



MATHS

BOOKS - KUMAR PRAKASHAN KENDRA MATHS (GUJRATI ENGLISH)

CONTINUITY AND DIFFERENTIABILITY

Practice Work

1. Examine the continuity of the function f(x) = 2x + 3 at x = 1

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2. Prove that the function $f(x) = x^2$ is continuous at x=0.

3. Find the points of discontinuity of the following function.

(a)
$$f(x) = \frac{3x+7}{x^2-5x+6}$$
 (b) $f(x) = \frac{1}{|x|-1} - \frac{x^2}{2}$ (c) $f(x) = \frac{\sqrt{x^2+1}}{1+\sin^2 x}$ (d) $f(x) = \tan\left(\frac{\pi}{2}x\right)$

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4. $f(x) = \frac{x^2 + 1}{x^2 - 1}$ and $g(x) = \tan x$. Examine the continuity of (fog) (x).

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5.
$$y = f(\mu)$$
, where $f(\mu) = \frac{3}{2\mu^2 + 5\mu - 3}$ and $\mu = \frac{1}{x + 2}$. Find the points of

discontinuity of y.

6. Show that
$$f(x) = \begin{cases} x^3 + 3, & x \neq 0 \\ 1, & x = 0 \end{cases}$$
 is a discontinuous function at x= 0.

7.
$$f(x) = \frac{1}{(x-1)(x-2)}$$
 and $g(x) = \frac{1}{x^2}$. Find the points of discontinuity of

the composite function f(g(x))?

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8.
$$f(x) = \begin{cases} \frac{|\sin x|}{x}, & x \neq 0\\ 1, & x = 0 \end{cases}$$
 Examine the continuity of f(x), x= 0

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$$\mathbf{9.} f(x) = \begin{cases} \frac{1}{e^{4x} + 1}, & x \neq 0\\ 0, & x = 0 \end{cases}$$
 Examine the continuity of f(x) at x= 0

10. f (x)= $\{x\}$ and g(x)= [x]. Where $\{ \}$ is a fractional part and [] is a greatest

integer function. Prove that f(x) + g(x) is a continuous function at x= 1.



11. Show that $f(x) = [x] + [-x], x \in R$ - {integer} is a continuous function.

Where [] is a greatest integer function



12. Discuss the continuity and differentiability for $f(x) = [\sin x]$ when $x \in [0, 2\pi]$, where $[\cdot]$ denotes the greatest integer function x.



13.
$$f(x) = \begin{cases} |x - 1|, & x \ge 0 \\ -|x|, & x < 0 \end{cases}$$
 Prove that $f(x)$ is continuous for $x \in R - \{0\}$.

14. Prove that
$$f(x) = \begin{cases} \frac{\sin x}{x} + \cos x, & x \neq 0\\ 2, & x = 0 \end{cases}$$
 is a continuous function at x= 0



15. Prove that f(x) = 2x - |x| is a continuous function at x= 0.

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16. For which value of x, the function $f(x) = \frac{e^{\sin x}}{4 - \sqrt{x^2 - 9}}$ is discontinuous?

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17. $f(x) = \frac{x^2 + 1}{x^2 - 1}$ and $g(x) = \tan x$. Examine the continuity of (fog) (x).

$$\mathbf{18.} f(x) = \begin{cases} \frac{1}{2} - x, & 0 \le x < \frac{1}{2} \\ 1, & x = \frac{1}{2} \\ \frac{3}{2} - x, & \frac{1}{2} < x \le 1 \end{cases}$$
 Discuss the continuity of f(x)

19.
$$f(x) = \begin{cases} 2 + \sqrt{1 - x^2}, & |x| \le 1 \\ 2e^{(1 - x)^2}, & |x| > 1 \end{cases}$$
 Discuss the continuity of f(x) at x=1

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20.
$$f(x) = \frac{\sqrt{2}\cos x - 1}{\cot x - 1}, x \neq \frac{\pi}{4}$$
. If the function $f(x)$ is continuous at $x = \frac{\pi}{4}$ then find $f\left(\frac{\pi}{4}\right)$

21.
$$f(x) = \begin{cases} \frac{\tan 2x}{x}, & x \neq 0\\ K, & x = 0 \end{cases}$$
 If a function f is continuous at x=0 then find k.

22.
$$f(x) = \begin{cases} \frac{1 - \cos 4x}{8x^2}, & x \neq 0\\ k, & x = 0 \end{cases}$$
 If the function f(x) is continuous at x= 0, then

find k.



23. Show that,
$$f(x) = \begin{cases} \frac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1}, & x \neq 0\\ 0, & x = 0 \end{cases}$$
. Is discontinuous at x= 0

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.

24. The give functions is continuous at x=0. Find a, b, c

$$f(x) = \begin{cases} \frac{\sin(a+1)x + \sin x}{x}, & x < 0\\ C, & x = 0\\ \frac{\sqrt{x+bx^2} - \sqrt{x}}{bx^{\frac{3}{2}}}, & x > 0 \end{cases}$$

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25.
$$f(x) = \begin{cases} \frac{x(1+a\cos x) - b\sin x}{x^3}, & x \neq 0\\ 0, & x = 0 \end{cases}$$
. If f is continuous at x=0 then find the

value of a and b.



26. The give functions is continuous at x=0. Find a, b, c

$$f(x) = \begin{cases} \frac{\sin(a+1)x + \sin x}{x}, & x < 0\\ C, & x = 0\\ \frac{\sqrt{x+bx^2} - \sqrt{x}}{bx^{\frac{3}{2}}}, & x > 0 \end{cases}$$

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27.
$$f(x) = \begin{cases} -4\sin x + \cos x, & x \le -\frac{\pi}{2} \\ a\sin x + b, & -\frac{\pi}{2} < x < \frac{\pi}{2}. & \text{If } f(x) \text{ is continuous for } x \in R, \\ \cos x + 2, & \frac{\pi}{2} \le x \end{cases}$$

then find the value of a and b.



28.
$$f(x) = \begin{cases} |x+1|, & x < -2 \\ 2x+3, & -2 \le x < 0 \\ x^2+3, & 0 \le x < 3 \\ x^3-15 & 3 \le x \end{cases}$$
. Find at which points, the function f(x) is

discontinuous ?

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29.
$$f(x) = \frac{1 - \tan x}{4x - \pi}, x \neq \frac{\pi}{4}$$
. If the function $f(x), x \in \left[0, \frac{\pi}{2}\right)$ is continuous then find $f\left(\frac{\pi}{4}\right)$.

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30.
$$f(x) = \begin{cases} \frac{2^{x+2} - 16}{4^x - 16}, & x \neq 2\\ k, & x = 2 \end{cases}$$
 f(x) is continuous at x=2 then find k

31. Find the derivative of the following functions with respect to x

 $\tan(2x + 3)$

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32. Find the derivative of the following functions with respect to x

sin 3*x*. sin3*x*

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33. Find the derivative of the following functions with respect to x

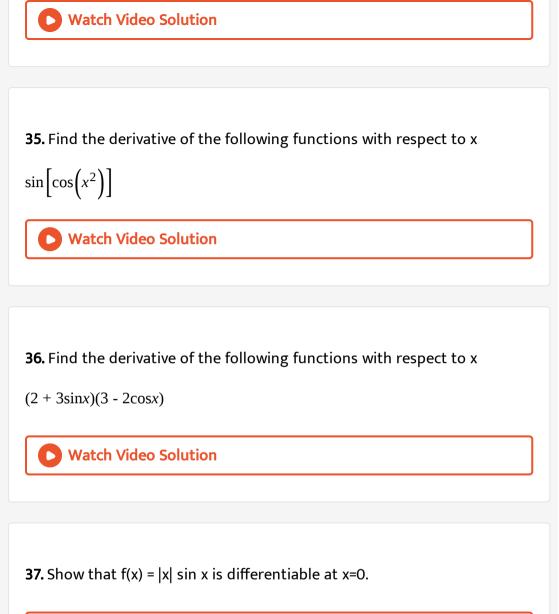
$$\sqrt{\frac{x+1}{x-1}}$$



34. Find the derivative of the following functions with respect to x

sec*x* - 1

 $\sec x + 1$



38. The right hand derivative of $f(x) = [x] \tan (\pi x)$ at a point x=7 is $k\pi$ then

find the value of k. where [.] is the greatest integer function.

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39.
$$f(x) \begin{cases} \left| x - \frac{1}{2} \right|, & 0 \le x < 1 \\ x[x], & 1 \le x < 2 \end{cases}$$
 where [.] denotes the greatest integer

function. Show that f(x) is continuous at x=1 but not differentiable at x=1.

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40. $f(x) = x^3 \operatorname{sgn}(x)$. Show that f(x) is differentiable at x=0.



41. The left hand derivative of $f(x) = [x]\sin(\pi x)$ at x = k is an integer, is

42. Differentiate the following functions with respect to x:

$$x^2 + y^2 = xy$$



43. Differentiate the following functions with respect to x:

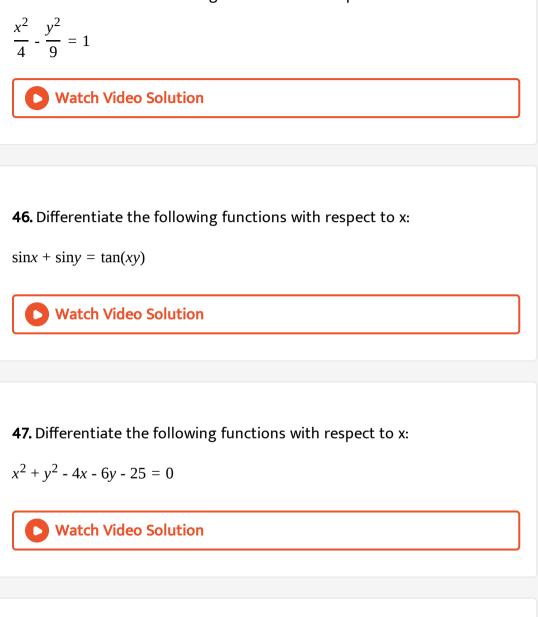
$$x^3 + y^3 = \sin(x + y)$$

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44. Differentiate the following functions with respect to x:

$$\sqrt{x} + \sqrt{y} = \sqrt{a}$$

45. Differentiate the following functions with respect to x:



48. Differentiate the following functions with respect to x:

 $x + \sin x = \sin y$

49. Find
$$\frac{dy}{dx}$$
 in the following:
 $y = \sin^{-1}(3x - 4x^3), 0 < x < \frac{1}{2}$

50. Find
$$\frac{dy}{dx}$$
 in the following:

$$y = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right), |x| < \frac{1}{\sqrt{3}}$$

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51. Find
$$\frac{dy}{dx}$$
 in the following:
 $y = \sin^{-1}\left(2x\sqrt{1-x^2}\right), \frac{1}{\sqrt{2}} < x < 1$

52. Find
$$\frac{dy}{dx}$$
 in the following:
 $y = \sec^{-1}\left(\frac{x^2+1}{x^2-1}\right)$

53. Find
$$\frac{dy}{dx}$$
 in the following:
 $y = \tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right)$

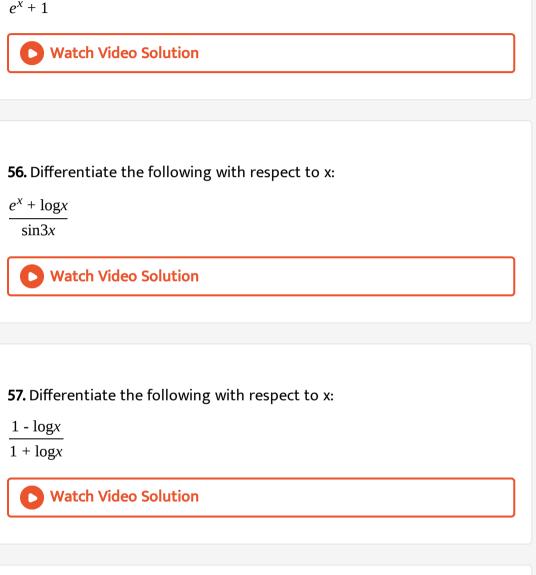


54. Differentiate the following with respect to x:

 $\frac{e^{x} \log x}{x^{2}}$

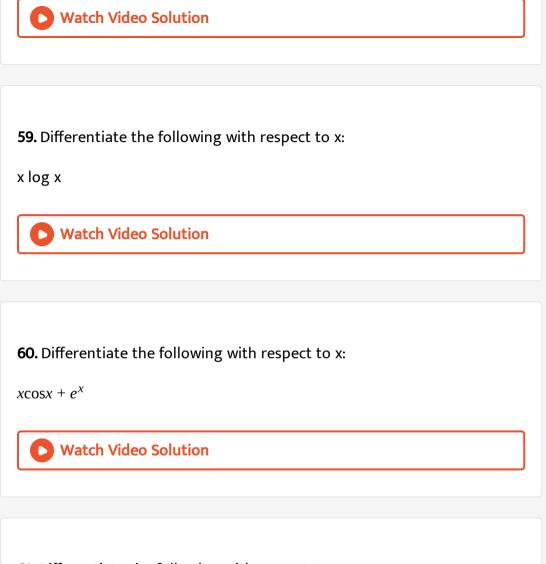
55. Differentiate the following with respect to x:

 $\frac{e^x - 1}{e^x + 1}$



58. Differentiate the following with respect to x:

$$\cos^{-1}(e^x)$$



61. Differentiate the following with respect to x:

 $\log \left[\log \left(\log x^5 \right) \right]$

62. Differentiate the following with respect to x:

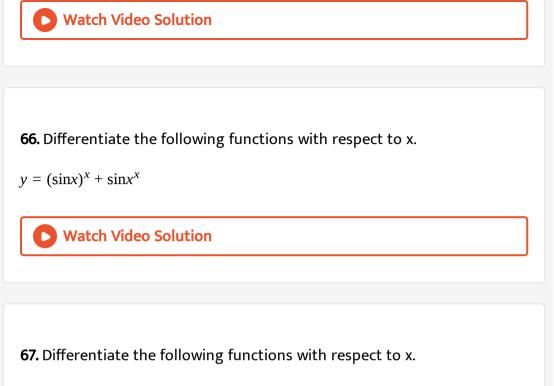
logx $1 + x \log x$ Watch Video Solution 63. Differentiate the following with respect to x: $\sin\left[\log(e^{\chi})\right]$ Watch Video Solution 64. Differentiate the following functions with respect to x.

$$y = (\sin x)^{x} + \left(\frac{1}{x}\right)^{\cos x}$$

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65. Differentiate the following functions with respect to x.

 $y = x^{x} . \sin x + (\sin x)^{x}$



$$x^y + y^x = 1000$$

68. Differentiate the following functions with respect to x.

$$y = \cos\left(x^{x}\right) + \sin\left(x^{x}\right)$$

69. If
$$y = x^{x^x}$$
 then find $\frac{dy}{dx}$



70. If
$$y = x^{x} + x^{a} + a^{x}$$
 then find $\frac{dy}{dx}$

71.
$$e^x + e^y = e^{x+y}$$
 then prove that, $\frac{dy}{dx} + \frac{e^x(e^y - 1)}{e^y(e^x - 1)} = 0$

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72. If
$$y = x^{\tan x} + \sqrt{\frac{x^2 + 1}{x}}$$
 then find $\frac{dy}{dx}$

73. If
$$f(x) = |\cos x - \sin x|$$
 then find $f'\left(\frac{\pi}{6}\right)$



74. Find
$$\frac{dy}{dx}$$
:

 $x = \cos^2 \theta$ and $y = \sin^2 \theta$

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75. Find
$$\frac{dy}{dx}$$
:
 $x = t + \frac{1}{t}$ and $y = t - \frac{1}{t}$

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76. Find
$$\frac{dy}{dx}$$
:
 $x = te^t, y = 1 + \log t$

77. Find
$$\frac{dy}{dx}$$
:
 $x = a\sec^3\theta, y = a\tan^3\theta$

78. Find
$$\frac{dy}{dx}$$
:
 $x = a\sin^2\theta\cos\theta$, $y = 2b\cos^2\theta$ (- sin θ)

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79. Find
$$\frac{dy}{dx}$$
:
 $x = 2\cos\theta - \cos^2\theta$ and $y = 2\sin\theta - \sin^2\theta$
Show that $\frac{dy}{dx} = -1$ when $\theta = \frac{\pi}{2}$

80. Find
$$\frac{dy}{dx}$$
:
 $x = e^{\cos 2t}$ and $y = e^{\sin 2t}$ show that, $\frac{dy}{dx} = \frac{-y \log x}{x \log y}$

81. Find
$$\frac{dy}{dx}$$
:
 $x = a\sin 2t(1 + \cos 2t)$ and $y = b\cos 2t$
(1 - cos2t) show that, $\left(\frac{dy}{dx}\right)_{t=\frac{\pi}{4}} = \frac{b}{a}$

82. Find the second order derivatives of the following functions:

 e^{ax}

83. Find the second order derivatives of the following functions:

$$x^3 + \tan x$$

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84. Find the second order derivatives of the following functions:

 $\sin^2 x$

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85. Find the second order derivatives of the following functions:

 $\tan^{-1}3x$



86. Find the second order derivatives of the following functions:

 $\log e^{\chi^{x^{x}}}$



87. Find the second order derivatives of the following functions:

 $3\sin 4x - 4\sin^3 4x$

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88. Find the second order derivatives of the following functions:

 $e^{-2\log x}$

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89. Find the second order derivatives of the following functions:

 $\sin\left(x^2+5\right)$

90. If $y = \sin^{-1}x$ then prove that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$



91. If
$$y = e^{\tan x}$$
 then show that, $\left(\cos^2 x\right) \frac{d^2 y}{dx^2} - (1 + \sin^2 x) \frac{dy}{dx} = 0$

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92. If
$$y = \sin(\sin x)$$
 then show that, $\frac{d^2y}{dx^2} + (\tan x)\frac{dy}{dx} + y\cos^2 x = 0$

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93. If
$$x^m$$
. $y^n = (x + y)^{m+n}$ then show that, $\frac{d^2y}{dx^2} = 0$

94. If
$$y = \log\left(x + \sqrt{x^2 + 1}\right)$$
 then show that, $\left(x^2 + 1\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$

95. If
$$y = e^{ax} \sin bx$$
 then show that, $y_2 - 2ay_1 + (a^2 + b^2)y = 0$

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96. If
$$y = (x + \sqrt{x^2 + 1})^m$$
 then prove that, $(x^2 + 1)y_2 + xy_1 = m^2y$

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97. If
$$2x = y^{\frac{1}{m}} + y^{-\frac{1}{m}} (n \ge 1)$$
 then prove that, $(x^2 - 1)y_2 + xy_1 = m^2 y$

98. Verify Rolle's theorem for the following functions:

$$f(x) = \sqrt{9 - x^2}, x \in [-3, 3]$$

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99. Verify Rolle's theorem for the following functions:

 $f(x) = x^3 - 6x^2 + 11x - 6, x \in [2, 3]$

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100. Verify Rolle's theorem for the following functions:

 $f(x) = x(x - 3)^2, x \in [0, 3]$

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101. Verify Rolle's theorem for the following functions:

$$f(x) = \sin x + \cos x - 1, x \in \left[0, \frac{\pi}{2}\right]$$



102. Verify Rolle's theorem for the following functions:

 $f(x) = a^{\sin x}, x \in [0, \pi], a > 0$

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103. Vertify mean value theorem for the following functions:

 $f(x) = \log_e x, x \in [1, 2]$

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104. Vertify mean value theorem for the following functions:

 $f(x) = x - 2\sin x, x \in [-\pi, \pi]$

105. Vertify mean value theorem for the following functions:

$$f(x) = x + \frac{1}{x}, x \in [1, 3]$$
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106. Vertify mean value theorem for the following functions:
$$f(x) = \tan^{-1}x, x \in [0, 1]$$

107. Vertify mean value theorem for the following functions:

 $f(x) = x^2 + 2x + 3, x \in [4, 6]$

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108. Prove that
$$0 < a < b < \frac{\pi}{2}$$
, $\sec^2 a < \frac{\tan b - \tan a}{b - a} < \sec^2 b$

109.
$$y = e^{x + e^{x + e^{x + \dots \infty}}}$$
 then find $\frac{dy}{dx}$

110. Find the derivative of
$$\tan^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right]$$
 with respect to $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$
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111.
$$y = x + \frac{1}{x + \frac{1}{x + \frac{1}{x + \dots \infty}}}$$
 then find $\frac{dy}{dx}$

112. Find the derivative of
$$\sin^{-1}\left[\frac{2^{x+1} \cdot 3^x}{1+(36)^x}\right]$$
 with respect to x.

113.
$$x = a \sin t$$
 and $y = a \left(\cos t + \log \tan \frac{t}{2} \right)$ then find $\frac{d^2 y}{dx^2}$.

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114. If
$$y = \frac{ax^2}{(x-a)(x-b)(x-c)} + \frac{bx}{(x-b)(x-c)} + \frac{c}{(x-c)} + 1$$
 then prove that
 $\frac{y'}{y} = \frac{1}{x} \left[\frac{a}{a-x} + \frac{b}{b-x} + \frac{c}{c-x} \right]$

115.
$$y = \sin^{-1} \left[\frac{5x + 12\sqrt{1 - x^2}}{13} \right]$$
 then find $\frac{dy}{dx}$

116. If
$$y = x \log\left(\frac{x}{a+bx}\right)$$
 prove that $\frac{d^2y}{dx^2} = \frac{1}{x}\left(\frac{a}{a+bx}\right)^2$

117. If
$$y = \left\{x + \sqrt{x^2 + a^2}\right\}^n$$
 prove that $\frac{dy}{dx} = \frac{ny}{\sqrt{x^2 + a^2}}$. $n > 1 \neq N$

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118.
$$y = \sqrt{\frac{1 - \sin 2x}{1 + \sin 2x}}$$
 then prove that $\frac{dy}{dx} + \sec^2\left(\frac{\pi}{4} - x\right) = 0$

119. Show that
$$\frac{d}{dx}e^{ax}\cos(bx+c) = re^{ax}\cos(bx+c+\alpha)$$
 where $r = \sqrt{a^2 + b^2}, \cos\alpha = \frac{a}{r}, \sin\alpha = \frac{b}{r}$ and $\frac{d^2}{dx^2}e^{ax}\cos(ax+c) = r^2e^{ax}\cos(bx+c+2\alpha)$



120. If
$$(a - b\cos y)(a + b\cos x) = a^2 - b^2$$
 show that $\frac{dy}{dx} = \frac{\sqrt{a^2 - b^2}}{a + b\cos x}, 0 < x < \frac{\pi}{2}$

121. Find
$$\frac{d}{dx} \left[\operatorname{cosec}^{-1} x \right]_{x=-2}$$

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122.
$$\frac{d}{dx}$$
tan⁻¹(secx - tanx)- Find

123. Prove that,
$$\frac{d}{dx} \left[\log \frac{x^2 + x + 1}{x^2 - x + 1} + \frac{2}{\sqrt{3}} \tan^{-1} \frac{\sqrt{3}x}{1 - x^2} \right] = \frac{4}{x^4 + x^2 + 1}$$
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124. Examine the continuity of the function $f(x) = 2x + 3$ at $x = 1$
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125. Prove that the function $f(x) = x^2$ is continuous at x=0.
126. Find the points of discontinuity of the following function.

(a)
$$f(x) = \frac{3x+7}{x^2-5x+6}$$
 (b) $f(x) = \frac{1}{|x|-1} - \frac{x^2}{2}$ (c) $f(x) = \frac{\sqrt{x^2+1}}{1+\sin^2 x}$ (d)
 $f(x) = \tan\left(\frac{\pi}{2}x\right)$

127.
$$f(x) = \frac{x^2 + 1}{x^2 - 1}$$
 and $g(x) = \tan x$. Examine the continuity of (fog) (x).

128.
$$y = f(\mu)$$
, where $f(\mu) = \frac{3}{2\mu^2 + 5\mu - 3}$ and $\mu = \frac{1}{x+2}$. Find the points of

discontinuity of y.

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129. Show that
$$f(x) = \begin{cases} x^3 + 3, & x \neq 0 \\ 1, & x = 0 \end{cases}$$
 is a discontinuous function at x= 0.

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130. $f(x) = \frac{1}{(x-1)(x-2)}$ and $g(x) = \frac{1}{x^2}$. Find the points of discontinuity of

the composite function f(g(x))?



131.
$$f(x) = \begin{cases} \frac{|\sin x|}{x}, & x \neq 0\\ 1, & x = 0 \end{cases}$$
 Examine the continuity of f(x), x= 0

132.
$$f(x) = \begin{cases} \frac{1}{e^{4x} + 1}, & x \neq 0\\ 0, & x = 0 \end{cases}$$
 Examine the continuity of f(x) at x= 0

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133. $f(x) = \{x\}$ and g(x) = [x]. Where $\{ \}$ is a fractional part and [] is a greatest

integer function. Prove that f(x) + g(x) is a continuous function at x= 1.

134. Show that $f(x) = [x] + [-x], x \in \mathbb{R}$ - {integer} is a continuous

function. Where [] is a greatest integer function



135. $f(x) = [\sin x], x \in [0, 2\pi]$ At which points, f(x) is discontinuous?

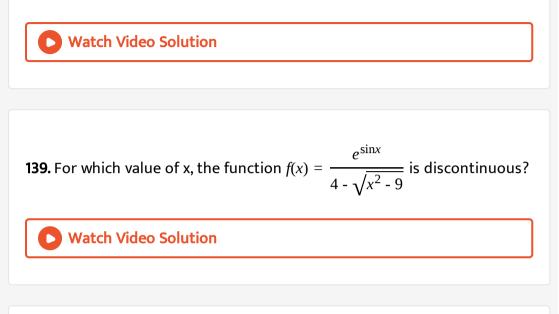
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136.
$$f(x) = \begin{cases} |x-1|, & x \ge 0 \\ -|x|, & x < 0 \end{cases}$$
 Prove that $f(x)$ is continuous for $x \in R - \{0\}$.

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137. Prove that
$$f(x) = \begin{cases} \frac{\sin x}{x} + \cos x, & x \neq 0\\ 2, & x = 0 \end{cases}$$
 is a continuous function at x= 0

138. Prove that f(x) = 2x - |x| is a continuous function at x= 0.



140.
$$f(x) = \frac{x^2 + 1}{x^2 - 1}$$
 and $g(x) = \tan x$. Examine the continuity of the

composite function (fog) (x)

141.
$$f(x) = \begin{cases} \frac{1}{2} - x, & 0 \le x < \frac{1}{2} \\ 1, & x = \frac{1}{2} \\ \frac{3}{2} - x, & \frac{1}{2} < x \le 1 \end{cases}$$
 Discuss the continuity of f(x)

142.
$$f(x) = \begin{cases} 2 + \sqrt{1 - x^2}, & |x| \le 1 \\ 2e^{(1 - x)^2} & |x| > 1 \end{cases}$$
 Discuss the continuity of f(x) at x=1

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143.
$$f(x) = \frac{\sqrt{2}\cos x - 1}{\cot x - 1}, x \neq \frac{\pi}{4}$$
. If the function $f(x)$ is continuous at $x = \frac{\pi}{4}$ then find $f\left(\frac{\pi}{4}\right)$

144.
$$f(x) = \begin{cases} \frac{\tan 2x}{x}, & x \neq 0\\ K, & x = 0 \end{cases}$$
 If a function f is continuous at x=0 then find k.

145.
$$f(x) = \begin{cases} \frac{1 - \cos 4x}{8x^2}, & x \neq 0\\ k, & x = 0 \end{cases}$$
. If the function f(x) is continuous at x= 0, then

find k.

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146. Show that,
$$f(x) = \begin{cases} \frac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1}, & x \neq 0\\ 0, & x = 0 \end{cases}$$
. Is discontinuous at x= 0

147. The give functions is continuous at x=0. Find a, b, c

$$f(x) = \begin{cases} \frac{\sin(a+1)x + \sin x}{x}, & x < 0\\ C, & x = 0\\ \frac{\sqrt{x+bx^2} - \sqrt{x}}{bx^{\frac{3}{2}}}, & x > 0 \end{cases}$$

148.
$$f(x) = \begin{cases} \frac{x(1+a\cos x) - b\sin x}{x^3}, & x \neq 0\\ 0, & x = 0 \end{cases}$$
. If f is continuous at x=0 then find the

value of a and b.



149.
$$f(x) = \begin{cases} \frac{\sin(p+1)x + \sin x}{x}, & x < 0\\ q, & x = 0\\ \frac{\sqrt{x+x^2} - \sqrt{x}}{x}, & \frac{\sqrt{x+x^2} - \sqrt{x}}{x} \end{cases}$$
 If f(x) is continuous for $x \in R$ then

find the value of p and q.

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150.
$$f(x) = \begin{cases} -4\sin x + \cos x, & x \le -\frac{\pi}{2} \\ a\sin x + b, & -\frac{\pi}{2} < x < \frac{\pi}{2}. & \text{If } f(x) \text{ is continuous for } x \in R, \\ \cos x + 2, & \frac{\pi}{2} \le x \end{cases}$$

then find the value of a and b.



151.
$$f(x) = \begin{cases} |x+1|, & x < -2 \\ 2x+3, & -2 \le x < 0 \\ x^2+3, & 0 \le x < 3 \\ x^3-15 & 3 \le x \end{cases}$$
. Find at which points, the function f(x) is

discontinuous ?

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152.
$$f(x) = \frac{1 - \tan x}{4x - \pi}, x \neq \frac{\pi}{4}$$
. If the function $f(x), x \in \left[0, \frac{\pi}{2}\right)$ is continuous then find $f\left(\frac{\pi}{4}\right)$.

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153.
$$f(x) = \begin{cases} \frac{2^{x+2} - 16}{4^x - 16}, & x \neq 2\\ k, & x = 2 \end{cases}$$
 f(x) is continuous at x=2 then find k

154. Find the derivative of the following functions with respect to x

 $\tan(2x + 3)$

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155. Find the derivative of the following functions with respect to x

sin 3*x*. sin3*x*

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156. Find the derivative of the following functions with respect to x

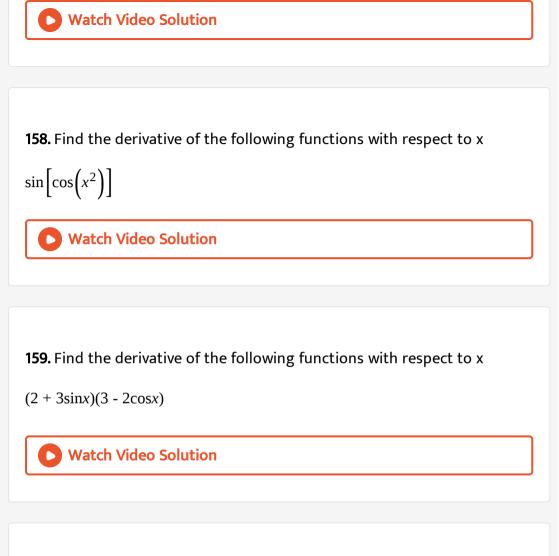
$$\sqrt{\frac{x+1}{x-1}}$$



157. Find the derivative of the following functions with respect to x

sec*x* - 1

 $\sec x + 1$



160. Show that $f(x) = |x| \sin x$ is differentiable at x=0.

161. The right hand derivative of $f(x) = [x] \tan (\pi x)$ at a point x=7 is $k\pi$ then

find the value of k. where [.] is the greatest integer function.

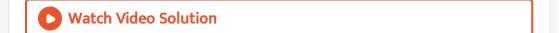
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162.
$$f(x) \begin{cases} \left| x - \frac{1}{2} \right|, & 0 \le x < 1 \\ x[x], & 1 \le x < 2 \end{cases}$$
 where [.] denotes the greatest integer

function. Show that f(x) is continuous at x=1 but not differentiable at x=1.

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163. $f(x) = x^3 \operatorname{sgn}(x)$. Show that f(x) is differentiable at x=0.



164. Find the left hand derivative of $f(x) = [x]\sin(\pi x)$ at x=k. where k is an

integer and [.] denotes the greatest integer function.

165. Differentiate the following functions with respect to x:

$$x^2 + y^2 = xy$$

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166. Differentiate the following functions with respect to x:

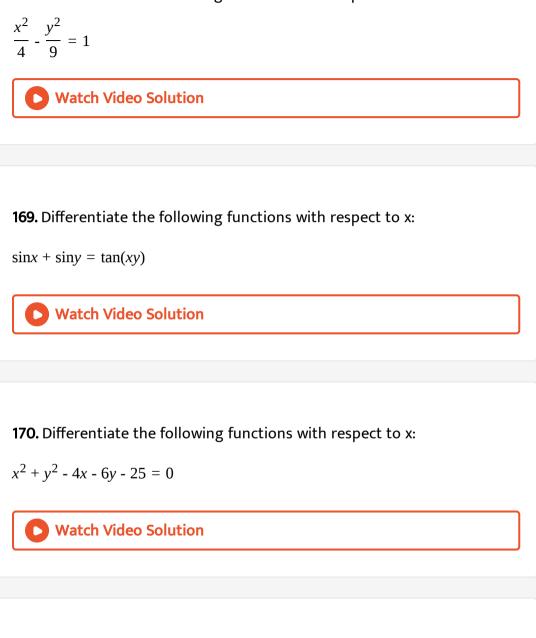
$$x^3 + y^3 = \sin(x + y)$$

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167. Differentiate the following functions with respect to x:

$$\sqrt{x} + \sqrt{y} = \sqrt{a}$$

168. Differentiate the following functions with respect to x:



171. Differentiate the following functions with respect to x:

 $x + \sin x = \sin y$

172. Find
$$\frac{dy}{dx}$$
 in the following:
 $y = \sin^{-1}(3x - 4x^3), 0 < x < \frac{1}{2}$

173. Find
$$\frac{dy}{dx}$$
 in the following:

$$y = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right), |x| < \frac{1}{\sqrt{3}}$$

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174. Find
$$\frac{dy}{dx}$$
 in the following:
 $y = \sin^{-1}\left(2x\sqrt{1-x^2}\right), \frac{1}{\sqrt{2}} < x < 1$

175. Find
$$\frac{dy}{dx}$$
 in the following:
 $y = \sec^{-1}\left(\frac{x^2+1}{x^2-1}\right)$

176. Find
$$\frac{dy}{dx}$$
 in the following:
 $y = \tan^{-1} \left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x} \right)$



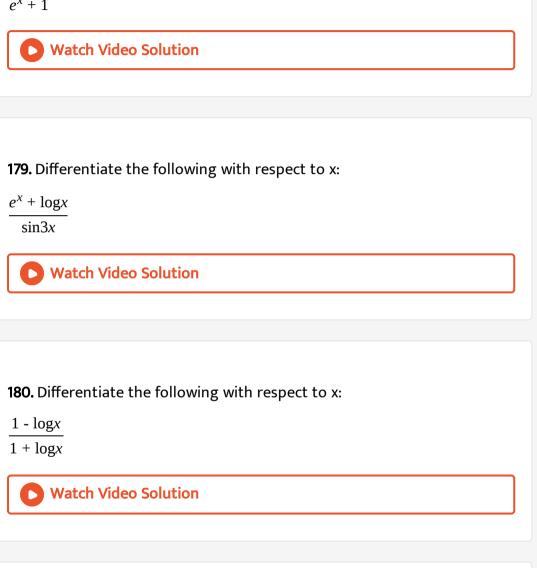
177. Differentiate the following with respect to x:

 $\frac{e^{x}\log x}{2}$

 x^2

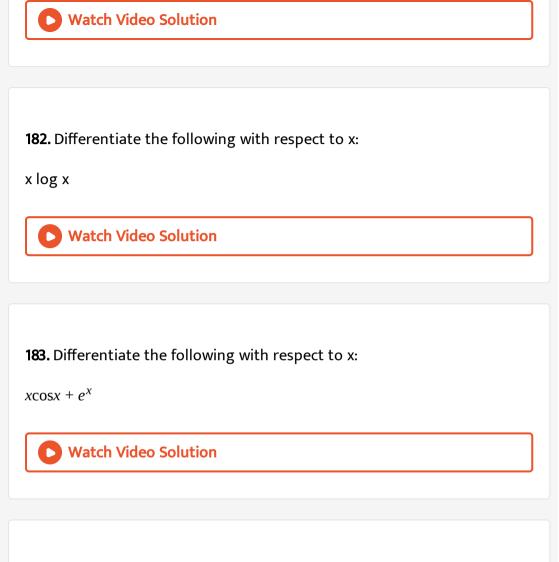
178. Differentiate the following with respect to x:

 $\frac{e^{x}-1}{e^{x}+1}$



181. Differentiate the following with respect to x:

$$\cos^{-1}(e^x)$$



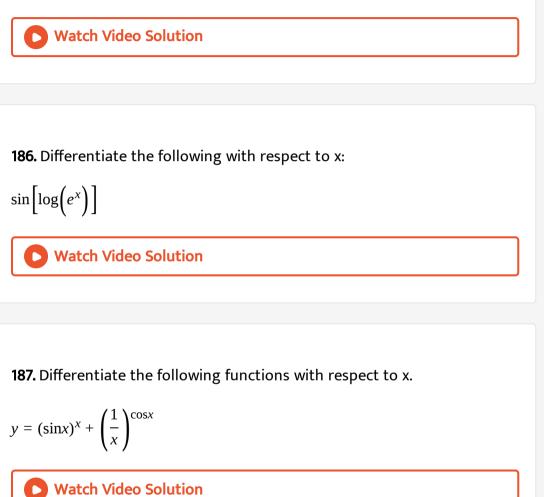
184. Differentiate the following with respect to x:

 $\log \left[\log \left(\log x^5 \right) \right]$

185. Differentiate the following with respect to x:

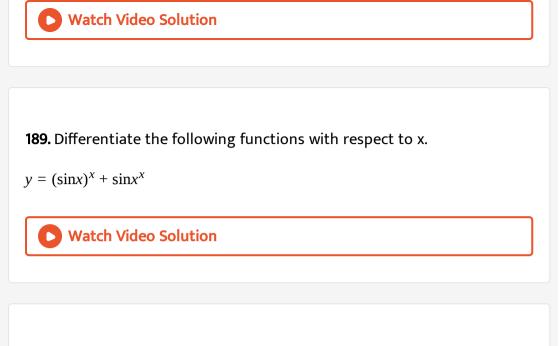
logx

 $1 + x \log x$



188. Differentiate the following functions with respect to x.

$$y = x^{\chi} . \sin x + (\sin x)^{\chi}$$



190. Differentiate the following functions with respect to x.

$$x^y + y^x = 1000$$

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191. Differentiate the following functions with respect to x.

$$y = \cos\left(x^{x}\right) + \sin\left(x^{x}\right)$$

192. If
$$y = x^{x^x}$$
 then find $\frac{dy}{dx}$



193. If
$$y = x^{x} + x^{a} + a^{x}$$
 then find $\frac{dy}{dx}$

194.
$$e^{x} + e^{y} = e^{x+y}$$
 then prove that, $\frac{dy}{dx} + \frac{e^{x}(e^{y}-1)}{e^{y}(e^{x}-1)} = 0$

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195. If
$$y = x^{\tan x} + \sqrt{\frac{x^2 + 1}{x}}$$
 then find $\frac{dy}{dx}$

196. If
$$f(x) = |\cos x - \sin x|$$
 then find $f\left(\frac{\pi}{6}\right)$



197. Find
$$\frac{dy}{dx}$$
:
 $x = \cos^2 \theta$ and $y = \sin^2 \theta$

198. Find
$$\frac{dy}{dx}$$
:
 $x = t + \frac{1}{t}$ and $y = t - \frac{1}{t}$

199. Find
$$\frac{dy}{dx}$$
:
 $x = te^{t}, y = 1 + \log t$

200. Find
$$\frac{dy}{dx}$$
:
 $x = a\sec^3\theta, y = a\tan^3\theta$

201. Find
$$\frac{dy}{dx}$$
:

$$x = a\sin^2\theta\cos\theta, y = 2b\cos^2\theta(-\sin\theta)$$

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202. Find
$$\frac{dy}{dx}$$
:
 $x = 2\cos\theta - \cos^2\theta$ and $y = 2\sin\theta - \sin^2\theta$
Show that $\frac{dy}{dx} = -1$ when $\theta = \frac{\pi}{2}$

203. Find
$$\frac{dy}{dx}$$
:
 $x = e^{\cos 2t}$ and $y = e^{\sin 2t}$ show that, $\frac{dy}{dx} = \frac{-y \log x}{x \log y}$

204. Find
$$\frac{dy}{dx}$$
:
 $x = a\sin 2t(1 + \cos 2t)$ and $y = b\cos 2t$
 $(1 - \cos 2t)$ show that, $\left(\frac{dy}{dx}\right)_{t=\frac{\pi}{4}} = \frac{b}{a}$

205. Find the second order derivatives of the following functions:

 e^{ax}

206. Find the second order derivatives of the following functions:

$$x^3 + \tan x$$

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207. Find the second order derivatives of the following functions:

 $\sin^2 x$

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208. Find the second order derivatives of the following functions:

 $\tan^{-1}3x$

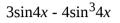


209. Find the second order derivatives of the following functions:

 $\log e^{\chi^{x^{x}}}$



210. Find the second order derivatives of the following functions:



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211. Find the second order derivatives of the following functions:

 $e^{-2\log x}$

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212. Find the second order derivatives of the following functions:

 $\sin\left(x^2+5\right)$

213. If
$$y = \sin^{-1}x$$
 then prove that $\left(1 - x^2\right) \frac{d^2y}{dx^2} - x \frac{dy}{dx} = 0$

214. If
$$y = e^{\tan x}$$
 then show that, $\left(\cos^2 x\right) \frac{d^2 y}{dx^2} - (1 + \sin 2x) \frac{dy}{dx} = 0$

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215. If
$$y = \sin(\sin x)$$
 then show that, $\frac{d^2y}{dx^2} + (\tan x)\frac{dy}{dx} + y\cos^2 x = 0$

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216. If
$$x^m$$
. $y^n = (x + y)^{m+n}$ then show that, $\frac{d^2y}{dx^2} = 0$

217. If
$$y = \log\left(x + \sqrt{x^2 + 1}\right)$$
 then show that, $\left(x^2 + 1\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$

218. If
$$y = e^{ax} \sin bx$$
 then show that, $y_2 - 2ay_1 + (a^2 + b^2)y = 0$

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219. If
$$y = (x + \sqrt{x^2 + 1})^m$$
 then prove that, $(x^2 + 1)y_2 + xy_1 = m^2y$

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220. If
$$2x = y\frac{1}{m} + y^{-1}\frac{1}{m}$$
 $(n \ge 1)$ then prove that, $(x^2 - 1)y_2 + xy_1 = m^2y$

221. Verify Rolle's theorem for the following functions:

$$f(x) = \sqrt{9 - x^2}, x \in [-3, 3]$$

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222. Verify Rolle's theorem for the following functions:

 $f(x) = x^3 - 6x^2 + 11x - 6, x \in [2, 3]$

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223. Verify Rolle's theorem for the following functions:

 $f(x) = x(x - 3)^2, x \in [0, 3]$



224. Verify Rolle's theorem for the following functions:

$$f(x) = \sin x + \cos x - 1, x \in \left[0, \frac{\pi}{2}\right]$$



225. Verify Rolle's theorem for the following functions:

 $f(x) = a^{\sin x}, x \in [0, \pi], a > 0$

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226. Vertify mean value theorem for the following functions:

 $f(x) = \log_e x, x \in [1, 2]$

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227. Vertify mean value theorem for the following functions:

 $f(x) = x - 2\sin x, x \in [-\pi, \pi]$

228. Vertify mean value theorem for the following functions:

$$f(x) = x + \frac{1}{x}, x \in [1, 3]$$
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229. Vertify mean value theorem for the following functions:
$$f(x) = \tan^{-1}x, x \in [0, 1]$$
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230. Vertify mean value theorem for the following functions:

 $f(x) = x^2 + 2x + 3, x \in [4, 6]$

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231. Prove that
$$0 < a < b < \frac{\pi}{2}$$
, $\sec^2 a < \frac{\tan b - \tan a}{b - a} < \sec^2 b$

232.
$$y = e^{x + e^{x + e^{x + \dots \infty}}}$$
 then find $\frac{dy}{dx}$

233. Find the derivative of
$$\tan^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right]$$
 with respect to $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$
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234.
$$y = x + \frac{1}{x + \frac{1}{x + \frac{1}{x + \dots \infty}}}$$
 then find $\frac{dy}{dx}$

235. Find the derivative of
$$\sin^{-1}\left[\frac{2^{x+1} \cdot 3^x}{1+(36)^x}\right]$$
 with respect to x.

236.
$$x = a \sin t$$
 and $y = a \left(\cos t + \log \tan \frac{t}{2} \right)$ then find $\frac{d^2 y}{dx^2}$.

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237. If
$$y = \frac{ax^2}{(x-a)(x-b)(x-c)} + \frac{bx}{(x-b)(x-c)} + \frac{c}{(x-c)} + 1$$
 then prove that

$$\frac{y'}{y} = \frac{1}{x} \left[\frac{a}{a-x} + \frac{b}{b-x} + \frac{c}{c-x} \right]$$

$$238. y = \sin^{-1} \left[\frac{5x + 12\sqrt{1 - x^2}}{13} \right] \text{ then find } \frac{dy}{dx}$$

239. If
$$y = x \log\left(\frac{x}{a+bx}\right)$$
 prove that $\frac{d^2y}{dx^2} = \frac{1}{x}\left(\frac{a}{a+bx}\right)^2$

240. If
$$y = \left\{x + \sqrt{x^2 + a^2}\right\}^n$$
 prove that $\frac{dy}{dx} = \frac{ny}{\sqrt{x^2 + a^2}}$. $n > 1 \neq N$

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241.
$$y = \sqrt{\frac{1 - \sin 2x}{1 + \sin 2x}}$$
 then prove that $\frac{dy}{dx} + \sec^2\left(\frac{\pi}{4} - x\right) = 0$

242. Show that
$$\frac{d}{dx}e^{ax}\cos(bx+c) = re^{ax}\cos(bx+c+\alpha)$$
 where $r = \sqrt{a^2 + b^2}$, $\cos\alpha = \frac{a}{r}$, $\sin\alpha = \frac{b}{r}$ and $\frac{d^2}{dx^2}e^{ax}\cos(ax+c) = r^2e^{ax}\cos(bx+c+2\alpha)$



243. If
$$(a - b\cos y)(a + b\cos x) = a^2 - b^2$$
 show that $\frac{dy}{dx} = \frac{\sqrt{a^2 - b^2}}{a + b\cos x}, 0 < x < \frac{\pi}{2}$

244. Find
$$\frac{d}{dx} \left[\operatorname{cosec}^{-1} x \right]_{x=-2}$$

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245.
$$\frac{d}{dx}$$
tan⁻¹(secx - tanx)- Find

246. Prove that,
$$\frac{d}{dx} \left[\log \frac{x^2 + x + 1}{x^2 - x + 1} + \frac{2}{\sqrt{3}} \tan^{-1} \frac{\sqrt{3}x}{1 - x^2} \right] = \frac{4}{x^4 + x^2 + 1}$$

Exercise-5.1

1. Prove that the function f(x) = 5x - 3 is continuous at x=0, at x = -3 and

at x= 5



2. Examine the continuity of the function $f(x) = 2x^2 - 1$ at x = 3



3. Examine the following functions for continuity.

$$f(x) = x - 5$$

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4. Examine the following functions for continuity.

$$f(x)=\frac{1}{x-5}, x\neq 5$$

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5. Examine the following functions for continuity.

$$f(x) = \frac{x^2 - 25}{x + 5}, x \neq -5$$

6. Examine the following functions for continuity.

f(x) = |x - 5|

• Watch Video Solution 7. Prove that the function $f(x) = x^n$ is continuous at x = n, where n is a positive integer • Watch Video Solution

8. Is the function f defined by $f(x) = \begin{cases} x, & \text{if } x \le 1 \\ 5, & \text{if } x > 1 \end{cases}$ continuous at x = 0? At

x=1? At x=2?

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$$9. f(x) = \begin{cases} 2x + 3, & \text{if } x \le 2\\ 2x - 3, & \text{if } x > 2 \end{cases}$$

$$\mathbf{10.} f(x) = \begin{cases} |x| + 3, & \text{if } x \le -3 \\ -2x, & \text{if } -3 < x < 3 \\ 6x + 2, & \text{if } x \ge 3 \end{cases}$$

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$$\mathbf{11.} f(x) = \begin{cases} \frac{|x|}{x}, & \text{if } x \neq 0\\ 0, & \text{if } x = 0 \end{cases}$$

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$$\mathbf{12.} f(x) = \begin{cases} \frac{x}{|x|}, & \text{if } x < 0\\ -1, & \text{if } x \ge 0 \end{cases}$$

13.
$$f(x) = \begin{cases} x+1, & \text{if } x \ge 1 \\ x^2+1, & \text{if } x < 1 \end{cases}$$

14.
$$f(x) = \begin{cases} x^3 - 3, & \text{if } x \le 2 \\ x^2 + 1, & \text{if } x > 2 \end{cases}$$

15.
$$f(x) = \begin{cases} x^{10} - 1, & \text{if } x \le 1 \\ x^2, & \text{if } x > 1 \end{cases}$$

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16. Is the function defined by $f(x) = \begin{cases} x+5, & \text{if } x \le 1 \\ x-5, & \text{if } x > 1 \end{cases}$ a continuous

function?

$$\mathbf{17.} f(x) = \begin{cases} 3, & \text{if} 0 \le x \le 1\\ 4, & \text{if} 1 \le x \le 3\\ 5, & \text{if} 3 \le x \le 10 \end{cases}$$

$$\mathbf{18.} f(x) = \begin{cases} 2x, & \text{if } x < 0\\ 0, & \text{if } 0 \le x \le 1\\ 4x, & \text{if } x > 1 \end{cases}$$

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$$\mathbf{19.} f(x) = \begin{cases} -2, & \text{if } x \le -1 \\ 2x, & \text{if } -1 < x \le 1 \\ 2, & \text{if } x > 1 \end{cases}$$

20. Find the relationship between a and b so that the function f defined

by

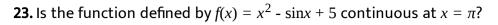
$$f(x) = \begin{cases} ax + 1 & \text{if } x \le 3\\ bx + 3 & \text{if } x > 3 \end{cases} \text{ is continuous at x=3.}$$

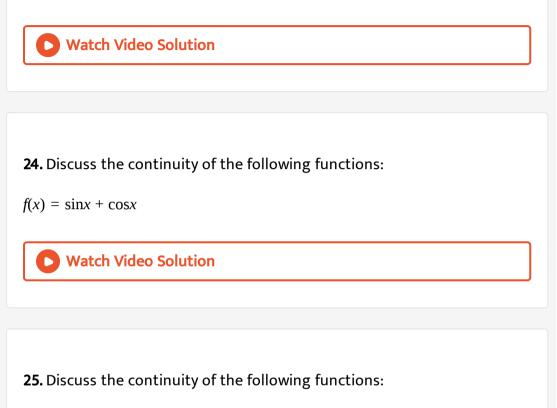


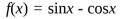
21. For what value of
$$\lambda$$
 is the function defined by
$$f(x) = \begin{cases} \lambda (x^2 - 2x), & \text{if } x \leq 0 \\ 4x + 1, & \text{if } x > 0 \end{cases}$$
continuous at x=0 ? What about continuity at x=1?

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22. Show that the function defined by g(x) = x - [x] is discontinuous at all integral points. Here [x] denotes the greatest integer less than or equal to x.









26. Discuss the continuity of the following functions:

 $f(x) = \sin x \cdot \cos x$



27. Discuss the continuity of

 $f(x) = \cos x, x \in R$

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28. Discuss the continuity of the cosine, cosecant, secant and cotangent

functions:

$$f(x) = \operatorname{cosec} x = \frac{1}{\sin x}, x \in R - \{n\pi\}, n \in I$$

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29. Discuss the continuity of the cosine, cosecant, secant and cotangent

functions:

$$f(x) = \sec x = \frac{1}{\cos x}, x \in R - \left\{ (2n+1)\frac{\pi}{2}, n \in I \right\}$$

30. Discuss the continuity of the cosine, cosecant, secant and cotangent

functions:

$$f(x) = \cot x = \frac{1}{\tan x}, x \in R - \{n\pi, n \in I\}$$

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31. Find all points of discontinuity of f, where
$$f(x) = \begin{cases} \frac{\sin x}{x}, & \text{if } x < 0 \\ x + 1, & \text{if } x \ge 0 \end{cases}$$

32. Determine if f defined by
$$f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$
 is a continuous

function?

$$f(x) = \begin{cases} \sin x - \cos x, & \text{if } x \neq 0\\ -1 & \text{if } x = 0 \end{cases}$$

34. Find the values of k so that the function f is continuous at the indicated point

$$f(x) = \begin{cases} \frac{k\cos x}{\pi - 2x}, & \text{if } x \neq \frac{\pi}{2} \\ 3 & \text{if } x = \frac{\pi}{2} \end{cases} \text{ at } x = \frac{\pi}{2}$$

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35. Find the values of k so that the function f is continuous at the indicated point

$$f(x) = \begin{cases} kx^2, & \text{if } x \le 2\\ 3, & \text{if } x > 2 \end{cases} \text{ at } x=2$$



36. Find the values of k so that the function f is continuous at the

indicated point

$$f(x) = \begin{cases} kx + 1, & \text{if } x \le \pi \\ \cos x & \text{if } x > \pi \end{cases} \text{ at } x = \pi$$

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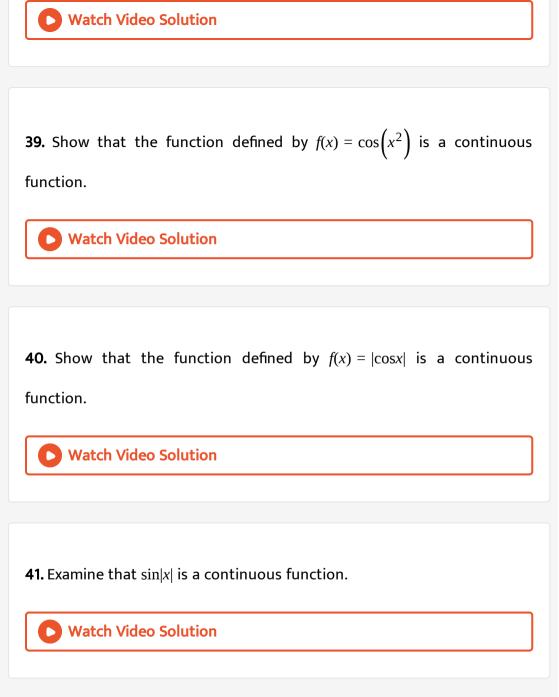
37. Find the values of k so that the function f is continuous at the indicated point

$$f(x) = \begin{cases} kx + 1, & \text{if } x \le 5\\ 3x - 5, & \text{if } x > 5 \end{cases} \text{ at } x = 5$$

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38. Find the values of a and b such that the function defined by

$$f(x) = \begin{cases} 5 & \text{if } x \le 2\\ ax + b, & \text{If } 2 < x < 10 \text{ is a continuous function}\\ 21 & \text{If } x \ge 10 \end{cases}$$



42. Find all the points of discontinuity of f defined by f(x) = |x| - |x + 1|



43. Prove that the function f(x) = 5x - 3 is continuous at x=0, at x = -3

and at x= 5

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44. Examine the continuity of the function $f(x) = 2x^2 - 1$ at x = 3

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45. Examine the following functions for continuity.

f(x) = x - 5

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$$f(x)=\frac{1}{x-5}, x\neq 5$$

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47. Examine the following functions for continuity.

$$f(x) = \frac{x^2 - 25}{x + 5}, x \neq -5$$

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48. Examine the following functions for continuity.

$$f(x) = |x - 5|$$



49. Prove that the function $f(x) = x^n$ is continuous at x= n, where n is a

positive integer

50. Is the function f defined by $f(x) = \begin{cases} x, & \text{if } x \le 1 \\ 5, & \text{if } x > 1 \end{cases}$ continuous at x = 0? At

x=1? At x=2?

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51.
$$f(x) = \begin{cases} 2x + 3, & \text{if } x \le 2\\ 2x - 3, & \text{if } x > 2 \end{cases}$$

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52.
$$f(x) = \begin{cases} |x| + 3, & \text{if } x \le -3 \\ -2x, & \text{if } -3 < x < 3 \\ 6x + 2, & \text{if } x \ge 3 \end{cases}$$

53.
$$f(x) = \begin{cases} \frac{|x|}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

54.
$$f(x) = \begin{cases} \frac{x}{|x|}, & \text{if } x < 0 \\ -1, & \text{if } x \ge 0 \end{cases}$$

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55.
$$f(x) = \begin{cases} x+1, & \text{if } x \ge 1 \\ x^2+1, & \text{if } x < 1 \end{cases}$$

56.
$$f(x) = \begin{cases} x^3 - 3, & \text{if } x \le 2 \\ x^2 + 1, & \text{if } x > 2 \end{cases}$$

57.
$$f(x) = \begin{cases} x^{10} - 1, & \text{if } x \le 1 \\ x^2, & \text{if } x > 1 \end{cases}$$

58. Is the function defined by
$$f(x) = \begin{cases} x+5, & \text{if } x \le 1 \\ x-5, & \text{if } x > 1 \end{cases}$$
 a continuous

function?

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59.
$$f(x) = \begin{cases} 3, & \text{if } 0 \le x \le 1 \\ 4, & \text{if } 1 < x < 3 \\ 5, & \text{if } 3 \le x \le 10 \end{cases}$$

$$60. f(x) = \begin{cases} 2x, & \text{if } x < 0\\ 0, & \text{if } 0 \le x \le 1\\ 4x, & \text{if } x > 1 \end{cases}$$

$$\mathbf{61.} f(x) = \begin{cases} -2, & \text{if } x \le -1 \\ 2x, & \text{if } -1 < x \le 1 \\ 2, & \text{if } x > 1 \end{cases}$$

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62. Find the relationship between a and b so that the function f defined

by

$$f(x) = \begin{cases} ax + 1 & \text{if } x \le 3\\ bx + 3 & \text{if } x > 3 \end{cases} \text{ is continuous at x=3.}$$

63. For what value of
$$\lambda$$
 is the function defined by
$$f(x) = \begin{cases} \lambda (x^2 - 2x), & \text{if } x \leq 0 \\ 4x + 1, & \text{if } x > 0 \end{cases}$$
continuous at x=0? What about continuity at x=1?

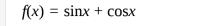
64. Show that the function defined by g(x) = x - [x] is discontinuous at all integral points. Here [x] denotes the greatest integer less than or equal

to x.



65. Is the function defined by $f(x) = x^2 - \sin x + 5$ continuous at $x = \pi$?

66. Discuss the continuity of the following functions:



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67. Discuss the continuity of the following functions:

 $f(x) = \sin x - \cos x$

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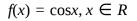
68. Discuss the continuity of the following functions:

 $f(x) = \sin x \cdot \cos x$



69. Discuss the continuity of the cosine, cosecant, secant and cotangent

functions:





70. Discuss the continuity of the cosine, cosecant, secant and cotangent

functions:

$$f(x) = \operatorname{cosec} x = \frac{1}{\sin x}, x \in R - \{n\pi\}, n \in I$$

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71. Discuss the continuity of the cosine, cosecant, secant and cotangent

functions:

$$f(x) = \sec x = \frac{1}{\cos x}, x \in R - \left\{ (2n+1)\frac{\pi}{2}, n \in I \right\}$$

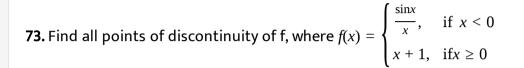
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72. Discuss the continuity of the cosine, cosecant, secant and cotangent

functions:

$$f(x) = \cot x = \frac{1}{\tan x}, x \in R - \{n\pi, n \in I\}$$





74. Determine if f defined by
$$f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$
 is a continuous

function?

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75. Examine the continuity of f, where f is defined by $f(x) = \begin{cases} \sin x - \cos x, & \text{if } x \neq 0 \\ -1, & \text{if } x = 0 \end{cases}$

76. Find the values of k so that the function f is continuous at the indicated point

$$f(x) = \begin{cases} \frac{k\cos x}{\pi - 2x}, & \text{if } x \neq \frac{\pi}{2} \\ 3 & \text{if } x = \frac{\pi}{2} \end{cases} \text{ at } x = \frac{\pi}{2}$$

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77. Find the values of k so that the function f is continuous at the indicated point

$$f(x) = \begin{cases} kx^2, & \text{if } x \le 2\\ 3, & \text{if } x > 2 \end{cases} \text{ at } x=2$$

78. Find the values of k so that the function f is continuous at the

indicated point

$$f(x) = \begin{cases} kx + 1, & \text{if } x \le \pi \\ \cos x & \text{if } x > \pi \end{cases} \text{ at } x = \pi$$

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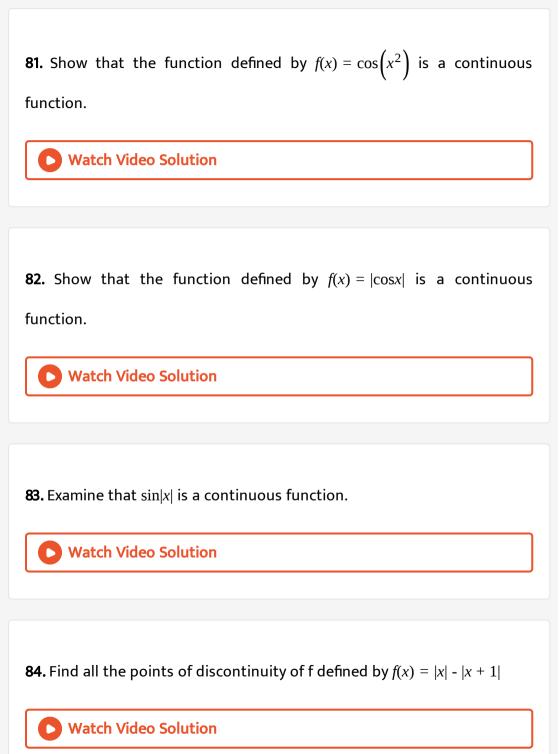
79. Find the values of k so that the function f is continuous at the indicated point

$$f(x) = \begin{cases} kx + 1, & \text{if } x \le 5\\ 3x - 5, & \text{if } x > 5 \end{cases} \text{ at } x = 5$$

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80. Find the values of a and b such that the function defined by

$$f(x) = \begin{cases} 5 & \text{if } x \le 2\\ ax + b, & \text{If } 2 < x < 10 \text{ is a continuous function}\\ 21 & \text{If } x \ge 10 \end{cases}$$





 $\sin\left(x^2+5\right)$

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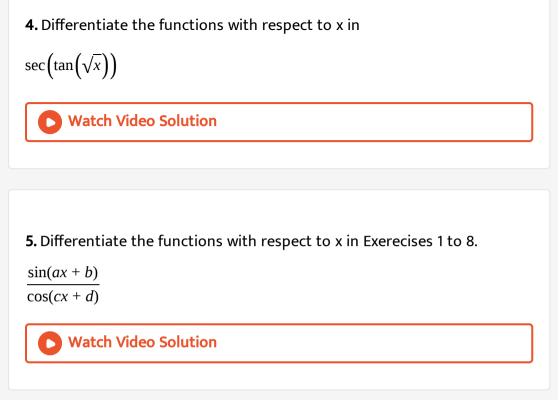
2. Differentiate the functions with respect to x in

cos(sinx)

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3. Differentiate the functions with respect to x in

sin(ax + b)



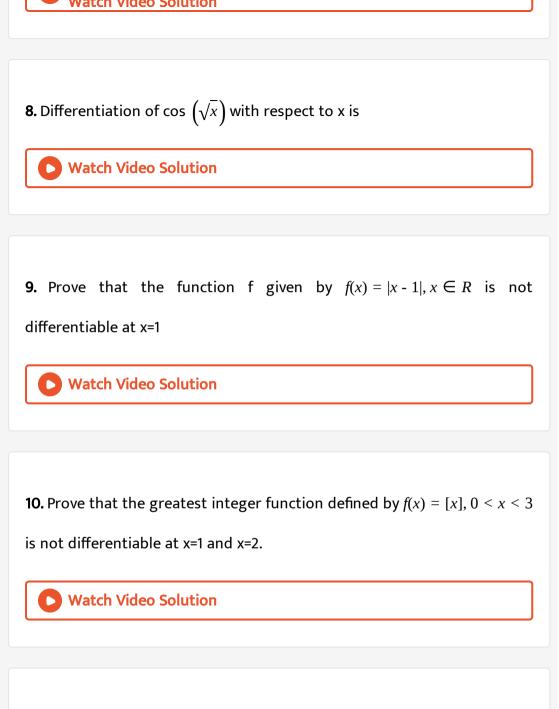
6. Differentiate the functions with respect to x in Exerecises 1 to 8.

 $\cos x^3 \cdot \sin^2(x^5)$.

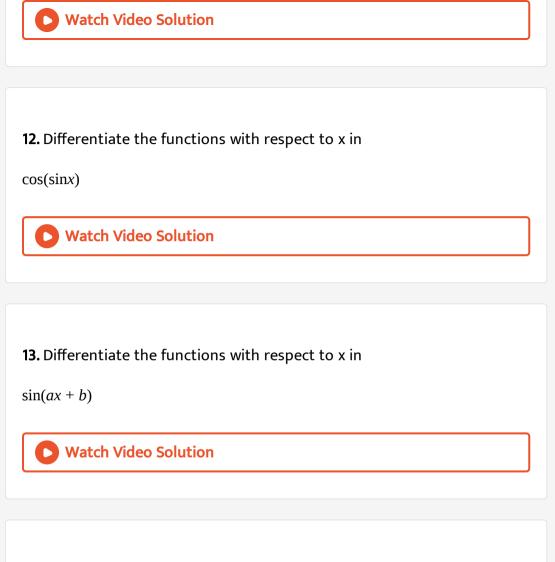
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7. Differentiate the functions with respect to x in

$$2\sqrt{\cot(x^2)}$$



 $\sin\left(x^2+5\right)$



 $\operatorname{sec}(\operatorname{tan}(\sqrt{x}))$

 $\frac{\sin(ax+b)}{\cos(cx+d)}$



16. Differentiate the functions with respect to x in

$$\cos\left(x^3\right)$$
. $\sin^2\left(x^5\right)$

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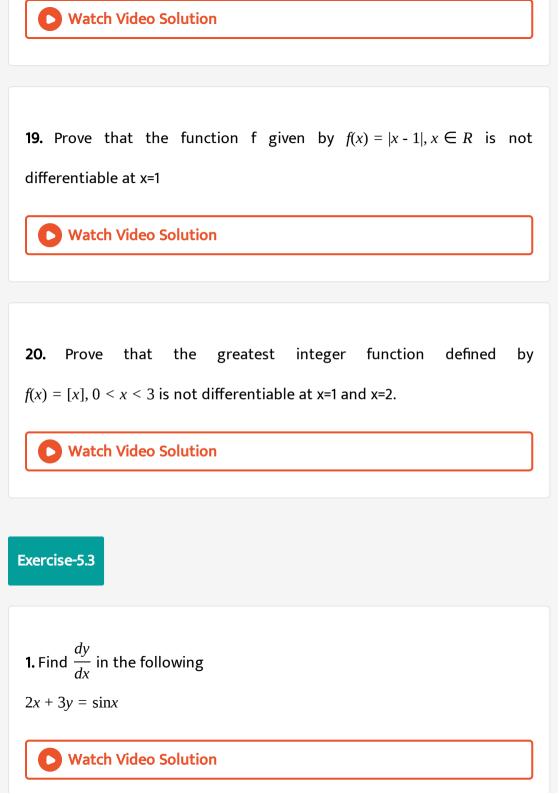
17. Differentiate the functions with respect to x in

$$2\sqrt{\cot(x^2)}$$



18. Differentiate the functions with respect to x in

$$\cos\left(\sqrt{x}\right)$$



2. Find
$$\frac{dy}{dx}$$
 in the following

 $2x + 3y = \sin y$

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3. Find
$$\frac{dy}{dx}$$
 in the following $ax + by^2 = \cos y$

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4. Find
$$\frac{dy}{dx}$$
 in the following $xy + y^2 = \tan x + y$

5. Find
$$\frac{dy}{dx}$$
 in the following $x^2 + xy + y^2 = 100$

6. Find
$$\frac{dy}{dx}$$
 in the following
 $x^3 + x^2y + xy^2 + y^3 = 81$

7. Find
$$\frac{dy}{dx}$$
 in the following $\sin^2 y + \cos xy = k$



8. Find
$$\frac{dy}{dx}$$
 in the following $\sin^2 x + \cos^2 y = 1$

9. Find
$$\frac{dy}{dx}$$
 in the following

$$y = \sin^{-1} \left(\frac{2x}{1 + x^2} \right)$$

10. Find
$$\frac{dy}{dx}$$
 in the following:
 $y = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right), |x| < \frac{1}{\sqrt{3}}$

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11. Find
$$\frac{dy}{dx}$$
 in the following

$$y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), 0 < x < 1$$

12. Find
$$\frac{dy}{dx}$$
 in the following

$$y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right), 0 < x < 1$$

13. Find
$$\frac{dy}{dx}$$
 in the following
 $y = \cos^{-1}\left(\frac{2x}{1+x^2}\right)$, $-1 < x < 1$

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14. Find
$$\frac{dy}{dx}$$
 in the following
 $y = \sin^{-1}\left(2x\sqrt{1-x^2}\right), -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$

15. Find
$$\frac{dy}{dx}$$
 in the following
 $y = \sec^{-1}\left(\frac{1}{2x^2 - 1}\right), \ 0 < x < \frac{1}{\sqrt{2}}$

16. Find
$$\frac{dy}{dx}$$
 in the following

 $2x + 3y = \sin x$

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17. Find $\frac{dy}{dx}$ in the following $2x + 3y = \sin y$

18. Find
$$\frac{dy}{dx}$$
 in the following $ax + by^2 = \cos y$

19. Find
$$\frac{dy}{dx}$$
 in the following $xy + y^2 = \tan x + y$

20. Find
$$\frac{dy}{dx}$$
 in the following $x^2 + xy + y^2 = 100$

21. Find
$$\frac{dy}{dx}$$
 in the following
 $x^3 + x^2y + xy^2 + y^3 = 81$

22. Find
$$\frac{dy}{dx}$$
 in the following

 $\sin^2 y + \cos xy = k$

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23. Find
$$\frac{dy}{dx}$$
 in the following $\sin^2 x + \cos^2 y = 1$

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24. Find
$$\frac{dy}{dx}$$
 in the following
 $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

25. Find
$$\frac{dy}{dx}$$
 in the following:
 $y = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right), |x| < \frac{1}{\sqrt{3}}$

26. Find
$$\frac{dy}{dx}$$
 in the following
 $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), 0 < x < 1$

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27. Find
$$\frac{dy}{dx}$$
 in the following
 $y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right), 0 < x < 1$

28. Find
$$\frac{dy}{dx}$$
 in the following

$$y = \cos^{-1}\left(\frac{2x}{1+x^2}\right), -1 < x < 1$$

29. Find
$$\frac{dy}{dx}$$
 in the following
 $y = \sin^{-1}\left(2x\sqrt{1-x^2}\right), -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$

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30. Find
$$\frac{dy}{dx}$$
 in the following

$$y = \sec^{-1}\left(\frac{1}{2x^2 - 1}\right), \ 0 < x < \frac{1}{\sqrt{2}}$$



 e^{x}

sinx

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2. Differentiate the following w.r.t. x:

 $e^{\sin^{-1}x}$

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3. Differentiate the following w.r.t. x :

 e^{x^3} .

$$\sin\left(\tan^{-1}e^{-x}\right)$$

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5. Differentiate the following w.r.t. x:

 $\log(\cos e^{\chi})$

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6. Differentiate the following w.r.t. x:

$$e^{x} + e^{x^{2}} + \dots + e^{x^{5}}$$



7. Differentiate the following w.r.t. x:

$$\sqrt{e^{\sqrt{x}}}, x > 0$$

log(log x), x > 1

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9. Differentiate the following w.r.t. x:

 $\frac{\cos x}{\log x}, x > 0, x \neq 1$



10. Differentiate the following w.r.t. x:

$$\cos\left(\log x + e^x\right), x > 0$$

 e^{x}

sin*x*



12. Differentiate the following w.r.t. x:

 $e^{\sin^{-1}x}$

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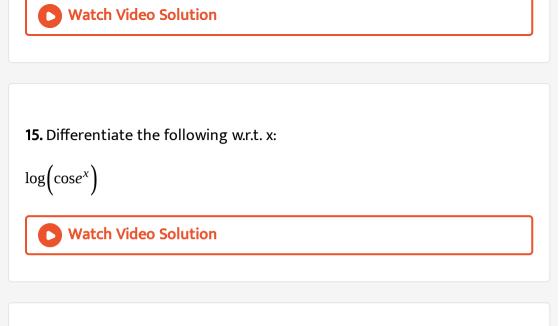
13. Differentiate the following w.r.t. x :





14. Differentiate the following w.r.t. x:

 $\sin\left(\tan^{-1}e^{-x}\right)$



$$e^{x} + e^{x^{2}} + \dots + e^{x^{5}}$$



17. Differentiate the following w.r.t. x:

$$\sqrt{e^{\sqrt{x}}}, x > 0$$

log(log x), x > 1



19. Differentiate the following w.r.t. x:

 $\frac{\cos x}{\log x}, x > 0, x \neq 1$

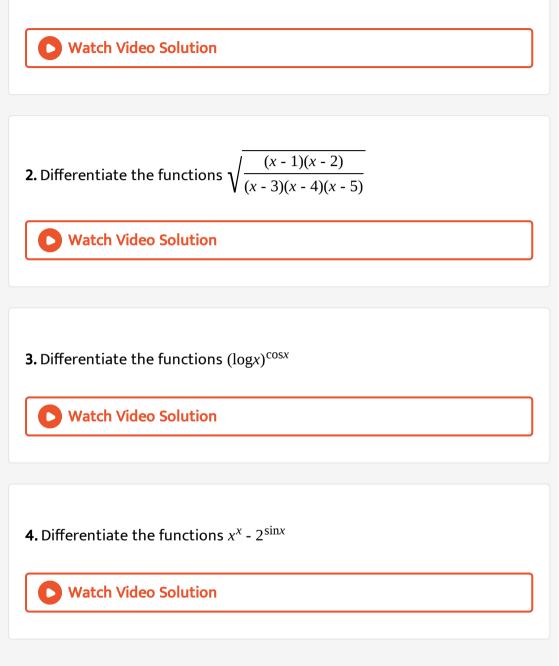
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20. Differentiate the following w.r.t. x:

$$\cos\left(\log x + e^x\right), x > 0$$



1. Differentiate the functions $\cos x$. $\cos 2x$. $\cos 3x$



5. Differentiate the functions $(x + 3)^2$. $(x + 4)^3$. $(x + 5)^4$



6. Differentiate the functions
$$\left(x + \frac{1}{x}\right)^x + x \left(1 + \frac{1}{x}\right)^x$$

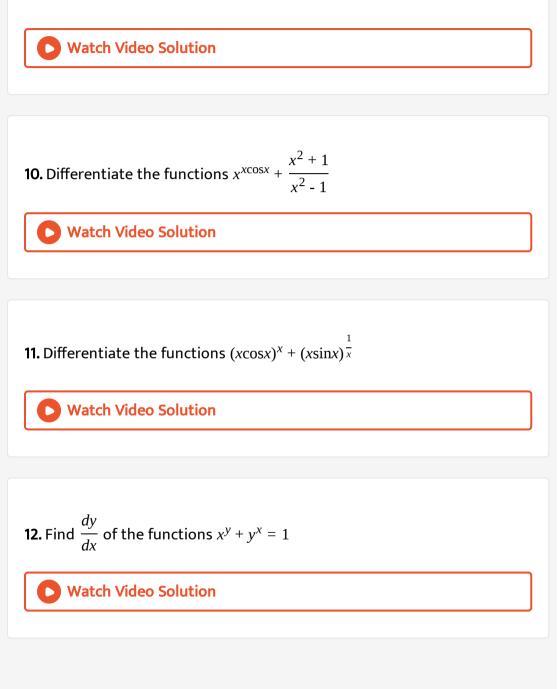
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7. Differentiate the functions $(\log x)^{x} + x^{\log x}$

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8. Differentiate the functions $(\sin x)^x + \sin^{-1}\sqrt{x}$

9. Differentiate the functions $x^{\sin x} + (\sin x)^{\cos x}$



13. Find
$$\frac{dy}{dx}$$
 of the functions $y^x = x^y$



14. Find
$$\frac{dy}{dx}$$
 of the functions $(\cos x)^y = (\cos y)^x$

15. Find
$$\frac{dy}{dx}$$
 of the functions given in Exercises 12 to 15.
 $xy = e^{(x-y)}$.

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16. Find the derivative of the function given by
$$f(x) = (1+x)(1+x^2)(1+x^4)(1+x^8) \text{ and hence find f'(1)}$$

17. Differentiate
$$(x^2 - 5x + 8)(x^3 + 7x + 9)$$

By using product rule



18. Differentiate $(x^2 - 5x + 8)(x^3 + 7x + 9)$ in three ways mentioned

below:

By expanding the product to obtain a single polynomial

Do they all give the same answer?

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19. Differentiate
$$(x^2 - 5x + 8)(x^3 + 7x + 9)$$
 in three ways mentioned

below:

By logarithmic differentiation.

Do they all give the same answer?

20. If u, v and w are functions of x, then show that $\frac{d}{dx}(u.v.w) = \frac{du}{dx}v.w+u.\frac{dv}{dx}.w+u.v.\frac{dw}{dx}$ in two ways- first by repeated application of product rule, second by logarithmic differentiation.

Using product rule

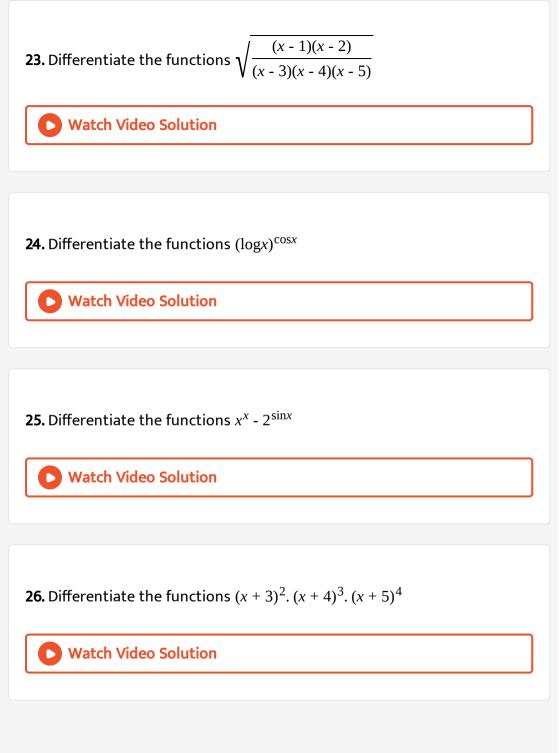
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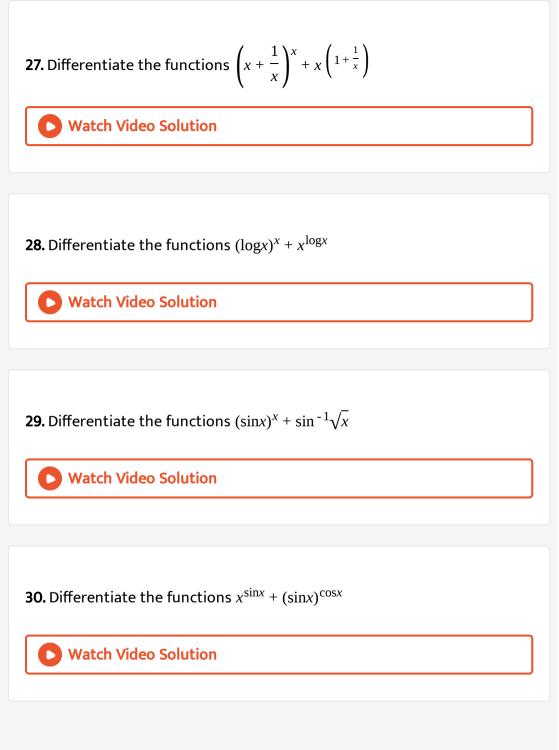
21. If u, v and w are functions of x, then show that $\frac{d}{dx}(u.v.w) = \frac{du}{dx}v.w+u.\frac{dv}{dx}.w+u.v.\frac{dw}{dx}$ in two ways- first by repeated application of product rule, second by logarithmic differentiation.

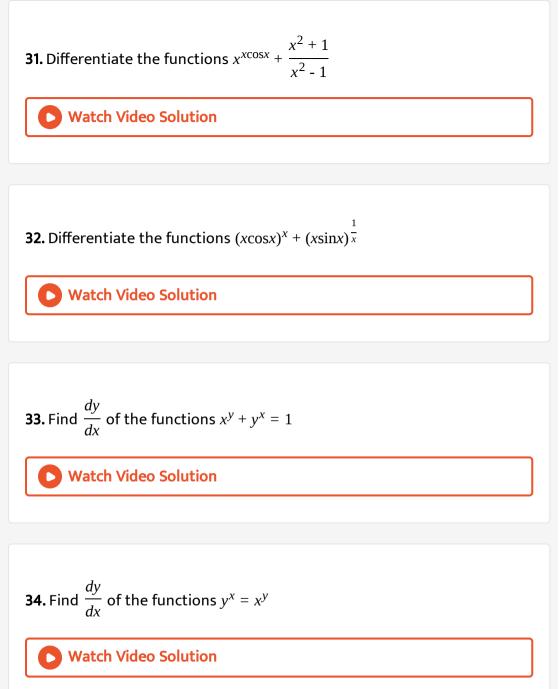
Using product rule



22. Differentiate the functions cos*x*. cos2*x*. cos3*x*







35. Find
$$\frac{dy}{dx}$$
 of the functions $(\cos x)^y = (\cos y)^x$



36. Find
$$\frac{dy}{dx}$$
 of the functions given in Exercises 12 to 15.
 $xy = e^{(x-y)}$.



37. Find the derivative of the function given by
$$f(x) = (1+x)\left(1+x^2\right)\left(1+x^4\right)\left(1+x^8\right)$$
 and hence find f'(1)

38. Differentiate
$$(x^2 - 5x + 8)(x^3 + 7x + 9)$$

By using product rule

39. Differentiate $(x^2 - 5x + 8)(x^3 + 7x + 9)$ in three ways mentioned below:

By expanding the product to obtain a single polynomial

Do they all give the same answer?

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40. Differentiate
$$(x^2 - 5x + 8)(x^3 + 7x + 9)$$
 in three ways mentioned

below:

By logarithmic differentiation.

Do they all give the same answer?

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41. If u, v and w are functions of x, then show that $\frac{d}{dx}(u.v.w) = \frac{du}{dx}v.w+u.\frac{dv}{dx}.w+u.v.\frac{dw}{dx}$ in two ways- first by repeated

application of product rule, second by logarithmic differentiation.

Using product rule



42. If u, v and w are functions of x, then show that $\frac{d}{dx}(u.v.w) = \frac{du}{dx}v.w+u.\frac{dv}{dx}.w+u.v.\frac{dw}{dx}$ in two ways- first by repeated application of product rule, second by logarithmic differentiation.

Using product rule

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1. If x and y are connected parametrically by the equations without eliminating the parameter, find $\frac{dy}{dx}$

$$x = 2at^2$$
, $y = at^4$

2. If x and y are connected parametrically by the equations without

eliminating the parameter, find $\frac{dy}{dx}$

 $x = a\cos\theta, y = b\cos\theta$

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3. If x and y are connected parametrically by the equations without eliminating the parameter, find $\frac{dy}{dx}$ $x = \sin t, y = \cos 2t$

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4. If x and y are connected parametrically by the equations without eliminating the parameter, find $\frac{dy}{dx}$

$$x = 4t, y = \frac{4}{t}$$

5. If x and y are connected parametrically by the equations without

eliminating the parameter, find $\frac{dy}{dx}$

 $x = \cos\theta - \cos 2\theta, y = \sin\theta - \sin 2\theta$

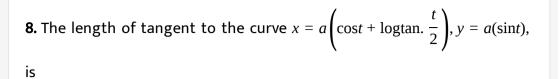
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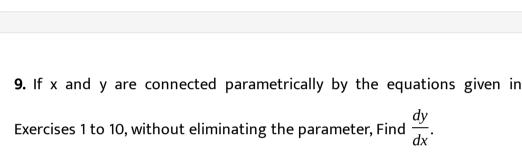
6. If x and y are connected parametrically by the equations given in Exercises 1 to 10, without eliminating the parameter, Find $\frac{dy}{dx}$. $x = a(\theta - \sin\theta), y = a(1 + \cos\theta).$

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7. If x and y are connected parametrically by the equations given in Exercises 1 to 10, without eliminating the parameter, Find $\frac{dy}{dx}$.

$$x = \frac{\sin^3 t}{\sqrt{\cos 2t}}, y = \frac{\cos^3 t}{\sqrt{\cos 2t}}.$$





 $x = a \sec \theta, y = b \tan \theta.$



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10. If x and y are connected parametrically by the equations without eliminating the parameter, find $\frac{dy}{dx}$ $x = a(\cos\theta + \theta\sin\theta), y = a(\sin\theta - \theta\cos\theta)$

11. If
$$x = \sqrt{a^{\sin^{-1}t}}, y = \sqrt{a^{\cos^{-1}t}}, a > 0$$
 and $-1 < t < 1$. show that
 $\frac{dy}{dx} = -\frac{y}{x},$
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12. If x and y are connected parametrically by the equations without eliminating the parameter, find $\frac{dy}{dx}$ $x = 2at^2$, $y = at^4$

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13. If x and y are connected parametrically by the equations without eliminating the parameter, find $\frac{dy}{dx}$ $x = a\cos\theta, y = b\cos\theta$

14. If x and y are connected parametrically by the equations without

eliminating the parameter, find $\frac{dy}{dx}$

 $x = \sin t, y = \cos 2t$



15. If x and y are connected parametrically by the equations without

eliminating the parameter, find $\frac{dy}{dx}$

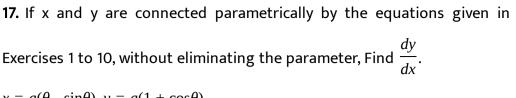
$$x = 4t, y = \frac{2}{4}$$

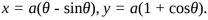
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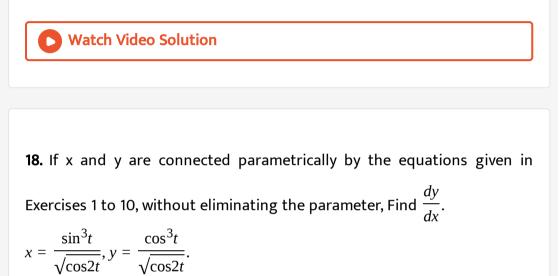
16. If x and y are connected parametrically by the equations without

eliminating the parameter, find
$$\frac{dy}{dx}$$

$$x = \cos\theta - \cos2\theta, y = \sin\theta - \sin2\theta$$





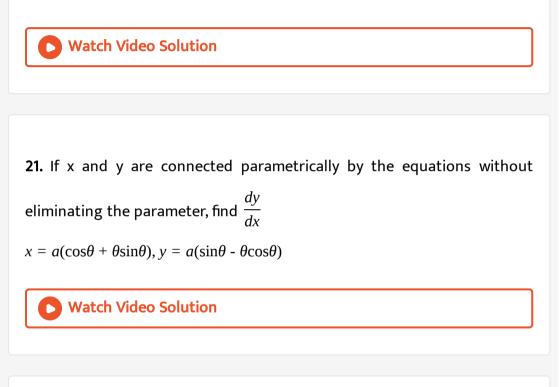


19. If x and y are connected parametrically by the equations without

eliminating the parameter, find $\frac{dy}{dx}$

$$x = a\left(\cos t + \log \tan \frac{t}{2}\right), y = a\sin t$$

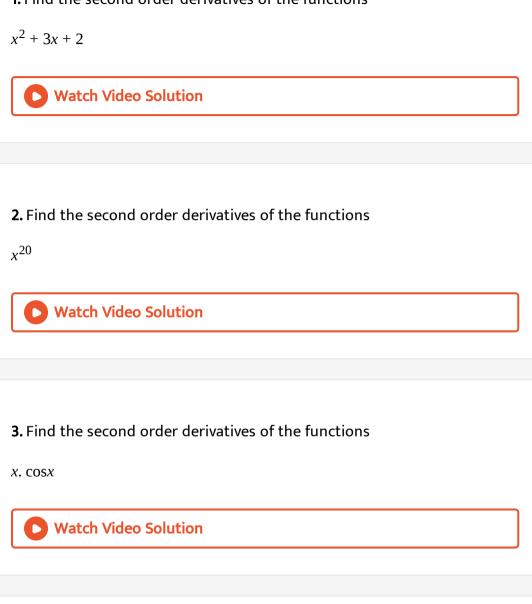
20. If x and y are connected parametrically by the equations given in Exercises 1 to 10, without eliminating the parameter, Find $\frac{dy}{dx}$. $x = a\sec\theta, y = b\tan\theta$.



22. If
$$x = \sqrt{a^{\sin^{-1}t}}$$
, $y = \sqrt{a^{\cos^{-1}t}}$, show that $\frac{dy}{dx} = -\frac{y}{x}$



1. Find the second order derivatives of the functions



4. Find the second order derivatives of the functions

log x



5. Find the second order derivatives of the functions given in Exercises 1

to 10.

 $x^3 \log x$.

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6. Find the second order derivatives of the functions

e^xsin5x

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7. Find the second order derivatives of the functions given in Exercises 1

to 10.

 $e^{6x}\cos 3x$.

8. Find the second order derivatives of the functions

 $\tan^{-1}x$

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9. Find the second order derivatives of the functions

log (log x)

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10. Find the second order derivatives of the functions

sin (log x)

11. If
$$y = 5\cos x - 3\sin x$$
, prove that $\frac{d^2y}{dx^2} + y = 0$

12. If
$$y = \cos^{-1}x$$
, Find $\frac{d^2y}{dx^2}$ in terms of y alone

13. If $y = 3\cos(\log x) + 4\sin(\log x)$, show that $x^2y_2 + xy_1 + y = 0$

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14. If
$$y = Ae^{mx} + Be^{nx}$$
, show that $\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = 0$

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15. If
$$y = 500e^{7x} + 600e^{-7x}$$
, show that $\frac{d^2y}{dx^2} = 49y$

16. If
$$e^{y}(x+1) = 1$$
, show that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$

17. If
$$y = (\tan^{-1}x)^2$$
 show that $(x^2 + 1)^2 y_2 + 2x(x^2 + 1)y_1 = 2$

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18. Find the second order derivatives of the functions

$$x^2 + 3x + 2$$

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19. Find the second order derivatives of the functions

 x^{20}

20. Find the second order derivatives of the functions

x. cos*x*

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21. Find the second order derivatives of the functions
log x
Watch Video Solution
22. Find the second order derivatives of the functions
x ³ logx
Vatch Video Solution
23. Find the second order derivatives of the functions
$e^x \sin 5x$

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24. Find the second order derivatives of the functions $e^{6x}\cos 3x$
e ^w cos3x Watch Video Solution

25. Find the second order derivatives of the functions

 $\tan^{-1}x$

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26. Find the second order derivatives of the functions

log (log x)

27. Find the second order derivatives of the functions

sin (log x)



28. If
$$y = 5\cos x - 3\sin x$$
, prove that $\frac{d^2y}{dx^2} + y = 0$

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29. If
$$y = \cos^{-1}x$$
, Find $\frac{d^2y}{dx^2}$ in terms of y alone

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30. If $y = 3\cos(\log x) + 4\sin(\log x)$, show that $x^2y_2 + xy_1 + y = 0$

31. If
$$y = Ae^{mx} + Be^{nx}$$
, show that $\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = 0$

32. If
$$y = 500e^{7x} + 600e^{-7x}$$
, show that $\frac{d^2y}{dx^2} = 49y$

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33. If
$$e^{y}(x+1) = 1$$
, show that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$

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34. If
$$y = (\tan^{-1}x)^2$$
 show that $(x^2 + 1)^2 y_2 + 2x(x^2 + 1)y_1 = 2$



1. Verify Rolle's theorem for the function $f(x) = x^2 + 2x - 8, x \in [-4, 2]$



2. Examine if Rolle's theorem is applicable to any of the following functions.

 $f(x) = [x], x \in [5, 9]$

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3. Examine if Rolle's theorem is applicable to any of the following

functions.

 $f(x) = [x], x \in [-2, 2]$

4. Examine if Rolle's theorem is applicable to any of the following functions. Can you say some thing about the converse of Rolle's theorem from these example?

 $f(x) = x^2 - 1$ for $x \in [1, 2]$.

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5. If $f: [-5, 5] \rightarrow R$ is a differentiable function and if f'(x) does not vanish

anywhere, then prove that $f(-5) \neq f(5)$

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6. Verify Mean Value Theorem, if $f(x) = x^2 - 4x - 3$ in the interval [a, b]

where a=1 and b= 4.

7. Verify Mean Value Theorem, if $f(x) = x^3 - 5x^2 - 3x$ in the interval [a, b], where a=1 and b=3. Find all $c \in (1, 3)$ for which f'(c) = 0



8. Examine the applicability of Mean Value Theorem for all three functions

 $f(x) = [x], x \in [5, 9]$

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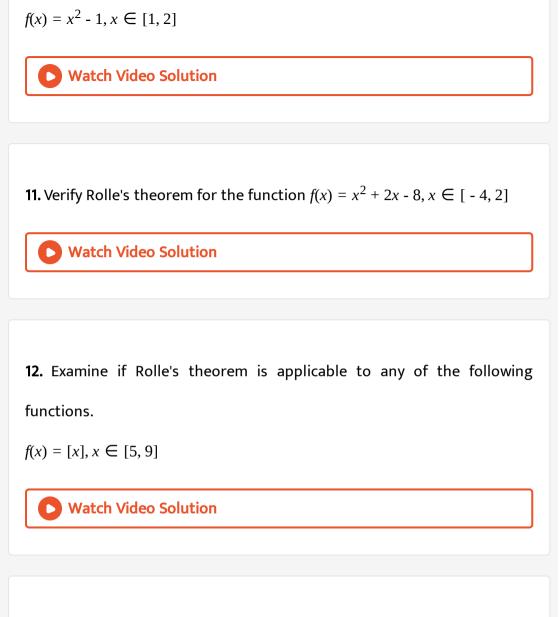
9. Examine the applicability of Mean Value Theorem for all three functions

 $f(x) = [x], x \in [-2, 2]$

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10. Examine the applicability of Mean Value Theorem for all three

functions



13. Examine if Rolle's theorem is applicable to any of the following functions.

 $f(x) = [x], x \in [-2, 2]$

14. Examine if Rolle's theorem is applicable to any of the following functions. Can you say some thing about the converse of Rolle's theorem from these example?

 $f(x) = x^2 - 1$ for $x \in [1, 2]$.

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15. If *f*: [-5, 5] → *R* is a differentiable function and if f'(x) does not vanish anywhere, then prove that $f(-5) \neq f(5)$

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16. Verify Mean Value Theorem, if $f(x) = x^2 - 4x - 3$ in the interval [a, b]

where a=1 and b= 4.

17. Verify Mean Value Theorem, if $f(x) = x^3 - 5x^2 - 3x$ in the interval [a, b],

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 $f(x) = [x], x \in [5, 9]$

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19. Examine the applicability of Mean Value Theorem for all three

functions

 $f(x) = [x], x \in [-2, 2]$

20. Examine the applicability of Mean Value Theorem for all three

functions

 $f(x) = x^2 - 1, x \in [1, 2]$

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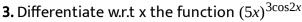
Miscellaneous Exercise - 5

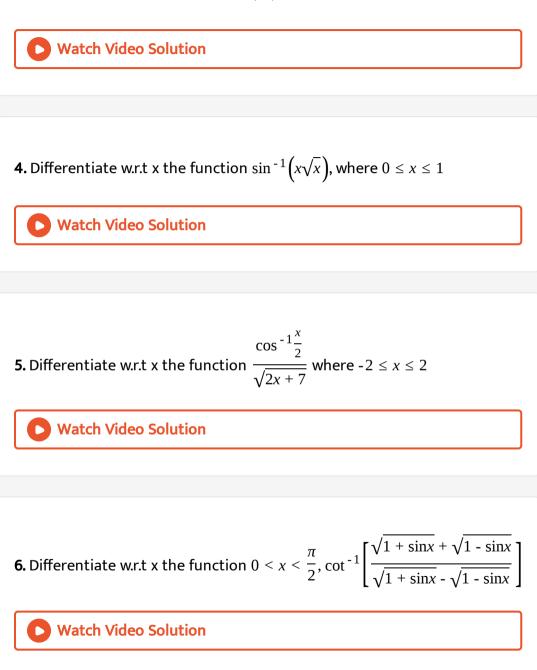
1. Differentiate w.r.t.x the function in Exercises 1 to 11.

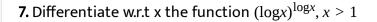
$$\left(3x^2 - 9x + 5\right)^9$$

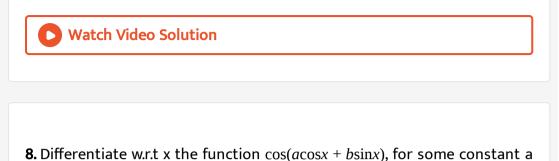
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2. Differentiate w.r.t x the function $\sin^3 x + \cos^6 x$



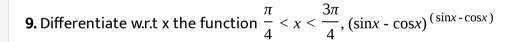






and b

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10. Differentiate w.r.t x the function $x^{x} + x^{a} + a^{x} + a^{a}$, for some fixed

a > 0 and x > 0.

11. Differentiate w.r.t x the function $x^{x^2-3} + (x-3)^{x^2}$, for x > 3



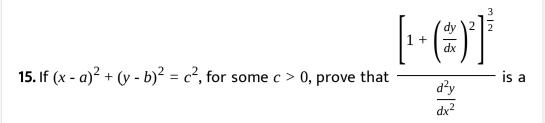
12. Find
$$\frac{dy}{dx}$$
, if $y = 12(1 - \cos t)$, $x = 10(t - \sin t)$, $-\frac{\pi}{2} < t < \frac{\pi}{2}$

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13. Find
$$\frac{dy}{dx}$$
, If $y = \sin^{-1}x + \sin^{-1}\sqrt{1 - x^2}$, $0 \le x \le 1$

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14. If
$$x\sqrt{1+y+y}\sqrt{1+x} = 0$$
 then $\frac{dy}{dx}$ equals.



constant independent of a and b.

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16. If $\cos y = x\cos(a + y)$, with $\cos a \neq \pm 1$, prove that $\frac{dy}{dx} = \frac{\cos^2(a + y)}{\sin a}$

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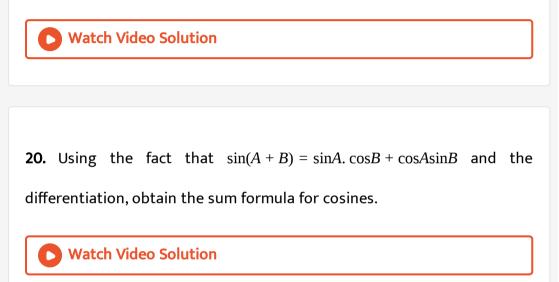
17. If
$$x = a(\cos t + t\sin t)$$
 and $y = a(\sin t - t\cos t)$, find $\frac{d^2y}{dx^2}$.



18. If $f(x) = |x|^3$, show that f''(x) exists for all real x and find it

19. Using mathematical induction prove that $\frac{d}{dx}(x^n) = nx^{n-1}$ for all

positive integers n.



21. Does there exist a function which is continuous everywhere but not

differentiable at exactly two points ? Justify your answer.



22. If
$$y = \begin{vmatrix} f(x) & g(x) & h(x) \\ l & m & n \\ a & b & c \end{vmatrix}$$
, prove that $\frac{dy}{dx} = \begin{vmatrix} f(x) & g'(x) & h'(x) \\ l & m & n \\ a & b & c \end{vmatrix}$

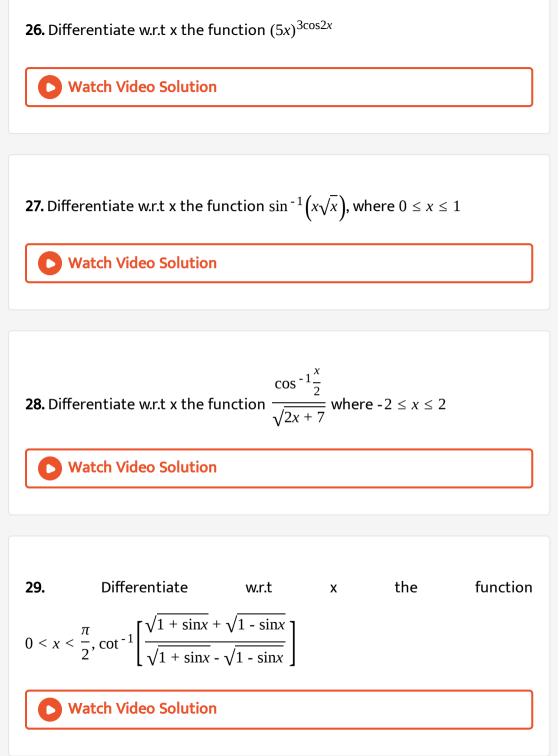
23. If
$$y = e^{a\cos^{-1}x}$$
 show that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$. Where $-1 \le x \le 1$

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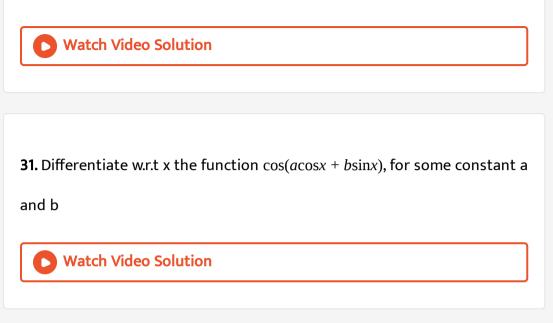
24. Differentiate w.r.t x the function $(3x^2 - 9x + 5)^9$

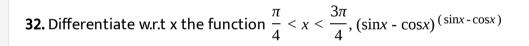
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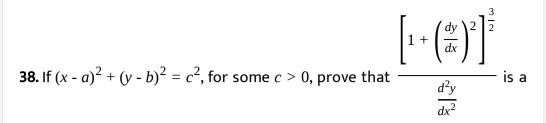


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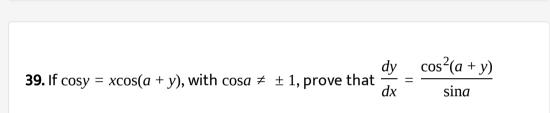
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37. If
$$x\sqrt{1+y} + y\sqrt{1+x} = 0$$
, for $-1 < x < 1$, prove that $\frac{dy}{dx} = -\frac{-1}{(1+x)^2}$



constant independent of a and b.

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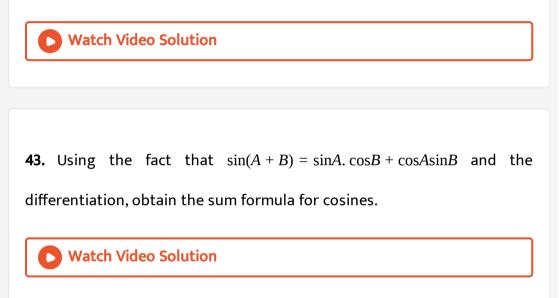
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Textbook based MCQs

1. If
$$y = \log\left(x + \sqrt{x^2 + a^2}\right)$$
 then $\frac{dy}{dx} = \dots$
A. (a) $\sqrt{x^2 + a^2}$
B. (b) $\frac{1}{\sqrt{x^2 + a^2}}$
C. (c) $x^2 + a^2$

D. (d)
$$\frac{1}{x^2 + a^2}$$

Answer: B



2. If
$$y = \log_{10} \sin x$$
 then $\frac{dy}{dx} = \dots$

A. cot x

B. cotx. $\log_e 10$

C. cot x. $\log_{10}e$

 $D. \log_{10} \cot x$

Answer: C

3.
$$y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \dots \infty}}}$$
 then $\frac{dy}{dx}$ =.....

A.
$$\frac{\cos x}{2y - 1}$$

B. cosx(2y + 1)

$$\mathsf{C}.\,\frac{\cos x}{2y+1}$$

D. give not

Answer: A

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4. $f(x) = x^2 e^{2(x-1)}, 0 < x < 1 = a \operatorname{sgn}(x+1) \cos(2x-2) + bx^2, 1 < x \le 2$. If a

function f(x) is differentiable at x=1 then.

A. a = -1, b = 2

B. a = 1, b = -2

C. a = -3, b = 4

D. a = 3, b = -4

Answer: A

5. The value of f(0), so that
$$f(x) = \frac{\sqrt{a^2 - ax + x^2} - \sqrt{a^2 + ax + x^2}}{\sqrt{a + x} - \sqrt{a - x}}$$
 becomes

continuous for all x, is given by

A.
$$a\sqrt{a}$$

B. \sqrt{a}

D. -
$$a\sqrt{a}$$

Answer: C

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6. f(x + y) = f(x)f(y), For $\forall x$ and y. If f(3)=3 and f'(0)=11 then f'(3)=...

B.44

C. 28

D. None of these

Answer: D

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7. $f(x) = [x] + \sqrt{x - [x]}$, where [.] is a greatest integer function then (a) f(x) is continuous in R+ (b) f(x) is continuous in R (C) f(x) is continuous in R - 1 (d) None of these

A. f(x) is continuous in R^+

B. f(x) is continuous in R

C. f(x) is continuous in R - 1

D. None of these

Answer: B



8. The function $f(x) = (\sin 3x)^{\tan^2 3x}$ is continuous at $x = \frac{\pi}{6}$ then $f\left(\frac{\pi}{6}\right) = \dots$ A. $e^{-\frac{1}{2}}$ B. $\frac{1}{e}$ C. $e^{(d)}$ D. $e^{\frac{1}{2}}$

Answer: A



9.
$$y^2 = ax^2 + bx + c$$
 then $y^3 \frac{d^2y}{dx^2}$ is a function

A. constant

B. only for x

C. only for y

D. for x and y

Answer: A

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10. $f(x) = x + \tan x$ and f is an inverse function of g then g'(x)=

A. (a)
$$\frac{1}{1 + (g(x) - x)^2}$$

B. (b)
$$\frac{1}{1 - (g(x) - x)^2}$$

C. (c)
$$\frac{1}{2 + (g(x) - x)^2}$$

D. (d)
$$\frac{1}{2 - (g(x) - x)^2}$$

Answer: C

11. If $y^2 = p(x)$ the is polynomial of order 3, then $2\frac{d}{dx}\left[y^3\frac{d^2y}{dx^2}\right] = \dots$

A. p'''(x) + p'(x)B. p''(x). p'''(x)C. p(x). p'''(x)

D. p(x) + p'''(x)

Answer: C

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12. If
$$x^2 + y^2 = t - \frac{1}{t}$$
 and $x^4 + y^4 = t^2 + \frac{1}{t^2}$ then $x^3 y \frac{dy}{dx} = \dots$

A. - 1

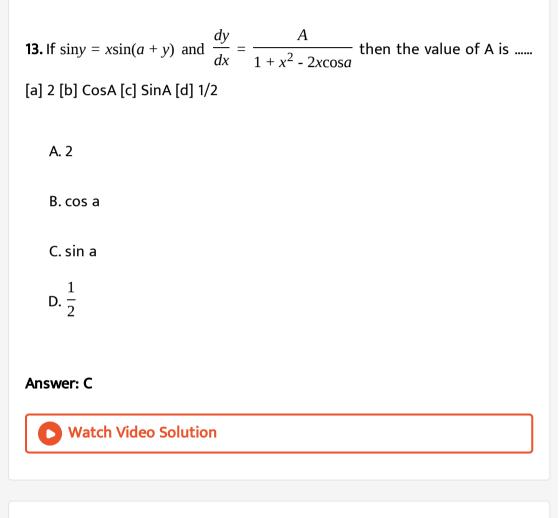
B. 0

C. 1

D. None of these

Answer: C





14.
$$f(x + y) = f(x) + f(y)$$
, for $\forall x$ and y and $f(x) = (2x^2 + 3x)g(x)$. For $\forall x$. If

g(x) is a continuous function and g(0)=3 then $f'(x) = \dots$

A. 9

B. 3

C. 6

D. None of these

Answer: A

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15.
$$y = \sin x - \cos x$$
 and $f(x) = \frac{d^{17}y}{dx^{17}}$ then $f\left(\frac{\pi}{4}\right) = \dots$ [a] $\sqrt{2}$ [b] $1/\sqrt{2}$ [c] $(\sqrt{2})^{17}$ [d] 0

A.
$$\sqrt{2}$$

B. $\frac{1}{\sqrt{2}}$
C. $(\sqrt{2})^{17}$

D. 0

Answer: A

16.
$$y = \tan^{-1}\left(\frac{ax - b}{bx + a}\right)$$
 then $\frac{dy}{dx}|_{x=-1}$ =
A. $\frac{1}{2}$
B. a
C. ab

Answer: A

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17.
$$u = f(\tan x), v = g(\sec x), f'(1) = 2$$
 and $g'(\sqrt{2}) = 4$ then $\frac{du}{dv}\Big|_{x=\frac{\pi}{4}} = \dots$

A. $\sqrt{2}$

B. 2

C.
$$\frac{1}{\sqrt{2}}$$

D. $\frac{1}{2}$

Answer: C



18.
$$f(x) = \sin^2 x + \sin^2 \left(x + \frac{\pi}{3} \right) + \cos x \cos \left(x + \frac{\pi}{3} \right)$$
 and g(5 / 4)' =1 then (gof)

(x) =

A. 1

B. $\cos^2 x$

C. 0

D. sin 2x

Answer: C

19.
$$y = e^{3x+7}$$
 then $y_n(0) = \dots$

A. 1 B. 3^n C. $3^n e^7$ D. 3^n . $(e^7.7)$

Answer: C

20.
$$x = f(t), y = \phi(t)$$
 then $\frac{d^2y}{dx^2} = \dots$
A. $\frac{f_1(t)\phi_2(t) - \phi_1(t)f_2(t)}{(f_1(t))^2}$
B. $\frac{f_1(t)\phi_2(t) - \phi_1(t)f_2(t)}{(f_1(t))^3}$
C. $\frac{\phi_1(t)f_2(t) - f_1(t)\phi_2(t)}{(f_1(t))^3}$

D. None of these

Answer: B

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21. In the function $f(x) = 2x^3 + bx^2 + qx$ satisfies conditions of Rolle's theorem in [-1, 1] and $c = \frac{1}{2}$ then the value of 2b + q is.....

A. 0

B. 1

 $C. -\frac{1}{2}$

D. - 1

Answer: D

22. If $y = \frac{a + bx}{c + dx}$, where a, b, c, d are constants and $\lambda y_1 y_3 = \mu y_2^2$ then the value of μ^{λ^2} is where y_1, y_2, y_3 are respectively. First, second and third derivatives of y.

A. 42 B. 81 C. 64 D. 27

Answer: B



23. The function
$$f(x) = \frac{2 - \sqrt[4]{x^2 + 16}}{\cos 2x - 1}$$
 is continuous at x=0 then f(0)=
A. $\frac{1}{8}$
B. $\frac{1}{64}$

C.
$$\frac{1}{32}$$

D. $\frac{1}{2}$

Answer: B



24. Let
$$f(x) = x^3 - x^2 + x + 1$$
 $g(x) = max$

L

{ $f(t), 0 \le t \le x$ }, $0 \le x \le 1 = 3 - x, 1 < x \le 2$

Then in [0, 2] the points where g(x) is not differentiable is......

A. 0

B. 1

C. 2

D. None of these

Answer: B

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25.
$$f(x) = \begin{vmatrix} \sin x & \cos x \\ \tan x & \cot x \end{vmatrix}$$
 then $f'\left(\frac{\pi}{4}\right) = \dots$
A. 0
B. $-\sqrt{2}$
C. $-2\sqrt{2}$
D. $\sqrt{2}$

Answer: D

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26.
$$x = t^2 + 3t - 8$$
, $y = 2t^2 - 2t - 4$. If at point (2, -1), $\lambda = \frac{dy}{dx}$ then the value of λ =

B.
$$\frac{6}{7}$$

C. - 6

Answer: B

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27.
$$x = 2 + t^3$$
, $y = 2t^2$. If $\frac{\frac{d^2y}{dx^2}}{\left(\frac{dy}{dx}\right)^n}$ is constant then n=

A. 4

B. 1

C. 0

D. 3

Answer: A

28. If
$$F(x) = \frac{1}{x^2} \int_{-\infty}^{x} 4(4t^2 - 2F'(t)) dt$$
 then F'(4) equals to
A. $\frac{32}{9}$
B. $\frac{64}{3}$
C. $\frac{64}{9}$
D. $\frac{32}{3}$

Answer: A

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29. If
$$\sqrt{x^2 + y^2} = a \cdot e^{\tan^{-1}\left(\frac{y}{x}\right)}$$
, $a > 0$ then the value of y''(0) is.....

A.
$$\frac{a}{2}e^{-\frac{\pi}{2}}$$

B. $ae^{\frac{\pi}{2}}$
C. $\frac{-2}{a}e^{-\frac{\pi}{2}}$

D. Does not exist



30. If f(x - y), f(x). f(y) and f(x + y) are in arithmatic progression and $f(0) \neq 0$ then (for $\forall x$ and y......

A. f(2) + f'(2) = 0

B.f(2) + f(-2) = 0

C. f(2) - f'(-2) = 0

$$\mathsf{D}.f(2) - f(-2) = 0$$

Answer: B

31.
$$\left[\frac{d}{dx}\sec^{-1}x\right]_{x=-3}$$
=.....

A.
$$\frac{1}{\sqrt{x^2 - 1}}$$

B.
$$-\frac{1}{\sqrt{x^2 - 1}}$$

C.
$$\frac{1}{6\sqrt{2}}$$

D.
$$-\frac{1}{6\sqrt{2}}$$

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32.
$$\frac{d}{dx}(x^{x}) = \dots (x > 0)$$

A. x^{x-1}
B. x^{x}
C. 0

 $\mathsf{D.}\,x^{x}(1+\log x)$

Answer: D



33.
$$\frac{d}{dx} \left(\sin^{-1}x + \cos^{-1}x \right) = \dots (|x| < 1)$$

A. 0

B.
$$\frac{2}{\sqrt{1-x^2}}$$
C.
$$\frac{1}{\sqrt{1-x^2}}$$

D. Does not exist

Answer: A

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$$\mathbf{34.} \ \frac{d}{dx} \left(a^a \right) = \dots (a > 0)$$

A. $a^{a}(1 + \log a)$

B. 0

C. *a*^{*a*}

D. Does not exist

Answer: B



35.
$$\frac{d}{dx} (e^{5x}) = \dots$$

A. e^{5x}
B. $5e^{5x}$
C. $5x. e^{5x-1}$

D. 0

Answer: B

36.
$$\frac{d}{dx}(\log|x|) = \dots (x \neq 0)$$

A. $\frac{1}{|x|}$ B. $\frac{1}{x}$

C. does not exist

D. e^{x}

Answer: B

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37. $\frac{d}{dx}(\sin^3 x)$ =

A. $3\sin^2 x$

B. $3\cos^2 x$

C. $3\sin^2 x. \cos x$

D. $-3\cos^2 x \sin x$



38.
$$\frac{d}{dx}(\tan^n x)$$
=

A. $n \tan^{n-1} x$

B. $n \tan^{n-1} x$. $\sec^2 x$

C. $n \sec^{2n} x$

D. $n \tan^{n-1} x$. $\sec^{n-1} x$

Answer: B

39. If
$$f(x) = \begin{cases} ax + b & 1 \le x < 5 \\ 7x - 5 & 5 \le x < 10 \text{ is continuous, (a,b)} = \dots \\ bx + 3a & x \ge 10 \end{cases}$$

A. (5, 10)

B. (5,5)

C. (10, 5)

D. (0, 0)

Answer: B

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40. If
$$f(x) = \begin{cases} \frac{x^2}{a} - a, & x < a \\ 0, & x = a \text{ then,} \\ a - \frac{x^2}{a}, & x > a \end{cases}$$

A. $\lim x \to a^+ f(x) = a$

.

B. $\lim x \to af(x) = -a$

C. f is continuous at x=a

D. f is differentiable at x=a

Answer: C



41. If
$$f(x) = \begin{cases} x, & x \in \{0, 1\} \\ 1, & x \ge 1 \end{cases}$$
 then,

A. f is continuous at x=1 only

B. f is discontinuous at x=1 only

C. f is continuous on R^+

D. f is not defined for x=1

Answer: C

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42.
$$\frac{d}{dx} \cdot \left(\frac{1}{\log|x|}\right) = \dots$$
A.
$$\frac{1}{|x|}$$
B.
$$\frac{1}{(\log x)^2}$$
C.
$$\frac{-1}{x(\log|x|)^2}$$
D.
$$e^x$$

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43. If
$$y = a \sin x + b \cos x$$
 then, $y^2 + (y_1)^2 = \dots (a^2 + b^2 \neq 0)$

A. *a*cos*x* - *b*sin*x*

 $\mathsf{B.} (a \sin x - b \cos x)^2$

C. $a^2 + b^2$

D. 0



44.
$$\frac{d}{dx}(x^2 + \sin^2 x)^3 = \dots$$

A.
$$3(x^2 + \sin^2 x)$$

B. $3(x^2 + \sin^2 x)^2(2x + \sin^2 x)$

C. $2x + 2\sin x \cos x$

Answer: B



45.
$$\frac{d}{dx}\left(\sqrt{x \sin x}\right) = \dots (0 < x < \pi)$$

A.
$$\frac{x \sin x + \cos x}{\sqrt{x \sin x}}$$

B.
$$\frac{x\cos x}{2\sqrt{x\sin x}}$$

C.
$$\frac{x\cos x + \sin x}{2\sqrt{x\sin x}}$$

D.
$$\frac{1}{2\sqrt{x\sin x}}$$

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46.
$$\frac{d}{dx}\left(e^{\sin^{-1}x + \cos^{-1}x}\right) = \dots (|x| < 1)$$

A.
$$\frac{2}{\sqrt{1-x^2}}$$

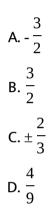
B. 0

$$\mathsf{C}.\,\frac{1}{\sqrt{1-x^2}}$$

D. $e^{\sin^{-1}x + \cos^{-1}x}$

Answer: B

47. If function f(x) is continuous at x= 0, $f(x) = \begin{cases} \frac{\sin(4x)}{9x}, & x \neq 0\\ k^2, & x = 0 \end{cases}$ then k=



.

Answer: C



48. If
$$x = at^2$$
, $y = 2at$ then $\frac{dy}{dx} = \dots$, where $t \neq 0$
A. $\frac{1}{t}$

B.t

C. - *t*

D. a

Answer: A

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49.
$$\frac{d}{dx} \left(\log_5 x^2 \right) = \dots$$
A.
$$\frac{1}{(\log 5)x}$$
B.
$$\frac{1}{x^2}$$
C.
$$\frac{2}{(\log 5)x}$$
D.
$$\frac{1}{(\log 5)x^2}$$

Answer: C

50. Derivative of $\tan^{-1}x$ w.r. to $\cot^{-1}x$ is Where $x \in R$

A. - 1

B. 1

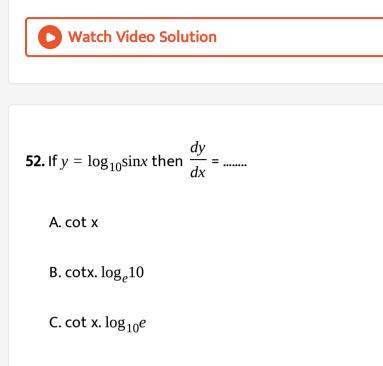
C.
$$\frac{1}{1 + x^2}$$

D. $-\frac{1}{1 + x^2}$

Answer: A

51. If
$$y = \log\left(x + \sqrt{x^2 + a^2}\right)$$
 then $\frac{dy}{dx} = \dots$
A. $\sqrt{x^2 + a^2}$
B. $\frac{1}{\sqrt{x^2 + a^2}}$
C. $x^2 + a^2$
D. $\frac{1}{x^2 + a^2}$

Answer: B



 $D. \log_{10} \cot x$

Answer: C

53.
$$y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \dots \infty}}}$$
 then $\frac{dy}{dx}$ =.....

A.
$$\frac{\cos x}{2y - 1}$$

B. cosx(2y + 1)

C.
$$\frac{\cos x}{2y+1}$$

D. give not

Answer: A



54.

 $f(x) = x^2 e^{2(x-1)}, 0 < x < 1$ and $f(x) = a \operatorname{sgn}(x + 1) \cos(2x - 2) + bx^2, 1 < x \le 2$

. If a function f(x) is differentiable at x=1 then.

B. a = 1, b = -2

$$C. a = -3, b = 4$$

D. a = 3, b = -4

Answer: A



55. The value of f(0), so that
$$f(x) = \frac{\sqrt{a^2 - ax + x^2} - \sqrt{a^2 + ax + x^2}}{\sqrt{a + x} - \sqrt{a - x}}$$
 becomes

continuous for all x, is given by

A. $a\sqrt{a}$ B. \sqrt{a} C. $-\sqrt{a}$

D. -*a√a*

Answer: C

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56. f(x + y) = f(x)f(y), For $\forall x$ and y. If f(3)=3 and f'(0)=11 then f'(3)=...

A. 22

B.44

C. 28

D. None of these

Answer: D

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57. $f(x) = [x] + \sqrt{x - [x]}$, where [.] is a greatest integer function then (a)

f(x) is continuous in R+ (b) f(x) is continuous in R (C) f(x) is continuous in

R - 1 (d) None of these

A. f(x) is continuous in R^+

B. f(x) is continuous in R

C. f(x) is continuous in R - 1

D. None of these

Answer: B

58. The function $f(x) = (\sin 3x)^{\tan^2 3x}$ is continuous at $x = \frac{\pi}{6}$ then $f\left(\frac{\pi}{6}\right) = \dots$

A.
$$e^{-\frac{1}{2}}$$

B. $\frac{1}{e}$
C. $e^{(d)}$
D. $e^{\frac{1}{2}}$

Answer: A

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59.
$$y^2 = ax^2 + bx + c$$
 then $y^3 \frac{d^2y}{dx^2}$ is a function

A. constant

B. only for x

C. only for y

D. for x and y

Answer: A



60. $f(x) = x + \tan x$ and f is an inverse function of g then g'(x)=

A.
$$\frac{1}{1 + (g(x) - x)^2}$$

B.
$$\frac{1}{1 - (g(x) - x)^2}$$

C.
$$\frac{1}{2 + (g(x) - x)^2}$$

D.
$$\frac{1}{2 - (g(x) - x)^2}$$

Answer: C

61. If
$$y^2 = p(x)$$
 the is polynomial of order 3, then $2\frac{d}{dx}\left[y^3\frac{d^2y}{dx^2}\right] = \dots$

A.
$$p'''(x) + p'(x)$$

B. $p''(x)$. $p'''(x)$
C. $p(x)$. $p'''(x)$
D. $p(x) + p'''(x)$

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62. If
$$x^2 + y^2 = t - \frac{1}{t}$$
 and $x^4 + y^4 = t^2 + \frac{1}{t^2}$ then $x^3 y \frac{dy}{dx} = \dots$

A. - 1

Β.Ο

C. 1

D. None of these

Answer: C

63. If siny =
$$xsin(a + y)$$
 and $\frac{dy}{dx} = \frac{A}{1 + x^2 - 2xcosa}$ then the value of A is
[a] 2 [b] CosA [c] SinA [d] 1/2

A. 2

B. cos a

C. sin a

D. $\frac{1}{2}$

Answer: C

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64. f(x + y) = f(x) + f(y), for $\forall x$ and y and $f(x) = (2x^2 + 3x)g(x)$. For $\forall x$. If g(x) is a continuous function and g(0)=3 then $f'(x) = \dots$

A. 9

B. 3

C. 6

D. None of these

Answer: A

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65.
$$y = \sin x - \cos x$$
 and $f(x) = \frac{d^{17}y}{dx^{17}}$ then $f\left(\frac{\pi}{4}\right) = \dots$ [a] $\sqrt{2}$ [b] $1/\sqrt{2}$ [c]

A.
$$\sqrt{2}$$

B. $\frac{1}{\sqrt{2}}$
C. $(\sqrt{2})^{17}$

D. 0

Answer: A

66.
$$y = \tan^{-1}\left(\frac{ax - b}{bx + a}\right)$$
 then $\frac{dy}{dx}|_{x = -1}$ =
A. $\frac{1}{2}$
B. a
C. ab
D. $\frac{b}{a}$

Answer: A

D Watch Video Solution

67.
$$u = f(\tan x), v = g(\sec x), f'(1) = 2$$
 and $g'(\sqrt{2}) = 4$ then $\frac{du}{dv}|_{x=\frac{\pi}{4}} = \dots$

A.
$$\sqrt{2}$$

B. 2

C.
$$\frac{1}{\sqrt{2}}$$

D. $\frac{1}{2}$



68.
$$f(x) = \sin^2 x + \sin^2 \left(x + \frac{\pi}{3} \right) + \cos x \cos \left(x + \frac{\pi}{3} \right)$$
 then f'(x) =

A. 1

B. $\cos^2 x$

C. 0

D. sin 2x

Answer: C

69.
$$y = e^{3x+7}$$
 then $y_n(0) = \dots$

B. 3^{*n*}

C.
$$3^{n}e^{7}$$

D. 3^{n} . $(e^{7}.7)$

Answer: C



70.
$$x = f(t), y = \phi(t)$$
 then $\frac{d^2y}{dx^2} = \dots$
A. $\frac{f_1(t)\phi_2(t) - \phi_1(t)f_2(t)}{(f_1(t))^2}$
B. $\frac{f_1(t)\phi_2(t) - \phi_1(t)f_2(t)}{(f_1(t))^3}$
C. $\frac{\phi_1(t)f_2(t) - f_1(t)\phi_2(t)}{(f_1(t))^3}$

D. None of these

Answer: B

71. In the function $f(x) = 2x^3 + bx^2 + qx$ satisfies conditions of Rolle's theorem in [-1, 1] and $c = \frac{1}{2}$ then the value of 2b + q is.....

A. 0

B. 1

C. $-\frac{1}{2}$

D. - 1

Answer: D

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72. If $y = \frac{a+bx}{c+dx}$, where a, b, c, d are constants and $\lambda y_1 y_3 = \mu y_2^2$ then the value of μ^{λ^2} is where y_1, y_2, y_3 are respectively. First, second and third derivatives of y.

B. 81

C. 64

D. 27

Answer: B

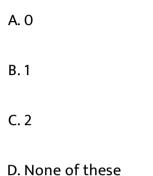
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73. The function
$$f(x) = \frac{2 - \sqrt[4]{x^2 + 16}}{\cos 2x - 1}$$
 is continuous at x=0 then f(0)=
A. $\frac{1}{8}$
B. $\frac{1}{64}$
C. $\frac{1}{32}$
D. $\frac{1}{2}$

Answer: B

 $g(x) = \{ \max(f(t)), 0 \le t \le x \}, 0 \le x \le 1 \} \{3 - x, 1 < x \le 2 \}$

Then in [0, 2] the points where g(x) is not differentiable is.....



Answer: B

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75.
$$f(x) = \begin{vmatrix} \sin x & \cos x \\ \tan x & \cot x \end{vmatrix}$$
 then $f'\left(\frac{\pi}{4}\right) = \dots$

A. 0

B. $-\sqrt{2}$

C. $-2\sqrt{2}$

D. $\sqrt{2}$

Answer: D



76.
$$x = t^2 + 3t - 8$$
, $y = 2t^2 - 2t - 4$. If at point (2, -1), $\lambda = \frac{dy}{dx}$ then the value of λ =
A. 2
B. $\frac{6}{7}$
C. -6
D. 7

Answer: B

77.
$$x = 2 + t^3$$
, $y = 2t^2$. If $\frac{\frac{d^2y}{dx^2}}{\left(\frac{dy}{dx}\right)^n}$ is constant then n=
A. 4
B. 1
C. 0
D. 3

Answer: A

78. If
$$F(x) = \frac{1}{x^2} \int_{-\infty}^{x} 4(4t^2 - 2F'(t)) dt$$
 then F'(4) equals to
A. $\frac{32}{9}$
B. $\frac{64}{3}$
C. $\frac{64}{9}$

D. $\frac{32}{3}$

Answer: A

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79. If
$$\sqrt{x^2 + y^2} = a \cdot e^{\tan^{-1}\left(\frac{y}{x}\right)}$$
, $a > 0$ then the value of y''(0) is.....

B.
$$ae^{\frac{\pi}{2}}$$

C.
$$\frac{-2}{a}e^{-\frac{\pi}{2}}$$

D. Does not exist

Answer: C

80. If f(x - y), f(x). f(y) and f(x + y) are in arithmatic progression and $f(0) \neq 0$ then (for $\forall x$ and y......

A. f(2) + f'(2) = 0

 $\mathsf{B}.f(2) + f(-2) = 0$

C.f(2) - f'(-2) = 0

D.f(2) - f(-2) = 0

Answer: B

81.
$$\left[\frac{d}{dx}\sec^{-1}x\right]_{x=-3}$$
=.....
A. $\frac{1}{\sqrt{x^2 - 1}}$
B. $-\frac{1}{\sqrt{x^2 - 1}}$
C. $\frac{1}{6\sqrt{2}}$

$$\mathsf{D.} - \frac{1}{6\sqrt{2}}$$

Answer: C



82.
$$\frac{d}{dx}(x^{x}) = \dots (x > 0)$$

A. x^{x-1}
B. x^{x}
C. 0

D. $x^{x}(1 + \log x)$

Answer: D

83.
$$\frac{d}{dx}\left(\sin^{-1}x + \cos^{-1}x\right) = \dots (|x| < 1)$$

A. 0

B.
$$\frac{2}{\sqrt{1-x^2}}$$
C.
$$\frac{1}{\sqrt{1-x^2}}$$

D. Does not exist

Answer: A

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84.
$$\frac{d}{dx}(a^a) = \dots (a > 0)$$

A. $a^{a}(1 + \log a)$

B. 0

C. *a*^{*a*}

D. Does not exist

Answer: B





85. $\frac{d}{dx} (e^{5x}) = \dots$ A. e^{5x} B. $5e^{5x}$ C. 5x. e^{5x-1}

D. 0

Answer: B

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86.
$$\frac{d}{dx}(\log|x|) = \dots (x \neq 0)$$

A. $\frac{1}{|x|}$
B. $\frac{1}{x}$

C. does not exist

Answer: B



87.
$$\frac{d}{dx}\left(\sin^3 x\right) = \dots$$

A. $3\sin^2 x$

B. $3\cos^2 x$

C. $3\sin^2 x. \cos x$

D. $-3\cos^2 x \sin x$

Answer: C



88.
$$\frac{d}{dx}(\tan^n x)$$
=

A. $n \tan^{n-1} x$

B. $n \tan^{n-1} x$. $\sec^2 x$

C. $n \sec^{2n} x$

D. $n \tan^{n-1} x$. $\sec^{n-1} x$

Answer: B

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89. If
$$f(x) = \begin{cases} ax + b & 1 \le x \le 5 \\ 7x - 5 & 5 \le x \le 10 \text{ is continuous, (a,b)} = \dots \\ bx + 3a & x \ge 10 \end{cases}$$

A. (5, 10)
B. (5,5)
C. (10, 5)
D. (0, 0)

Answer: B



90. If
$$f(x) = \begin{cases} \frac{x^2}{a} - a, & x < a \\ 0, & x = a \text{ then,} \\ a - \frac{x^2}{a}, & x > 0 \end{cases}$$

A.
$$\lim x \to a^+ f(x) = a$$

-

- $\mathsf{B.} \lim x \to af(x) = -a$
- C. f is continuous at x=a
- D. f is differentiable at x=a

Answer: C



91. If
$$f(x) = \begin{cases} x, & x \in \{0, 1\} \\ 1, & x \ge 1 \end{cases}$$
 then,

A. f is continuous at x=1 only

B. f is discontinuous at x=1 only

C. f is continuous on R^+

D. f is not defined for x=1

Answer: C

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92.
$$\frac{d}{dx} \cdot \left(\frac{1}{\log|x|}\right) = \dots$$
A.
$$\frac{1}{|x|}$$
B.
$$\frac{1}{(\log x)^2}$$
C.
$$\frac{-1}{x(\log|x|)^2}$$
D.
$$e^x$$

Answer: C

93. If
$$y = a \sin x + b \cos x$$
 then, $y^2 + (y_1)^2 = \dots (a^2 + b^2 \neq 0)$

A. *a*cos*x* - *b*sin*x*

B. $(a \sin x - b \cos x)^2$

C. $a^2 + b^2$

D. 0

Answer: C

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94.
$$\frac{d}{dx} \left(x^2 + \sin^2 x \right)^3 = \dots$$

A. $3 \left(x^2 + \sin^2 x \right)$
B. $3 \left(x^2 + \sin^2 x \right)^2 (2x + \sin^2 x)$

C. $2x + 2\sin x \cos x$

Answer: B



95.
$$\frac{d}{dx} \left(\sqrt{x \sin x} \right) = \dots (0 < x < \pi)$$

A.
$$\frac{x \sin x + \cos x}{\sqrt{x \sin x}}$$

B.
$$\frac{x \cos x}{2\sqrt{x \sin x}}$$

C.
$$\frac{x \cos x + \sin x}{2\sqrt{x \sin x}}$$

D.
$$\frac{1}{2\sqrt{x \sin x}}$$

Answer: C

96.
$$\frac{d}{dx}\left(e^{\sin^{-1}x + \cos^{-1}x}\right) = \dots (|x| < 1)$$

A.
$$\frac{2}{\sqrt{1-x^2}}$$

B. 0

$$\mathsf{C}.\,\frac{1}{\sqrt{1-x^2}}$$

D.
$$e^{\sin^{-1}x + \cos^{-1}x}$$

Answer: B



97. If function f(x) is continuous at x= 0,
$$f(x) = \begin{cases} \frac{\sin(4x)}{9x}, & x \neq 0\\ k^2, & x = 0 \end{cases}$$
 then k=

A.
$$-\frac{3}{2}$$

B. $\frac{3}{2}$
C. $\pm \frac{2}{3}$
D. $\frac{4}{9}$

•••••

Answer: C



98. If
$$x = at^2$$
, $y = 2at$ then $\frac{dy}{dx} =$, where $t \neq 0$
A. $\frac{1}{t}$
B. t
C. $-t$
D. a

Answer: A



$$99. \frac{d}{dx} \left(\log_5 x^2 \right) = \dots$$

A. $\frac{1}{(\log 5)x}$

B.
$$\frac{1}{x^2}$$

C. $\frac{2}{(\log 5)x}$
D. $\frac{1}{(\log 5)x^2}$

Answer: C



100. Derivative of $\tan^{-1}x$ w.r. to $\cot^{-1}x$ is Where $x \in R$

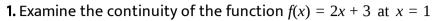
A. - 1

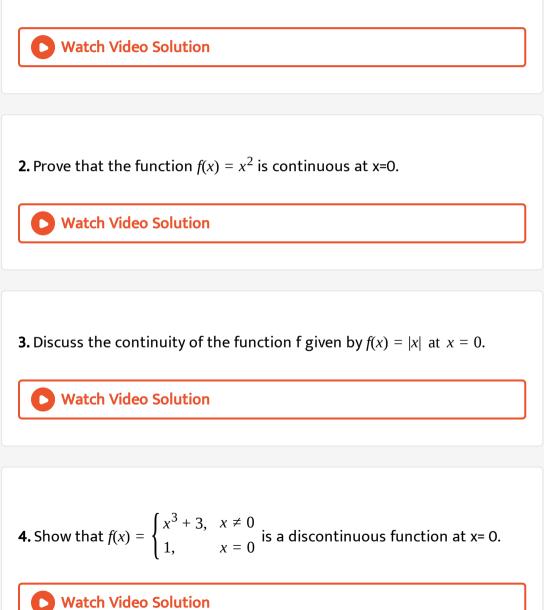
B. 1

C.
$$\frac{1}{1 + x^2}$$

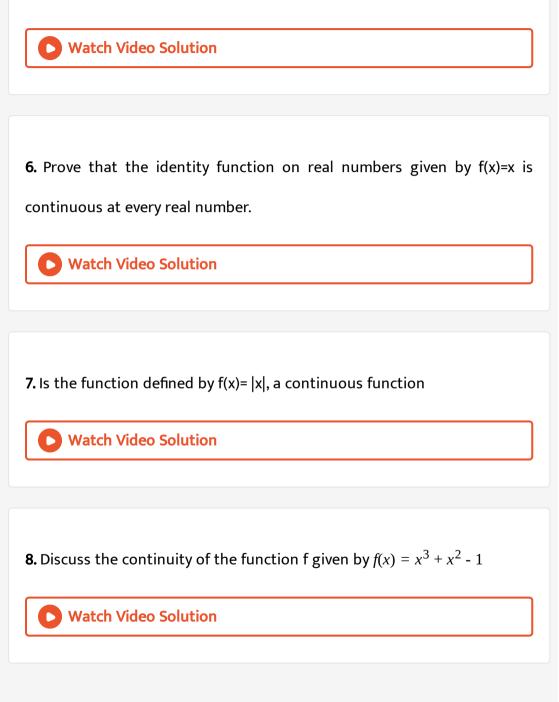
D. $-\frac{1}{1 + x^2}$

Answer: A

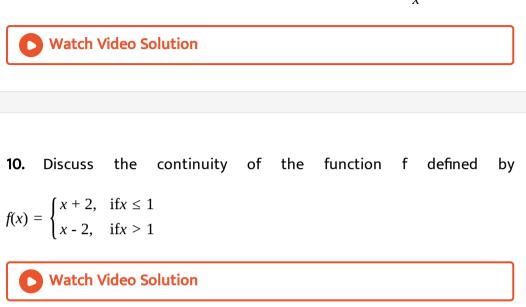




5. Check the points where the constant function f(x)= k is continuous

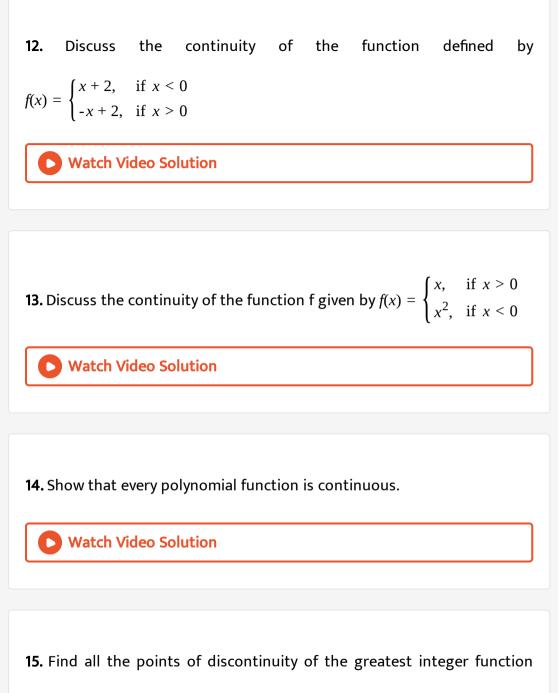


9. Discuss the continuity of the function f defined by $f(x) = \frac{1}{x}, x \neq 0$



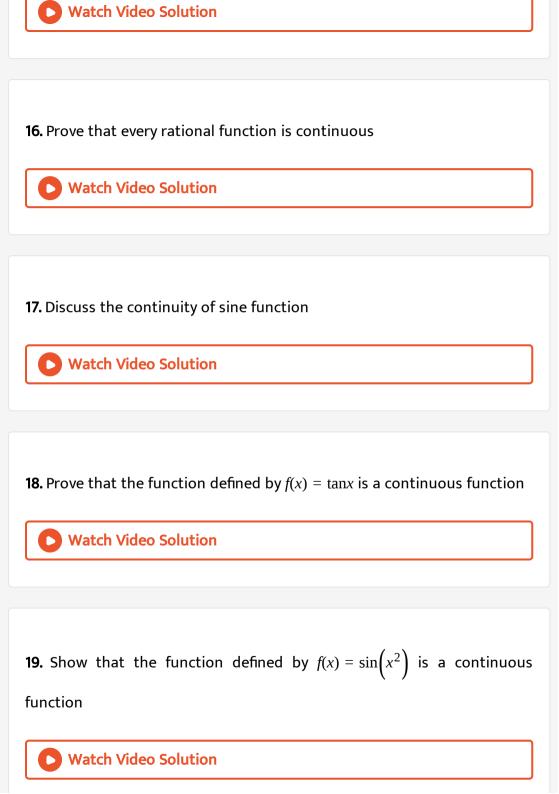
11. Find all the points of discontinuity of the function f defined by

$$f(x) = \begin{cases} x+2, & \text{if } x < 1 \\ 0 & \text{if } x = 1 \\ x-2, & \text{if } x > 1 \end{cases}$$



defined by f(x)=[x], where [x] denotes the greatest integer less than or

equal to x.



20. Show that the function f defined by f(x) = |1 - x + |x|| where x is any real number is continous. Watch Video Solution **21.** Find the derivative of the function given by $f(x) = \sin x^2$ Watch Video Solution **22.** Find the derivative of tan(2x + 3)Watch Video Solution **23.** Differentiate $sin(cosx^2)$ with respect to x Watch Video Solution

24. Find
$$\frac{dy}{dx}$$
 if $x - y = \pi$



25. Find
$$\frac{dy}{dx}$$
 if $y + \sin y = \cos x$

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26. Find the derivative of f given by $f(x) = \sin^{-1}x$ assuming it exists.



27. Find the derivative of f given by $f(x) = \tan^{-1}x$ assuming it exists

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28. Is it true that $x = e^{\log x}$ for all real x?



29. Differentiate the following w.r.t x:

(i) e^{-x} (ii) $\sin(\log x)$, x > 0 (iii) $\cos^{-1}(e^x)$ (iv) $e^{\cos x}$

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30. Differentiate
$$\frac{\sqrt{(x-3)(x^2+4)}}{\sqrt{(3x^2+4x+5)}}$$
 w.r.t x.

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31. Differentiate a^{x} w.r.t x, where a is a positive constant



32. Differentiate $x^{\sin x}$, x > 0 w.r.t x

33. Find
$$\frac{dy}{dx}$$
, If $y^x + x^y + x^x = a^b$



34. Find
$$\frac{dy}{dx}$$
, if $x = a\cos\theta$, $y = a\sin\theta$

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35. Find
$$\frac{dy}{dx}$$
, if $x = at^2$, $y = 2at$

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36. Find
$$\frac{dy}{dx}$$
, if $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$

37. Find
$$\frac{dy}{dx}$$
, if $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$



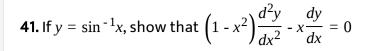
38. Find
$$\frac{d^2y}{dx^2}$$
, if $y = x^3 + \tan x$

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39. If
$$y = A\sin x + B\cos x$$
, then prove that $\frac{d^2y}{dx^2} + y = 0$

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40. If
$$y = 3e^{2x} + 2e^{3x}$$
, prove that $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$



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42. Verify Rolle's theorem for the function $y = x^2 + 2$, a = -2 and b = 2

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43. Verify Mean value Theorem for the function $f(x) = x^2$ in the interval [2,4]

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44. Differentiate w.r.t x, the following function:

(i)
$$\sqrt{3x+2} + \frac{1}{\sqrt{2x^2+4}}$$
 (ii) $e^{\sec^2 x} + 3\cos^{-1} x$ (iii) $\log_7(\log x)$

45. Differentiate the following w.r.t x.

(i)
$$\cos^{-1}(\sin x)$$
 (ii) $\tan^{-1}\left(\frac{\sin x}{1 + \cos x}\right)$ (iii) $\sin^{-1}\left(\frac{2^{x+1}}{1 + 4^x}\right)$

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46. Find f'(x) if $f(x) = (\sin x)^{\sin x}$ for all $0 < x < \pi$.

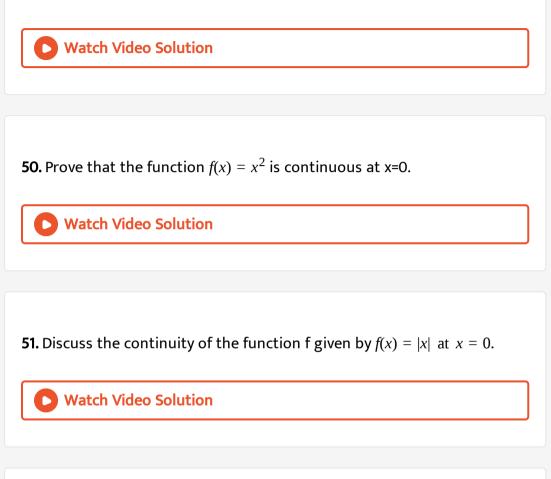
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47. For a positive constant a find
$$\frac{dy}{dx}$$
, where $y = a^{t+\frac{1}{t}}$, and $x = \left(t + \frac{1}{t}\right)^a$

48. Differentiate $\sin^2 x$ w.r.t $e^{\cos x}$

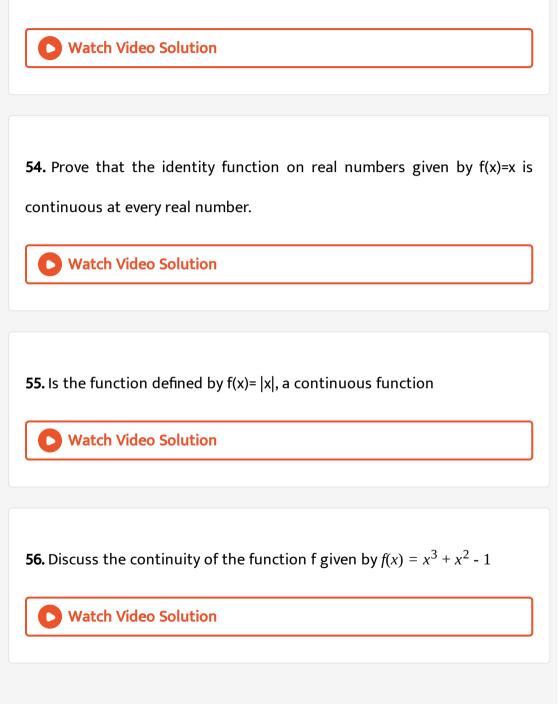


49. Examine the continuity of the function f(x) = 2x + 3 at x = 1

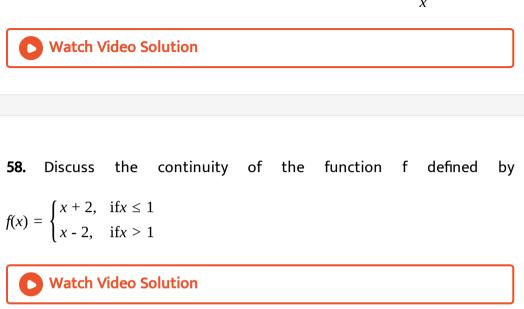


52. Show that
$$f(x) = \begin{cases} x^3 + 3, & x \neq 0 \\ 1, & x = 0 \end{cases}$$
 is a discontinuous function at x= 0.

53. Check the points where the constant function f(x) = k is continuous

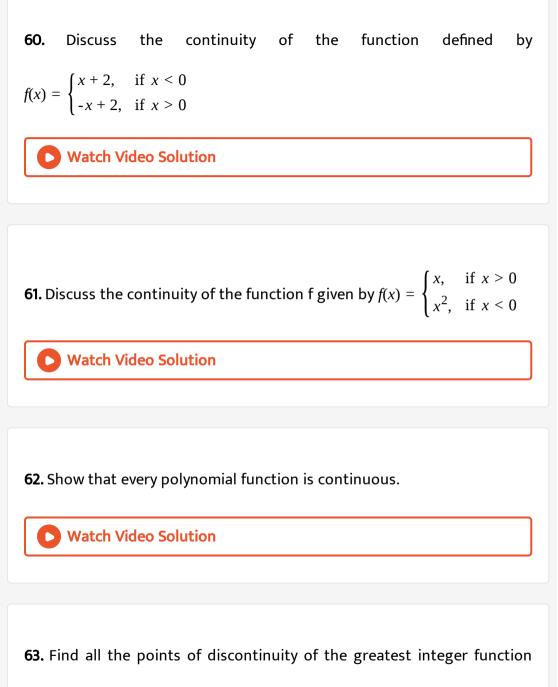


57. Discuss the continuity of the function f defined by $f(x) = \frac{1}{x}, x \neq 0$



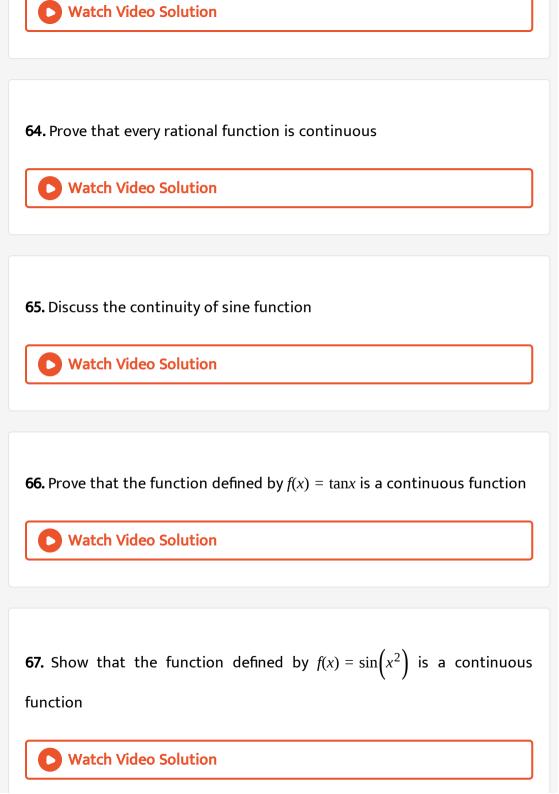
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 w.r.t x.

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79. Differentiate a^{x} w.r.t x, where a is a positive constant



80. Differentiate $s^{\sin x}$, x > 0 w.r.t x

81. Find
$$\frac{dy}{dx}$$
, If $y^{x} + x^{y} + x^{x} = a^{b}$



82. Find
$$\frac{dy}{dx}$$
, if $x = a\cos\theta$, $y = a\sin\theta$

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83. Find
$$\frac{dy}{dx}$$
, if $x = at^2$, $y = 2at$

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84. If x and y are connected parametrically by the equations without eliminating the parameter, find $\frac{dy}{dx}$ $x = a(\theta - \sin\theta), y = a(1 + \cos\theta)$

85. Find
$$\frac{dy}{dx}$$
, if $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$



86. Find
$$\frac{d^2y}{dx^2}$$
, if $y = x^3 + \tan x$

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87. If $y = A\sin x + B\cos x$, then prove that $\frac{d^2y}{dx^2} + y = 0$

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88. If
$$y = 3e^{2x} + 2e^{3x}$$
, prove that $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$

89. If
$$y = \sin^{-1}x$$
, show that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$
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90. Verify Rolle's theorem for the function $y = x^2 + 2$, $a = -2$ and $b = 2$
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91. Verify Mean value Theorem for the function $f(x) = x^2$ in the interval [2,4]
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92. Differentiate w.r.t x, the following function:

(i)
$$\sqrt{3x+2} + \frac{1}{\sqrt{2x^2+4}}$$
 (ii) $e^{\sec^2 x} + 3\cos^{-1} x$ (iii) $\log_7(\log x)$

93. Differentiate the following w.r.t x.

(i)
$$\cos^{-1}(\sin x)$$
 (ii) $\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$ (iii) $\sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$

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94. Find f'(x) if $f(x) = (\sin x)^{\sin x}$ for all $0 < x < \pi$.

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95. For a positive constant a find
$$\frac{dy}{dx}$$
, where $y = a^{t+\frac{1}{t}}$, and $x = \left(t + \frac{1}{t}\right)^a$

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96. Differentiate $\sin^2 x$ w.r.t $e^{\cos x}$

$$f(x) = x^3 + 2x^2 - 1$$
 at $x = 1$

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2. Examine the continuity of the function

$$f(x) = \begin{cases} 3x + 5, & \text{if } x \ge 2\\ x^2 & \text{if } x < 2 \end{cases} \text{ at } x = 2$$

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3. Examine the continuity of the function

$$f(x) = \begin{cases} \frac{1 - \cos(2x)}{x^2} & \text{if } x \neq 0\\ 5, & \text{if } x = 0 \end{cases} \text{ at } x=0$$



$$f(x) = \begin{cases} \frac{2x^2 - 3x - 2}{x - 2}, & \text{if } x \neq 2\\ 5, & \text{if } x = 2 \end{cases} \text{ at } x = 2$$



5. Examine the continuity of the function

$$f(x) = \begin{cases} \frac{|x-4|}{2(x-4)}, & \text{if } x \neq 4\\ 0, & \text{if } x = 4 \end{cases} \text{ at } x=4$$

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6. Examine the continuity of the function

$$f(x) = \begin{cases} |x| \cdot \cos\left(\frac{1}{x}\right), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases} \text{ at } x = 0$$

$$f(x) = \begin{cases} |x - a| \sin\left(\frac{1}{x - a}\right), & \text{if } x \neq a \\ 0, & \text{if } 'x = a \end{cases} \text{ at } x=a$$

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8. Examine the continuity of the function

$$f(x) = \begin{cases} \frac{e^{\frac{1}{x}}}{1+e^{\frac{1}{x}}}, & \text{if } x \neq 0\\ 0, & \text{if } x = 0 \end{cases} \text{ at } x = 0$$

$$f(x) = \begin{cases} \frac{x^2}{2}, & \text{if } 0 \le x \le 1\\ 2x^2 - 3x + \frac{3}{2}, & \text{if } 1 < x \le 2 \end{cases}$$

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10. Examine the continuity of the function

f(x) = |x| + |x - 1| at x=1

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11. Find the values of k so that the function f is continuous at the

indicated point
$$f(x) = \begin{cases} 3x - 8, & \text{if } x \le 5\\ 2k, & \text{if } x > 5 \end{cases}$$
 at x= 5

12.
$$f(x) = \begin{cases} \frac{2^{x+2} - 16}{4^x - 16}, & x \neq 2\\ k, & x = 2 \end{cases}$$
 f(x) is continuous at x=2 then find k



13. Find the values of k so that the function f is continuous at the

indicated point
$$f(x) = \begin{cases} \frac{\sqrt{1+kx} - \sqrt{1-kx}}{x}, & \text{if } -1 \le x < 0\\ \frac{2x+1}{x-1}, & \text{if } 0 \le x < 1 \end{cases}$$
 at x= 0

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14. Find the values of k so that the function f is continuous at the

indicated point
$$f(x) = \begin{cases} \frac{1 - \cos(kx)}{x^2}, & \text{if } x \neq 0\\ \frac{1}{2}, & \text{if } x = 0 \end{cases}$$
 at $x = 0$

15. Prove that the function f defined by $f(x) = \begin{cases} \frac{x}{|x| + 2x^2}, & \text{if } x \neq 0\\ k & \text{if } x = 0 \end{cases}$ remains

discontinuous at x=0, regardless the choice of k

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16. Find the values of a and b such that the function f defined by

$$f(x) = \begin{cases} \frac{x-4}{|x-4|} + a, & \text{if } x < 4\\ a+b & \text{if } x = 4 \text{ is a continuous function at } x=4\\ \frac{x-4}{|x-4|} + b & \text{if } x > 4 \end{cases}$$

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17. If the function $f(x) = \frac{1}{x+2}$, then find the points of discontinuity of the composite function y= f {f(x)}



18. Find all points of discontinuity of the function $f(t) = \frac{1}{t^2 + t - 2}$, where

$$t = \frac{1}{x - 1}$$

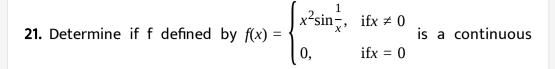
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19. Show that the function $f(x) = |\sin x + \cos x|$ is continuous at $x = \pi$

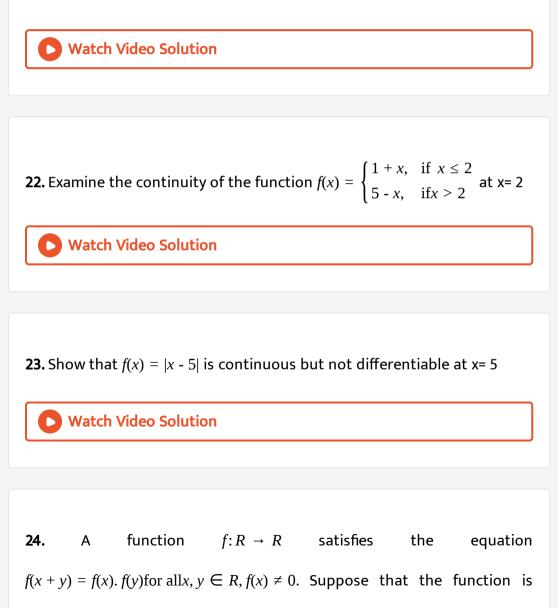
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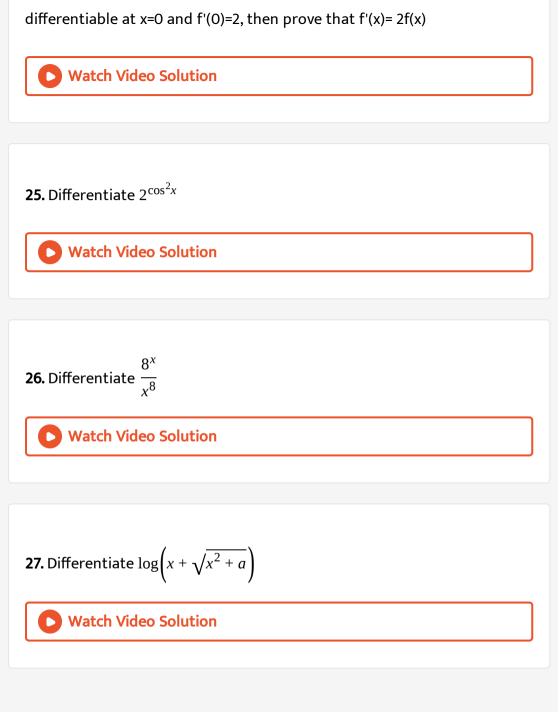
20. Examine the differentiability of f, where f is defined by

$$f(x) = \begin{cases} x. [x], & \text{if } 0 \le x < 2\\ (x - 1)x & \text{if } 2 \le x < 3 \end{cases} \text{ at } x = 2$$

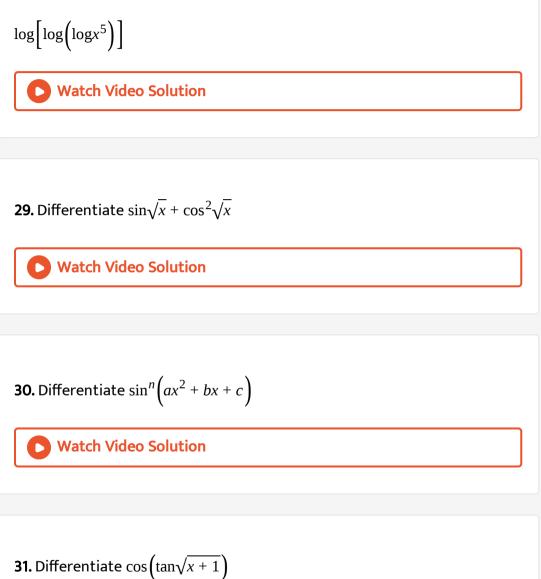


function?





28. Differentiate the following with respect to x:



32. Differentiate
$$\sin x^2 + \sin^2 x + \sin^2 \left(x^2\right)$$



33. Differentiate
$$\sin^{-1}\left(\frac{1}{\sqrt{x+1}}\right)$$

34. Differentiate (sinx)^{cosx}

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35. Differentiate $\sin^m x \cdot \cos^n x$

36. Differentiate
$$(x + 1)^2(x + 2)^3(x + 3)^4$$



37. Differentiate
$$\cos^{-1}\left(\frac{\sin x + \cos x}{\sqrt{2}}\right)$$
, $-\frac{\pi}{4} < x < \frac{\pi}{4}$

38. Differentiate
$$\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$$
, $-\frac{\pi}{4} < x < \frac{\pi}{4}$

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39. Differentiate
$$\tan^{-1}(\sec x + \tan x)$$
, $-\frac{\pi}{2} < x < \frac{\pi}{2}$

40. Differentiate,
$$\tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right)$$
, $\frac{-\pi}{2} < x < \frac{\pi}{2}$ and $\frac{a}{b}\tan x > -1$

41. Differentiate sec⁻¹
$$\left(\frac{1}{4x^3 - 3x}\right)$$
, $0 < x < \frac{1}{\sqrt{2}}$

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42. Write the following functions in the simplest form :

$$\tan^{-1}\left(\frac{3a^2x - x^3}{a^3 - 3ax^2}\right), a > 0, \frac{-a}{\sqrt{3}} < x < \frac{a}{\sqrt{3}}$$

43. Prove that :

$$\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right) = \frac{\pi}{4} + \frac{1}{2}\cos^{-1}x^2.$$

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44. Find
$$\frac{dy}{dx}$$
:
 $x = t + \frac{1}{t}$ and $y = t - \frac{1}{t}$

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45. Find $\frac{dy}{dx}$ of each of the functions expressed in parametric form:

$$x = e^{\theta} \left(\theta + \frac{1}{\theta} \right), y = e^{-\theta} \left(\theta - \frac{1}{\theta} \right)$$

46. Find $\frac{dy}{dx}$ of each of the functions expressed in parametric form:

 $x = 3\cos\theta - 2\cos^3\theta, y = 3\sin\theta - 2\sin^3\theta$

47. Find $\frac{dy}{dx}$ of each of the functions expressed in parametric form: $\sin x = \frac{2t}{1+t^2}$, $\tan y = \frac{2t}{1-t^2}$, $t \in R$

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48. Find $\frac{dy}{dx}$ of each of the functions expressed in parametric form: $x = \frac{1 + \log t}{t^2}, y = \frac{3 + 2\log t}{t}$

49. Find
$$\frac{dy}{dx}$$
:
 $x = e^{\cos 2t}$ and $y = e^{\sin 2t}$ show that, $\frac{dy}{dx} = \frac{-y \log x}{x \log y}$

50. Find
$$\frac{dy}{dx}$$
:
 $x = a\sin 2t(1 + \cos 2t)$ and $y = b\cos 2t$
 $(1 - \cos 2t)$ show that, $\left(\frac{dy}{dx}\right)_{t=\frac{\pi}{4}} = \frac{b}{a}$

51. If
$$x = 3\sin t - \sin(3t)$$
, $y = 3\cos t - \cos 3t$, then find $\left(\frac{dy}{dx}\right)$ at $t = \frac{\pi}{3}$

52. Differentiate
$$\frac{x}{\sin x}$$
 w.r.t sin x

53. Differentiate
$$\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$$
 w.r.t $\tan^{-1}x$, where $x \neq 0$

54. Find
$$\frac{dy}{dx}$$
 when x and y are connected by the relation given:
 $sin(xy) + \frac{x}{y} = x^2 - y$

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55. Find $\frac{dy}{dx}$ when x and y are connected by the relation given:

 $\sec(x + y) = xy$

56. Find $\frac{dy}{dx}$ when x and y are connected by the relation given:

$$\tan^{-1}\left(x^2 + y^2\right) = a$$

57. Find $\frac{dy}{dx}$ when x and y are connected by the relation given: $(x^2 + y^2)^2 = xy$

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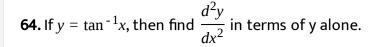
58. If
$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$
, then show that $\frac{dy}{dx} \cdot \frac{dx}{dy} = 1$

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59. If
$$x = e^{\frac{x}{y}}$$
, then prove that $\frac{dy}{dx} = \frac{x - y}{x \cdot \log x}$

60. If
$$y^x = e^{y-x}$$
, then prove that $\frac{dy}{dx} = \frac{(1 + \log y)^2}{\log y}$
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61. If $y = (\cos x)^{(\cos x) \cdots \infty}$, then show that $\frac{dy}{dx} = \frac{y^2 \tan x}{y \cdot \log \cos x - 1}$
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62. If $x \sin(a + y) + \sin a \cos(a + y) = 0$, then prove that $\frac{dy}{dx} = \frac{\sin^2(a + y)}{\sin a}$
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63. If $\sqrt{1 - x^2} + \sqrt{1 - y^2} = a(x - y)$, then prove that $\frac{dy}{dx} = \sqrt{\frac{1 - y^2}{1 - x^2}}$. (Where

 $|x| \le 1, |y| \le 1$)



65. Verify the Rolle's theorem for each of the function in following questions:

 $f(x) = x(x - 1)^2$, in $x \in [0, 1]$

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66. Verify the Rolle's theorem for each of the function in following questions:

$$f(x) = \sin^4 x + \cos^4 x, \text{ in } x \in \left[0, \frac{\pi}{2}\right]$$

67. Verify the Rolle's theorem for each of the function in following questions:

$$f(x) = \log(x^2 + 2) - \log 3$$
, in $x \in [-1, 1]$

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68. Verify the Rolle's theorem for each of the function in following questions:

$$f(x) = x(x + 3). e^{-\frac{x}{2}}, \text{ in } x \in [-3, 0]$$

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69. Verify the Rolle's theorem for each of the function in following questions:

$$f(x) = \sqrt{4 - x^2}$$
, in $x \in [-2, 2]$

70. Discuss the applicability of Rolle's theorem on the function given by

$$f(x) = \begin{cases} x^2 + 1, & \text{if } 0 \le x < 1\\ 3 - x, & \text{if } 1 \le x \le 2 \end{cases}$$



71. Find the points on the curve $y = \cos x - 1$ in $x \in [0, 2\pi]$, where the tangent is parallel to X-axis.



72. Using Rolle's theorem, find the point on the curve $y = x(x - 4), x \in [0, 4]$, where the tangent is parallel to X-axis



73. Verify mean value theorem for each of the functions:

$$f(x) = \frac{1}{4x - 1}, x \in [1, 4]$$

74. Verify mean value theorem for each of the functions:

$$f(x) = x^3 - 2x^2 - x + 3, x \in [0, 1]$$

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75. Verify mean value theorem for each of the functions:

$$f(x) = \sin x - \sin(2x)$$
, in $x \in [0, \pi]$

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76. Verify mean value theorem for each of the functions:

$$f(x) = \sqrt{25 - x^2}$$
, in $x \in [1, 5]$

77. Find a point on the curve $y = (x - 3)^2$, where the tangent is parallel to

the chord joining the points (3,0) and (4,1).



78. Using mean value theorem, prove that there is a point on the curve $y = 2x^2 - 5x + 3$ between the points P(1,0) and B(2,1), where tangent is parallel to the chord AB. Also, find the point.

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79. Examine the continuity of the function

 $f(x) = x^3 + 2x^2 - 1$ at x = 1

$$f(x) = \begin{cases} 3x + 5, & \text{if } x \ge 2\\ x^2 & \text{if } x < 2 \end{cases} \text{ at } x = 2$$

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81. Examine the continuity of the function

$$f(x) = \begin{cases} \frac{1 - \cos(2x)}{x^2} & \text{if } x \neq 0\\ 5, & \text{if } x = 0 \end{cases} \text{ at } x=0$$

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82. Examine the continuity of the function

$$f(x) = \begin{cases} \frac{2x^2 - 3x - 2}{x - 2}, & \text{if } x \neq 2\\ 5, & \text{if } x = 2 \end{cases} \text{ at } x = 2$$

$$f(x) = \begin{cases} \frac{|x-4|}{2(x-4)}, & \text{if } x \neq 4\\ 0, & \text{if } x = 4 \end{cases} \text{ at } x=4$$



84. Examine the continuity of the function

$$f(x) = \begin{cases} |x| \cdot \cos\left(\frac{1}{x}\right), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases} \text{ at } x = 0$$

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85. Examine the continuity of the function

$$f(x) = \begin{cases} |x - a| \sin\left(\frac{1}{x - a}\right), & \text{if } x \neq a \\ 0, & \text{if } 'x = a \end{cases} \text{ at } x = a$$

$$f(x) = \begin{cases} \frac{e^{\frac{1}{x}}}{1 + e^{\frac{1}{x}}}, & \text{if } x \neq 0\\ 0, & \text{if } x = 0 \end{cases} \text{ at } x = 0$$

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87. Examine the continuity of the function

$$f(x) = \begin{cases} \frac{x^2}{2}, & \text{if } 0 \le x \le 1\\ 2x^2 - 3x + \frac{3}{2}, & \text{if } 1 \le x \le 2 \end{cases}$$
 at x=1

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88. Examine the continuity of the function

f(x) = |x| + |x - 1| at x=1

89. Find the values of k so that the function f is continuous at the

indicated point
$$f(x) = \begin{cases} 3x - 8, & \text{if } x \le 5\\ 2k, & \text{if } x > 5 \end{cases}$$
 at x= 5

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90.
$$f(x) = \begin{cases} \frac{2^{x+2} - 16}{4^x - 16}, & x \neq 2\\ k, & x = 2 \end{cases}$$
 f(x) is continuous at x=2 then find k

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91. Find the values of k so that the function f is continuous at the

indicated point
$$f(x) = \begin{cases} \frac{\sqrt{1+kx} - \sqrt{1-kx}}{x}, & \text{if } -1 \le x < 0\\ \frac{2x+1}{x-1}, & \text{if } 0 \le x < 1 \end{cases}$$
 at x= 0

92. Find the values of k so that the function f is continuous at the

indicated point
$$f(x) = \begin{cases} \frac{1 - \cos(kx)}{x^2}, & \text{if } x \neq 0\\ \frac{1}{2}, & \text{if } x = 0 \end{cases}$$
 at x= 0

-



93. Prove that the function f defined by
$$f(x) = \begin{cases} \frac{x}{|x|+2x^2}, & \text{if } x \neq 0\\ k, & \text{if } x = 0 \end{cases}$$
 remains

discontinuous at x=0, regardless the choice of k



94. Find the values of a and b such that the function f defined by

$$f(x) = \begin{cases} \frac{x-4}{|x-4|} + a, & \text{if } x < 4\\ a+b & \text{if } x = 4 \text{ is a continuous function at } x=4\\ \frac{x-4}{|x-4|} + b & \text{if } x > 4 \end{cases}$$

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95. If the function $f(x) = \frac{1}{x+2}$, then find the points of discontinuity of the composite function y= f {f(x)}

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96. Find all points of dicontinuity of the function $f(t) = \frac{1}{t^2 + t - 2}$, where $t = \frac{1}{x - 1}$

97. Show that the function $f(x) = |\sin x + \cos x|$ is continuous at $x = \pi$



98. Examine the differentiability of f, where f is defined by

$$f(x) = \begin{cases} x. [x], & \text{if } 0 \le x < 2\\ (x - 1)x & \text{if } 2 \le x < 3 \end{cases} \text{ at } x = 2$$

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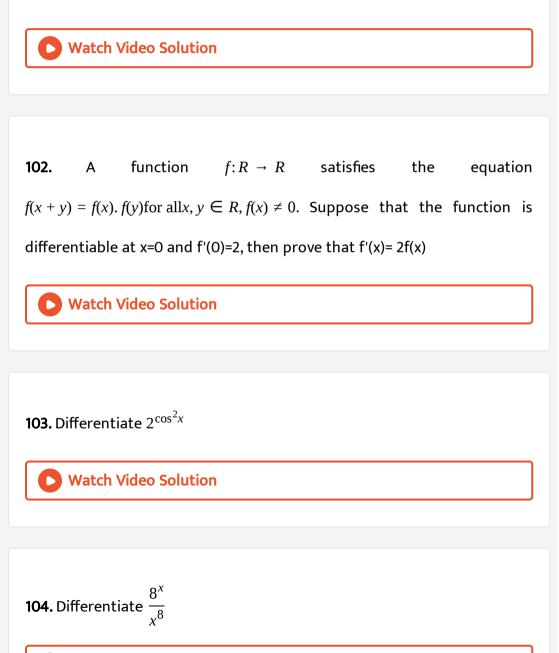
99. Determine if f defined by
$$f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$
 is a continuous

function?

100. Examine the continuity of the function $f(x) = \begin{cases} 1+x, & \text{if } x \le 2\\ 5-x, & \text{if } x > 2 \end{cases}$ at x=2



101. Show that f(x) = |x - 5| is continuous but not differentiable at x= 5



105. Differentiate
$$\log\left(x + \sqrt{x^2 + a}\right)$$

106. Differentiate the following with respect to x:

 $\log\left[\log\left(\log x^{5}\right)\right]$

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107. Differentiate
$$\sin\sqrt{x} + \cos^2\sqrt{x}$$



108. Differentiate
$$\sin^n \left(ax^2 + bx + c \right)$$

109. Differentiate
$$\cos(\tan\sqrt{x+1})$$



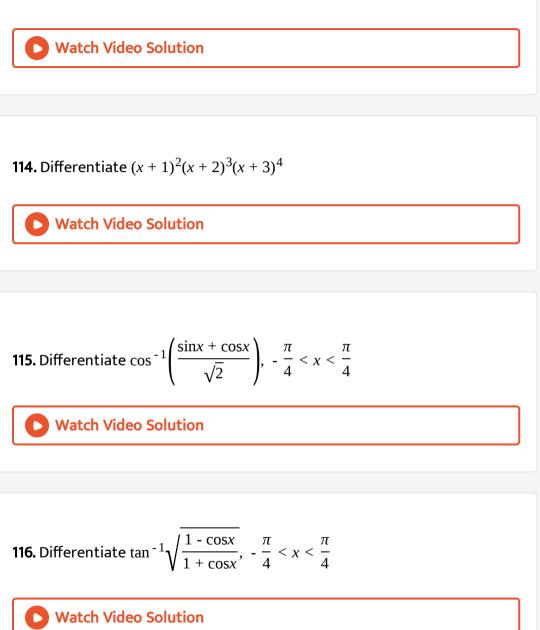
110. Differentiate
$$\sin x^2 + \sin^2 x + \sin^2 \left(x^2\right)$$

111. Differentiate
$$\sin^{-1}\left(\frac{1}{\sqrt{x+1}}\right)$$

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112. Differentiate (sinx)^{cosx}

113. Differentiate $\sin^m x. \cos^n x$



117. Differentiate
$$\tan^{-1}(\sec x + \tan x)$$
, $-\frac{\pi}{2} < x < \frac{\pi}{2}$



118. Differentiate,
$$\tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right)$$
, $\frac{-\pi}{2} < x < \frac{\pi}{2}$ and $\frac{a}{b}\tan x > -1$

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119. Differentiate
$$\sec^{-1}\left(\frac{1}{4x^3 - 3x}\right), 0 < x < \frac{1}{\sqrt{2}}$$

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120. Write the following functions in the simplest form :

$$\tan^{-1}\left(\frac{3a^2x - x^3}{a^3 - 3ax^2}\right), a > 0, \ \frac{-a}{\sqrt{3}} < x < \frac{a}{\sqrt{3}}$$

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121. Prove that :

$$\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right) = \frac{\pi}{4} + \frac{1}{2}\cos^{-1}x^2.$$

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122. Find
$$\frac{dy}{dx}$$
:
 $x = t + \frac{1}{t}$ and $y = t - \frac{1}{t}$

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123. Find $\frac{dy}{dx}$ of each of the functions expressed in parametric form:

$$x = e^{\theta} \left(\theta + \frac{1}{\theta} \right), y = e^{-\theta} \left(\theta - \frac{1}{\theta} \right)$$

124. Find $\frac{dy}{dx}$ of each of the functions expressed in parametric form: $x = 3\cos\theta - 2\cos^{3}\theta, y = 3\sin\theta - 2\sin^{3}\theta$

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125. Find $\frac{dy}{dx}$ of each of the functions expressed in parametric form: $\sin x = \frac{2t}{1+t^2}$, $\tan y = \frac{2t}{1-t^2}$, $t \in R$

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126. Find $\frac{dy}{dx}$ of each of the functions expressed in parametric form: $x = \frac{1 + \log t}{t^2}, y = \frac{3 + 2\log t}{t}$

127. Find
$$\frac{dy}{dx}$$
:
 $x = e^{\cos 2t}$ and $y = e^{\sin 2t}$ show that, $\frac{dy}{dx} = \frac{-y \log x}{x \log y}$

128. Find
$$\frac{dy}{dx}$$
:
 $x = a\sin 2t(1 + \cos 2t)$ and $y = b\cos 2t$
(1 - cos2t) show that, $\left(\frac{dy}{dx}\right)_{t=\frac{\pi}{4}} = \frac{b}{a}$

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129. If
$$x = 3\sin t - \sin(3t)$$
, $y = 3\cos t - \cos 3t$, then find $\left(\frac{dy}{dx}\right)$ at $t = \frac{\pi}{3}$

130. Differentiate
$$\frac{x}{\sin x}$$
 w.r.t sin x

131. Differentiate
$$\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$$
 w.r.t $\tan^{-1}x$, where $x \neq 0$

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132. Find
$$\frac{dy}{dx}$$
 when x and y are connected by the relation given:
 $sin(xy) + \frac{x}{y} = x^2 - y$

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133. Find
$$\frac{dy}{dx}$$
 when x and y are connected by the relation given:

 $\sec(x + y) = xy$

134. Find $\frac{dy}{dx}$ when x and y are connected by the relation given:

$$\tan^{-1}\left(x^2 + y^2\right) = a$$

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135. Find
$$\frac{dy}{dx}$$
 when x and y are connected by the relation given:
 $(x^2 + y^2)^2 = xy$

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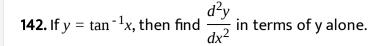
136. If
$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$
, then show that $\frac{dy}{dx} \cdot \frac{dx}{dy} = 1$

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137. If
$$x = e^{\frac{x}{y}}$$
, then prove that $\frac{dy}{dx} = \frac{x - y}{x \cdot \log x}$

138. If
$$y^x = e^{y - x}$$
, then prove that $\frac{dy}{dx} = \frac{(1 + \log y)^2}{\log y}$
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139. If $y = (\cos x)^{(\cos x) \cdots \infty}$, then show that $\frac{dy}{dx} = \frac{y^2 \tan x}{y \cdot \log \cos x - 1}$
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140. If $x \sin(a + y) + \sin a \cos(a + y) = 0$, then prove that $\frac{dy}{dx} = \frac{\sin^2(a + y)}{\sin a}$
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141. If $\sqrt{1 - x^2} + \sqrt{1 - y^2} = a(x - y)$, then prove that $\frac{dy}{dx} = \sqrt{\frac{1 - y^2}{1 - x^2}}$. (Where

 $|x|\leq 1, |y|\leq 1)$



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143. Verify the Rolle's theorem for each of the function in following questions:

 $f(x) = x(x - 1)^2$, in $x \in [0, 1]$

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144. Verify the Rolle's theorem for each of the function in following questions:

$$f(x) = \sin^4 x + \cos^4 x, \text{ in } x \in \left[0, \frac{\pi}{2}\right]$$

145. Verify the Rolle's theorem for each of the function in following questions:

$$f(x) = \log(x^2 + 2) - \log 3$$
, in $x \in [-1, 1]$

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146. Verify the Rolle's theorem for each of the function in following questions:

$$f(x) = x(x+3). e^{-\frac{x}{2}}, \text{ in } x \in [-3, 0]$$

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147. Verify the Rolle's theorem for each of the function in following questions:

$$f(x) = \sqrt{4 - x^2}$$
, in $x \in [-2, 2]$

148. Discuss the applicability of Rolle's theorem on the function given by

$$f(x) = \begin{cases} x^2 + 1, & \text{if } 0 \le x < 1\\ 3 - x, & \text{if } 1 \le x \le 2 \end{cases}$$

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149. Find the points on the curve $y = \cos x - 1$ in $x \in [0, 2\pi]$, where the tangent is parallel to X-axis.

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150. Using Rolle's theorem, find the point on the curve $y = x(x - 4), x \in [0, 4]$, where the tangent is parallel to X-axis



151. Verify mean value theorem for each of the functions:

$$f(x) = \frac{1}{4x - 1}, x \in [1, 4]$$

152. Verify mean value theorem for each of the functions:

$$f(x) = x^3 - 2x^2 - x + 3, x \in [0, 1]$$

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153. Verify mean value theorem for each of the functions:

$$f(x) = \sin x - \sin(2x), \text{ in } x \in [0, \pi]$$

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154. Verify mean value theorem for each of the functions:

$$f(x) = \sqrt{25 - x^2}$$
, in $x \in [1, 5]$

155. Find a point on the curve $y = (x - 3)^2$, where the tangent is parallel to

the chord joining the points (3,0) and (4,1).



156. Using mean value theorem, prove that there is a point on the curve $y = 2x^2 - 5x + 3$ between the points P(1,0) and B(2,1), where tangent is parallel to the chord AB. Also, find the point.

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NCERT Exemplar Problems and Solution (Long Answer Type Questions)

1. Find the values of p and q, so that
$$f(x) = \begin{cases} x^2 + 3x + p & x \le 1 \\ qx + 2 & x > 1 \end{cases}$$
 is

differentiable at x=1.

2. If
$$x^m$$
. $y^n = (x + y)^{m+n}$, prove that

$$\frac{dy}{dx} = \frac{y}{x}$$

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3. If
$$x^m$$
. $y^n = (x + y)^{m+n}$ then show that, $\frac{d^2y}{dx^2} = 0$

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4. If
$$y = \sin(pt)$$
, $x = \sin t$, then prove that $(1 - x^2)y_2 - xy_1 + p^2y = 0$

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5. Find the value of
$$\frac{dy}{dx}$$
 if $y = x^{\tan x} + \sqrt{\frac{x^2 + 1}{2}}$

6. Find the values of p and q, so that $f(x) = \begin{cases} x^2 + 3x + p & x \le 1 \\ qx + 2 & x > 1 \end{cases}$ is

differentiable at x=1.



7. If
$$x^m$$
. $y^n = (x + y)^{m+n}$, prove that
 $\frac{dy}{dx} = \frac{y}{x}$



8. If x^m . $y^n = (x + y)^{m+n}$ then show that, $\frac{d^2y}{dx^2} = 0$

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9. If $y = \sin(pt)$, $x = \sin t$, then prove that $(1 - x^2)y_2 - xy_1 + p^2y = 0$

10. Find the value of
$$\frac{dy}{dx}$$
 if $y = x^{\tan x} + \sqrt{\frac{x^2 + 1}{2}}$

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NCERT Exemplar Problems and Solution (Objective Type Questions)

1. If
$$f(x) = 2x$$
 and $g(x) = \frac{x^2}{2} + 1$, then which of the following can be a

discontinuous function?

A. f(x) + g(x)

B. f(x) - g(x)

C. f(x). g(x)

D.
$$\frac{g(x)}{f(x)}$$

Answer: D

2. The function $f(x) = \frac{4 - x^2}{4x - x^3}$ is

A. discontinuous at only one point

B. discontinuous at exactly two points

C. discontinuous at exactly three points

D. none of the above

Answer: C

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3. The set of points where the function f given by $f(x) = |2x - 1|\sin x$ is differentiable is $x \in \dots$

A. R

$$\mathsf{B}.\,\mathsf{R}-\left\{\frac{1}{2}\right\}$$

C. (0, ∞)

D. none of these

Answer: B



4. The function $f(x) = \cot x$ is discontinuous on the set

A.
$$\{x \mid x = n\pi, n \in Z\}$$

B. { $x \mid x = 2n\pi, n \in Z$ }

C.
$$\left\{ x \mid x = (2n+1)\frac{\pi}{2}, n \in Z \right\}$$

D.
$$\left\{ x \mid x = \frac{n\pi}{2}, n \in Z \right\}$$

Answer: A

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5. The function $f(x) = e^{|x|}$ is

A. continuous everywher but not differentiable at x= 0

B. continuous and differentiable everywhere

C. Not continuous at x= 0

D. none of the above

Answer: A

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6. If
$$f(x) = x^2 \sin\left(\frac{1}{x}\right)$$
, where $x \neq 0$, then the value of the function f at x=0,

so that the function is continuous at x= 0, is

A. 0

B. - 1

C. 1

D. None of these

Answer: A

7. If
$$f(x)$$

$$\begin{cases}
mx + 1 & x \le \frac{\pi}{2} \\
(\sin x) + n & x > \frac{\pi}{2}
\end{cases}$$
is continuous at $x = \frac{\pi}{2}$, then
A. $m = 1, n = 0$
B. $m = \frac{n\pi}{2} + 1$
C. $n = \frac{m\pi}{2}$
D. $m = n = \frac{\pi}{2}$

Answer: C



8. If $f(x) = |\sin x|$, then

A. f is everywhere differentiable

B. f is everywhere continuous but not differentiable at $x = n\pi$, $n \in Z$

C.f is everywhere continuous but not differentiable at

$$x = \left(2n + \frac{\pi}{2}\right), n \in \mathbb{Z}$$

D. none of the above

Answer: B

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9. If
$$y = \log\left(\frac{1-x^2}{1+x^2}\right)$$
, then $\frac{dy}{dx}$ is equal to
A. $\frac{4x^3}{1-x^4}$
B. $\frac{-4x}{1-x^4}$
C. $\frac{1}{4-x^4}$
D. $\frac{-4x^3}{1-x^4}$

Answer: B

10. If
$$y = \sqrt{\sin x + y}$$
, then $\frac{dy}{dx}$ is equal to
A. $\frac{\cos x}{2y - 1}$
B. $\frac{\cos x}{1 - 2y}$
C. $\frac{\sin x}{1 - 2y}$
D. $\frac{\sin x}{2y - 1}$

Answer: A

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11. The derivative of
$$\cos^{-1}(2x^2 - 1)$$
 w.r.t $\cos^{-1}x$ is

A. 2

$$B. \frac{-1}{2\sqrt{1-x^2}}$$

C.
$$\frac{2}{x}$$

D. 1 - x^2

Answer: A



12. If
$$x = t^2$$
 and $y = t^3$, then $\frac{d^2y}{dx^2}$ is equal to

A.
$$\frac{3}{2}$$

B. $\frac{3}{4t}$
C. $\frac{3}{2t}$
D. $\frac{3t}{2}$

Answer: B

13. The value of c in Rolle's theorem for the function $f(x) = x^3$ in the interval $x \in [0, \sqrt{3}]$

A. 1

B. -1 C. $\frac{3}{2}$ D. $\frac{1}{3}$

Answer: A

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14. For the function $f(x) = x + \frac{1}{x}$, $x \in [1, 3]$ the value of c for mean value

theorem is

A. 1

B. $\sqrt{3}$

C. 2

D. None of these

Answer: B

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15. If f(x) = 2x and $g(x) = \frac{x^2}{2} + 1$, then which of the following can be a discontinuous function?

A. f(x) + g(x)B. f(x) - g(x)C. f(x). g(x)

D.
$$\frac{g(x)}{f(x)}$$

Answer: D

16. The function $f(x) = \frac{4 - x^2}{4x - x^3}$ is

A. discontinuous at only one point

B. discontinuous at exactly two points

C. discontinuous at exactly three points

D. none of the above

Answer: C

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17. The set of points where the function f given by $f(x) = |2x - 1|\sin x$ is differentiable is $x \in$

A. R

$$B. R - \left\{\frac{1}{2}\right\}$$
$$C. (0, \infty)$$

D. none of these

Answer: B



18. The function $f(x) = \cot x$ is discontinuous on the set

A.
$$\{x \mid x = n\pi, n \in Z\}$$

B. {
$$x \mid x = 2n\pi, n \in Z$$
}

C.
$$\left\{ x \mid x = (2n+1)\frac{\pi}{2}, n \in Z \right\}$$

D.
$$\left\{ x \mid x = \frac{n\pi}{2}, n \in Z \right\}$$

Answer: A

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19. The function $f(x) = e^{|x|}$ is

A. continuous everywher but not differentiable at x= 0

B. continuous and differentiable everywhere

C. Not continuous at x= 0

D. none of the above

Answer: A

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20. If
$$f(x) = x^2 \sin\left(\frac{1}{x}\right)$$
, where $x \neq 0$, then the value of the function f at x=0,

so that the function is continuous at x= 0, is

A. 0

B. - 1

C. 1

D. None of these

Answer: A

21. If
$$f(x)$$

$$\begin{cases}
mx + 1 & x \le \frac{\pi}{2} \\
(\sin x) + n & x > \frac{\pi}{2}
\end{cases}$$
is continuous at $x = \frac{\pi}{2}$, then
A. $m = 1, n = 0$
B. $m = \frac{n\pi}{2} + 1$
C. $n = \frac{m\pi}{2}$
D. $m = n = \frac{\pi}{2}$

Answer: C

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22. If
$$f(x) = |\cos x|$$
, then $f'\left(\frac{\pi}{4}\right)$ is equal to $0 < x < \frac{\pi}{2}$

A. f is everywhere differentiable

B. f is everywhere continuous but not differentiable at $x = n\pi$, $n \in Z$

C.f is everywhere continuous but not differentiable at

$$x = \left(2n + \frac{\pi}{2}\right), n \in \mathbb{Z}$$

D. none of the above

Answer: B

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23. If
$$y = \log\left(\frac{1-x^2}{1+x^2}\right)$$
, then $\frac{dy}{dx}$ is equal to
A. $\frac{4x^3}{1-x^4}$
B. $\frac{-4x}{1-x^4}$
C. $\frac{1}{4-x^4}$
D. $\frac{-4x^3}{1-x^4}$

Answer: B

24. If
$$y = \sqrt{\sin x + y}$$
, then $\frac{dy}{dx}$ is equal to
A. $\frac{\cos x}{2y - 1}$
B. $\frac{\cos x}{1 - 2y}$
C. $\frac{\sin x}{1 - 2y}$
D. $\frac{\sin x}{2y - 1}$

Answer: A



25. The derivative of
$$\cos^{-1}(2x^2 - 1)$$
 w.r.t $\cos^{-1}x$ is

A. 2

$$B. \frac{-1}{2\sqrt{1-x^2}}$$

C.
$$\frac{2}{x}$$

D. 1 - x^2

Answer: A



26. If
$$x = t^2$$
 and $y = t^3$, then $\frac{d^2y}{dx^2}$ is equal to

A.
$$\frac{3}{2}$$

B. $\frac{3}{4t}$
C. $\frac{3}{2t}$
D. $\frac{3t}{2}$

Answer: B

27. The value of c in Rolle's theorem for the function $f(x) = x^3$ in the interval $x \in [0, \sqrt{3}]$

A. 1

B. -1 C. $\frac{3}{2}$ D. $\frac{1}{3}$

Answer: A

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28. For the function $f(x) = x + \frac{1}{x}$, $x \in [1, 3]$ the value of c for mean value

theorem is

A. 1

B. $\sqrt{3}$

C. 2

D. None of these

Answer: B

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NCERT Exemplar Problems and Solution (Fillers)

1. Does there exist a function which is continuous everywhere but not

differentiable at exactly two points ? Justify your answer.

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2. Derivative of x^2 w.r.t x^3 is.....

3. If
$$f(x) = |\cos x|$$
, then $f'\left(\frac{\pi}{4}\right)$ is equal to $0 < x < \frac{\pi}{2}$

4. If
$$f(x) = |\cos x - \sin x|$$
, then $f\left(\frac{\pi}{3}\right)$ is equal to

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5. For the curve
$$\sqrt{x} + \sqrt{y} = 1$$
, $\frac{dy}{dx}$ at $\left(\frac{1}{4}, \frac{1}{4}\right)$ is

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6. Does there exist a function which is continuous everywhere but not

differentiable at exactly two points ? Justify your answer.



7. Derivative of
$$x^2$$
 w.r.t x^3 is.....

8. If
$$f(x) = |\cos x|$$
, then $f'\left(\frac{\pi}{4}\right)$ is equal to $0 < x < \frac{\pi}{2}$

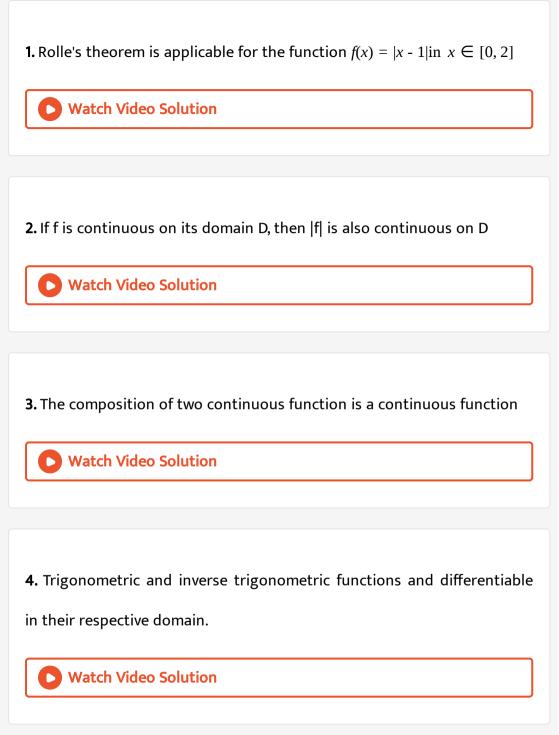
9. If
$$f(x) = |\cos x - \sin x|$$
, then $f'\left(\frac{\pi}{3}\right)$ is equal to

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10. For the curve
$$\sqrt{x} + \sqrt{y} = 1$$
, $\frac{dy}{dx}$ at $\left(\frac{1}{4}, \frac{1}{4}\right)$ is

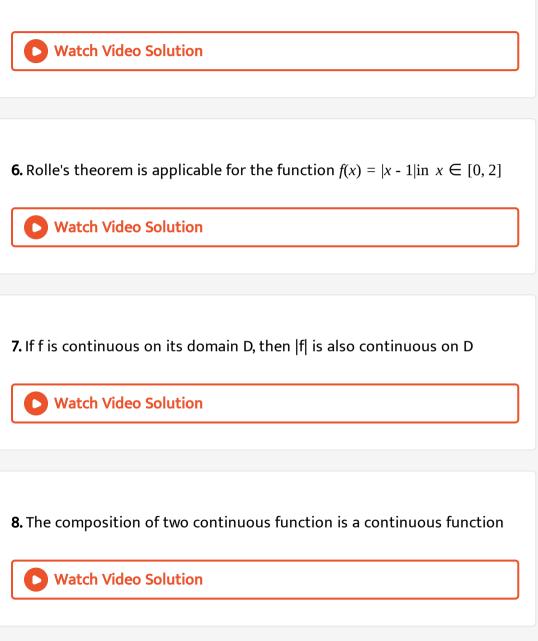
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NCERT Exemplar Problems and Solution (True/False)



5. If f.g is continuous at x=a, then f and g are separately continuous at x=





9. Trigonometric and inverse trigonometric functions and differentiable in

their respective domain.

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10. If f.g is continuous at x=a, then f and g are separately continuous at x=

a.

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Practice Paper - 5 (Section - A)

$$1. \frac{d}{dx} \left(\sqrt{x \sin x} \right) = \dots \quad (0 < x < \pi)$$

A.
$$\frac{x \sin x + \cos x}{\sqrt{x \sin x}}$$

B.
$$\frac{x \cos x}{2\sqrt{x \sin x}}$$

C.
$$\frac{x \cos x + \sin x}{2\sqrt{x \sin x}}$$

D.
$$\frac{1}{2\sqrt{x\sin x}}$$

$$2. \frac{d}{dx} \left(\tan^{-1}x + \cot^{-1}x \right) = \dots$$

A. 0

B.
$$\frac{1}{1+x^2}$$

C. $-\frac{1}{1+x^2}$

D. Does not exist

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3.
$$\frac{d}{dx}(a^a) = \dots (a > 0)$$

A. $a^{a}(1 + \log a)$

B. 0

C. *a*^{*a*}

D. Does not exist

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4.
$$\sqrt{x} + \sqrt{y} = \sqrt{a}$$
 then $\frac{dy}{dx} = \dots$

A.
$$\frac{y}{x}$$

B. $\sqrt{\frac{y}{x}}$
C. $-\sqrt{\frac{y}{x}}$
D. $-\sqrt{\frac{x}{y}}$

5. If
$$2t = v^2$$
 then $\frac{dv}{dt} = \dots$
A. 0
B. $\frac{1}{v}$
C. $\frac{1}{2}$
D. $-\frac{1}{v^2}$

$$\mathbf{6.} \ \frac{d}{dx} \left(\sqrt{x \sin x} \right) = \dots (0 < x < \pi)$$

A.
$$\frac{x\sin x + \cos x}{\sqrt{x\sin x}}$$

B.
$$\frac{x\cos x}{2\sqrt{x\sin x}}$$

C.
$$\frac{x\cos x + \sin x}{2\sqrt{x\sin x}}$$

D.
$$\frac{1}{2\sqrt{x\sin x}}$$

7.
$$\frac{d}{dx} \left(\tan^{-1}x + \cot^{-1}x \right) = \dots$$

B.
$$\frac{1}{1+x^2}$$

C. $-\frac{1}{1+x^2}$

D. Does not exist

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$$\mathbf{8.} \frac{d}{dx} \left(a^a \right) = \dots (a > 0)$$

A. $a^{a}(1 + \log a)$

B. 0

C. *a*^{*a*}

D. Does not exist



9.
$$\sqrt{x} + \sqrt{y} = \sqrt{a}$$
 then $\frac{dy}{dx} = \dots$

A.
$$-\frac{1}{x}$$

B. $\sqrt{\frac{y}{x}}$
C. $-\sqrt{\frac{y}{x}}$
D. $-\sqrt{\frac{x}{y}}$

10. If
$$2t = v^2$$
 then $\frac{dv}{dt} = \dots$.

A. 0
B.
$$\frac{1}{v}$$

C. $\frac{1}{2}$
D. $-\frac{1}{v^2}$

Practice Paper - 5 (Section- B)

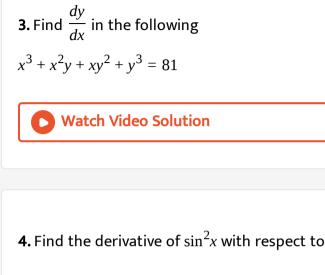
1. If
$$x^y = e^{x-y}$$
, then find $\frac{dy}{dx}$

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2. $f(x) = \begin{cases} kx + 1, & x \le \pi \\ \cos x & x > \pi \end{cases}$ If the function f(x) is continuous at $x = \pi$, then

find the value of k.





4. Find the derivative of $\sin^2 x$ with respect to $e^{\cos x}$

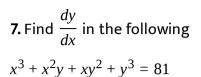
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5. If
$$x^y = e^{x-y}$$
, then find $\frac{dy}{dx}$

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6. $f(x) = \begin{cases} kx + 1, & x \le \pi \\ \cos x & x > \pi \end{cases}$ If the function f(x) is continuous at $x = \pi$, then

find the value of k.



8. Find the derivative of $\sin^2 x$ with respect to $e^{\cos x}$

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Practice Paper -5 (Section-C)

1. Verify Mean Value Theorem, if $f(x) = x^3 - 5x^2 - 3x$ in the interval [a, b],

where a=1 and b=3. Find all $c \in (1, 3)$ for which f'(c)= 0

2. Prove that the greatest integer function defined by f(x) = [x], 0 < x < 3

is not differentiable at x=1 and x=2.



3. If x and y are connected parametrically by the equations without eliminating the parameter, find $\frac{dy}{dx}$ $x = a(\cos\theta + \theta\sin\theta), y = a(\sin\theta - \theta\cos\theta)$

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4. Find
$$\frac{dy}{dx}$$
 in the following
 $y = \sin^{-1}\left(2x\sqrt{1-x^2}\right), -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$

5. Find
$$\frac{dy}{dx}$$
, $y = x^{\sin x} + (\sin x)^{\cos x}$

6. Verify Mean Value Theorem, if $f(x) = x^3 - 5x^2 - 3x$ in the interval [a, b],

where a=1 and b=3. Find all $c \in (1, 3)$ for which f'(c)= 0

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7. Prove that the greatest integer function defined by f(x) = [x], 0 < x < 3

is not differentiable at x=1 and x=2.

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8. If x and y are connected parametrically by the equations without eliminating the parameter, find $\frac{dy}{dx}$ $x = a(\cos\theta + \theta \sin\theta), y = a(\sin\theta - \theta \cos\theta)$

9. Find
$$\frac{dy}{dx}$$
 in the following
 $y = \sin^{-1}\left(2x\sqrt{1-x^2}\right), -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$

10. Find
$$\frac{dy}{dx}$$
, $y = x^{\sin x} + (\sin x)^{\cos x}$

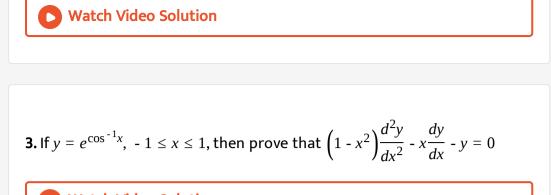
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Practice Paper - 5 (Section-D)

1. If
$$y = e^{\cos^{-1}x}$$
, $-1 \le x \le 1$, then prove that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - y = 0$

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2. Differentiate w.r.t x the function $0 < x < \frac{\pi}{2}$, $\cot^{-1}\left[\frac{\sqrt{1 + \sin x} + \sqrt{1 - \sin x}}{\sqrt{1 + \sin x} - \sqrt{1 - \sin x}}\right]$





4. Prove that :

$$\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right) = \frac{x}{2}, x \in \left(0, \frac{\pi}{4}\right)$$