Example

1. If (x+1, y-2)=(3,1), find the values of x and y.

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2. If P={a,b,c} and Q={r}, form the sets $P \times Q$ and $Q \times P$.

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3. Let A={1,2,3}, B={3,4} and C={4,5,6}. Find $A \cup (B \cup C)$

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4. If P={1,2}, form the set $P \times P \times P$

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5. If R is the set of all real numbers, what do the cartesian products $R \times R$ and $R \times R \times R$ represent?

6. If $A \times B = \left\{ \begin{array}{cc} (p, q) & (p, r) \\ (m, q) & (m, r) \end{array} \right\}$, find A and B.

7. Let A={1,2,3,4,5,6}. Define a relation R form A to A by

R= {(x,y) : y=x+1}

(i) Depict this relation using an arrow diagram.

(ii) Write down the domain, codmain and range of R.

8. Let A={1,2} and B={3,4}. Find the number of relations from A to B.

9. Examine each of the following relations given below and state in each case, giving resons whether it is function or not?

(i) R={(2,1), (3,1), (4,2)}, (ii) R={(2,2), (2,4) ,(3,3),(4,4)}

(iii) R={(1,2),(2,3),(3,4),(4,5),(5,6),(6,7)}

10. Let N be the set of natural numbers. Define a real valued function $f: N \rightarrow n$ by f(x)=2x+1. Using this defination, complete the table given below,

x	1	2	3	4	5	6	7
y	$f(1) = ...$	$f(2) = ...$	$f(3) = ...$	$f(4) = ...$	$f(5) = ...$	$f(6) = ...$	$f(7) = ...$

11. Define the function $f: R \to R$ by $y = f(x) = x^2$, $x \in R$. Complete the Table given below by using this defination. What is the domain and range of this function? Draw the graph of f.

x		-4	-3	-2	-1	0	1	2	3	4
$y = f(x) = x^2$										

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12. Draw the graph of the function $f: R \to R$ defined by $f(x) = x^3$, $x \in R$.

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13. Define the real valued function $f: R - \{0\} \to R$ defined by $f(x) = \dfrac{1}{x}; x \in R - \{0\}$. Complete the Table given below using this definition. What is the domain and range of this function?

x		-2	-1.5	-1	-0.5	0.25	0.5	1	1.5	2
$y = \dfrac{1}{x}$		...	...	...	...	...	...	...	...	...

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14. Let $f(x) = x^2$ and $g(x) = 2x + 1$ be two real functions. Find $(f + g)(x), (f - g)(x), (fg)(x), \left(\dfrac{f}{g}\right)(x).$

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15. Let $f(x) = \sqrt{x}$ and $g(x) = x$ be two functions defined over the set of non-negative real numbers. Find $(f + g)(x)$, $(f - g)$, $(fg)(x)$ and $\left(\dfrac{f}{g}\right)(x)$.

16. Let R be the set of real numbers. Define the real function $f: R \to R$ by $f(x) = x + 10$ and sketch the graph of this function.

17. Let R be a relation from Q to Q defined by $R = \{(a, b) : a, b \in Q \text{ and } a - b \in Z\}$. Show that

(i) $(a, a) \in R$ for all $a \in Q$ `

18. Let $f = \{(1,1),(2,3),(0,-1),(,-1,-3)\}$ be a linear function from Z into Z. Find f(x).

19. Find the domain of the function
$$f(x) = \frac{x^2 + 3x + 5}{x^2 - 5x + 4}$$

20. The function f is defined by
$$f(x) = \begin{cases} 1 - x & x < 0 \\ 1 & x = 0 \\ x + 1 & x > 0 \end{cases}$$

Draw the graph of f(x).

Exercise 2 1

1. If $\left(x + 3 + 1, y - \dfrac{2}{3}\right) = \left(\dfrac{5}{3}, \dfrac{1}{3}\right)$, find the values of x and y.

2. If the set A has 3 elements and the set B =(3, 4, 5), then find the number of elements in $(A \times B)$.

3. If G =(7, 8) and H=(5,4,2), find $G \times H$ and $H \times G$.

4. State whether each of the following statements are true or false. If the statement is false, rewrite the given statement correctly.

(i) If P= {m, n} and Q = {n, m}, then $P \times Q = \{(m, n), (n, m)\}$.

(ii) If A and B are non-empty sets, then $A \times B$ is a non-empty set of ordered pairs (x, y) such that $x \in A$ and $y \in B$.

(iii) If A={1,2}, B={3,4} then $A \times (B \cap \phi) = \phi$

5. If A={-1,1}, find $A \times A \times A$

6. If $A \times B = \{(a, x), (a, y), (b, x), (b, y)\}$. Find A and B.

7. Let A={1,2}, B={1,2,3,4}, C={5,6} and D={5,6,7,8}. Verify that (i) $A \times (B \cap C) = (A \times B) \cap (A \times C)$, $(ii) A \times C$ is a subset of $B \times D$

8. Let A={1,2} and B={3,4}. Write $A \times B$. How many subsets will $A \times B$ have? List them.

9. Let A and B be two sets such that n(A) =3 and n(B) =2. If (x, 1), (y, 2), (z,1) are in $A \times B$, find A and B, where x, y and z are distinct elements.

10. The Cartesian product $A \times A$ has 9 elements among which are found (-1,0) and (0,1). Find the set A and the remaining elements of $A \times A$.

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Exercise 2 2

1. Let A={1,2,3....14}. Define a relation R from A to A by

$$R = \{(x, y) : 3x - y = 0, \quad \text{where} \quad x, y \in A\}.$$

Write down its domain, condomain and range.

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2. Define a relation R on the set N of natural numbers by $R = \{(x, y) : y = x + 5,$ x is a natural number less than $4, x, y \in N)$. Depict this relationship using roster form. Write down the domain and the range.
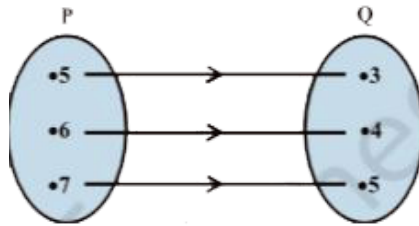
3. A =(1, 2, 3, 5) and B= {4, 6, 9). Define a relation R from A to B by R= {(x, y): the difference between x and y is odd, $x \in A, y \in B\}$. Write R in roster form.

4. The Fig 2.7 shows a relationship between the sets P and Q. Write this relation

(i) in set-builder form (ii) roster form. What is its

domain and range?

5. Let A= (1, 2, 3, 4, 6). Let R be the relation on A defined by $\{(a, b)a, b \in A, b$ is exactly divisible by a]

(i) Write R in roster form

(ii) Find the domain of R

(iii) Find the range of R.

6. Determine the domain and range of the relation R

defined by $R = \{(x, x+5): x \in \{0, 1, 2, 3, 4, 5\}\}$.

7. Write the relation $R = \{(x, x^3): x$ is a prime

number less than 10) in roster form.

8. Let A = {x, y, z) and B = {1, 2}. Find the number of relations from A to B.

9. Let R be the relation on Z defined by $R = \{(a, b) : a, b \in Z, a - b \text{ is an integer}\}$. Find the domain and range of R.

Exercise 2 3

1. Which of the following relations are functions? Give reasons. If it is a function, determine its domain and range.

(i) {(2,1),(5,1),(8,1),(11,1),(14,1),(17,1)}

(ii) {(2,1),(4,2),(6,3),(8,4),(10,5),(12,6),(14,7)}

(iii) {(1,3),(1,5),(2,5)}

2. Find the domain and range of the following real functions:

(i) f(x)=-|x| (ii) $f(x) = \sqrt{9 - x^2}$

3. A function f is defined by f(x)=2x-5. Write down the values of (i) f(0), (ii) f(7), (iii) f(-3)

4. The function 't' which maps temperature in degree Celsius into temperature in degree Fahrenheit is defined by $t(C) = \dfrac{9C}{5} + 32$

Find (i) t(0) (ii) t(28) (iii) t(-10) (iv) The value of C, when t(C)=212.

5. Find the range of each of the following functions.

(i) $f(x) = 2 - 3x, x \in R, x > 0$

(ii) $f(x) = x^2 + 2x,$ x is a real number.

(iii) f(x) = x, x is a real number

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Miscellaneous Exercise On Chapter 2

1. The relation f is defined by

$$f(x) = \begin{cases} x^2 & 0 \le x \le 3 \\ 3x & 3 \le x \le 10 \end{cases}$$

The relation g is defined by

$$g(x) = \begin{cases} x^2 & 0 \leq x \leq 2 \\ 3x & 2 \leq x \leq 10 \end{cases}$$

Show that f is a function and g is not a function.

2. If $f(x) = x^2$, find $\dfrac{f(1.1) - f(1)}{(1.1 - 1)}$

3. Find the domain of the function

$$f(x) = \dfrac{x^2 + 2x + 1}{x^2 - 8x + 12}$$

4. Find the domain and the range of the real function f defined by $f(x) = \sqrt{(x-1)}$

5. Find the domain and the range of the real function f defined by $f(x) = |x - 1|$

6. Let $f = \left\{ \left(x, \dfrac{x^2}{1 + x^2} \right), x \in R \right\}$ be a function from R into R. Determine the range of f.

7. Let $f, g: R \to R$ be defined, respectively byt f(x)=x+1, g(x)=2x-3. Find $f + g$, $f - g$ and $\dfrac{f}{g}$.

8. Let f={(1,1),(2,3),(0,-1),(-1,-3)} be a function from Z to Z defined by f(x)= ax + b, for some integers a, b. Determine a, b,

9. Let R be a relation from N to N defined by $R = \{(a, b) : a, b \in N \text{ and } a = b^2\}$. Are the following true?

(i) $(a, a) \in R$, for all $a \in N$ (ii) $(a, b) \in R$, implies $(b, a) \in R$

(iii) $(a, b) \in R, (b, c) \in R$ implies $(a, c) \in R$.

Justify your answer in each case.

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10. Let A={1,2,3,4}, B={1,5,9,11,15,16} and f={(1,5),(2,9),(3,1), (4,5),(2,11)} Are the following true?

(i) f is a relation from A to B (ii) f is a function from A to B. Justify your answer in each case.

11. Let f be the subset of $Z \times Z$ defined by $f = \{(ab, a + b) : a, b \in Z\}$. Is f a function from Z to Z? Justify your answer.

12. Let A={9,10,11,12,13} and let $f : A \rightarrow N$ be defined by f(n)= the highest prime factor of n. Find the range

of f.

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