



MATHS

BOOKS - VIKAS GUPTA MATHS (HINGLISH)

AREA UNDER CURVES

Axercise Single Choice Problems

1. The area enclosed by the curve

[x+3y]=[x-2] where $x\in[3,4]$ is :

(where[.] denotes greatest integer function)

A.
$$\frac{2}{3}$$

B. $\frac{1}{3}$
C. $\frac{1}{4}$

D. 1



2. The area of the region enclosed by $y=x^2 \; ext{and} \; y=\sqrt{|x|}$ is

A.
$$\frac{1}{3}$$

B. $\frac{2}{3}$
C. $\frac{4}{3}$
D. $\frac{16}{3}$

Answer: B



3. Find the area enclosed by the figure described by the equation $x^4+1=2x^2+y^2.$

A. 2

B.
$$\frac{16}{3}$$

C. $\frac{8}{3}$
D. $\frac{4}{3}$

Answer: C

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4. The area defined by $|y| \leq e^{|x|} - rac{1}{2}$ in cartesian co-ordinate system, is :

A. $(2-2\ln 2)$

B. $(4 - \ln 2)$

 $\mathsf{C.}\left(2-\ln 2\right)$

D. $(2 - 2 \ln 2)$

Answer: D

5. For each positive integer $n > a, A_n$ represents the area of the region restricted to the following two inequalities : $\frac{x^2}{n^2} + y^2$ and $x^2 + \frac{y^2}{n^2} < 1$. Find $\lim_{n \to \infty} A_n$.

A. 4

- B. 1
- C. 2

D. 3

Answer: A



6. Find the ratio in which the area bounded by the curves $y^2 = 12xandx^2 = 12y$ is divided by the line x = 3.

A. 7:15

B. 15:49

C.1:3

D. 17: 49

Answer: B

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7. The value of positive real parameter 'a' such that area of region blunded by parabolas $y = x - ax^2$, $ay = x^2$ attains its maximum value is equal to :

A.
$$\frac{1}{2}$$

B. 2
C. $\frac{1}{3}$

D. 1

Answer: D



8. For 0 < r < 1, let n_r denotes the line that is normal to the curve $y = x^{\mathbb{R}}$ at the point (1, 1) Let S_r denotes the region in the first quadrant bounded by the curve $y = x^r$, the x-axis and the line n_r ' Then the value of r the minimizes the area of S_r is :

A.
$$\frac{1}{\sqrt{2}}$$

B. $\sqrt{2} - 1$
C. $\frac{\sqrt{2} - 1}{2}$
D. $\sqrt{2} - \frac{1}{2}$

Answer: B

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9. The area bounded by $|x|=1-y^2 \, ext{ and } \, |x|+|y|=1$ is:

A.
$$\frac{1}{3}$$

B. $\frac{1}{2}$
C. $\frac{2}{3}$
D. 1

Answer: C

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10. Point A lies on the curve $y = e^{x^2}$ and has the coordinate (x, e^{-x^2}) where x > 0. Point B has the coordinates (x, 0). If 'O' is the origin, then the maximum area of the ΔAOB is

A.
$$\frac{1}{\sqrt{8}e}$$

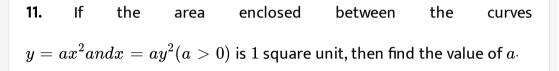
B.
$$\frac{1}{\sqrt{4}e}$$

C.
$$\frac{1}{\sqrt{2}e}$$

D.
$$\frac{1}{\sqrt{e}}$$

Answer: A





A.
$$\frac{1}{\sqrt{3}}$$

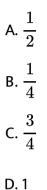
B. $\frac{1}{2}$
C. 1

D.
$$\frac{1}{3}$$

Answer: D

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12. Let $f(x) = x^3 - 3x^2 + 3x + 1$ and g be the inverse of it , then area bounded by the curve y = g(x) wirth x-axis between x = 1 to x = 2 is (in square units):



Answer: B



13. Area bounded by $x^2y^2 + y^4 - x^2 - 5y^2 + 4 = 0$ is equal to :

A.
$$rac{4\pi}{2}+\sqrt{2}$$

B. $rac{4\pi}{3}-\sqrt{2}$

$$\mathsf{C}.\,\frac{4\pi}{3}-\sqrt{2}$$

D. none of these

Answer: C

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14. Let $R41QEESf(x): RR^{\to +}$ is an invertible function such that f'(x) > 0 and $f^x > 0 \forall x \in [1, 5]$. If f(1) = 1 and f(5) = 5 and area under the curve y = f(x) on x-axis from $x = 1 \to x = 5is8$ sq. units, then area bounded by $y = f^{-1}(x)$ on x-axis from $x = 1 \to x = 5$ is 8 b. 12 c. 16 d. 20

A. 12

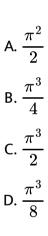
B. 16

C. 18

D. 20



15. A circel centered at origin and having radius π units is divided by the curve $y = \sin x$ in two parts. Then area of the upper part equals to



:

Answer: C

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16. The area of the loop formed by $y^2 = xig(1-x^3ig)$ dx is:

A.
$$\int_{0}^{1} \sqrt{x - x^{4}} dx$$

B. $2\int_{0}^{1} \sqrt{x - x^{4}} dx$
C. $\int_{-1}^{1} \sqrt{x - x^{4}} dx$
D. $4\int_{0}^{1/2} \sqrt{x - x^{4}} dx$

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17. If
$$f(x) = \min \left[x^2, \sin \frac{x}{2}, (x - 2\pi)^2\right]$$
, the area bounded by the curve $y = f(x)$, x-axis, $x = 0$ and $x = 2\pi$ is given by
Note: x_1 is the point of intersection of the curves x^2 and $\sin \frac{x}{2}, x_2$ is

the point of intersection of the curves sin $rac{x}{2}$ and $(x-2\pi)^2 \Big)$

A.

$$\int_{0}^{x_{1}} \Bigl(\sin rac{x}{2} \Bigr) dx + \int_{x_{1}}^{\pi} x^{2} dx + \int_{\pi}^{x_{2}} (x - 2\pi)^{2} dx + \int_{x_{2}}^{2\pi} \Bigl(\sin rac{x}{2} \Bigr) dx$$

$$\begin{array}{ll} \mathsf{B}. \int_{0}^{x_{1}}x^{2}dx + \int_{x_{1}}^{x_{3}}\left(\sin\frac{x}{2}\right)dx + \int_{x_{2}}^{2\pi}(x-2\pi)^{2}dx, \qquad \text{where} \\ x_{1} \in \left(0, \frac{\pi}{3}\right) \text{ and } x_{2} \in \left(\frac{5\pi}{3}, 2\pi\right) \\ \mathsf{C}. \int_{0}^{x_{1}}x^{2}dx + \int_{x_{1}}^{x_{2}}\sin\left(\frac{x}{2}\right)dx + \int_{x_{2}}^{2\pi}(x-2\pi)^{2}dx, \qquad \text{where} \\ x_{1} \in \left(\frac{\pi}{3}, \frac{\pi}{2}\right) \text{ and } x_{2} \in \left(\frac{3\pi}{2}, 2\pi\right) \\ \mathsf{D}. \int_{0}^{x_{1}}x^{2}dx + \int_{x_{1}}^{x_{2}}\sin\left(\frac{x}{2}\right)dx + \int_{x_{2}}^{2\pi}(x-2\pi)^{2}dx, \qquad \text{where} \\ x_{1} \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right) \text{ and } x_{2} \in (\pi, 2\pi) \end{array}$$

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18. The area enclosed between the curves
$$|x| + |y| \ge 2$$
 and $y^2 = 4\left(1 - \frac{x^2}{9}\right)$ is :
A. $(6\pi - 4)$ sq. units
B. $(6\pi - 8)$ se. units
C. $(3\pi - 4)$ se. units

D. $(3\pi - 2)$ sq. units

Answer: B

Axercise One Or More Than One Answer Is Are Correct

1.
$$T_n = \sum_{r=2n}^{3n-1} rac{r}{r^2 + n^2}, S_n = \sum_{r=2n+1}^{3n} rac{r}{r^2 + n^2}, ext{ then } orall n \in \{1, 2, 3...\}:$$

A. $T_n > rac{1}{2} \ln 2$
B. $S_n < rac{1}{2} \ln 2$
C. $T_n < rac{1}{2} \ln 2$
D. $S_n > rac{1}{2} \ln 2$

Answer: A::D

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2. If a curve $y = a\sqrt{x} + bx$ passes through point (1,2) and the area

bounded by curve, line x = 4 and x-axis is 6, then :

A.
$$a=rac{15}{4}$$

B. $b=3$
C. $a=-1$
D. $b=-rac{7}{4}$

Answer: A::D

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3. Area enclosed by the curves $y=x^{2+1}$ and a normal drawn to it with

gradient -1, is equal to:

A.
$$\frac{2}{3}$$

B. $\frac{1}{3}$

C.
$$\frac{3}{4}$$

D. $\frac{4}{3}$

Answer: D

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Axercise Comprehension Type Problems

1. Let
$$f: A \to Bf(x) = \frac{x+a}{bx^2 + cx + 2}$$
, where A represent domain set
and B represent range set of function $f(x)$ a,b,c
 $\in R, f(-1) = 0$ and $y = 1$ is an asymptote of
 $y = f(x)$ and $y = g(x)$ is the inverse of $f(x)$.

g (0) is equal to :

 $\mathsf{A.}-1$

B.-3

$$C. -\frac{5}{2}$$

$$\mathsf{D.}-rac{3}{2}$$

Answer: A

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2. Let
$$f: A \to Bf(x) = \frac{x+a}{bx^2 + cx + 2}$$
, where A represent domain set
and B represent range set of function $f(x)$ a,b,c
 $\in R, f(-1) = 0$ and $y = 1$ is an asymptote of
 $y = f(x)$ and $y = g(x)$ is the inverse of $f(x)$.

Area bounded between the curves y = f(x) and y = g(x) is:

A.
$$2\sqrt{5} + \ln\left(\frac{3-\sqrt{5}}{5+\sqrt{5}}\right)$$

B. $2\sqrt{5} + 2\ln\left(\frac{3+\sqrt{5}}{3-\sqrt{5}}\right)$
C. $3\sqrt{5} + 4\ln\left(\frac{3-\sqrt{5}}{3+\sqrt{5}}\right)$
D. $3\sqrt{5} + 2\ln\left(\frac{3-\sqrt{5}}{3+\sqrt{5}}\right)$

Answer: D

3. Let
$$f: A \to Bf(x) = \frac{x+a}{bx^2 + cx + 2}$$
, where A represent domain set
and B represent range set of function $f(x)$ a,b,c
 $\in R, f(-1) = 0$ and $y = 1$ is an asymptote of
 $y = f(x)$ and $y = g(x)$ is the inverse of $f(x)$.
Area of region enclosed by asymptotes of curves
 $y = f(x)$ and $y = g(x)$ is:

A. 4

B. 9

C. 12

D. 25

Answer: B

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4. For $j=0,1,2\ldots n$ let S_j be the area of region bounded by the x-axis and the curve $ye^x=\sin x$ for $f\pi\leq x\leq (j+1)\pi$

The value of S_o is :

A.
$$rac{1}{2}(1+e^x)$$

B. $rac{1}{2}(1+e^{-\pi})$
C. $rac{1}{2}(1-e^{-\pi})$
D. $rac{1}{2}(e^{\pi}-1)$

Answer: B



5. For $j=0,1,2\ldots n$ let S_j be the area of region bounded by the x-axis and the curve $ye^x=\sin x$ for $j\pi\leq x\leq (j+1)\pi$

The ratio ${S_{2009}\over S_{2010}}$ equals :

A. e^{-x}

 $\mathsf{B.}\,e^{\pi}$

$$\mathsf{C}.\,\frac{1}{2}e^x$$

D.
$$2e^{\pi}$$

Answer: B

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6. For $j=0,1,2{\dots}n$ let S_j be the area of region bounded by the x-axis

and the curve $ye^x=\sin x$ for $j\pi\leq x\leq (j+1)\pi$

The value of $\sum_{j=0}^\infty S_j$ equals to :

A.
$$rac{e^x(1+e^x)}{2(e^\pi-1)}$$

B. $rac{1+e^\pi}{2(e^\pi-1)}$
C. $rac{1+e^\pi}{e^\pi-1}$
D. $rac{e^\pi(1+e^\pi)}{(e^\pi-1)}$

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Axercise Subjective Type Problems

1. Let f be a differentiable function satisfying the condition $f\left(rac{x}{y}
ight)=rac{f(x)}{f(y)}(y
eq 0,f(y)
eq 0) \, orall x,y\in R ext{ and } f'(1)=2.$ If the

smaller area enclosed by $y=f(x), x^2+y^2=2$ is A, then findal [A],

where [.] represents the greatest integer function.

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2. Let f (x) be a function which satisfy the equation f(xy) = f(x) + f(y) for all x > 0, y > 0 such that f'(1) = 2. Let A be the area of the region bounded by the curves

$$y=f(x), y=\left|x^3-6x^2+11x-6
ight|$$
 and $x=0,$ then find value of $rac{28}{17}A.$



3. If the area bounded by circle $x^2 + y^2 = 4$, the parabola $y = x^2 + x + 1$ and the curve $y = \left[\sin^2 \frac{x}{4} + \cos \frac{x}{4}\right]$, (where [] denotes the greats integer function) and x-axis is $\left(\sqrt{3} + \frac{2\pi}{3} - \frac{1}{k}\right)$, then the numerical quantitity is should be :



4. Let the function $f: [-4, 4] \rightarrow [-1, 1]$ be defined impicity by the equatin $x + 5y - y^5 = 0$ If the area of triangle formed by tangent and normal to f(x)atx = 0 and the line y = 5 is A, find $\frac{A}{13}$.



5. Area of the region bounded by $\left[x
ight]^2=\left[y
ight]^2, ~~ ext{if}~~x\in[1,5]$, where []

denotes the greatest integer function is:



6. Consider $y = x^2$ and f(x) where f (x), is a differentiable function

satisfying

 $f(x+1)+f(z-1)=f(x+z)\,orall x,z\in R\,\, ext{and}\,\,f(0)=0,\,f^{\,\prime}(0)=4.$

If area bounded by curve $y=x^2 \, ext{ and } \, y=f(x)$ is $\Delta, \,$ find the value of

 $\left(\frac{3}{16}\Delta\right).$

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7. The least integer which is greater than or equal to the area of region

in
$$x-y$$
 plane satisfying $x^6-x^2+y^2\leq 0$ is:

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8. The set of points (x,y) in the plnae satisfying $x^{2/5} + |y| = 1$ form a curve enclosing a region of area $\frac{p}{q}$ square units, when p and q are relatively prime positive intergers. Find p - q.

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