MATHS

BOOKS - KUMAR PRAKASHAN KENDRA MATHS (GUJRATI ENGLISH)

RELATIONS AND FUNCTIONS

Exercise 1 1

1. Determine whether each of the following relations are reflexive , symmetric and transitive :

Relation R in the set $A = \{1, 2, 3, \dots .13, 14\}$ defined as $R = \{(x, y) : 3x - y = 0\}$

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2. Determine whether each of the following relations are reflexive , symmetric and transitive :

Relation R is the set N of natural numbers defined as R = {(x,y): y = x +5 and x $<$ 4 } .

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3. Determine whether each of the following relations are reflexive , symmetric and transitive :

Relation R in the set A = {1,2,3,4,5,6} as R = {(x,y):y is divisible by x}.

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4. Determine whether each of the following relations are reflexive , symmetric and transitive :

Relation R in the set Z of all integers defined as ={$(x, y) : x - y$ is an integers }

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5. Determine whether each of the following relations are reflexive , symmetric and transitive :

Relation R in the set A of human beings in a town at a particular time given by

(a) $r = \{(x, y) : x \text{ and } y$ works at the same place }

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6. Determine whether each of the following relations are reflexive , symmetric and transitive :

Relation R in the set A of human beings in a town at a particular time given by

R = {(x,y)} : x and y live in the same locality }

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7. Determine whether each of the following relations are reflexive , symmetric and transitive :

Relation R in the set A of human beings in a town at a particular time given by

R = {(x,y)} : x is exactly 7 cm taller than y }

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8. Determine whether each of the following relations are reflexive , symmetric and transitive :

Relation R in the set A of human beings in a town at a particular time given by

$R = \{(x, y)\!: x$ is wife of y }

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9. Determine whether each of the following relations are reflexive , symmetric and transitive :

Relation R in the set A of human beings in a town at a particular time given by

$R = \{(x, y) : x$ is father of y }

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10. Show that the relation R in the set R of real number , defined as $R = \{(a, b) : a \leq b^2\}$ is neither reflexive nor symmetric nor transitive.

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11. Check whether the relation R defined in the set {1,2,3,4,5,6} as R = {(a,b) : b = a +1 } is reflexive , symmetric or transitive.

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12. Show that the relation R is R defined as $R = \{(a, b) : a \leq b\}$ is reflexive and transitive but not symmetric.

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13. Check whether the relation R defined by $R = \{(a, b) : a \leq b^3\}$ is reflexive , symmetric or transitive.

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14. Show that the relation R in the set $\{1, 2, 3\}$ given by $R = \{(1, 2), (2, 1)\}$ is symmetric but neither reflexive nor transitive.

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15. Show that the relation R in the set A of all the books in a library of a college , given by $R = \{(x, y)$: x and y have same number of pages} is an equivalence relation.

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16. Show that the relation R in the set $A = \{1, 2, 3, 4, 5\}$ given by $R\{(a, b) : |a - b|$ is even } , is an equivalence relation . Show that all the elements of {1,3,5} are related to each other all the elements of {2,4} are related to each other . But no element of {1,3,5} is related to any element of {2,4} .

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17. Show that each of the relation R in the set $A = \{x \in Z : 0 \leq x \leq 12\}$ given by

$R = \{(a, b) : |a - b|$ is multiple of 4} is in equivelance.

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18. Show that each of the relation R in the set $A = \{x \in Z : 0 \leq x \leq 12\}$ given by

$R = \{(a, b) : a = b\}$ is an equivalence relation . Find the set of all elements related to 1 each case.

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19. Give an example of relation . Which is

Symmetric but neither reflexive nor transitive.

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20. Give an example of relation . Which is

Transitive but neither reflexive nor symmetric .

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21. Give an example of relation . Which is

Reflexive and symmetric but not transitive .

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22. Give an example of relation . Which is

Reflexive and transitive but not symmetric.

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23. Give an example of relation . Which is

Symmetric and transitive but not reflexive.

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24. Show that the relation R in the set A of points in a plane give by R = {(P,Q) : distance of the point P from the origin is same as the distance of the point Q from the origin} , is an equivalence relation. Further , show that the set equivalence relation . Further , show that the set of all points related to a point $P \neq (0, 0)$ is the circle passing through P with origin as centre.

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25. Show that the relation R defined in the set A of all triangles as $R = \{(T_1, T_2\} T_1$ is similar to T_2 } , is equivalence relation . Consider three right angle triangles T_1 with sides 3,4,5 , T_2 with sides 5,12, 13 and T_3 with sides 6, 8, 10 . Which triangles among T_1, T_2 and T_3 are related ?

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26. Show that the relation R defined in the set A of all polygons as $R = \{(P_1, P_2) : P_1$ and P_2 have same number of sides} , is an equivalence relation . What is the set of all elements in A related to the right angle triangle T with sides 3,4 and 5 ?

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27. Let L be the set of all lines in XY plane and R be the relation in L defined as $R = \{(L_1, L_2\} : L_1$ is parallel to L_2 } . Show that R is an equivalence relation . Find the set of all lines related to the line y = 2x + 4.

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28. Let R be the relation in the set $\{(1, 2, 3, 4\}$ given by $R = \{(1, 2), (2, 2), (1, 1)(4, 4), (1, 3), (3, 3), (3, 2)\}$. Choose the correct answer.

A. R is reflexive and symmetric but not transitive .

B. R is reflexive and transitive but not symmetric.

C. R is symmetric and transitive but not reflexive.

D. R is an equivalence relation.

Answer: B

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29. Let R be the relation on the set N given by $R = \{(a, b) : a = b - 2, b > 6\}$. Choose the correct answer.

A. $(2, 4) \in R$

B. $(3, 8) \in R$

C. $(6, 8) \in R$

D. $(8, 7) \in R$

Answer: C

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Exercise 1 2

1. Show that the function f : $R \rightarrow R$, defined by $f(x) = \frac{1}{x}$ is one - one and onto , where R is the set of all non - zero real number . is the result true, if the domain R is replaced by N with co-domain being same as R ?

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2. Check the injectiveity and surjectivity of the following functions :

$f : N \rightarrow N$ given by $f(x) = x^2$

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3. Check the injectiveity and surjectivity of the following functions :

$f: Z \rightarrow Z$ given by $f(x) = x^2$

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4. Check the injectiveity and surjectivity of the following functions :

$f: R \rightarrow R$ given by $f(x) = x^2$

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5. Check the injectiveity and surjectivity of the following functions :

$f: N \rightarrow N$ given by $f(x) = x^3$

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6. Check the injectiveity and surjectivity of the following functions :

$f: Z \to Z$ given by $f(x) = x^3$

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7. Prove that the Greatest Integer Function $f: R \to R$, given by f(x) = [x] , is neither one - one nor onto , where [x] denotes the greatest integer less than or equal to x.

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8. Show that the Modulus Function $f: R \to R$, given by f(x) = |x| , is neither oneone nor onto , where |x| is x, if x is positive or 0 and |x| is - x , if x is negative.

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9. Show that the Signum Function $f: R \rightarrow R$ given by $f(x) = \begin{cases} 1 & \text{if } \ x > 0 \\ 0 & \text{if } \ x = 0 \\ -1 & \text{if } \ x < 0 \end{cases}$ is neither one - one nor onto.

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10. Let $A = \{1, 2, 3\}, B = \{4, 5, 6, 7\}$ and let $f = \{(1, 4), (2, 5), (3, 6)\}$ be a function from A to B. Show that f is one - one.

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11. In each of the following cases , state whether the function is one - one , onto or bijective. Justify your answer.

$f: R \rightarrow R$ defined by f(x) = $3 - 4x$.

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12. In each of the following cases , state whether the function is one - one , onto or bijective. Justify your answer.

$f: R \rightarrow R$ defined by f(x) = $1 + x^2$

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13. Let A and B be sets. Show that $f: A \times B \rightarrow B \times A$ such that f(a,b) = (b,a) is bijecive function.

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14. Let : $f: N \rightarrow N$ be defined by $f(n) = \begin{cases} \frac{n+1}{2} & \text{if n is odd} \\ \frac{n}{2} & \text{if n is even} \end{cases}$ for all $n \in N$.

State whether the function f is bijective . Justify your answer.

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15. Let A = R - {3} and B = R - {1}. Consider the function $f: A \rightarrow B$ defined by , $f(x) = \left(\frac{x-2}{x-3}\right)$ is f one - one and onto ?

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16. Let $f: R \rightarrow R$ be defined as $f(x) = x^4$. Choose the correct answer.

A. f is one - one onto

B. f is many - one onto

C. f is one - one but not onto

D. f is neither one - one nor onto

Answer: D

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17. Let $f: R \rightarrow R$ be defined as $f(x) = 3x$. Choose the correct answer.

A. f is one - one onto

B. f is many - one onto

C. f is one - one but not onto

D. f is neither one - one nor onto

Answer: A

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Exercise 1 3

1. Let $f: \{1, 3, 4\} \to \{1, 2, 5\}$ and $g: \{1, 2, 5\} \to \{1, 3\}$ be given by $f = \{(1, 2), (3, 5), (4, 1)$ and $g\{(1, 3), (2, 3), (5, 1)\}$. Write down gof.

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2. Let f , g and h be functions from R to R . Show that , (f +g) oh = foh + goh (f.g) oh = (foh)+ (goh)

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3. Find gof and fog , if

$f(x) = |x|$ and $g(x) = |5x - 2|$.

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4. Find gof and fog , if

$f(x) = 8x^3 \text{ and } g(x) = x^{\frac{1}{3}}$

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5. If $f(x) = \frac{4x+3}{6x-4}, x \neq \frac{2}{3}$, show that fof (x) = x, for all $x \neq \frac{2}{3}$. What is the inverse of f ?

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6. State with reason whether following functions have inverse :

$f:\{1, 2, 3, 4\} \rightarrow \{10\}$ with $f:\{(1, 10), (2, 10), (3, 10), (4, 10)\}$

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7. State with reason whether following functions have inverse :

$g:\{5, 6, 7, 8\} \rightarrow \{1, 2, 3, 4\}$ with $g:\{(5, 4), (6, 3), (7, 4), (8, 2)\}$

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8. State with reason whether following functions have inverse :

$h:\{2, 3, 4, 5\} \rightarrow \{7, 9, 11, 13\}$ with $h\{(2, 7), (3, 9), (4, 11), (5, 13)\}$

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9. Show that f : [-1,1] $\rightarrow R$, given by $f(x) = \dfrac{x}{x+2}$ is one - one . Find the inverse of the function $f:[-1, 1] \rightarrow$ Range f.

(Hint: For $y \in$ Range $f, y = f(x) = \frac{x}{x+2}$, for some x in [-1,1] , i.e., $x = \frac{2y}{1-y}$).

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10. Consider $f: R \to R$ given by f(x) = 4x +3. Show that f is invertible. Find inverse of f .

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11. Consider $f: R^{+} \to [4, \infty]$ given by $f(x) = x^2 + 4$ show that f is f invertible with the inverse f^{-1} of given by $f^{-1}(y) = \sqrt{y-4}$ where R^{+} is set of all non - negative real numbers .

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12. Consider $f: R_{+} \to [-5, \infty)$ given by f(x) = $9x^2 + 6x - 5$. Show that f is invertible with $f^{-1}(y) = \left(\frac{\sqrt{y+6}-1}{3}\right)$

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13. Let $f: X \rightarrow Y$ be an invertible function . Show that f has unique inverse .

(Hint : Suppose g_1 and g_2 are two inverse of f. Then for all $y \in Y, (fog_1)(y) = I_Y(y) = (fog_2)(y)$. Use one - one ness of f).

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14. Consider $f: \{1, 2, 3\} \rightarrow \{a, b, c\}$ given by f(1) = a, f(2) and f(3) = c . Find f^{-1} and show that $(f^{-1})^{-1} = f$.

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15. Let $f: X \rightarrow Y$ be an invertible function . Show that the inverse of f^{-1} is f . i.e., $(f^{-1})^{-1} = f$.

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16. If $f: R \to R$ be given by $f(x) = \left(3 - x^3\right)^{\frac{1}{3}}$ then fof (x) is

A. $x^{\frac{1}{3}}$

B. x^3

C. x

D. $\left(3 - x^2\right)$

Answer: C

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17. Let $f: R - \left\{ - \frac{4}{3} \right\} \to R$ be a function defined as $f(x) = \frac{4x}{3x + 4}$.

The inverse of f is the map g : Range $f \to R - \left\{ - \frac{4}{3} \right\}$ given by

A. $g(y) = \frac{3y}{3 - 4y}$

B. $g(y) = \frac{4y}{4 - 3y}$

C. $g(y) = \frac{4y}{3 - 4y}$

D. $g(y) = \frac{3y}{4-3y}$

Answer: B

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18. Determine whether or not each of the definition of $*$ given below gives a binary operation. In the even that $*$ is not a binary operation , give justification for this .

On Z^+ , define $*$ by $a * b = a - b$

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19. Determine whether or not each of the definition of $*$ given below gives a binary operation. In the even that $*$ is not a binary operation , give justification for this .

On Z^+ , define $*$ by $a * b = ab$

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20. Determine whether or not each of the definition of $*$ given below gives a binary operation. In the even that $*$ is not a binary operation , give justification for this .

On R , define $*$ by $a \quad * \quad b = ab^2$

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21. Determine whether or not each of the definition of $*$ given below gives a binary operation. In the even that $*$ is not a binary operation , give justification for this .

On Z^+ , define $*$ by $a \quad * \quad b = |a - b|$

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22. Determine whether or not each of the definition of $*$ given below gives a binary operation. In the even that $*$ is not a binary operation , give justification for this .

On Z^{+} , define $*$ by $a \quad * \quad b = a$

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23. For each opertion $*$ difined below, determine whether $*$ isw binary, commutative or associative.

(i) On Z, define $a * b = a - b$

(ii) On Q, define $a * b = ab + 1$

(iii) On Q, define $a * b = \frac{ab}{2}$

(iv) On Z^{+}, define $a * b = 2^{ab}$

(v) On Z^{+}, define $a * b = a^{b}$

(vi) On $R - \{-1\}$, define $a * b = \frac{a}{b+1}$

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24. For each opertion $*$ difined below, determine whether $*$ isw binary, commutative or associative.

(i) On Z, define $a * b = a - b$

(ii) On Q, define $a * b = ab + 1$

(iii) On Q, define $a * b = \frac{ab}{2}$

(iv) On Z^+, define $a * b = 2^{ab}$

(v) On Z^+, define $a * b = a^b$

(vi) On $R - \{-1\}$, define $a * b = \frac{a}{b+1}$

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25. For each opertion $*$ difined below, determine whether $*$ isw binary, commutative or associative.

(i) On Z, define $a * b = a - b$

(ii) On Q, define $a * b = ab + 1$

(iii) On Q, define $a * b = \frac{ab}{2}$

(iv) On Z^+, define $a * b = 2^{ab}$

(v) On Z^+, define $a * b = a^b$

(vi) On $R - \{-1\}$, define $a * b = \frac{a}{b+1}$

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26. For each opertion $*$ difined below, determine whether $*$ isw binary, commutative or associative.

(i) On Z, define $a*b=a-b$

(ii) On Q, define $a*b=ab+1$

(iii) On Q, define $a*b=\frac{ab}{2}$

(iv) On Z^+, define $a*b=2^{ab}$

(v) On Z^+, define $a*b=a^b$

(vi) On $R-\{-1\}$, define $a*b=\frac{a}{b+1}$

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27. For each operation $*$ defined below, determine , whether $*$ is binary , commutative or associative.

On Z^+ , define $a*b=ab$

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28. For each opertion $*$ difined below, determine whether $*$ isw binary, commutative or associative.

(i) On Z, define $a * b = a - b$

(ii) On Q, define $a * b = ab + 1$

(iii) On Q, define $a * b = \frac{ab}{2}$

(iv) On Z^+, define $a * b = 2^{ab}$

(v) On Z^+, define $a * b = a^b$

(vi) On $R - \{-1\}$, define $a * b = \frac{a}{b+1}$

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29. Consider the binary operation $\wedge$ on the set {1,2,3,4,5} defined by $a \wedge b$ = min {a,b} . Write the operation table of the operation $\wedge$.

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Exercise 1 4

1. Consider a binary operation $*$ on the set {1,2,3,4,5} given by the following multiplication table.

(i) Compute $(2*3)*4$ and $2*(3*4)$

(ii) Is $*$ commutative ?

(iii) Compute $(2*3)*(4*5)$

(Hint: use the following table)

*	1	2	3	4	5
1	1	1	1	1	1
2	1	2	1	2	1
3	1	1	3	1	1
4	1	2	1	4	1
5	1	1	1	1	5

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2. Let $*$ be the binary operation on the set {1,2,3,4,5} defined a^*b = H.C.F of a and b Is the operation $*$ same as the operation $*$ defined in Exercise 4 above ? Justify your answer .

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3. Let $*$ be the binary operation on N given by $a * b$ = L.C.M. of a and b. Find

$5^*7, 20^*16$

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4. Let $*$ be the binary operation on N given by $a * b$ = L.C.M. of a and b. Find

Is $*$ commutative ?

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5. Let $*$ be the binary operation on N given by $a * b$ = L.C.M. of a and b.

Find

Is $*$ associative ?

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6. Let $*$ be the binary operation on N given by $a * b$ = L.C.M. of a and b.

Find

Find the identity of $*$ in N.

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7. Let $*$ be the binary operation on N given by $a * b$ = L.C.M. of a and b.

Find

Which elements of N are invertible for the operation $*$?

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8. Is $*$ defined on the set $\{1, 2, 3, 4, 5\}$ by $a^{*}b$ = L.C.M. of a and b a binary operation ? Justify your answer.

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9. Let $*$ be the binary operation on N defined by $a^{*}b$ = H.C.F of a and b . Is $*$ commutative ? Is $*$ associative ? Does there exist identity for this binary operation on N ?

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10. Let $*$ be a binary operation on the set Q of rational numbers as follows :

$a^{*}b = a - b$

Find which of the binary operations are commutative and which are associative.

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11. Let $*$ be a binary operation on the set Q of rational numbers as follows :

$a^*b = a^2 + b^2$

Find which of the binary operations are commutative and which are associative.

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12. Let $*$ be a binary operation on the set Q of rational numbers as follows :

$a^*b = a + ab$

Find which of the binary operations are commutative and which are associative.

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13. Let $*$ be a binary operation on the set Q of rational numbers as follows :

$a^*b = (a-b)^2$

Find which of the binary operations are commutative and which are associative.

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14. Let $*$ be a binary operation on the set Q of rational numbers as follows :

$a^*b = \frac{ab}{4}$

Find which of the binary operations are commutative and which are associative.

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15. Let $*$ be a binary operation on the set Q of rational numbers as follows :

$a^*b = ab^2$

Find which of the binary operations are commutative and which are associative.

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16. Find which of the operations given above has identity.

$a*b = a - b$

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17. Find which of the operations given above has identity.

$a*b = a^2 + b^2$

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18. For which values of p does the pair of equations given below has unique solution ?

$4x + py + 8 = 0$

$2x + 2y + 2 = 0$

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19. Find which of the operations given above has identity.

$a*b = (a - b)^2$

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20. Find which of the operations given above has identity.

$a*b = \frac{ab}{4}$

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21. Find which of the operations given above has identity.

$a*b = ab^2$

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22. L $A = N \times N$ and $*$ be the binary operation on A defined by $(a, b)^*(c, d) = (a + c, b + d)$ Show that $*$ is commutative and associative . Find the identity element for $*$ on A , if any.

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23. State whether the following statements are true or false . Justify .

For an arbitrary binary operation $*$ on a set N, $a^*a = a \forall a \in N$.

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24. State whether the following statements are true or false . Justify .

If $*$ is commutative binary operation on N, then $a^*(b^*c) = (c^*b)^*a$.

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25. Consider a binary operation $*$ on N defined as $a*b = a^3 + b^3$.

Choose the correct answer.

A. Is $*$ both associative and commutative ?

B. Is $*$ commutative but not associative ?

C. Is $*$ associative but not commutative ?

D. Is $*$ neither commutative nor associative ?

Answer: B

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Miscellaneous Exercise 1

1. Let $f: R \rightarrow R$ be defined as $f(x) = 10x + 7$. Find the function $g: R \rightarrow R$ such that gof = fog = I_g

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2. Let $f: W \to W$ be defined as f(n) = n - 1, if n is odd and f(n) = n + 1 , if n even. Show that f is invertible. Find the inverse of f. Here, W is the set all whole numbers.

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3. If $f: R \to R$ is defined by $f(x) = x^2 - 3x + 2$, find f(f(x)).

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4. Show that the function $f: R \to \{x \in R: -1 < x < 1\}$ defined by $f(x) = \frac{x}{1 + |x|}, x \in R$ is one one and onto function.

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5. Show that the function $f: R \to R$ given by $f(x) = x^3$ is injective.

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6. Give examples of two functions $f: N \rightarrow Z$ and $g: Z \rightarrow Z$ such that gof is injective but g is not injective.

(Hint : Consider f(x) = x and g(x) = |x|) .

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7. Give examples of two function $f: N \rightarrow N$ and $g: N \rightarrow N$ such that gof is onto but f is not onto. (Hint: Consider f(x) = x+1 " and "g(x) = {x-1 if x>1 1 if x=1.

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8. Given a non empty set X , consider P(X) which is the set of all subsets of X. Define the relation R in P(X) as follows : For subsets A , B in P(X) ARB if and only if $A \subset B$. Is R an equivalence relation on P(X) ? Justify your answer.

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9. Given a non - empty set, X , consider the binary operation $*: P(X) \times P(X) \rightarrow P(X)$ given by $A * B = A \cap B, \forall A, B$ in P(X) , where P(X) is the power set X. Show that X is the identity element for this operation and X is the only invertible element in P(X) with respect to the operation $*$.

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10. Find the number of all onto functions from the set {1,2,3,.......,n} to itself.

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11. Let S = {a,b,c} and T = {1,2,3} . Find F^{-1} of the following functions F from S to T , if it exists .

$F = \{(a, 3), (b, 2), (c, 1)\}$

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12. Let S = {a,b,c} and T = {1,2,3} . Find F^{-1} of the following functions F from S to T , if it exists .

$F = \{(a, 2), (b, 1), (c, 1)\}$

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13. Consider the binary operations $* R \times R \to R$ and $o: R \times R \to R$ defined as $a * b|a - b|$ and $\text{a o b} = a, \forall a, b \in R.$ Show that $*$ is commutative but not associative , o is associative but not commutative. Further, show that $\forall a, b, c \in R, a * (\text{b o c}) = (a * b)o(a * c)$. [If it is so , we say that the operation $*$ distributes over the operation o] . Does o distribute over $*$? Justify your answer.

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14. Given a non - empty set X , let $* : P(X) \times P(X) \to P(X)$ be defined as $A * B = (A - B) \cup (B - A), \forall A, B \in P(X)$. Show that the empty set ϕ is the identity for the operation $*$ and all the elements A of P(X) are invertible with $A^{-1} = A$. (Hint : $(A - \phi) \cup (\phi - A) = a$ and $(A - A) \cup (A - A) = A * A = \phi$)

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15. Define a binary operation $*$ on the set $\{0, 1, 2, 3, 4, 5\}$ as $a * b = \begin{cases} a + b, & \text{if } a + b < 6 \\ a + b - 6, & \text{If } a + b \geq 6 \end{cases}$ Show that zero is the identity for this operation and each element $a \neq 0$ of the set is invertible with 6 - a being the inverse of a.

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16. Let $A = \{-1, 0, 1, 2\}, B = \{-4, -2, 0, 2\}$ and $f, g : A \to B$ be functions defined $f(x) = x^2 - x, x \in R$ and $g(x) = 2\left|x - \frac{1}{2}\right| - 1, x \in R$. Are f and g equal ? Justify your answer.

(Hint : One may note that two functions f : $A \rightarrow B$ and $g: A \rightarrow B$ such that f (a) = g(a) $Aa \in A,$ are called equal functions).

Watch Video Solution

17. Let A = {1,2,3}. Then number of relations containing (1,2) and (1,3) which are reflexive and symmetric but not transitive is

A. 1

B. 2

C. 3

D. 4

Answer: A

Watch Video Solution

18. Let A = {1,2,3}. Then number of equivalence relations containing (1,2) is

A. 1

B. 2

C. 3

D. 4

Answer: B

Watch Video Solution

19. Let $f: R \rightarrow R$ be the Signum Function defined as $f(x) = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}$ and $g: R \rightarrow R$ be the Greatest Integer Function given by g(x) = [x] , where [x] is greatest integer less than or equal to x. Then , does fog and gof coincide in (0,1] ?

A. Yes

B. No

C. Nothing can be said

D. Composite function does not exists

Answer: B

Watch Video Solution

20. Number of binary operations on the set {a,b} are

A. 10

B. 16

C. 20

D. 8

Answer: B

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Practice Work

1. The relation R defined in the set of real number R is as follow :

$R\{(x, y) : x - y + \sqrt{2}$ is an irrational number}

Is R transitive relation ?

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2. Let R be relation defined on the set of natural number N as follows : $R = \{(x, y) : x \in N, y \in N, 2x + y = 41\}$. Find the domian and range of the relation R . Also verify whether R is reflexive, symmetric and transitive.

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3. $A = \{(1, 2, 3, \ldots\ldots 10\}$ The relation R defined in the set A as R $= \{(x, y) : y = 2x\}$. Show that R is not an equivalence relation.

Watch Video Solution

4. The relation R difined the set Z as $R = \{(x, y) : x - y \in Z\}$ show that R is an equivalence relation.

Watch Video Solution

5. Show that the relation R defined by $(a, b)R(c, d) \Rightarrow a + d = b + c$ on the set $N \times N$ is an equivalence relation.

Watch Video Solution

6. R is relation in $N \times N$ as (a,b) R (c,d) $\Leftrightarrow ad = bc$. Show that R is an equivalence relation.

Watch Video Solution

7. The relation R defined in the set N of natural number as $\forall n, \min N$ if on division by 5 each of the integers n and m leaves the remainder less than 5. Show that R is equivalence relation. Also obtain the pairwise disjoint subset determined by R.

Watch Video Solution

8. Find the domain and range of the following function :

$f: R \to R, f(x) = -|x|$

Watch Video Solution

9. Find the domain and range of the following function :

$f: R \to R, f(x) = \frac{x^2 - 1}{x - 1}, x \neq 1$

Watch Video Solution

10. Find the domain and range of the following function :

$f: R \to R, f(x) = \frac{1}{1 - x^2}, x \neq \pm 1$

Watch Video Solution

11. Find the domain and range of the following function :

$f: R \to R, f(x) = \sqrt{9 - x^2}$

Watch Video Solution

12. Find the domain and range of the following function :

$f: R \to R, f(x) = \frac{2}{x - 2}$

Watch Video Solution

13. $f: R \to R, f(x) = \begin{cases} 12x + 5 & x > 1 \\ x - 4 & x \leq 1 \end{cases}$ then find $f(0), f\left(-\frac{1}{2}\right), f(3), f(-5)$.

Watch Video Solution

14. Check the injectivity and surjectivity of the following functions .

$f: R \to R, f(x) = x^2 + 7$

Watch Video Solution

15. Check the injectivity and surjectivity of the following functions .

$f: R \to R, f(x) = x^3$

Watch Video Solution

16. Check the injectivity and surjectivity of the following functions .

$f: R \to R, f(x) = x^2 - 2$

Watch Video Solution

17. Show that the function $f: R \to \{x \in R: -1 < x < 1\}$ defined by $f(x) = \frac{x}{1 + |x|}, x \in R$ is one one and onto function.

Watch Video Solution

18. $f: Z \to Z, f(n) = \begin{cases} (n+2) & \text{if n is even} \\ (2n+1) & \text{if n is odd} \end{cases}$

State whether the function f is one - one and onto .

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19. $f: N \times N \to N, f((\text{m,n})) = m + n$. If f one one and onto ?

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20. Show that $f: R \to R, f(x) = \frac{x}{x^2+1}$ is not one one and onto function.

Watch Video Solution

21. $f: R \to R, f(x) = x^2 + 1$. Find the preimage of 17 and -3.

Watch Video Solution

22. $f: R \to R$, $f(x) = \begin{cases} 2x & x > 3 \\ x^2 & 1 < x \leq 3 \\ 3x & x \leq 1 \end{cases}$ then find f (-1) + f(2) + f(4) .

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23. Is $g = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$ a function, justify. If this is described by the relation, $g(x) = \alpha x + \beta$, then what values should be assigned to α and β ?

Watch Video Solution

24. The functions f and g are defined as follow : $f = \{(1, 2), (3, 5), (4, 1)\}$ and $g = \{(2, 3), (5, 1), (1, 3)\}$. Find the range of f and g . Also find the composition function fog and gof.

Watch Video Solution

25. For functions $f: A \to B$ and $g: B \to A, gof = I_A$. Prove that f is one one and g onto functions .

Watch Video Solution

26. $f: R \to R, f(x) = x^2 + 2$ and $g: R \to R, g(x) = \frac{x}{x-1}$ then find fog and gof.

Watch Video Solution

27. $f: N \to R, f(x) = 4x^2 + 12x + 5$. Show that $f: N \to R$ is invertible function . Find the inverse of f.

Watch Video Solution

28. f and g are real valued function $f(x) = x^2 + x + 7, x \in R$ and $g(x) = 5x - 3, x \in R$. Find fog and gof. Also find $(fog)(2)$ and $(gof)(1)$.

Watch Video Solution

29. If f is greatest integer function and g is a modulus functions the find .

$(gof)\left(-\frac{1}{3}\right)-(fog)\left(-\frac{1}{3}\right)$.

Watch Video Solution

30. $f:R\to R, f(x)=\frac{x}{\sqrt{1+x^2}}, \forall x\in R$. Then find (fofof) (x).

Watch Video Solution

31. $f:Z\to Z$ and $g:Z\to Z$. Defined as $f(n)=3n$ and $g(n)=\begin{cases}\frac{n}{3}, & \text{If n is a multiple of 3}\\ 0, & \text{If n is not a multiple of 3}\end{cases}$ $\forall n\in Z$ Then show that $gof=I_z$ but $fog\neq I_z$

Watch Video Solution

32. $f: R \to R$ be defined by $f(x) = \frac{x}{2} + 3$, $g: R \to R$ be defined by g(x) = $2x - K$. If fog = gof then find the value of K.

Watch Video Solution

33. $f: Z \to Z$ and $g: Z \to Z$ are defined as follow :

$f(n) = \begin{cases} n+2 & \text{n even} \\ 2n-1 & \text{n odd} \end{cases}$, $g(n) = \begin{cases} 2n & \text{n even} \\ \frac{n-1}{2} & \text{n odd} \end{cases}$ Find fog and gof.

Watch Video Solution

34. $*$ is a binary operation on the set Q.

$a*b = \frac{2a+b}{4}$ then find $2*3$.

Watch Video Solution

35. $*$ is a binary operation on the set Q.

$a^*b = a + 12b + ab$ then find $2^*\frac{1}{3}$

Watch Video Solution

36. $*$ is a binary operation on the set Q.

$a^*b = \frac{a}{2} + \frac{b}{3}$ then find $\frac{1}{2}*\frac{4}{5}$.

Watch Video Solution

37. $*$ is a binary operation o Z. If $x^*y = x^2 + y^2 + xy$ then find $[(1^*2) + (0^*3)]^2$.

Watch Video Solution

38. $*$ be a binary operation on R defined by

$a^*b = \frac{a}{4} + \frac{b}{7}, a, b \in R.$

Show that $*$ is not commutative and associative.

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39. Show that addition and multiplication are associative binary operation on R. But subtraction and division is not associative on R.

Watch Video Solution

40. Find the identity element , if it exists for the following operation . Also find the inverse if it exists.

On R defined $a^*b = \sqrt{a^2 - b^2}, |a| > |b|$.

Watch Video Solution

41. Find the identity element , if it exists for the following operation . Also find the inverse if it exists.

On Z defined $a^*b = a + b - 2$.

Watch Video Solution

42. Find the identity element , if it exists for the following operation . Also find the inverse if it exists.

On R - {1} defined $a^{*}b = a + b - ab.$

Watch Video Solution

43. Find the identity element , if it exists for the following operation . Also find the inverse if it exists.

On Q -{0} defined $a^{*}b = \frac{ab}{2}$.

Watch Video Solution

44. Find the identity element , if it exists for the following operation . Also find the inverse if it exists.

On Q - {-1} defined $a^{*}b = a + b + ab.$

Watch Video Solution

45. Find the identity element , if it exists for the following operation . Also find the inverse if it exists.

On P(X) defined $A^*B = A \cap B$, where $X \ne \phi$.

Watch Video Solution

46. Find the identity element , if it exists for the following operation . Also find the inverse if it exists.

On P(X) defined $A^*B = A \cup B$, where $X \ne \phi$.

Watch Video Solution

47. On R - {-1}, a binary operation $*$ defined by $a^*b = a + b + ab$ then find a^{-1}.

Watch Video Solution

48. $*$ be a binary operation on a set $\{0, 1, 2, 3, 4\}$ defined by

$$a * b = \begin{cases} a+b & \text{if } \ a+b < 6 \\ a+b-6 & \text{if } \ a+b \geq 6 \end{cases}$$

Then find identity element of $*$.

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49. On Z $*$ defined by $a * b = a + b + 1$. Is $*$ associative ? Find identity element and inverse if it exists.

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50. $*$ be binary operation defined on a set R by $a * b = a + b - (ab)^2$. Show that $*$ is commutative, but it is not associative. Find the identity element for $*$.

Watch Video Solution

51. A binary operation $*$ be defined on the set R by $a * b = a + b + ab$. Show that $*$ is commutative, and it is also Associative.

Watch Video Solution

52. Show that if $f: A \to B$ and $g: B \to C$ are onto, then $gof: A \to C$ is also onto.

Watch Video Solution

53. Show that if $f: A \to B$ and $g: B \to C$ are one- one, then gof: $A \to C$ is also one-one.

Watch Video Solution

54. $f: R \to R,\ f(x) = \cos x$ and $g: R \to R,\ g(x) = 3x^2$ then find the composite functions gof and fog.

Watch Video Solution

55. Check the injectivity and surjectivity of the following function .

$$f: R \to R,\ f(x) = \begin{cases} -x+1 & x \geq 0 \\ x^2 & x < 0 \end{cases}$$

Watch Video Solution

56. Check the injectivity and surjectivity of the following function .

$$f: R \to R,\ f(x) = \begin{cases} 2x+1 & x \geq 0 \\ x^2 & x < 0 \end{cases}$$

Watch Video Solution

57. Check the injectivity and surjectivity of the following function .

$$f: R \times R - \{0\} \to R,\ f(x, y) = \frac{x}{y}$$

Watch Video Solution

58. Check the injectivity and surjectivity of the following function .

$f: [-1, 1] \rightarrow [-1, 1], f(x) = x|x|$

Watch Video Solution

59. Check the injectivity and surjectivity of the following function .

$f: N \rightarrow N \cup \{0\}, f(n) = n + (-1)^n.$

Watch Video Solution

60. Check the injectivity and surjectivity of the following function .

$f: N - \{1\} \rightarrow N,$ f(n) = Greatest prime factor of n .

Watch Video Solution

61. $f: R \rightarrow (-1, 1), f(x) = \frac{10^x - 10^x}{10^x + 10^{-x}}$. If inverse of f^{-1} exists then find it .

Watch Video Solution

62. $f: R^{+} \cup \{0\} \rightarrow R^{+} \cup \{0\}, f(x) = \sqrt{x}$.

$g: R \rightarrow R, g(x) = x^2 - 1$ then find fog .

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63. $f: R - \left\{\frac{2}{3}\right\} \rightarrow R, f(x) = \frac{4x+3}{6x-4}$. Prove that (fof) (x) = x , what is about f^{-1} ?

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64. A = {1,2,3,4} , B = {1,5,9,11,15,16}

$f = \{(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)\}$

Is f a relation from A to B ?

Give reason for your answer.

Watch Video Solution

65. A = {1,2,3,4} , B = {1,5,9,11,15,16}

$f = \{(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)\}$

Is f a function from A to B ?

Give reason for your answer.

Watch Video Solution

66. Let f be the subset of $Z \times Z$ defined by $f = \{(ab, a + b) : a, b \in Z\}$.

Is f a function from Z to Z? Justify your answer.

Watch Video Solution

Textbook Based Mcqs

1. If a set A has m elements and a set B has n elements then the number of relation from a to B is ....

A. 2^{m+n}

B. 2^{mn}

C. $m+n$

D. mn

Answer: B

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2. A relation R on a finite set having n elements is reflexive. If R has m pairs then ....

A. $m \geq n$

B. $m \leq n$

C. $m=n$

D. None of these

Answer: A

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3. x and y are real numbers . If $xRy \Leftrightarrow x - y + \sqrt{5}$ is on irrational number then R is .... Relation .

A. Reflexive

B. Symmetric

C. Transitive

D. None of these

Answer: A

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4. A = {1,2,3,4} . A relation R is A is given by $F = \{(2, 2), (3, 3), (4, 4), (1, 2)\}$. Then R is relation.

A. Reflexive

B. Symmetric

C. Transitive

D. None of these

Answer: C

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5. A relation R is form set A to B , and a relation S is from set B to C . Then relation SOR is from ....

A. Set C to A

B. Set A to C

C. Does not exist

D. None of these

Answer: B

Watch Video Solution

6. Relation $R = \{(4, 5), (1, 4), (4, 6), (7, 6), (3, 7)\}$ then $R^{-1}OR =$ ....

A. $\{(1, 1), (4, 4), (7, 4), (4, 7), (7, 7)\}$

B. $\{(1, 1), (4, 4), (4, 7), (7, 4), (7, 7), (3, 3)\}$

C. $\{(1, 5), (1, 6), (3, 6)\}$

D. None of these

Answer: B

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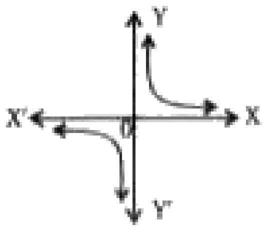
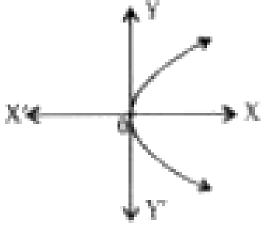
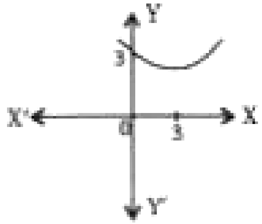
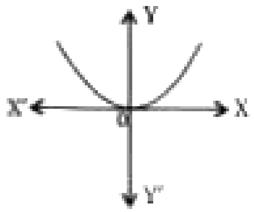
7. Which of the graphs is not a graph of functions ?



A.

B. 

C. 

D. 

Answer: B

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8. If $f(1) = 1, f(n+1) = 2f(n) + 1, n \geq 1$ then f(n) = ...

A. $2^n + 1$

B. 2^n

C. $2^n - 1$

D. $2^{n-1} - 1$

Answer: C

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9. A function $y = f(x)$ satisfies the condition $f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2} (x \ne 0)$ then f(x) = ...

A. $-x^2 + 2$

B. $x^2 - 2$

C. $x^2 - 2, x \in R - \{0\}$

D. $x^2 - 2, |x| \in [2, \infty)$

Answer: D

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10. If $f(x+ay, x-ay) = axy$ then f(x,y) = ...

A. xy

B. $x^2 - a^2y^2$

C. $\dfrac{x^2 - y^2}{4}$

D. $\dfrac{x^2 - y^2}{a^2}$

Answer: C

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11. For function $f(x) = \dfrac{\alpha x}{x+1}, x \neq -1$ if fof(x) = x then $\alpha =$ ....

A. $\sqrt{2}$

B. -1

C. $\dfrac{1}{2}$

D. $-\sqrt{2}$

Answer: B

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12. For real valued functions f and g, f(x) = 2sin $\left(\frac{\pi}{x}\right)$ and $g(x) = \sqrt{x}$.

Then fog(4) - gof (6) = ...

A. 0

B. $\frac{1}{2}$

C. 1

D. $\frac{\sqrt{3}}{2}$

Answer: C

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13. The domain of the real value function

$f(x) = \sqrt{5 - 4x - x^2} + x^2 \log(x + 4)$ is ....

A. $-5 \le x \le 1$

B. $-5 \le 4 \text{ and } n \ge 1$

C. $-4 < x \le 1$

D. ϕ

Answer: C

Watch Video Solution

14. The domian of $\sin^{-1}\left[\log_3\left(\frac{x}{3}\right)\right]$ is ....

A. [1,9]

B. $[-1, 9]$

C. $[-9, 1]$

D. $[-9, -1]$

Answer: A

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15. Range of the function $f(x) = \frac{x^2 + x + 2}{x^2 + x + 1}$ is......

A. $(1, \infty)$

B. $\left(1, \frac{11}{7}\right)$

C. $\left(1, \frac{7}{3}\right)$

D. $\left(1, \frac{7}{5}\right)$

Answer: C

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16. If g(x) $= x^2 + x - 2$ and $\frac{1}{2}(gof)(x) = 2x^2 - 5x + 2$ then f(X) =........

A. $2x - 3$

B. $2x + 3$

C. $2x^2 + 3x + 1$

D. $2x^2 - 3x - 1$

Answer: B

Watch Video Solution

17. $g(x) = 1 + \sqrt{x}$ and $f(g(x)) = 3 + 2\sqrt{x} + x$ then f(x) = ...

A. $1 + 2x^2$

B. $2 + x^2$

C. $1 + x$

D. $2 + x$

Answer: B

Watch Video Solution

18. If real function f(x) $= (x+1)^2$ and $g(x) = x^2 + 1$ then (fog) (-3) = ...

A. 121

B. 112

C. 211

D. 111

Answer: A

Watch Video Solution

19. $f(x) = \cot^{-1} x : R^+ \to (0, \pi)$ and $g(x) = 2x - x^2 : R \to R$ then the range of f(g(x)) is ....

A. $\left(0, \frac{\pi}{2}\right)$

B. $\left(0, \frac{\pi}{4}\right)$

C. $\left[\frac{\pi}{4}, \frac{\pi}{2}\right)$

D. $\left\{\frac{\pi}{4}\right\}$

Answer: C

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20. The domian of f is [-5,7] and g(x) = |2x+5| then the domian of (fog) (x) is ....

A. $[-4, 1]$

B. $[-5, 1]$

C. $[-6, 1]$

D. None of these

Answer: C

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21. A set A has 3 elements and a set B has 4 elements . The number of one one function defined from set A to B is ....

A. 144

B. 12

C. 24

D. 64

Answer: C

Watch Video Solution

22. $f: R \to R, f(x) = (x-1)(x-2)(x-3)$ then f is ....

A. One - one but not onto.

B. Onto but not one - one

C. One - one and onto.

D. Neither one one nor onto.

Answer: B

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23. $f: N \to N, f(n) = (n+5)^2, n \in N$, then the function f is ....

A. Neither one one nor onto

B. One one and onto

C. One one but not onto

D. Onto but not one one.

Answer: B

Watch Video Solution

24. $f: [0, \infty) \to [0, \infty), f(x) = \dfrac{x}{1+x}$ then the function f is ....

A. One one and onto

B. One one but not onto

C. Onto but not one one

D. Neither one one nor onto.

Answer: B

Watch Video Solution

25. $f(x)=\frac{x^3}{3}+\frac{x^2}{2}+ax+b, \forall x \in R.$ If (x) is one one function then the minimum value of a is ....

A. $\frac{1}{4}$

B. 1

C. $\frac{1}{2}$

D. $\frac{1}{8}$

Answer: A

Watch Video Solution

26. $f(x) = x^2 - 2x - 1, \forall x \in R, f : (-\infty, \infty] \to [b, \infty)$ is one one and onto function then $b =$ ....

A. -2

B. -1

C. 0

D. 1

Answer: B

Watch Video Solution

27. $f(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}} + 2$. The inverse of f(x) is ....

A. $\log_e\left(\dfrac{x-2}{x-1}\right)^{\frac{1}{2}}$

B. $\log_e\left(\dfrac{x-1}{3-x}\right)^{\frac{1}{2}}$

C. $\log_e\left(\dfrac{x}{2-x}\right)^{\frac{1}{2}}$

D. $\log_e\left(\frac{x-1}{x+1}\right)^{-\frac{1}{2}}$

Answer: B

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28. $f:(2,4)\to(1,3),\ f(x)=x-\left[\frac{x}{2}\right]$, where [.] is a greatest integer function then $f^{-1}(x)$ = ...

A. 2x

B. $x+\left[\frac{x}{2}\right]$

C. $x+1$

D. does not exist

Answer: C

Watch Video Solution

29. $f:[2,\infty)\to y, f(x)=x^2-4x+5$ is a one and Onto function . If $y\in[a,\infty)$ then the value of a is ....

A. 2

B. 1

C. $-\infty$

D. -1

Answer: B

Watch Video Solution

30. $f:N\to N, f(x)=x+(-1)^{x-1}$ then $f^{-1}(x)=$ ....

A. xy

B. $x-1$

C. $x-(-1)^{x-1}$

D. $x+(-1)^{x-1}$

Answer: D

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31. $a > 1$ is a real number $f(x) = \log_a x^2$, where $x > 0$ If $f^{-1}(x)$ is a inverse of f(x) and b and c are real numbers then $f^{-1}(b + c)$ = ...

A. $f^{-1}(b) . f^{-1}(c)$

B. $f^{-1}(b) + f^{-1}(c)$

C. $\frac{1}{f(b +)}$

D. None of these

Answer: A

Watch Video Solution

32. $f : R \to R, f(x) = 2x + |\cos x|$ then f is .... function .

A. One one and onto

B. One one but not onto

C. Neither one one nor onto

D. Not one one but onto

Answer: A

Watch Video Solution

33. The number of onto function from set $\{1, 2, 3, 4\}$ to $\{3, 4, 7\}$ is ....

A. 18

B. 36

C. 64

D. None of these

Answer: B

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34. Match the Section (A) with the Section (B) properly.

Section (A)		Section (B)	
(1)	$f(x) = \sin(\tan^{-1} x)$	(A)	$f^{-1}(x) = -\log_2(1-x)$
(2)	$f(x) = 1 - 2^{-x}$	(B)	$f^{-1}(x) = (5 - x^2)^{\frac{1}{2}}$
(3)	$f(x) = 2^{\frac{x}{x-1}}$	(C)	$f^{-1}(x) = \frac{x}{\sqrt{1-x^2}}$
(4)	$f(x) = (5 - x^2)^{\frac{1}{2}}$	(D)	$f^{-1}(x) = \frac{\log_2 x}{\log_2 x - 1}$

A. $1 \rightarrow A, 2 \rightarrow D, 3 \rightarrow B, 4 \rightarrow C$

B. $1 \rightarrow C, 2 \rightarrow A, 3 \rightarrow D, 4 \rightarrow B$

C. $1 \rightarrow A, 2 \rightarrow C, 3 \rightarrow B, 4 \rightarrow D$

D. $1 \rightarrow C, 2 \rightarrow B, 3 \rightarrow D, 4 \rightarrow A$

Answer: B

View Text Solution

35. $f:[0,3] \to [1,29]$, $f(x) = 2x^3 - 15x^2 + 36x + 1$ then f is .... function.

A. One one and onto

B. One one but not onto

C. Neither one one nor onto

D. Not one one but onto

Answer: B

Watch Video Solution

36. f(x,y) = $(\max(x, y))^{(\min(x,y))}$ and g(x,y)= max (x,y) - min (x,y) then $f\left(g\left(-1, -\frac{3}{2}\right), g(-4, -1.75)\right) = \ldots\ldots\ldots$

A. 0.5

B. -0.5

C. 1

D. 1.5

Answer: D

Watch Video Solution

37. Let A = {1,2,3}. Then number of equivalence relations containing (1,2) is

A. 1

B. 2

C. 3

D. 8

Answer: B

Watch Video Solution

38. S is defined in Z by $(x, y) \in S \Leftrightarrow |x - y| \leq 1.$ S is ....

A. Reflexive and transitive but not symmetric.

B. Reflexive and symmetric but not transitive.

C. symmetric and transitive but not reflexive.

D. an equivalence relation

Answer: B

Watch Video Solution

39. If S is defined on R by (x,y) $\in R \Leftrightarrow xy \geq 0$. Then S is ....

A. an equivalence relation

B. reflexive only

C. symmetric only

D. transitive only

Answer: A

Watch Video Solution

40. Which of the following defined on Z is not an equivalence relation ?

A. $(x, y) \in S \Leftrightarrow x \geq y$

B. $(x, y) \in S \Leftrightarrow x = y$

C. $(x, y) \in S \leftrightarrow x - y$ is a multiple of 3

D. $(x, y) \in S$ if |x-y| is even

Answer: A

Watch Video Solution

41. If $a * b = \frac{ab}{3}$ on Q^{+} then the inverse of $a (a \neq 0)$ for $*$ is ......

A. $\frac{3}{a}$

B. $\frac{9}{a}$

C. $\frac{1}{a}$

D. $\frac{2}{a}$

Answer: B

Watch Video Solution

42. The number of binary operation on $\{1, 2, 3, \ldots\ldots, n\}$ is ....

A. 2^n

B. n^{n^2}

C. n^3

D. n^{2n}

Answer: B

Watch Video Solution

43. If $a * b = a + b$ on R - {1} , then a^{-1} is ....

A. a^3

B. $\frac{1}{a}$

C. $\frac{-a}{a+1}$

D. $\frac{1}{a^2}$

Answer: C

Watch Video Solution

44. For $a * b = a + b + 10$ on Z , the identity element is ....

A. 0

B. -5

C. -10

D. 1

Answer: C

Watch Video Solution

45. $f: R - \{q\} \rightarrow R - \{1\}, f(x) = \frac{x-p}{x-q}, p \neq q$, then f is ....

A. one - one and onto .

B. many - one and not onto .

C. one - one and not onto .

D. many - one and onto .

Answer: A

Watch Video Solution

46. Check the injectivity and surjectivity of the following function .

$f: [-1, 1] \rightarrow [-1, 1], f(x) = x|x|$

A. one - one and onto .

B. many - one and onto .

C. many - one and not onto .

D. one - one and not onto.

Answer: A

Watch Video Solution

47. $f:\left[-\frac{\pi}{2},\frac{\pi}{2}\right]\to[-1,1]$ is a bijection , if ......

A. $f(x)=|x|$

B. $f(x)=\sin x$

C. $f(x)=x^2$

D. $f(x)=\cos x$

Answer: B

Watch Video Solution

48. $f:R\to R, f(x)=x^2+2x+3$ is ....

A. one - one but not onto.

B. onto but not one - one

C. onto but not one one

D. many - one and not onto .

Answer: D

Watch Video Solution

49. If $a * b = a^2 + b^2$ on Z , then $*$ is ....

A. commutative and associative.

B. commutative and not associative.

C. not commutative and associative.

D. neither commutative nor associative.

Answer: B

Watch Video Solution

50. If $a * b = a + b - ab$ on Q^{+} , then the identity and the inverse of a for $*$ are respectively ....

A. $0 \text{ and } \frac{a}{a-1}$

B. $1 \text{ and } \frac{a-1}{a}$

C. $-1 \text{ and } a$

D. $0, \frac{1}{a}$

Answer: A

Watch Video Solution

51. If $a * b = \frac{ab}{3}$ on Q^{+} , then $3 * \left(\frac{1}{5} * \frac{1}{2}\right)$ is ....

A. $\frac{5}{160}$

B. $\frac{1}{30}$

C. $\frac{3}{160}$

D. $\frac{3}{60}$

Answer: B

Watch Video Solution

52. If Δ is defined on $P(X)(X \neq \phi)$ by , $A\delta B = (A \cup B) - (A \cap B)$, then ....

A. identity for Δ is ϕ and inverse of A is A.

B. identity for Δ is A and inverse of A is ϕ .

C. identity for Δ is A' and inverse of A is A.

D. identity for Δ is X and inverse of A is ϕ .

Answer: A

Watch Video Solution

53. S is defined on $N \times N$ by $((a, b), (c, d) \in S \Leftrightarrow a + d = b + c.......$

A. S is reflexive , but not symmetric

B. S is reflexive , and transitive only

C. S is an equivalence relation

D. S is transitive only

Answer: C

Watch Video Solution

54. If $f : R^{+} \rightarrow R, f(x) = \frac{x}{x+1}$ is ....

A. one - one and onto .

B. one - one and not onto .

C. not one - one and not onto.

D. Onto but not one - one.

Answer: B

Watch Video Solution

55. If

$f: R \rightarrow R, f(x)=[x], g: R \rightarrow R, g(x)=\sin x, h: R \rightarrow R, h(x)=2x$

, then ho(gof) = ...

A. $\sin[x]$

B. $[\sin 2x]$

C. $2(\sin[x])$

D. $\sin 2[x]$

Answer: C

Watch Video Solution

56. $f: R \rightarrow R, f(x) = \begin{cases} -1 & x < 0 \\ 0 & x = 0 \\ 1 & x > 0 \end{cases}$ $g: R \rightarrow R, g(x) =$ 1 + x - [x]

then for all x, f (g(x)) = ...

A. 1

B. 2

C. 0

D. -1

Answer: A

Watch Video Solution

57. If $f: \{x \mid x \geq 1, \text{x} \in R\} \rightarrow \{x \mid x \geq 2, x \in R\}$ f(x) = $x + \frac{1}{x}$ then $f^{-1}(x)$ = ...

A. $\dfrac{x + \sqrt{x^2 - 4}}{2}$

B. $\dfrac{x - \sqrt{x}^2 - 4}{2}$

C. $\dfrac{x^2+1}{x}$

D. $\sqrt{x^2-4}$

Answer: A

Watch Video Solution

58. $f: R \to R,\ f(x) = \dfrac{x}{\sqrt{1+x^2}},\ \forall x \in R$. Then find (fofof) (x).

A. $\dfrac{x}{1+x^2}$

B. $\dfrac{1+x^2}{x}$

C. $\dfrac{x}{\sqrt{1+2x^2}}$

D. $\dfrac{x}{\sqrt{1+3x^2}}$

Answer: D

Watch Video Solution

59. $f: R \to R, f(x) = x^2, g: R \to R, g(x) = 2^x$, then $\{x \mid (fog)(x) = (gof)(x)\}$ = ...

A. $\{0\}$

B. $\{0, 1\}$

C. R

D. $\{0, 2\}$

Answer: D

Watch Video Solution

60. The relation S on set $\{1, 2, 3, 4, 5\}$ is $S = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5)\}$. The S is ....

A. Only symmetric

B. Only reflexive

C. Only transitive

D. Equivalence relation

Answer: D

Watch Video Solution

61. The function $f: R \to R, f(x) = 5x + 7$ then the function f is ....

A. One one and onto

B. One one and not onto

C. Onto but not one one

D. Neither one one nor onto.

Answer: A

Watch Video Solution

62. The number of binary operation on set $\{1, 2\}$ is ....

A. 8

B. 16

C. 2

D. 4

Answer: B

Watch Video Solution

63. The function $f: R^{+} \rightarrow R^{+}, f(x) = x^{3}, g: R^{+} \rightarrow R^{+}, g(x) = x^{\frac{1}{3}}$

then (fog)(x) = ...

A. x^{3}

B. $\frac{1}{x}$

C. $\sqrt[3]{x}$

D. x

Answer: D

Watch Video Solution

64. $a * b = a^2 + b^2 + ab + 2$ on Z then $3 * 4$ = ...

A. 39

B. 40

C. 25

D. 41

Answer: A

Watch Video Solution

Textbook Illustrations For Practice Work

1. Let A be the set of all students of a boys school. Show that the relation R in A given by $R = \{(a, b) : a$ is sister of b} is the empty relation and

$R' = \{(a, b):$ the difference between heights of a and b is less than 3 meters } is the universal relation.

Watch Video Solution

2. Let T be the set of all triangles in a plane with R a relation in T given by $R = \{(T_1, T_2) : T_1$ is congruent to $T_2\}$ Show that R is an equivalence relation.

Watch Video Solution

3. Let L be the set of all lines in a plane and R be the relation in L defined as $R = \{(L_1, L_2) : L_1$ is perpendicular to L_2 }. Show that R is symmetric but neither reflexive nor transitive.

Watch Video Solution

4. Show that the relation R in the set $\{1, 2, 3\}$ given by $R = \{(1, 1), (2, 2), (3, 3), (1, 2)., (2, 3)\}$ is reflexive but neither symmetric nor transitive.

Watch Video Solution

5. Show that the relation R in the set Z of intergers given by

$R = \{(a, b) : 2$ divides a-b }

is an equivalence relation.

Watch Video Solution

6. Let R be the realtion defined in the set $A = \{1, 2, 3, 4, 5, 6, 7\}$ by $R = \{(a, b) :$ both a and b are either odd or even}. Show that R is an equivalance relation. further, show that all the elements of the subset $\{1, 3, 5, 7\}$ are related to each other and all elements of subset {2, 4, 6} are related to each other, but no element of the subset $\{1, 3, 5, 7\}$ is related to any element of the subset $\{2, 4, 6\}$.

Watch Video Solution

7. Let A be the set of all 50 students of Class X in a school. Let f : $A \rightarrow N$ be function defined by f(x) = roll number of the student x. Show that f is one-one but not onto.

Watch Video Solution

8. Show that the function $f: N \rightarrow N$, given by f(x) = 2x, is one-one but not onto.

Watch Video Solution

9. Prove that the function $f: R \rightarrow R$, given by f(x) = 2x, is one-one and onto.

Watch Video Solution

10. Show that the function $f: N \rightarrow N$, given by f(1) = f(2) = 1 and f(x) = x - 1, for every $x > 2$, is onto but not one-one.

Watch Video Solution

11. Show that the function f: $R \rightarrow R$, defined as $f(x) = x^2$, is neither one-one nor onto.

Watch Video Solution

12. Show that $f: N \rightarrow N$, given by

$f(x) = \begin{cases} x+1 & \text{if x is odd} \\ x-1 & \text{if x is even} \end{cases}$ is both one - one and onto .

Watch Video Solution

13. Show that an onto function $f: \{1, 2, 3\} \rightarrow \{1, 2, 3\}$ is always one-one.

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14. Show that a one-one function $f:\{1, 2, 3\} \to \{1, 2, 3\}$ must be onto.

Watch Video Solution

15. Let $f:\{2, 3, 4, 5\} \to \{3, 4, 5, 9\}$ and $g:\{3, 4, 5, 9\} \to \{7, 11, 15\}$ be functions defined as f (2) = 3, f(3) = 4, f(4) = f(5) = 5 and g(3) = g(4) = 7 and g(5) = g(9) = 11. Find gof.

Watch Video Solution

16. Find gof and fog, if $f:R \to R$ and $g:R \to R$ are given by $f(x) = \cos x$ and $g(x) = 3x^2$. Show that gof $\neq$ fog.

Watch Video Solution

17. Show that if $f:R - \left\{\frac{7}{5}\right\} \to R - \left\{\frac{3}{5}\right\}$ is defined by $f(x) = \frac{3x+4}{5x-7}$ and $g:R - \left\{\frac{3}{5}\right\} \to R - \left\{\frac{7}{5}\right\}$ is defined by

$g(x) = \frac{7x+4}{5x-3}$, then fog $= I_A$ and $gof = I_B$, where $A = R - \left\{\frac{3}{5}\right\}, B = R - \left\{\frac{7}{5}\right\}, I_A(x) = x, \forall x \in A, I_B(x) = x, \forall x \in B$

are called identity functions on sets A and B , respectively .

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18. Show that if $f: A \to B$ and $g: B \to C$ are one- one, then gof: $A \to C$ is also one-one.

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19. Show that if $f: A \to B$ and $g: B \to C$ are onto, then $gof: A \to C$ is also onto.

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20. Consider functions f and g such that composite gof is defined and is oneone. Are f and g both necessarily one-one.

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21. Are fand g both necessarily onto, if gof is onto?

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22. Let $f:\{1,2,3\} \rightarrow \{a,b,c\}$ be one-one and onto function given by f(1) = a, f(2) = b and f(3) = C. Show that there exists a function $g:\{a,b,c\} \rightarrow \{1,2,3\}$ such that gof $= I_x$ and $fog = I_Y$, where X = {1,2,3} and Y = {a,b,c} ,

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23. Let $f: N \rightarrow Y$ be a function defined as f(x) = 4x + 3, where, Y = { $y \in N : y = 4x + 3$ for some $x \in N$}. Show that f is invertible. Find the inverse.

Watch Video Solution

24. Let Y = $\{n^2 : n \in N\} \subset N$ Consider $f : N \to Y$ as f(n) = n^2 . Show that f is invertible. Find the inverse of f.

Watch Video Solution

25. Let $f' : N \to R$ be a function defined as $f'(x) = 4x^2 + 12x + 15$. Show that $f : N \to S$, where, S is the range of f, is invertible. Find the inverse of f.

Watch Video Solution

26. Consider $f : N \to N, g : N \to N$ and $h : N \to R$ defined as $f(x) = 2x, g(y) = 3y + 4$ and $h(z) = \sin z, \forall x, y$ and z in N. Show that $ho(gof) = (hog)of.$

Watch Video Solution

27. Consider $f:\{1, 2, 3\} \rightarrow \{a, b, c\}$ and $g:\{a, b, c\} \rightarrow$ {apple, ball, cat} defined as $f(1) = a$, $f(2) = b$, $f(3) = c$, $g(a)$ = apple,$g(b) =$ ball and g (c)= cat. Show that f, g and gof are invertible. Find out f^{-1}, g^{-1} and $(\text{gof})^{-1}$ and show that $(\text{gof})^{-1} = f^{-1}og^{-1}$.

Watch Video Solution

28. Let S = {1, 2, 3}. Determine whether the functions $f:S \rightarrow S$ defined as below have inverses. Find f^{-1}, if it exists.

Note : Here we accept that inverse at function is unique.

$f = \{(1, 1), (2, 2), (3, 3)\}$

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29. Let S = {1, 2, 3}. Determine whether the functions $f:S \rightarrow S$ defined as below have inverses. Find f^{-1}, if it exists.

Note : Here we accept that inverse at function is unique.

$f = \{(1, 2), (2, 1), (3, 1)\}$

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30. Let S = {1, 2, 3}. Determine whether the functions $f: S \to S$ defined as below have inverses. Find f^{-1}, if it exists.

Note : Here we accept that inverse at function is unique.

$f = \{(1, 3), (3, 2), (2, 1)\}$

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31. Show that addition, subtraction and multiplication are binary operations on R, but division is not a binary operation on R. Further, show that division is a binary operation on the set R^* of nonzero real numbers.

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32. Show that subtraction and division are not binary operations on N.

Watch Video Solution

Watch Video Solution

33. Show that $* : R \times R \to R$ given by $(a, b) \to a + 4b^2$ is a binary operation.

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34. Let P be the set of all subsets of a given set X. Show that $\cup : P \times P \to P$ given by $(A, B) \to A \cup B$ and $\cap : P \times P \to P$ given by (A, B) $\to rA \cap B$ are binary operations on the set P.

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35. Show that the $VV : R \times R \to R$ given by $(a, b) \to$ max {a, b} and the $\wedge : R \times R \to R$ given by $(a, b) \to$ min {a, b} are binary operations.

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36. Show that $+ : R \times R \to R$ and $\times : R \times R \to R$ are commutative binary operations, but -: $R \times R \to R$ and $\div id : R^* \times R^* \to R^*$ are not commutative.

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37. Show that $* : R \times R \to R$ defined by $a^* b = a + 2b$ is not commutative.

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38. Show that addition and multiplication are associative binary operation on R. But subtraction is not associative on R . Division is not associative on R^*

Watch Video Solution

39. Show that $^* : R \times R \rightarrow R$ given by $a^* b \rightarrow a + 2b$ is not associative.

Watch Video Solution

40. Show that zero is the identity for addition on R and 1 is the identity for multiplication on R. But there is no identity element for the operations $- : R \times R \rightarrow R$ and $\div idR^{\cdot} \times R^{\cdot} \rightarrow R.$

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41. Show that $-a$ is the inverse of a for the addition operation '+' on R and $\frac{1}{a}$ is the inverse of $a \neq 0$ for the multiplication operation x on R.

Watch Video Solution

42. Show that -a is not the inverse of $a \in N$ for the addition operation $+$ on N and $\frac{1}{a}$ not the inverse of $a \in N$ for multiplication operation on

N, for $a \neq 1$.

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43. If R_1 and R_2 are equivalence relations in a set A, show that $R_1 \cap R_2$ is also an equivalence relation.

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44. Let R be a relation on the set A of ordered pairs of positive integers defined by (x, y) R (u, v) if and only if xv= yu. Show that R is an equivalence relation.

Watch Video Solution

45. Let X = {1, 2, 3, 4, 5, 6, 7, 8, 9). Let R_1 be a relation in X given by $R_1 = \{(x, y) : x - y$ is divisible by 3} and R, be another relation on X given by $R_2 = \{(x, y) : \{x, y\} \subset \{1, 4, 7\}\}$ or $\{x, y\} \subset \{2, 5, 8\}$ or $\{x, y\} \subset \{3, 6, 9\}$. Show that $R_1 = R_2$.

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46. Let $f : X \to Y$ be a function. Define a relation R in X given by $R = \{(a, b) : f(a) = f(b)\}$. Examine whether R is an equivalence relation or not.

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47. Determine which of the following binary operations on the set R are associative and which are commutative :

$a^* b = 1, \forall a, b \in R$

Watch Video Solution

48. Determine which of the following binary operations on the set R are associative and which are commutative :

$a^* b = \frac{(a+b)}{2}, \forall a, b \in R$

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49. Find the number of all one-one functions from set A = {1, 2, 3} to itself.

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50. Let $A = \{1, 2, 3\}$ Then show that the number of relations containing $(1, 2)$ and $(2, 3)$ which are reflexive and transitive but not symmetric is three.

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51. Show that the number of equivalence relation in the set $\{1, 2, 3\}$ containing $(1, 2)$ and $(2, 1)$ is two.

Watch Video Solution

52. Show that the number of binary operations on $\{1, 2\}$ having 1 as identity and having 2 as the inverse. of 2 is exactly one.

Watch Video Solution

53. Consider the identity function $I_N : N \rightarrow N$ defined as $I_N(x) = x, \forall x \in N$. Show that although I_N is onto but $I_N + I_N : N \rightarrow N$ defined as $(I_N + I_N)(x) = I_N(x) + I_N(x) = x + x = 2x$ is not onto.

Watch Video Solution

54. Consider a function f : $\left[0, \frac{\pi}{2}\right] \rightarrow R$ given by f(x) = sin x and g : $\left[0, \frac{\pi}{2}\right] \rightarrow R$ given by g(x) = cos x. Show that f and g are one-one, but f+g is not one-one.

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Solutions Of Ncert Exemplar Problems Short Answer Type Questions

1. Let $A = \{a, b, c\}$ and the relation R be defined on A as follows : $R = \{(a, a), (b, c), (a, b)\}$ Then , write minimum number of ordered pairs to be added in R to make R reflexive and transitive.

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2. Let D be the domain of the real valued function f defined by $f(x) = \sqrt{25 - x^2}$. Then , write D .

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3. If $f, g: R \rightarrow R$ be defined by f(x) = 2x + 1 and g(x) $= x^2 - 2, \forall x \in R$, respectively . Find gof .

Watch Video Solution

4. Let $f: R \rightarrow R$ be the function defined by $f(x) = 2x - 2, \forall x \in R$. Write f^{-1} .

Watch Video Solution

5. If A = {a,b,c,d} and the function f $= \{(a, b), (b, d), (c, a), (d, c)\}$. Write f^{-1}.

Watch Video Solution

6. If $f: R \rightarrow R$ is defined by $f(x) = x^2 - 3x + 2$, find f(f(x)).

Watch Video Solution

Watch Video Solution

7. Is $g = \{(1,1),(2,3),(3,5),(4,7)\}$ a function, justify. If this is described by the relation, $g(x) = \alpha x + \beta$, then what values should be assigned to α and β ?

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8. Are the following set of ordered pairs functions ? If so examine whether the mapping is injective or surjective .

{(x,y) : x is a person, y is the mother of x }

Watch Video Solution

9. Are the following set of ordered pairs functions ? If so examine whether the mapping is injective or surjective .

{(x,y) : x is a person, y is the mother of x }

Watch Video Solution

10. If the mappings f and g are given be $f = \{(1, 2), (3, 5), (4, 1)\}$ and $g = \{(2, 3), (5, 1), (1, 3)\}$, write fog .

Watch Video Solution

11. Let C be the set of complex numbers . Prove that the mapping $f : C \rightarrow R$ given by $f(z) = |z|, \forall z \in C$, is neither one - one nor onto .

Watch Video Solution

12. Let the function $f : R \rightarrow R$ be defined by $f(x) = \cos x$, AA x in R \` .

Show that f is nether one - one nor onto .

Watch Video Solution

13. Let X = {1,2,3} and Y = {4,5} . Find whether the following subsets of $X \times Y$ are functions form X to Y or not .

$f = \{(1, 4), (1, 5), (2, 4), (3, 5)\}$

Watch Video Solution

14. Let X = {1,2,3} and Y = {4,5} . Find whether the following subsets of $X \times Y$ are functions form X to Y or not .

$g = \{(1, 4), (2, 4), (3, 4)\}$

Watch Video Solution

15. Let X = {1,2,3} and Y = {4,5} . Find whether the following subsets of $X \times Y$ are functions form X to Y or not .

$h = \{(1, 4), (2, 5), (3, 5)\}$

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16. Let X = {1,2,3} and Y = {4,5} . Find whether the following subsets of $X \times Y$ are functions form X to Y or not .

$k = \{(1, 4), (2, 5)\}$

Watch Video Solution

17. If functions $f: A \rightarrow B$ and $g: B \rightarrow A$ satisfy $gof = I_A$, then show that f is one one and g is onto.

Watch Video Solution

18. Let $f: R \rightarrow R$ be the function defined by $f(x) = \frac{1}{2 - \cos x}, \forall x \in R$. Then , find the range of f .

Watch Video Solution

19. Let n be a fixed positive integer. Defiene a relation R in Z as follows : $\forall a, b \in Z, aRb$ if and only if a - b divisible by n. Show that R is equivalance relation.

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Solutions Of Ncert Exemplar Problems Long Answer Type Questions

1. If $A = \{1, 2, 3, 4\}$, define relations on A which have properties of being :

Reflexive , transitive but not symmetric

Watch Video Solution

2. If $A = \{1, 2, 3, 4\}$, define relations on A which have properties of being :

Symmetric but neither reflexive nor transitive

Watch Video Solution

3. If $A = \{1, 2, 3, 4\}$, define relations on A which have properties of being :

Reflexive , symmetric and transitive .

Watch Video Solution

4. Let R be relation defined on the set of natural number N as follows : $R = \{(x, y) : x \in N, y \in N, 2x + y = 41\}$. Find the domian and range of the relation R . Also verify whether R is reflexive, symmetric and transitive.

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5. Given A = {2,3,4} , B = {2,5,6,7} . Construct an example of each of the following :

An injective mapping from A to B .

Watch Video Solution

6. Given A = {2,3,4} , B = {2,5,6,7} . Construct an example of each of the following :

A mapping from A to B which is not injective.

Watch Video Solution

7. Given A = {2,3,4} , B = {2,5,6,7} . Construct an example of each of the following :

A mapping from B to A .

Watch Video Solution

8. Give an example of a map

Which is one - one but not onto

Watch Video Solution

9. Give an example of a map

Which is not one - one but onto

Watch Video Solution

10. Give an example of a map

Which is neither one - one nor onto.

Watch Video Solution

11. Let A = R - {3} , B = R - {1} . If $f: A \rightarrow B$ be defined $f(x) = \frac{x-2}{x-3} \forall x \in A$. Then show that f is bijective.

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12. Let A = [-1,1] . Then , discuss whether the following functions defined on A are one - one , onto or bijective.

$f(x) = \frac{x}{2}$

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13. Let A = [-1,1] . Then , discuss whether the following functions defined on A are one - one , onto or bijective.

$g(x) = |x|$

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14. Check the injectivity and surjectivity of the following function .

$f : [-1, 1] \rightarrow [-1, 1], f(x) = x|x|$

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15. Let A = [-1,1] . Then , discuss whether the following functions defined on A are one - one , onto or bijective.

$k(x) = x^2$

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16. Each of the following defines a relation of N :

x is greater than y,x, y $\in N$.

Determine which of the above relations are reflexive , symmetric and transitive .

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17. Each of the following defines a relation of N :

$x + y = 10, x, y \in N$

Determine which of the above relations are reflexive , symmetric and transitive .

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18. Each of the following defines a relation of N :

x. y is square of an integer $x, y \in N.$

Determine which of the above relations are reflexive , symmetric and transitive .

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19. Each of the following defines a relation of N :

$x + 4y = 10, x, y \in N$

Determine which of the above relations are reflexive , symmetric and transitive .

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20. Let $A = \{1, 2, 3.......9\}$ and R be the relation in $A \times A$ defined by (a,b) R , (c,d) if a + d = b + c for (a,b) , (c,d) in $A \times A$. Prove that R is an equivalence relation and also obtain the equivalent class [(2,5)].

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21. Using the definition ,prove that the function F : $A \to B$ is invertible if and only if f is both one -one and onto.

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22. Functions $f, g: R \to R$ are defined ,respectively, by $f(x) = x^2 + 3x + 1, g(x) = 2x - 3,$ find

fog

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23. Functions $f, g: R \to R$ are defined ,respectively, by $f(x) = x^2 + 3x + 1, g(x) = 2x - 3,$ find gof.

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24. Functions $f, g: R \to R$ are defined ,respectively, by $f(x) = x^2 + 3x + 1, g(x) = 2x - 3,$ find fof.

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25. Functions $f, g: R \rightarrow R$ are defined ,respectively, by $f(x) = x^2 + 3x + 1, g(x) = 2x - 3,$ find gog.

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26. Let $*$ be the binary operation defined on Q. Find which of the following binary operations are commutative.

$a * b = a - b, \forall a, b \in Q$

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27. Let $*$ be the binary operation defined on Q. Find which of the following binary operations are commutative.

$a * b = a^2 + b^2, \forall a, b \in Q$

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28. Let $*$ be the binary operation defined on Q. Find which of the following binary operations are commutative.

$a * b = a + ab, \forall a, b \in Q$

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29. Let $*$ be the binary operation defined on Q. Find which of the following binary operations are commutative.

$a * b = (a - ab)^2, \forall a, b \in Q$

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30. If $*$ be binary operation defined on R by $a * b = 1 + ab, \forall a, b \in R$. Then the operation $*$ is

(i) Commutative but not associative.

(ii) Associative but not commutative .

(iii) Neither commutative nor associative .

(iv) Both commutative and associative.

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Solutions Of Ncert Exemplar Problems Objective Type Questions

1. Let T be set of all triangle in the Euclidean plane , and let a relation R on T be defined as aRb if a is congruent to$b, \forall \mathrm{a}, b \in T$. Then, R is ....

A. Reflexive but not transitive

B. Transitive but not symmetric

C. Equivalence

D. None of these

Answer: C

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2. Consider the non-empty set consisting of children in a family and a relation R defined as aRb, if a is brother of b. Then , R is ....

A. Symmetric but not transitive

B. Transitive but not symmetric

C. Neither symmetric not transitive

D. Both symmetric and transitive

Answer: B

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3. The maximum number of equivalence relations on the set A = {1,2,3} are

....

A. 1

B. 2

C. 3

D. 7

Answer: D

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4. If the relation R on the set {1,2,3} be defined by R = {(1,2)}. Then , R is ....

A. Reflexive

B. Transitive

C. Symmetric

D. None of these

Answer: B

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5. Let us define a relation R in R as aRb if $a \geq b$. Then, R is ....

A. an equivalence relation

B. reflexive , Transitive but not symmetric

C. symmetric , transitive but not reflexive

D. neither transitive nor reflexive but symmetric .

Answer: B

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6. If A = {1,2,3} and consider the relation R = {(1,1), (2,2) , (3,3) , (1,2) ,(2,3), (1,3)} . Then R is ....

A. reflexive but not symmetric

B. reflexive but not transitive

C. symmetric and transitive

D. neither symmetric, nor transitive

Answer: A

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7. The identity element for the binary operation $*$ defined on Q~$\{0\}$ as $a * b = \frac{ab}{2}$, $\forall a, b \in Q - \{0\}$ is ....

A. 1

B. 0

C. 2

D. None of these

Answer: C

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8. If the set A contains 5 elements and the set B contains 6 elements , then the number of one -one and onto mapping from A to B is ....

A. 720

B. 120

C. 0

D. None of these

Answer: C

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9. If $A = \{1, 2, 3..., n\}$ and $B = \{a, b\}$ Then , the number of subjection from A into B is ....

A. ${}^{n}P_{2}$

B. $2^{n} - 2$

C. $2^{n} - 1$

D. None of these

Answer: D

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10. If $f: R \rightarrow R$ be defined by $f(x) = \frac{1}{x}, \forall x \in R$. Then , f is ....

A. one-one

B. onto

C. bijective

D. f is not defined

Answer: D

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11. Let $f: R \rightarrow R$ be defined by $f(x) = 3x^2 - 5$ and $g: R \rightarrow R, g(x) = \frac{x}{x^2 + 1}$ Then gof is ....

A. $\frac{3x^2 - 5}{9x^4 - 30x^2 + 26}$

B. $\frac{3x^2 - 5}{9x^4 - 6x^2 + 26}$

C. $\frac{3x^2}{x^4 + 2x^2 - 4}$

D. $\frac{3x^2}{9x^4 + 30x^2 - 2}$

Answer: A

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12. Which of the following functions from Z into Z are bijections ?

A. $f(x) = x^3$

B. $f(x) = x + 2$

C. $f(x) = 2x + 1$

D. $f(x) = x^2 + 1$

Answer: B

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13. If $f : R \rightarrow R$ be the functions defined by $f(x) = x^3 + 5$, then $f^{-1}(x)$ is ....

A. $(x+5)^{\frac{1}{3}}$

B. $(x-5)^{\frac{1}{3}}$

C. $(5-x)^{\frac{1}{3}}$

D. $5-x$

Answer: B

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14. If $f: A \rightarrow B$ and $g: B \rightarrow C$ be the bijective functions , then $(gof)^{-1}$ is ....

A. $f^{-1} \text{og}^{-1}$

B. fog

C. $g^{-1} \text{of}^{-1}$

D. gof

Answer: A

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15. If $f: R - \left\{\frac{3}{5}\right\} \rightarrow R$ be defined by $f(x) = \frac{3x+2}{5x-3}$, then ....

A. $f^{-1}(x) = f(x)$

B. $f^{-1}(x) = -f(x)$

C. $\text{fof}(x) = -x$

D. $f^{-1}(x) = \frac{1}{19}f(x)$

Answer: A

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16. If $f: [0, 1] \rightarrow [0, 1]$ be difined by $f(x) = \begin{cases} x & \text{if x is rational} \\ 1-x & \text{if x is irrational} \end{cases}$

then fof(x) is ....

A. constant

B. 1 + x

C. x

D. None of these

Answer: C

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17. If $f:[(2,\infty)\to R$ be the function defined by $f(x)=x^2-4x+5$, then the range of f is ....

A. R

B. $[1,\infty)$

C. $[4,\infty)$

D. $[5,\infty)$

Answer: B

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18. If $f: N \rightarrow R$ be the function defined by $f(x) = \frac{2x-1}{2}$ and $g: Q \rightarrow R$ be another function defined by $g(x) = x + 2$. Then , gof $\left(\frac{3}{2}\right)$ is ....

A. 1

B. -1

C. 3

D. None of these

Answer: D

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19. $f: R \rightarrow R, f(x) = \begin{cases} 2x & x > 3 \\ x^2 & 1 < x \leq 3 \\ 3x & x \leq 1 \end{cases}$ then find f (-1) + f(2) + f(4) .

A. 9

B. 14

C. 5

D. None of these

Answer: A

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20. If $f: R \rightarrow R$ be given by f(x) = tan x , then $f^{-1}(1)$ is ....

A. $\frac{\pi}{4}$

B. $\left\{n\pi + \frac{\pi}{4} : n \in Z\right\}$

C. Does not exist

D. None of these

Answer: A

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Solutions Of Ncert Exemplar Problems Fillers

1. Let the relation R be defined in N by aRb , if 2a + 3b = 30 . Then , R = ...

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2. If the relation R be defined on the set $A = \{1, 2, 3, 4, 5\}$ by $R = \{(a, b) : |a^2 - b^2| < 8\}$. Then , R is given by ....

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3. The functions f and g are defined as follow : $f = \{(1, 2), (3, 5), (4, 1)\}$ and $g = \{(2, 3), (5, 1), (1, 3)\}$. Find the range of f and g . Also find the composition function fog and gof.

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4. $f: R \to R, f(x) = \frac{x}{\sqrt{1+x^2}}, \forall x \in R$. Then find (fofof) (x).

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5. If $f(x) = \left[4 - (x-7)^3\right]$, then $f^{-1}(x) = \ldots\ldots\ldots$

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Solutions Of Ncert Exemplar Problems True False

1. Let $R = \{(3, 1), (1, 3), (3, 3)\}$ be a relation defined on the set $A = \{1, 2, 3\}$. Then , R is symmetric , transitive but not reflexive.

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2. If $f: R \to R$ be the function defined by $f(x) = \sin(3x+2)\,\forall x \in R$. Then , f is invertible .

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3. Every relation which is symmetric and transitive is also reflexive.

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4. An integer m is said to be related to another integer n if m is a integral multiple of n. This relation in Z is reflexive, symmetric and transitive.

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5. If $A = \{0, 1\}$ and N be the set of natural numbers. Then , the mapping $f: N \to A$ defined by $f(2n-1) = 0, f(2n) = 1, \forall n \in N$, is onto.

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6. The relation R on the set $A = \{1, 2, 3\}$ defined as R = $\{(1, 1), (1, 2), (2, 1), (3, 3)\}$ is reflexive , symmetric and transitive.

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7. The composition of function is commutative .

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8. The composition of function is associative.

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9. Every function is invertible.

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10. A binary operation on a set has always the identity element.

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Practice Paper 1 Section A

1. Which of the following defined on Z is not an equivalence relation ?

A. $(x, y) \in S \Leftrightarrow x \geq y$

B. $(x, y) \in S \Leftrightarrow x = y$

C. $(x, y) \in S \leftrightarrow x - y$ is a multiple of 3

D. If $|x - y|$ is even $\Leftrightarrow (x, y) \in S$

Answer:

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2. The number of binary operation on $\{1, 2, 3, \ldots\ldots, n\}$ is ....

A. 2^n

B. n^{n^2}

C. n^3

D. n^{2n}

Answer:

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3. If $a * b = a^2 + b^2$ is on Z then , $(2 * 3) * 4 = \ldots\ldots$

A. 13

B. 16

C. 185

D. 31

Answer:

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4. A = {1,2} , the number of one - one functions on $A \to A$ is ....

A. 1

B. 2

C. 3

D. 4

Answer:

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5. $*$ is defined by $a * b = a + b - 1$ on Z , then identity element for $*$ is

....

A. 1

B. 0

C. -1

D. 2

Answer:

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Practice Paper 1 Section B

1. If f(x) = $8x^3$ and $g(x) = x^{\frac{1}{3}}$ then find gof and fog .

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2. Let $*$ be the binary operation on Q define $a * b = a + ab$. Is $*$ commutative ? Is $*$ associative ?

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3. Let $f: R \to R$ be defined as $f(x) = 10x + 7$. Find the function $g: R \to R$ such that gof = fog = I_g

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4. Let A = {1,2,3}. Then number of relations containing (1,2) and (1,3) which are reflexive and symmetric but not transitive is

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Practice Paper 1 Section C

1. Let T be the set of all triangles in a plane with R a relation in T given by $R = \{(T_1, T_2) : T_1$ is congruent to $T_2\}$ Show that R is an equivalence relation.

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2. Prove that binary operation on set R defined as $a * b = a + 2b$ does not obey associative rule.

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3. Let $f' : N \rightarrow R$ be a function defined as $f'(x) = 4x^2 + 12x + 15$. Show that $f : N \rightarrow S$, where, S is the range of f, is invertible. Find the inverse of f.

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4. If $f : A \rightarrow B$ and $g : B \rightarrow C$ be the bijective functions, then $(gof)^{-1}$ is ....

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5. Show that $f: R_+ \rightarrow R_+, f(x) = \frac{1}{x}$ is one to one and onto function.

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Practice Paper 1 Section D

1. Let : $f: N \rightarrow N$ be defined by $f(n) = \begin{cases} \frac{n+1}{2} & \text{if n is odd} \\ \frac{n}{2} & \text{if n is even} \end{cases}$ for all $n \in N$.

State whether the function f is bijective . Justify your answer.

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2. $f: Z \rightarrow Z$ and $g: Z \rightarrow Z$ are defined as follow :

$f(n) = \begin{cases} n+2 & \text{n even} \\ 2n-1 & \text{n odd} \end{cases}, g(n) = \begin{cases} 2n & \text{n even} \\ \frac{n-1}{2} & \text{n odd} \end{cases}$ Find fog and gof.

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