x) = ax + b$, where $a > 0$ is strictly increasing function $\forall x \in R$.

Watch Video Solution

20. $f : R^{+} \to R^{+}, f(x) = \frac{\log x}{\sqrt{x}}$. Find the intervals in which f(x) is increasing or decreasing.

Watch Video Solution

21. Find the equation of the tangent to the curve $y^2 = 16x$ which is parallel to the line $4x - y = 1$.

Watch Video Solution

22. Find the equation of the normal to the curve y = 8x which is perpendicular to the line $2x - y - 1 = 0$.

Watch Video Solution

23. Prove that the curves $x^2 + y^2 = ax$ and $x^2 + y^2 = by$ are cuts orthogonally.

Watch Video Solution

24. Find the equation of the tangent and normal to the curve $y^2(a+x)=x^2(3a-x)$ at x = a.

Watch Video Solution

25. Show that the normal to the curve $5x^5-10x^3+x+2y+6=0$ at $P(0,\ -3)$ intersects the curve again in two points. Also find these points.

Watch Video Solution

26. Show that a point at which the line $\frac{x}{a} + \frac{y}{b} = 1$ touches to the curve $y = be^{-\frac{x}{a}}$ lies on Y-axis.

Watch Video Solution

27. A line $x\cos\alpha + y\sin\alpha = P$ is a tangent to the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Prove that $a^2\cos^2\alpha + b^2\sin^2\alpha = P^2$.

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28. Find the equations of the tangent to the curve $y = \cos(x + y)$ which is parallel to the line $x + 2y = 0$.

Watch Video Solution

29. Find the points on the curve $x^2y^2 + xy = 2$ at which the slope of the tangent is -1.

Watch Video Solution

30. The slope of the tangent to the curve $xy + ax + by = 2$ at a point (1, 1) then find the value of a and b.

Watch Video Solution

31. Prove that all points on the curve $y^2 = 4a\left[x + a\sin\left(\frac{x}{a}\right)\right]$ at which the tangent is parallel to the X - axis lie on the parabola $y^2 = 4ax$.

Watch Video Solution

32. Find the equation of the normal to the curve $y=(1+x)^{y}+\sin^{-1}(\sin^{2}x)$ at x = 0.

Watch Video Solution

33. If the curves $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{4}=1$ and $y^{3}=16x$ intersect at right angles, show that $a^{2}=\frac{4}{3}$.

Watch Video Solution

34. Find the equations of the tangents drawn to the curve $y^2 - 2x^3 - 4y + 8 = 0$ from the point (1, 2).

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35. Find the approximate value of each of the following :

$\sqrt{399}$

Watch Video Solution

36. Using differentials, find the approzimate value of each of the following up to 3 places of decimal :

$(0.999)^{\frac{1}{10}}$

Watch Video Solution

37. Using differentials, find the approximate value of each of the up to 3 places of decimal.

$(255)^{\frac{1}{4}}$

Watch Video Solution

38. Find the approximate value of each of the following :

$\cos 29^\circ$

Watch Video Solution

39. Find the approximate value of each of the following :

$\tan 31^\circ$

Watch Video Solution

40. Find the approximate value of each of the following :

$\log_e(100.1)$

Watch Video Solution

41. Find the approximate value of each of the following :

$(15.5)^{\frac{1}{4}}$

Watch Video Solution

42. Using differentials, find the approximate value of each of the up to 3 places of decimal.

$(0.009)^{\frac{1}{3}}$

Watch Video Solution

43. Find the approximate value of each of the following :

$(65)^{\frac{1}{3}}$

Watch Video Solution

44. Find the approximate value of each of the following :

$\log_e(10.02)$

Watch Video Solution

45. If $f(x) = 2x^2 - 3x + 5$ then find the approximate value of $f(1.05)$.

Watch Video Solution

46. The radius of a cone is twice of its height and the radius is 10 cm. If there is an error of 0.01 cm in measure of radius, then find the approximate error in calculating its volume.

Watch Video Solution

47. Find the approximate change in the surface area of a cube of side x m caused by decreasing the side by 1% ?

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48. The angle of elevation of a tower from a point 200 m away from the tower is 30°. The height of the tower is 200 m. But actually, the angle of elevation is 30° 12'. Find the approximate error in the measure of the height of the tower.

View Text Solution

49. Area of the triangle is calculated by the formula $A = \frac{1}{2} ab \sin C$. If $C = \frac{\pi}{6}$ and error in the measure of C is x%, find the approximate change in the area of the triangle. Where a and b are constant.

Watch Video Solution

50. Find the maximum and minimum values in the following functions :

$x^3 - 9x^2 + 15x - 1,$

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51. Find the maximum and minimum values in the following functions :

$x^5 - 5x^4 + 5x^3 - 10$

Watch Video Solution

52. Find the maximum and minimum values in the following functions :

$(x-8)^4(x-9)^5$

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53. Prove that for $\theta = \tan^{-1}\sqrt{\frac{p}{q}}$, $\sin^p \theta \cos^q \theta$ is maximum.

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54. For which value of x, function $f(x) = \frac{40}{3x^4 + 8x^3 - 18x^2 + 60}$ has maximum and minimum ?

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55. A function $y = \frac{ax+b}{(x-1)(x-4)}$ has maximum or minimum at $P(2, -1)$ then find a and b. Also show that y has maximum value at P.

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56. Critical points of $y = a\log x + bx^2 + x$ are $x = -1$ and x = 2. Then find a and b.

Watch Video Solution

57. Find maximum and minimum value of the following functions in the given interval.

$f(x) = 2x^3 - 3x^2 - 12x + 1, x \in \left[-2, \frac{5}{2}\right]$

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58. Find maximum and minimum value of the following functions in the given interval.

$f(x) = x^2 \log x, x \in [0, 2]$

Watch Video Solution

59. Find maximum and minimum value of the following functions in the given interval.

$f(x) = xe^{-x}, x \in [0, \infty]$

Watch Video Solution

60. A function $y = f(x)$ is given by

$x = \phi(t) = t^5 - 5t^3 - 20t + 7$

$y = \Psi(t) = 4t^3 - 3t^2 - 18t + 3, \ -2 < t < 2$

Find the maximum and minimum value of the function.

Watch Video Solution

61. For which value of a, A function,

$$f(x) = x^3 + 3(a-7)x^2 + 3\left(a^2 - 9\right)x - 1$$

attains its maximum value point.

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62. Prove that the minimum value of

$$\frac{(a+x)(b+x)}{(c+x)}, x > -c$$ is

$$\left[\sqrt{a-c} + \sqrt{b-c}\right]^2.$$

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63. At what points, $f(x) = 1 + 2\sin x + 3\cos^2 x, 0 < x < \frac{2\pi}{3}$ has maximum or minimum ? Find the maximum and minimum value of the function.

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64. Divide 20 into two parts such that the product of one part and the cube of the other part is maximum.

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65. Find two positive numbers x and y, such that $x + y = 64$ and $x^3 + y^3$ is minimum.

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66. What is the maximum slope of the curve $y = -x^3 + 3x^2 + 2x - 27$? Also find the point.

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67. Find the co-ordinates of a point on the parabola $y = x^2 + 7x + 2$ which is closed to the straight line $y = 3x - 3$.

Watch Video Solution

68. The total area of a page is 150 square inchese. The combined width of the margin at the top and bottom is 3" and the side is 2". What must be the dimension of the page in order that the area of the printed matter may be maximum.

Watch Video Solution

69. The three sides of a trapezium are equal, each being 6 cm long. Find the area of trapezium when it is maximum.

Watch Video Solution

70. In a right angle triangle, the sum of hypotenuous and one side is given, Prove that when the angle between them is $\frac{\pi}{3}$, the area of the triangle is maximum.

Watch Video Solution

71. Show that a triangle of maximum area that can be inscribed in a circle of radius a is an equilateral triangle.

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72. The circle $x^2 + y^2 = 1$ cats the X - axis at P and Q. Another circle with centre at Q and veriable radius intersects the first circle at R above the X - axis and the line segment PQ at S. Find the maximum area of the triangle QSR.

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73. Two roads OA and OB intersect at an angle 60°. A car driver approaches O from A, where OA = 800 metres, at a uniform speed of 20 m/sec. Simultaneasly, O runner starts running from O towards B at uniform speed of 5 m/sec. Find the time when the car and the runner are closest.

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74. Find the interval in which the function $f(x) = x^4 + 32x$ is increasing or decreasing. $x \in R$

Watch Video Solution

75. Show that the curves $xy = a^2$ and $x^2 + y^2 = 2a^2$ touch each other.

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76. Prove that the slope of tangent at any point of the curve $y = 6x\,?^3 + 15x + 10$ can not 12. $x \in R$.

Watch Video Solution

77. Prove that $f(x) = \left(\frac{1}{x}\right)^x$ has local maximum at $x = \frac{1}{e}$. $x \in R^+$

Watch Video Solution

78. For simple pendulum, $T = 2\pi\sqrt{\frac{l}{g}}$ where T is the periodic time and l is the length of pendulum. If there is 4% error in the measure of periodic time then find the error percentage in the length of pendulum.

Watch Video Solution

79. Find the equation of normal to the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ at $\left(\frac{a}{2\sqrt{2}}, \frac{a}{2\sqrt{2}}\right)$.

Watch Video Solution

80. Prove that the function $f(x) = 2|x-1| + 3|x-4|$ is decreasing in the intrerval (2, 4).

Watch Video Solution

81. The radius of the cone is increasing at the rate of 4 cm/se. Its height is decreasing at the rate of 3 cm/se. When its radius is 3 cm and height is 4 cm. Find the rate of change of its curved surface area.

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82. Prove that in all rectangles with given area, the perimeter of square is minimum.

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83. The equation of a particle in motion is $S = \frac{1}{4}t^4 - 2t^3 + 4t^2 - 7.$ When its acceleration become minimum ?

Watch Video Solution

84. At which point the curve $y = -x^3 + 3x^2 + 2x - 27$ has maximum slope ? How much ?

Watch Video Solution

85. Water comes out from a conical funnel at the rate of $5cm^3 / s$. When the slant height of a water cone is 4 cm, find the rate of decrease of slant height of a water cone. The semi vertical angle of a conical funnel is $\frac{\pi}{3}$.

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Textbook Based Mcqs

1. If x and y are the sides of two squares such that $y = x - x^2$. Then the rate of change, if the area of second square with respect to the first square, when x = 2, is ....

A. 1

B. 4

C. 3

D. 6

Answer: C

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2. A point is moving along the curve $y^3 = 27x$. The interval in which the abscissa changes at slower rate then the ordinate is ....

A. $(-3, 3)$

B. $(-\infty, \infty)$

C. $(-1, 1)$

D. $(-\infty, -3) \cup (3, \infty)$

Answer: C

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3. A particle moves along the curve $y = x^{\frac{3}{2}}$ in the first quadrant in such a way that its distance from the origin increases at the rate of 11 units/sec. The value of $\frac{dx}{dt}$ when x = 3 is ....

A. 4

B. $\frac{9}{2}$

C. $\frac{3\sqrt{3}}{2}$

D. None of these

Answer: A

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4. A balloon is filled in such a way that its volume increases at the rate of $40cm^3$/min. when radius is 8 cm, the rate of increases in the surface area is .... cm^2/min.

A. 8

B. 10

C. 20

D. None of these

Answer: B

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5. Match the following :

[A]	[B]
(1) Circular plate is expanded by heat from radius 5 cm. to 5.06 cm, then approximate increases in area is	(A) 4
(2) If the rate of decrease of $\frac{x^2}{2} - 2x + 5$ is twice the rate of decrease of x, then x is equal to	(B) 3
(3) If an edge of cube is increased by 1%, then percentage increase in volume is	(C) $\frac{3\sqrt{3}}{4}$
(4) The rate of increase in area of an equilateral triangle of side 15 cm. when each side is increasing at the rate 0.1 cm/s. is	(D) 0.6π

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6. The equation of normal to the curve $x+y=x^{y}$ where it cuts X -axis is ....

A. $y=x+1$

B. $y=-x+1$

C. $y=x-1$

D. $y=-x-1$

Answer: C

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7. The intercept on X - axis made by tangent to the curve $y = \int_0^x |t| dt, x \in R$ which are parallel to the line y = 2x are equal to ....

A. ± 1

B. ± 2

C. ± 3

D. ± 4

Answer: A

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8. The point on the curve $y^3 + 3x^2 = 12y$ where the tangent is vertical is ....

A. $\left(\pm\frac{4}{\sqrt{3}}, -2\right)$

B. $\left(\pm\frac{\sqrt{11}}{3}, 1\right)$

C. (0,0)

D. $\left(\pm\frac{4}{\sqrt{3}}, 2\right)$

Answer: D

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9. The function $f(x) = x^2 f(1) - x f'(2) + f''(3)$ such that $f(0) = 2$. The equation of tangent to $y = f(x)$ at x = 3 is ....

A. $y = x - 7$

B. $y = \frac{x}{4} - 7$

C. $y = 4x - 7$

D. None of these

Answer: C

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10. For .... value of a, function $f(x) = \sin x - \cos x - ax + b \forall x \in R$ is a decreasing function.

A. $a \geq -\sqrt{2}$

B. $a \leq -\sqrt{2}$

C. $a \leq \sqrt{2}$

D. $a \geq \sqrt{2}$

Answer: D

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11. Out of the following which statement is true for $x \in (0, 1)$?

A. $e^x < 1 + x$

B. $\log_e(1 + x) < x$

C. $\sin x > x$

D. $\log_e x > x - 1$

Answer: B

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12. $f(x) = \int \left(2 - \frac{1}{1+x^2} - \frac{1}{\sqrt{1+x^2}}\right) dx$

then f is ....

A. Increasing in $(0, \infty)$ and decreasing $(-\infty, 0)$

B. Decreasing in $(0, \infty)$ and increasing in $(-\infty, 0)$

C. Increasing in $(-\infty, \infty)$

D. Decreasing $(-\infty, \infty)$

Answer: C

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13. If

$f(x) = x^3 + ax^2 + bx + 5\sin^2 x, \forall x \in R$ is

an increasing function, then ....

A. $a^2 - 3b - 15 > 0$

B. $a^2 - 3b + 15 < 0$

C. $a^2 - 3b - 15 < 0$

D. $a^2 + 3b + 15 < 0$

Answer: B

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14. If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$, where $a > 0$, attains its maximum and minimum at p and q respectivelt such that $p^2 = q$ then a is ....

A. $\frac{1}{2}$

B. 3

C. 1

D. 2

Answer: D

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15. A rectangle has one side on the positive side of Y - axis and one side on the positive side of X - axis. The upper right hand vertiex is on the curve $y = \frac{\log x}{x^2}$. The maximum area of the rectangle is .... sq. unit.

A. $\frac{1}{e}$

B. $\frac{1}{2\sqrt{e}}$

C. 1

D. $\sqrt{2}$

Answer: B

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16. The function $f(x) = 2|x| + |x+2| - ||x+2| - 2|x||$ has a local maximum at .... and local minimum ....

A. $-\frac{2}{3}, \ -2$

B. $-\frac{2}{3}, 2$

C. $\frac{2}{3}, \ -2$

D. $\frac{2}{3}, 2$

Answer: A

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17. Equation of line passing through a point (1, 4) and the sum of the intercept on the positive axes is minimum is ....

A. $2x + y - 6 = 0$

B. $x + 2y - 9 = 0$

C. $y + 2x + 6 = 0$

D. None of these

Answer: A

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18.

$p(x) = a_0 + a_1x^2 + a_2x^4 + \ldots\ldots\ldots\ldots + a_nx^{2n}$

is a polynomial with real variable x. If

$0 < a_0 < a_1 < a_2 < \ldots\ldots\ldots\ldots < a_n$ then

p(x) has ....

A. Neither maximum nor minimum

B. Only one minimum

C. Only one minimum

D. None of these

Answer: C

Watch Video Solution

19. A tangent drawn to the curve $x = e^t \cos t, y = e^t \sin t$ at $t = \frac{\pi}{4}$ makes an angle .... with positive side of X - axis.

A. 0

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

Answer: D

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20. The approximate value of $\sin^{-1}(0.51)$ is

....

A. $\frac{\pi}{6}-\frac{1}{50\sqrt{3}}$

B. $\frac{\pi}{3}+\frac{1}{50\sqrt{3}}$

C. $\frac{\pi}{3}-\frac{1}{50\sqrt{3}}$

D. $\frac{\pi}{6}+\frac{1}{50\sqrt{3}}$

Answer: D

Watch Video Solution

21. A stone is thrown in upward direction. Its equation is $S = 80t - 16t^2$. The time to attain maximum height is .... second.

A. 2.5

B. 2

C. 3.5

D. 4

Answer: A

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22. If there is an error 4% in the area of circle, then the error in radius is ....

A. 1%

B. 2%

C. 3%

D. 4%

Answer: B

Watch Video Solution

23. An iron ball having 10 cm radius is covered equally with Ice. Ice is melted at the rate of 50 cm^3/min. when the thickness of ice is 5 cm, rate of decrease in radius is .... cm/min.

A. $\frac{1}{36\pi}$

B. $\frac{1}{18\pi}$

C. $\frac{1}{54\pi}$

D. $\frac{5}{6\pi}$

Answer: B

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Watch Video Solution

24. If x is a real number then maximum value of $\frac{3x^2+9x+17}{3x^2+9x+7}$ is ....

A. $\frac{1}{4}$

B. 41

C. 1

D. $\frac{17}{7}$

Answer: B

Watch Video Solution

25. The diagonal of a suare is increasing at the rate of 0.5 cm/sec. when the area of a square is $400cm^2$, the area is increasing at the rate of

....

A. $10cm^2 / s$

B. $10\sqrt{2}cm^2 / s$

C. $20cm^2 / s$

D. $5\sqrt{2}cm^2 / s$

Answer: B

Watch Video Solution

26. If the curves $ax^2 + by^2 = 1$ and $a'x^2 + b'y^2 = 1$ are orthogonally then ....

A. $\frac{1}{a} - \frac{1}{b} = \frac{1}{a'} - \frac{1}{b'}$

B. $\frac{1}{a} + \frac{1}{b} = \frac{1}{a'} + \frac{1}{b'}$

C. $\frac{1}{b} - \frac{1}{b} = \frac{1}{a'} + \frac{1}{b'}$

D. $\frac{1}{a} + \frac{1}{b} = \frac{1}{a'} - \frac{1}{b'}$

Answer: B

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27. The tangent to the cureve $xy + ax + by = 0$ at (1, 1) makes an angle with X - axis is $\tan^{-1} 2$ then ....

A. a = 1, b = 2

B. a = 1, b = -2

C. a = -1, b = 2

D. $a = -1, b = -2$

Answer: B

Watch Video Solution

28. The maximum value of $27^{\cos(2x)} \times 81^{\sin(2x)}$ is ....

A. -5

B. $\frac{1}{5}$

C. $\frac{1}{25}$

D. 243

Answer: D

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29. A balloon, which always remains spherical, has a variable diameter $\frac{3}{2}(2x+1)$. Find he rate of change of its volume with respect to x.

A. $\frac{\pi}{8}(2x+1)^2$

B. $\frac{27\pi}{8}(2x+1)^2$

C. $\frac{27\pi}{8}(2x+1)$

D. None of these

Answer: B

Watch Video Solution

30. The function f (x) is differentiable for every x. $f(1) = -2$ and $f'(x) \geq 2, \forall x \in [1, 6]$. Then minimum value of f (6) is ....

A. 2

B. 4

C. 6

D. 8

Answer: D

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31. The side of an equilateral triangle expands at the rate of $\sqrt{3}$ cm/sec. When the side is 12 cm, the rate of increase of its area is ....

A. $12cm^2 / \sec$

B. $18cm^2 / \sec$

C. $3\sqrt{3}cm^2 / \sec$

D. $10cm^2 / \sec$

Answer: B

Watch Video Solution

32. The distance s moved by a particle in time t is given by $s = t^3 - 6t^2 + 6t + 8$. When the acceleration is zero, the velocity is ....

A. 5 cm/sec

B. 2 cm/sec

C. 6 cm/sec

D. -6 cm/sec

Answer: D

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33. The volume of a sphere is increasing at the rate of $\pi cm^2 / \sec$. The rate at which the radius is increasing is ...., when the radius is 3 cm.

A. $\frac{1}{36}$ cm/sec

B. 36 cm/sec

C. 9 cm/sec

D. 27 cm/sec

Answer: A

Watch Video Solution

34. There is 4% error in measuring the period of a simple pendulum. The approximate percentage error in length is ....

Hint : $T = 2\pi\sqrt{\frac{l}{g}}$

A. 4%

B. 8%

C. 2%

D. 6%

Answer: B

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35. The height and radius of a cylinder are equal. An error of 2% is made in measuring height. The approximate percentage error in volume is ....

A. 6%

B. 4%

C. 3%

D. 2%

Answer: A

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36. The tangent to $(at^2, 2at)$ is perpendicular to X - axis at

A. (4a, 4a)

B. (a, 2a)

C. (0, 0)

D. (a, -2a)

Answer: C

Watch Video Solution

37. The line $y = mx + 1$ touches $y^2 = 4x$, if m = ...

A. 0

B. 1

C. -1

D. 2

Answer: B

Watch Video Solution

38. Find the equation of normal to the curve $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$ at $\left(\frac{a}{2\sqrt{2}}, \frac{a}{2\sqrt{2}}\right)$.

A. $2x+y=0$

B. y = 1

C. x = 0

D. x = y

Answer: D

Watch Video Solution

39. $f(x) = x^x$ decreases in ....

A. (0, e)

B. $\left(0, \frac{1}{e}\right)$

C. (0, 1)

D. $(0, \infty)$

Answer: B

Watch Video Solution

40. $f(x) = 2|x-2| + 3|x-4|$ is .... in (2, 4).

A. decreasing

B. increasing

C. constant

D. cannot be decided

Answer: A

Watch Video Solution

41. The local maximum value of $f(x) = x + \frac{1}{x}$ is ....

A. 2

B. -2

C. 4

D. -4

Answer: B

Watch Video Solution

42. The local minimum value of $\frac{x}{\log x}$ is ....

A. -1

B. 0

C. $\frac{1}{e}$

D. e

Answer: D

Watch Video Solution

43. If $\log_e 4 = 1.3868$, then approximate value of $\log_e 4.01 =$ ....

A. 1.3867

B. 1.3869

C. 1.3879

D. 1.3893

Answer: D

Watch Video Solution

44. The circumference of a circle is 20 cm and there is an error of 0.02 cm in its measurement. The approximate percentage error in area is ....

A. 0.02

B. 0.2

C. π

D. $\frac{1}{\pi}$

Answer: B

Watch Video Solution

45. If the line y = x touches the curve $y = x^2 + bx + c$ at (1, 1), then ....

A. b = 1, c = 2

B. $b = -1, c = 1$

C. b = 1, c = 1

D. b = 0, c = 1

Answer: B

Watch Video Solution

46. $y = ae^{x}, y = be^{-x}$ intersect at right angles if .... $(a \neq 0, b \neq 0)$

A. $a = \frac{1}{b}$

B. a = b

C. $a = -\frac{1}{b}$

D. $a + b = 0$

Answer: A

Watch Video Solution

47. Tangent to $y = 5x^5 + 10x + 15$ ....

A. is always vertical

B. is always horizontal

C. makes acute angle with the positive X - axis

D. makes obtuse angle with the positive X -

axis

Answer: C

Watch Video Solution

48.

$f(x) = 2x + \cot^{-1} x - \log\left|x + \sqrt{1 + x^2}\right|$ is

....

A. decreasing on $(-\infty, 0)$

B. decreasing on $(0, \infty)$

C. constant

D. increasing on R

Answer: D

Watch Video Solution

49. The sum of two non - zero numbers in 12. The minimum sum of their reciprocals is ....

A. $\frac{1}{10}$

B. $\frac{1}{4}$

C. $\frac{1}{2}$

D. $\frac{1}{3}$

Answer: D

Watch Video Solution

50. The local minimum value of $f(x) = x^2 + 4x + 5$ is ....

A. 2

B. 4

C. 1

D. -1

Answer: C

Watch Video Solution

51. The maximum value of $f(x) = 5\cos x + 12\sin x$ is ....

A. 13

B. 12

C. 5

D. 17

Answer: A

Watch Video Solution

52. The minimum value of $f(x) = 3\cos x + 4\sin x$ is ....

A. 7

B. 5

C. -5

D. 4

Answer: C

Watch Video Solution

53. $f(x) = x \log x$ has minimum value ....

A. 1

B. 0

C. e

D. $\frac{1}{e}$

Answer: D

Watch Video Solution

54. $f(x) = \sqrt{3}\cos x + \sin x, x \in \left[0, \frac{\pi}{2}\right]$ is maximum for x = ...

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{2}$

D. 0

Answer: A

Watch Video Solution

55. $f(x) = (x-a)^2 + (x-b)^2 + (x-c)^2$

has minimum value at x = ...

A. $\sqrt[3]{abc}$

B. $a+b+c$

C. $\dfrac{a+b+c}{3}$

D. 0

Answer: C

Watch Video Solution

56. $f(x) = (x+2)e^{-x}$ is increasing in ....

A. $(-\infty, -1)$

B. $(-1, \infty)$

C. $(2, \infty)$

D. R^{+}

Answer: A

Watch Video Solution

57. The measure of the angle of intersection between $y^2 = x$ and $x^2 = y$ other than one at (0, 0) is ....

A. $\tan^{-1}\frac{4}{3}$

B. $\tan^{-1}\frac{3}{4}$

C. $\frac{\pi}{4}$

D. $\frac{\pi}{2}$

Answer: B

Watch Video Solution

58. The point where normal to $y = x^2 - 2x + 3$ is parallel to Y-axis is ....

A. (0, 3)

B. $(-1, 2)$

C. (1,2)

D. (3,6)

Answer: C

Watch Video Solution

59. The slope of normal to $(3t^2+1, t^3-1)$ at t = 1 is ....

A. $\frac{1}{2}$

B. -2

C. 2

D. $-\frac{1}{2}$

Answer: B

Watch Video Solution

60. The equation of normal to $3x^2 - y^2 = 8$ at $(2, -2)$ is ....

A. $x + 2y = -2$

B. $x - 3y = 8$

C. $3x + y = 4$

D. $x + y = 0$

Answer: B

Watch Video Solution

61. The angle made by the tangent with the + ve direction of X-axis to $x = e^t \cos t, y = e^t \sin t$ at $t = \frac{\pi}{4}$ is ....

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. 0

D. $\frac{\pi}{3}$

Answer: B

Watch Video Solution

62. The equation of tangent to $y = \cos x$ at (0, 1) is ....

A. x = 0

B. y = 0

C. x = 1

D. y = 1

Answer: D

Watch Video Solution

63. The equation of normal to y = sin x at $\left(\frac{\pi}{2}, 1\right)$ is ....

A. x = 1

B. x = 0

C. $y = \frac{\pi}{2}$

D. $x = \frac{\pi}{2}$

Answer: D

Watch Video Solution

64. Find the points on the curve $x^2 + y^2 - 2x - 3 = 0$ at which the tangents are parallel to the X-axis.

A. $\left(0, \pm\sqrt{3}\right)$

B. $\left(2, \pm\sqrt{3}\right)$

C. $(1, 2), (1, -2)$

D. (3, 0)

Answer: C

Watch Video Solution

65. The point on $y^2 = x$ where tangent makes angle of measure $\frac{\pi}{4}$ with the positive X-axis is ....

A. $\left(\frac{1}{4}, \frac{1}{2}\right)$

B. (2, 1)

C. (0, 0)

D. $(-1, 1)$

Answer: A

Watch Video Solution

66. A cone with its height equal to the diameter of the base is expanding in volume at the rate of $50 cm^3 / \sec$. If the base has area $1 m^2$, the radius is increasing at the rate ....

A. 0.0025 cm/sec

B. 0.25 cm/sec

C. 1 cm/sec

D. 4 cm/sec

Answer: A

Watch Video Solution

67. The rate of increase of $f(x) = x^3 - 5x^2 + 5x + 25$ is twice the rate of increase of x for x = ...

A. $-3, \ -\frac{1}{3}$

B. $3, \frac{1}{3}$

C. $-3, \frac{1}{3}$

D. $3, \ -\frac{1}{3}$

Answer: B

Watch Video Solution

68. The radius of the cone is increasing at the rate of 4 cm/se. Its height is decreasing at the rate of 3 cm/se. When its radius is 3 cm and height is 4 cm. Find the rate of change of its curved surface area.

A. $30\pi cm^2 / \sec$

B. $10cm^2 / \sec$

C. $20\pi cm^2 / \sec$

D. $22\pi cm^2 / \sec$

Answer: C

Watch Video Solution

69. The rate of change of surface area of a sphere w.r.t. radius is ....

A. 8π (diameter)

B. 3π (diameter)

C. 4π (radius)

D. 8π (radius)

Answer: D

Watch Video Solution

70. The rate of change of volume of a cylinder w.r.t. radius whose radius is equal to its height is ....

A. 4 (area of base)

B. 3 (area of base)

C. 2 (area of base)

D. (area of base)

Answer: B

Watch Video Solution

71. $f(x) = \tan^{-1} x - x$, is ....

A. increasing on R

B. decreasing on R

C. increasing on R^{+}

D. increasing on $(- \infty, 0)$

Answer: B

Watch Video Solution

72.

$f(x) = 2x - \tan^{-1} x - \log\left|x + \sqrt{1 + x^2}\right|$ is .... $(x \in R)$

A. increasing on R

B. decreasing on R

C. has a minimum at x = 1

D. has a maximum at x = 1

Answer: A

Watch Video Solution

73. If ...., then $f(x) = x^2 - kx + 20, [0, 3]$

is strictly increasing.

A. $k < 0$

B. $0 < k < 1$

C. $1 < k < 2$

D. $2 < k < 3$

Answer: A

Watch Video Solution

74. $f(x) = |x-1| + |x-2|$ is increasing if

....

A. $x > 2$

B. $x < 1$

C. $x < 0$

D. $x < -2$

Answer: A

Watch Video Solution

75. The points on the curve $9y^2 = x^3$, where the normal to the curve makes equal intercepts with the axes are ....

A. $\left(-4, -\frac{8}{3}\right)$

B. $\left(4, \pm\frac{8}{3}\right)$

C. $\left(\pm 4, \frac{8}{3}\right)$

D. $\left(8, \frac{8}{3}\right)$

Answer: B

Watch Video Solution

76. $y = mx + 4$ touches $y^2 = 8x$, if m = ...

A. $\frac{1}{2}$

B. $-\frac{1}{2}$

C. 2

D. -2

Answer: A

Watch Video Solution

77. The measure of the angle between the curves $y = 2\sin^2 x$ and $y = \cos 2x$ at $x = \frac{\pi}{6}$ is ....

A. $\frac{\pi}{2}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{4}$

D. $\frac{\pi}{6}$

Answer: B

Watch Video Solution

78. The normal to $x^2 = 4y$ passing through (1, 2) has equation ....

A. 2x = y

B. $x + y - 3 = 0$

C. $2x + 3y - 8 = 0$

D. $x - y + 1 = 0$

Answer: B

Watch Video Solution

79. The local minimum value of $x^2 + \frac{16}{x} (x \neq 0)$ is ....

A. 12

B. 22

C. -12

D. 2

Answer: A

Watch Video Solution

80. The minimum value of $\sec x, x \in \left[\frac{2\pi}{3}, \pi\right]$

is ....

A. 1

B. -2

C. 2

D. π

Answer: B

Watch Video Solution

81. The maximum value of cosec x, $x \in \left[\frac{\pi}{6}, \frac{\pi}{3}\right]$ is ....

A. 2

B. $\frac{2}{\sqrt{3}}$

C. $\frac{\pi}{6}$

D. $\frac{\pi}{3}$

Answer: A

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82. $(31)^{\frac{1}{5}}$ has approximate value ....

A. 2.1

B. 2.01

C. 2.0125

D. 1.9875

Answer: D

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83. $f(x) = x^2 + 4x + 5$ has minimum value .... $(x \in R)$

A. 4

B. 2

C. 1

D. -1

Answer: C

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Textbook Illustrations For Practice Work

1. Find the rate of change of the area of a circle per second with respect to its radius r when r = 5 cm.

Watch Video Solution

2. The volume of a cube is increasing at a rate of 9 cubic centimetres per second. How fast is the surface area increasing when the length of an edge is 10 centimeters ?

View Text Solution

3. A stone is dropped into a quiet lake and waves move in circles at the speed of 5 cm/s. At the instant when the radius of the circular wave is 8 cm, how fast is the enclosed area increasing ?

Watch Video Solution

4. The length x of a rectangle is decreasing at the rate of 5 cm/minute and the width y is increasing at the rate of 4 cm/minute. When x = 8 cm and y = 6 cm, find the rates of change of (a) the [erimeter, and (b) the area of the rectangle.

Watch Video Solution

5. The total cost C(x) in Rupees associated with the production of x units of an item is given by

$C(x) = 0.005x^3 - 0.02x^2 + 30x + 5000$. Find the marginal cost when 3 units are produced, where by marginal cost we mean the instantaneous rate of change of total cost at ant level of output.

Watch Video Solution

6. The total revenue in Rupees received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$. The marginal revenue, when x = 15 is ....

Watch Video Solution

7. Show that the function given by $f(x) = 7x - 3$ s increasing on R.

Watch Video Solution

8. Show that the function f given by $f(x) = x^3 - 3x^2 + 4x, x \in R$ is increasing on R.

Watch Video Solution

9. Prove that the function given by f (x)=cos x

(a) Decreasing in $(0, \pi)$

(b) Increasing in $(\pi, 2\pi)$ and

(c) Neither increasing nor decreasing in $(0, 2\pi)$

Watch Video Solution

10. Find the intervals in which the function f given by $f(x) = x^2 - 4x + 6$ is

(a) Increasing

(b) Decreasing

Watch Video Solution

11. Find the intervals in which the function f given by $f(x) = 4x^3 - 6x^2 - 72x + 30$ is

(a) Increasing

(b) Decreasing.

Watch Video Solution

12. Find intervals in which the function given by $f(x) = \sin 3x, \, x \in \left[0, \frac{\pi}{2}\right]$ is

(a) Increasing

(b) Decreasing.

Watch Video Solution

13. Find the intervals in which the function f given by $f(x) = \sin x + \cos x, \, 0 \leq x \leq 2\pi$ is Increasing or Decreasing.

Watch Video Solution

14. Find the slope of the tangent to the curve $y = x^3 - x$ at x = 2.

Watch Video Solution

15. Find the point at which the tangent to the curve $y = \sqrt{4x - 3} - 1$ has its slope $\frac{2}{3}$.

Watch Video Solution

16. Find the equation of all lines having slope 2 and being tangent to the curve $y + \frac{2}{x-3} = 0$.

Watch Video Solution

17. Find points on the curve $\frac{x^2}{4} + \frac{y^2}{25} = 1$ at which the tangents are

(i) Parallel to X - axis

(ii) Parallel to Y - axis.

Watch Video Solution

18. Find the equation of the tangent to the curve $y = \frac{x-7}{(x-2)(x-3)}$ at the point where it cuts the X-axis.

Watch Video Solution

19. Find the equations of the tangent and normal to the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 2$ at (1, 1).

Watch Video Solution

20. Find the equation of tangent to the curve given by $x = a\sin^3 t, y = b\cos^3 t$ at a point where $t = \frac{\pi}{2}$.

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21. Use differential to approximate $\sqrt{36.6}$.

Watch Video Solution

22. Use differential to approximate $(25)^{\frac{1}{3}}$.

Watch Video Solution

23. Find the approximate value of $f(3.02)$, where $f(x) = 3x^2 + 5x + 3$.

Watch Video Solution

24. Find the approximate change in the volume V of a cube of side x meters caused by increasing the side by 2%.

Watch Video Solution

25. If the radius of a sphere is measured as 9m with an error of 0.03m, then find the approximte error in calculating its surface area.

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26. Find the maximum and the minimum values, if any, of the function f given by $f(x) = x^2, x \in R$.

Watch Video Solution

27. Find the maximum and minimum values of f, if any, of the function given by $f(x) = |x|, x \in R.$

Watch Video Solution

28. Find the maximum and the minimum values, if any, of the function given by $f(x) = x, x \in (0, 1).$

Watch Video Solution

29. Find all points of local maxima and local minima of the function f given by $f(x) = x^3 - 3x + 3.$

Watch Video Solution

30. Find all the points of local maxima and local minima of the function f given by $f(x) = 2x^3 - 6x^2 + 6x + 5.$

Watch Video Solution

31. Find local minimum value of the function f given by $f(x) = 3 + |x|, x \in R$.

Watch Video Solution

32. Find local maximum and local minimum values of the function f given by $f(x) = 3x^4 + 4x^3 - 12x^2 + 12$.

Watch Video Solution

33. Find all the points of local maxima and local minima of the function f given by $f(x) = 2x^3 - 6x^2 + 6x + 5.$

Watch Video Solution

34. Find two positive numbers whose sum is 15 and the sum of whose squares is minimum.

Watch Video Solution

35. Find the shortest distance of the point (0, c) from the parabola $y = x^2$, where $\frac{1}{2} \leq c \leq 5$.

Watch Video Solution

36. Let AP and BQ be two vertical poles at points A and B respectively. If AP = 16 m, BQ = 22m and AB = 20 m, then find the distance of a point R on AB from the point A such that $RP^2 + RQ^2$ is minimum.

Watch Video Solution

37. If length of three sides of a trapezium other than base are equal to 10 cm, then find the area of the trapezium when it is maximum.

Watch Video Solution

38. Prove that the radius of the right circular cylinder of greatest curved area which can be inscribed in a given cone is half of that of the cone.

Watch Video Solution

39. Find the global maximum and minimum values of the function f given by $f(x) = 2x^3 - 15x^2 + 36x + 1, x \in [1, 5]$.

Watch Video Solution

40. Find absolute maximum and minimum values of a function f given by $f(x) = 12x^{\frac{4}{3}} - 6x^{\frac{1}{3}}, x \in [-1, 1]$

Watch Video Solution

41. An Apache helicopter of enemy is flying along the curve given by $y = x^2 + 7$. A solider, placed at (3, 7), wants to shoot down the helicopter when it is nearest to him. Find the nearest distance.

Watch Video Solution

42. A car starts from a point P at time t = 0 seconds and stops at point Q. The distance x, in metres, covered by it, in t seconds is given by

$$x = t^2\left(2 - \frac{t}{3}\right)$$

Find the time taken by it to reach Q and also find distance between P and Q.

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43. A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lowermost. Its semi-vertical angle is $\tan^{-1}(0.5)$. Water is poured into it at a constant rate of 5 cubic meter per hour. Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is 4 m.

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44. A man of height 2 metres walks at a uniform speed of 5 km/h away from a lamp post which is 6 metres high. Find the rate at which the length of his shadow increases.

Watch Video Solution

45. Find the equation of the normal to the curve $x^2 = 4y$ which passes through the point (1, 2).

Watch Video Solution

46. Find the equations of the tangent to the curve $y = \cos(x + y)$ which is parallel to the line $x + 2y = 0$.

Watch Video Solution

47. Find intervals in which the function given by $f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$ is (a) Increasing (b) Decreasing.

Watch Video Solution

48. Show that the function f given by $f(x) = \tan^{-1}(\sin x + \cos x), x > 0$ is always an increasing function in $\left(0, \frac{\pi}{4}\right)$.

Watch Video Solution

49. A circular disc of radius 3 cm is being heated. Due to expansion, its radius increases at the rate of 0.05 cm/s. Find the rate at which its area is increasing when radius is 3.2 cm.

Watch Video Solution

50. An open topped box is to be constructed by removing equal squares from each corner of a 3 metre by 8 metre rectangular sheet of aluminimum and folding up the sides. Find the volume of the largest such box.

Watch Video Solution

51. Manufacturer can sell x items at a price of rupees $\left(5-\frac{x}{100}\right)$ each. The cost price of x items is Rs. $\left(\frac{x}{5}+500\right)$. Find the number of items he should sell to earn maximum prodit.

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Solutions Of Ncert Exemplar Problems Short Answer Type Questions

1. A spherical ball of salt is dissolving in water in such a manner that the rate of decrease of the volume at any instant is propotional to the surface. Prove that the radius is decreasing at a constant rate.

Hint for solution : Take volume V and curve surface are S of spherical ball. Calculate by taking $\frac{dV}{dt} \alpha S$.

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2. If the area of a circle increases at a uniform rate, then prove that perimeter varies inversely as the radius.

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3. A kite is moving horizontally at a height of 151.5 meters. If the speed of kite is 10 m/s, how fast is the string being let out , when the kite is 250 m away from the boy who is flying the kite ? The height of boy is 1.5 m.

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4. Two men A and B start with velocities v at the same time from the junction of two roads inclined at 45° to each other. If they travel by different roads, find the rate at which they are being seperated.

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5. Find an angle θ, where $0 < \theta < \frac{\pi}{2}$ which increases twice as fast as its sine.

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6. Find the approximate value of $(1.999)^5$.

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7. Find the approximate volume of metal in a hollow spherical shell whose internal and external radii are 3 cm and 3.0005 cm, respectively.

Watch Video Solution

8. A man, 2m tall, walks at the rate of $1\frac{2}{3}$ m/s towards a street light which is $5\frac{1}{3}$ m above the ground. At what rate is the tip of his shadow moving and at what rate is the length of the shadow changing when he is $3\frac{1}{3}m$ from the base of the light ?

Watch Video Solution

9. A swimming pool is to be drained for cleaning. If L represents the number of litres of water in the pool t seconds after the pool has been plugged off to drain and $L = 200(10 - t)^2$. How fast is the water running out at the end of 5 seconds ? What is the average rate at which the water flows out during the first 5 seconds ?

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10. The volume of a cube increases at a consant rate. Prove that the increase in its surface area varies inversely as the length of the side.

Watch Video Solution

11. x and y are the sides of two squares such that $y = x - x^2$. Find the rate of change of the area of second square with respect to the area of first square.

Watch Video Solution

12. Find the condition that the curves $2x = y^2$ and 2xy = k intersect orthogonally.

Watch Video Solution

13. Show that the curves $xy = a^2$ and $x^2 + y^2 = 2a^2$ touch each other.

Watch Video Solution

14. Find the co-ordinates of the point on the curve $\sqrt{x} + \sqrt{y} = 4$ at which tangent is equally inclined to the axes.

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15. Find the angle of intersection of the curves $y = 4 - x^2$ and $y = x^2$.

Watch Video Solution

16. Prove that the curves $y^2 = 4x$ and $x^2 + y^2 - 6x + 1 = 0$ touch each other at the point (1, 2).

Watch Video Solution

17. Find the equation of the normal lines to the curve $3x^2 - y^2 = 8$ which are parallel to the line $x + 3y = 4$.

Watch Video Solution

18. At what points on the curve $x^2 + y^2 - 2x - 4y + 1 = 0$, the tangents are parallel to the Y-axis ?

Watch Video Solution

19. Show that a point at which the line $\frac{x}{a} + \frac{y}{b} = 1$ touches to the curve $y = be^{-\frac{x}{a}}$ lies on Y-axis.

Watch Video Solution

20. Show that

$$f(x) = 2x + \cot^{-1} x + \log\left(\sqrt{1+x^2} - x\right)$$

is increasing in R.

Watch Video Solution

21. Show that for

$a \geq 1, f(x) = \sqrt{3} \sin x - \cos x - 2ax + b$ is

decreasing in R.

Watch Video Solution

22. Show that $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in $\left(0, \frac{\pi}{4}\right)$.

Watch Video Solution

23. At what point, the slope of the curve $y = -x^3 + 3x^2 + 9x - 27$ is maximum ? Also find the maximum slope.

Watch Video Solution

24. Prove that $f(x) = \sin x + 3^{\frac{1}{2}} \cos x$ has maximum value at $x = \frac{\pi}{6}$

Watch Video Solution

Solutions Of Ncert Exemplar Problems Long Answer Type Questions

1. If the sum of the lengths of the hypotenuse and a side of a right angled triangle is given, show that the area of the triangle is maximum when the angle between them is $\frac{\pi}{3}$.

View Text Solution

2. Find the points of local maxima, local minima and the points of inflection of the function $f(x) = x^5 - 5x^4 + 5x^3 - 1$. Also find the corresponding local maximum and local minimum values.

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3. A telephone company in a town has 500 subscribers on its list and collects fixed charges of Rs. 300/- per subscriber per year. The company proposes to increase the annual subscription and it is beloeved that for every increase of Rs. 1/- one subscriber will discontinue the service. Find what increase will bring maximum profit ?

Watch Video Solution

4. If the straight line $x\cos\alpha + y\sin\alpha = p$ touches the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ then prove that $a^2\cos^2\alpha + b^2\sin^2\alpha = p^2$.

Watch Video Solution

5. An open box with square base is to be made of a given quantity of card board of area c^2. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cubic units.

Watch Video Solution

6. Find the dimensions of the rectangle of perimeter 36 cm which will sweep out a volume as large as possible, when revolved about one of its sides. Also find the maximum volume.

Watch Video Solution

7. If the sum of the surface areas of cube and a sphere is constant, what is the ratio of an edge of the cube to the diameter of the sphere, when the sum of their volumes is minimum ?

Watch Video Solution

8. AB is a diameter of a circle and C is any point on the circle. Show that the area of ΔABC is maximum, when it is isosceles.

Watch Video Solution

9. A metal box with a square base and vertical sides is to contain $1024cm^3$. The materical for the top and bottom costs Rs. $5/cm^2$ and the material for the sides costs Rs. $2.50/cm^2$. Find the least cost of the box.

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10. The sum of the surface areas of a rectangular parallelopiped with sides x, 2x and $\frac{x}{3}$ and a sphere is given to be constant. Prove that the sum of their volumes is minimum, if x is equal to three times the radius of the sphere. Also find the minimum value of the sum of their volumes.

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Solutions Of Ncert Exemplar Problems Objective Type Type Questions

1. The sides of an equilateral triangle are increasing at the rate of 2cm/sec. The rate at which the area increases, when side is 10 cm is

....

A. $10cm^2 / s$

B. $\sqrt{3}cm^2 / s$

C. $10\sqrt{3}cm^2 / s$

D. $\frac{10}{3}cm^2 / s$

Answer: C

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2. A ladder, 5 meter long, standing on a horizontal floor, leans against a vertical wall. If the top of the ladder slides downwards at the rate of 10 cm/sec, then the rate at which the angle between the floor and the ladder is decreasing when lower end of ladder is 2 metres from the wall is ....

A. $\frac{1}{10}$ radian/sec.

B. $\frac{1}{20}$ radian/sec.

C. 20 radian/sec.

D. 10 radian/sec.

Answer: B

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3. At point (0, 0) the curve $y = x^{\frac{1}{5}}$ has ....

A. a vertical tangent (parallel to Y = axis)

B. a horizontal tangent (parallel to X-axis)

C. an oblique tangent

D. no tangent

Answer: A

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4. The equation of normal to the curve $3x^2 - y^2 = 8$ which is parallel to the line $x + 3y = 8$ is ....

A. $3x - y = 8$

B. $3x + y + 8 = 0$

C. $x + 3y \pm 8 = 0$

D. $x + 3y = 0$

Answer: C

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5. If the curve $ay + x^2 = 7$ and $x^3 = y$, cuts orthogonall at (1, 1), then the value of a is ....

A. 1

B. 0

C. -6

D. 6

Answer: D

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6. If $y = x^4 - 10$ and if x change from 2 to 1.99, what is the change in y ....

A. 0.32

B. 0.032

C. 5.68

D. 5.968

Answer: A

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7. The equation of tangent to the curve $y(1+x^2)=2-x$, where it crosses X-axis is ....

A. $x+5y=2$

B. $x-5y=2$

C. $5x-y=2$

D. $5x+y=2$

Answer: A

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8. The points at which the tangents to the curve $y = x^3 - 12x + 18$ are parallel to X-axis are :

A. $(2, -2), (-2, -34)$

B. $(2, 34), (-2, 0)$

C. $(0, 34), (-2, 0)$

D. $(2, 2), (-2, 34)$

Answer: D

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9. The tangent to the curve $y = e^{2x}$ at the point (0, 1) meets X-axis at :

A. $(-1, 0)$

B. $\left(-\frac{1}{2}, 0\right)$

C. (2, 0)

D. $(-2, 0)$

Answer: B

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10. The slope of tangent to the curve $x = t^2 + 3t - 8, y = 2t^2 - 2t - 5$ at the point $(2, -1)$ is ....

A. $\frac{22}{7}$

B. $\frac{6}{7}$

C. $-\frac{6}{7}$

D. -6

Answer: B

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11. The two curve $x^3 - 3xy^2 + 2 = 0$ and $3x^2y - y^3 - 2 = 0$ intersect at an angle of

....

A. $\frac{\pi}{4}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{2}$

D. $\frac{\pi}{3}$

Answer: C

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12. The interval on which the function $f(x) = 2x^3 + 9x^2 + 12x - 1$ is decreasing is :

A. $[-1, \infty]$

B. $[-2, -1]$

C. $(-\infty, -2]$

D. $[-1, 1]$

Answer: B

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13. Let the f : R $\rightarrow$ R be defined by $f(x) = 2x + \cos x$, then f ....

A. has a minimum at $x = \pi$

B. has a maximum, at x = 0

C. is a decreasing function

D. is an increasing function

Answer: D

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14. $y = x(x-3)^2$ decreases for the values of x given by ....

A. $1 < x < 3$

B. $x < 0$

C. $x > 0$

D. $0 < x < \frac{3}{2}$

Answer: A

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15. The function $f(x) = 4\sin^3 x - 6\sin^2 x + 12\sin x + 100$ is strictly ....

A. increasing in $\left(\pi, \frac{3\pi}{2}\right)$

B. decreasing in $\left(\frac{\pi}{2}, \pi\right)$

C. decreasing in $\left[-\frac{\pi}{2}, \pi\right]$

D. decreasing in $\left[0, \frac{\pi}{2}\right]$

Answer: B

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16. Which of the following functions is decreasing on $\left(0, \frac{\pi}{2}\right)$.

A. sin (2x)

B. tan x

C. cos x

D. cos (3x)

Answer: C

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17. The function f (x) = tan $x - x$ ....

A. always increases

B. always decreases

C. never increases

D. sometimes increases and sometimes decreases

Answer: A

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18. If x is real, the minimum value of $x^2 - 8x + 17$ is .... (Where $x \in R$)

A. -1

B. 0

C. 1

D. 2

Answer: C

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19. The smallest value of the polynomial $x^3 - 18x^2 + 96x$ in [0, 9] is ....

A. 126

B. 0

C. 135

D. 160

Answer: B

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20. The function $f(x) = 2x^3 - 3x^2 - 12x + 4$

, has ....

A. two points of local maximum

B. two points of local minimum

C. one maxima and one minima

D. no maxima or minima

Answer: C

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21. The maximum value of sin x.cos x is ....

A. $\frac{1}{4}$

B. $\frac{1}{2}$

C. $\sqrt{2}$

D. $2\sqrt{2}$

Answer: B

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22. At $x = \frac{5\pi}{6}$, $f(x) = 2\sin 3x + 3\cos(3x)$ is

....

A. maximum

B. minimum

C. Zero

D. neither maximum nor minimum

Answer: D

23. Maximum slope of the curve $y = -x^3 + 3x^2 + 9x - 27$ is ....

A. 0

B. 12

C. 16

D. 32

Answer: B

24. $f(x) = x^x$ has a stationary point at ....

A. x = e

B. $x = \frac{1}{e}$

C. x = 1

D. $x = \sqrt{e}$

Answer: B

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25. The maximum value of $f(x) = \left(\frac{1}{x}\right)^{x}$ is

....

A. e

B. e^{e}

C. $e^{\frac{1}{e}}$

D. $\left(\frac{1}{e}\right)^{\frac{1}{e}}$

Answer: C

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Solutions Of Ncert Exemplar Problems Fillers

1. The curves $y = 4x^2 + 2x - 5$ and $y = x^3 - x + 13$ touch each other at the point ....

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2. The equation of normal to the curve y = tan x at (0, 0) is ....

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3. The values of a for which the function $f(x) = \sin x - ax + b$ increases on R are ....

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4. The function $f(x) = \frac{2x^2 - 1}{x^4}, x > 0,$ decreases in the interval ....

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5. The least value of the function $f(x) = ax + \frac{b}{x}$ is .... (Where $a > 0, b > 0, c > 0)$

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Practice Paper 6 Section A

1. A particle covers distance S in time t is given by $S = t^3 - 6t^2 + 6t + 8.$ When the acceleration is 0, the velocity is ....

A. 5 cm/sec

B. 2 cm/sec

C. 6 cm/sec

D. -6 cm/sec

Answer: D

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2. Local maximum value of $f(x) = x + \frac{1}{x}x \neq 0$ is ....

A. 2

B. -2

C. 4

D. -4

Answer: A

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3. If a line y = x touches the curve $y = x^2 + bx + c$ at (1, 1) then ....

A. b= 1, c = 2

B. $b = -1, c = 1$

C. b = 1, c = 1

D. b = 0, c = 1

Answer: B

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4. Rate of increase in surface area of a sphere w.r.t radius is ....

A. 8π (diameter)

B. 3π (diameter)

C. 4π (diameter)

D. 8π (radius)

Answer: D

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5. If ...., then $f(x) = x^2 - kx + 20, [0, 3]$ is strictly increasing.

A. $k < 0$

B. $0 < k < 1$

C. $1 < k < 2$

D. $2 < k < 3$

Answer: A

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Practice Paper 6 Section B

1. Find points on the curve $\frac{x^2}{9} + \frac{y^2}{16} = 1$ at which the tangents are

(i) parallel to X - axis

(ii) parallel to Y-axis.

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2. Find approminate value of $(255)^{\frac{1}{4}}$.

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3. Find maximum value of $y = 2x^3 - 24x + 107$ in [1, 3].

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4. Radius of an air bubble increases at the rate of $\frac{1}{2}$ cm/sec. At what rate volume at bubble increases when radius is 1 cm.

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Practice Paper 6 Section C

1. Show that of all rectangles inscribed in a given fixed circle, the square has the maximum area.

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2. Find local maximum and minimum value of $f(x) = \sin^4 x + \cos^4 x, x \in \left[0, \frac{\pi}{2}\right]$.

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3. Obtain equation of tangent and normal at point (1, 1) to the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 2$.

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4. Find intervals in which the function given by $f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$ is

(a) Increasing (b) Decreasing.

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5. Equation of the motion of the particle is $S = t^3 - 6t^2 + 9t$, where S is in mets and t is in seconds.

(i) Find the instantaneous velocity when t = 2.

(ii) When particle is at rest ?

(iii) Find distance travelled by particle in first 5 second.

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Practice Paper 6 Section D

1. If the length of three sides of a trapezium other than base are equal to 10 cm. Than find the area of trapezium when it is maximum.

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2. Find point on $y = 2x - 3$ nearest to origin.

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