MATHS

BOOKS - KUMAR PRAKASHAN KENDRA MATHS (GUJRATI ENGLISH)

CONTINUITY AND DIFFERENTIABILITY

Practice Work

1. Examine the continuity of the function $f(x) = 2x + 3$ at $x = 1$

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2. Prove that the function $f(x) = x^2$ is continuous at x=0.

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3. Find the points of discontinuity of the following function.

(a) $f(x) = \frac{3x + 7}{x^2 - 5x + 6}$ (b) $f(x) = \frac{1}{|x| - 1} - \frac{x^2}{2}$ (c) $f(x) = \frac{\sqrt{x^2 + 1}}{1 + \sin^2 x}$ (d) $f(x) = \tan\left(\frac{\pi}{2}x\right)$

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4. $f(x) = \frac{x^2 + 1}{x^2 - 1}$ and $g(x) = \tan x$. Examine the continuity of (fog) (x).

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5. $y = f(\mu)$, where $f(\mu) = \frac{3}{2\mu^2 + 5\mu - 3}$ and $\mu = \frac{1}{x + 2}$. Find the points of discontinuity of y.

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6. Show that $f(x) = \begin{cases} x^3 + 3, & x \neq 0 \\ 1, & x = 0 \end{cases}$ is a discontinuous function at x= 0.

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7. $f(x) = \frac{1}{(x - 1)(x - 2)}$ and $g(x) = \frac{1}{x^2}$. Find the points of discontinuity of the composite function f(g(x))?

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8. $f(x) = \begin{cases} \frac{|\sin x|}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$ Examine the continuity of f(x), x= 0

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9. $f(x) = \begin{cases} \frac{1}{e^{4x} + 1}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ Examine the continuity of f(x) at x= 0

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10. f (x)= {x} and g(x)= [x]. Where { } is a fractional part and [ ] is a greatest integer function. Prove that $f(x) + g(x)$ is a continuous function at x= 1.

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11. Show that $f(x) = [x] + [-x], x \in R -$ {integer} is a continuous function. Where [ ] is a greatest integer function

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12. Discuss the continuity and differentiability for $f(x) = [\sin x]$ when $x \in [0, 2\pi]$, where [·] denotes the greatest integer function x.

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13. $f(x) = \begin{cases} |x-1|, & x \geq 0 \\ -|x|, & x < 0 \end{cases}$ Prove that f(x) is continuous for $x \in R - \{0\}$.

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14. Prove that $f(x) = \begin{cases} \frac{\sin x}{x} + \cos x, & x \neq 0 \\ 2, & x = 0 \end{cases}$ is a continuous function at x= 0

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15. Prove that $f(x) = 2x - |x|$ is a continuous function at x= 0.

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16. For which value of x, the function $f(x) = \frac{e^{\sin x}}{4 - \sqrt{x^2 - 9}}$ is discontinuous?

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17. $f(x) = \frac{x^2 + 1}{x^2 - 1}$ and $g(x) = \tan x$. Examine the continuity of (fog) (x).

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18. $f(x) = \begin{cases} \frac{1}{2} - x, & 0 \le x < \frac{1}{2} \\ 1, & x = \frac{1}{2} \\ \frac{3}{2} - x, & \frac{1}{2} < x \le 1 \end{cases}$ Discuss the continuity of f(x)

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19. $f(x) = \begin{cases} 2 + \sqrt{1 - x^2}, & |x| \le 1 \\ 2e^{(1-x)^2} & |x| > 1 \end{cases}$ Discuss the continuity of f(x) at x=1

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20. $f(x) = \dfrac{\sqrt{2}\cos x - 1}{\cot x - 1}, x \ne \dfrac{\pi}{4}$. If the function f(x) is continuous at $x = \dfrac{\pi}{4}$ then find $f\left(\dfrac{\pi}{4}\right)$

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21. $f(x)=\begin{cases}\dfrac{\tan 2x}{x}, & x\neq 0\\ K, & x=0\end{cases}$ If a function f is continuous at x=0 then find k.

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22. $f(x)=\begin{cases}\dfrac{1-\cos 4x}{8x^2}, & x\neq 0\\ k, & x=0\end{cases}$. If the function f(x) is continuous at x= 0, then find k.

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23. Show that, $f(x)=\begin{cases}\dfrac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}, & x\neq 0\\ 0, & x=0\end{cases}$. Is discontinuous at x= 0

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24. The give functions is continuous at x=0. Find a, b, c

$$f(x) = \begin{cases} \dfrac{\sin(a+1)x + \sin x}{x}, & x < 0 \\ C, & x = 0 \\ \dfrac{\sqrt{x+bx^2} - \sqrt{x}}{bx^{\frac{3}{2}}}, & x > 0 \end{cases}$$

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25. $f(x) = \begin{cases} \dfrac{x(1 + a\cos x) - b\sin x}{x^3}, & x \neq 0 \\ 0, & x = 0 \end{cases}$. If f is continuous at x=0 then find the value of a and b.

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26. The give functions is continuous at x=0. Find a, b, c

$$f(x) = \begin{cases} \frac{\sin(a+1)x + \sin x}{x}, & x < 0 \\ C, & x = 0 \\ \frac{\sqrt{x+bx^2} - \sqrt{x}}{bx^{\frac{3}{2}}}, & x > 0 \end{cases}$$

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27. $f(x) = \begin{cases} -4\sin x + \cos x, & x \le -\frac{\pi}{2} \\ a\sin x + b, & -\frac{\pi}{2} < x < \frac{\pi}{2} \\ \cos x + 2, & \frac{\pi}{2} \le x \end{cases}$. If f(x) is continuous for $x \in R$,

then find the value of a and b.

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28. $f(x) = \begin{cases} |x+1|, & x < -2 \\ 2x+3, & -2 \le x < 0 \\ x^2+3, & 0 \le x < 3 \\ x^3-15 & 3 \le x \end{cases}$. Find at which points, the function f(x) is discontinuous ?

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29. $f(x) = \dfrac{1-\tan x}{4x-\pi}, x \ne \dfrac{\pi}{4}$. If the function f(x), $x \in \left[0, \dfrac{\pi}{2}\right)$ is continuous then find $f\left(\dfrac{\pi}{4}\right)$.

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30. $f(x) = \begin{cases} \dfrac{2^{x+2}-16}{4^x-16}, & x \ne 2 \\ k, & x = 2 \end{cases}$ f(x) is continuous at x=2 then find k

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31. Find the derivative of the following functions with respect to x

$\tan(2x + 3)$

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32. Find the derivative of the following functions with respect to x

$\sin 3x . \sin 3x$

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33. Find the derivative of the following functions with respect to x

$$\sqrt{\frac{x + 1}{x - 1}}$$

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34. Find the derivative of the following functions with respect to x

$$\frac{\sec x - 1}{\sec x + 1}$$

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35. Find the derivative of the following functions with respect to x

$\sin\left[\cos\left(x^2\right)\right]$

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36. Find the derivative of the following functions with respect to x

$(2 + 3\sin x)(3 - 2\cos x)$

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37. Show that f(x) = |x| sin x is differentiable at x=0.

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38. The right hand derivative of f(x)= [x] tan (πx) at a point x=7 is $k\pi$ then find the value of k. where [.] is the greatest integer function.

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39. $f(x)\begin{cases} \left|x - \frac{1}{2}\right|, & 0 \le x < 1 \\ x[x], & 1 \le x < 2 \end{cases}$ where [.] denotes the greatest integer function. Show that f(x) is continuous at x=1 but not differentiable at x=1.

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40. $f(x) = x^3$ sgn(x). Show that f(x) is differentiable at x=0.

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41. The left hand derivative of $f(x) = [x]\sin(\pi x)$at $\mathrm{x} = k$ is an integer, is

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42. Differentiate the following functions with respect to x:

$x^2 + y^2 = xy$

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43. Differentiate the following functions with respect to x:

$x^3 + y^3 = \sin(x + y)$

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44. Differentiate the following functions with respect to x:

$\sqrt{x} + \sqrt{y} = \sqrt{a}$

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45. Differentiate the following functions with respect to x:

$$\frac{x^2}{4} - \frac{y^2}{9} = 1$$

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46. Differentiate the following functions with respect to x:

$$\sin x + \sin y = \tan(xy)$$

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47. Differentiate the following functions with respect to x:

$$x^2 + y^2 - 4x - 6y - 25 = 0$$

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48. Differentiate the following functions with respect to x:

$$x + \sin x = \sin y$$

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49. Find $\frac{dy}{dx}$ in the following:

$$y = \sin^{-1}\left(3x - 4x^3\right), 0 < x < \frac{1}{2}$$

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50. Find $\frac{dy}{dx}$ in the following:

$$y = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right), |x| < \frac{1}{\sqrt{3}}$$

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51. Find $\frac{dy}{dx}$ in the following:

$$y = \sin^{-1}\left(2x\sqrt{1 - x^2}\right), \frac{1}{\sqrt{2}} < x < 1$$

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52. Find $\frac{dy}{dx}$ in the following:

$$y = \sec^{-1}\left(\frac{x^2+1}{x^2-1}\right)$$

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53. Find $\frac{dy}{dx}$ in the following:

$$y = \tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right)$$

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54. Differentiate the following with respect to x:

$$\frac{e^x \log x}{x^2}$$

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55. Differentiate the following with respect to x:

$$\frac{e^{x} - 1}{e^{x} + 1}$$

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56. Differentiate the following with respect to x:

$$\frac{e^{x} + \log x}{\sin 3x}$$

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57. Differentiate the following with respect to x:

$$\frac{1 - \log x}{1 + \log x}$$

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58. Differentiate the following with respect to x:

$$\cos^{-1}\left(e^{x}\right)$$

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59. Differentiate the following with respect to x:

x log x

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60. Differentiate the following with respect to x:

$x\cos x + e^x$

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61. Differentiate the following with respect to x:

$\log\left[\log\left(\log x^5\right)\right]$

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62. Differentiate the following with respect to x:

$$\frac{\log x}{1 + x\log x}$$

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63. Differentiate the following with respect to x:

$$\sin\left[\log\left(e^{x}\right)\right]$$

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64. Differentiate the following functions with respect to x.

$$y = (\sin x)^{x} + \left(\frac{1}{x}\right)^{\cos x}$$

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65. Differentiate the following functions with respect to x.

$$y = x^{x}.\sin x + (\sin x)^{x}$$

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66. Differentiate the following functions with respect to x.

$y = (\sin x)^x + \sin x^x$

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67. Differentiate the following functions with respect to x.

$x^y + y^x = 1000$

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68. Differentiate the following functions with respect to x.

$y = \cos\left(x^x\right) + \sin\left(x^x\right)$

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69. If $y = x^{x^x}$ then find $\frac{dy}{dx}$

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70. If $y = x^x + x^a + a^x$ then find $\frac{dy}{dx}$

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71. $e^x + e^y = e^{x+y}$ then prove that, $\frac{dy}{dx} + \frac{e^x\left(e^y - 1\right)}{e^y\left(e^x - 1\right)} = 0$

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72. If $y = x^{\tan x} + \sqrt{\frac{x^2 + 1}{x}}$ then find $\frac{dy}{dx}$

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73. If $f(x) = |\cos x - \sin x|$ then find $f'\left(\frac{\pi}{6}\right)$

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74. Find $\frac{dy}{dx}$:

$x = \cos^2\theta$ and $y = \sin^2\theta$

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75. Find $\frac{dy}{dx}$:

$x = t + \frac{1}{t}$ and $y = t - \frac{1}{t}$

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76. Find $\frac{dy}{dx}$:

$x = te^t, y = 1 + \log t$

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77. Find $\frac{dy}{dx}$:

$x = a\sec^3\theta, y = a\tan^3\theta$

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78. Find $\frac{dy}{dx}$:

$x = a\sin^2\theta\cos\theta, y = 2b\cos^2\theta(-\sin\theta)$

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79. Find $\frac{dy}{dx}$:

$x = 2\cos\theta - \cos^2\theta$ and $y = 2\sin\theta - \sin 2\theta$

Show that $\frac{dy}{dx} = -1$ when $\theta = \frac{\pi}{2}$

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80. Find $\frac{dy}{dx}$:

$x = e^{\cos 2t}$ and $y = e^{\sin 2t}$ show that, $\frac{dy}{dx} = \frac{-y\log x}{x\log y}$

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81. Find $\frac{dy}{dx}$:

$x = a\sin 2t(1 + \cos 2t)$ and $y = b\cos 2t$

$(1 - \cos 2t)$ show that, $\left(\frac{dy}{dx}\right)_{t = \frac{\pi}{4}} = \frac{b}{a}$

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82. Find the second order derivatives of the following functions:

e^{ax}

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83. Find the second order derivatives of the following functions:

$x^3 + \tan x$

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84. Find the second order derivatives of the following functions:

$\sin^2 x$

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85. Find the second order derivatives of the following functions:

$\tan^{-1} 3x$

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86. Find the second order derivatives of the following functions:

$\log e^{x^{x^x}}$

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87. Find the second order derivatives of the following functions:

$3\sin 4x - 4\sin^3 4x$

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88. Find the second order derivatives of the following functions:

$e^{-2\log x}$

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89. Find the second order derivatives of the following functions:

$\sin\left(x^2 + 5\right)$

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90. If $y = \sin^{-1}x$ then prove that $\left(1 - x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$

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91. If $y = e^{\tan x}$ then show that, $\left(\cos^2 x\right)\frac{d^2y}{dx^2} - (1 + \sin 2x)\frac{dy}{dx} = 0$

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92. If $y = \sin(\sin x)$ then show that, $\frac{d^2y}{dx^2} + (\tan x)\frac{dy}{dx} + y\cos^2 x = 0$

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93. If $x^m . y^n = (x + y)^{m+n}$ then show that, $\frac{d^2y}{dx^2} = 0$

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94. If $y=\log\left(x+\sqrt{x^2+1}\right)$ then show that, $\left(x^2+1\right)\frac{d^2y}{dx^2}+x\frac{dy}{dx}=0$

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95. If $y=e^{ax}\sin bx$ then show that, $y_2-2ay_1+\left(a^2+b^2\right)y=0$

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96. If $y=\left(x+\sqrt{x^2+1}\right)^m$ then prove that, $\left(x^2+1\right)y_2+xy_1=m^2y$

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97. If $2x=y^{\frac{1}{m}}+y^{-\frac{1}{m}}(n\geq 1)$ then prove that, $\left(x^2-1\right)y_2+xy_1=m^2y$

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98. Verify Rolle's theorem for the following functions:

$f(x) = \sqrt{9 - x^2}, x \in [-3, 3]$

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99. Verify Rolle's theorem for the following functions:

$f(x) = x^3 - 6x^2 + 11x - 6, x \in [2, 3]$

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100. Verify Rolle's theorem for the following functions:

$f(x) = x(x - 3)^2, x \in [0, 3]$

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101. Verify Rolle's theorem for the following functions:

$f(x) = \sin x + \cos x - 1, x \in \left[0, \frac{\pi}{2}\right]$

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102. Verify Rolle's theorem for the following functions:

$f(x) = a^{\sin x}, x \in [0, \pi], a > 0$

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103. Vertify mean value theorem for the following functions:

$f(x) = \log_e x, x \in [1, 2]$

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104. Vertify mean value theorem for the following functions:

$f(x) = x - 2\sin x, x \in [-\pi, \pi]$

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105. Vertify mean value theorem for the following functions:

$f(x) = x + \frac{1}{x}, x \in [1, 3]$

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106. Vertify mean value theorem for the following functions:

$f(x) = \tan^{-1}x, x \in [0, 1]$

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107. Vertify mean value theorem for the following functions:

$f(x) = x^2 + 2x + 3, x \in [4, 6]$

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108. Prove that $0 < a < b < \frac{\pi}{2}, \sec^2 a < \frac{\tan b - \tan a}{b - a} < \sec^2 b$

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109. $y = e^{x + e^{x + e^{x + \dots \infty}}}$ then find $\frac{dy}{dx}$

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110. Find the derivative of $\tan^{-1}\left[\frac{\sqrt{1 + x^2} - 1}{x}\right]$ with respect to $\tan^{-1}\left(\frac{2x}{1 - x^2}\right)$

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111. $y = x + \cfrac{1}{x + \cfrac{1}{x + \cfrac{1}{x + \dots \infty}}}$ then find $\frac{dy}{dx}$

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112. Find the derivative of $\sin^{-1}\left[\frac{2^{x+1}.3^{x}}{1+(36)^{x}}\right]$ with respect to x.

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113. $x = a\sin t$ and $y = a\left(\cos t + \log\tan\frac{t}{2}\right)$ then find $\frac{d^2y}{dx^2}$.

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114. If $y = \frac{ax^2}{(x-a)(x-b)(x-c)} + \frac{bx}{(x-b)(x-c)} + \frac{c}{(x-c)} + 1$ then prove that $\frac{y'}{y} = \frac{1}{x}\left[\frac{a}{a-x} + \frac{b}{b-x} + \frac{c}{c-x}\right]$

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115. $y = \sin^{-1}\left[\frac{5x + 12\sqrt{1-x^2}}{13}\right]$ then find $\frac{dy}{dx}$

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116. If $y = x\log\left(\frac{x}{a+bx}\right)$ prove that $\frac{d^2y}{dx^2} = \frac{1}{x}\left(\frac{a}{a+bx}\right)^2$

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117. If $y = \left\{x + \sqrt{x^2 + a^2}\right\}^n$ prove that $\frac{dy}{dx} = \frac{ny}{\sqrt{x^2 + a^2}}. n > 1 \neq N$

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118. $y = \sqrt{\frac{1 - \sin 2x}{1 + \sin 2x}}$ then prove that $\frac{dy}{dx} + \sec^2\left(\frac{\pi}{4} - x\right) = 0$

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119. Show that $\frac{d}{dx}e^{ax}\cos(bx + c) = re^{ax}\cos(bx + c + \alpha)$ where $r = \sqrt{a^2 + b^2}, \cos\alpha = \frac{a}{r}, \sin\alpha = \frac{b}{r}$ and $\frac{d^2}{dx^2}e^{ax}\cos(ax + c) = r^2e^{ax}\cos(bx + c + 2\alpha$

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120. If $(a - b\cos y)(a + b\cos x) = a^2 - b^2$ show that $\frac{dy}{dx} = \frac{\sqrt{a^2 - b^2}}{a + b\cos x}, 0 < x < \frac{\pi}{2}$

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121. Find $\frac{d}{dx}\left[\operatorname{cosec}^{-1}x\right]_{x = -2}$

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122. $\frac{d}{dx}\tan^{-1}(\sec x - \tan x)$- Find

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123. Prove that, $\frac{d}{dx}\left[\log\frac{x^2+x+1}{x^2-x+1}+\frac{2}{\sqrt{3}}\tan^{-1}\frac{\sqrt{3}x}{1-x^2}\right]=\frac{4}{x^4+x^2+1}$

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124. Examine the continuity of the function $f(x) = 2x + 3$ at $x = 1$

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125. Prove that the function $f(x) = x^2$ is continuous at x=0.

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126. Find the points of discontinuity of the following function.

(a) $f(x) = \frac{3x+7}{x^2-5x+6}$ (b) $f(x) = \frac{1}{|x|-1} - \frac{x^2}{2}$ (c) $f(x) = \frac{\sqrt{x^2+1}}{1+\sin^2 x}$ (d) $f(x) = \tan\left(\frac{\pi}{2}x\right)$

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127. $f(x) = \dfrac{x^2 + 1}{x^2 - 1}$ and $g(x) = \tan x$. Examine the continuity of (fog) (x).

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128. $y = f(\mu)$, where $f(\mu) = \dfrac{3}{2\mu^2 + 5\mu - 3}$ and $\mu = \dfrac{1}{x + 2}$. Find the points of discontinuity of y.

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129. Show that $f(x) = \begin{cases} x^3 + 3, & x \neq 0 \\ 1, & x = 0 \end{cases}$ is a discontinuous function at x= 0.

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130. $f(x) = \dfrac{1}{(x - 1)(x - 2)}$ and $g(x) = \dfrac{1}{x^2}$. Find the points of discontinuity of the composite function f(g(x))?

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131. $f(x) = \begin{cases} \frac{|\sin x|}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$ Examine the continuity of f(x), x= 0

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132. $f(x) = \begin{cases} \frac{1}{e^{4x}+1}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ Examine the continuity of f(x) at x= 0

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133. f (x)= {x} and g(x)= [x]. Where { } is a fractional part and [ ] is a greatest integer function. Prove that $f(x) + g(x)$ is a continuous function at x= 1.

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134. Show that $f(x) = [x] + [-x], x \in R -$ {integer} is a continuous function. Where [ ] is a greatest integer function

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135. $f(x) = [\sin x], x \in [0, 2\pi]$ At which points, f(x) is discontinuous?

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136. $f(x) = \begin{cases} |x-1|, & x \geq 0 \\ -|x|, & x < 0 \end{cases}$ Prove that f(x) is continuous for $x \in R - \{0\}$.

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137. Prove that $f(x) = \begin{cases} \dfrac{\sin x}{x} + \cos x, & x \neq 0 \\ 2, & x = 0 \end{cases}$ is a continuous function at x= 0

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138. Prove that $f(x) = 2x - |x|$ is a continuous function at x= 0.

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139. For which value of x, the function $f(x) = \frac{e^{\sin x}}{4 - \sqrt{x^2 - 9}}$ is discontinuous?

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140. $f(x) = \frac{x^2 + 1}{x^2 - 1}$ and $g(x) = \tan x$. Examine the continuity of the composite function (fog) (x)

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141. $f(x)=\begin{cases}\frac{1}{2}-x, & 0\le x<\frac{1}{2}\\ 1, & x=\frac{1}{2}\\ \frac{3}{2}-x, & \frac{1}{2}<x\le 1\end{cases}$ Discuss the continuity of f(x)

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142. $f(x)=\begin{cases}2+\sqrt{1-x^2}, & |x|\le 1\\ 2e^{(1-x)^2} & |x|>1\end{cases}$ Discuss the continuity of f(x) at x=1

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143. $f(x)=\dfrac{\sqrt{2}\cos x-1}{\cot x-1}, x\ne\dfrac{\pi}{4}$. If the function f(x) is continuous at $x=\dfrac{\pi}{4}$ then find $f\left(\dfrac{\pi}{4}\right)$

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144. $f(x)=\begin{cases}\dfrac{\tan 2x}{x}, & x\neq 0\\ K, & x=0\end{cases}$ If a function f is continuous at x=0 then find k.

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145. $f(x)=\begin{cases}\dfrac{1-\cos 4x}{8x^2}, & x\neq 0\\ k, & x=0\end{cases}$. If the function f(x) is continuous at x= 0, then find k.

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146. Show that, $f(x)=\begin{cases}\dfrac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}, & x\neq 0\\ 0, & x=0\end{cases}$. Is discontinuous at x= 0

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147. The give functions is continuous at x=0. Find a, b, c

$$f(x) = \begin{cases} \dfrac{\sin(a+1)x + \sin x}{x}, & x < 0 \\ C, & x = 0 \\ \dfrac{\sqrt{x+bx^2} - \sqrt{x}}{bx^{\frac{3}{2}}}, & x > 0 \end{cases}$$

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148. $f(x) = \begin{cases} \dfrac{x(1+a\cos x) - b\sin x}{x^3}, & x \neq 0 \\ 0, & x = 0 \end{cases}$. If f is continuous at x=0 then find the value of a and b.

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149. $f(x) = \begin{cases} \dfrac{\sin(p+1)x + \sin x}{x}, & x < 0 \\ q, & x = 0 \\ \dfrac{\sqrt{x+x^2} - \sqrt{x}}{x^{\frac{3}{2}}}, & x > 0 \end{cases}$. If f(x) is continuous for $x \in R$ then find the value of p and q.

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150. $f(x) = \begin{cases} -4\sin x + \cos x, & x \leq -\dfrac{\pi}{2} \\ a\sin x + b, & -\dfrac{\pi}{2} < x < \dfrac{\pi}{2} \\ \cos x + 2, & \dfrac{\pi}{2} \leq x \end{cases}$. If f(x) is continuous for $x \in R$, then find the value of a and b.

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151. $f(x)=\begin{cases}|x+1|, & x< -2\\ 2x+3, & -2\le x<0\\ x^2+3, & 0\le x<3\\ x^3-15 & 3\le x\end{cases}$. Find at which points, the function f(x) is discontinuous ?

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152. $f(x)=\dfrac{1-\tan x}{4x-\pi}, x\ne\dfrac{\pi}{4}$. If the function f(x), $x\in\left[0,\dfrac{\pi}{2}\right)$ is continuous then find $f\left(\dfrac{\pi}{4}\right)$.

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153. $f(x)=\begin{cases}\dfrac{2^{x+2}-16}{4^x-16}, & x\ne 2\\ k, & x=2\end{cases}$ f(x) is continuous at x=2 then find k

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154. Find the derivative of the following functions with respect to x

$\tan(2x + 3)$

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155. Find the derivative of the following functions with respect to x

sin $3x$. $\sin 3x$

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156. Find the derivative of the following functions with respect to x

$\sqrt{\frac{x+1}{x-1}}$

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157. Find the derivative of the following functions with respect to x

$\frac{\sec x - 1}{\sec x + 1}$

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158. Find the derivative of the following functions with respect to x

$\sin\left[\cos\left(x^2\right)\right]$

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159. Find the derivative of the following functions with respect to x

$(2 + 3\sin x)(3 - 2\cos x)$

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160. Show that f(x) = |x| sin x is differentiable at x=0.

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161. The right hand derivative of f(x)= [x] tan (πx) at a point x=7 is $k\pi$ then find the value of k. where [.] is the greatest integer function.

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162. $f(x)\begin{cases} \left|x - \frac{1}{2}\right|, & 0 \le x < 1 \\ x[x], & 1 \le x < 2 \end{cases}$ where [.] denotes the greatest integer function. Show that f(x) is continuous at x=1 but not differentiable at x=1.

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163. $f(x) = x^3$ sgn(x). Show that f(x) is differentiable at x=0.

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164. Find the left hand derivative of $f(x) = [x]\sin(\pi x)$ at x=k. where k is an integer and [.] denotes the greatest integer function.

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165. Differentiate the following functions with respect to x:

$x^2 + y^2 = xy$

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166. Differentiate the following functions with respect to x:

$x^3 + y^3 = \sin(x + y)$

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167. Differentiate the following functions with respect to x:

$\sqrt{x} + \sqrt{y} = \sqrt{a}$

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168. Differentiate the following functions with respect to x:

$$\frac{x^2}{4} - \frac{y^2}{9} = 1$$

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169. Differentiate the following functions with respect to x:

$$\sin x + \sin y = \tan(xy)$$

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170. Differentiate the following functions with respect to x:

$$x^2 + y^2 - 4x - 6y - 25 = 0$$

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171. Differentiate the following functions with respect to x:

$$x + \sin x = \sin y$$

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172. Find $\frac{dy}{dx}$ in the following:

$$y = \sin^{-1}\left(3x - 4x^3\right), 0 < x < \frac{1}{2}$$

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173. Find $\frac{dy}{dx}$ in the following:

$$y = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right), |x| < \frac{1}{\sqrt{3}}$$

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174. Find $\frac{dy}{dx}$ in the following:

$$y = \sin^{-1}\left(2x\sqrt{1 - x^2}\right), \frac{1}{\sqrt{2}} < x < 1$$

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175. Find $\frac{dy}{dx}$ in the following:

$$y = \sec^{-1}\left(\frac{x^2+1}{x^2-1}\right)$$

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176. Find $\frac{dy}{dx}$ in the following:

$$y = \tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right)$$

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177. Differentiate the following with respect to x:

$$\frac{e^x \log x}{x^2}$$

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178. Differentiate the following with respect to x:

$$\frac{e^x - 1}{e^x + 1}$$

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179. Differentiate the following with respect to x:

$$\frac{e^x + \log x}{\sin 3x}$$

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180. Differentiate the following with respect to x:

$$\frac{1 - \log x}{1 + \log x}$$

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181. Differentiate the following with respect to x:

$$\cos^{-1}\left(e^x\right)$$

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182. Differentiate the following with respect to x:

x log x

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183. Differentiate the following with respect to x:

$x\cos x + e^x$

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184. Differentiate the following with respect to x:

$\log\left[\log\left(\log x^5\right)\right]$

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185. Differentiate the following with respect to x:

$$\frac{\log x}{1 + x\log x}$$

Watch Video Solution

186. Differentiate the following with respect to x:

$$\sin\left[\log\left(e^{x}\right)\right]$$

Watch Video Solution

187. Differentiate the following functions with respect to x.

$$y = (\sin x)^{x} + \left(\frac{1}{x}\right)^{\cos x}$$

Watch Video Solution

188. Differentiate the following functions with respect to x.

$$y = x^{x}.\sin x + (\sin x)^{x}$$

Watch Video Solution

189. Differentiate the following functions with respect to x.

$y = (\sin x)^x + \sin x^x$

Watch Video Solution

190. Differentiate the following functions with respect to x.

$x^y + y^x = 1000$

Watch Video Solution

191. Differentiate the following functions with respect to x.

$y = \cos\left(x^x\right) + \sin\left(x^x\right)$

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192. If $y = x^{x^{x}}$ then find $\frac{dy}{dx}$

Watch Video Solution

193. If $y = x^{x} + x^{a} + a^{x}$ then find $\frac{dy}{dx}$

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194. $e^{x} + e^{y} = e^{x+y}$ then prove that, $\frac{dy}{dx} + \frac{e^{x}\left(e^{y} - 1\right)}{e^{y}\left(e^{x} - 1\right)} = 0$

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195. If $y = x^{\tan x} + \sqrt{\frac{x^{2} + 1}{x}}$ then find $\frac{dy}{dx}$

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196. If $f(x) = |\cos x - \sin x|$ then find $f'\left(\frac{\pi}{6}\right)$

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197. Find $\frac{dy}{dx}$:

$x = \cos^2\theta$ and $y = \sin^2\theta$

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198. Find $\frac{dy}{dx}$:

$x = t + \frac{1}{t}$ and $y = t - \frac{1}{t}$

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199. Find $\frac{dy}{dx}$:

$x = te^t, y = 1 + \log t$

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200. Find $\frac{dy}{dx}$:

$x = a\sec^3\theta, y = a\tan^3\theta$

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201. Find $\frac{dy}{dx}$:

$x = a\sin^2\theta\cos\theta, y = 2b\cos^2\theta(-\sin\theta)$

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202. Find $\frac{dy}{dx}$:

$x = 2\cos\theta - \cos^2\theta$ and $y = 2\sin\theta - \sin 2\theta$

Show that $\frac{dy}{dx} = -1$ when $\theta = \frac{\pi}{2}$

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203. Find $\frac{dy}{dx}$:

$x = e^{\cos 2t}$ and $y = e^{\sin 2t}$ show that, $\frac{dy}{dx} = \frac{-y\log x}{x\log y}$

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204. Find $\frac{dy}{dx}$:

$x = a\sin 2t(1 + \cos 2t)$ and $y = b\cos 2t$

$(1 - \cos 2t)$ show that, $\left(\frac{dy}{dx}\right)_{t = \frac{\pi}{4}} = \frac{b}{a}$

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205. Find the second order derivatives of the following functions:

e^{ax}

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206. Find the second order derivatives of the following functions:

$x^3 + \tan x$

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207. Find the second order derivatives of the following functions:

$\sin^2 x$

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208. Find the second order derivatives of the following functions:

$\tan^{-1} 3x$

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209. Find the second order derivatives of the following functions:

$\log e^{x^{x^x}}$

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210. Find the second order derivatives of the following functions:

$3\sin 4x - 4\sin^3 4x$

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211. Find the second order derivatives of the following functions:

$e^{-2\log x}$

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212. Find the second order derivatives of the following functions:

$\sin\left(x^2+5\right)$

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213. If $y = \sin^{-1}x$ then prove that $\left(1 - x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$

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214. If $y = e^{\tan x}$ then show that, $\left(\cos^2 x\right)\frac{d^2y}{dx^2} - (1 + \sin 2x)\frac{dy}{dx} = 0$

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215. If $y = \sin(\sin x)$ then show that, $\frac{d^2y}{dx^2} + (\tan x)\frac{dy}{dx} + y\cos^2 x = 0$

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216. If $x^m . y^n = (x + y)^{m+n}$ then show that, $\frac{d^2y}{dx^2} = 0$

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217. If $y = \log\left(x + \sqrt{x^2 + 1}\right)$ then show that, $\left(x^2 + 1\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$

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218. If $y = e^{ax}\sin bx$ then show that, $y_2 - 2ay_1 + \left(a^2 + b^2\right)y = 0$

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219. If $y = \left(x + \sqrt{x^2 + 1}\right)^m$ then prove that, $\left(x^2 + 1\right)y_2 + xy_1 = m^2y$

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220. If $2x = y^{\frac{1}{m}} + y^{-\frac{1}{m}}(n \geq 1)$ then prove that, $\left(x^2 - 1\right)y_2 + xy_1 = m^2y$

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221. Verify Rolle's theorem for the following functions:

$f(x) = \sqrt{9 - x^2}, x \in [-3, 3]$

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222. Verify Rolle's theorem for the following functions:

$f(x) = x^3 - 6x^2 + 11x - 6, x \in [2, 3]$

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223. Verify Rolle's theorem for the following functions:

$f(x) = x(x - 3)^2, x \in [0, 3]$

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224. Verify Rolle's theorem for the following functions:

$f(x) = \sin x + \cos x - 1, x \in \left[0, \frac{\pi}{2}\right]$

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225. Verify Rolle's theorem for the following functions:

$f(x) = a^{\sin x}, x \in [0, \pi], a > 0$

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226. Vertify mean value theorem for the following functions:

$f(x) = \log_e x, x \in [1, 2]$

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227. Vertify mean value theorem for the following functions:

$f(x) = x - 2\sin x, x \in [-\pi, \pi]$

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228. Vertify mean value theorem for the following functions:

$f(x) = x + \frac{1}{x}, x \in [1, 3]$

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229. Vertify mean value theorem for the following functions:

$f(x) = \tan^{-1}x, x \in [0, 1]$

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230. Vertify mean value theorem for the following functions:

$f(x) = x^2 + 2x + 3, x \in [4, 6]$

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231. Prove that $0 < a < b < \frac{\pi}{2}, \sec^2 a < \frac{\tan b - \tan a}{b - a} < \sec^2 b$

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232. $y = e^{x+e^{x+e^{x+\dots\infty}}}$ then find $\frac{dy}{dx}$

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233. Find the derivative of $\tan^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right]$ with respect to $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$

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234. $y = x + \cfrac{1}{x + \cfrac{1}{x + \cfrac{1}{x + \dots\infty}}}$ then find $\frac{dy}{dx}$

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235. Find the derivative of $\sin^{-1}\left[\frac{2^{x+1}.3^{x}}{1+(36)^{x}}\right]$ with respect to x.

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236. $x = a\sin t$ and $y = a\left(\cos t + \log\tan\frac{t}{2}\right)$ then find $\frac{d^2y}{dx^2}$.

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237. If $y = \frac{ax^2}{(x-a)(x-b)(x-c)} + \frac{bx}{(x-b)(x-c)} + \frac{c}{(x-c)} + 1$ then prove that $\frac{y'}{y} = \frac{1}{x}\left[\frac{a}{a-x} + \frac{b}{b-x} + \frac{c}{c-x}\right]$

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238. $y = \sin^{-1}\left[\frac{5x+12\sqrt{1-x^2}}{13}\right]$ then find $\frac{dy}{dx}$

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239. If $y = x\log\left(\frac{x}{a+bx}\right)$ prove that $\frac{d^2y}{dx^2} = \frac{1}{x}\left(\frac{a}{a+bx}\right)^2$

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240. If $y = \left\{x + \sqrt{x^2 + a^2}\right\}^n$ prove that $\frac{dy}{dx} = \frac{ny}{\sqrt{x^2 + a^2}}.\, n > 1 \neq N$

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241. $y = \sqrt{\frac{1 - \sin 2x}{1 + \sin 2x}}$ then prove that $\frac{dy}{dx} + \sec^2\left(\frac{\pi}{4} - x\right) = 0$

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242. Show that $\frac{d}{dx}e^{ax}\cos(bx + c) = re^{ax}\cos(bx + c + \alpha)$ where $r = \sqrt{a^2 + b^2}, \cos\alpha = \frac{a}{r}, \sin\alpha = \frac{b}{r}$ and $\frac{d^2}{dx^2}e^{ax}\cos(ax + c) = r^2e^{ax}\cos(bx + c + 2\alpha$

.

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243. If $(a - b\cos y)(a + b\cos x) = a^2 - b^2$ show that $\frac{dy}{dx} = \frac{\sqrt{a^2 - b^2}}{a + b\cos x}, 0 < x < \frac{\pi}{2}$

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244. Find $\frac{d}{dx}\left[\operatorname{cosec}^{-1}x\right]_{x=-2}$

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245. $\frac{d}{dx}\tan^{-1}(\sec x - \tan x)$- Find

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246. Prove that, $\frac{d}{dx}\left[\log\frac{x^2+x+1}{x^2-x+1}+\frac{2}{\sqrt{3}}\tan^{-1}\frac{\sqrt{3}x}{1-x^2}\right]=\frac{4}{x^4+x^2+1}$

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Exercise-5.1

1. Prove that the function $f(x)=5x-3$ is continuous at x=0, at $x=-3$ and at x= 5

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2. Examine the continuity of the function $f(x)=2x^2-1$ at $x=3$

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3. Examine the following functions for continuity.

$f(x) = x - 5$

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4. Examine the following functions for continuity.

$f(x) = \dfrac{1}{x - 5}, x \neq 5$

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5. Examine the following functions for continuity.

$f(x) = \dfrac{x^2 - 25}{x + 5}, x \neq -5$

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6. Examine the following functions for continuity.

$f(x) = |x - 5|$

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7. Prove that the function $f(x) = x^n$ is continuous at x= n, where n is a positive integer

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8. Is the function f defined by $f(x) = \begin{cases} x, & \text{if} x \leq 1 \\ 5, & \text{if} x > 1 \end{cases}$ continuous at $x = 0$? At x= 1 ? At x=2 ?

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9. $f(x) = \begin{cases} 2x + 3, & \text{if} x \leq 2 \\ 2x - 3, & \text{if} x > 2 \end{cases}$

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10. $f(x) = \begin{cases} |x| + 3, & \text{if} x \le -3 \\ -2x, & \text{if} -3 < x < 3 \\ 6x + 2, & \text{if } x \ge 3 \end{cases}$

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11. $f(x) = \begin{cases} \frac{|x|}{x}, & \text{if} x \ne 0 \\ 0, & \text{if} x = 0 \end{cases}$

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12. $f(x) = \begin{cases} \frac{x}{|x|}, & \text{if} x < 0 \\ -1, & \text{if} x \ge 0 \end{cases}$

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13. $f(x) = \begin{cases} x + 1, & \text{if} x \ge 1 \\ x^2 + 1, & \text{if } x < 1 \end{cases}$

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14. $f(x) = \begin{cases} x^3 - 3, & \text{if} x \leq 2 \\ x^2 + 1, & \text{if } x > 2 \end{cases}$

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15. $f(x) = \begin{cases} x^{10} - 1, & \text{if} x \leq 1 \\ x^2, & \text{if} x > 1 \end{cases}$

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16. Is the function defined by $f(x) = \begin{cases} x + 5, & \text{if} x \leq 1 \\ x - 5, & \text{if} x > 1 \end{cases}$ a continuous function?

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17. $f(x) = \begin{cases} 3, & \text{if} 0 \leq x \leq 1 \\ 4, & \text{if} 1 < x < 3 \\ 5, & \text{if} 3 \leq x \leq 10 \end{cases}$

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18. $f(x) = \begin{cases} 2x, & \text{if} x < 0 \\ 0, & \text{if} 0 \leq x \leq 1 \\ 4x, & \text{if} x > 1 \end{cases}$

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19. $f(x) = \begin{cases} -2, & \text{if} x \leq -1 \\ 2x, & \text{if} -1 < x \leq 1 \\ 2, & \text{if } x > 1 \end{cases}$

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20. Find the relationship between a and b so that the function f defined by

$f(x) = \begin{cases} ax + 1 & \text{if } x \leq 3 \\ bx + 3 & \text{if } x > 3 \end{cases}$ is continuous at x=3.

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21. For what value of λ is the function defined by

$f(x) = \begin{cases} \lambda\left(x^2 - 2x\right), & \text{if} x \leq 0 \\ 4x + 1, & \text{if} x > 0 \end{cases}$ continuous at x=0 ? What about continuity at x=1?

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22. Show that the function defined by $g(x) = x - [x]$ is discontinuous at all integral points. Here [x] denotes the greatest integer less than or equal to x.

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23. Is the function defined by $f(x) = x^2 - \sin x + 5$ continuous at $x = \pi$?

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24. Discuss the continuity of the following functions:

$f(x) = \sin x + \cos x$

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25. Discuss the continuity of the following functions:

$f(x) = \sin x - \cos x$

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26. Discuss the continuity of the following functions:

$f(x) = \sin x. \cos x$

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27. Discuss the continuity of

$f(x) = \cos x, x \in R$

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28. Discuss the continuity of the cosine, cosecant, secant and cotangent functions:

$f(x) = \operatorname{cosec}\ x = \frac{1}{\sin x}, x \in R - \{n\pi\}, n \in I$

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29. Discuss the continuity of the cosine, cosecant, secant and cotangent functions:

$f(x) = \sec x = \frac{1}{\cos x}, x \in R - \left\{(2n+1)\frac{\pi}{2}, n \in I\right\}$

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30. Discuss the continuity of the cosine, cosecant, secant and cotangent functions:

$f(x) = \cot x = \frac{1}{\tan x}, x \in R - \{n\pi, n \in I\}$

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31. Find all points of discontinuity of f, where $f(x) = \begin{cases} \frac{\sin x}{x}, & \text{if } x < 0 \\ x + 1, & \text{if} x \geq 0 \end{cases}$

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32. Determine if f defined by $f(x) = \begin{cases} x^2 \sin\frac{1}{x}, & \text{if} x \neq 0 \\ 0, & \text{if} x = 0 \end{cases}$ is a continuous function?

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33. Examine the continuity of f, where f is defined by

$$f(x) = \begin{cases} \sin x - \cos x, & \text{if} x \neq 0 \\ -1 & \text{if} x = 0 \end{cases}$$

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34. Find the values of k so that the function f is continuous at the indicated point

$$f(x) = \begin{cases} \frac{k\cos x}{\pi - 2x}, & \text{if} x \neq \frac{\pi}{2} \\ 3 & \text{if} x = \frac{\pi}{2} \end{cases} \text{ at } x = \frac{\pi}{2}$$

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35. Find the values of k so that the function f is continuous at the indicated point

$$f(x) = \begin{cases} kx^2, & \text{if} x \leq 2 \\ 3, & \text{if} x > 2 \end{cases}$$ at x=2

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36. Find the values of k so that the function f is continuous at the indicated point

$$f(x) = \begin{cases} kx + 1, & \text{if} x \leq \pi \\ \cos x & \text{if} x > \pi \end{cases} \text{ at } x = \pi$$

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37. Find the values of k so that the function f is continuous at the indicated point

$$f(x) = \begin{cases} kx + 1, & \text{if} x \leq 5 \\ 3x - 5, & \text{if} x > 5 \end{cases}$$ at x=5

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38. Find the values of a and b such that the function defined by

$$f(x) = \begin{cases} 5 & \text{if} x \leq 2 \\ ax + b, & \text{If } 2 < x < 10 \\ 21 & \text{If} x \geq 10 \end{cases}$$ is a continuous function

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39. Show that the function defined by $f(x) = \cos\left(x^2\right)$ is a continuous function.

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40. Show that the function defined by $f(x) = |\cos x|$ is a continuous function.

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41. Examine that $\sin|x|$ is a continuous function.

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42. Find all the points of discontinuity of f defined by $f(x) = |x| - |x + 1|$

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43. Prove that the function $f(x) = 5x - 3$ is continuous at x=0, at $x = -3$ and at x= 5

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44. Examine the continuity of the function $f(x) = 2x^2 - 1$ at $x = 3$

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45. Examine the following functions for continuity.

$f(x) = x - 5$

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46. Examine the following functions for continuity.

$$f(x) = \frac{1}{x - 5}, x \neq 5$$

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47. Examine the following functions for continuity.

$$f(x) = \frac{x^2 - 25}{x + 5}, x \neq -5$$

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48. Examine the following functions for continuity.

$$f(x) = |x - 5|$$

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49. Prove that the function $f(x) = x^n$ is continuous at x= n, where n is a positive integer

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50. Is the function f defined by $f(x) = \begin{cases} x, & \text{if} x \leq 1 \\ 5, & \text{if} x > 1 \end{cases}$ continuous at $x = 0$? At x= 1 ? At x=2 ?

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51. $f(x) = \begin{cases} 2x + 3, & \text{if} x \leq 2 \\ 2x - 3, & \text{if} x > 2 \end{cases}$

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52. $f(x) = \begin{cases} |x| + 3, & \text{if} x \leq -3 \\ -2x, & \text{if} -3 < x < 3 \\ 6x + 2, & \text{if } x \geq 3 \end{cases}$

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53. $f(x) = \begin{cases} \dfrac{|x|}{x}, & \text{if} x \neq 0 \\ 0, & \text{if} x = 0 \end{cases}$

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54. $f(x) = \begin{cases} \dfrac{x}{|x|}, & \text{if} x < 0 \\ -1, & \text{if} x \geq 0 \end{cases}$

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55. $f(x) = \begin{cases} x + 1, & \text{if} x \geq 1 \\ x^2 + 1, & \text{if } x < 1 \end{cases}$

Watch Video Solution

56. $f(x) = \begin{cases} x^3 - 3, & \text{if} x \leq 2 \\ x^2 + 1, & \text{if } x > 2 \end{cases}$

Watch Video Solution

57. $f(x) = \begin{cases} x^{10} - 1, & \text{if} x \le 1 \\ x^2, & \text{if} x > 1 \end{cases}$

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58. Is the function defined by $f(x) = \begin{cases} x + 5, & \text{if} x \le 1 \\ x - 5, & \text{if} x > 1 \end{cases}$ a continuous function?

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59. $f(x) = \begin{cases} 3, & \text{if} 0 \le x \le 1 \\ 4, & \text{if} 1 < x < 3 \\ 5, & \text{if} 3 \le x \le 10 \end{cases}$

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60. $f(x) = \begin{cases} 2x, & \text{if} x < 0 \\ 0, & \text{if} 0 \le x \le 1 \\ 4x, & \text{if} x > 1 \end{cases}$

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61. $f(x) = \begin{cases} -2, & \text{if} x \le -1 \\ 2x, & \text{if} -1 < x \le 1 \\ 2, & \text{if } x > 1 \end{cases}$

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62. Find the relationship between a and b so that the function f defined by

$f(x) = \begin{cases} ax + 1 & \text{if } x \le 3 \\ bx + 3 & \text{if } x > 3 \end{cases}$ is continuous at x=3.

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63. For what value of λ is the function defined by

$f(x) = \begin{cases} \lambda\left(x^2 - 2x\right), & \text{if} x \leq 0 \\ 4x + 1, & \text{if} x > 0 \end{cases}$ continuous at x=0 ? What about continuity at x=1?

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64. Show that the function defined by $g(x) = x - [x]$ is discontinuous at all integral points. Here [x] denotes the greatest integer less than or equal to x.

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65. Is the function defined by $f(x) = x^2 - \sin x + 5$ continuous at $x = \pi$?

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66. Discuss the continuity of the following functions:

$f(x) = \sin x + \cos x$

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67. Discuss the continuity of the following functions:

$f(x) = \sin x - \cos x$

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68. Discuss the continuity of the following functions:

$f(x) = \sin x . \cos x$

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69. Discuss the continuity of the cosine, cosecant, secant and cotangent functions:

$f(x) = \cos x, x \in R$

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70. Discuss the continuity of the cosine, cosecant, secant and cotangent functions:

$f(x) = \operatorname{cosec}\ x = \frac{1}{\sin x}, x \in R - \{n\pi\}, n \in I$

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71. Discuss the continuity of the cosine, cosecant, secant and cotangent functions:

$f(x) = \sec x = \frac{1}{\cos x}, x \in R - \left\{(2n+1)\frac{\pi}{2}, n \in I\right\}$

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72. Discuss the continuity of the cosine, cosecant, secant and cotangent functions:

$f(x) = \cot x = \frac{1}{\tan x}, x \in R - \{n\pi, n \in I\}$

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73. Find all points of discontinuity of f, where $f(x) = \begin{cases} \frac{\sin x}{x}, & \text{if } x < 0 \\ x + 1, & \text{if} x \geq 0 \end{cases}$

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74. Determine if f defined by $f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & \text{if} x \neq 0 \\ 0, & \text{if} x = 0 \end{cases}$ is a continuous function?

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75. Examine the continuity of f, where f is defined by

$f(x) = \begin{cases} \sin x - \cos x, & \text{if} x \neq 0 \\ -1 & \text{if} x = 0 \end{cases}$

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76. Find the values of k so that the function f is continuous at the indicated point

$$f(x)=\begin{cases}\frac{k\cos x}{\pi-2x}, & \text{if} x\neq\frac{\pi}{2}\\ 3 & \text{if} x=\frac{\pi}{2}\end{cases} \text{ at } x=\frac{\pi}{2}$$

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77. Find the values of k so that the function f is continuous at the indicated point

$$f(x)=\begin{cases}kx^2, & \text{if} x\leq 2\\ 3, & \text{if} x>2\end{cases} \text{ at x=2}$$

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78. Find the values of k so that the function f is continuous at the indicated point

$$f(x) = \begin{cases} kx + 1, & \text{if} x \leq \pi \\ \cos x & \text{if} x > \pi \end{cases} \text{ at } x = \pi$$

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79. Find the values of k so that the function f is continuous at the indicated point

$$f(x) = \begin{cases} kx + 1, & \text{if} x \leq 5 \\ 3x - 5, & \text{if} x > 5 \end{cases}$$ at x=5

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80. Find the values of a and b such that the function defined by

$$f(x) = \begin{cases} 5 & \text{if} x \leq 2 \\ ax + b, & \text{If } 2 < x < 10 \\ 21 & \text{If} x \geq 10 \end{cases}$$ is a continuous function

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81. Show that the function defined by $f(x) = \cos(x^2)$ is a continuous function.

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82. Show that the function defined by $f(x) = |\cos x|$ is a continuous function.

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83. Examine that $\sin|x|$ is a continuous function.

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84. Find all the points of discontinuity of f defined by $f(x) = |x| - |x+1|$

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Exercise-5.2

1. Differentiate the functions with respect to x in

$\sin\left(x^2 + 5\right)$

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2. Differentiate the functions with respect to x in

$\cos(\sin x)$

Watch Video Solution

3. Differentiate the functions with respect to x in

$\sin(ax + b)$

Watch Video Solution

4. Differentiate the functions with respect to x in

$\sec\left(\tan\left(\sqrt{x}\right)\right)$

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5. Differentiate the functions with respect to x in Exerecises 1 to 8.

$\frac{\sin(ax+b)}{\cos(cx+d)}$

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6. Differentiate the functions with respect to x in Exerecises 1 to 8.

$\cos x^3 . \sin^2\left(x^5\right).$

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7. Differentiate the functions with respect to x in

$2\sqrt{\cot\left(x^2\right)}$

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8. Differentiation of cos $\left(\sqrt{x}\right)$ with respect to x is

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9. Prove that the function f given by $f(x) = |x - 1|, x \in R$ is not differentiable at x=1

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10. Prove that the greatest integer function defined by $f(x) = [x], 0 < x < 3$ is not differentiable at x=1 and x=2.

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11. Differentiate the functions with respect to x in

$\sin\left(x^2 + 5\right)$

Watch Video Solution

12. Differentiate the functions with respect to x in

$\cos(\sin x)$

Watch Video Solution

13. Differentiate the functions with respect to x in

$\sin(ax + b)$

Watch Video Solution

14. Differentiate the functions with respect to x in

$\sec\left(\tan\left(\sqrt{x}\right)\right)$

Watch Video Solution

15. Differentiate the functions with respect to x in

$$\frac{\sin(ax+b)}{\cos(cx+d)}$$

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16. Differentiate the functions with respect to x in

$$\cos\left(x^3\right).\sin^2\left(x^5\right)$$

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17. Differentiate the functions with respect to x in

$$2\sqrt{\cot\left(x^2\right)}$$

Watch Video Solution

18. Differentiate the functions with respect to x in

$$\cos\left(\sqrt{x}\right)$$

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19. Prove that the function f given by $f(x) = |x - 1|, x \in R$ is not differentiable at x=1

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20. Prove that the greatest integer function defined by $f(x) = [x], 0 < x < 3$ is not differentiable at x=1 and x=2.

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Exercise-5.3

1. Find $\frac{dy}{dx}$ in the following

$2x + 3y = \sin x$

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2. Find $\frac{dy}{dx}$ in the following

$2x + 3y = \sin y$

Watch Video Solution

3. Find $\frac{dy}{dx}$ in the following

$ax + by^2 = \cos y$

Watch Video Solution

4. Find $\frac{dy}{dx}$ in the following

$xy + y^2 = \tan x + y$

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5. Find $\frac{dy}{dx}$ in the following

$x^2 + xy + y^2 = 100$

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6. Find $\frac{dy}{dx}$ in the following

$x^3 + x^2y + xy^2 + y^3 = 81$

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7. Find $\frac{dy}{dx}$ in the following

$\sin^2 y + \cos xy = k$

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8. Find $\frac{dy}{dx}$ in the following

$\sin^2 x + \cos^2 y = 1$

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9. Find $\frac{dy}{dx}$ in the following

$$y = \sin^{-1}\left(\frac{2x}{1 + x^2}\right)$$

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10. Find $\frac{dy}{dx}$ in the following:

$$y = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right), |x| < \frac{1}{\sqrt{3}}$$

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11. Find $\frac{dy}{dx}$ in the following

$$y = \cos^{-1}\left(\frac{1 - x^2}{1 + x^2}\right), 0 < x < 1$$

Watch Video Solution

12. Find $\frac{dy}{dx}$ in the following

$$y = \sin^{-1}\left(\frac{1 - x^2}{1 + x^2}\right), 0 < x < 1$$

Watch Video Solution

13. Find $\frac{dy}{dx}$ in the following

$$y = \cos^{-1}\left(\frac{2x}{1 + x^2}\right), \ -1 < x < 1$$

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14. Find $\frac{dy}{dx}$ in the following

$$y = \sin^{-1}\left(2x\sqrt{1 - x^2}\right), \ -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$$

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15. Find $\frac{dy}{dx}$ in the following

$$y = \sec^{-1}\left(\frac{1}{2x^2 - 1}\right), 0 < x < \frac{1}{\sqrt{2}}$$

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16. Find $\frac{dy}{dx}$ in the following

$2x + 3y = \sin x$

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17. Find $\frac{dy}{dx}$ in the following

$2x + 3y = \sin y$

Watch Video Solution

18. Find $\frac{dy}{dx}$ in the following

$ax + by^2 = \cos y$

Watch Video Solution

19. Find $\frac{dy}{dx}$ in the following

$xy + y^2 = \tan x + y$

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20. Find $\frac{dy}{dx}$ in the following

$x^2 + xy + y^2 = 100$

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21. Find $\frac{dy}{dx}$ in the following

$x^3 + x^2y + xy^2 + y^3 = 81$

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22. Find $\frac{dy}{dx}$ in the following

$\sin^2 y + \cos xy = k$

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23. Find $\frac{dy}{dx}$ in the following

$\sin^2 x + \cos^2 y = 1$

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24. Find $\frac{dy}{dx}$ in the following

$y = \sin^{-1}\left(\frac{2x}{1 + x^2}\right)$

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25. Find $\frac{dy}{dx}$ in the following:

$$y = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right), |x| < \frac{1}{\sqrt{3}}$$

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26. Find $\frac{dy}{dx}$ in the following

$$y = \cos^{-1}\left(\frac{1 - x^2}{1 + x^2}\right), 0 < x < 1$$

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27. Find $\frac{dy}{dx}$ in the following

$$y = \sin^{-1}\left(\frac{1 - x^2}{1 + x^2}\right), 0 < x < 1$$

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28. Find $\frac{dy}{dx}$ in the following

$$y = \cos^{-1}\left(\frac{2x}{1 + x^2}\right), \ -1 < x < 1$$

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29. Find $\frac{dy}{dx}$ in the following

$$y = \sin^{-1}\left(2x\sqrt{1 - x^2}\right), \ -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$$

Watch Video Solution

30. Find $\frac{dy}{dx}$ in the following

$$y = \sec^{-1}\left(\frac{1}{2x^2 - 1}\right), 0 < x < \frac{1}{\sqrt{2}}$$

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Exercise-5.4

1. Differentiate the following w.r.t. x:

$$\frac{e^x}{\sin x}$$

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2. Differentiate the following w.r.t. x:

$e^{\sin^{-1} x}$

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3. Differentiate the following w.r.t. x :

e^{x^3}.

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4. Differentiate the following w.r.t. x:

$\sin\left(\tan^{-1}e^{-x}\right)$

Watch Video Solution

5. Differentiate the following w.r.t. x:

$\log\left(\cos e^{x}\right)$

Watch Video Solution

6. Differentiate the following w.r.t. x:

$e^{x}+e^{x^{2}}+\ldots.+e^{x^{5}}$

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7. Differentiate the following w.r.t. x:

$\sqrt{e^{\sqrt{x}}}, x>0$

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8. Differentiate the following w.r.t. x:

$\log(\log x), x > 1$

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9. Differentiate the following w.r.t. x:

$\frac{\cos x}{\log x}, x > 0, x \neq 1$

Watch Video Solution

10. Differentiate the following w.r.t. x:

$\cos\left(\log x + e^x\right), x > 0$

Watch Video Solution

11. Differentiate the following w.r.t. x:

$$\frac{e^x}{\sin x}$$

Watch Video Solution

12. Differentiate the following w.r.t. x:

$$e^{\sin^{-1}x}$$

Watch Video Solution

13. Differentiate the following w.r.t. x :

$$e^{x^3}.$$

Watch Video Solution

14. Differentiate the following w.r.t. x:

$$\sin\left(\tan^{-1}e^{-x}\right)$$

Watch Video Solution

15. Differentiate the following w.r.t. x:

$\log\left(\cos e^{x}\right)$

Watch Video Solution

16. Differentiate the following w.r.t. x:

$e^{x}+e^{x^{2}}+\ldots.+e^{x^{5}}$

Watch Video Solution

17. Differentiate the following w.r.t. x:

$\sqrt{e^{\sqrt{x}}}, x>0$

Watch Video Solution

18. Differentiate the following w.r.t. x:

$\log(\log x), x > 1$

Watch Video Solution

19. Differentiate the following w.r.t. x:

$\frac{\cos x}{\log x}, x > 0, x \neq 1$

Watch Video Solution

20. Differentiate the following w.r.t. x:

$\cos\left(\log x + e^x\right), x > 0$

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Exercise-5.5

1. Differentiate the functions $\cos x.\ \cos 2x.\ \cos 3x$

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2. Differentiate the functions $\sqrt{\dfrac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$

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3. Differentiate the functions $(\log x)^{\cos x}$

Watch Video Solution

4. Differentiate the functions $x^{x} - 2^{\sin x}$

Watch Video Solution

5. Differentiate the functions $(x+3)^2.(x+4)^3.(x+5)^4$

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6. Differentiate the functions $\left(x+\frac{1}{x}\right)^x + x^{\left(1+\frac{1}{x}\right)}$

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7. Differentiate the functions $(\log x)^x + x^{\log x}$

Watch Video Solution

8. Differentiate the functions $(\sin x)^x + \sin^{-1}\sqrt{x}$

Watch Video Solution

9. Differentiate the functions $x^{\sin x} + (\sin x)^{\cos x}$

Watch Video Solution

10. Differentiate the functions $x^{x\cos x} + \frac{x^2 + 1}{x^2 - 1}$

Watch Video Solution

11. Differentiate the functions $(x\cos x)^x + (x\sin x)^{\frac{1}{x}}$

Watch Video Solution

12. Find $\frac{dy}{dx}$ of the functions $x^y + y^x = 1$

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13. Find $\frac{dy}{dx}$ of the functions $y^x = x^y$

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14. Find $\frac{dy}{dx}$ of the functions $(\cos x)^y = (\cos y)^x$

Watch Video Solution

15. Find $\frac{dy}{dx}$ of the functions given in Exercises 12 to 15.

$xy = e^{(x-y)}$.

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16. Find the derivative of the function given by $f(x) = (1 + x)\left(1 + x^2\right)\left(1 + x^4\right)\left(1 + x^8\right)$ and hence find f'(1)

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17. Differentiate $\left(x^2 - 5x + 8\right)\left(x^3 + 7x + 9\right)$

By using product rule

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18. Differentiate $\left(x^2 - 5x + 8\right)\left(x^3 + 7x + 9\right)$ in three ways mentioned below:

By expanding the product to obtain a single polynomial

Do they all give the same answer?

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19. Differentiate $\left(x^2 - 5x + 8\right)\left(x^3 + 7x + 9\right)$ in three ways mentioned below:

By logarithmic differentiation.

Do they all give the same answer?

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20. If u, v and w are functions of x, then show that $\frac{d}{dx}(u.v.w)=\frac{du}{dx}v.w+u.\frac{dv}{dx}.w+u.v.\frac{dw}{dx}$ in two ways- first by repeated application of product rule, second by logarithmic differentiation.

Using product rule

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21. If u, v and w are functions of x, then show that $\frac{d}{dx}(u.v.w)=\frac{du}{dx}v.w+u.\frac{dv}{dx}.w+u.v.\frac{dw}{dx}$ in two ways- first by repeated application of product rule, second by logarithmic differentiation.

Using product rule

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22. Differentiate the functions $\cos x.\cos 2x.\cos 3x$

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23. Differentiate the functions $\sqrt{\frac{(x - 1)(x - 2)}{(x - 3)(x - 4)(x - 5)}}$

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24. Differentiate the functions $(\log x)^{\cos x}$

Watch Video Solution

25. Differentiate the functions $x^{x} - 2^{\sin x}$

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26. Differentiate the functions $(x + 3)^{2}.(x + 4)^{3}.(x + 5)^{4}$

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27. Differentiate the functions $\left(x+\frac{1}{x}\right)^{x}+x^{\left(1+\frac{1}{x}\right)}$

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28. Differentiate the functions $(\log x)^{x}+x^{\log x}$

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29. Differentiate the functions $(\sin x)^{x}+\sin^{-1}\sqrt{x}$

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30. Differentiate the functions $x^{\sin x}+(\sin x)^{\cos x}$

Watch Video Solution

31. Differentiate the functions $x^{x\cos x}+\frac{x^2+1}{x^2-1}$

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32. Differentiate the functions $(x\cos x)^x+(x\sin x)^{\frac{1}{x}}$

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33. Find $\frac{dy}{dx}$ of the functions $x^y+y^x=1$

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34. Find $\frac{dy}{dx}$ of the functions $y^x=x^y$

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35. Find $\frac{dy}{dx}$ of the functions $(\cos x)^y = (\cos y)^x$

Watch Video Solution

36. Find $\frac{dy}{dx}$ of the functions given in Exercises 12 to 15.

$xy = e^{(x-y)}$.

Watch Video Solution

37. Find the derivative of the function given by $f(x) = (1 + x)\left(1 + x^2\right)\left(1 + x^4\right)\left(1 + x^8\right)$ and hence find f'(1)

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38. Differentiate $\left(x^2 - 5x + 8\right)\left(x^3 + 7x + 9\right)$

By using product rule

Watch Video Solution

39. Differentiate $\left(x^2 - 5x + 8\right)\left(x^3 + 7x + 9\right)$ in three ways mentioned below:

By expanding the product to obtain a single polynomial

Do they all give the same answer?

Watch Video Solution

40. Differentiate $\left(x^2 - 5x + 8\right)\left(x^3 + 7x + 9\right)$ in three ways mentioned below:

By logarithmic differentiation.

Do they all give the same answer?

Watch Video Solution

41. If u, v and w are functions of x, then show that $\frac{d}{dx}(u.v.w) = \frac{du}{dx}v.w + u.\frac{dv}{dx}.w + u.v.\frac{dw}{dx}$ in two ways- first by repeated

application of product rule, second by logarithmic differentiation.

Using product rule

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42. If u, v and w are functions of x, then show that $\frac{d}{dx}(u.\,v.\,w) = \frac{du}{dx}v.\,w + u.\,\frac{dv}{dx}.\,w + u.\,v.\,\frac{dw}{dx}$ in two ways- first by repeated application of product rule, second by logarithmic differentiation.

Using product rule

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Exercise-5.6

1. If x and y are connected parametrically by the equations without eliminating the parameter, find $\frac{dy}{dx}$

$x = 2\text{at}^2, y = \text{at}^4$

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2. If x and y are connected parametrically by the equations without eliminating the parameter, find $\frac{dy}{dx}$

$x = a\cos\theta, y = b\cos\theta$

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3. If x and y are connected parametrically by the equations without eliminating the parameter, find $\frac{dy}{dx}$

$x = \sin t, y = \cos 2t$

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4. If x and y are connected parametrically by the equations without eliminating the parameter, find $\frac{dy}{dx}$

$x = 4t, y = \frac{4}{t}$

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5. If x and y are connected parametrically by the equations without eliminating the parameter, find $\frac{dy}{dx}$

$x = \cos\theta - \cos 2\theta, y = \sin\theta - \sin 2\theta$

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6. If x and y are connected parametrically by the equations given in Exercises 1 to 10, without eliminating the parameter, Find $\frac{dy}{dx}$.

$x = a(\theta - \sin\theta), y = a(1 + \cos\theta)$.

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7. If x and y are connected parametrically by the equations given in Exercises 1 to 10, without eliminating the parameter, Find $\frac{dy}{dx}$.

$x = \frac{\sin^3 t}{\sqrt{\cos 2t}}, y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$.

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8. The length of tangent to the curve $x = a\left(\cos t + \log\tan.\frac{t}{2}\right), y = a(\sin t)$, is

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9. If x and y are connected parametrically by the equations given in Exercises 1 to 10, without eliminating the parameter, Find $\frac{dy}{dx}$.

$x = a\sec\theta, y = b\tan\theta$.

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10. If x and y are connected parametrically by the equations without eliminating the parameter, find $\frac{dy}{dx}$

$x = a(\cos\theta + \theta\sin\theta), y = a(\sin\theta - \theta\cos\theta)$

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11. If $x = \sqrt{a^{\sin^{-1}t}}, y = \sqrt{a^{\cos^{-1}t}}, a > 0$ and $-1 < t < 1.$ show that $\frac{dy}{dx} = -\frac{y}{x},$

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12. If x and y are connected parametrically by the equations without eliminating the parameter, find $\frac{dy}{dx}$

$x = 2\mathrm{at}^2, y = \mathrm{at}^4$

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13. If x and y are connected parametrically by the equations without eliminating the parameter, find $\frac{dy}{dx}$

$x = a\cos\theta, y = b\cos\theta$

Watch Video Solution

14. If x and y are connected parametrically by the equations without eliminating the parameter, find $\frac{dy}{dx}$

$x = \sin t, y = \cos 2t$

Watch Video Solution

15. If x and y are connected parametrically by the equations without eliminating the parameter, find $\frac{dy}{dx}$

$x = 4t, y = \frac{4}{t}$

Watch Video Solution

16. If x and y are connected parametrically by the equations without eliminating the parameter, find $\frac{dy}{dx}$

$x = \cos\theta - \cos 2\theta, y = \sin\theta - \sin 2\theta$

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17. If x and y are connected parametrically by the equations given in Exercises 1 to 10, without eliminating the parameter, Find $\frac{dy}{dx}$.

$x = a(\theta - \sin\theta), y = a(1 + \cos\theta)$.

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18. If x and y are connected parametrically by the equations given in Exercises 1 to 10, without eliminating the parameter, Find $\frac{dy}{dx}$.

$x = \frac{\sin^3 t}{\sqrt{\cos 2t}}, y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$.

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19. If x and y are connected parametrically by the equations without eliminating the parameter, find $\frac{dy}{dx}$

$x = a\left(\cos t + \log\tan\frac{t}{2}\right), y = a\sin t$

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20. If x and y are connected parametrically by the equations given in Exercises 1 to 10, without eliminating the parameter, Find $\frac{dy}{dx}$.

$x = a\sec\theta, y = b\tan\theta.$

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21. If x and y are connected parametrically by the equations without eliminating the parameter, find $\frac{dy}{dx}$

$x = a(\cos\theta + \theta\sin\theta), y = a(\sin\theta - \theta\cos\theta)$

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22. If $x = \sqrt{a^{\sin^{-1}t}}, y = \sqrt{a^{\cos^{-1}t}}$, show that $\frac{dy}{dx} = -\frac{y}{x}$

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Exercise-5.7

1. Find the second order derivatives of the functions

$x^2 + 3x + 2$

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2. Find the second order derivatives of the functions

x^{20}

Watch Video Solution

3. Find the second order derivatives of the functions

$x.\cos x$

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4. Find the second order derivatives of the functions

log x

Watch Video Solution

5. Find the second order derivatives of the functions given in Exercises 1 to 10.

$x^3 \log x.$

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6. Find the second order derivatives of the functions

$e^x \sin 5x$

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7. Find the second order derivatives of the functions given in Exercises 1 to 10.

$e^{6x} \cos 3x.$

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8. Find the second order derivatives of the functions

$\tan^{-1} x$

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9. Find the second order derivatives of the functions

log (log x)

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10. Find the second order derivatives of the functions

sin (log x)

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11. If $y = 5\cos x - 3\sin x$, prove that $\frac{d^2y}{dx^2} + y = 0$

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12. If $y = \cos^{-1}x$, Find $\frac{d^2y}{dx^2}$ in terms of y alone

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13. If $y = 3\cos(\log x) + 4\sin(\log x)$, show that $x^2y_2 + xy_1 + y = 0$

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14. If $y = Ae^{mx} + Be^{nx}$, show that $\frac{d^2y}{dx^2} - (m + n)\frac{dy}{dx} + mny = 0$

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15. If $y = 500e^{7x} + 600e^{-7x}$, show that $\frac{d^2y}{dx^2} = 49y$

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16. If $e^{y}(x+1)=1$, show that $\frac{d^2y}{dx^2}=\left(\frac{dy}{dx}\right)^2$

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17. If $y=\left(\tan^{-1}x\right)^2$ show that $\left(x^2+1\right)^2y_2+2x\left(x^2+1\right)y_1=2$

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18. Find the second order derivatives of the functions

x^2+3x+2

Watch Video Solution

19. Find the second order derivatives of the functions

x^{20}

Watch Video Solution

20. Find the second order derivatives of the functions

$x.\cos x$

Watch Video Solution

21. Find the second order derivatives of the functions

log x

Watch Video Solution

22. Find the second order derivatives of the functions

$x^3 \log x$

Watch Video Solution

23. Find the second order derivatives of the functions

$e^x \sin 5x$

Watch Video Solution

Watch Video Solution

24. Find the second order derivatives of the functions

$e^{6x}\cos 3x$

Watch Video Solution

25. Find the second order derivatives of the functions

$\tan^{-1}x$

Watch Video Solution

26. Find the second order derivatives of the functions

log (log x)

Watch Video Solution

27. Find the second order derivatives of the functions

sin (log x)

Watch Video Solution

28. If $y = 5\cos x - 3\sin x$, prove that $\frac{d^2y}{dx^2} + y = 0$

Watch Video Solution

29. If $y = \cos^{-1}x$, Find $\frac{d^2y}{dx^2}$ in terms of y alone

Watch Video Solution

30. If $y = 3\cos(\log x) + 4\sin(\log x)$, show that $x^2y_2 + xy_1 + y = 0$

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31. If $y = Ae^{mx} + Be^{nx}$, show that $\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = 0$

Watch Video Solution

32. If $y = 500e^{7x} + 600e^{-7x}$, show that $\frac{d^2y}{dx^2} = 49y$

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33. If $e^y(x+1) = 1$, show that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$

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34. If $y = \left(\tan^{-1}x\right)^2$ show that $\left(x^2+1\right)^2 y_2 + 2x\left(x^2+1\right)y_1 = 2$

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Exercise-5.8

1. Verify Rolle's theorem for the function $f(x) = x^2 + 2x - 8, x \in [-4, 2]$

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2. Examine if Rolle's theorem is applicable to any of the following functions.

$f(x) = [x], x \in [5, 9]$

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3. Examine if Rolle's theorem is applicable to any of the following functions.

$f(x) = [x], x \in [-2, 2]$

Watch Video Solution

4. Examine if Rolle's theorem is applicable to any of the following functions. Can you say some thing about the converse of Rolle's theorem from these example?

$f(x) = x^2 - 1$ for $x \in [1, 2]$.

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5. If $f : [-5, 5] \to R$ is a differentiable function and if f'(x) does not vanish anywhere, then prove that $f(-5) \neq f(5)$

Watch Video Solution

6. Verify Mean Value Theorem, if $f(x) = x^2 - 4x - 3$ in the interval [a, b] where a=1 and b= 4.

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7. Verify Mean Value Theorem, if $f(x) = x^3 - 5x^2 - 3x$ in the interval [a, b], where a=1 and b=3. Find all $c \in (1, 3)$ for which f'(c)= 0

Watch Video Solution

8. Examine the applicability of Mean Value Theorem for all three functions

$f(x) = [x], x \in [5, 9]$

Watch Video Solution

9. Examine the applicability of Mean Value Theorem for all three functions

$f(x) = [x], x \in [-2, 2]$

Watch Video Solution

10. Examine the applicability of Mean Value Theorem for all three functions

$f(x) = x^2 - 1, x \in [1, 2]$

Watch Video Solution

11. Verify Rolle's theorem for the function $f(x) = x^2 + 2x - 8, x \in [-4, 2]$

Watch Video Solution

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$f(x) = [x], x \in [-2, 2]$

Watch Video Solution

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$f(x) = x^2 - 1$ for $x \in [1, 2]$.

Watch Video Solution

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Watch Video Solution

17. Verify Mean Value Theorem, if $f(x) = x^3 - 5x^2 - 3x$ in the interval [a, b], where a=1 and b=3. Find all $c \in (1, 3)$ for which f'(c)= 0

Watch Video Solution

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$f(x) = [x], x \in [5, 9]$

Watch Video Solution

19. Examine the applicability of Mean Value Theorem for all three functions

$f(x) = [x], x \in [-2, 2]$

Watch Video Solution

20. Examine the applicability of Mean Value Theorem for all three functions

$f(x) = x^2 - 1, x \in [1, 2]$

Watch Video Solution

Miscellaneous Exercise - 5

1. Differentiate w.r.t.x the function in Exercises 1 to 11.

$\left(3x^2 - 9x + 5\right)^9$

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2. Differentiate w.r.t x the function $\sin^3 x + \cos^6 x$

Watch Video Solution

3. Differentiate w.r.t x the function $(5x)^{3\cos 2x}$

Watch Video Solution

4. Differentiate w.r.t x the function $\sin^{-1}\left(x\sqrt{x}\right)$, where $0 \le x \le 1$

Watch Video Solution

5. Differentiate w.r.t x the function $\dfrac{\cos^{-1}\frac{x}{2}}{\sqrt{2x+7}}$ where $-2 \le x \le 2$

Watch Video Solution

6. Differentiate w.r.t x the function $0 < x < \dfrac{\pi}{2}, \cot^{-1}\left[\dfrac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]$

Watch Video Solution

7. Differentiate w.r.t x the function $(\log x)^{\log x}, x > 1$

Watch Video Solution

8. Differentiate w.r.t x the function $\cos(a\cos x + b\sin x)$, for some constant a and b

Watch Video Solution

9. Differentiate w.r.t x the function $\frac{\pi}{4} < x < \frac{3\pi}{4}, (\sin x - \cos x)^{(\sin x - \cos x)}$

Watch Video Solution

10. Differentiate w.r.t x the function $x^x + x^a + a^x + a^a$, for some fixed $a > 0$ and $x > 0$.

Watch Video Solution

11. Differentiate w.r.t x the function $x^{x^2-3}+(x-3)^{x^2}$, for $x>3$

Watch Video Solution

12. Find $\frac{dy}{dx}$, if $y=12(1-\cos t), x=10(t-\sin t), -\frac{\pi}{2}<t<\frac{\pi}{2}$

Watch Video Solution

13. Find $\frac{dy}{dx}$, If $y=\sin^{-1}x+\sin^{-1}\sqrt{1-x^2}, 0\le x\le 1$

Watch Video Solution

14. If $x\sqrt{1+y}+y\sqrt{1+x}=0$ then $\frac{dy}{dx}$ equals.

Watch Video Solution

15. If $(x - a)^2 + (y - b)^2 = c^2$, for some $c > 0$, prove that $\dfrac{\left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\dfrac{d^2y}{dx^2}}$ is a constant independent of a and b.

Watch Video Solution

16. If $\cos y = x\cos(a + y)$, with $\cos a \neq \pm 1$, prove that $\dfrac{dy}{dx} = \dfrac{\cos^2(a + y)}{\sin a}$

Watch Video Solution

17. If $x = a(\cos t + t\sin t)$ and $y = a(\sin t - t\cos t)$, find $\dfrac{d^2y}{dx^2}$.

Watch Video Solution

18. If $f(x) = |x|^3$, show that f''(x) exists for all real x and find it

Watch Video Solution

19. Using mathematical induction prove that $\frac{d}{dx}\left(x^n\right) = nx^{n-1}$ for all positive integers n.

Watch Video Solution

20. Using the fact that $\sin(A + B) = \sin A.\cos B + \cos A \sin B$ and the differentiation, obtain the sum formula for cosines.

Watch Video Solution

21. Does there exist a function which is continuous everywhere but not differentiable at exactly two points ? Justify your answer.

Watch Video Solution

22. If $y = \begin{vmatrix} f(x) & g(x) & h(x) \\ l & m & n \\ a & b & c \end{vmatrix}$, prove that $\frac{dy}{dx} = \begin{vmatrix} f'(x) & g'(x) & h'(x) \\ l & m & n \\ a & b & c \end{vmatrix}$

Watch Video Solution

23. If $y = e^{a\cos^{-1}x}$ show that $\left(1 - x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$. Where $-1 \le x \le 1$

Watch Video Solution

24. Differentiate w.r.t x the function $\left(3x^2 - 9x + 5\right)^9$

Watch Video Solution

25. Differentiate w.r.t x the function $\sin^3 x + \cos^6 x$

Watch Video Solution

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Watch Video Solution

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Watch Video Solution

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Watch Video Solution

36. Find $\frac{dy}{dx}$, If $y = \sin^{-1}x + \sin^{-1}\sqrt{1-x^2}, 0 \le x \le 1$

Watch Video Solution

37. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, for $-1 < x < 1$, prove that $\frac{dy}{dx} = -\frac{-1}{(1+x)^2}$

Watch Video Solution

38. If $(x-a)^2+(y-b)^2=c^2$, for some $c>0$, prove that $\dfrac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$ is a constant independent of a and b.

Watch Video Solution

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Watch Video Solution

40. If $x=a(\cos t + t\sin t)$ and $y=a(\sin t - t\cos t)$, find $\dfrac{d^2y}{dx^2}$.

Watch Video Solution

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Watch Video Solution

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Watch Video Solution

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Watch Video Solution

46. If $y = e^{a\cos^{-1}x}$ show that $\left(1 - x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$. Where $-1 \leq x \leq 1$

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Textbook based MCQs

1. If $y = \log\left(x + \sqrt{x^2 + a^2}\right)$ then $\frac{dy}{dx}$ = ...

A. (a) $\sqrt{x^2 + a^2}$

B. (b) $\frac{1}{\sqrt{x^2 + a^2}}$

C. (c) $x^2 + a^2$

D. (d) $\dfrac{1}{x^2+a^2}$

Answer: B

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2. If $y = \log_{10}\sin x$ then $\dfrac{dy}{dx}$ = ...

A. cot x

B. cotx. $\log_e 10$

C. cot x. $\log_{10} e$

D. $\log_{10}\cot x$

Answer: C

Watch Video Solution

3. $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \ldots\ldots \infty}}}$ then $\dfrac{dy}{dx}$=.........

A. $\dfrac{\cos x}{2y - 1}$

B. $\cos x(2y + 1)$

C. $\dfrac{\cos x}{2y + 1}$

D. give not

Answer: A

Watch Video Solution

4. $f(x) = x^2 e^{2(x-1)}, 0 < x < 1 = a\operatorname{sgn}(x + 1)\cos(2x - 2) + bx^2, 1 < x \leq 2.$ If a function f(x) is differentiable at x=1 then.

A. $a = -1, b = 2$

B. $a = 1, b = -2$

C. $a = -3, b = 4$

D. $a = 3, b = -4$

Answer: A

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5. The value of f(0), so that $f(x)=\dfrac{\sqrt{a^2-ax+x^2}-\sqrt{a^2+ax+x^2}}{\sqrt{a+x}-\sqrt{a-x}}$ becomes continuous for all x, is given by ....

A. $a\sqrt{a}$

B. $\sqrt{a}$

C. $-\sqrt{a}$

D. $-a\sqrt{a}$

Answer: C

Watch Video Solution

6. $f(x+y)=f(x)f(y)$, For $\forall x$ and y. If f(3)= 3 and f'(0) =11 then f'(3)= ....

A. 22

B. 44

C. 28

D. None of these

Answer: D

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7. $f(x) = [x] + \sqrt{x - [x]}$, where [.] is a greatest integer function then .... (a) f(x) is continuous in R+ (b) f(x) is continuous in R (C) f(x) is continuous in R - 1 (d) None of these

A. f(x) is continuous in R^+

B. f(x) is continuous in R

C. f(x) is continuous in $R - 1$

D. None of these

Answer: B

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8. The function $f(x) = (\sin 3x)^{\tan^2 3x}$ is continuous at $x = \frac{\pi}{6}$ then $f\left(\frac{\pi}{6}\right)$= ......

A. $e^{-\frac{1}{2}}$

B. $\frac{1}{e}$

C. $e^{(d)}$

D. $e^{\frac{1}{2}}$

Answer: A

Watch Video Solution

9. $y^2 = ax^2 + bx + c$ then $y^3 \frac{d^2y}{dx^2}$ is a .... function

A. constant

B. only for x

C. only for y

D. for x and y

Answer: A

Watch Video Solution

10. $f(x) = x + \tan x$ and f is an inverse function of g then g'(x)= ....

A. (a) $\dfrac{1}{1 + (g(x) - x)^2}$

B. (b) $\dfrac{1}{1 - (g(x) - x)^2}$

C. (c) $\dfrac{1}{2 + (g(x) - x)^2}$

D. (d) $\dfrac{1}{2 - (g(x) - x)^2}$

Answer: C

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11. If $y^2 = p(x)$ the is polynomial of order 3, then $2\frac{d}{dx}\left[y^3\frac{d^2y}{dx^2}\right] =$ ....

A. $p'''(x) + p'(x)$

B. $p''(x).\ p'''(x)$

C. $p(x).\ p'''(x)$

D. $p(x) + p'''(x)$

Answer: C

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12. If $x^2 + y^2 = t - \frac{1}{t}$ and $x^4 + y^4 = t^2 + \frac{1}{t^2}$ then $x^3y\frac{dy}{dx} =$ ......

A. -1

B. 0

C. 1

D. None of these

Answer: C

Watch Video Solution

13. If $\sin y = x\sin(a + y)$ and $\frac{dy}{dx} = \frac{A}{1 + x^2 - 2x\cos a}$ then the value of A is ......

[a] 2 [b] CosA [c] SinA [d] 1/2

A. 2

B. cos a

C. sin a

D. $\frac{1}{2}$

Answer: C

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14. $f(x + y) = f(x) + f(y)$, for $\forall x$ and y and $f(x) = \left(2x^2 + 3x\right)g(x)$. For $\forall x$. If g(x) is a continuous function and g(0)= 3 then f'(x) = ...

A. 9

B. 3

C. 6

D. None of these

Answer: A

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15. $y = \sin x - \cos x$ and $f(x) = \dfrac{d^{17}y}{dx^{17}}$ then $f\left(\dfrac{\pi}{4}\right)$= .... [a] √2 [b] 1/√2 [c] (√2)^17 [d] 0

A. $\sqrt{2}$

B. $\dfrac{1}{\sqrt{2}}$

C. $\left(\sqrt{2}\right)^{17}$

D. 0

Answer: A

Watch Video Solution

16. $y = \tan^{-1}\left(\frac{ax - b}{bx + a}\right)$ then $\frac{dy}{dx}|_{x=-1}$= ....

A. $\frac{1}{2}$

B. a

C. ab

D. $\frac{b}{a}$

Answer: A

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17. $u = f(\tan x), v = g(\sec x), f'(1) = 2$ and $g'\left(\sqrt{2}\right) = 4$ then $\frac{du}{dv}|_{x=\frac{\pi}{4}}$=..........

A. $\sqrt{2}$

B. 2

C. $\frac{1}{\sqrt{2}}$

D. $\frac{1}{2}$

Answer: C

Watch Video Solution

18. $f(x) = \sin^2 x + \sin^2\left(x + \frac{\pi}{3}\right) + \cos x \cos\left(x + \frac{\pi}{3}\right)$ and g(5 / 4)' =1 then (gof) (x) = ...

A. 1

B. $\cos^2 x$

C. 0

D. sin 2x

Answer: C

Watch Video Solution

19. $y = e^{3x+7}$ then $y_n(0) =$ ....

A. 1

B. 3^n

C. $3^n e^7$

D. $3^n.\left(e^7.7\right)$

Answer: C

Watch Video Solution

20. $x = f(t), y = \phi(t)$ then $\dfrac{d^2y}{dx^2}$ = ...

A. $\dfrac{f_1(t)\phi_2(t) - \phi_1(t)f_2(t)}{\left(f_1(t)\right)^2}$

B. $\dfrac{f_1(t)\phi_2(t) - \phi_1(t)f_2(t)}{\left(f_1(t)\right)^3}$

C. $\dfrac{\phi_1(t)f_2(t) - f_1(t)\phi_2(t)}{\left(f_1(t)\right)^3}$

D. None of these

Answer: B

Watch Video Solution

21. In the function $f(x) = 2x^3 + bx^2 + qx$ satisfies conditions of Rolle's theorem in $[-1, 1]$ and $c = \frac{1}{2}$ then the value of 2b + q is.......

A. 0

B. 1

C. $-\frac{1}{2}$

D. -1

Answer: D

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22. If $y = \frac{a + bx}{c + dx}$, where a, b, c, d are constants and $\lambda y_1 y_3 = \mu y_2^2$ then the value of μ^{λ^2} is .... where y_1, y_2, y_3 are respectively. First, second and third derivatives of y.

A. 42

B. 81

C. 64

D. 27

Answer: B

Watch Video Solution

23. The function $f(x) = \frac{2 - \sqrt[4]{x^2 + 16}}{\cos 2x - 1}$ is continuous at x=0 then f(0)= ....

A. $\frac{1}{8}$

B. $\frac{1}{64}$

C. $\frac{1}{32}$

D. $\frac{1}{2}$

Answer: B

Watch Video Solution

24. Let $f(x) = x^3 - x^2 + x + 1$ g(x) = max I $\{f(t), 0 \le t \le x\}, 0 \le x \le 1 = 3 - x, 1 < x \le 2$

Then in [0, 2] the points where g(x) is not differentiable is.........

A. 0

B. 1

C. 2

D. None of these

Answer: B

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25. $f(x) = \begin{vmatrix} \sin x & \cos x \\ \tan x & \cot x \end{vmatrix}$ then $f'\left(\frac{\pi}{4}\right)$= ....

A. 0

B. $-\sqrt{2}$

C. $-2\sqrt{2}$

D. $\sqrt{2}$

Answer: D

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26. $x = t^2 + 3t - 8, y = 2t^2 - 2t - 4$. If at point $(2, -1), \lambda = \frac{dy}{dx}$ then the value of λ= ....

A. 2

B. $\frac{6}{7}$

C. -6

D. 7

Answer: B

View Text Solution

27. $x = 2 + t^3, y = 2t^2$. If $\dfrac{\frac{d^2y}{dx^2}}{\left(\frac{dy}{dx}\right)^n}$ is constant then n= ....

A. 4

B. 1

C. 0

D. 3

Answer: A

Watch Video Solution

28. If $F(x) = \frac{1}{x^2}\int^{x} 4\left(4t^2 - 2F'(t)\right)dt$ then F'(4) equals to ....

A. $\frac{32}{9}$

B. $\frac{64}{3}$

C. $\frac{64}{9}$

D. $\frac{32}{3}$

Answer: A

Watch Video Solution

29. If $\sqrt{x^2 + y^2} = a.\, e^{\tan^{-1}\left(\frac{y}{x}\right)}, a > 0$ then the value of y''(0) is.......

A. $\frac{a}{2}e^{-\frac{\pi}{2}}$

B. $ae^{\frac{\pi}{2}}$

C. $\frac{-2}{a}e^{-\frac{\pi}{2}}$

D. Does not exist

Answer: C

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30. If $f(x-y), f(x).f(y)$ and $f(x+y)$ are in arithmatic progression and $f(0)\neq 0$ then (for $\forall x$ and y.........

A. $f(2)+f'(2)=0$

B. $f'(2)+f'(-2)=0$

C. $f(2)-f'(-2)=0$

D. $f'(2)-f'(-2)=0$

Answer: B

Watch Video Solution

31. $\left[\dfrac{d}{dx}\sec^{-1}x\right]_{x=-3}$ =.......

A. $\frac{1}{\sqrt{x^2 - 1}}$

B. $-\frac{1}{\sqrt{x^2 - 1}}$

C. $\frac{1}{6\sqrt{2}}$

D. $-\frac{1}{6\sqrt{2}}$

Answer: C

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32. $\frac{d}{dx}\left(x^x\right) = \ldots. (x > 0)$

A. x^{x-1}

B. x^x

C. 0

D. $x^x(1 + \log x)$

Answer: D

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33. $\frac{d}{dx}\left(\sin^{-1}x + \cos^{-1}x\right) = \text{........}(|x| < 1)$

A. 0

B. $\frac{2}{\sqrt{1 - x^2}}$

C. $\frac{1}{\sqrt{1 - x^2}}$

D. Does not exist

Answer: A

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34. $\frac{d}{dx}\left(a^a\right) = \ldots\ldots(a > 0)$

A. $a^a(1 + \log a)$

B. 0

C. a^a

D. Does not exist

Answer: B

Watch Video Solution

35. $\frac{d}{dx}\left(e^{5x}\right) = \ldots\ldots\ldots$

A. e^{5x}

B. $5e^{5x}$

C. $5x.\ e^{5x-1}$

D. 0

Answer: B

Watch Video Solution

36. $\frac{d}{dx}(\log|x|)=$ ....$(x \neq 0)$

A. $\frac{1}{|x|}$

B. $\frac{1}{x}$

C. does not exist

D. e^{x}

Answer: B

Watch Video Solution

37. $\frac{d}{dx}\left(\sin^{3}x\right)=$ ....

A. $3\sin^{2}x$

B. $3\cos^{2}x$

C. $3\sin^{2}x.\cos x$

D. $-3\cos^{2}x\sin x$

Answer: C

Watch Video Solution

38. $\frac{d}{dx}\left(\tan^n x\right)$= ....

A. $n\tan^{n-1}x$

B. $n\tan^{n-1}x.\sec^2 x$

C. $n\sec^{2n}x$

D. $n\tan^{n-1}x.\sec^{n-1}x$

Answer: B

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39. If $f(x) = \begin{cases} ax+b & 1 \le x < 5 \\ 7x-5 & 5 \le x < 10 \\ bx+3a & x \ge 10 \end{cases}$ is continuous, (a,b)= ....

A. (5, 10)

B. (5,5)

C. (10, 5)

D. (0, 0)

Answer: B

Watch Video Solution

40. If $f(x) = \begin{cases} \frac{x^2}{a} - a, & x < a \\ 0, & x = a \\ a - \frac{x^2}{a}, & x > a \end{cases}$ then, ...

A. $\lim_{x \to a^+} f(x) = a$

B. $\lim_{x \to a} f(x) = -a$

C. f is continuous at x=a

D. f is differentiable at x=a

Answer: C

Watch Video Solution

41. If $f(x) = \begin{cases} x, & x \in \{0, 1\} \\ 1, & x \geq 1 \end{cases}$ then, ...

A. f is continuous at x=1 only

B. f is discontinuous at x=1 only

C. f is continuous on R^{+}

D. f is not defined for x=1

Answer: C

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42. $\frac{d}{dx}.\left(\frac{1}{\log|x|}\right)=$ ....

A. $\frac{1}{|x|}$

B. $\frac{1}{(\log x)^2}$

C. $\frac{-1}{x(\log|x|)^2}$

D. e^x

Answer: C

Watch Video Solution

43. If $y = a\sin x + b\cos x$ then, $y^2 + \left(y_1\right)^2 =$ .... $\left(a^2 + b^2 \neq 0\right)$

A. $a\cos x - b\sin x$

B. $(a\sin x - b\cos x)^2$

C. $a^2 + b^2$

D. 0

Answer: C

Watch Video Solution

44. $\frac{d}{dx}\left(x^2 + \sin^2 x\right)^3$ =.......

A. $3\left(x^2 + \sin^2 x\right)$

B. $3\left(x^2 + \sin^2 x\right)^2(2x + \sin 2x)$

C. $2x + 2\sin x\cos x$

D. 0

Answer: B

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45. $\frac{d}{dx}\left(\sqrt{x\sin x}\right)$= ...... $(0 < x < \pi)$

A. $\frac{x\sin x + \cos x}{\sqrt{x\sin x}}$

B. $\dfrac{x\cos x}{2\sqrt{x\sin x}}$

C. $\dfrac{x\cos x + \sin x}{2\sqrt{x\sin x}}$

D. $\dfrac{1}{2\sqrt{x\sin x}}$

Answer: C

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46. $\dfrac{d}{dx}\left(e^{\sin^{-1}x+\cos^{-1}x}\right) = \ldots\ldots.(|x| < 1)$

A. $\dfrac{2}{\sqrt{1-x^2}}$

B. 0

C. $\dfrac{1}{\sqrt{1-x^2}}$

D. $e^{\sin^{-1}x+\cos^{-1}x}$

Answer: B

Watch Video Solution

47. If function f(x) is continuous at x= 0, $f(x) = \begin{cases} \frac{\sin(4x)}{9x}, & x \neq 0 \\ k^2 & x = 0 \end{cases}$ then k= ....

A. $-\frac{3}{2}$

B. $\frac{3}{2}$

C. $\pm\frac{2}{3}$

D. $\frac{4}{9}$

Answer: C

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48. If $x = at^2, y = 2at$ then $\frac{dy}{dx}$ = ..., where $t \neq 0$

A. $\frac{1}{t}$

B. t

C. $-t$

D. a

Answer: A

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49. $\frac{d}{dx}\left(\log_5 x^2\right)$= ....

A. $\frac{1}{(\log 5)x}$

B. $\frac{1}{x^2}$

C. $\frac{2}{(\log 5)x}$

D. $\frac{1}{(\log 5)x^2}$

Answer: C

Watch Video Solution

50. Derivative of $\tan^{-1}x$ w.r. to $\cot^{-1}x$ is .... Where $x \in R$

A. -1

B. 1

C. $\frac{1}{1 + x^2}$

D. $-\frac{1}{1 + x^2}$

Answer: A

Watch Video Solution

51. If $y = \log\left(x + \sqrt{x^2 + a^2}\right)$ then $\frac{dy}{dx}$ = ...

A. $\sqrt{x^2 + a^2}$

B. $\frac{1}{\sqrt{x^2 + a^2}}$

C. $x^2 + a^2$

D. $\frac{1}{x^2 + a^2}$

Answer: B

Watch Video Solution

52. If $y = \log_{10}\sin x$ then $\frac{dy}{dx}$ = ...

A. cot x

B. cotx. $\log_e 10$

C. cot x. $\log_{10} e$

D. $\log_{10}\cot x$

Answer: C

Watch Video Solution

53. $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \ldots\ldots \infty}}}$ then $\frac{dy}{dx}$=.........

A. $\frac{\cos x}{2y - 1}$

B. $\cos x(2y+1)$

C. $\dfrac{\cos x}{2y+1}$

D. give not

Answer: A

Watch Video Solution

54.

$f(x) = x^2 e^{2(x-1)}, 0 < x < 1$ and $f(x) = a\operatorname{sgn}(x+1)\cos(2x-2) + bx^2, 1 < x \le 2$

. If a function f(x) is differentiable at x=1 then.

A. $a = -1, b = 2$

B. $a = 1, b = -2$

C. $a = -3, b = 4$

D. $a = 3, b = -4$

Answer: A

View Text Solution

55. The value of f(0), so that $f(x) = \dfrac{\sqrt{a^2 - ax + x^2} - \sqrt{a^2 + ax + x^2}}{\sqrt{a + x} - \sqrt{a - x}}$ becomes continuous for all x, is given by ....

A. $a\sqrt{a}$

B. $\sqrt{a}$

C. $-\sqrt{a}$

D. $-a\sqrt{a}$

Answer: C

Watch Video Solution

56. $f(x + y) = f(x)f(y)$, For $\forall x$ and y. If f(3)= 3 and f'(0) =11 then f'(3)= ....

A. 22

B. 44

C. 28

D. None of these

Answer: D

Watch Video Solution

57. $f(x) = [x] + \sqrt{x - [x]}$, where [.] is a greatest integer function then .... (a) f(x) is continuous in R+ (b) f(x) is continuous in R (C) f(x) is continuous in R - 1 (d) None of these

A. f(x) is continuous in R^+

B. f(x) is continuous in R

C. f(x) is continuous in $R - 1$

D. None of these

Answer: B

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58. The function $f(x) = (\sin 3x)^{\tan^2 3x}$ is continuous at $x = \frac{\pi}{6}$ then $f\left(\frac{\pi}{6}\right) =$ ......

A. $e^{-\frac{1}{2}}$

B. $\frac{1}{e}$

C. $e^{(d)}$

D. $e^{\frac{1}{2}}$

Answer: A

Watch Video Solution

59. $y^2 = ax^2 + bx + c$ then $y^3 \frac{d^2y}{dx^2}$ is a .... function

A. constant

B. only for x

C. only for y

D. for x and y

Answer: A

Watch Video Solution

60. $f(x) = x + \tan x$ and f is an inverse function of g then g'(x)= ....

A. $\dfrac{1}{1 + (g(x) - x)^2}$

B. $\dfrac{1}{1 - (g(x) - x)^2}$

C. $\dfrac{1}{2 + (g(x) - x)^2}$

D. $\dfrac{1}{2 - (g(x) - x)^2}$

Answer: C

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61. If $y^2 = p(x)$ the is polynomial of order 3, then $2\dfrac{d}{dx}\left[y^3\dfrac{d^2y}{dx^2}\right] =$ ....

A. $p'''(x) + p'(x)$

B. $p''(x).\ p'''(x)$

C. $p(x).\ p'''(x)$

D. $p(x) + p'''(x)$

Answer: C

Watch Video Solution

62. If $x^2 + y^2 = t - \frac{1}{t}$ and $x^4 + y^4 = t^2 + \frac{1}{t^2}$ then $x^3 y \frac{dy}{dx}$ = ...

A. -1

B. 0

C. 1

D. None of these

Answer: C

Watch Video Solution

63. If $\sin y = x\sin(a+y)$ and $\frac{dy}{dx} = \frac{A}{1+x^2 - 2x\cos a}$ then the value of A is ......

[a] 2 [b] CosA [c] SinA [d] 1/2

A. 2

B. cos a

C. sin a

D. $\frac{1}{2}$

Answer: C

Watch Video Solution

64. $f(x+y) = f(x) + f(y)$, for $\forall x$ and y and $f(x) = \left(2x^2 + 3x\right)g(x)$. For $\forall x$. If g(x) is a continuous function and g(0)= 3 then f'(x) = ...

A. 9

B. 3

C. 6

D. None of these

Answer: A

Watch Video Solution

65. $y = \sin x - \cos x$ and $f(x) = \frac{d^{17}y}{dx^{17}}$ then $f\left(\frac{\pi}{4}\right)$= .... [a] √2 [b] 1/√2 [c] (√2)^17 [d] 0

A. $\sqrt{2}$

B. $\frac{1}{\sqrt{2}}$

C. $\left(\sqrt{2}\right)^{17}$

D. 0

Answer: A

Watch Video Solution

66. $y = \tan^{-1}\left(\frac{ax - b}{bx + a}\right)$ then $\frac{dy}{dx}\Big|_{x = -1}$= ....

A. $\frac{1}{2}$

B. a

C. ab

D. $\frac{b}{a}$

Answer: A

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67. $u = f(\tan x), v = g(\sec x), f'(1) = 2$ and $g'\left(\sqrt{2}\right) = 4$ then $\frac{du}{dv}\Big|_{x = \frac{\pi}{4}}$=..........

A. $\sqrt{2}$

B. 2

C. $\frac{1}{\sqrt{2}}$

D. $\frac{1}{2}$

Answer: C

Watch Video Solution

68. $f(x) = \sin^2 x + \sin^2\left(x + \frac{\pi}{3}\right) + \cos x \cos\left(x + \frac{\pi}{3}\right)$ then f'(x) = ...

A. 1

B. $\cos^2 x$

C. 0

D. sin 2x

Answer: C

Watch Video Solution

69. $y = e^{3x+7}$ then $y_n(0) = $ ....

A. 1

B. 3^n

C. $3^n e^7$

D. $3^n.\left(e^7.7\right)$

Answer: C

Watch Video Solution

70. $x = f(t), y = \phi(t)$ then $\dfrac{d^2y}{dx^2}$ = ...

A. $\dfrac{f_1(t)\phi_2(t) - \phi_1(t)f_2(t)}{\left(f_1(t)\right)^2}$

B. $\dfrac{f_1(t)\phi_2(t) - \phi_1(t)f_2(t)}{\left(f_1(t)\right)^3}$

C. $\dfrac{\phi_1(t)f_2(t) - f_1(t)\phi_2(t)}{\left(f_1(t)\right)^3}$

D. None of these

Answer: B

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71. In the function $f(x) = 2x^3 + bx^2 + qx$ satisfies conditions of Rolle's theorem in $[-1, 1]$ and $c = \frac{1}{2}$ then the value of 2b + q is.......

A. 0

B. 1

C. $-\frac{1}{2}$

D. -1

Answer: D

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72. If $y = \frac{a + bx}{c + dx}$, where a, b, c, d are constants and $\lambda y_1 y_3 = \mu y_2^2$ then the value of μ^{λ^2} is .... where y_1, y_2, y_3 are respectively. First, second and third derivatives of y.

A. 42

B. 81

C. 64

D. 27

Answer: B

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73. The function $f(x) = \dfrac{2 - \sqrt[4]{x^2 + 16}}{\cos 2x - 1}$ is continuous at x=0 then f(0)= ....

A. $\dfrac{1}{8}$

B. $\dfrac{1}{64}$

C. $\dfrac{1}{32}$

D. $\dfrac{1}{2}$

Answer: B

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74. Let $f(x) = x^3 - x^2 + x + 1$

$g(x) = \{ \max (f(t)), 0 \le t \le x\}, 0 \le x \le 1\}\{3 - x, 1 < x \le 2\}$

Then in [0, 2] the points where g(x) is not differentiable is.........

A. 0

B. 1

C. 2

D. None of these

Answer: B

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75. $f(x) = \begin{vmatrix} \sin x & \cos x \\ \tan x & \cot x \end{vmatrix}$ then $f'\left(\frac{\pi}{4}\right) =$ ....

A. 0

B. $-\sqrt{2}$

C. $-2\sqrt{2}$

D. $\sqrt{2}$

Answer: D

Watch Video Solution

76. $x = t^2 + 3t - 8, y = 2t^2 - 2t - 4$. If at point $(2, -1), \lambda = \frac{dy}{dx}$ then the value of λ= ....

A. 2

B. $\frac{6}{7}$

C. -6

D. 7

Answer: B

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77. $x = 2 + t^3, y = 2t^2$. If $\dfrac{\dfrac{d^2y}{dx^2}}{\left(\dfrac{dy}{dx}\right)^n}$ is constant then n= ....

A. 4

B. 1

C. 0

D. 3

Answer: A

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78. If $F(x) = \dfrac{1}{x^2}\int_4^x \left(4t^2 - 2F'(t)\right)dt$ then F'(4) equals to ....

A. $\dfrac{32}{9}$

B. $\dfrac{64}{3}$

C. $\dfrac{64}{9}$

D. $\frac{32}{3}$

Answer: A

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79. If $\sqrt{x^2+y^2}=a.e^{\tan^{-1}\left(\frac{y}{x}\right)}, a>0$ then the value of y''(0) is.......

A. $\frac{a}{2}e^{-\frac{\pi}{2}}$

B. $ae^{\frac{\pi}{2}}$

C. $\frac{-2}{a}e^{-\frac{\pi}{2}}$

D. Does not exist

Answer: C

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80. If $f(x-y), f(x).f(y)$ and $f(x+y)$ are in arithmatic progression and $f(0) \neq 0$ then (for $\forall x$ and y.........

A. $f(2) + f'(2) = 0$

B. $f'(2) + f'(-2) = 0$

C. $f(2) - f'(-2) = 0$

D. $f'(2) - f'(-2) = 0$

Answer: B

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81. $\left[\frac{d}{dx}\sec^{-1}x\right]_{x=-3}$ =.......

A. $\frac{1}{\sqrt{x^2-1}}$

B. $-\frac{1}{\sqrt{x^2-1}}$

C. $\frac{1}{6\sqrt{2}}$

D. $-\frac{1}{6\sqrt{2}}$

Answer: C

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82. $\frac{d}{dx}\left(x^{x}\right) = \ldots . (x > 0)$

A. x^{x-1}

B. x^{x}

C. 0

D. $x^{x}(1 + \log x)$

Answer: D

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83. $\frac{d}{dx}\left(\sin^{-1}x + \cos^{-1}x\right) = \ldots\ldots\ldots(|x| < 1)$

A. 0

B. $\frac{2}{\sqrt{1 - x^2}}$

C. $\frac{1}{\sqrt{1 - x^2}}$

D. Does not exist

Answer: A

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84. $\frac{d}{dx}\left(a^a\right) = \ldots\ldots(a > 0)$

A. $a^a(1 + \log a)$

B. 0

C. a^a

D. Does not exist

Answer: B

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85. $\frac{d}{dx}\left(e^{5x}\right)$= ....

A. e^{5x}

B. $5e^{5x}$

C. $5x.\ e^{5x-1}$

D. 0

Answer: B

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86. $\frac{d}{dx}(\log|x|)$= ....$(x \neq 0)$

A. $\frac{1}{|x|}$

B. $\frac{1}{x}$

C. does not exist

D. e^{x}

Answer: B

Watch Video Solution

87. $\dfrac{d}{dx}\left(\sin^3 x\right)=$ ....

A. $3\sin^2 x$

B. $3\cos^2 x$

C. $3\sin^2 x.\ \cos x$

D. $-3\cos^2 x\sin x$

Answer: C

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88. $\dfrac{d}{dx}\left(\tan^n x\right)=$ ....

A. $n\tan^{n-1}x$

B. $n\tan^{n-1}x.\sec^2 x$

C. $n\sec^{2n}x$

D. $n\tan^{n-1}x.\sec^{n-1}x$

Answer: B

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89. If $f(x) = \begin{cases} ax+b & 1 \le x < 5 \\ 7x-5 & 5 \le x < 10 \\ bx+3a & x \ge 10 \end{cases}$ is continuous, (a,b)= ....

A. (5, 10)

B. (5,5)

C. (10, 5)

D. (0, 0)

Answer: B

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90. If $f(x) = \begin{cases} \frac{x^2}{a} - a, & x < a \\ 0, & x = a \\ a - \frac{x^2}{a}, & x > 0 \end{cases}$ then, ...

A. $\lim_{x \to a^+} f(x) = a$

B. $\lim_{x \to a} f(x) = -a$

C. f is continuous at x=a

D. f is differentiable at x=a

Answer: C

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91. If $f(x) = \begin{cases} x, & x \in \{0, 1\} \\ 1, & x \geq 1 \end{cases}$ then, ...

A. f is continuous at x=1 only

B. f is discontinuous at x=1 only

C. f is continuous on R^{+}

D. f is not defined for x=1

Answer: C

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92. $\frac{d}{dx}.\left(\frac{1}{\log|x|}\right)$= ....

A. $\frac{1}{|x|}$

B. $\frac{1}{(\log x)^2}$

C. $\frac{-1}{x(\log|x|)^2}$

D. e^x

Answer: C

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93. If $y = a\sin x + b\cos x$ then, $y^2 + \left(y_1\right)^2 =$ .... $\left(a^2 + b^2 \neq 0\right)$

A. $a\cos x$ - $b\sin x$

B. $(a\sin x$ - $b\cos x)^2$

C. $a^2 + b^2$

D. 0

Answer: C

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94. $\frac{d}{dx}\left(x^2 + \sin^2 x\right)^3 =$.......

A. $3\left(x^2 + \sin^2 x\right)$

B. $3\left(x^2 + \sin^2 x\right)^2(2x + \sin 2x)$

C. $2x + 2\sin x\cos x$

D. 0

Answer: B

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95. $\frac{d}{dx}\left(\sqrt{x\sin x}\right)=$ ...... $(0 < x < \pi)$

A. $\frac{x\sin x + \cos x}{\sqrt{x\sin x}}$

B. $\frac{x\cos x}{2\sqrt{x\sin x}}$

C. $\frac{x\cos x + \sin x}{2\sqrt{x\sin x}}$

D. $\frac{1}{2\sqrt{x\sin x}}$

Answer: C

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96. $\frac{d}{dx}\left(e^{\sin^{-1}x + \cos^{-1}x}\right) =$ ....$(|x| < 1)$

A. $\dfrac{2}{\sqrt{1-x^2}}$

B. 0

C. $\dfrac{1}{\sqrt{1-x^2}}$

D. $e^{\sin^{-1}x+\cos^{-1}x}$

Answer: B

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97. If function f(x) is continuous at x= 0, $f(x) = \begin{cases} \dfrac{\sin(4x)}{9x}, & x \neq 0 \\ k^2 & x = 0 \end{cases}$ then k= ....

A. $-\dfrac{3}{2}$

B. $\dfrac{3}{2}$

C. $\pm\dfrac{2}{3}$

D. $\dfrac{4}{9}$

Answer: C

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98. If $x = at^2, y = 2at$ then $\frac{dy}{dx}$ = ..., where $t \neq 0$

A. $\frac{1}{t}$

B. t

C. $-t$

D. a

Answer: A

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99. $\frac{d}{dx}\left(\log_5 x^2\right)$= ....

A. $\frac{1}{(\log 5)x}$

B. $\frac{1}{x^2}$

C. $\frac{2}{(\log 5)x}$

D. $\frac{1}{(\log 5)x^2}$

Answer: C

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100. Derivative of $\tan^{-1}x$ w.r. to $\cot^{-1}x$ is .... Where $x \in R$

A. -1

B. 1

C. $\frac{1}{1+x^2}$

D. $-\frac{1}{1+x^2}$

Answer: A

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Textbook Illustrations for Practice Work

1. Examine the continuity of the function $f(x) = 2x + 3$ at $x = 1$

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2. Prove that the function $f(x) = x^2$ is continuous at x=0.

Watch Video Solution

3. Discuss the continuity of the function f given by $f(x) = |x|$ at $x = 0$.

Watch Video Solution

4. Show that $f(x) = \begin{cases} x^3 + 3, & x \neq 0 \\ 1, & x = 0 \end{cases}$ is a discontinuous function at x= 0.

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5. Check the points where the constant function f(x)= k is continuous

Watch Video Solution

6. Prove that the identity function on real numbers given by f(x)=x is continuous at every real number.

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7. Is the function defined by f(x)= |x|, a continuous function

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8. Discuss the continuity of the function f given by $f(x) = x^3 + x^2 - 1$

Watch Video Solution

9. Discuss the continuity of the function f defined by $f(x) = \frac{1}{x}, x \neq 0$

Watch Video Solution

10. Discuss the continuity of the function f defined by

$$f(x) = \begin{cases} x + 2, & \text{if} x \leq 1 \\ x - 2, & \text{if} x > 1 \end{cases}$$

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11. Find all the points of discontinuity of the function f defined by

$$f(x) = \begin{cases} x + 2, & \text{if } x < 1 \\ 0 & \text{if } x = 1 \\ x - 2, & \text{if } x > 1 \end{cases}$$

Watch Video Solution

12. Discuss the continuity of the function defined by

$$f(x) = \begin{cases} x + 2, & \text{if } x < 0 \\ -x + 2, & \text{if } x > 0 \end{cases}$$

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13. Discuss the continuity of the function f given by $f(x) = \begin{cases} x, & \text{if } x > 0 \\ x^2, & \text{if } x < 0 \end{cases}$

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14. Show that every polynomial function is continuous.

Watch Video Solution

15. Find all the points of discontinuity of the greatest integer function defined by f(x)= [x], where [x] denotes the greatest integer less than or equal to x.

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16. Prove that every rational function is continuous

Watch Video Solution

17. Discuss the continuity of sine function

Watch Video Solution

18. Prove that the function defined by $f(x) = \tan x$ is a continuous function

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19. Show that the function defined by $f(x) = \sin\left(x^2\right)$ is a continuous function

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20. Show that the function f defined by $f(x) = |1 - x + |x||$ where x is any real number is continous.

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21. Find the derivative of the function given by $f(x) = \sin x^2$

Watch Video Solution

22. Find the derivative of $\tan(2x + 3)$

Watch Video Solution

23. Differentiate $\sin\left(\cos x^2\right)$ with respect to x

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24. Find $\frac{dy}{dx}$ if $x - y = \pi$

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25. Find $\frac{dy}{dx}$ if $y + \sin y = \cos x$

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26. Find the derivative of f given by $f(x) = \sin^{-1}x$ assuming it exists.

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27. Find the derivative of f given by $f(x) = \tan^{-1}x$ assuming it exists

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28. Is it true that $x = e^{\log x}$ for all real x?

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29. Differentiate the following w.r.t x:

(i) e^{-x} (ii) $\sin(\log x), x > 0$ (iii) $\cos^{-1}\left(e^x\right)$ (iv) $e^{\cos x}$

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30. Differentiate $\dfrac{\sqrt{(x-3)\left(x^2+4\right)}}{\sqrt{\left(3x^2+4x+5\right)}}$ w.r.t x.

Watch Video Solution

31. Differentiate a^x w.r.t x, where a is a positive constant

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32. Differentiate $x^{\sin x}, x > 0$ w.r.t x

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33. Find $\frac{dy}{dx}$, If $y^x + x^y + x^x = a^b$

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34. Find $\frac{dy}{dx}$, if $x = a\cos\theta, y = a\sin\theta$

Watch Video Solution

35. Find $\frac{dy}{dx}$, if $x = at^2, y = 2at$

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36. Find $\frac{dy}{dx}$, if $x = a(\theta + \sin\theta), y = a(1 - \cos\theta)$

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37. Find $\frac{dy}{dx}$, if $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$

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38. Find $\frac{d^2y}{dx^2}$, if $y = x^3 + \tan x$

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39. If $y = A\sin x + B\cos x$, then prove that $\frac{d^2y}{dx^2} + y = 0$

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40. If $y = 3e^{2x} + 2e^{3x}$, prove that $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$

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41. If $y = \sin^{-1}x$, show that $\left(1 - x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$

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42. Verify Rolle's theorem for the function $y = x^2 + 2, a = -2 \text{ and } b = 2$

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43. Verify Mean value Theorem for the function $f(x) = x^2$ in the interval [2,4]

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44. Differentiate w.r.t x, the following function:

(i) $\sqrt{3x + 2} + \frac{1}{\sqrt{2x^2 + 4}}$ (ii) $e^{\sec^2 x} + 3\cos^{-1}x$ (iii) $\log_7(\log x)$

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45. Differentiate the following w.r.t x.

(i) $\cos^{-1}(\sin x)$ (ii) $\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$ (iii) $\sin^{-1}\left(\frac{2^{x+1}}{1+4^{x}}\right)$

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46. Find f'(x) if $f(x) = (\sin x)^{\sin x}$ for all $0 < x < \pi$.

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47. For a positive constant a find $\frac{dy}{dx}$, where $y = a^{t+\frac{1}{t}}$, and $x = \left(t+\frac{1}{t}\right)^{a}$

Watch Video Solution

48. Differentiate $\sin^2 x$ w.r.t $e^{\cos x}$

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49. Examine the continuity of the function $f(x) = 2x + 3$ at $x = 1$

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56. Discuss the continuity of the function f given by $f(x) = x^3 + x^2 - 1$

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57. Discuss the continuity of the function f defined by $f(x) = \frac{1}{x}, x \neq 0$

Watch Video Solution

58. Discuss the continuity of the function f defined by

$$f(x) = \begin{cases} x + 2, & \text{if} x \leq 1 \\ x - 2, & \text{if} x > 1 \end{cases}$$

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59. Find all the points of discontinuity of the function f defined by

$$f(x) = \begin{cases} x + 2, & \text{if} \quad x < 1 \\ 0 & \text{if} \quad x = 1 \\ x - 2, & \text{if} \quad x > 1 \end{cases}$$

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60. Discuss the continuity of the function defined by

$$f(x) = \begin{cases} x + 2, & \text{if } x < 0 \\ -x + 2, & \text{if } x > 0 \end{cases}$$

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61. Discuss the continuity of the function f given by $f(x) = \begin{cases} x, & \text{if } x > 0 \\ x^2, & \text{if } x < 0 \end{cases}$

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62. Show that every polynomial function is continuous.

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63. Find all the points of discontinuity of the greatest integer function defined by f(x)= [x], where [x] denotes the greatest integer less than or equal to x.

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64. Prove that every rational function is continuous

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65. Discuss the continuity of sine function

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66. Prove that the function defined by $f(x) = \tan x$ is a continuous function

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67. Show that the function defined by $f(x) = \sin\left(x^2\right)$ is a continuous function

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68. Show that the function f defined by $f(x) = |1 - x + |x||$ where x is any real number is continous.

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69. Find the derivative of the function given by $f(x) = \sin x^2$

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70. Find the derivative of $\tan(2x + 3)$

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71. Differentiate $\sin\left(\cos x^2\right)$ with respect to x

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72. Find $\frac{dy}{dx}$ if $x - y = \pi$

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73. Find $\frac{dy}{dx}$ if $y + \sin y = \cos x$

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74. Find the derivative of f given by $f(x) = \sin^{-1}x$ assuming it exists.

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75. Find the derivative of f given by $f(x) = \tan^{-1}x$ assuming it exists

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76. Is it true that $x = e^{\log x}$ for all real x?

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77. Differentiate the following w.r.t x:

(i) e^{-x} (ii) $\sin(\log x), x > 0$ (iii) $\cos^{-1}\left(e^x\right)$ (iv) $e^{\cos x}$

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78. Differentiate $\dfrac{\sqrt{(x-3)\left(x^2+4\right)}}{\sqrt{\left(3x^2+4x+5\right)}}$ w.r.t x.

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79. Differentiate a^x w.r.t x, where a is a positive constant

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80. Differentiate $s^{\sin x}, x > 0$ w.r.t x

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81. Find $\frac{dy}{dx}$, If $y^x + x^y + x^x = a^b$

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82. Find $\frac{dy}{dx}$, if $x = a\cos\theta, y = a\sin\theta$

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83. Find $\frac{dy}{dx}$, if $x = at^2, y = 2at$

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84. If x and y are connected parametrically by the equations without eliminating the parameter, find $\frac{dy}{dx}$

$x = a(\theta - \sin\theta), y = a(1 + \cos\theta)$

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85. Find $\frac{dy}{dx}$, if $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$

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86. Find $\frac{d^2y}{dx^2}$, if $y = x^3 + \tan x$

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87. If $y = A\sin x + B\cos x$, then prove that $\frac{d^2y}{dx^2} + y = 0$

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88. If $y = 3e^{2x} + 2e^{3x}$, prove that $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$

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89. If $y = \sin^{-1}x$, show that $\left(1 - x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$

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90. Verify Rolle's theorem for the function $y = x^2 + 2, a = -2 \text{ and } b = 2$

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91. Verify Mean value Theorem for the function $f(x) = x^2$ in the interval [2,4]

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92. Differentiate w.r.t x, the following function:

(i) $\sqrt{3x + 2} + \frac{1}{\sqrt{2x^2 + 4}}$ (ii) $e^{\sec^2 x} + 3\cos^{-1}x$ (iii) $\log_7(\log x)$

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93. Differentiate the following w.r.t x.

(i) $\cos^{-1}(\sin x)$ (ii) $\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$ (iii) $\sin^{-1}\left(\frac{2^{x+1}}{1+4^{x}}\right)$

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94. Find f'(x) if $f(x) = (\sin x)^{\sin x}$ for all $0 < x < \pi$.

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95. For a positive constant a find $\frac{dy}{dx}$, where $y = a^{t+\frac{1}{t}}$, and $x = \left(t + \frac{1}{t}\right)^{a}$

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96. Differentiate $\sin^2 x$ w.r.t $e^{\cos x}$

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NCERT Exemplar Problems and Solution (Short Answer Type Questions)

1. Examine the continuity of the function

$f(x) = x^3 + 2x^2 - 1$ at $x = 1$

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2. Examine the continuity of the function

$$f(x) = \begin{cases} 3x + 5, & \text{if } x \geq 2 \\ x^2 & \text{if } x < 2 \end{cases} \text{ at x= 2}$$

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3. Examine the continuity of the function

$$f(x) = \begin{cases} \frac{1 - \cos(2x)}{x^2} & \text{if } x \neq 0 \\ 5, & \text{if} x = 0 \end{cases} \text{ at x=0}$$

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4. Examine the continuity of the function

$$f(x) = \begin{cases} \frac{2x^2 - 3x - 2}{x - 2}, & \text{if } x \neq 2 \\ 5, & \text{if } x = 2 \end{cases} \text{ at } x = 2$$

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5. Examine the continuity of the function

$$f(x) = \begin{cases} \frac{|x-4|}{2(x-4)}, & \text{if} x \neq 4 \\ 0, & \text{if} x = 4 \end{cases} \text{ at x=4}$$

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6. Examine the continuity of the function

$$f(x) = \begin{cases} |x|.\cos\left(\frac{1}{x}\right), & \text{if} x \neq 0 \\ 0, & \text{if} x = 0 \end{cases} \text{ at x= 0}$$

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7. Examine the continuity of the function

$$f(x) = \begin{cases} |x - a|\sin\left(\frac{1}{x-a}\right), & \text{if } x \neq a \\ 0, & \text{if } 'x = a \end{cases}$$ at x=a

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8. Examine the continuity of the function

$$f(x) = \begin{cases} \frac{e^{\frac{1}{x}}}{1+e^{\frac{1}{x}}}, & \text{if } x \neq 0 \\ 0, & \text{if} x = 0 \end{cases}$$ at x= 0

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9. Examine the continuity of the function

$$f(x) = \begin{cases} \frac{x^2}{2}, & \text{if } 0 \le x \le 1 \\ 2x^2 - 3x + \frac{3}{2}, & \text{if } 1 < x \le 2 \end{cases}$$ at x=1

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10. Examine the continuity of the function

$f(x) = |x| + |x - 1|$ at x=1

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11. Find the values of k so that the function f is continuous at the indicated point $f(x) = \begin{cases} 3x - 8, & \text{if} x \le 5 \\ 2k, & \text{if} x > 5 \end{cases}$ at x= 5

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12. $f(x) = \begin{cases} \frac{2^{x+2}-16}{4^x-16}, & x \neq 2 \\ k, & x = 2 \end{cases}$ f(x) is continuous at x=2 then find k

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13. Find the values of k so that the function f is continuous at the indicated point $f(x) = \begin{cases} \frac{\sqrt{1+kx}-\sqrt{1-kx}}{x}, & \text{if } -1 \leq x < 0 \\ \frac{2x+1}{x-1}, & \text{if } 0 \leq x < 1 \end{cases}$ at x= 0

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14. Find the values of k so that the function f is continuous at the indicated point $f(x) = \begin{cases} \frac{1-\cos(kx)}{x^2}, & \text{if } x \neq 0 \\ \frac{1}{2}, & \text{if } x = 0 \end{cases}$ at x= 0

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15. Prove that the function f defined by $f(x) = \begin{cases} \dfrac{x}{|x| + 2x^2}, & \text{if} x \neq 0 \\ k & \text{if} x = 0 \end{cases}$ remains discontinuous at x=0, regardless the choice of k

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16. Find the values of a and b such that the function f defined by

$f(x) = \begin{cases} \dfrac{x-4}{|x-4|} + a, & \text{if } x < 4 \\ a + b & \text{if } x = 4 \\ \dfrac{x-4}{|x-4|} + b & \text{if } x > 4 \end{cases}$ is a continuous function at x= 4

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17. If the function $f(x) = \dfrac{1}{x+2}$, then find the points of discontinuity of the composite function y= f {f(x)}

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18. Find all points of dicontinuity of the function $f(t)=\frac{1}{t^2+t-2}$, where $t=\frac{1}{x-1}$

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19. Show that the function $f(x)=|\sin x+\cos x|$ is continuous at $x=\pi$

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20. Examine the differentiability of f, where f is defined by

$f(x)=\begin{cases} x.[x], & \text{if } 0 \le x < 2 \\ (x-1)x & \text{if } 2 \le x < 3 \end{cases}$ at x= 2

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21. Determine if f defined by $f(x) = \begin{cases} x^2 \sin\frac{1}{x}, & \text{if} x \neq 0 \\ 0, & \text{if} x = 0 \end{cases}$ is a continuous function?

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22. Examine the continuity of the function $f(x) = \begin{cases} 1 + x, & \text{if } x \leq 2 \\ 5 - x, & \text{if} x > 2 \end{cases}$ at x= 2

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23. Show that $f(x) = |x - 5|$ is continuous but not differentiable at x= 5

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24. A function $f: R \rightarrow R$ satisfies the equation $f(x + y) = f(x).f(y) \text{for all} x, y \in R, f(x) \neq 0.$ Suppose that the function is differentiable at x=0 and f'(0)=2, then prove that f'(x)= 2f(x)

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25. Differentiate $2^{\cos^2 x}$

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26. Differentiate $\frac{8^x}{x^8}$

Watch Video Solution

27. Differentiate $\log\left(x+\sqrt{x^2+a}\right)$

Watch Video Solution

28. Differentiate the following with respect to x:

$$\log\left[\log\left(\log x^5\right)\right]$$

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29. Differentiate $\sin\sqrt{x} + \cos^2\sqrt{x}$

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30. Differentiate $\sin^n\left(ax^2 + bx + c\right)$

Watch Video Solution

31. Differentiate $\cos\left(\tan\sqrt{x+1}\right)$

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32. Differentiate $\sin x^2 + \sin^2 x + \sin^2\left(x^2\right)$

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33. Differentiate $\sin^{-1}\left(\frac{1}{\sqrt{x+1}}\right)$

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34. Differentiate $(\sin x)^{\cos x}$

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35. Differentiate $\sin^m x. \cos^n x$

Watch Video Solution

36. Differentiate $(x+1)^2(x+2)^3(x+3)^4$

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37. Differentiate $\cos^{-1}\left(\frac{\sin x + \cos x}{\sqrt{2}}\right), -\frac{\pi}{4} < x < \frac{\pi}{4}$

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38. Differentiate $\tan^{-1}\sqrt{\frac{1 - \cos x}{1 + \cos x}}, -\frac{\pi}{4} < x < \frac{\pi}{4}$

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39. Differentiate $\tan^{-1}(\sec x + \tan x), -\frac{\pi}{2} < x < \frac{\pi}{2}$

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40. Differentiate, $\tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right), \frac{-\pi}{2} < x < \frac{\pi}{2}$ and $\frac{a}{b}\tan x > -1$

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41. Differentiate $\sec^{-1}\left(\frac{1}{4x^3 - 3x}\right), 0 < x < \frac{1}{\sqrt{2}}$

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42. Write the following functions in the simplest form :

$\tan^{-1}\left(\frac{3a^2x - x^3}{a^3 - 3ax^2}\right), a > 0, \frac{-a}{\sqrt{3}} < x < \frac{a}{\sqrt{3}}$

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43. Prove that :

$$\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)=\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x^2.$$

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44. Find $\frac{dy}{dx}$:

$x = t + \frac{1}{t}$ and $y = t - \frac{1}{t}$

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45. Find $\frac{dy}{dx}$ of each of the functions expressed in parametric form:

$x = e^{\theta}\left(\theta + \frac{1}{\theta}\right), y = e^{-\theta}\left(\theta - \frac{1}{\theta}\right)$

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46. Find $\frac{dy}{dx}$ of each of the functions expressed in parametric form:

$x = 3\cos\theta - 2\cos^3\theta, y = 3\sin\theta - 2\sin^3\theta$

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47. Find $\frac{dy}{dx}$ of each of the functions expressed in parametric form:

$\sin x = \frac{2t}{1 + t^2}, \tan y = \frac{2t}{1 - t^2}, t \in R$

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48. Find $\frac{dy}{dx}$ of each of the functions expressed in parametric form:

$x = \frac{1 + \log t}{t^2}, y = \frac{3 + 2\log t}{t}$

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49. Find $\frac{dy}{dx}$:

$x = e^{\cos 2t}$ and $y = e^{\sin 2t}$ show that, $\frac{dy}{dx} = \frac{-y\log x}{x\log y}$

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50. Find $\frac{dy}{dx}$:

$x = a\sin 2t(1 + \cos 2t)$ and $y = b\cos 2t$

$(1 - \cos 2t)$ show that, $\left(\frac{dy}{dx}\right)_{t = \frac{\pi}{4}} = \frac{b}{a}$

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51. If $x = 3\sin t - \sin(3t), y = 3\cos t - \cos 3t$, then find $\left(\frac{dy}{dx}\right)$ at $t = \frac{\pi}{3}$

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52. Differentiate $\frac{x}{\sin x}$ w.r.t sin x

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53. Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ w.r.t $\tan^{-1}x$, where $x \neq 0$

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54. Find $\frac{dy}{dx}$ when x and y are connected by the relation given:

$\sin(xy) + \frac{x}{y} = x^2 - y$

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55. Find $\frac{dy}{dx}$ when x and y are connected by the relation given:

$\sec(x+y) = xy$

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56. Find $\frac{dy}{dx}$ when x and y are connected by the relation given:

$\tan^{-1}\left(x^2+y^2\right)=a$

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57. Find $\frac{dy}{dx}$ when x and y are connected by the relation given:

$\left(x^2+y^2\right)^2=xy$

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58. If $ax^2+2hxy+by^2+2gx+2fy+c=0$, then show that $\frac{dy}{dx}\cdot\frac{dx}{dy}=1$

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59. If $x=e^{\frac{x}{y}}$, then prove that $\frac{dy}{dx}=\frac{x-y}{x.\log x}$

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60. If $y^x = e^{y-x}$, then prove that $\frac{dy}{dx} = \frac{(1+\log y)^2}{\log y}$

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61. If $y = (\cos x)^{(\cos x)^{(\cos x)\ldots\ldots\infty}}$, then show that $\frac{dy}{dx} = \frac{y^2 \tan x}{y.\log\cos x - 1}$

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62. If $x\sin(a+y) + \sin a\cos(a+y) = 0$, then prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$

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63. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, then prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$. (Where $|x| \leq 1, |y| \leq 1$)

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64. If $y = \tan^{-1}x$, then find $\frac{d^2y}{dx^2}$ in terms of y alone.

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65. Verify the Rolle's theorem for each of the function in following questions:

$f(x) = x(x-1)^2, \quad \text{in} x \in [0, 1]$

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66. Verify the Rolle's theorem for each of the function in following questions:

$f(x) = \sin^4 x + \cos^4 x, \text{ in } x \in \left[0, \frac{\pi}{2}\right]$

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67. Verify the Rolle's theorem for each of the function in following questions:

$f(x) = \log\left(x^2 + 2\right) - \log 3, \quad \text{in } x \in [-1, 1]$

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68. Verify the Rolle's theorem for each of the function in following questions:

$f(x) = x(x + 3).\, e^{-\frac{x}{2}}, \text{ in} x \in [-3, 0]$

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69. Verify the Rolle's theorem for each of the function in following questions:

$f(x) = \sqrt{4 - x^2}, \text{ in } x \in [-2, 2]$

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70. Discuss the applicability of Rolle's theorem on the function given by

$$f(x) = \begin{cases} x^2 + 1, & \text{if} 0 \le x < 1 \\ 3 - x, & \text{if} 1 \le x \le 2 \end{cases}$$

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71. Find the points on the curve $y = \cos x - 1$ in $x \in [0, 2\pi]$, where the tangent is parallel to X-axis.

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72. Using Rolle's theorem, find the point on the curve $y = x(x - 4), x \in [0, 4]$, where the tangent is parallel to X-axis

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73. Verify mean value theorem for each of the functions:

$$f(x) = \frac{1}{4x - 1}, x \in [1, 4]$$

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74. Verify mean value theorem for each of the functions:

$f(x) = x^3 - 2x^2 - x + 3, x \in [0, 1]$

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75. Verify mean value theorem for each of the functions:

$f(x) = \sin x - \sin(2x)$, in $x \in [0, \pi]$

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76. Verify mean value theorem for each of the functions:

$f(x) = \sqrt{25 - x^2}$, in $x \in [1, 5]$

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77. Find a point on the curve $y = (x - 3)^2$, where the tangent is parallel to the chord joining the points (3,0) and (4,1).

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78. Using mean value theorem, prove that there is a point on the curve $y = 2x^2 - 5x + 3$ between the points P(1,0) and B(2,1), where tangent is parallel to the chord AB. Also, find the point.

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79. Examine the continuity of the function

$f(x) = x^3 + 2x^2 - 1$ at $x = 1$

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80. Examine the continuity of the function

$$f(x) = \begin{cases} 3x + 5, & \text{if } x \geq 2 \\ x^2 & \text{if } x < 2 \end{cases} \text{ at x= 2}$$

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81. Examine the continuity of the function

$$f(x) = \begin{cases} \frac{1 - \cos(2x)}{x^2} & \text{if } x \neq 0 \\ 5, & \text{if} x = 0 \end{cases} \text{ at x=0}$$

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82. Examine the continuity of the function

$$f(x) = \begin{cases} \frac{2x^2 - 3x - 2}{x - 2}, & \text{if } x \neq 2 \\ 5, & \text{if } x = 2 \end{cases} \text{ at } x = 2$$

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83. Examine the continuity of the function

$$f(x) = \begin{cases} \frac{|x-4|}{2(x-4)}, & \text{if} x \neq 4 \\ 0, & \text{if} x = 4 \end{cases} \quad \text{at x=4}$$

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84. Examine the continuity of the function

$$f(x) = \begin{cases} |x|.\cos\left(\frac{1}{x}\right), & \text{if} x \neq 0 \\ 0, & \text{if} x = 0 \end{cases} \quad \text{at x= 0}$$

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85. Examine the continuity of the function

$$f(x) = \begin{cases} |x - a|\sin\left(\frac{1}{x-a}\right), & \text{if } x \neq a \\ 0, & \text{if } 'x = a \end{cases} \quad \text{at x=a}$$

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86. Examine the continuity of the function

$$f(x) = \begin{cases} \dfrac{e^{\frac{1}{x}}}{1+e^{\frac{1}{x}}}, & \text{if } x \neq 0 \\ 0, & \text{if} x = 0 \end{cases} \text{ at x= 0}$$

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87. Examine the continuity of the function

$$f(x) = \begin{cases} \dfrac{x^2}{2}, & \text{if } 0 \le x \le 1 \\ 2x^2 - 3x + \dfrac{3}{2}, & \text{if } 1 < x \le 2 \end{cases} \text{ at x=1}$$

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88. Examine the continuity of the function

$f(x) = |x| + |x - 1|$ at x=1

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89. Find the values of k so that the function f is continuous at the indicated point $f(x) = \begin{cases} 3x - 8, & \text{if} x \leq 5 \\ 2k, & \text{if} x > 5 \end{cases}$ at x= 5

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90. $f(x) = \begin{cases} \dfrac{2^{x+2} - 16}{4^x - 16}, & x \neq 2 \\ k, & x = 2 \end{cases}$ f(x) is continuous at x=2 then find k

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91. Find the values of k so that the function f is continuous at the indicated point $f(x) = \begin{cases} \dfrac{\sqrt{1 + kx} - \sqrt{1 - kx}}{x}, & \text{if } -1 \leq x < 0 \\ \dfrac{2x + 1}{x - 1}, & \text{if } 0 \leq x < 1 \end{cases}$ at x= 0

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92. Find the values of k so that the function f is continuous at the indicated point $f(x) = \begin{cases} \frac{1 - \cos(kx)}{x^2}, & \text{if } x \neq 0 \\ \frac{1}{2}, & \text{if } x = 0 \end{cases}$ at x= 0

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93. Prove that the function f defined by $f(x) = \begin{cases} \frac{x}{|x| + 2x^2}, & \text{if} x \neq 0 \\ k & \text{if} x = 0 \end{cases}$ remains discontinuous at x=0, regardless the choice of k

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94. Find the values of a and b such that the function f defined by

$$f(x) = \begin{cases} \frac{x-4}{|x-4|} + a, & \text{if } x < 4 \\ a + b & \text{if } x = 4 \\ \frac{x-4}{|x-4|} + b & \text{if } x > 4 \end{cases}$$ is a continuous function at x= 4

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95. If the function $f(x) = \frac{1}{x+2}$, then find the points of discontinuity of the composite function y= f {f(x)}

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96. Find all points of dicontinuity of the function $f(t) = \frac{1}{t^2 + t - 2}$, where $t = \frac{1}{x - 1}$

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97. Show that the function $f(x) = |\sin x + \cos x|$ is continuous at $x = \pi$

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98. Examine the differentiability of f, where f is defined by

$$f(x) = \begin{cases} x.[x], & \text{if } 0 \le x < 2 \\ (x-1)x & \text{if } 2 \le x < 3 \end{cases}$$ at x= 2

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99. Determine if f defined by $f(x) = \begin{cases} x^2 \sin\frac{1}{x}, & \text{if} x \ne 0 \\ 0, & \text{if} x = 0 \end{cases}$ is a continuous function?

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100. Examine the continuity of the function $f(x) = \begin{cases} 1 + x, & \text{if } x \le 2 \\ 5 - x, & \text{if} x > 2 \end{cases}$ at x= 2

Watch Video Solution

101. Show that $f(x) = |x - 5|$ is continuous but not differentiable at x= 5

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102. A function $f: R \rightarrow R$ satisfies the equation $f(x+y) = f(x).\, f(y) \text{for all} x, y \in R, f(x) \neq 0.$ Suppose that the function is differentiable at x=0 and f'(0)=2, then prove that f'(x)= 2f(x)

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103. Differentiate $2^{\cos^2 x}$

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104. Differentiate $\frac{8^x}{x^8}$

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105. Differentiate $\log\left(x + \sqrt{x^2 + a}\right)$

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106. Differentiate the following with respect to x:

$\log\left[\log\left(\log x^5\right)\right]$

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107. Differentiate $\sin\sqrt{x} + \cos^2\sqrt{x}$

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108. Differentiate $\sin^n\left(ax^2 + bx + c\right)$

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109. Differentiate $\cos\left(\tan\sqrt{x+1}\right)$

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110. Differentiate $\sin x^2 + \sin^2 x + \sin^2\left(x^2\right)$

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111. Differentiate $\sin^{-1}\left(\frac{1}{\sqrt{x+1}}\right)$

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112. Differentiate $(\sin x)^{\cos x}$

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113. Differentiate $\sin^m x.\cos^n x$

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114. Differentiate $(x+1)^2(x+2)^3(x+3)^4$

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115. Differentiate $\cos^{-1}\left(\frac{\sin x+\cos x}{\sqrt{2}}\right), -\frac{\pi}{4}<x<\frac{\pi}{4}$

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116. Differentiate $\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}, -\frac{\pi}{4}<x<\frac{\pi}{4}$

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117. Differentiate $\tan^{-1}(\sec x + \tan x),\ -\frac{\pi}{2} < x < \frac{\pi}{2}$

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118. Differentiate, $\tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right), \frac{-\pi}{2} < x < \frac{\pi}{2}$ and $\frac{a}{b}\tan x > -1$

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119. Differentiate $\sec^{-1}\left(\frac{1}{4x^3 - 3x}\right), 0 < x < \frac{1}{\sqrt{2}}$

Watch Video Solution

120. Write the following functions in the simplest form :

$\tan^{-1}\left(\frac{3a^2x - x^3}{a^3 - 3ax^2}\right), a > 0, \frac{-a}{\sqrt{3}} < x < \frac{a}{\sqrt{3}}$

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121. Prove that :

$$\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)=\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x^2.$$

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122. Find $\frac{dy}{dx}$:

$x=t+\frac{1}{t}$ and $y=t-\frac{1}{t}$

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123. Find $\frac{dy}{dx}$ of each of the functions expressed in parametric form:

$$x=e^{\theta}\left(\theta+\frac{1}{\theta}\right), y=e^{-\theta}\left(\theta-\frac{1}{\theta}\right)$$

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124. Find $\frac{dy}{dx}$ of each of the functions expressed in parametric form:

$x = 3\cos\theta - 2\cos^3\theta, y = 3\sin\theta - 2\sin^3\theta$

Watch Video Solution

125. Find $\frac{dy}{dx}$ of each of the functions expressed in parametric form:

$\sin x = \frac{2t}{1 + t^2}, \tan y = \frac{2t}{1 - t^2}, t \in R$

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126. Find $\frac{dy}{dx}$ of each of the functions expressed in parametric form:

$x = \frac{1 + \log t}{t^2}, y = \frac{3 + 2\log t}{t}$

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127. Find $\frac{dy}{dx}$:

$x = e^{\cos 2t}$ and $y = e^{\sin 2t}$ show that, $\frac{dy}{dx} = \frac{-y\log x}{x\log y}$

Watch Video Solution

128. Find $\frac{dy}{dx}$:

$x = a\sin 2t(1 + \cos 2t)$ and $y = b\cos 2t$

$(1 - \cos 2t)$ show that, $\left(\frac{dy}{dx}\right)_{t = \frac{\pi}{4}} = \frac{b}{a}$

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129. If $x = 3\sin t - \sin(3t)$, $y = 3\cos t - \cos 3t$, then find $\left(\frac{dy}{dx}\right)$ at $t = \frac{\pi}{3}$

Watch Video Solution

130. Differentiate $\frac{x}{\sin x}$ w.r.t sin x

Watch Video Solution

131. Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ w.r.t $\tan^{-1}x$, where $x \neq 0$

Watch Video Solution

132. Find $\frac{dy}{dx}$ when x and y are connected by the relation given:

$\sin(xy) + \frac{x}{y} = x^2 - y$

Watch Video Solution

133. Find $\frac{dy}{dx}$ when x and y are connected by the relation given:

$\sec(x+y) = xy$

Watch Video Solution

134. Find $\frac{dy}{dx}$ when x and y are connected by the relation given:

$\tan^{-1}\left(x^2 + y^2\right) = a$

Watch Video Solution

135. Find $\frac{dy}{dx}$ when x and y are connected by the relation given:

$\left(x^2 + y^2\right)^2 = xy$

Watch Video Solution

136. If $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, then show that $\frac{dy}{dx} \cdot \frac{dx}{dy} = 1$

Watch Video Solution

137. If $x = e^{\frac{x}{y}}$, then prove that $\frac{dy}{dx} = \frac{x - y}{x. \log x}$

Watch Video Solution

138. If $y^x = e^{y-x}$, then prove that $\frac{dy}{dx} = \frac{(1 + \log y)^2}{\log y}$

Watch Video Solution

139. If $y = (\cos x)^{(\cos x)^{(\cos x) \dots \dots \infty}}$, then show that $\frac{dy}{dx} = \frac{y^2 \tan x}{y. \log \cos x - 1}$

Watch Video Solution

140. If $x\sin(a + y) + \sin a\cos(a + y) = 0$, then prove that $\frac{dy}{dx} = \frac{\sin^2(a + y)}{\sin a}$

Watch Video Solution

141. If $\sqrt{1 - x^2} + \sqrt{1 - y^2} = a(x - y)$, then prove that $\frac{dy}{dx} = \sqrt{\frac{1 - y^2}{1 - x^2}}$. (Where

$|x| \leq 1, |y| \leq 1)$

Watch Video Solution

142. If $y = \tan^{-1}x$, then find $\frac{d^2y}{dx^2}$ in terms of y alone.

Watch Video Solution

143. Verify the Rolle's theorem for each of the function in following questions:

$f(x) = x(x-1)^2, \ \ \text{in} x \in [0, 1]$

Watch Video Solution

144. Verify the Rolle's theorem for each of the function in following questions:

$f(x) = \sin^4 x + \cos^4 x, \text{ in } x \in \left[0, \frac{\pi}{2}\right]$

Watch Video Solution

145. Verify the Rolle's theorem for each of the function in following questions:

$f(x) = \log\left(x^2 + 2\right) - \log 3, \text{ in } x \in [-1, 1]$

Watch Video Solution

146. Verify the Rolle's theorem for each of the function in following questions:

$f(x) = x(x+3).\, e^{-\frac{x}{2}}, \text{ in} x \in [-3, 0]$

Watch Video Solution

147. Verify the Rolle's theorem for each of the function in following questions:

$f(x) = \sqrt{4 - x^2}, \text{ in } x \in [-2, 2]$

Watch Video Solution

148. Discuss the applicability of Rolle's theorem on the function given by

$$f(x) = \begin{cases} x^2 + 1, & \text{if} 0 \le x < 1 \\ 3 - x, & \text{if} 1 \le x \le 2 \end{cases}$$

Watch Video Solution

149. Find the points on the curve $y = \cos x - 1$ in $x \in [0, 2\pi]$, where the tangent is parallel to X-axis.

Watch Video Solution

150. Using Rolle's theorem, find the point on the curve $y = x(x - 4), x \in [0, 4]$, where the tangent is parallel to X-axis

Watch Video Solution

151. Verify mean value theorem for each of the functions:

$$f(x) = \frac{1}{4x - 1}, x \in [1, 4]$$

Watch Video Solution

152. Verify mean value theorem for each of the functions:

$f(x) = x^3 - 2x^2 - x + 3, x \in [0, 1]$

Watch Video Solution

153. Verify mean value theorem for each of the functions:

$f(x) = \sin x - \sin(2x)$, in $x \in [0, \pi]$

Watch Video Solution

154. Verify mean value theorem for each of the functions:

$f(x) = \sqrt{25 - x^2}$, in $x \in [1, 5]$

Watch Video Solution

155. Find a point on the curve $y = (x - 3)^2$, where the tangent is parallel to the chord joining the points (3,0) and (4,1).

Watch Video Solution

156. Using mean value theorem, prove that there is a point on the curve $y = 2x^2 - 5x + 3$ between the points P(1,0) and B(2,1), where tangent is parallel to the chord AB. Also, find the point.

Watch Video Solution

NCERT Exemplar Problems and Solution (Long Answer Type Questions)

1. Find the values of p and q, so that $f(x) = \begin{cases} x^2 + 3x + p & x \le 1 \\ qx + 2 & x > 1 \end{cases}$ is differentiable at x=1.

Watch Video Solution

2. If $x^m . y^n = (x+y)^{m+n}$, prove that

$$\frac{dy}{dx} = \frac{y}{x}$$

Watch Video Solution

3. If $x^m . y^n = (x+y)^{m+n}$ then show that, $\frac{d^2y}{dx^2} = 0$

Watch Video Solution

4. If $y = \sin(pt), x = \sin t$, then prove that $\left(1 - x^2\right)y_2 - xy_1 + p^2y = 0$

Watch Video Solution

5. Find the value of $\frac{dy}{dx}$ if $y = x^{\tan x} + \sqrt{\frac{x^2+1}{2}}$

Watch Video Solution

6. Find the values of p and q, so that $f(x) = \begin{cases} x^2 + 3x + p & x \le 1 \\ qx + 2 & x > 1 \end{cases}$ is differentiable at x=1.

Watch Video Solution

7. If $x^m . y^n = (x + y)^{m+n}$, prove that

$\frac{dy}{dx} = \frac{y}{x}$

Watch Video Solution

8. If $x^m . y^n = (x + y)^{m+n}$ then show that, $\frac{d^2y}{dx^2} = 0$

Watch Video Solution

9. If $y = \sin(pt), x = \sin t$, then prove that $\left(1 - x^2\right)y_2 - xy_1 + p^2y = 0$

Watch Video Solution

10. Find the value of $\frac{dy}{dx}$ if $y = x^{\tan x} + \sqrt{\frac{x^2 + 1}{2}}$

Watch Video Solution

NCERT Exemplar Problems and Solution (Objective Type Questions)

1. If $f(x) = 2x$ and $g(x) = \frac{x^2}{2} + 1$, then which of the following can be a discontinuous function?

A. $f(x) + g(x)$

B. $f(x) - g(x)$

C. $f(x).\, g(x)$

D. $\frac{g(x)}{f(x)}$

Answer: D

Watch Video Solution

2. The function $f(x) = \frac{4 - x^2}{4x - x^3}$ is

A. discontinuous at only one point

B. discontinuous at exactly two points

C. discontinuous at exactly three points

D. none of the above

Answer: C

Watch Video Solution

3. The set of points where the function f given by $f(x) = |2x - 1|\sin x$ is differentiable is $x \in$ ....

A. R

B. $R - \left\{\frac{1}{2}\right\}$

C. $(0, \infty)$

D. none of these

Answer: B

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4. The function $f(x) = \cot x$ is discontinuous on the set

A. $\{x \mid x = n\pi, n \in Z\}$

B. $\{x \mid x = 2n\pi, n \in Z\}$

C. $\left\{x \mid x = (2n+1)\frac{\pi}{2}, n \in Z\right\}$

D. $\left\{x \mid x = \frac{n\pi}{2}, n \in Z\right\}$

Answer: A

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5. The function $f(x) = e^{|x|}$ is

A. continuous everywher but not differentiable at x= 0

B. continuous and differentiable everywhere

C. Not continuous at x= 0

D. none of the above

Answer: A

Watch Video Solution

6. If $f(x) = x^2 \sin\left(\frac{1}{x}\right)$, where $x \neq 0$, then the value of the function f at x=0, so that the function is continuous at x= 0, is

A. 0

B. -1

C. 1

D. None of these

Answer: A

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7. If $f(x)\begin{cases} mx+1 & x \le \frac{\pi}{2} \\ (\sin x)+n & x > \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$, then ......

A. $m = 1, n = 0$

B. $m = \frac{n\pi}{2} + 1$

C. $n = \frac{m\pi}{2}$

D. $m = n = \frac{\pi}{2}$

Answer: C

Watch Video Solution

8. If $f(x) = |\sin x|$, then

A. f is everywhere differentiable

B. f is everywhere continuous but not differentiable at $x = n\pi, n \in Z$

C. f is everywhere continuous but not differentiable at $x = \left(2n + \frac{\pi}{2}\right), n \in Z$

D. none of the above

Answer: B

Watch Video Solution

9. If $y = \log\left(\frac{1 - x^2}{1 + x^2}\right)$, then $\frac{dy}{dx}$ is equal to

A. $\frac{4x^3}{1 - x^4}$

B. $\frac{-4x}{1 - x^4}$

C. $\frac{1}{4 - x^4}$

D. $\frac{-4x^3}{1 - x^4}$

Answer: B

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10. If $y=\sqrt{\sin x+y}$, then $\frac{dy}{dx}$ is equal to

A. $\frac{\cos x}{2y-1}$

B. $\frac{\cos x}{1-2y}$

C. $\frac{\sin x}{1-2y}$

D. $\frac{\sin x}{2y-1}$

Answer: A

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11. The derivative of $\cos^{-1}\left(2x^2-1\right)$ w.r.t $\cos^{-1}x$ is

A. 2

B. $\frac{-1}{2\sqrt{1-x^2}}$

C. $\frac{2}{x}$

D. $1 - x^2$

Answer: A

Watch Video Solution

12. If $x = t^2$ and $y = t^3$, then $\frac{d^2y}{dx^2}$ is equal to

A. $\frac{3}{2}$

B. $\frac{3}{4t}$

C. $\frac{3}{2t}$

D. $\frac{3t}{2}$

Answer: B

Watch Video Solution

13. The value of c in Rolle's theorem for the function $f(x) = x^3$ in the interval $x \in \left[0, \sqrt{3}\right]$

A. 1

B. -1

C. $\frac{3}{2}$

D. $\frac{1}{3}$

Answer: A

Watch Video Solution

14. For the function $f(x) = x + \frac{1}{x}, x \in [1, 3]$ the value of c for mean value theorem is

A. 1

B. $\sqrt{3}$

C. 2

D. None of these

Answer: B

Watch Video Solution

15. If $f(x) = 2x$ and $g(x) = \frac{x^2}{2} + 1$, then which of the following can be a discontinuous function?

A. $f(x) + g(x)$

B. $f(x) - g(x)$

C. $f(x).\, g(x)$

D. $\frac{g(x)}{f(x)}$

Answer: D

Watch Video Solution

16. The function $f(x) = \frac{4 - x^2}{4x - x^3}$ is

A. discontinuous at only one point

B. discontinuous at exactly two points

C. discontinuous at exactly three points

D. none of the above

Answer: C

Watch Video Solution

17. The set of points where the function f given by $f(x) = |2x - 1|\sin x$ is differentiable is $x \in$ ....

A. R

B. $R - \left\{\frac{1}{2}\right\}$

C. $(0, \infty)$

D. none of these

Answer: B

Watch Video Solution

18. The function $f(x) = \cot x$ is discontinuous on the set

A. $\{x \mid x = n\pi, n \in Z\}$

B. $\{x \mid x = 2n\pi, n \in Z\}$

C. $\left\{x \mid x = (2n+1)\frac{\pi}{2}, n \in Z\right\}$

D. $\left\{x \mid x = \frac{n\pi}{2}, n \in Z\right\}$

Answer: A

Watch Video Solution

19. The function $f(x) = e^{|x|}$ is

A. continuous everywher but not differentiable at x= 0

B. continuous and differentiable everywhere

C. Not continuous at x= 0

D. none of the above

Answer: A

Watch Video Solution

20. If $f(x) = x^2 \sin\left(\frac{1}{x}\right)$, where $x \neq 0$, then the value of the function f at x=0, so that the function is continuous at x= 0, is

A. 0

B. -1

C. 1

D. None of these

Answer: A

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21. If $f(x)\begin{cases} mx+1 & x \le \frac{\pi}{2} \\ (\sin x)+n & x > \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$, then ......

A. $m = 1, n = 0$

B. $m = \frac{n\pi}{2} + 1$

C. $n = \frac{m\pi}{2}$

D. $m = n = \frac{\pi}{2}$

Answer: C

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22. If $f(x) = |\cos x|$, then $f'\left(\frac{\pi}{4}\right)$ is equal to ...... $0 < x < \frac{\pi}{2}$

A. f is everywhere differentiable

B. f is everywhere continuous but not differentiable at $x = n\pi, n \in Z$

C. f is everywhere continuous but not differentiable at $x = \left(2n + \frac{\pi}{2}\right), n \in Z$

D. none of the above

Answer: B

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23. If $y = \log\left(\frac{1 - x^2}{1 + x^2}\right)$, then $\frac{dy}{dx}$ is equal to

A. $\frac{4x^3}{1 - x^4}$

B. $\frac{-4x}{1 - x^4}$

C. $\frac{1}{4 - x^4}$

D. $\frac{-4x^3}{1 - x^4}$

Answer: B

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24. If $y = \sqrt{\sin x + y}$, then $\frac{dy}{dx}$ is equal to

A. $\frac{\cos x}{2y - 1}$

B. $\frac{\cos x}{1 - 2y}$

C. $\frac{\sin x}{1 - 2y}$

D. $\frac{\sin x}{2y - 1}$

Answer: A

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25. The derivative of $\cos^{-1}\left(2x^2 - 1\right)$ w.r.t $\cos^{-1}x$ is

A. 2

B. $\frac{-1}{2\sqrt{1 - x^2}}$

C. $\frac{2}{x}$

D. $1 - x^2$

Answer: A

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26. If $x = t^2$ and $y = t^3$, then $\frac{d^2y}{dx^2}$ is equal to

A. $\frac{3}{2}$

B. $\frac{3}{4t}$

C. $\frac{3}{2t}$

D. $\frac{3t}{2}$

Answer: B

Watch Video Solution

27. The value of c in Rolle's theorem for the function $f(x) = x^3$ in the interval $x \in \left[0, \sqrt{3}\right]$

A. 1

B. -1

C. $\frac{3}{2}$

D. $\frac{1}{3}$

Answer: A

Watch Video Solution

28. For the function $f(x) = x + \frac{1}{x}, x \in [1, 3]$ the value of c for mean value theorem is

A. 1

B. $\sqrt{3}$

C. 2

D. None of these

Answer: B

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NCERT Exemplar Problems and Solution (Fillers)

1. Does there exist a function which is continuous everywhere but not differentiable at exactly two points ? Justify your answer.

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2. Derivative of x^2 w.r.t x^3 is......

Watch Video Solution

3. If $f(x) = |\cos x|$, then $f'\left(\frac{\pi}{4}\right)$ is equal to ...... $0 < x < \frac{\pi}{2}$

Watch Video Solution

4. If $f(x) = |\cos x - \sin x|$, then $f'\left(\frac{\pi}{3}\right)$ is equal to ....

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5. For the curve $\sqrt{x} + \sqrt{y} = 1$, $\frac{dy}{dx}$ at $\left(\frac{1}{4}, \frac{1}{4}\right)$ is ....

Watch Video Solution

6. Does there exist a function which is continuous everywhere but not differentiable at exactly two points ? Justify your answer.

Watch Video Solution

7. Derivative of x^2 w.r.t x^3 is......

Watch Video Solution

8. If $f(x) = |\cos x|$, then $f'\left(\frac{\pi}{4}\right)$ is equal to ...... $0 < x < \frac{\pi}{2}$

Watch Video Solution

9. If $f(x) = |\cos x - \sin x|$, then $f'\left(\frac{\pi}{3}\right)$ is equal to ....

Watch Video Solution

10. For the curve $\sqrt{x} + \sqrt{y} = 1$, $\frac{dy}{dx}$ at $\left(\frac{1}{4}, \frac{1}{4}\right)$ is ....

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NCERT Exemplar Problems and Solution (True/False)

1. Rolle's theorem is applicable for the function $f(x) = |x - 1| \text{in } x \in [0, 2]$

Watch Video Solution

2. If f is continuous on its domain D, then |f| is also continuous on D

Watch Video Solution

3. The composition of two continuous function is a continuous function

Watch Video Solution

4. Trigonometric and inverse trigonometric functions and differentiable in their respective domain.

Watch Video Solution

5. If f.g is continuous at x=a, then f and g are separately continuous at x= a.

Watch Video Solution

6. Rolle's theorem is applicable for the function $f(x) = |x - 1| \text{in } x \in [0, 2]$

Watch Video Solution

7. If f is continuous on its domain D, then |f| is also continuous on D

Watch Video Solution

8. The composition of two continuous function is a continuous function

Watch Video Solution

9. Trigonometric and inverse trigonometric functions and differentiable in their respective domain.

Watch Video Solution

10. If f.g is continuous at x=a, then f and g are separately continuous at x= a.

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Practice Paper - 5 (Section - A)

1. $\frac{d}{dx}\left(\sqrt{x\sin x}\right)$= ...... $(0 < x < \pi)$

A. $\frac{x\sin x + \cos x}{\sqrt{x\sin x}}$

B. $\frac{x\cos x}{2\sqrt{x\sin x}}$

C. $\frac{x\cos x + \sin x}{2\sqrt{x\sin x}}$

D. $\frac{1}{2\sqrt{x\sin x}}$

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2. $\frac{d}{dx}\left(\tan^{-1}x + \cot^{-1}x\right)$= ......

A. 0

B. $\frac{1}{1+x^2}$

C. $-\frac{1}{1+x^2}$

D. Does not exist

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3. $\frac{d}{dx}\left(a^a\right) = \ldots\ldots(a > 0)$

A. $a^a(1 + \log a)$

B. 0

C. a^a

D. Does not exist

Watch Video Solution

4. $\sqrt{x}+\sqrt{y}=\sqrt{a}$ then $\dfrac{dy}{dx}$= ....

A. $\dfrac{y}{x}$

B. $\sqrt{\dfrac{y}{x}}$

C. $-\sqrt{\dfrac{y}{x}}$

D. $-\sqrt{\dfrac{x}{y}}$

Watch Video Solution

5. If $2t = v^2$ then $\frac{dv}{dt}$= ....

A. 0

B. $\frac{1}{v}$

C. $\frac{1}{2}$

D. - $\frac{1}{v^2}$

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6. $\frac{d}{dx}\left(\sqrt{x\sin x}\right)$= ...... $(0 < x < \pi)$

A. $\frac{x\sin x + \cos x}{\sqrt{x\sin x}}$

B. $\frac{x\cos x}{2\sqrt{x\sin x}}$

C. $\frac{x\cos x + \sin x}{2\sqrt{x\sin x}}$

D. $\frac{1}{2\sqrt{x\sin x}}$

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7. $\frac{d}{dx}\left(\tan^{-1}x+\cot^{-1}x\right)$= ......

A. 0

B. $\frac{1}{1+x^2}$

C. - $\frac{1}{1+x^2}$

D. Does not exist

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8. $\frac{d}{dx}\left(a^a\right) = \ldots\ldots(a > 0)$

A. $a^a(1+\log a)$

B. 0

C. a^a

D. Does not exist

Watch Video Solution

9. $\sqrt{x}+\sqrt{y}=\sqrt{a}$ then $\frac{dy}{dx}$= ....

A. $\frac{y}{x}$

B. $\sqrt{\frac{y}{x}}$

C. $-\sqrt{\frac{y}{x}}$

D. $-\sqrt{\frac{x}{y}}$

Watch Video Solution

10. If $2t=v^2$ then $\frac{dv}{dt}$= ....

A. 0

B. $\frac{1}{v}$

C. $\frac{1}{2}$

D. $-\frac{1}{v^2}$

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Practice Paper - 5 (Section- B)

1. If $x^y = e^{x-y}$, then find $\frac{dy}{dx}$

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2. $f(x) = \begin{cases} kx + 1, & x \leq \pi \\ \cos x & x > \pi \end{cases}$ If the function f(x) is continuous at $x = \pi$, then find the value of k.

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3. Find $\frac{dy}{dx}$ in the following

$x^3 + x^2y + xy^2 + y^3 = 81$

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4. Find the derivative of $\sin^2 x$ with respect to $e^{\cos x}$

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5. If $x^y = e^{x-y}$, then find $\frac{dy}{dx}$

Watch Video Solution

6. $f(x) = \begin{cases} kx + 1, & x \leq \pi \\ \cos x & x > \pi \end{cases}$ If the function f(x) is continuous at $x = \pi$, then find the value of k.

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7. Find $\frac{dy}{dx}$ in the following

$x^3 + x^2y + xy^2 + y^3 = 81$

Watch Video Solution

8. Find the derivative of $\sin^2 x$ with respect to $e^{\cos x}$

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Practice Paper -5 (Section-C)

1. Verify Mean Value Theorem, if $f(x) = x^3 - 5x^2 - 3x$ in the interval [a, b], where a=1 and b=3. Find all $c \in (1, 3)$ for which f'(c)= 0

Watch Video Solution

2. Prove that the greatest integer function defined by $f(x) = [x], 0 < x < 3$ is not differentiable at x=1 and x=2.

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3. If x and y are connected parametrically by the equations without eliminating the parameter, find $\frac{dy}{dx}$

$x = a(\cos\theta + \theta\sin\theta), y = a(\sin\theta - \theta\cos\theta)$

Watch Video Solution

4. Find $\frac{dy}{dx}$ in the following

$y = \sin^{-1}\left(2x\sqrt{1 - x^2}\right), \ -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$

Watch Video Solution

5. Find $\frac{dy}{dx}, y = x^{\sin x} + (\sin x)^{\cos x}$

Watch Video Solution

6. Verify Mean Value Theorem, if $f(x) = x^3 - 5x^2 - 3x$ in the interval [a, b], where a=1 and b=3. Find all $c \in (1, 3)$ for which f'(c)= 0

Watch Video Solution

7. Prove that the greatest integer function defined by $f(x) = [x], 0 < x < 3$ is not differentiable at x=1 and x=2.

Watch Video Solution

8. If x and y are connected parametrically by the equations without eliminating the parameter, find $\frac{dy}{dx}$

$x = a(\cos\theta + \theta\sin\theta), y = a(\sin\theta - \theta\cos\theta)$

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9. Find $\frac{dy}{dx}$ in the following

$$y = \sin^{-1}\left(2x\sqrt{1 - x^2}\right), \ -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$$

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10. Find $\frac{dy}{dx}, y = x^{\sin x} + (\sin x)^{\cos x}$

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Practice Paper - 5 (Section-D)

1. If $y = e^{\cos^{-1}x}, \ -1 \le x \le 1$, then prove that $\left(1 - x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - y = 0$

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2. Differentiate w.r.t x the function $0 < x < \frac{\pi}{2}, \cot^{-1}\left[\frac{\sqrt{1 + \sin x} + \sqrt{1 - \sin x}}{\sqrt{1 + \sin x} - \sqrt{1 - \sin x}}\right]$

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3. If $y = e^{\cos^{-1}x}, -1 \leq x \leq 1$, then prove that $\left(1 - x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - y = 0$

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4. Prove that :

$$\cot^{-1}\left(\frac{\sqrt{1 + \sin x} + \sqrt{1 - \sin x}}{\sqrt{1 + \sin x} - \sqrt{1 - \sin x}}\right) = \frac{x}{2}, x \in \left(0, \frac{\pi}{4}\right)$$

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