MATHS

NCERT - NCERT MATHEMATICS(BENGALI)

LIMITS AND DERIVATIVES

Example

1. Find the limits : (i) $\lim_{x \to 1} \left[x^3 - x^2 + 1\right]$

(ii) $\lim_{x \to 3} \left[x(x+1)\right]$

(iii) $\lim_{x \to -1} \left[1 + x + x^2 + \ldots + x^{10}\right]$

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2. Find the limits :

$\lim_{x \to 1} \left[\frac{x^2 + 1}{x + 100}\right]$

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3. Evaluate :

$\lim_{x \to 1} \frac{x^{15} - 1}{x^{10} - 1}$

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4. Evaluate :

$$\lim_{x \to 0} \frac{\sqrt{1+x}-1}{x}$$

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5. Evaluate :

$$\lim_{x \to 0} \frac{\sin 4x}{\sin 2x}$$

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6. Evaluate :

$$\lim_{x \to 0} \frac{\tan x}{x}$$

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7. Find the derivative at $x = 2$ of the function

$f(x) = 3x.$

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8. Find the derivative of the function $f(x) = 2x^2 + 3x - 5$ at $x = -1$. Also prove that $f'(0) + 3f'(-1) = 0$.

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9. Find the derivative of $\sin x$ at $x = 0$.

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10. Find the derivative of $f(x) = 3atx = 0$ and $atx = 3$.

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11. Find the derivative of $f(x) = 10x$.

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12. Find the derivative of $f(x) = x^2$.

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13. Find the derivative of the constant function $f(x) = a$ for a fixed real number a.

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14. Find the derivative of $f(x) = \frac{1}{x}$

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15. Compute the derivative of $6x^{100} - x^{55} + x$.

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16. Find the derivative of $f(x) = 1 + x + x^2 + x^3 + \ldots + x^{50}$ at $x = 1$.

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17. Find the derivative of $f(x) = \dfrac{x+1}{x}$

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18. Compute the derivative of sin x.

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19. Compute the derivative of tan x.

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20. Compute the derivative of $f(x) = \sin^2 x$.

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Miscellaneous Examples

1. Find the derivative of f , where f is given by

$$f(x) = \frac{2x + 3}{x - 2}$$

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2. Find the derivative of f from the first principle, where f is given by

$$f(x) = x + \frac{1}{x}$$

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3. Find the derivative of f(x) from the first principle, where f(x) is

$$\sin x + \cos x$$

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4. Find the derivative of f(x) from the first principle, where f(x) is

$x \sin x$

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5. Compute derivative of

$f(x) = \sin 2x$

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6. Compute derivative of

$$g(x) = \cot x$$

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7. Find the derivative of

$$\frac{x^5 - \cos x}{\sin x}$$

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8. Find the derivative of

$$\frac{x + \cos x}{\tan x}$$

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Exercise 13 1

1. Evaluate the following limits in

$$\lim_{x \to 3} x + 3$$

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2. Evaluate the following limits in

$$\lim_{x \to \pi} \left(x - \frac{22}{77} \right)$$

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3. Evaluate the following limits in

$$\lim_{\to 1} \pi r^2$$

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4. Evaluate the following limits in

$$\lim_{x \to 4} \frac{4x + 3}{x - 2}$$

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5. Evaluate the following limits in

$$\lim_{x \to -1} \frac{x^{10} + x^5 + 1}{x - 1}$$

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6. Evaluate the following limits in

$$\lim_{x \to 0} \frac{(x+1)^5 - 1}{x}$$

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7. Evaluate the following limits in

$$\lim_{x \to 2} \frac{3x^2 - x - 10}{x^2 - 4}$$

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8. Evaluate the following limits in

$$\lim_{x \to 3} \frac{x^4 - 81}{2x^2 - 5x - 3}$$

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9. Evaluate the following limits in

$$\lim_{x \to 0} \frac{ax + b}{cx + 1}$$

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10. Evaluate the following limits in

$$\lim_{z \to 1} \frac{z^{\frac{1}{3}} - 1}{z^{\frac{1}{6}} - 1}$$

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11. Evaluate the following limits in

$$\lim_{x \to 1} \frac{ax^2 + bx + c}{cx^2 + bx + a}, a + b + c \neq 0$$

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12. Evaluate the following limits in

$$\lim_{x \to 2} \frac{\frac{1}{x} + \frac{1}{2}}{x + 2}$$

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13. Evaluate the following limits in

$$\lim_{x \to 0} \frac{\sin ax}{bx}$$

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14. Evaluate the following limits in

$$\lim_{x \to 0} \frac{\sin ax}{\sin bx}, a, b \neq 0$$

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15. Evaluate the following limits in

$$\lim_{x \to \pi} \frac{\sin(\pi - x)}{\pi(\pi - x)}$$

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16. Evaluate the following limits in

$$\lim_{x \to 0} \frac{\cos x}{\pi - x}$$

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17. Evaluate the following limits in

$$\lim_{x \to 0} \frac{\cos 2x - 1}{\cos x - 1}$$

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18. Evaluate the following limits in

$$\lim_{x \to 0} \frac{ax + x \cos x}{b \sin x}$$

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19. Evaluate the following limits in

$$\lim_{x \to 0} x \sec x$$

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20. Evaluate the following limits in

$$\lim_{x \to 0} \frac{\sin ax + bx}{ax + \sin bx} a, b, a + b \neq -$$

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21. Evaluate the following limits in

$$\lim_{x \to 0} (\text{cosec } x - \cot x)$$

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22. Evaluate the following limits in

$$\lim_{x \to \frac{\pi}{2}} \frac{\tan 2x}{x - \frac{\pi}{2}}$$

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23. Find $\lim_{x \to 0} f(x)$ and $\lim_{x \to 1} f(x)$, where

$$f(x) = \begin{cases} 2x + 3, & x \leq 0 \\ 3(x + 1), & x > 0 \end{cases}$$

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24. Find $\lim_{x \to 1} f(x)$, where

$$f(x) = \begin{cases} x^2 - 1, & x \leq 1 \\ -x^2 - 1, & x > 1 \end{cases}$$

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25. Evaluate

$$\lim_{x \to 0} f(x), \quad \text{where} \quad f(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

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26. Find $\lim_{x \to 0} f(x)$, where

$$f(x) = \begin{cases} \frac{x}{|x|}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

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27. Find $\lim_{x \to 5} f(x)$, where $f(x) = |x| - 5$

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28. Suppose $f(x) = \begin{cases} a + bx, & x < 1 \\ 4, & x - 1 \\ b - ax, & x > 1 \end{cases}$

and if $\lim_{x \to 1} f(x) = f(1)$ what are possible values of a and b?

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29. Let $a_1, a_2, \ldots, a_n$ be fixed real numbers and define a function $f(x) = (x - a_1)(x - a_2)\ldots(x - a_n).$

What is $\lim_{x \to a_1} f(x)$? For some $a \neq a_1, a_2, \ldots\ldots, a_n,$ compute $\lim_{x \to a} (f(x).$

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30. If $f(x) = \begin{cases} |x| + 1, & x < 0 \\ 0, & x = 0 \\ |x| - 1, & x > 0 \end{cases}$

For what value(s) of a does $\lim_{x \to a} f(x)$ exists?

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31. If the function $f(x)$ satisfies $\lim_{x \to 1} \frac{f(x) - 2}{x^2 - 1} = \pi$, evaluate $\lim_{x \to 1} f(x)$.

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32. If $f(x) = \begin{cases} mx^2 + n, & x < 0 \\ nx + m, & 0 \leq x \leq 1 \\ nx^3 + m, & x > 1 \end{cases}$. For what integers m and n does both $\lim_{x \to 0} f(x)$ and $\lim_{x \to 1} f(x)$ exist?

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Exercise 13 2

1. Find the derivative of $x^2 - 2$ at $x = 10$.

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2. Find the derivative of x at $x = 1$.

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3. Find the derivative of 99x at x = l00.

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4. Find the derivative of the following functions from first principle.

$x^3 - 27$

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5. Find the derivative of the following functions from first principle.

$(x - 1)(x - 2)$

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6. Find the derivative of the following functions from first principle.

$$\frac{1}{x^2}$$

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7. Find the derivative of the following functions from first principle.

$$\frac{x+1}{x-1}$$

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8. For the function

$$f(x) = \frac{x^{100}}{100} + \frac{x^{99}}{99} + \ldots + \frac{x^2}{2} + x + 1.$$

Prove that

$f'(1) = 100f'(0).$

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9. Find the derivative of

$x^n + ax^{n-1} + a^2x^{n-2} + \ldots + a^{n-1}x + a^n$

for some fixed real number a.

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10. For some constants a and b, find the derivative of

$(x - a)(x - b)$

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11. For some constants a and b, find the derivative of

$(ax^2 + b)^2$

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12. For some constants a and b, find the derivative of

$$\frac{x-a}{x-b}$$

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13. Find the derivative of $\frac{x^n - a^n}{x-a}$ for some constant a.

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14. Find the derivative of

$$2x - \frac{3}{4}$$

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15. Find the derivative of

$$\left(5x^3 + 3x - 1\right)(x - 1)$$

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16. Find the derivative of

$$x^{-3}(5+3x)$$

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17. Find the derivative of

$$x^{5}\left(3-6x^{-9}\right)$$

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18. Find the derivative of

$$x^{-4}\left(3-4x^{-5}\right)$$

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19. Find the derivative of

$$\frac{2}{x+1}-\frac{x^2}{3x-1}$$

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20. Find the derivative of $\cos\left(x^3\right)$

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21. Find the derivative of the following functions:

$\sin x \cos x$

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22. Find the derivative of the following functions:

$\sec\left(x^2+3\right)$

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23. Find the derivative of the following functions:

$5 \sec x + 4 \cos x$

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24. Find the derivative of the following functions:

$\operatorname{cosec}(x + 2)$

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25. Find the derivative of the following functions:

$3 \cot x + 5 \quad \text{cosec} \quad x$

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26. Find the derivative of the following functions:

$5 \sin x - 6 \cos x + 7$

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27. Find the derivative of the following functions:

$2\tan x - 7\sec x$

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Miscellaneous Exercise On Chapter 13

1. Find the derivative of the following functions :

$-x+\sin^2 x$

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2. Find the derivative of the following functions :

$(-x)^{-1}$

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3. Find the derivative of the following functions :

$\sin(x+1)$

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4. Find the derivative of the following functions :

$\cos\left(x-\frac{\pi}{8}\right)$

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5. Find the derivative of the following functions (it is to be understood that a, b, c, d,

p, q, r and s are fixed non-zero constants and m and n are integers):

$(x + a)$

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6. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):

$(px + q)\left(\frac{r}{x} + s\right)$

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7. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):

$(ax + b)(vx + d)^2$

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8. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and

m and n are integers):

$$\frac{ax+b}{cx+d}$$

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9. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):

$$\left(\frac{1+\frac{1}{x}}{1-\frac{1}{x}}\right)$$

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10. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):

$$\frac{1}{ax^2 + bx + c}$$

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11. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and

m and n are integers):

$$\frac{ax+b}{px^2+qx+r}$$

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12. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):

$$\frac{px^2+qx+r}{ax+b}$$

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13. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):

$$\frac{a}{x^4} - \frac{b}{x^2} + \cos x$$

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14. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and

m and n are integers):

$4\sqrt{x} - 2$

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15. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):

$(ax + b)^n$

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16. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):

$(ax+b)^n(cx+d)^n$

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17. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and

m and n are integers):

$\sin(x + a)$

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18. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):

$\text{cosec} \quad x \cot x$

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19. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):

$$\frac{\cos x}{1+\sin x}$$

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20. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and

m and n are integers):

$$\frac{\sin x + \cos x}{\sin x - \cos x}$$

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21. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):

$$\frac{\sec x - 1}{\sec x + 1}$$

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22. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):

$\sin^n x$

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23. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and

m and n are integers):

$$\frac{a + b\sin x}{c + d\cos x}$$

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24. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):

$$\frac{\sin(x + a)}{\cos x}$$

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25. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):

$x^4(5\sin x - 3\cos x)$

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26. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and

m and n are integers):

$$\left(x^3+1\right)\cos x$$

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27. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):

$$\left(ax^2+\sin x\right)\left(p+q\cos x\right)$$

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28. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):

$(x + \cos x)(x - \tan x)$

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29. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and

m and n are integers):

$$\frac{4x + 5\sin x}{3x + 7\cos x}$$

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30. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):

$$\frac{x^2 \cos\left(\frac{\pi}{4}\right)}{\sin x}$$

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31. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):

$$\frac{x}{1+\tan x}$$

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32. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and

m and n are integers):

$$(x + \sec x)(x - \tan x)$$

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33. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):

$$\frac{x}{\sin^n x}$$

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