



## MATHS

### BOOKS - TELUGU ACADEMY MATHS (TELUGU ENGLISH)

#### PROPERTIES OF TRIANGLES

Spq

1. Prove that  $r + r_1 + r_2 - r_3 = 4R \cos C$

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2. In  $\triangle ABC$ , if  $AD$ ,  $BE$ ,  $CF$  are the perpendiculars drawn from the vertices  $A, B, C$  to the opposite sides, show that

$$\frac{1}{AD} + \frac{1}{BE} + \frac{1}{CF} = \frac{1}{r} \quad \text{and} \quad (ii) AD \cdot BE \cdot CF = \frac{(abc)^2}{8R^3} = \frac{8\Delta^3}{abc}$$

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3. If  $\sin \theta = \frac{a}{b+c}$  then show that  $\cos \theta = \frac{2\sqrt{bc}}{b+c} \cos\left(\frac{A}{2}\right)$

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4. In  $\triangle ABC$  show that

$$\sin^2 \frac{A}{2} + \sin^2 \frac{B}{2} + \sin^2 \frac{C}{2} = 1 - \frac{r}{2R} \quad (\text{ii})$$

$$\cos A + \cos B + \cos C = 1 + \frac{r}{R}$$

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5. Let an object be placed at some height  $h$  cm and let  $P$  and  $Q$  two points of observation which are at a distance 10 cm apart on a line inclined at angle  $15^\circ$  to the horizontal. If the angles of elevation of the object from  $P$  and  $Q$  are  $30^\circ$  and  $60^\circ$  respectively then find  $h$ .

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1. Show that  $\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c} = \frac{a^2 + b^2 + c^2}{2abc}$

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2. Prove that  $\frac{a}{bc} + \frac{\cos A}{a} = \frac{b}{ca} + \frac{\cos B}{b} = \frac{c}{ab} + \frac{\cos C}{c}$

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3. Prove that  $\cot A + \cot B + \cot C = \frac{a^2 + b^2 + c^2}{4\Delta}$

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4. In  $\Delta ABC$ , if  $\frac{1}{a+c} + \frac{1}{b+c} = \frac{3}{a+b+c}$  then show that  $C = 60^\circ$

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5. If  $C = 60^\circ$ , then show that  $\frac{a}{b+c} + \frac{b}{c+a} = 1$

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6. In a  $\Delta ABC$  if  $a:b:c = 7:8:9$  then show that  $\cos A : \cos B : \cos C = 14:11:6$

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7. Show that  $a^2 \cot A + b^2 \cot B + c^2 \cot C = \frac{abc}{R}$

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8. Prove that  $\frac{1 + \cos(A - B)\cos C}{1 + \cos(A - C)\cos B} = \frac{a^2 + b^2}{a^2 + c^2}$

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9. Show that  $b^2 \sin 2C + c^2 \sin 2B = 2bc \sin A$

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10. In a  $\triangle ABC$  if  $a \cos A = b \cos B$  the prove that triangle is either isosceles or right angled .

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11. In  $\triangle ABC$  show that  $(a + b + c) \left( \tan. \frac{A}{2} + \tan. \frac{B}{2} \right) = 2 \cot. \frac{C}{2}$

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12. If  $\frac{\cot A}{2} : \cot \frac{B}{2} : \cot \frac{C}{2} = 3 : 5 : 7$  then show that  $a : b : c = 6 : 5 : 4$ .

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13. Prove that  $\cot. \frac{A}{2} + \cot. \frac{B}{2} + \cot. \frac{C}{2} = \frac{s^2}{\Delta}$

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14. Prove that  $\tan. \frac{A}{2} + \tan. \frac{B}{2} + \tan. \frac{C}{2} = \frac{bc + ca + ab - s^2}{\Delta}$

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15. If  $\cot. \frac{A}{2} + \cot. \frac{B}{2} + \cot. \frac{C}{2}$  are in A.P , then prove that a, b, c are in A.P

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16. If  $\sin^2. \frac{A}{2} + \sin^2. \frac{B}{2} + \sin^2. \frac{C}{2}$  are in H.P , show that a , b, c are in H.P

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17. In  $\triangle ABC$  prove that  $\tan\left(\frac{B - C}{2}\right) = \frac{b - c}{b + c} \cot \frac{A}{2}$

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18. Show that  $\frac{1}{r^2} + \frac{1}{r_1^2} + \frac{1}{r_2^2} + \frac{1}{r_3^2} = \frac{a^2 + b^2 + c^2}{\Delta^2}$

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## Laq

1. Show that  $r(r_1 + r_2 + r_3) = ab + bc + ca - s^2$ .

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2. Prove that  $\frac{r_1(r_2 + r_3)}{\sqrt{r_1 r_2 + r_2 r_3 + r_3 r_1}} = a$

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3. Prove that  $4(r_1r_2 + r_2r_3 + r_3r_1) = (a + b + c)^2$

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4. Prove that  $\left(\frac{1}{r} - \frac{1}{r_1}\right)\left(\frac{1}{r} - \frac{1}{r_2}\right)\left(\frac{1}{r} - \frac{1}{r_3}\right) = \frac{abc}{\Delta^3} = \frac{4R}{r^2s^2}$

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5. Show that  $(r_1 + r_2)\sec^2 \frac{C}{2} = (r_2 + r_3)\sec^2 \frac{A}{2} = (r_3 + r_1)\sec^2 \frac{B}{2}$

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6. Show that  $r_1 + r_2 + r_3 - r = 4R$

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7. Show that  $r + r_3 + r_1 - r_2 = 4R \cos B$ .





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8. If  $A, A_1, A_2, A_3$  are the areas of incircle and ex-circle of a triangle respectively then prove that  $\frac{1}{\sqrt{A_1}} + \frac{1}{\sqrt{A_2}} + \frac{1}{\sqrt{A_3}} = \frac{1}{\sqrt{A}}$



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9. In a  $\Delta ABC$  if  $a = 13, b = 14, c = 15$  then show that  $R = \frac{65}{8}, r = 4, r_1 =$



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10. In a  $\Delta ABC$  if  $r_1 = 8, r_2 = 12, r_3 = 24$  find  $a, b, c$ .



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11. If  $r_1 = 2, r_2 = 3, r_3 = 6$  and  $r = 1$ , prove that  $a=3, b=4$  and  $c=5$ .



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12. In  $\Delta ABC$  prove that  $\frac{r_1}{bc} + \frac{r_2}{ca} + \frac{r_3}{ab} = \frac{1}{r} - \frac{1}{2R}$



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13. In  $\Delta ABC$ , show that  $\frac{ab - r_1 r_2}{r_3} = \frac{bc - r_2 r_3}{r_1} = \frac{ca - r_3 r_1}{r_2}$



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14. If  $p_1, p_2, p_3$  are altitudes of a  $\Delta ABC$  then show that

$$\frac{1}{p_1} + \frac{1}{p_2} + \frac{1}{p_3} = \frac{1}{r}$$



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15. If  $P_1, P_2, P_3$  are altitudes of a  $\Delta ABC$  then show that

$$\frac{1}{P_1} + \frac{1}{P_2} - \frac{1}{P_3} = \frac{1}{r_3}$$

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16. If  $P_1, P_2, P_3$  are altitudes of a  $\Delta ABC$  then show that

$$P_1 P_2 P_3 = \frac{(abc)^2}{8R^3} = \frac{8\Delta^3}{abc}$$

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17. In  $\Delta ABC$ , with usual notation show that

$$\frac{(a+b+c)^2}{a^2+b^2+c^2} = \frac{\cot \frac{A}{2} + \cot \frac{B}{2} + \cot \frac{C}{2}}{\cot A + \cot B + \cot C}$$

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18. If  $a = (b+c)\cos\theta$ , then prove that  $\sin\theta = \frac{2\sqrt{bc}}{b+c} \cos\left(\frac{A}{2}\right)$

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19. If  $a = (b-c)\sec\theta$ , then prove that  $\tan\theta = \frac{2\sqrt{bc}}{b-c} \frac{\sin A}{2}$ .



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20. In  $\triangle ABC$  prove that  $a \cos^2 \frac{A}{2} \cdot b \cos^2 \frac{B}{2} \cdot c \cos^2 \frac{C}{2} = s + \frac{A}{R}$



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Laq Saq Vsaq 2 Dhardq 3 Dmis Q

1. If  $\frac{a^2 + b^2}{a^2 - b^2} = \frac{\sin C}{\sin(A - B)}$ , then S.T.  $\triangle ABC$  is either isocoles or right angled triangle .



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2. In a  $\triangle ABC$  if  $a^2 + b^2 + c^2 = 8R^2$  then show that  $\triangle ABC$  is a right angled triangle.



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3. In  $\triangle ABC$  show that  $\sum a^3 \cos(B - C) = 3abc$

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4. In  $\triangle ABC$  prove that  $\cos^2 \frac{A}{2} + \cos^2 \frac{B}{2} + \cos^2 \frac{C}{2} = 2 + \frac{r}{2R}$

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5. If  $P_1, P_2, P_3$  are the altitudes of the  $\triangle ABC$  then, show that

$$\frac{1}{P_1^2} + \frac{1}{P_2^2} + \frac{1}{P_3^2} = \frac{\cot A + \cot B + \cot C}{\Delta}$$

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6. In  $\triangle ABC$  prove that  $(b - c)^2 \cos^2 \frac{A}{2} + (b + c)^2 \sin^2 \frac{A}{2} = a^2$

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7. Show that  $\frac{c - b \cos A}{b - c \cos A} = \frac{\cos B}{\cos C}$

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8. If  $b + c = 3a$ , then find the value of  $\cot \frac{B}{2} \cot \frac{C}{2}$

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9. Show that  $2(bc \cos A + ca \cos B + ab \cos C) = a^2 + b^2 + c^2$ .

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10. Prove that  $a(b \cos C - c \cos B) = b^2 - c^2$

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11. If  $a = 26$ ,  $b = 30$ ,  $\cos C = 63/65$ , then find  $c$



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12. If  $\tan. \frac{A}{2} = \frac{5}{6}$  and  $\tan. \frac{C}{2} = \frac{2}{5}$  then determine the relation between a, b, c



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13. Prove that  $\frac{a^2 + b^2 - c^2}{c^2 + a^2 - b^2} = \frac{\tan B}{\tan C}$



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14. What is the value of  $r/R$  in an equilateral triangle .



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15. If  $rr_2 = r_1r_3$  then find B.



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16. Express  $\sum r_1 \cot. \frac{A}{2}$  in terms of  $s$ .

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17. Show that  $rr_1 \cot. \frac{A}{2} = \Delta$ .

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18. In  $\triangle ABC$ , show that the sides  $a, b, c$  are in A.P., if and only if  $r_1, r_2, r_3$  are in H.P.

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19. If the lengths of the sides of a triangle are 3, 4, 5, find the circumradius of the triangle.

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20. If the sides of a triangle are 13,14,15 , then find circum diameter .

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21. If  $a=6$  ,  $b=5$  , $c=9$  then angle A .

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22. If  $a =2$  ,  $b = 3$   $c=4$  then find  $\cos A$  .

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23. Show that  $\sum (b + c)\cos A = 2s$  Prove that  $(b + c)\cos A + (c + a)\cos B + (a + b)\cos C = a + b + c$

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24. If  $(a+b+c)(b+c-a) = 3bc$  then find angle A.

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25. In  $\triangle ABC$  prove that  $(b-c)^2 \cos^2 \frac{A}{2} + (b+c)^2 \sin^2 \frac{A}{2} = a^2$

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26. If  $\tan \frac{C-A}{2} = k \cot \frac{B}{2}$  then find k.

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27. Show that  $\frac{c - b \cos A}{b - c \cos A} = \frac{\cos B}{\cos C}$

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28. In  $\triangle ABC$ , find  $b \cos^2 \frac{C}{2} + c \cos^2 \frac{B}{2}$

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29. If  $\cot. \frac{A}{2} = \frac{b+c}{a}$  find angle B

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30. Show that  $b^2 \sin 2C + c^2 \sin 2B = 2bc \sin A$

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31. In  $\triangle ABC$  show that  $\frac{b^2 - c^2}{a^2} = \frac{\sin(B - C)}{\sin(B + C)}$

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32. Show that  $a \cos A + b \cos B + c \cos C = \frac{2\Delta}{R}$

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33. Show that  $a^2 \sin 2C + c^2 \sin 2A = 4\Delta$

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34. In  $\Delta ABC$ , prove that  $\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{1}{r}$ .

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35. Show that  $rr_1r_2r_3 = \Delta^2$

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36. If  $A = 90^\circ$ , show that  $2(r + R) = b + c$

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37. If  $s = 12$  and  $A = 90^\circ$  then find the value of  $r_1$



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38. If  $a = 18$ ,  $b=24$ ,  $c=30$ , find  $r_1$ .



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39. If  $a, b, c$  are in A.P., then show that  $3 \tan. \frac{A}{2} \tan. \frac{C}{2} = 1$



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40. If  $a \cos^2. \frac{C}{2} + c \cos^2 \frac{A}{2} = \frac{3b}{2}$ , then show that  $a, b, c$  are in A.P



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41. Two trees A and B are on the same side of a river. From a point C in the river the distance of the trees A and B are 250m and 300m respectively

. If the angle C is  $45^\circ$  find the distances between the trees (use  $\sqrt{2} = 1.414$ )

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42. The upper  $3/4^{th}$  portion of a vertical pole subtends an angle  $\tan^{-1}(3/5)$  at a point in the horizontal plane through its foot and at a distance 40 m from the foot . Given that the vertical pole is at a height less than 100m from the ground ,find its height .

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43. AB is a vertical pole with B at the ground level and and A at the top .A man finds that the angle of elevation of the point A from a certain point C on the ground is  $60^\circ$  . He moves away from the pole along the line BC to a point D such that  $CD = 7$  m .From D . the angle of elevation of the point A is  $45^\circ$  .Find the height of the pole .

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**44.** The angle of elevation of the top point P of the vertical tower PQ of height h from a point A is  $45^\circ$  and from a point B is  $60^\circ$ , where B is a point at a distance 30 meters from the point A measured along the line AB which makes an angle  $30^\circ$  with AQ. Then the height of the tower is.



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