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## MATHS

# BOOKS - VIKAS GUPTA MATHS (HINGLISH) 

## APPLICATION OF DERIVATIVES

## Exercise Single Choice Problems

1. The difference between the maximum and minimum value of the function $f(x)=3 \sin ^{4} x-\cos ^{6} x$ is :
A. $\frac{3}{2}$
B. $\frac{5}{2}$
C. 3
D. 4

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2. A function $y=f(x)$ has a second order derivative $f(x)=6(x-1)$. If its graph passes through the point $(2,1)$ and at that point the tangent to the graph is $y=3 x-5$ then the function is
A. $(x-1)^{2}$
B. $(x-1)^{3}$
C. $(x+1)^{3}$
D. $(x+1)^{2}$

## Answer: B

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3. If the subnormal at any point on the curve $y=3^{1-k} . x^{k}$ is of constant length the $k$ equals to :
A. $\frac{1}{2}$
B. 1
C. 2
D. 0

## Answer: A

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4. If $x^{5}-5 q x+4 r$ is divisible by $(x-c)^{2}$ then which of the following must hold true
A. $q=r$
B. $q+r=0$
C. $q^{5}+r=0$
D. $q^{4}=r^{5}$

## Answer: C

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5. A spherical iron ball 10 cm in radius is coated with a layer of ice of unirform thichness that melts at a rate of $50 \mathrm{~cm}^{3} / \mathrm{min}$. when the thickness of ice is 5 cm , then the rate at which the thickness of ice decreases, is
A. $\frac{1}{36 \pi} \mathrm{~cm} / \mathrm{min}$
B. $\frac{1}{18 \pi} \mathrm{~cm} / \min$
C. $\frac{1}{54 \pi} \mathrm{~cm} / \mathrm{min}$
D. $\frac{5}{6 \pi} \mathrm{~cm} / \mathrm{min}$

## Answer: B

6. If $f(x)=\frac{(x-1)(x-2)}{(x-3)(x-4)}$, then number of local extemas for $\mathrm{g}(\mathrm{x})$, where $g(x)=f(|x|)$ :
A. 3
B. 4
C. 5
D. None of these

## Answer: C

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7. Two straight roads OA and OB intersect at an angle $60^{\circ}$. $A$ car approaches O from A, where $O A=700 \mathrm{~m}$ at a uniform speed of $20 \mathrm{~m} / \mathrm{s}$, Simultaneously, a runner starts running from O towards B at a uniform speed of $5 \mathrm{~m} / \mathrm{s}$. The time after start when the car and the runner are closest is :
A. 10 sec
B. 15 sec
C. 20 sec
D. 30 sec

## Answer: D

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8. Let $f(x)=\left\{\begin{array}{ll}a-3 x & -2 \leq x<0 \\ 4 x \pm 3 & 0 \leq x<1\end{array}\right.$, if $f(x)$ has smallest
valueat $x=0$, then range of a, is
A. $(-\infty, 3)$
B. $(-\infty, 3]$
C. $(3, \infty)$
D. $(-3, \infty)$

## Answer: D

9. If $f(x)=\left\{3+|x-k|, x \leq k a^{2}-2+\frac{s n(x-k)}{x-k}, x>k \quad\right.$ has minimum at $x=k$, then $a \in R$ b. $|a|<2 \mathrm{c} .|a|>2 \mathrm{~d} .1<|a|<2$
A. $a \in R$
B. $|a|<2$
C. $|a|>2$
D. $1<|a|<2$

## Answer: C

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10. For a certain curve $\frac{d^{2} y}{d x^{2}}=6 x-4$ and curve has local minimum values $5 a t x=1$, Let the global maximum and global minimum vlaues, where $0 \leq x \leq 2$, are $M$ and $m$. Then the vlaue of ( $M-m$ ) equals to :
A. -2
B. 2
C. 12
D. -12

## Answer: B

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11. The tangent to $y=a x^{2}+b x+\frac{7}{2} a t(1,2)$ is parallel to the normal at the point $(-2,2)$ on the curve $y=x^{2}+6 x+10$. Then the vlaue of $\frac{a}{2}-b$ is:
A. 2
B. 0
C. 3
D. 1

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12. If ( $\mathrm{a}, \mathrm{b}$ ) be the point on the curve $9 y^{2}=x^{3}$ where normal to the curve make equal intercepts with the axis, then the value of $(a+b)$ is:
A. 0
B. $\frac{10}{3}$
C. $\frac{20}{3}$
D. None of these

## Answer: C

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13. For a certain curve $y=f(x)$ satisfying
$\frac{d^{2} y}{d x^{2}}=6 x-4, \mathrm{f}(\mathrm{x})$ has a local minimum value 5 when $\mathrm{x}=1$, Find the
equation of the curve and also the gobal maximum and global minimum values of $\mathrm{f}(\mathrm{x})$ given that $0 \leq x \leq 2$.
A. 1
B. 0
C. 5
D. None of these

## Answer: C

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14. Let $A$ be the point where the curve $5 \alpha^{2} x^{3}+10 \alpha x^{2}+x+2 y-4=0(\alpha \in R, \alpha \neq 0)$ meets the $y$-axis, then the equation of tangent to the curve at the point where normal at

A meets the curve again, is:
A. $x-\alpha y+2 \alpha=0$
B. $\alpha x+y-2=0$
C. $2 x-y+2=0$
D. $x+2 y-4=0$

## Answer: C

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15. The difference between the greatest and the least value of the function $f(x)=\cos x+\frac{1}{2} \cos 2 x-\frac{1}{3} \cos 3 x$
A. $\frac{11}{5}$
B. $\frac{13}{6}$
C. $\frac{9}{4}$
D. $\frac{7}{3}$

## Answer: C

16. The ordinate of point on the curve $y=\sqrt{x}$ which is closest to the point $(2,1)$ is
A. $\frac{2+\sqrt{3}}{2}$
B. $\frac{1+\sqrt{2}}{2}$
C. $\frac{-1+\sqrt{3}}{2}$
D. 1

## Answer: A

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17. The tangent at a point $P$ on the curve $y=\ln \left(\frac{2+\sqrt{4-x^{2}}}{2-\sqrt{4-x^{2}}}\right)-\sqrt{4-x^{2}}$ meets the $y$-axis at T , then $P T^{2}$ equals to :
A. 2
B. 4
C. 8
D. 16

## Answer: B

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18. Let

$$
f(x)=\int_{x^{2}}^{x^{3}} \frac{d t}{\ln t}
$$

$x>1$ and $g(x)=\int_{1}^{x}\left(2 t^{2}-\ln t\right) f(t) d t(x>1)$, then:
A. $g$ is increasing on $(1, \infty)$
B. $g$ is decreasing on $(1, \infty)$
C. g is increasing on ( 1,20 and decreasing on $(2,00)$
D. $g$ is decreasing on $(1,2)$ and increasing on $(2, \infty)$

## Answer: A

19. Let $f(x)=x^{3}+6 x^{2}+a x+2$, if $(-3,-1)$ is the largest possible interval for which $f(x)$ is decreasing function, then $a=$
A. 3
B. 9
C. -2
D. 1

## Answer: B

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20. Let $f\left(x-=\tan ^{-1}\left(\frac{1-x}{1+x}\right)\right.$. Then difference of the greatest and least value of $f(x)$ on $[0,1]$ is:
A. $\pi / 2$
B. $\pi / 4$
C. $\pi$
D. $\pi / 3$

## Answer: B

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21. The number of integral values of $a$ for which
$f(x)=x^{3}+(a+2) x^{2}+3 a x+5$ is monotonic in $\forall x \in R$
A. 2
B. 4
C. 6
D. 7

## Answer: B

22. The number of of critical
$f(x)=\left(\int_{0}^{x}\left(\cos ^{2} t-\sqrt[3]{t}\right) d t\right)+\frac{3}{4} x^{4 / 3}-\frac{x+1}{2}$ in $(0,6 \pi]$ is:
A. 10
B. 8
C. 6
D. 12

## Answer: D

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23. Let $f(x)=\min \left[\frac{1}{2}-3 \frac{x^{2}}{4}, 5 \frac{x^{2}}{4}\right]$, for $0 \leq x \leq 1$ then maximum value of $f(x)$ is
A. 0
B. $\frac{5}{64}$
C. $\frac{5}{4}$
D. $\frac{5}{16}$

## Answer: D

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24. Let $f(x)= \begin{cases}2-\left|x^{2}+5 x+6\right| & x \neq-2 \\ b^{2}+1 & x=-2\end{cases}$

Has relative maximum at $x=-2$, then complete set of values b can take is:
A. $|b| \geq 1$
B. $|b|<1$
C. $b>1$
D. $b<1$

## Answer: A

25. Let for function $f(x)=\left[\begin{array}{ll}\cos ^{-1} x & -1 \leq x \leq 0 \\ m x+c & 0<x \leq 1\end{array}\right.$, Lagrange's mean value theorem is applicable in $[-1,1]$ then ordered pair $(m, c)$ is:
A. $\left(1,-\frac{\pi}{2}\right)$
B. $\left(1, \frac{\pi}{2}\right)$
C. $\left(-1,-\frac{\pi}{2}\right)$
D. $\left(-1, \frac{\pi}{2}\right)$

## Answer: D

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26. Tangents are drawn from the origin to the curve $y=\cos X$. Their points of contact lie on
A. $\frac{1}{x^{2}}=\frac{1}{y^{2}}+1$
B. $\frac{1}{x^{2}}=\frac{1}{y^{2}}-2$
C. $\frac{1}{y^{2}}=\frac{1}{x^{2}}+1$
D. $\frac{1}{y^{2}}=\frac{1}{x^{2}}-2$

## Answer: C

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27. The least natural number $a$ for which $x+a x^{-2}>2 \forall x \in(0, \infty)$ is

1 (b) 2 (c) 5 (d) none of these
A. 1
B. 2
C. 5
D. None of these

Answer: B

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28. Angle between the tangents to the curve $y=x^{2}-5 x+6$ at the points $(2,0)$ and $(3,0)$ is: (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{4}$
A. $\frac{\pi}{6}$
B. $\frac{\pi}{4}$
C. $\frac{\pi}{3}$
D. $\frac{\pi}{2}$

## Answer: D

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29. Difference between the greatest and least values opf the function $f(x)=\int_{0}^{x}\left(\cos ^{2} t+\cos t+2\right) \mathrm{dt}$ in the interval $[0,2 \pi]$ is $K \pi$, then K is equal to:
A. 1
B. 3
C. 5
D. None of these

## Answer: C

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30. The range of the function $f(\theta)=\frac{\sin \theta}{\theta}+\frac{\theta}{\tan \theta}, \theta \in\left(0, \frac{\pi}{2}\right)$ is equal to :
A. $(0, \infty)$
B. $\left(\frac{1}{\pi}, 2\right)$
C. $(2, \infty 0$
D. $\left(\frac{2}{\pi}, 2\right)$

## Answer: D

31. Number of integers in the range of ' $c$ ' so that the equation $x^{3}-3 x+c=0$ has all its roots real and distinct is
A. 2
B. 3
C. 4
D. 5

## Answer: B

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32. Let $f(x)=\int e^{x}(x-1)(x-2) d x$, then $\mathrm{f}(\mathrm{x})$ decrease in the interval
A. $(2, \infty)$
B. $(-2,-1)$
C. $(1,2)$
D. $(-\infty, 1) i i(2, \infty)$

## Answer: C

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33. If the cubic polymomial $y=a x^{3}+b x^{2}+c x+d(a, b, c, d \in R)$ has only one critical point in its entire domain and $a c=2$, then the value of $|b|$ is:
A. $\sqrt{2}$
B. $\sqrt{3}$
C. $\sqrt{5}$
D. $\sqrt{6}$

## Answer: D

34. On the curve $y=\frac{1}{1+x^{2}}$, the point at which $\left|\frac{d y}{d x}\right|$ is greatest in the first quadrant is :
A. $\left(\frac{1}{2}, \frac{4}{5}\right)$
B. $\left(1, \frac{1}{4}\right)$
C. $\left(\frac{1}{\sqrt{2}}, \frac{2}{3}\right)$
D. $\left(\frac{1}{\sqrt{3}}, \frac{3}{4}\right)$

## Answer: D

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35. If $f_{1}(x)=2 x, f_{2}(x)=3 \sin x-x \cos x$ then for $x \in\left(0, \frac{\pi}{2}\right)$
A. $f(x)>g(x)$
B. $f(x)<g(x)$
C. $f(x)=g(x)$ has exactly one real root.
D. $f(x)=g(x)$ has exactly two real roots

## Answer: A

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36. let $f(x)=\sin ^{-1}\left(\frac{2 g(x)}{1+g(x)^{2}}\right)$, then which are correct ?
(i) $\mathrm{f}(\mathrm{x})$ is decreasing if $g(x)$ is increasig and $\mid g(x)>1$
(ii) $f(x)$ is an increasing function if $g(x)$ is increasing and $|g(x)| \leq 1$
(iii) $\mathrm{f}(\mathrm{x})$ is decreasing function if $f(x)$ is decreasing and $|g(x)|>1$
A. (i) and (iii)
B. (i) and (ii)
C. (i) (ii) and (iii)
D. (iii)

## Answer: B

37. The graph of the function $y=f(x)$ has a unique tangent at the point $\left(e^{a}, 0\right)$ through which the graph passes then $\underline{\lim }\left(x \rightarrow e^{a}\right) \frac{\log _{e}\{1+7 f(x)\}-\sin f(x)}{3 f(x)}$
A. 1
B. 3
C. 2
D. 7

## Answer: C

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38. Let $f(x)$ be a function such that $f^{\prime}(x)=\log _{1 / 3}\left(\log _{3}(\sin x+a)\right)$. The complete set of values of 'a' for which $f(x)$ is strictly decreasing for all real values of $x$ is:
A. $[4, \infty)$
B. $[3,4]$
C. $(-\infty, 4)$
D. $[2, \infty)$

## Answer: A

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39. If $f(x)=a \ln |x|+b x^{2}+x$ has extremas at $x=1$ and $x=3$ then:
A. $a=\frac{3}{4}, b=-\frac{1}{8}$
B. $a=\frac{3}{4}, b=\frac{1}{8}$
C. $a=-\frac{3}{4}, b=-\frac{1}{8}$
D. $a=-\frac{3}{4}, b=\frac{1}{8}$

## Answer: C

40. Let $f(x)= \begin{cases}1+\sin x & x<0 \\ x^{2}-x+1 & x \geq 0\end{cases}$
A. f has a local maximum at $x=0$
B. f has a local minimum at $x=0$
C. $f$ is increasing everywhere
D. f is decreasing everywhere

## Answer: A

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41. If $m$ and $n$ are positive integers and
$f(x)=\int_{1}^{x}(t-a)^{2 n}(t-b)^{2 m+1} d t, a \neq b$, then
A. $x=b$ is a point of local minimum
B. $x=b$ is a point of local maximum
C. $x=a$ is a point of local minimum
D. $x=a$ is a point of local maximum

## Answer: A

## D Watch Video Solution

42. For any the real $\theta$ the maximum value of $\cos ^{2}(\cos \theta)+\sin ^{2}(\sin \theta)$ is
A. 1
B. $1+\sin ^{2} 1$
C. $1+\cos ^{2} 1$
D. Does not exist

## Answer: B

43. If the tangentat P of the curve $y^{2}=x^{3}$ intersect the curve again at Q and the straigta line $O P, O Q$ have inclinations $\alpha$ and $\beta$ where O is origin, then $\frac{\tan \alpha}{\tan \beta}$ has the value equals to
A. -1
B. -2
C. 2
D. $\sqrt{2}$

## Answer: B

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44. If $x+4 y=14$ is a normal to the curve $y^{2}=\alpha x^{3}-\beta$ at $(2,3)$, then the value of $\alpha+\beta$ is 9 (b) -5 (c) 7 (d) -7
A. 9
B. -5
C. 7
D. -7

## Answer: A

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45. The tangent to the curve $y=e^{k x}$ at a point $(0,1)$ meets the $x$-axis at (a,0), where $a \in[-2,-1]$. Then $k \in\left[-\frac{1}{2}, 0\right]$ (b) $\left[-1,-\frac{1}{2}\right]$
$[0,1]$ (d) $\left[\frac{1}{2}, 1\right]$
A. $\left[-\frac{1}{2}, 0\right]$
B. $\left[-1-\frac{1}{2}\right]$
C. $[0,1]$
D. $\left[\frac{1}{2}, 1\right]$

Answer: D
46. Which of the following graph represent the function
$f(x)=\int_{0}^{\sqrt{x}} e^{\frac{u^{2}}{x}} \mathrm{du}$, for $x>0$ and $f(0)=o$

A.

B.
C.


D.
47. Let $f(x)=(x-a)(x-b)(x-c)$ be a ral vlued function where $a<b c(a, b, c \in R)$ such that $f^{\prime \prime}(\alpha)=0$. Then if $\alpha \in\left(c_{1}, c_{2}\right)$, which one of the following is correct?
A. $\alpha<c_{1}<b$ and $b<c_{2}<c$
B. $\alpha<c_{1}, c_{2}<b$
C. $b<c_{1}, c_{2}<c$
D. None of these

## Answer: A

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48. $f(x)=x^{6}-x-1, x \in[1,2]$. Consider the following statements :
A. $f$ is increasing on $[1,2]$
B. $f$ has a root in $[1,2]$
C. $f$ is decreasing on $[1,2]$
D. $f$ has no root in $[1,2]$

## Answer: A

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49. Which one of the following curves is the orthogonal trajectory of straight lines passing through a fixed point $(\mathrm{a}, \mathrm{b})$ ?
A. $x-a=k(y-b)$
B. $(x-a)(y-b)=k$
C. $(x-a)^{2}=k(y-b)$
D. $(x-a)^{2}+(y-b)^{2}=k$

## Answer: D

50. The function $f(x)=\sin ^{3} x-m \sin x$ is defined on open interval ( $-\frac{\pi}{2}, \frac{\pi}{2}$ ) and if assumes only 1 maximum value and only 1 minimum value on this interval. Then, which one of the must be correct?
A. $0<m<3$
B. $-3<m<0$
C. $m>3$
D. $m<-3$

## Answer: A

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51. The greatest of the numbers $2^{\frac{1}{2}}, 3^{\frac{1}{3}}, 4^{\frac{1}{4}}, 5^{\frac{1}{5}}, 6^{\frac{1}{6}}$ and $7^{\frac{1}{7}}$ is
A. $2^{1 / 2}$
B. $3^{1 / 3}$
C. $7^{1 / 7}$
D. $6^{1 / 6}$

## Answer: B

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52. Let I be the line through $(0,0)$ an tangent to the curve $y=x^{3}+x+16$. Then the slope of I equal to :
A. 10
B. 11
C. 17
D. 13

## Answer: D

53. The slope of the tangent at the point of inflection of $y=x^{3}-3 x^{2}+6 x+2009$ is equal to :
A. 2
B. 3
C. 1
D. 4

## Answer: B

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54. Let f be a real valued function with $(n+1)$ derivatives at each point of R. For each pair of real numbers $a, b, a<b$, such that
$\ln \left[\frac{f(b)+f(b)+\ldots .+f^{(n)}(b)}{f(a)+f^{\prime}(a)+\ldots .+f^{(n)}(a)}\right]$
Statement-1 : There is a number $c \in h(a, b)$ for which $f^{(n+1)}(c)=f(c)$

Statement-2: If $h(x)$ be a derivable function such that $h(p)=h(q)$ then by Rolle's theorem $h^{\prime}(d)=9, d \in(p, q)$
A. Statement-1 is true, statemet-2 is true and statement-2 is correct explanation for statement-1
B. Statement- 1 is true, statement- 2 is true and statement- 2 is not correct explanation for statement-1
C. Statement-1 is true, statement-2 is false
D. Statement-1 is false, statement-2 is true

## Answer: A

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55. If $f(x)$ is a differentiable real valued function satisfying
$f^{\prime \prime}(x)-3 f^{\prime}(x)>3 \forall x \geq 0$ and $f^{\prime}(0)=-1$, then
$f(x)+x \forall x>0$ is
A. strictly increasing
B. strictly decreasing
C. non monotonic
D. data insufficient

## Answer: A

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56. If the line joining the points $(0,3) \operatorname{and}(5,-2)$ is a tangent to the curve $y=\frac{C}{x+1}$, then the value of $c$ is 1 (b) -2 (c) 4 (d) none of these
A. 2
B. 3
C. 4
D. 5
57. Number of solutions (s) of in $|\sin x|=-x^{2}$ if $x \in\left[-\frac{\pi}{2}, \frac{3 \pi}{2}\right]$ is/are:
A. 2
B. 4
C. 6
D. 8

## Answer: B

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58. Find the value of a for which $\sin ^{-1} x=|x-a|$ will have at least one solution.
A. $[-1,1]$
B. $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
C. $\left[1-\frac{\pi}{2}, 1+\frac{\pi}{2}\right]$
D. $\left[\frac{\pi}{2}-1, \frac{\pi}{2}+1\right]$

## Answer: C

## (D) Watch Video Solution

59. For any pal number $b$, let $f(b)$ denotes the maximum of $\left|\sin x+\frac{2}{3+\sin x}+b\right| \forall \times x \in R$. Then the minimum valur of $f(b) \forall b \in R$ is:
A. $\frac{1}{2}$
B. $\frac{3}{4}$
C. $\frac{1}{4}$
D. 1
60. Which of the following are correct
A. $x^{4}+2 x^{2}-6 x+2=0$ has exactly four real solution
B. $x^{3}+5 x+1=0$ has exactly three real solutions
C. $x^{n}+a x+b=0$ where n is an even natural number has atmost two real solution $a, b$, in R .
D. $x^{3}-3 x+c=0, x>0$ has two real solutin for $x \in(0,1)$

## Answer: C

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61. For any real number $b$, let $f(b)$ denotes the maximum of $\left|\sin x+\frac{2}{3+\sin x}+b\right| \forall x \in R$. Then the minimum vlaue of $f(b) \forall b \in R$ is:
A. $\frac{1}{2}$
B. $\frac{3}{4}$
C. $\frac{1}{4}$
D. 1

## Answer: B

## D Watch Video Solution

62. Find the coordinates of the point on the curve $y=\frac{x}{1+x^{2}}$ where the tangent to the curve has the greatest slope.
A. $(0,0)$
B. $\left(\sqrt{3}, \frac{\sqrt{3}}{4}\right)$
C. $\left(-\sqrt{3},-\frac{\sqrt{3}}{4}\right)$
D. $\left(1, \frac{1}{2}\right)$
63. Let $f:[0,2 p] \rightarrow[-3,3]$ be a given function defined at $f(x)=\cos \frac{\pi}{2}$. The slope of the tangent to the curve $y=f^{-1}(x)$ at the point where the curve crosses the $y$-axis is:
A. -1
B. $-\frac{2}{3}$
C. $-\frac{1}{6}$
D. $-\frac{1}{3}$

## Answer: B

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64. Number of stationary points in $[0, p o]$ for the function
$f(x)=\sin x+\tan x-2 x$ is:
A. 0
B. 1
C. 2
D. 3

## Answer: C

## D Watch Video Solution

65. If $\mathrm{a}, \mathrm{b}, \mathrm{c} \mathrm{d} \in R$ such that $\frac{a+2 c}{b+3 d}+\frac{4}{3}=0$, then the equation $a x^{3}+b x^{3}+c x+d=0$ has
A. atleast one root in $(-1,0)$
B. atleast one root in $(0,1)$
C. no root in $(-1,1)$
D. no root in $(0,2)$
66. If $f^{\prime}(x) \phi(x)(x-2)^{2}$. Were $\phi(2) \neq 0$ and $\phi(x)$ is continuous at $x=2$ then in the neighbouhood of $x=2$
A. $f$ is increasing if $\phi(2)<0$
B. f is decreasing if $\phi(2)>0$
C. f is neither increasing nor decreasing
D. f is increasin if $\phi(2)>0$

## Answer: D

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67. If the functio $f(x)^{3}-6 x^{2}+a x+b$ satisfies Rolle's theorem in the interval $[1,3]$ and $f^{\prime}\left(\frac{2 \sqrt{3}+1}{\sqrt{3}}\right)=0$, then

$$
\text { A. } a=-11, b=5
$$

B. $a=-11, b=-6$
C. $a=11, b \in R$
D. $1=22, b=-6$

## Answer: C

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68. For which of the following function 9s) Lagrange's mean value theorem is not applicable in $[1,2]$ ?
A. $f(x)= \begin{cases}\frac{3}{2}-x, & x<\frac{3}{2} \\ \left(\frac{3}{2}-x\right)^{2}, & x \geq \frac{3}{2}\end{cases}$
B. $f(x)= \begin{cases}\frac{\sin (x-1)}{x-1}, & x \neq 1 \\ 1, & x=1\end{cases}$
C. $f(x)=(x-1)|x+1|$
D. $f(x)=|x-1|$
69. If the curves $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{4}=1$ and $y^{2}=16 x$ intersect at right angles, then:
A. $a= \pm 1$
B. $a= \pm \sqrt{3}$
C. $a= \pm \sqrt{3}$
D. $a= \pm \sqrt{2}$

## Answer: D

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70. If the line $x \cos \alpha+y \sin \alpha=P$ touches the curve $4 x^{3}=27 a y^{2}$, then $\frac{P}{a}=$
A. $\cot ^{2} \alpha \cos \alpha$
B. $\cot ^{2} \alpha \sin \alpha$
C. $\operatorname{tna}^{2} \alpha \cos \alpha$
D. $\tan ^{2} \alpha \sin \alpha$

## Answer: A

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## Exercise One Or More Than Answer Is Are Correct

1. common tangent to $y=x^{3}$ and $x=y^{3}$
A. $x-y=\frac{1}{\sqrt{3}}$
B. $x-y=-\frac{1}{\sqrt{3}}$
C. $x-y=\frac{2}{3 \sqrt{3}}$
D. $x-y=\frac{-2}{3 \sqrt{3}}$

## (D) Watch Video Solution

2. Let $f:[0,8] \rightarrow R$ be differentiable function such that $f(0)=0, f(4)=1, f(8)=1$, then which of the following hold(s) good ?
A. There exist some $c_{1} \in(0,8)$ where $f\left(c_{1}\right)=\frac{1}{4}$
B. There exist some $x \in(0,8)$ where $f^{\prime}(c)=\frac{1}{12}$
C. There exist $c_{1}, c_{2} \in[0,8]$ where $8 f^{\prime}\left(c_{1}\right) f\left(c_{2}\right)=1$
D. There exist some $\alpha, \beta=(0,2) \quad$ such that

$$
\int_{0}^{8} f(t) d t=3\left(\alpha^{2} f\left(\alpha^{3}\right)+\beta^{2}\left(\beta^{3}\right)\right)
$$

## Answer: A::C::D

## - Watch Video Solution

3. If $f(x)=\left\{\sin ^{-1}(\sin x), x>0\right.$
$\frac{\pi}{2}, x=0$, then $\cos ^{-1}(\cos x), x<0$
A. $x=0$ is a point of maxima
B. $f(x)$ is continous $\forall x \in R$
C. glolab maximum vlaue of $f(x) \forall x \in R$ is $\pi$
D. global minimum vlaue of $f(x) \forall x \in R$ is 0

## Answer: A::C::D

## - Watch Video Solution

4. A function $f: R \rightarrow R$ is given by $f(x)=\left\{\begin{array}{ll}x^{4}\left(2+\sin \frac{1}{x}\right) & x \neq 0 \\ 0 & x=0\end{array}\right.$, then
A. f has a continous derivative $\forall x \in R$
B. $f$ is a bounded function
C. f has an global minimum at $x=0$
D. f " is continous $\forall x \in R$

## Answer: A::C::D

## D View Text Solution

5. If $f^{\prime \prime}(x) \mid \leq 1 \forall x \in R$, and $f(0)=0=f^{\prime}(0)$, then which of the following can not be true ?
A. $f\left(-\frac{1}{2}\right)=\frac{1}{6}$
B. $f(2)=-4$
C. $f(-2)=3$
D. $f\left(\frac{1}{2}\right)=\frac{1}{5}$

## Answer: A::B::C::D

6. Let $f:[-3,4] \rightarrow R$ such that $f^{\prime \prime}(x)>0$ for all $x \in[-3,4]$, then which of the following are always true?
A. $\mathrm{f}(\mathrm{x})$ has a relative minimum on $(-3,4)$
B. $f(x)$ has a minimum on $[-3,4]$
C. $f(x)$ has a maximum on $[-3,4]$
D. if $f(3)=f(4)$, then $f(x)$ has a critical point on $[-3,4]$

## Answer: $\mathrm{B}:: \mathrm{C}:: \mathrm{D}$

## - Watch Video Solution

7. Let $\mathrm{f}(\mathrm{x})$ be twice differentialbe function such that $f^{\prime \prime}(x)>0$ in $[0,2]$. Then :
A. $f(0)+f(2)=2 f(x)$, for atleast one $c, c \in(0,2)$
B. $f(0)+f(2)<2 f(1)$
C. $f(0)+f(2)>2 f(1)$
D. $2 f(0)+f(2)>3 f\left(\frac{2}{3}\right)$

## Answer: C::D

## (D) Watch Video Solution

8. Let $g(x)$ be a cubic polnomial having local maximum at $x=-1$ and g ${ }^{\prime}(x)$ has a local minimum at $x=1, \operatorname{Ifg}(-1)=10 g,(3)=-22$, then
A. perpendicular distance between its two horizontal tangents is 12
B. perpendicular distance betweent its two horizontal tangents is 32
C. $g(x)=0$ has atleast one real root lying in interval $(-2,0)$
D. $g(x)=0$, has 3 distinict real roots

## Answer: B::D

9. Let S be the set of real values of parameter $\lambda$ for which the equation $\mathrm{f}(\mathrm{x})=2 x^{3}-3(2+\lambda) x^{2}+12 \lambda \mathrm{x}$ has exactly one local maximum and exactly one local minimum. Then $S$ is a subset of
A. $\lambda \in(-4, \infty)$
B. $\lambda \in(-\infty, 0)$
C. $\lambda \in(-3,3)$
D. $\lambda \in(1, \infty)$

## Answer: A::B::C::D

## - Watch Video Solution

10. The function $f(x)=1+x \ln \left(x+\sqrt{1+x^{2}}\right)-\sqrt{1-x^{2}}$ is:
A. strictly increasing $A x \in(0,1)$
B. strictly decrreasing $\forall x \in(-1,0)$
C. strictly decreasing for $x \in(-1,0)$
D. strictly decreasing for $x \in(0,1)$

## Answer: A::C::D

## - Watch Video Solution

11. Let m and n bwe positive integers and $x, y>0$ and $x+y=k$, where k is constnat. Let $f(x, y)=x^{m} y^{n}$, then:
A. $f(x, y)$ is maximum when $x=\frac{m k}{m+n}$
B. $f(x, y)$ is maximuim wheere $x=y$
C. maximum value of $f(x, y) i s \frac{m^{n} n^{m} k^{m+n}}{(m+n)^{m+n}}$
D. maximum vlaue of $f(x, y) i s \frac{k^{m+n} m^{m} n^{n}}{(m+n)^{m+n}}$

## Answer: A::D

12. Determine the equation of straight line which is tangent at one point and normal at any point of the curve $x=3 t^{2}, y=2 t^{3}$
A. $y+\sqrt{3}(x-1)=0$
B. $y-\sqrt{3}(x-1)=0$
C. $y+\sqrt{2}(x-2)=0$
D. $y-\sqrt{2}(x-2)=0$

## Answer: C::D

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13. A curve is such that the ratio of the subnomal at any point to the sum of its co-ordinates is equal tothe ratio of the ordinate of this point to its abscissa. If the curve passes through $M(1,0)$, then possible equation of the curve is(are)
A. $y=x \ln x$
B. $y=\frac{\ln x}{x}$
C. $y=\frac{2(x-1)}{x^{2}}$
D. $y=\frac{1-x^{2}}{2 x}$

## Answer: A: D

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14. Number of A parabola of the form $y=a x^{2}+b x+c$ with $a>0$ intersection (s)of these graph of $f(x)=\frac{1}{x^{2}-4}$.number of a possible distinct intersection(s) of these graph is
A. 0
B. 2
C. 3
D. 4
15. Find the gradient of the line passing through the point $(2,8)$ and touching the curve $y=x^{3}$.
A. 3
B. 6
C. 9
D. 12

## Answer: A::D

## - Watch Video Solution

16. The equation $x+\cos x=a$ has exactly one positive root. Complete set of values of ' $a$ ' is

$$
\text { A. } a \in(0,1)
$$

B. $a \in(2,3)$
C. $a \in(1, \infty)$
D. $a \in(-\infty, 1)$

## Answer: B::C

## - Watch Video Solution

17. Given that $f(x)$ is a non-constant linear function. Then the curves:
A. $y=f(x)$ and $y=f^{-1}(x)$ are orthogonal
B. $y=f(x)$ and $y=f^{-1}(-x)$ are orthogonal
C. $y=f(-x)$ and $y=f^{-1}(x)$ are orthogonal
D. $y=f(-x)$ and $y=f^{-1}(-x)$ are orthogonal

## Answer: B::C

18. $f(x)=\int_{0}^{x} e^{t^{3}}\left(t^{2}-1\right)(t+1)^{2011} d t(x>0)$ then :
A. The number of point iof inflections is atleast 1
B. The number of point of inflectins is 0
C. The number of point of local maxima is 1
D. The number of point of local minima is 1

## Answer: A: D

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19. Let $f(x)=\sin x+a x+b$. Then which of the following is/are true? $f(x)=0$ has only one real root which is positive if $a>1, b<0$. $f(x)=0$ has only one real root which is negative if $a>1, b<0$. $f(x)=0$ has only one real root which is negative if $a>1, b>0$. noneofthese
A. only one real root which is positive if $a>1, b<0$
B. only one real root which is negative if $a>1, b>0$
C. only one real root which is negative if $a<-1, b<0$
D. only one real root which is positive if $a<-1, b<0$

## Answer: A::B::C

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20. Which of the following graphs represent function whose derivatives have a maximum in the interval $(0,1)$ ?
A.


B.
C.

D.


## Answer: A::B

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21. Consider $f(x)=\sin ^{5} x-1, x \in\left[0, \frac{\pi}{2}\right]$, which of the following is/are correct ?
A. f is strictly decreasing in $\left[0, \frac{\pi}{4}\right]$
B. f is strictly increasing in $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$
C. There exist a numbe 'c' in $\left(0, \frac{\pi}{2}\right)$ such that $f(c)=0$
D. The equation $f(x)=0$ has only two roots in $\left[0, \frac{\pi}{2}\right]$

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22. If $f(x)=x^{a} \log x$ and $f(0)=0$ then the value of $\alpha$ for which Rolle's theorem can be applied in $[0,1]$ is
A. $-\frac{1}{2}$
B. $-\frac{1}{3}$
C. $-\frac{1}{4}$
D. -1

## Answer: B::C

## D Watch Video Solution

23. Which of the following is/are true for the function
$f(x)=\int_{0}^{x} \frac{\operatorname{cost}}{t} d t(x>0) ?$
A. $f$

$$
\left((4 n-1), \frac{\pi}{2},(4 n+1) \frac{\pi}{2}\right) \forall n \in N
$$

B. $\mathrm{f}(\mathrm{x})$ has a local minima at $x=(4 n-1) \frac{\pi}{2} \forall n \in N$
C. The point of infection of the curve $y=f(x)$ lie on the curve $x \tan x+1=0$
D. Number of critiacal points of $y=f(x)$ in $(0,10 \pi)$ are 19

## Answer: A::B::C

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24. Let $F(x)=(f(x))^{2}+\left(f^{\prime}(x)\right)^{2}, F(0)=6$, where $\mathrm{f}(\mathrm{x})$ is a thrice differentiable function such that $|f(x)| \mid \leq 1 \forall x \in[-1,1]$, then choose the correct statement (s)
A. there is atleast one point in each of the intervals

$$
(-1,0) \text { and }(0,1) \text { where } \mid f^{\prime}(x) \leq 2
$$

B. there is atleast one point in each of the intervals $(-1,0)$ and $(0,1)$ where $F(x) \leq 5$
C. there is no poin tof local maxima of $F(x)$ in $(-1,1)$
D. for some $c \in(-1,1), F(c) \geq 6, F^{\prime}(c)=0$ and $f^{\prime \prime}(c) \leq 0$

## Answer: A::B::D

## - View Text Solution

25. Let $\mathrm{f}(\mathrm{x})= \begin{cases}x^{3}+x^{2}-10 x & -1 \leq x<0 \\ \sin x & 0 \leq x<x / 2 \text { then } \mathrm{f}(\mathrm{x}) \text { has } \\ 1+\cos x & \pi / 2 \leq x \leq \pi\end{cases}$
A. local maximum at $x=\frac{\pi}{2}$
B. local minimum at $x=\frac{\pi}{2}$
C. absolute minimum at $x=0, \pi$
D. absolute maximum at $x=\frac{\pi}{2}$
26. Minimum distnace between the curves $y^{2}=x-1$ and $x^{2}=x-1$ and $x^{2}=y-1$ is equal to :
A. $\frac{\sqrt{2}}{4}$
B. $\frac{3 \sqrt{2}}{4}$
C. $\frac{5 \sqrt{2}}{4}$
D. $\frac{7 \sqrt{2}}{4}$

## Answer: B

27. For the equation $\frac{e^{-x}}{x+1}$ which of the following statement(s) is/are correct ?
A. When $\lambda \in(0, \infty)$ equation has 2 real and distinct roots
B. When $\lambda, \in\left(-\infty,-e^{2}\right)$ equation has 2 real and istinct roots
C. When $\lambda \in(0, \infty)$ equation hs 1 real root
D. When $\lambda \in(-e, 0)$ equation has no real root

## Answer: B::C::D

## - Watch Video Solution

28. If $y=m x+5$ is a tangent to the curve
$x^{3} y^{3}=a x^{3}+b y^{3} a t P(1,2)$, then
A. $a+b=\frac{18}{5}$
B. $a>b$
C. $a<b$
D. $a+b=\frac{19}{5}$

## Answer: B::D

29. If $(f(x)-1)\left(x^{2}+x+1\right)^{2}-(f(x)+1)\left(x^{4}+x^{2}+1\right)=0$ $\forall x \in R-\{0\}$ and $f(x) \neq \pm 1$, then which of the following statement (s) is/are correct ?
A. $\mid f(x) \geq 2 \forall x \in R-\{0\}$
B. $f(x)$ has a local maximum at $x=-1$
C. $f(x)$ has a local minimum at $x=1$
D. $\int_{-\pi}^{\pi}(\cos x) f(x) d x=0$

## Answer: A::B::C::D

## - Watch Video Solution

Exercise Comprehension Type Problems
$x y=x+y+1, x \in R-\{1\}$ and $g(x)=x f(x)$

The minimum value of $g(x)$ is:
A. $3-\sqrt{2}$
B. $3+\sqrt{2}$
C. $3-2 \sqrt{2}$
D. $3+2 \sqrt{2}$

## Answer: D

## - Watch Video Solution

2. Let
$y=f(x)$
such
that
$x y=x+y+1, x \in R-\{1\}$ and $g(x)=x f(x)$
There exist two values of $x, x_{1}$ and $x_{2}$ where $g^{\prime}(x)=\frac{1}{2}$, then $\left|x_{1}\right|+\left|x_{2}\right|=$
A. 1
B. 2
C. 4
D. 5

## Answer: C

## - Watch Video Solution

3. Let $f(x)=\left[\begin{array}{ll}1-x & 0 \leq x \leq 1 \\ 0 & 1<x \leq 2 \text { and } g(x)=\int_{0}^{x} f(t) d t . \\ (2-x)^{2} & 2<x \leq 3\end{array}\right.$

Let the tangent to the curve $y=g(x)$ at point P whose abscissa is $\frac{5}{2}$ cuts $x$-axis in point $Q$.

Let the pependiculat from point Q on x -axis meets the curve $y=g(x)$ in point R.
$g(1)=$
A. 0
B. $\frac{1}{2}$
C. 1
D. 2

## Answer: B

## - Watch Video Solution

4. Let $f(x)=\left[\begin{array}{ll}1-x & 0 \leq x \leq 1 \\ 0 & 1<x \leq 2 \text { and } g(x)=\int_{0}^{x} f(t) d t . \\ (2-x)^{2} & 2<x \leq 3\end{array}\right.$

Let the tangent to the curve $y=g(x)$ at point P whose abscissa is $\frac{5}{2}$ cuts $x$-axis in point Q .

Let the pependiculat from point Q on x -axis meets the curve $y=g(x)$ in point R.

Rquation of tangent to the curve $y=g(x) a t P$ is:
A. $3 y=12 x-1$
B. $3 y=12 x-1$
C. $12 y=3 x-1$
D. $12 y=3 x+1$

## Answer: C

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5. Let $f(x)=\left[\begin{array}{ll}1-x & 0 \leq x \leq 1 \\ 0 & 1<x \leq 2 \text { and } g(x)=\int_{0}^{x} f(t) d t . \\ (2-x)^{2} & 2<x \leq 3\end{array}\right.$

Let the tangent to the curve $y=g(x)$ at point P whose abscissa is $\frac{5}{2}$ cuts $x$-axis in point Q .

Let the pependiculat from point Q on x -axis meets the curve $y=g(x)$ in point R.

If ' $\theta$ ' be the angle between tangents to the curve $y=g(x)$ at point P and R , then $\tan \theta$ equals to :
A. $\frac{5}{6}$
B. $\frac{5}{14}$
C. $\frac{5}{7}$
D. $\frac{5}{12}$

## Answer: B

## - Watch Video Solution

6. Let $f(x)<0 \forall x \in(\equiv \infty, 0)$ and $f(x)>0 \forall x \in(0, \infty)$ also $f(0)=o$,

Again
$f^{\prime}(x)<0 \forall x \in(-\infty,-1)$ and $f^{\prime}(x)>\forall x \in(-1, \infty) \quad$ also $f^{\prime}(-1)=0 \quad$ given $\quad \lim _{x \rightarrow \infty} f(x)=0$ and $\lim _{x \rightarrow \infty} f(x)=\infty \quad$ and function is twice differentiable.

If $f^{\prime}(x)<0 \forall x \in(0, \infty)$ and $f^{\prime}(0)=1$ then number of solutions of equatin $f(x)=x^{2}$ is :
A. 2
B. 3
C. 4
D. None of these

## Answer: D

## - Watch Video Solution

7. Let $f(x)<0 \forall x \in(\equiv \infty, 0)$ and $f(x)>0 \forall x \in(0, \infty)$ also
$f(0)=o$,
Again
$f^{\prime}(x)<0 \forall x \in(-\infty,-1)$ and $f^{\prime}(x)>\forall x \in(-1, \infty) \quad$ also
$f^{\prime}(-1)=0 \quad$ given $\quad \lim _{x \rightarrow \infty} f(x)=0$ and $\lim _{x \rightarrow \infty} f(x)=\infty \quad$ and function is twice differentiable.

If $f^{\prime}(x)<0 \forall x \in(0, \infty)$ and $f^{\prime}(0)=1$ then number of solutions of equatin $f(x)=x^{2}$ is:
A. 1
B. 2
C. 3
D. 4

## D Watch Video Solution

8. Let $f(x)<0 \forall x \in(-\infty, 0)$ and $f(x)>0 \forall x \in(0, \infty)$ also
$f(0)=0$,
Again
$f^{\prime}(x)<0 \forall x \in(-\infty,-1)$ and $f^{\prime}(x)>\forall x \in(-1, \infty) \quad$ also
$f^{\prime}(-1)=0 \quad$ given $\quad \lim _{x \rightarrow \infty} f(x)=0$ and $\lim _{x \rightarrow \infty} f(x)=\infty \quad$ and function is twice differentiable.

The minimum number of points where $f^{\prime}(x)$ is zero is:
A. 1
B. 2
C. 3
D. 4

## Answer: A

9. In the given figure graph of:
$y=p(x)=x^{n}+a_{1} x^{n-1}+a_{2} x^{n-2}+\ldots . .+a_{n}$ is given.


The product of all imaginary roots of $p(x)=0$ is:
A. -2
B. -1
C. $-1 / 2$
D. noen of these

## Answer: D

10. In the given figure graph of:
$y=p(x)=x^{n}+a_{1} x^{n-1}+a_{2} x^{n-2}+\ldots .+a_{n}$ is given.


If $p(x)+k=0$ has 4 distinct real roots $\alpha, \beta, \gamma, \delta$ then $[\alpha]+[\beta]+[\gamma]+[\delta]$, (where [.] denotes greatest integer function) is equal to:
A. -1
B. -2
C. 0
D. 1
11. In the given figure graph of:
$y=p(x)=x^{n}+a_{1} x^{n-1}+a_{2} x^{n-2}+\ldots .+a_{n}$ is given.


The minimum number of real roots of equation $\left(p^{\prime}(x)\right)^{2}+p(x) p^{\prime \prime}(x)=0$ are:
A. 3
B. 4
C. 5
D. 6

## - Watch Video Solution

12. The differentiable function $y=f(x)$ has a property that the chord joining any two points $A\left(x_{1}, f\left(x_{1}\right)\right.$ and $B\left(x_{2}, g\left(x_{2}\right)\right)$ always intersects $y$-axis at $\left(0,2 x_{1}, x_{2}\right)$. Given that $f(1)=-1$. then: $\int_{0}^{1 / 2} f(x) d x$ is equal to :
A. $\frac{1}{6}$
B. $\frac{1}{8}$
C. $\frac{1}{12}$
D. $\frac{1}{24}$

## Answer: D

## - Watch Video Solution

13. The differentiable function $y=f(x)$ has a property that the chord joining any two points $A\left(x_{1}, f\left(x_{1}\right)\right.$ and $B\left(x_{2}, g\left(x_{2}\right)\right)$ always intersects y -axis at $\left(0,2 x_{1}, x_{2}\right)$. Given that $f(1)=-1$. then:

The largest interval in whichy $f(x)$ is monotonically increasing, is :
A. $\left(-\infty, \frac{1}{2}\right]$
B. $\left[\frac{-1}{2}, \infty\right)$
C. $\left(-\infty, \frac{1}{4}\right]$
D. $\left[\frac{-1}{4}, \infty\right)$

## Answer: C

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14. The differentiable function $y=f(x)$ has a property that the chord joining any two points $A\left(x_{1}, f\left(x_{1}\right)\right.$ and $B\left(x_{2}, g\left(x_{2}\right)\right)$ always intersects $y$-axis at $\left(0,2 x_{1}, x_{2}\right)$. Given that $f(1)=-1$. then:

In which of the following intervals, the Rolle's theorem is applicable to the function $F 9 x)=f(x)+x$ ?
A. $0-1,0]$
B. $[0,1]$
C. $[-1,1]$
D. $[0,2]$

## Answer: B

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15. Let $f(x)=1+\int_{0}^{1}\left(x e^{y}+y e^{x}\right) f(y) d y$ where x and y are independent vartiables.

If complete solution set of ' x ' for which function $h(x)=f(x)+3 x$ is strictly increasing is $(-\infty, k)$ then $\left[(4) e^{\frac{k}{3}}\right]$ equals to: (where [.] denotes greatest integer function):
A. 1
B. 2
C. 3
D. 4

## Answer: C

## - View Text Solution

16. Let $f(x)=1+\int_{0}^{1}\left(x e^{y}+y e^{x}\right) f(y) d y$ where x and y are independent vartiables.

If acute ange of intersection of the curves
$\frac{x}{2}+\frac{y}{3}+\frac{1}{5}=0$ and $y=f(b) b e \theta$ then $\tan \theta$, equals to:
A. $\frac{8}{25}$
B. $\frac{16}{25}$
C. $\frac{14}{25}$
D. $\frac{4}{5}$

## D View Text Solution

## Exercise Mathcing Type Problems

1. The function $f(x)=\sqrt{\left(a x^{3}+b x^{2}+c x+a\right)}$ ha sits non-zero local minimum and local maximum values at $x-2$ and $x=2$, respectively. It 'a is a root of $x^{2}-x-6=0$

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## Exercise Subjective Type Problems

1. A conical vessel is to be prepared out of a circular sheet of gold of unit radius. How much sectorial area is to be removed from the sheet so that the vessel has maximum volume?
2. On $[1, e]$, then least and greatest vlaues of $f(x)=x^{2} \ln x$ are m and $M$ respectively, then $[\sqrt{M+m}]$ is : (where [] denotes greatest integer function)

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3. If $f(x)=\frac{p x}{e^{x}}-\frac{x^{2}}{2}+x$ is a decreasing function for every $x \leq 0$. Find the least value of $p^{2}$.

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4. Let $f(x)=\left\{\begin{array}{ll}x e^{a x}, & x \leq 0 \\ x+a x^{2}-x^{3}, & x>0\end{array}\right.$ Where a is a positive constnat .

The interval in which $\mathrm{f}^{\prime}(\mathrm{x})$ is increasing is $\left[\frac{k}{a}, \frac{a}{l}\right]$, Then $k+l$ is equal to
5. Find sum of all possible values of the real parameter $b$, if the difference between the largest and smallest values of the function $f(x)=x^{2}-2 b x+1$ in the interval $[0,1]$ is 4.

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6. Let ' $\theta$ ' be the angle in radians between the curves $\frac{x^{2}}{36}+\frac{y^{2}}{4}=1$ and $x^{2}+y^{2}=12$. If $\theta=\tan ^{-1}\left(\frac{a}{\sqrt{3}}\right)$, Find the value of $a$.

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7. Let set of all possible values of $\lambda$ such that $f(x)=e^{2 x}-(\lambda+1) e^{x}+2 x$ is monotonically increasing for $\forall x \in R$ is $(-\infty, k]$. Find the value of $k$.

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8. Let $a, b, c$ and $d$ be non-negative real number such that $a^{5}+b^{5} \leq 1$ and $c^{5}+d^{5} \leq 1$. Find the maximum value of $a^{2} c^{3}+b^{2} d^{3}$.

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9. There is a point ( $\mathrm{p}, \mathrm{q}$ ) on the graph of $f(x)=x^{2}$ and a point $(r, s)$ on the graph of $g(x)=\frac{-8}{x}$, wherep $>0 a n d r>0$. If the line through $(p, q) \operatorname{and}(r, s)$ is also tangent to both the curves at these points, respectively, then the value of $P+r$ is $\qquad$ .

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10. $f(x)=\max |2 \sin y-x|$ where $y \in R$ then determine the minimum value of $f(x)$.
11. Let $f(x)=\int_{0}^{x}\left((a-1)\left(t^{2}+t+1\right)^{2}-(a+1)\left(t^{4}+t^{2}+1\right)\right) \mathrm{dt}$. Then the total numbr of integral values of 'a' for which $f^{\prime}(x)=0$ has no rel roots is

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12. The numbr of real roots of the equation $x^{2013}+e^{20144 x}=0$ is

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13. Let the maximum value of expression $y=\frac{x^{4}-x^{2}}{x^{6}+2 x^{3}-1}$ for $x>1 i s \frac{p}{1}$, where p and 1 q are relatively prime natural numbers, then $p+q=$

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14. The least positive integral value of ' $k$ ' for which there exists at least one line that the tangent to the graph of the curve $y=x^{3}-k x$ at one point and normal to the graph at another point is

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15. Let $f(x)=x^{2}+2 x-t^{2}$ and $f(x)=0$ has two root $\alpha(t)$ and $\beta(t)(\alpha<\beta)$ where t is a real parameter. Let $I(t)=\int_{\alpha}^{\beta} f(x)$ dx . If the maximum value of $I(t)$ be $\lambda$ and $|\lambda|=\frac{p}{q}$ where p and q are relatively prime positive integers. Find the product (pq).

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16. A tank contains 100 litres of fresh water. A solution containing 1 $\mathrm{gm} /$ litre of soluble lawn fertilizeruns into the tank the of $1 \mathrm{lit} / \mathrm{min}$ and the mixture pumped out of the tank at the rate of at rate of f 3
litres/min. Find the time when the amount of fertilizer in the tank is maximum.

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17. If $f$ ( $x)$ is continous and differentiable in $[3,9)$ and $f^{\prime}(x) \in[-2,8] \forall x \in(-3,9)$. Let N be the number of divisors of the greatest possible value of $f(9)-f(-3)$, then find the sum of digits of $N$.

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18. It is given that f 9 x ) is difined on R satisfyinf $f(1)=1$ and for $\forall x \in R$,
$f(x+5) \geq f(x)+5$ and $f(x+1) \leq f(x)+1 . I f g(x)=f(x)+1-x$, then $g(2002)=$

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19. The number of normals to the curve $3 y^{3}=4 x$ which passes through the point $(0,1)$ is

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20. Find the number of real root (s) of the equation $a e^{x}=1+x+\frac{x^{2}}{2}$, where a is positive constant.

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21. Let $f(x)=a x+\cos 2 x+\sin x+\cos x$ is defined for $\forall x \in R$ and $a \in R$ and is strictely increasing function. If the range of a is $\left[\frac{m}{n}, \infty\right)$, then find the minimum vlaue of $(m-n)$.
22. If $p_{1}$ and $p_{2}$ are the lengths of the perpendiculars from origin on the tangent and normal drawn to the curve $x^{2 / 3}+y^{2 / 3}=6^{2 / 3}$ respectively. Find the vlaue of $\sqrt{4 p_{1}^{2}+p_{2}^{2}}$.
