MATHS

BOOKS - TELUGU ACADEMY MATHS (TELUGU ENGLISH)

APPLICATIONS OF DERIVATIVES

10 1 Errors And Approximations Vsaq

1. If $y = x^2 + 3x + 6$ then find Δy and dy when x=10, Δx=0.1.

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2. Find Δy and dy for the function $y = x^2 + x$, when x=10, $\Delta x = 0.1$

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3. Find Δy and dy for the function $y = 1/(x + 2)$, when x=8, $\Delta x = 0.02$.

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4. Find Δy and dy for the function $y = e^x + x$ when x=5, $\Delta x = 0.02$

5. If y=cosx then find Δy and dy when $x = 60°$ and $\Delta x = 1° = 0.0174$ rad

6. Find the approximate value of $\sqrt{82}$

7. Find the approximate value of $\sqrt[3]{999}$

8. Find the approximate value of $\sqrt[3]{65}$

9. Find the approximate value of $\sqrt[3]{7.8}$

10. Find the approximate value of $\sin 62°$

11. Find approximate value of $\cos(60° 5')$

12. If the increase in the side of a square is 4% then find the approximate percentage of increase in the area of the square.

13. If the increase in the side of a square is 2% then find the approximate percentage of increase in the area of the square.

14. The side of a square is increased from 3 cm to 3.01 cm. Find the approximate increase in the area of the square.

15. The radius of a sphere is measured as 14 cm. Later it was found that there is an error 0.02 cm in measuring the radius. Find the approximate error in surface of the sphere.

16. If the radius of a sphere is increased from 7 cm to 7.02 cm. then find the approximate increase in the volume of the sphere.

17. The diameter of a sphere is measured to be 40 cm. If an error of 0.02 cm is made in it, then find approximate errors in volume and surface area of the sphere.

18. The time t of a complete oscillation of a simple pendulum of length l is given by $t2\pi\sqrt{\dfrac{l}{g}}$ where g is gravitational constant. Find the approximate percentage of error in t when the percentage of error in l is 1%.

19. If $y = x^n$, then ratio of relative errors in y and x is

1. Find the equations of the tangent and the normal to the curve $y^4 = ax^3$ at (a,a)

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2. Find the equations of the tangent to the curve $y = 3x^2 - x^3$, where it meets the X-axis.

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3. Show that the equation of the tangent to the curve $\left(\dfrac{x}{a}\right)^n + \left(\dfrac{y}{b}\right)^n = 2\,(a \neq 0, b \neq 0)$ at the point (a,b) is $\dfrac{x}{a} + \dfrac{y}{b} = 2$

4. Show that the curves $x^2 + y^2 = 2, 3x^2 + y^2 = 4x$ have a common tangent at the point (1,1)

5. S.T the tangent at any point θ on the curve $x = c\sec\theta,\ y = c\tan\theta$ is $y\sin\theta = x - \cos\theta$.

6. Find the equation of tangent and normal to the curve $x = \cos t,\ y = \sin t,$ at $t = \dfrac{\pi}{4}$

7. Find lengths of normal and subnormal at a point on the curve $y = \dfrac{a}{2}\left(e^{\frac{x}{a}} + e^{-\frac{x}{a}}\right)$

8. Find the value of k, so that the length of the subnormal at any point on the curve $y = a^{1-k}x^k$ is a constant

9. Show that the length of subnormal at any point on the curve $xy = a^2$ varies as the cube of the ordinate of the point

1. Show that the tangent at $P(x_1, y_1)$ on the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ is $xx_1^{-\frac{1}{2}} + yy_1^{-\frac{1}{2}} = a^{\frac{1}{2}}$

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2. IF the tangent at a point on the curve $x^{2/3} + y^{2/3} = a^{2/3}$ intersects the coordinate axes in A and B then show that the length AB is a constant.

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3. IF the tangent at any point P on the curve $x^m y^n = a^{m+n}$, $mn \neq 0$ meets the coordinate axes in A.B then show that $AP:BP$ is a constant.

4. Show that the area of the triangle formed by the tangent at any point on the curve xy=c, (c $\neq$ 0), with the coordinate axes is constant.

5. At any point t on the curve x=a(t+sint), y=a(1-cost), find the lengths of tangent, normal, subtangent and subnormal.

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6. Find the angle between the curves xy=2 and $x^2 + 4y = 0$

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7. S.T the curves $y^2 = 4(x + 1), y^2 = 36(9 - x)$ intersect orthogonally.

8. If $ax^2 + by^2 = 1, a_1x^2 + b_1y^2 = 1$, then show that the condition for orthogonality of above curves is $\dfrac{1}{a} - \dfrac{1}{b} = \dfrac{1}{a_1} - \dfrac{1}{b_1}$

10 3 Rate Measure Saq

1. A particle moving along a straight line has the relation $s = t^3 + 2t + 3$, connecting the distance s

describe by the particle in time t. Find the velocity and acceleration of the particle at t=4 sec.

2. A particle is moving in a straight line so that after 't' seconds its distance is 'S' (in cms) from a fixed point of the line is given be S=f(t)=$8t + t^3$.

Find (i) the velocity at time t=2 (ii) the initial velocity (iii) acceleration at t=2 sec

3. The displacement s of a particle travelling in a straight line in t seconds is given by $s = 45t + 11t^2 - t^3$. Find the time when the particle comes to rest.

4. The distance-time formula for the motion of a particle along a straight line is $s = t^3 - 9t^2 + 24t - 18$. Find when and where the velocity is zero.

5. A particle is moving along a line according to $S = f(t) = 4t^3 - 3t^2 + 5t - 1,$ where S is measured in meters and t is measured in seconds. Find the velocity and acceleration at time t. At what time the acceleration is zero?

6. A stone is dropped into a quiet lake and ripples move in circles at the speed of 5 cm/sec. At the instant when the radius of circular ripple is 8cm, how fast is the enclosed area increases?

7. A balloon which always remains spherical on inflation is being inflated by pumping in 900 cubic centimeters of gas per second. Find the rate at which the radius of balloon increases when the radius in 15 cm.

8. The volume of a cube is increasing at a rate of 9 cubie centimeters per second. How fast is the surface area increasing when the length of edge is 10 cms?

9. Let a kind of bacteria grow by t^3 (t in sec). At what time the rate of growth of the bacteria is 300 bacteria per sec?

10. A container in the shape of an inverted cone has height 12 cm and radius 6cm at the top. If it is filled with water at the rate of $12cm^3/\sec$, what is the rate of change in the rate of change in the height of water level when the tank is filled 8 cm?

11. A point P is moving on the curve $y = 2x^2$. The x coordinate of P is increasing at the rate of 4 units per second . Find the rate at which y coordinate is incerasing when the point is at (2,8).

10 4 Mean Value Theorems Vsaq

1. Verify Rolle's theorem for the function $y = f(x) = x^2 + 4$ on [-3,3]

2. Verify Rolles's theorem for the function $f : [-3, 8] \to R$ be defined by $f(x) = x^2 - 5x + 6$.

3. Verify Rolle's theorem for the functions $f(x) = x(x + 3)e^{-x/2}$ on [-3,0]

4. Verify Rolle's theorem for the functions $\left(x^2 - 1\right)(x - 2)$ on [-1,2]. Find the point in the interval where the derivative vanishes.

5. Let $f(x) = (x - 1)(x - 2)(x - 3)$ then prove that there is more than one 'c' in (1,3) such that $f'(c) = 0$

6. The number of values of k for which the equation

$x^3 - 3x + k = 0$ has two distinct roots lying I the

interval (0,1) is

7. Verify the conditions of Lagrange's mean value

theorem for the function $x^2 - 1$ on [2,3]

8. Find c so that

$$f'(c) = \frac{f(b) - f(a)}{b - a} \quad \text{where} \quad f(x) = e^x, a = 0, b = 1$$

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10 5 Maxima And Minima Laq

1. Find the maximum area of the rectangle that can be formed with fixed perimeter 20.

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2. From a rectangular sheet of dimensions $30cm \times 80cm$, four squares of sides x cm are removed at the corners, and the sides are then turned up so as to form an open rectangular box. What is the value of x, so that the volume of the box is the greatest?

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3. A wire of length l is cut into two parts which are bent respectively in the form of a Square and a circle. What are the lengths of pieces of wire so that the sum of areas is least?

4. A window is in the shape of a rectangle surmounted by a semi-circle. If the perimeter of the window be 20 feet then find the maximum area.

5. Show that when the curved surface of a is right circular cylinder inscribed in a sphere of radius R is maximum , then the height of the cylinder is $\sqrt{2R}$.

6. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

7. Find the positive integers x and y such that $x + y = 60$ and xy^3 is maximum.

8. Find two positive numbers whose sum is 15 so that the sum of their squares is minimum.

1. Find the slope of the tangent to the curve $y = x^3 - x + 1$ at the point whose x co-ordinate is 2.

2. Find the slope of the tangent to the curve, $y = x^3 - 3x + 2$ at the point whose x co-ordinate is 3.

3. Find the slopr of the tangent to the curve $y = \dfrac{x-1}{x-2}$ at $x \neq 2$ and $x = 10$.

4. Find the equation of tangent and normal to the curve $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at (0,5).

5. Find the equations of the tangent and normal to the curve $y = \dfrac{1}{1 + x^2}$ at (0,1).

6. On the curve $y = x^2$, find a point at which the tangent is parallel to the chord joining (0,0) and (1,1).

7. Find a point on the graph of the curve $y = (x - 3)^2$, where the tangent is parallel to the chord joining (3,0) and (4,1)

8. Verify Rolle's theorem for the function sinx-sin2x

on $[0, \pi]$

9. Find the absolute extremum of $f(x) = x^2$ is

defined on [-2,2]

10. Find the points of local extrema for the function

f(x) = cos4x defined on $\left[0, \dfrac{\pi}{2}\right]$

11. Find the maxima and minima of

$$f(x) = \frac{x}{2} + \frac{2}{x}, \ (x > 0).$$

12. Determine the intervals in which

$$f(x) = \frac{2}{(x-1)} + 18x, \ \forall x \in R - \{0\} \quad \text{is} \quad \text{stricly}$$

increasing and decreasing.

13. The profit function p(x) of a company, selling x items per day is given by p(x)=(150-x)x-1600. Find the number of items that the company should sell to get maximum profit. Also find the maximum profit.

14. What is differential of y?

15. How do you define the angle between two intersecting curves?

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16. Define the strictly increasing function and strictly decreasing function on an interval.

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17. Define the increasing function and decreasing function on an interval.

18. What is a monotonic function?

19. Define Critical point of a function.

20. Define Stationery point of a function.

21. Define local maxima and local minima (or) What is second derivative test?

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22. Define Rolles mean value theorem.

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23. Define Lagrange's Mean value theorem.

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1. If $y = x^2 + 3x + 6$ then find Δy and dy when

x=10, Δx=0.1.

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2. Find the approximate value of $\sqrt{25.001}$

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3. Find the approximate value of $\sqrt[4]{17}$

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1. Find the equations of the tangent and the normal to the curve $y = x^3 + 4x^2$ at (-1,3)

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2. Find the equations of tangent and normal to the curve xy = 10 at (2 , 5)

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3. S.T the curves $6x^2 - 5x + 2y = 0, 4x^2 + 8y^2 = 3$

touch each other at $\left(\dfrac{1}{2}, \dfrac{1}{2} \right)$.

4. Find the value of k, so that the length of the

subnormal at any point on the curve $xy^k = a^{k+1}$ is

a constant.

5. Find the length of subtangent, subnormal at a

point on the curve

x=a(cost+sint), y=a(sint-tcost)

6. Find the lengths of subtangent and subnormal at a point on the curve $y = b\sin\left(\dfrac{x}{a}\right)$

7. Find the angle between the curves $x + y + 2 = 0$ and $x^2 + y^2 - 10y = 0$

8. Find the angle between the curve and line given below $:2y - 9x = 0, 3x^2 + 4y = 0$ (in the 4th quadrant)

9. Find the angle between the curve $y^2 = 4x$ and $x^2 + y^2 = 5$

10. Find the angle between the cures given below :

$y^2 = 8x, 4x^2 + y^2 = 32$

10 3 Rate Measure Saq Spq

1. The radius of an air bubble is increasing at the rate of 1/2 cm/sec. At what rate is the volume of the bubble increasing when the radius is 1 cm?

2. The volume of a cube is increasing at a rate of 8 cubie centimeters per second. How fast is the

surface area increasing when the length of the edge

is 12 cm?

3. Let a kind of bacteria grow in such a way that at

time t sec, there are $t^{3/2}$ bacteria.

Find the rate of growth at time t=4 hours.

4. A container is in the shape of an inverted cone

has height 8m and radius 6m at the top. If it is filled

with water at the rate of $2m^3$/minute, how fast is the height of water changing when the level is 4m?

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10 4 Mean Value Theorems Vsaq Spq

1. Verify Rolle's theroem for the function $x^2 - 1$ on [-1,1]`.

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2. Verify Rolle's theorem of the function $\log(x^2 + 2) - \log 3$ on (-1,1)

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3. Verify Rolle's theorem for the function sinx-sin2x on $[0, \pi]$

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4. Verify mean value theorem for the function $f(x) = x^2$ on [2,4]

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10 5 Maxima And Minima Laq Spq

1. Find two positive integers whose sum is 16 and the

sum of squares is minimum.

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10 5 Maxima And Minima Vsaq Saq Spq

1. Find a point on the curve $y = x^3$, when the

tangent is parallel to the chord joining (1,1), (3,27).

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