MATHS

BOOKS - VIKAS GUPTA MATHS (HINGLISH)

BIONMIAL THEOREM

Exercise 1 Single Problems

1. Let $N = 2^{1224} - 1, \alpha = 2^{153} + 2^{77} + 1 \text{ and } \beta = 2^{408} - 2^{204} + 1$. Then which of the following statement is correct ?

A. α divides N but β does not

B. β divides N but α does not

C. α and β both divide N

D. neither α nor β divides N

Answer: C

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2. If $\left(1+x+x^2\right)^n=\sum_{r=0}^{2n} a_4 x^r$, then $a_r-{}^nC_1\cdot a_{r-1}+{}^nC_2 a_{r-2}-{}^nC_2 a_{r-3}+\ldots\ldots\ldots+(-1)^{rn}C_r a_0$ is equal to : (r is not multiple of 3)

A. 0

B. ${}^nC_r$

C. a_r

D. 1

Answer: A

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3. The coefficient of the middle term in the binomial expansion in powers of x of$(1+\alpha x)^4$ and of$(1-\alpha x)^6$ is the same, if α equals $-\frac{5}{3}$ b. $\frac{10}{3}$ c.

$-\frac{3}{10}$ d. $\frac{3}{5}$

A. $-\frac{5}{3}$

B. $\frac{3}{5}$

C. $-\frac{3}{10}$

D. $\frac{10}{3}$

Answer: C

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4. If $(1+x)^{2010} = C_0 + C_1x + C_2x^2 + \ldots\ldots + C_{2010}x^{2010}$ then the sum of series $C_2 + C_5 + C_8 + \ldots\ldots\ldots + C_{2009}$ equals to :

A. $\frac{1}{2}\left(2^{2010} - 1\right)$

B. $\frac{1}{3}\left(2^{2010} - 1\right)$

C. $\frac{1}{2}\left(2^{2009} - 1\right)$

D. $\frac{1}{3}\left(2^{2009} - 1\right)$

Answer: B

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5. Let $\alpha_n = (2+\sqrt{3})^n$. Find $\lim_{n \to \infty} (\alpha_n - [\alpha_n])$([.] denotes greateset integer function)

A. 1

B. $\frac{1}{2}$

C. $\frac{1}{3}$

D. $\frac{2}{3}$

Answer: A

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6. The number $N = {}^{20}C_7 - {}^{20}C_8 + {}^{20}C_9 - {}^{20}C_{10} + \ldots\ldots - {}^{20}C_{20}$ is not divisible by :

A. 3

B. 7

C. 11

D. 19

Answer: C

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7. The value of the expression $\log_2\left(1+\frac{1}{2}\sum_{k=1}^{11}{}^{12}C_k\right)$:

A. 11

B. 12

C. 13

D. 14

Answer: A

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8. The constant term in the expansion of $\left(x+\frac{1}{x^3}\right)^{12}$ is :

A. 26

B. 169

C. 260

D. 220

Answer: D

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9. If $\frac{3}{4!}+\frac{4}{5!}+\frac{5}{6!}+\ldots.+50term=\frac{1}{3!}-\frac{1}{(k-3)!}$, then sum of coefficients in the expansion $(1+2x_1+3x_2+\ldots.+100x_{100})^k$ is:

(where x_1, x_2, x_3, ...x_{100} are independent variable)

A. $(5050)^{49}$

B. $(5050)^{51}$

C. $(5050)^{52}$

D. $(5050)^{50}$

Answer: D

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10. Statement-1: The remainder when $(128)^{(128)^{128}}$ is divided by 7 is 3. because Statement-2: $(128)^{128}$ when divided by 3 leaves the remainder 1.

A. Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

B. Statement-1 is true, statement-2 is true and statement-2 is not the correct explanation for statement-1.

C. Statement-1 is true, statement-2 is false.

D. Statement-1 is false, statement-2 is true.

Answer: D

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11. If $n > 3,$ then $xyz^{n}C_0 - (x-1)(y-1)(z-1)^{n}C_1 + (x-2)(y-2)(z-2)^{n}C_2 - (x-3)(y-3)(z-3)^{n}C_3 + \ldots.. + (-1)^{n}(x-n)(y-n)(z-n)^{n}C_n$ equals :

A. xyz

B. $x+y+z$

C. $xy+yz+zx$

D. 0

Answer: D

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12. If $\alpha_1, \alpha_2, \ldots\ldots\ldots, \alpha_n$ are the n, n^{th} roots of unity, $\alpha_r = e^{\frac{i2(r-1)\pi}{n}}, r = 1, 2, \ldots n$ then ${}^nC_1\alpha_1 + {}^nC_2\alpha_2 + \ldots\ldots + {}^nC_n\alpha_n$ is equal to :

A. $\left(1 + \frac{\alpha_2}{\alpha_1}\right)^n - 1$

B. $\frac{\alpha_1}{2}\left[(1+\alpha_1)^n - 1\right]$

C. $\frac{\alpha_1 + \alpha_{n-1} - 1}{2}$

D. $(\alpha_1 + \alpha_{n-1})^n - 1$

Answer: A

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13. The remainder when $2^{30} \cdot 3^{20}$ is divided by 7 is :

A. 1

B. 2

C. 4

D. 6

Answer: B

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14. ${}^{26}C_0 + {}^{26}C_1 + {}^{26}C_2 + \ldots.. + {}^{26}C_{13}$ is equal to :

A. $2^{25} - \frac{1}{2} \cdot {}^{26}C_{13}$

B. $2^{25} + \frac{1}{2} \cdot {}^{26}C_{13}$

C. 2^{13}

D. $2^{26} + \frac{1}{2} \cdot {}^{26}C_{13}$

Answer: B

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15. If a_r is the coefficient of x^r in the expansion of $(1 + x + x^2)^n (n \in N)$. Then the value of $(a_1 + 4a_4 + 7a_7 + 10a_{10} + \ldots\ldots.)$ is equal to :

A. 3^{n-1}

B. 2^n

C. $\frac{1}{3} \cdot 2^n$

D. $n \cdot 3^{n-1}$

Answer: D

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16. Let $\binom{n}{k}$ represents the combination of $'n'$ things taken $'k'$ at a time, then the value of the sum $\binom{99}{97} + \binom{98}{96} + \binom{97}{95} + \ldots\ldots\ldots + \binom{3}{1} + \binom{2}{0}$ equals-

A. $\binom{99}{97}$

B. $\binom{100}{98}$

C. $\binom{99}{98}$

D. $\binom{100}{97}$

Answer: D

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17. The last digit of $9! + 3^{9966}$ is :

A. 1

B. 3

C. 7

D. 9

Answer: D

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18. Let x be the 7^{th} term from the beginning and y be the 7^{th} term from the end in the expansion of $\left(3^{1/3}+\frac{1}{4^{1/3}}\right)^{n}$. If $y=12x$ then the value of n is :

A. 9

B. 8

C. 10

D. 11

Answer: A

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19. $^{10}(C_0)^2-{}^{10}(C_1)^2+{}^{10}(C_2)^2-......-\left({}^{10}C_9\right)^2+\left({}^{10}C_{10}\right)^2=$

A. $10!$

B. $\left({}^{10}C_5\right)^2$

C. $-{}^{10}C_5$

D. $^{10}C_5$

Answer: C

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20. Find the ratio of the coefficient of x^{15} to them independent of x in the expansion of $\left(x^2 + \frac{2}{x}\right)^{15}$.

A. $1:4$

B. $1:32$

C. $7:64$

D. $7:16$

Answer: B

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21. In the expansion of $(1+x)^2(1+y)^3(1+z)^4(1+w)^5$, the sum of the coefficient of the terms of degree 12 is :

A. 61

B. 71

C. 81

D. 91

Answer: D

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22. If $\sum_{r=0}^{n}\left(\frac{r^3+2r^2+3r+2}{r+1}\right)^n C_r = \frac{2^4+2^3+2^2-2}{3}$

A. 2

B. 2^2

C. 2^3

D. 2^4

Answer: A

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Exercise 2 One Or More Than One Answer Is Are Correct

1. The number $N = {}^{20}C_7 - {}^{20}C_8 + {}^{20}C_9 - {}^{20}C_{10} + \ldots.. - {}^{20}C_{20}$ is divisible by :

A. 3

B. 4

C. 7

D. 19

Answer: A::B::C::D

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2. If $\left(1+x+x^2+x^3\right)^{100}=a_0+a_1+a_2x^2+\ldots\ldots+a_{300}x^{300}$, then

A. $a_1=100$

B. $a_0+a_1+a_2+\ldots\ldots+a_{300}$ is divisible by 1024

C. coefficients equidistant from beginning and end are equal

D. $a_0+a_2+a_4+\ldots\ldots\ldots+a_{300}=a_1+a_3+a_5+\ldots\ldots+a_{299}$

Answer: A::B::C::D

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3. $\sum_{r=0}^{4}(-1)^{r\,16}C_r$ is divisible by :

A. 5

B. 7

C. 11

D. 13

Answer: A::B::D

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4. Arrange the expansion of $\left(x^{1/2}+\frac{1}{2x^{1/4}}\right)$ in decreasing powers of x. Suppose the coefficient of the first three terms form an arithemetic progression. Then the number of terms in the expression having integer powers of x is -

A. 0

B. 2

C. 4

D. 8

Answer: A::C::D

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5. Let $\left(1+x^{2}\right)^{2} \cdot (1+x)^{n}=\sum_{k=0}^{n+4} a_{k} \cdot x^{k}$ If a_{1}, a_{2} and a_{3} are iun AP , find n.

A. 6

B. 4

C. 3

D. 2

Answer: B::C::D

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6. $\sum_{i=0}^{n} \sum_{j=0}^{n} \sum_{k=0}^{n} \binom{n}{i}\binom{n}{j}\binom{n}{k}, \binom{n}{r}={ }^{n} C_{r}$:

A. is less than 500 if n = 3

B. is greater than 600 if n = 3

C. is less than 5000 if n = 4

D. is greater than 4000 if n = 4

Answer: C::D

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7. If $^{100}C_6 + 4.\ ^{100}C_7 + 6.\ ^{100}C_8 + 4.\ ^{100}C_9 + ^{100}C_{10}$ has the value equal to ^xC_y, then the possible value (s) of $x + y$ can be :

A. 112

B. 114

C. 196

D. 198

Answer: B::D

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8. If the co-efficient of x^{2r} is greater than half of the co-efficient of x^{2r+1} in the expansion of $(1+x)^{15}$, then the possible value of 'r' equal to :

A. 5

B. 6

C. 7

D. 8

Answer: A::B::C

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9. Let $f(x) = 1 + x^{111} + x^{222} + x^{333} \dots\dots\dots\dots\dots + x^{999}$ then $f(x)$ is divisible by

A. $x+1$

B. x

C. $x-1$

D. $1 + x^{222} + x^{444} + x^{666} + x^{888}$

Answer: A::D

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Exercise 4 Subjective Type Problems

1. The sum of series $3 \cdot {}^{2007}C_0 - 8 \cdot {}^{2007}C_1 + 13 \cdot {}^{2007}C_2 - 18 \cdot {}^{2007}C_3 + \ldots\ldots$ upto 2008 terms is K, then K is :

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2. In the polynomial function $f(x) = (x-1)(x^2-2)(x^3-3)\ldots\ldots(x^{11}-11)$ the coefficient of x^{60} is :

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3. If

$$\sum_{r=0}^{3n} a_r(x-4)^r = \sum_{r=0}^{3n} A_r(x-5)^r \text{ and } a_k = 1\,\forall K \geq 2n \text{ and } \sum_{r=0}^{3n} d_r(x-8$$

. Then find the value of $\dfrac{A_{2n}+D_{2n}}{B_{2n}}$.

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4. If $3^{101} - 2^{100}$ is divided by 11, the remainder is

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5. Find the hundred's digit in the co-efficient of x^{17} in the expansion of $\left(1+x^5+x^7\right)^{20}$.

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6. Let $n \in N, S_n = \sum_{r=0}^{3n} \hat{} (3n)C_r$ and $T_n = \sum_{r=0}^{n} \hat{} (3n)C_{3r}$, then $|S_n - 3T_n|$ equals

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7. Find the sum of possible real values of x for which the sixth term of $\left(3^{\log_3 \sqrt{9^{|x-2|}}} + 7^{\frac{1}{5}\log_7\left(3^{|x-2|-9}\right)}\right)^7$ equals 567.

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8. Let q be a positive with $q \leq 50$.

If the sum $^{98}C_{30} + 2\ ^{97}C_{30} + 3.\ ^{96}C_{30} + \ldots\ldots + 68.\ ^{31}C_{30} + 69.\ ^{30}C_{30} = {}^{100}$

Find the sum of the digits of q.

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9. The remainder when $\left(\sum_{k=1}^{5} {}^{20}C_{2k-1}\right)^{6}$ is divided by 11, is :

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10. Let $a = 3^{\frac{1}{223}} + 1$ and for all $n \geq 3$ let $f(n) = C(n,0)a^{n-1} - C(n,1).\,a^{n-2} - C(n,2)a^{n-3} - \ldots + (-1)^{n-1}.\,C($

If the value of $f(2007) + f(2008) = 3^{k}$ where $k \in N$ then the value of k is

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11. In the polynomial $(x-1)(x^{2}-2)(x^{3}-3)\ldots(x^{11}-11)$, the coefficient of x^{60} is :

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12. Let the sum of all divisors of the form $2^p \cdot 3^q$ (with p, q positive integers) of the number $19^{88} - 1$ be λ. Find the unit digit of λ.

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13. Find the sum of possible real values of x for which the sixth term of $\left(3^{\log_3 \sqrt{9^{|x-2|}}} + 7^{\frac{1}{5}\log_7\left(3^{|x-2|-9}\right)}\right)^7$ equals 567.

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14. Let $1 + \sum_{r=1}^{10} \left(3^r C(10, r) + rC(10, r)\right) = 2^{10}\left(\alpha 4^5 + \beta\right)$ where $\alpha, \beta \in N$ and $f(x) = x^2 - 2x - k^2 + 1$ If α, β lies betweenm the roots of $f(x) = 0$ then find the smalles positive integral value of k

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15. if $S_n = C_0C_1 + C_1C_2 + \ldots + C_{n-1}C_n$ and $\frac{S_{n+1}}{S_n} = \frac{15}{4}$ then n is

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