MATHS

BOOKS - VIKAS GUPTA MATHS (HINGLISH)

APPLICATION OF DERIVATIVES

Exercise Single Choice Problems

1. The difference between the maximum and minimum value of the function $f(x) = 3\sin^4 x - \cos^6 x$ is :

A. $\frac{3}{2}$

B. $\frac{5}{2}$

C. 3

D. 4

Answer: D

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2. A function $y = f(x)$ has a second order derivative $f(x) = 6(x - 1)$. If its graph passes through the point $(2, 1)$ and at that point the tangent to the graph is $y = 3x - 5$ then the function is

A. $(x - 1)^2$

B. $(x - 1)^3$

C. $(x + 1)^3$

D. $(x + 1)^2$

Answer: B

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3. If the subnormal at any point on the curve $y = 3^{1-k}.\,x^k$ is of constant length the k equals to :

A. $\frac{1}{2}$

B. 1

C. 2

D. 0

Answer: A

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4. If $x^5 - 5qx + 4r$ is divisible by $(x - c)^2$ then which of the following must hold true

A. $q = r$

B. $q + r = 0$

C. $q^5 + r = 0$

D. $q^4 = r^5$

Answer: C

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5. A spherical iron ball 10 cm in radius is coated with a layer of ice of unirform thichness that melts at a rate of $50cm^3$/min. when the thickness of ice is 5 cm, then the rate at which the thickness of ice decreases, is

A. $\frac{1}{36\pi}cm/\text{ min}$

B. $\frac{1}{18\pi}cm/\text{ min}$

C. $\frac{1}{54\pi}cm/\text{ min}$

D. $\frac{5}{6\pi}cm/\text{ min}$

Answer: B

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6. If $f(x) = \frac{(x-1)(x-2)}{(x-3)(x-4)}$, then number of local extemas for g (x), where $g(x) = f(|x|)$:

A. 3

B. 4

C. 5

D. None of these

Answer: C

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7. Two straight roads OA and OB intersect at an angle 60°. A car approaches O from A, where $OA = 700m$ at a uniform speed of 20 m/s, Simultaneously, a runner starts running from O towards B at a uniform speed of 5 m/s. The time after start when the car and the runner are closest is :

A. 10 sec

B. 15 sec

C. 20 sec

D. 30 sec

Answer: D

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8. Let $f(x) = \begin{cases} a - 3x & -2 \leq x < 0 \\ 4x \pm 3 & 0 \leq x < 1 \end{cases}$, if $f(x)$ has smallest valueat $x = 0$, then range of a, is

A. $(-\infty, 3)$

B. $(-\infty, 3]$

C. $(3, \infty)$

D. $(-3, \infty)$

Answer: D

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9. If $f(x)=\begin{cases}3+|x-k|, x\leq ka^2-2+\frac{sn(x-k)}{x-k}, x>k\end{cases}$ has minimum at $x=k$, then $a\in R$ b. $|a|<2$ c. $|a|>2$ d. $1<|a|<2$

A. $a\in R$

B. $|a|<2$

C. $|a|>2$

D. $1<|a|<2$

Answer: C

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10. For a certain curve $\frac{d^2y}{dx^2}=6x-4$ and curve has local minimum values $5atx=1$, Let the global maximum and global minimum vlaues, where $0\leq x\leq 2$, are M and m. Then the vlaue of (M-m) equals to :

A. -2

B. 2

C. 12

D. -12

Answer: B

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11. The tangent to $y = ax^2 + bx + \frac{7}{2}at(1, 2)$ is parallel to the normal at the point $(-2, 2)$ on the curve $y = x^2 + 6x + 10$. Then the vlaue of $\frac{a}{2} - b$ is:

A. 2

B. 0

C. 3

D. 1

Answer: C

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12. If (a,b) be the point on the curve $9y^2 = x^3$ where normal to the curve make equal intercepts with the axis, then the value of $(a + b)$ is:

A. 0

B. $\frac{10}{3}$

C. $\frac{20}{3}$

D. None of these

Answer: C

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13. For a certain curve y=f(x) satisfying $\frac{d^2y}{dx^2} = 6x - 4,$ f(x) has a local minimum value 5 when x=1, Find the equation of the curve and also the gobal maximum and global minimum values of f(x) given that $0 \leq x \leq 2$.

A. 1

B. 0

C. 5

D. None of these

Answer: C

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14. Let A be the point where the curve $5\alpha^2x^3 + 10\alpha x^2 + x + 2y - 4 = 0(\alpha \in R, \alpha \neq 0)$ meets the y-axis, then the equation of tangent to the curve at the point where normal at A meets the curve again, is:

A. $x - \alpha y + 2\alpha = 0$

B. $\alpha x + y - 2 = 0$

C. $2x - y + 2 = 0$

D. $x + 2y - 4 = 0$

Answer: C

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15. The difference between the greatest and the least value of the function $f(x) = \cos x + \frac{1}{2}\cos 2x - \frac{1}{3}\cos 3x$

A. $\frac{11}{5}$

B. $\frac{13}{6}$

C. $\frac{9}{4}$

D. $\frac{7}{3}$

Answer: C

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16. The ordinate of point on the curve $y = \sqrt{x}$ which is closest to the point $(2, 1)$ is

A. $\frac{2+\sqrt{3}}{2}$

B. $\frac{1+\sqrt{2}}{2}$

C. $\frac{-1+\sqrt{3}}{2}$

D. 1

Answer: A

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17. The tangent at a point P on the curve $y = \ln\left(\frac{2+\sqrt{4-x^2}}{2-\sqrt{4-x^2}}\right) - \sqrt{4-x^2}$ meets the y-axis at T, then PT^2 equals to :

A. 2

B. 4

C. 8

D. 16

Answer: B

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18. Let $f(x) = \int_{x^2}^{x^3} \frac{dt}{\ln t}$ for $x > 1$ and $g(x) = \int_1^x (2t^2 - \ln t) f(t) dt (x > 1)$, then:

A. g is increasing on $(1, \infty)$

B. g is decreasing on $(1, \infty)$

C. g is increasing on $(1, 20$ and decreasing on $(2, 00)$

D. g is decreasing on $(1, 2)$ and increasing on $(2, \infty)$

Answer: A

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19. Let $f(x) = x^3 + 6x^2 + ax + 2$, if $(-3, -1)$ is the largest possible interval for which $f(x)$ is decreasing function, then $a =$

A. 3

B. 9

C. -2

D. 1

Answer: B

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20. Let $f\left(x - = \tan^{-1}\left(\frac{1-x}{1+x}\right)\right.$. Then difference of the greatest and least value of $f(x)$ on $[0, 1]$ is:

A. $\pi/2$

B. $\pi/4$

C. π

D. $\pi/3$

Answer: B

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21. The number of integral values of a for which $f(x) = x^3 + (a+2)x^2 + 3ax + 5$ is monotonic in $\forall x \in R$

A. 2

B. 4

C. 6

D. 7

Answer: B

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22. The number of critical points of $f(x) = \left(\int_0^x \left(\cos^2 t - \sqrt[3]{t}\right) dt\right) + \frac{3}{4}x^{4/3} - \frac{x+1}{2}$ in $(0, 6\pi]$ is:

A. 10

B. 8

C. 6

D. 12

Answer: D

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23. Let $f(x) = \min\left[\frac{1}{2} - 3\frac{x^2}{4}, 5\frac{x^2}{4}\right]$, for $0 \le x \le 1$ then maximum value of f(x) is

A. 0

B. $\frac{5}{64}$

C. $\frac{5}{4}$

D. $\frac{5}{16}$

Answer: D

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24. Let $f(x) = \begin{cases} 2 - |x^2 + 5x + 6| & x \neq -2 \\ b^2 + 1 & x = -2 \end{cases}$

Has relative maximum at $x = -2,$ then complete set of values b can take is:

A. $|b| \geq 1$

B. $|b| < 1$

C. $b > 1$

D. $b < 1$

Answer: A

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25. Let for function $f(x) = \begin{bmatrix} \cos^{-1} x & -1 \leq x \leq 0 \\ mx + c & 0 < x \leq 1 \end{bmatrix}$, Lagrange's mean value theorem is applicable in $[-1, 1]$ then ordered pair (m, c) is:

A. $\left(1, -\frac{\pi}{2}\right)$

B. $\left(1, \frac{\pi}{2}\right)$

C. $\left(-1, -\frac{\pi}{2}\right)$

D. $\left(-1, \frac{\pi}{2}\right)$

Answer: D

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26. Tangents are drawn from the origin to the curve $y = \cos X$. Their points of contact lie on

A. $\frac{1}{x^2} = \frac{1}{y^2} + 1$

B. $\frac{1}{x^2} = \frac{1}{y^2} - 2$

C. $\frac{1}{y^2} = \frac{1}{x^2} + 1$

D. $\frac{1}{y^2} = \frac{1}{x^2} - 2$

Answer: C

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27. The least natural number a for which $x + ax^{-2} > 2 \forall x \in (0, \infty)$ is

1 (b) 2 (c) 5 (d) none of these

A. 1

B. 2

C. 5

D. None of these

Answer: B

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28. Angle between the tangents to the curve $y = x^2 - 5x + 6$ at the points $(2, 0)$ and $(3, 0)$ is : (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{4}$

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

Answer: D

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29. Difference between the greatest and least values opf the function $f(x) = \int_0^x (\cos^2 t + \cos t + 2)$ dt in the interval $[0, 2\pi]$ is $K\pi$, then K is equal to:

A. 1

B. 3

C. 5

D. None of these

Answer: C

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30. The range of the function $f(\theta)=\frac{\sin\theta}{\theta}+\frac{\theta}{\tan\theta}, \theta\in\left(0, \frac{\pi}{2}\right)$ is equal to :

A. $(0, \infty)$

B. $\left(\frac{1}{\pi}, 2\right)$

C. $(2, \infty 0$

D. $\left(\frac{2}{\pi}, 2\right)$

Answer: D

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31. Number of integers in the range of 'c' so that the equation $x^3 - 3x + c = 0$ has all its roots real and distinct is

A. 2

B. 3

C. 4

D. 5

Answer: B

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32. Let $f(x) = \int e^x (x-1)(x-2) dx,$ then f(x) decrease in the interval

A. $(2, \infty)$

B. $(-2, -1)$

C. $(1, 2)$

D. $(-\infty, 1)ii(2, \infty)$

Answer: C

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33. If the cubic polymomial $y = ax^3 + bx^2 + cx + d(a, b, c, d \in R)$ has only one critical point in its entire domain and $ac = 2$, then the value of $|b|$ is:

A. $\sqrt{2}$

B. $\sqrt{3}$

C. $\sqrt{5}$

D. $\sqrt{6}$

Answer: D

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34. On the curve $y = \dfrac{1}{1+x^2}$, the point at which $\left|\dfrac{dy}{dx}\right|$ is greatest in the first quadrant is :

A. $\left(\frac{1}{2}, \frac{4}{5}\right)$

B. $\left(1, \frac{1}{4}\right)$

C. $\left(\frac{1}{\sqrt{2}}, \frac{2}{3}\right)$

D. $\left(\frac{1}{\sqrt{3}}, \frac{3}{4}\right)$

Answer: D

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35. If $f_1(x) = 2x,\ f_2(x) = 3\sin x - x\cos x$ then for $x \in \left(0, \frac{\pi}{2}\right)$

A. $f(x) > g(x)$

B. $f(x) < g(x)$

C. $f(x) = g(x)$ has exactly one real root.

D. $f(x) = g(x)$ has exactly two real roots

Answer: A

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36. let $f(x) = \sin^{-1}\left(\frac{2g(x)}{1 + g(x)^2}\right)$, then which are correct ?

(i) f (x) is decreasing if $g(x)$ is increasig and $|g(x) > 1$

(ii) $f(x)$ is an increasing function if $g(x)$ is increasing and $|g(x)| \leq 1$

(iii) f (x) is decreasing function if $f(x)$ is decreasing and $|g(x)| > 1$

A. (i) and (iii)

B. (i) and (ii)

C. (i) (ii) and (iii)

D. (iii)

Answer: B

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37. The graph of the function $y = f(x)$ has a unique tangent at the point $(e^a, 0)$ through which the graph passes then $\underline{\lim}\ (x \to e^a) \dfrac{\log_e\{1 + 7f(x)\} - \sin f(x)}{3f(x)}$

A. 1

B. 3

C. 2

D. 7

Answer: C

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38. Let $f(x)$ be a function such that $f'(x) = \log_{1/3}(\log_3(\sin x + a))$. The complete set of values of 'a' for which $f(x)$ is strictly decreasing for all real values of x is:

A. $[4, \infty)$

B. $[3, 4]$

C. $(-\infty, 4)$

D. $[2, \infty)$

Answer: A

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39. If $f(x) = a\ln|x| + bx^2 + x$ has extremas at $x = 1$ and $x = 3$ then:

A. $a = \frac{3}{4}, b = -\frac{1}{8}$

B. $a = \frac{3}{4}, b = \frac{1}{8}$

C. $a = -\frac{3}{4}, b = -\frac{1}{8}$

D. $a = -\frac{3}{4}, b = \frac{1}{8}$

Answer: C

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40. Let $f(x)=\begin{cases}1+\sin x & x<0\\ x^2-x+1 & x\geq 0\end{cases}$

A. f has a local maximum at $x=0$

B. f has a local minimum at $x=0$

C. f is increasing everywhere

D. f is decreasing everywhere

Answer: A

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41. If m and n are positive integers and $f(x)=\int_1^x (t-a)^{2n}(t-b)^{2m+1}dt,\ a\neq b$, then

A. $x=b$ is a point of local minimum

B. $x=b$ is a point of local maximum

C. $x = a$ is a point of local minimum

D. $x = a$ is a point of local maximum

Answer: A

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42. For any the real $\theta$the maximum value of$\cos^2(\cos\theta) + \sin^2(\sin\theta)$ is

A. 1

B. $1 + \sin^2 1$

C. $1 + \cos^2 1$

D. Does not exist

Answer: B

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43. If the tangentat P of the curve $y^2 = x^3$ intersect the curve again at Q and the straigta line OP, OQ have inclinations α and β where O is origin, then $\frac{\tan\alpha}{\tan\beta}$ has the value equals to

A. -1

B. -2

C. 2

D. $\sqrt{2}$

Answer: B

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44. If $x + 4y = 14$ is a normal to the curve $y^2 = \alpha x^3 - \beta$ at (2,3), then the value of $\alpha + \beta$ is 9 (b) -5 (c) 7 (d) -7

A. 9

B. -5

C. 7

D. -7

Answer: A

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45. The tangent to the curve $y = e^{kx}$ at a point (0,1) meets the x-axis at (a,0), where $a \in [-2, -1]$. Then $k \in$ $\left[-\frac{1}{2}, 0\right]$ (b) $\left[-1, -\frac{1}{2}\right]$ $[0, 1]$ (d) $\left[\frac{1}{2}, 1\right]$

A. $\left[-\frac{1}{2}, 0\right]$

B. $\left[-1 - \frac{1}{2}\right]$

C. $[0, 1]$

D. $\left[\frac{1}{2}, 1\right]$

Answer: D

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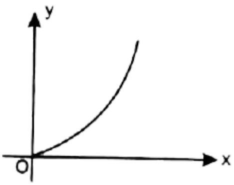
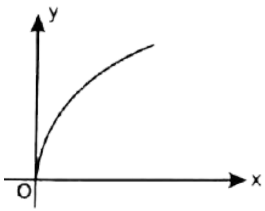
46. Which of the following graph represent the function

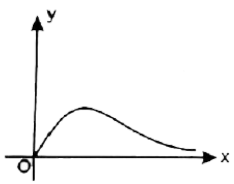
$f(x) = \int_0^{\sqrt{x}} e^{\frac{u^2}{x}}$ du, for $x > 0$ and $f(0) = o$



A.



B.



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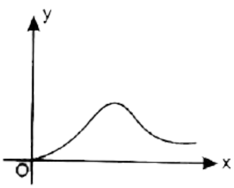

D.

Answer: B

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47. Let $f(x)=(x-a)(x-b)(x-c)$ be a ral vlued function where $a < bc(a, b, c \in R)$ such that $f''(\alpha)=0$. Then if $\alpha \in (c_1, c_2)$, which one of the following is correct ?

A. $\alpha < c_1 < b$ and $b < c_2 < c$

B. $\alpha < c_1, c_2 < b$

C. $b < c_1, c_2 < c$

D. None of these

Answer: A

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48. $f(x)=x^6-x-1, x \in [1, 2]$. Consider the following statements :

A. f is increasing on $[1, 2]$

B. f has a root in $[1, 2]$

C. f is decreasing on $[1, 2]$

D. f has no root in $[1, 2]$

Answer: A

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49. Which one of the following curves is the orthogonal trajectory of straight lines passing through a fixed point (a,b) ?

A. $x - a = k(y - b)$

B. $(x - a)(y - b) = k$

C. $(x - a)^2 = k(y - b)$

D. $(x - a)^2 + (y - b)^2 = k$

Answer: D

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50. The function $f(x) = \sin^3 x - m \sin x$ is defined on open interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and if assumes only 1 maximum value and only 1 minimum value on this interval. Then, which one of the must be correct?

A. $0 < m < 3$

B. $-3 < m < 0$

C. $m > 3$

D. $m < -3$

Answer: A

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51. The greatest of the numbers $2^{\frac{1}{2}}, 3^{\frac{1}{3}}, 4^{\frac{1}{4}}, 5^{\frac{1}{5}}, 6^{\frac{1}{6}}$ and $7^{\frac{1}{7}}$ is

A. $2^{1/2}$

B. $3^{1/3}$

C. $7^{1/7}$

D. $6^{1/6}$

Answer: B

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52. Let l be the line through $(0, 0)$ an tangent to the curve $y = x^3 + x + 16$. Then the slope of l equal to :

A. 10

B. 11

C. 17

D. 13

Answer: D

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53. The slope of the tangent at the point of inflection of $y = x^3 - 3x^2 + 6x + 2009$ is equal to :

A. 2

B. 3

C. 1

D. 4

Answer: B

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54. Let f be a real valued function with $(n+1)$ derivatives at each point of R. For each pair of real numbers $a, b, a < b,$ such that

$$\ln\left[\frac{f(b) + f(b) + \ldots. + f^{(n)}(b)}{f(a) + f'(a) + \ldots. + f^{(n)}(a)}\right]$$

Statement-1 : There is a number $c \in h(a, b)$ for which $f^{(n+1)}(c) = f(c)$

because

Statement-2: If $h(x)$ be a derivable function such that $h(p) = h(q)$ then by Rolle's theorem $h'(d) = 9, d \in (p, q)$

A. Statement-1 is true, statemet-2 is true and statement-2 is correct explanation for statement-1

B. Statement-1 is true, statement-2 is true and statement-2 is not correct explanation for statement-1

C. Statement-1 is true, statement-2 is false

D. Statement-1 is false, statement-2 is true

Answer: A

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55. If f(x) is a differentiable real valued function satisfying $f''(x) - 3f'(x) > 3 \forall x \geq 0$ and $f'(0) = -1$, then $f(x) + x \forall x > 0$ is

A. strictly increasing

B. strictly decreasing

C. non monotonic

D. data insufficient

Answer: A

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56. If the line joining the points $(0, 3) and (5, -2)$ is a tangent to the curve $y = \frac{C}{x+1}$, then the value of c is 1 (b) -2 (c) 4 (d) none of these

A. 2

B. 3

C. 4

D. 5

Answer: C

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57. Number of solutions (s) of in $|\sin x| = -x^2$ if $x \in \left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$ is/are:

A. 2

B. 4

C. 6

D. 8

Answer: B

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58. Find the value of a for which $\sin^{-1} x = |x - a|$ will have at least one solution.

A. $[-1, 1]$

B. $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

C. $\left[1-\frac{\pi}{2}, 1+\frac{\pi}{2}\right]$

D. $\left[\frac{\pi}{2}-1, \frac{\pi}{2}+1\right]$

Answer: C

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59. For any ral number b, let f (b) denotes the maximum of $\left|\sin x+\frac{2}{3+\sin x}+b\right| \forall \times x \in R.$ Then the minimum valur of $f(b) \forall b \in R$is:

A. $\frac{1}{2}$

B. $\frac{3}{4}$

C. $\frac{1}{4}$

D. 1

Answer: B

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60. Which of the following are correct

A. $x^4 + 2x^2 - 6x + 2 = 0$ has exactly four real solution

B. $x^3 + 5x + 1 = 0$ has exactly three real solutions

C. $x^n + ax + b = 0$where n is an even natural number has atmost two real solution $a, b,$ in R.

D. $x^3 - 3x + c = 0, x > 0$ has two real solutin for $x \in (0, 1)$

Answer: C

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61. For any real number b, let f (b) denotes the maximum of $\left|\sin x + \frac{2}{3 + \sin x} + b\right| \forall x \in R.$ Then the minimum vlaue of $f(b) \forall b \in R$ is:

A. $\frac{1}{2}$

B. $\frac{3}{4}$

C. $\frac{1}{4}$

D. 1

Answer: B

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62. Find the coordinates of the point on the curve $y = \frac{x}{1+x^2}$ where the tangent to the curve has the greatest slope.

A. $(0, 0)$

B. $\left(\sqrt{3}, \frac{\sqrt{3}}{4}\right)$

C. $\left(-\sqrt{3}, -\frac{\sqrt{3}}{4}\right)$

D. $\left(1, \frac{1}{2}\right)$

Answer: A

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63. Let $f:[0, 2p] \to [-3, 3]$ be a given function defined at $f(x) = \cos\frac{\pi}{2}$. The slope of the tangent to the curve $y = f^{-1}(x)$ at the point where the curve crosses the y-axis is:

A. -1

B. $-\frac{2}{3}$

C. $-\frac{1}{6}$

D. $-\frac{1}{3}$

Answer: B

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64. Number of stationary points in $[0, po]$ for the function $f(x) = \sin x + \tan x - 2x$ is:

A. 0

B. 1

C. 2

D. 3

Answer: C

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65. If a,b,c d $\in R$ such that $\frac{a+2c}{b+3d}+\frac{4}{3}=0,$ then the equation $ax^3+bx^3+cx+d=0$ has

A. atleast one root in $(-1, 0)$

B. atleast one root in $(0, 1)$

C. no root in $(-1, 1)$

D. no root in $(0, 2)$

Answer: B

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66. If $f'(x)\phi(x)(x-2)^2$. Were $\phi(2) \neq 0$ and $\phi(x)$ is continuous at $x=2$ then in the neighbouhood of $x=2$

A. f is increasing if $\phi(2) < 0$

B. f is decreasing if $\phi(2) > 0$

C. f is neither increasing nor decreasing

D. f is increasin if $\phi(2) > 0$

Answer: D

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67. If the functio $f(x)^3 - 6x^2 + ax + b$ satisfies Rolle's theorem in the interval [1,3] and $f'\left(\frac{2\sqrt{3}+1}{\sqrt{3}}\right) = 0$, then

A. $a = -11, b = 5$

B. $a = -11, b = -6$

C. $a = 11, b \in R$

D. $1 = 22, b = -6$

Answer: C

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68. For which of the following function 9s) Lagrange's mean value theorem is not applicable in $[1, 2]$?

A. $f(x) = \begin{cases} \frac{3}{2} - x, & x < \frac{3}{2} \\ \left(\frac{3}{2} - x\right)^2, & x \geq \frac{3}{2} \end{cases}$

B. $f(x) = \begin{cases} \frac{\sin(x-1)}{x-1}, & x \neq 1 \\ 1, & x = 1 \end{cases}$

C. $f(x) = (x - 1)|x + 1|$

D. $f(x) = |x - 1|$

Answer: A

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69. If the curves $\frac{x^2}{a^2}+\frac{y^2}{4}=1$ and $y^2=16x$ intersect at right angles, then:

A. $a= \pm 1$

B. $a= \pm \sqrt{3}$

C. $a= \pm \sqrt{3}$

D. $a= \pm \sqrt{2}$

Answer: D

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70. If the line $x\cos\alpha+y\sin\alpha=P$ touches the curve $4x^3=27ay^2$, then $\frac{P}{a}=$

A. $\cot^2\alpha\cos\alpha$

B. $\cot^2 \alpha \sin \alpha$

C. $tna^2 \alpha \cos \alpha$

D. $\tan^2 \alpha \sin \alpha$

Answer: A

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Exercise One Or More Than Answer Is Are Correct

1. common tangent to $y = x^3$ and $x = y^3$

A. $x - y = \frac{1}{\sqrt{3}}$

B. $x - y = -\frac{1}{\sqrt{3}}$

C. $x - y = \frac{2}{3\sqrt{3}}$

D. $x - y = \frac{-2}{3\sqrt{3}}$

Answer: C::D

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2. Let $f:[0,8]\to R$ be differentiable function such that $f(0)=0$, $f(4)=1$, $f(8)=1$, then which of the following hold(s) good ?

A. There exist some $c_1 \in (0,8)$ where $f(c_1)=\frac{1}{4}$

B. There exist some $x \in (0,8)$ where $f'(c)=\frac{1}{12}$

C. There exist $c_1, c_2 \in [0,8]$ where $8f'(c_1)f(c_2)=1$

D. There exist some $\alpha, \beta = (0,2)$ such that

$$\int_0^8 f(t)dt = 3\left(\alpha^2 f\left(\alpha^3\right) + \beta^2\left(\beta^3\right)\right)$$

Answer: A::C::D

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3. If $f(x) = \{\sin^{-1}(\sin x), x > 0$

$\frac{\pi}{2}, x = 0, then \cos^{-1}(\cos x), x < 0$

A. $x = 0$ is a point of maxima

B. $f(x)$ is continous $\forall x \in R$

C. glolab maximum vlaue of $f(x) \forall x \in R$ is π

D. global minimum vlaue of $f(x) \forall x \in R$ is 0

Answer: A::C::D

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4. A function $f: R \to R$ is given by $f(x) = \begin{cases} x^4\left(2 + \sin\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$,

then

A. f has a continous derivative $\forall x \in R$

B. f is a bounded function

C. f has an global minimum at $x = 0$

D. f" is continous $\forall x \in R$

Answer: A::C::D

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5. If $f''(x) \mid \leq 1 \forall x \in R, \text{ and } f(0) = 0 = f'(0),$ then which of the following can not be true ?

A. $f\left(-\frac{1}{2}\right) = \frac{1}{6}$

B. $f(2) = -4$

C. $f(-2) = 3$

D. $f\left(\frac{1}{2}\right) = \frac{1}{5}$

Answer: A::B::C::D

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6. Let $f:[-3, 4] \rightarrow R$ such that $f''(x) > 0$ for all $x \in [-3, 4]$, then which of the following are always true ?

A. f (x) has a relative minimum on $(-3, 4)$

B. f (x) has a minimum on $[-3, 4]$

C. f (x) has a maximum on $[-3, 4]$

D. if $f(3) = f(4)$, then $f(x)$ has a critical point on $[-3, 4]$

Answer: B::C::D

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7. Let f (x) be twice differentialbe function such that $f''(x) > 0$ in $[0, 2]$. Then :

A. $f(0) + f(2) = 2f(x)$, for atleast one c, $c \in (0, 2)$

B. $f(0) + f(2) < 2f(1)$

C. $f(0) + f(2) > 2f(1)$

D. $2f(0)+f(2)>3f\left(\frac{2}{3}\right)$

Answer: C::D

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8. Let $g(x)$ be a cubic polnomial having local maximum at $x=-1$ and g '(x) has a local minimum at $x=1, Ifg(-1)=10g, (3)=-22,$ then

A. perpendicular distance between its two horizontal tangents is 12

B. perpendicular distance betweent its two horizontal tangents is 32

C. $g(x)=0$ has atleast one real root lying in interval $(-2, 0)$

D. $g(x)=0,$ has 3 distinict real roots

Answer: B::D

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9. Let S be the set of real values of parameter λ for which the equation f(x) = $2x^3 - 3(2+\lambda)x^2 + 12\lambda$ x has exactly one local maximum and exactly one local minimum. Then S is a subset of

A. $\lambda \in (-4, \infty)$

B. $\lambda \in (-\infty, 0)$

C. $\lambda \in (-3, 3)$

D. $\lambda \in (1, \infty)$

Answer: A::B::C::D

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10. The function $f(x) = 1 + x\ln\left(x + \sqrt{1+x^2}\right) - \sqrt{1-x^2}$ is:

A. strictly increasing $Ax \in (0, 1)$

B. strictly decrreasing $\forall x \in (-1, 0)$

C. strictly decreasing for $x \in (-1, 0)$

D. strictly decreasing for $x \in (0, 1)$

Answer: A::C::D

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11. Let m and n bwe positive integers and $x, y > 0$ and $x + y = k$, where k is constnat. Let $f(x, y) = x^m y^n$, then:

A. $f(x, y)$ is maximum when $x = \frac{mk}{m+n}$

B. $f(x, y)$ is maximuim wheere $x = y$

C. maximum value of $f(x, y) is \frac{m^n n^m k^{m+n}}{(m+n)^{m+n}}$

D. maximum vlaue of $f(x, y) is \frac{k^{m+n} m^m n^n}{(m+n)^{m+n}}$

Answer: A::D

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12. Determine the equation of straight line which is tangent at one point and normal at any point of the curve $x = 3t^2, y = 2t^3$

A. $y + \sqrt{3}(x - 1) = 0$

B. $y - \sqrt{3}(x - 1) = 0$

C. $y + \sqrt{2}(x - 2) = 0$

D. $y - \sqrt{2}(x - 2) = 0$

Answer: C::D

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13. A curve is such that the ratio of the subnomal at any point to the sum of its co-ordinates is equal tothe ratio of the ordinate of this point to its abscissa. If the curve passes through M(1,0), then possible equation of the curve is(are)

A. $y = x \ln x$

B. $y = \frac{\ln x}{x}$

C. $y = \frac{2(x-1)}{x^2}$

D. $y = \frac{1-x^2}{2x}$

Answer: A::D

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14. Number of A parabola of the form $y = ax^2 + bx + c$ with $a > 0$ intersection (s)of these graph of $f(x) = \frac{1}{x^2 - 4}$ .number of a possible distinct intersection(s) of these graph is

A. 0

B. 2

C. 3

D. 4

Answer: B::C::D

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15. Find the gradient of the line passing through the point (2,8) and touching the curve $y = x^3$.

A. 3

B. 6

C. 9

D. 12

Answer: A::D

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16. The equation $x + \cos x = a$ has exactly one positive root. Complete set of values of $'a'$ is

A. $a \in (0, 1)$

B. $a \in (2, 3)$

C. $a \in (1, \infty)$

D. $a \in (-\infty, 1)$

Answer: B::C

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17. Given that $f(x)$ is a non-constant linear function. Then the curves :

A. $y = f(x)$ and $y = f^{-1}(x)$ are orthogonal

B. $y = f(x)$ and $y = f^{-1}(-x)$ are orthogonal

C. $y = f(-x)$ and $y = f^{-1}(x)$ are orthogonal

D. $y = f(-x)$ and $y = f^{-1}(-x)$ are orthogonal

Answer: B::C

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18. $f(x) = \int_0^x e^{t^3}(t^2-1)(t+1)^{2011} dt (x > 0)$ then :

A. The number of point iof inflections is atleast 1

B. The number of point of inflectins is 0

C. The number of point of local maxima is 1

D. The number of point of local minima is 1

Answer: A::D

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19. Let $f(x) = \sin x + ax + b$. Then which of the following is/are true?

$f(x) = 0$ has only one real root which is positive if $a > 1, b < 0$.

$f(x) = 0$ has only one real root which is negative if $a > 1, b < 0$.

$f(x) = 0$ has only one real root which is negative if $a > 1, b > 0$.

$none of these$

A. only one real root which is positive if $a > 1, b < 0$

B. only one real root which is negative if $a > 1, b > 0$

C. only one real root which is negative if $a < -1, b < 0$

D. only one real root which is positive if $a < -1, b < 0$

Answer: A::B::C

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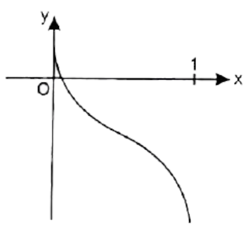
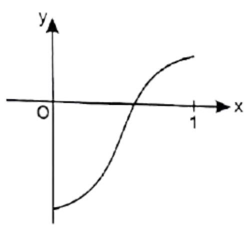
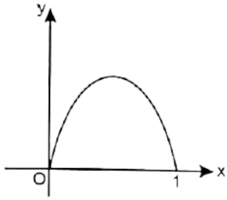
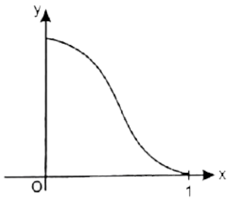
20. Which of the following graphs represent function whose derivatives have a maximum in the interval $(0, 1)$?

A. 

B. 

C. 

D. 

Answer: A::B

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21. Consider $f(x) = \sin^5 x - 1, x \in \left[0, \frac{\pi}{2}\right]$, which of the following is/are correct ?

A. f is strictly decreasing in $\left[0, \frac{\pi}{4}\right]$

B. f is strictly increasing in $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

C. There exist a numbe 'c' in $\left(0, \frac{\pi}{2}\right)$ such that $f(c) = 0$

D. The equation $f(x) = 0$ has only two roots in $\left[0, \frac{\pi}{2}\right]$

Answer: A::B::C::D

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22. If $f(x) = x^a \log x$ and $f(0) = 0$ then the value of α for which Rolle's theorem can be applied in [0,1] is

A. $-\frac{1}{2}$

B. $-\frac{1}{3}$

C. $-\frac{1}{4}$

D. -1

Answer: B::C

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23. Which of the following is/are true for the function $f(x) = \int_0^x \frac{\cos t}{t} dt (x > 0)$?

A. f (x) is monotonically increasing in $\left((4n-1),\frac{\pi}{2},(4n+1)\frac{\pi}{2}\right)\forall n\in N$

B. f (x) has a local minima at $x=(4n-1)\frac{\pi}{2}\forall n\in N$

C. The point of infection of the curve $y=f(x)$ lie on the curve $x\tan x+1=0$

D. Number of critiacal points of $y=f(x)$ in $(0,10\pi)$ are 19

Answer: A::B::C

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24. Let $F(x)=(f(x))^2+(f'(x))^2, F(0)=6,$ whtere f (x) is a thrice differentiable function such that $|f(x)|\,|\leq 1\forall x\in[-1,1],$ then choose the correct statement (s)

A. there is atleast one point in each of the intervals $(-1,0)$ and $(0,1)$ where $|f'(x)\leq 2$

B. there is atleast one point in each of the intervals

$(-1, 0)$ and $(0, 1)$ where $F(x) \leq 5$

C. there is no poin tof local maxima of $F(x)$ in $(-1, 1)$

D. for some $c \in (-1, 1)$, $F(c) \geq 6$, $F'(c) = 0$ and $f''(c) \leq 0$

Answer: A::B::D

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25. Let f(x) = $\begin{cases} x^3 + x^2 - 10x & -1 \leq x < 0 \\ \sin x & 0 \leq x < x/2 \\ 1 + \cos x & \pi/2 \leq x \leq \pi \end{cases}$ then f(x) has

A. local maximum at $x = \dfrac{\pi}{2}$

B. local minimum at $x = \dfrac{\pi}{2}$

C. absolute minimum at $x = 0, \pi$

D. absolute maximum at $x = \dfrac{\pi}{2}$

Answer: A::C::D

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26. Minimum distnace between the curves $y^2 = x - 1$ and $x^2 = x - 1$ and $x^2 = y - 1$ is equal to :

A. $\frac{\sqrt{2}}{4}$

B. $\frac{3\sqrt{2}}{4}$

C. $\frac{5\sqrt{2}}{4}$

D. $\frac{7\sqrt{2}}{4}$

Answer: B

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27. For the equation $\frac{e^{-x}}{x+1}$ which of the following statement(s) is/are correct ?

A. When $\lambda \in (0, \infty)$ equation has 2 real and distinct roots

B. When $\lambda, \in (-\infty, -e^2)$ equation has 2 real and istinct roots

C. When $\lambda \in (0, \infty)$ equation hs 1 real root

D. When $\lambda \in (-e, 0)$ equation has no real root

Answer: B::C::D

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28. If $y = mx + 5$ is a tangent to the curve $x^3y^3 = ax^3 + by^3 at P(1, 2)$, then

A. $a + b = \frac{18}{5}$

B. $a > b$

C. $a < b$

D. $a + b = \frac{19}{5}$

Answer: B::D

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29. If $(f(x)-1)(x^2+x+1)^2-(f(x)+1)(x^4+x^2+1)=0$ $\forall x \in R-\{0\}$ and $f(x)\neq \pm 1$, then which of the following statement (s) is/are correct ?

A. $|f(x) \geq 2 \forall x \in R-\{0\}$

B. $f(x)$ has a local maximum at $x=-1$

C. $f(x)$ has a local minimum at $x=1$

D. $\int_{-\pi}^{\pi}(\cos x) f(x) dx=0$

Answer: A::B::C::D

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Exercise Comprehension Type Problems

1. Let $y = f(x)$ such that $xy = x + y + 1, x \in R - \{1\}$ and $g(x) = xf(x)$

The minimum value of $g(x)$ is:

A. $3 - \sqrt{2}$

B. $3 + \sqrt{2}$

C. $3 - 2\sqrt{2}$

D. $3 + 2\sqrt{2}$

Answer: D

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2. Let $y = f(x)$ such that $xy = x + y + 1, x \in R - \{1\}$ and $g(x) = xf(x)$

There exist two values of x, x_1 and x_2 where $g'(x) = \frac{1}{2}$, then $|x_1| + |x_2| =$

A. 1

B. 2

C. 4

D. 5

Answer: C

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3. Let $f(x) = \begin{cases} 1-x & 0 \leq x \leq 1 \\ 0 & 1 < x \leq 2 \\ (2-x)^2 & 2 < x \leq 3 \end{cases}$ and $g(x) = \int_0^x f(t)dt.$

Let the tangent to the curve $y = g(x)$ at point P whose abscissa is $\frac{5}{2}$ cuts x-axis in point Q.

Let the pependiculat from point Q on x-axis meets the curve $y = g(x)$ in point R.

$g(1) =$

A. 0

B. $\frac{1}{2}$

C. 1

D. 2

Answer: B

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4. Let $f(x) = \begin{bmatrix} 1-x & 0 \le x \le 1 \\ 0 & 1 < x \le 2 \\ (2-x)^2 & 2 < x \le 3 \end{bmatrix}$ and $g(x) = \int_0^x f(t)dt.$

Let the tangent to the curve $y = g(x)$ at point P whose abscissa is $\frac{5}{2}$ cuts x-axis in point Q.

Let the pependiculat from point Q on x-axis meets the curve $y = g(x)$ in point R.

Rquation of tangent to the curve $y = g(x) at P$ is:

A. $3y = 12x - 1$

B. $3y = 12x - 1$

C. $12y = 3x - 1$

D. $12y = 3x + 1$

Answer: C

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5. Let $f(x) = \begin{bmatrix} 1-x & 0 \le x \le 1 \\ 0 & 1 < x \le 2 \\ (2-x)^2 & 2 < x \le 3 \end{bmatrix}$ and $g(x) = \int_0^x f(t)dt.$

Let the tangent to the curve $y = g(x)$ at point P whose abscissa is $\frac{5}{2}$ cuts x-axis in point Q.

Let the pependiculat from point Q on x-axis meets the curve $y = g(x)$ in point R.

If $'\theta'$ be the angle between tangents to the curve $y = g(x)$ at point P and R, then $\tan\theta$ equals to :

A. $\frac{5}{6}$

B. $\frac{5}{14}$

C. $\frac{5}{7}$

D. $\frac{5}{12}$

Answer: B

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6. Let $f(x) < 0 \forall x \in (\equiv \infty, 0)$ and $f(x) > 0 \forall x \in (0, \infty)$ also $f(0) = o,$ Again $f'(x) < 0 \forall x \in (-\infty, -1)$ and $f'(x) > \forall x \in (-1, \infty)$ also $f'(-1) = 0$ given $\lim_{x \to \infty} f(x) = 0$ and $\lim_{x \to \infty} f(x) = \infty$ and function is twice differentiable.

If $f'(x) < 0 \forall x \in (0, \infty)$ and $f'(0) = 1$ then number of solutions of equatin $f(x) = x^2$ is :

A. 2

B. 3

C. 4

D. None of these

Answer: D

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7. Let $f(x) < 0 \forall x \in (\equiv \infty, 0)$ and $f(x) > 0 \forall x \in (0, \infty)$ also $f(0) = o,$ Again $f'(x) < 0 \forall x \in (- \infty, - 1)$ and $f'(x) > \forall x \in (- 1, \infty)$ also $f'(- 1) = 0$ given $\lim_{x \to \infty} f(x) = 0$ and $\lim_{x \to \infty} f(x) = \infty$ and function is twice differentiable.

If $f'(x) < 0 \forall x \in (0, \infty)$ and $f'(0) = 1$ then number of solutions of equatin $f(x) = x^2$ is :

A. 1

B. 2

C. 3

D. 4

Answer: B

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8. Let $f(x) < 0 \forall x \in (-\infty, 0)$ and $f(x) > 0 \forall x \in (0, \infty)$ also $f(0) = 0$, Again $f'(x) < 0 \forall x \in (-\infty, -1)$ and $f'(x) > \forall x \in (-1, \infty)$ also $f'(-1) = 0$ given $\lim_{x \to \infty} f(x) = 0$ and $\lim_{x \to \infty} f(x) = \infty$ and function is twice differentiable.

The minimum number of points where $f'(x)$ is zero is:

A. 1

B. 2

C. 3

D. 4

Answer: A

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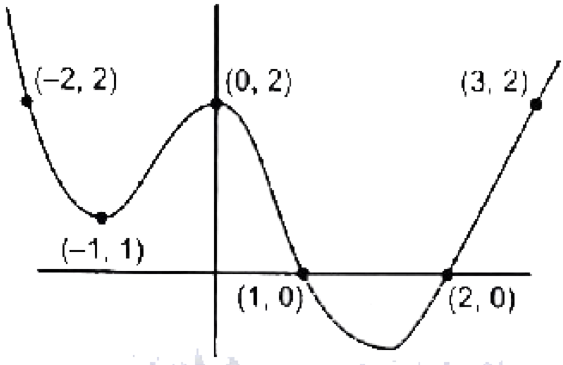
9. In the given figure graph of :

$y = p(x) = x^n + a_1 x^{n-1} + a_2 x^{n-2} + \ldots. + a_n$ is given.



The product of all imaginary roots of $p(x) = 0$ is:

A. -2

B. -1

C. $-1/2$

D. noen of these

Answer: D

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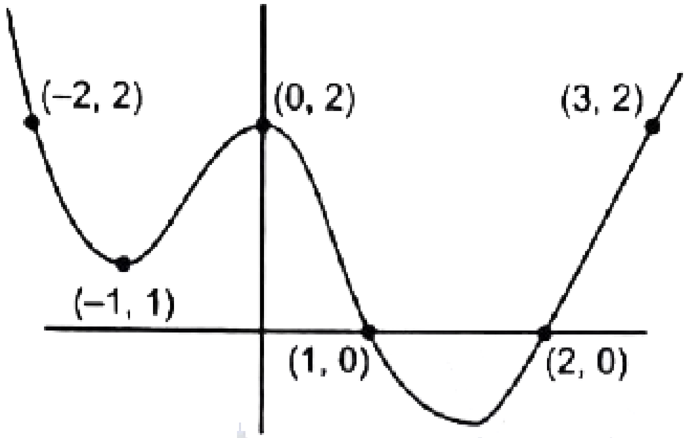
10. In the given figure graph of :

$y = p(x) = x^n + a_1 x^{n-1} + a_2 x^{n-2} + \ldots\ldots + a_n$ is given.



If $p(x) + k = 0$ has 4 distinct real roots $\alpha, \beta, \gamma, \delta$ then $[\alpha] + [\beta] + [\gamma] + [\delta]$, (where [.] denotes greatest integer function) is equal to:

A. -1

B. -2

C. 0

D. 1

Answer: A

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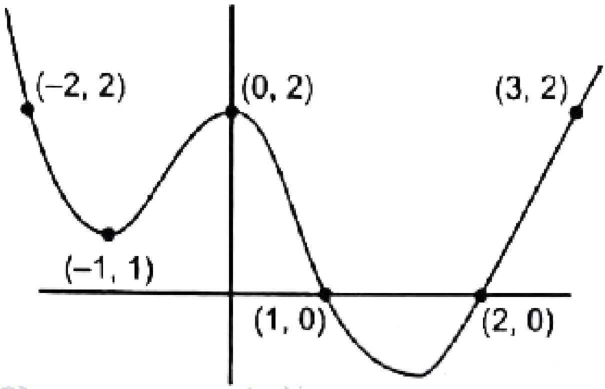
11. In the given figure graph of :

$y = p(x) = x^n + a_1 x^{n-1} + a_2 x^{n-2} + \ldots.. + a_n$ is given.



The minimum number of real roots of equation $(p'(x))^2 + p(x)p''(x) = 0$ are:

A. 3

B. 4

C. 5

D. 6

Answer: B

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12. The differentiable function $y = f(x)$ has a property that the chord joining any two points $A(x_1, f(x_1)$ and $B(x_2, g(x_2))$ always intersects y-axis at $(0, 2x_1, x_2)$. Given that $f(1) = -1$. then:

$\int_0^{1/2} f(x)dx$ is equal to :

A. $\frac{1}{6}$

B. $\frac{1}{8}$

C. $\frac{1}{12}$

D. $\frac{1}{24}$

Answer: D

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13. The differentiable function $y = f(x)$ has a property that the chord joining any two points $A(x_1, f(x_1)$ and $B(x_2, g(x_2))$ always intersects y-axis at $(0, 2x_1, x_2)$. Given that $f(1) = -1$. then:

The largest interval in whichy $f(x)$ is monotonically increasing, is :

A. $\left(-\infty, \frac{1}{2}\right]$

B. $\left[\frac{-1}{2}, \infty\right)$

C. $\left(-\infty, \frac{1}{4}\right]$

D. $\left[\frac{-1}{4}, \infty\right)$

Answer: C

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14. The differentiable function $y = f(x)$ has a property that the chord joining any two points $A(x_1, f(x_1)$ and $B(x_2, g(x_2))$ always intersects y-axis at $(0, 2x_1, x_2)$. Given that $f(1) = -1$. then:

In which of the following intervals, the Rolle's theorem is applicable to the function $F9x) = f(x) + x$?

A. $0 - 1, 0]$

B. $[0, 1]$

C. $[-1, 1]$

D. $[0, 2]$

Answer: B

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15. Let $f(x) = 1 + \int_0^1 (xe^y + ye^x) f(y) dy$ where x and y are independent vartiables.

If complete solution set of 'x' for which function $h(x) = f(x) + 3x$ is strictly increasing is $(-\infty, k)$ then $\left[(4)e^{\frac{k}{3}}\right]$ equals to: (where [.] denotes greatest integer function):

A. 1

B. 2

C. 3

D. 4

Answer: C

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16. Let $f(x) = 1 + \int_0^1 (xe^y + ye^x) f(y) dy$ where x and y are independent vartiables.

If acute ange of intersection of the curves $\frac{x}{2} + \frac{y}{3} + \frac{1}{5} = 0$ and $y = f(b) be\theta$ then $\tan\theta$, equals to:

A. $\frac{8}{25}$

B. $\frac{16}{25}$

C. $\frac{14}{25}$

D. $\frac{4}{5}$

Answer: A

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Exercise Mathcing Type Problems

1. The function $f(x) = \sqrt{(ax^3 + bx^2 + cx + a)}$ ha sits non-zero local minimum and local maximum values at $x - 2$ and $x = 2$, respectively. It 'a is a root of $x^2 - x - 6 = 0$

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Exercise Subjective Type Problems

1. A conical vessel is to be prepared out of a circular sheet of gold of unit radius. How much sectorial area is to be removed from the sheet so that the vessel has maximum volume?

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2. On $[1, e]$, then least and greatest vlaues of $f(x) = x^2 \ln x$ are m and M respectively, then $\left[\sqrt{M+m}\right]$ is : (where [] denotes greatest integer function)

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3. If $f(x) = \frac{px}{e^x} - \frac{x^2}{2} + x$ is a decreasing function for every $x \leq 0$. Find the least value of p^2.

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4. Let $f(x) = \begin{cases} xe^{ax}, & x \leq 0 \\ x + ax^2 - x^3, & x > 0 \end{cases}$ Where a is a positive constnat . The interval in which f '(x) is increasing is $\left[\frac{k}{a}, \frac{a}{l}\right]$, Then $k + l$ is equal to

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5. Find sum of all possible values of the real parameter b, if the difference between the largest and smallest values of the function $f(x) = x^2 - 2bx + 1$ in the interval $[0, 1]$ is 4.

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6. Let $'\theta'$ be the angle in radians between the curves $\frac{x^2}{36} + \frac{y^2}{4} = 1$ and $x^2 + y^2 = 12.$ If $\theta = \tan^{-1}\left(\frac{a}{\sqrt{3}}\right)$, Find the value of a.

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7. Let set of all possible values of λ such that $f(x) = e^{2x} - (\lambda + 1)e^x + 2x$ is monotonically increasing for $\forall x \in R$ is $(-\infty, k]$. Find the value of k.

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8. Let a,b,c and d be non-negative real number such that $a^5+b^5 \leq 1$ and $c^5+d^5 \leq 1$. Find the maximum value of $a^2c^3+b^2d^3$.

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9. There is a point (p,q) on the graph of $f(x)=x^2$ and a point (r, s) on the graph of $g(x)=\frac{-8}{x}, wherep > 0andr > 0$. If the line through $(p, q)and(r, s)$ is also tangent to both the curves at these points, respectively, then the value of $P+r$ is________.

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10. $f(x)=\max|2\sin y - x|$ where $y \in R$ then determine the minimum value of $f(x)$.

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11. Let $f(x)=\int_0^x \left((a-1)\left(t^2+t+1\right)^2-(a+1)\left(t^4+t^2+1\right)\right)$ dt.

Then the total numbr of integral values of 'a' for which $f'(x)=0$ has no rel roots is

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12. The numbr of real roots of the equation $x^{2013}+e^{20144x}=0$ is

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13. Let the maximum value of expression $y=\frac{x^4-x^2}{x^6+2x^3-1}$ for $x>1 is \frac{p}{1}$, where p and 1q are relatively prime natural numbers, then $p+q=$

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14. The least positive integral value of $'k'$ for which there exists at least one line that the tangent to the graph of the curve $y = x^3 - kx$ at one point and normal to the graph at another point is

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15. Let $f(x) = x^2 + 2x - t^2$ and $f(x) = 0$ has two root $\alpha(t)$ and $\beta(t)(\alpha < \beta)$ where t is a real parameter. Let $I(t) = \int_{\alpha}^{\beta} f(x)$ dx. If the maximum value of $I(t)$ be λ and $|\lambda| = \dfrac{p}{q}$ where p and q are relatively prime positive integers. Find the product (pq).

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16. A tank contains 100 litres of fresh water. A solution containing 1 gm/litre of soluble lawn fertilizeruns into the tank the of 1 lit/min and the mixture pumped out of the tank at the rate of at rate of f 3 litres/min. Find the time when the amount of fertilizer in the tank is maximum.

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17. If f (x) is continous and differentiable in $[3, 9)$ and $f'(x) \in [-2, 8] \forall x \in (-3, 9)$. Let N be the number of divisors of the greatest possible value of $f(9) - f(-3)$, then find the sum of digits of N.

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18. It is given that f 9x) is difined on R satisfyinf $f(1) = 1$ and for $\forall x \in R$,

$f(x+5) \geq f(x) + 5$ and $f(x+1) \leq f(x) + 1. If g(x) = f(x) + 1 - x,$

then g (2002)=

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19. The number of normals to the curve $3y^3 = 4x$ which passes through the point $(0, 1)$ is

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20. Find the number of real root (s) of the equation $ae^x = 1 + x + \frac{x^2}{2}$, where a is positive constant.

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21. Let $f(x) = ax + \cos 2x + \sin x + \cos x$ is defined for $\forall x \in R$ and $a \in R$ and is strictly increasing function. If the range of a is $\left[\frac{m}{n}, \infty\right)$, then find the minimum vlaue of $(m - n)$.

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22. If p_1 and p_2 are the lengths of the perpendiculars from origin on the tangent and normal drawn to the curve $x^{2/3} + y^{2/3} = 6^{2/3}$ respectively. Find the vlaue of $\sqrt{4p_1^2 + p_2^2}$.

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