MATHS

BOOKS - VIKAS GUPTA MATHS (HINGLISH)

COMPLEX NUMBERS

Exercise 1 Single Choice Problems

1. Let t_1, t_2, t_3 be the three distinct points on circle |t|=1. if θ_1, θ_2 and θ_3 be the arguments of t_1, t_2, t_3 respectively then $\cos(\theta_1 - \theta_2) + \cos(\theta_2 - \theta_3) + \cos(\theta_3 - \theta_1)$

A. $\geq -\frac{3}{2}$

B. $\leq -\frac{3}{2}$

C. $\geq \frac{3}{2}$

D. ≤ 2

Answer: A

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2. The number of points of intersection of the curves represented by $arg(z-2-7i) = \cot^{-1}(2)$ and arg $\left(\frac{z-5i}{z+2-i}\right) = \pm\frac{\pi}{2}$

A. 0

B. 1

C. 2

D. None of these

Answer: A

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3. All three roots of $az^3 + bz^2 + cz + d = 0$, z being complex number. Further , assume that the origin, z_1 and z_2 form an equilateral triangle, then :

A. All a,b,c,d have the same sign

B. a,b,c have same sign

C. a,b,d have same sign

D. b,c,d have same sign

Answer: C

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4. Let z_1 and z_2 be theroots of the equation $z^2 + az + b = 0$ z being compex. Further, assume that the origin z_1 and z_2 form an equilatrasl triangle then (A) $a^2 = 4b$ (B) $a^2 = b$ (C) $a^2 = 2b$ (D) `a^2=3b

A. $a^2 = b$

B. $a^2 = 2b$

C. $a^2 = 3b$

D. $a^2 = 4b$

Answer: C

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5. Let Z and w be two complex number such that $|zw| = 1$ and $arg(z) = \pi/2$ then

A. 1

B. -1

C. i

D. $-i$

Answer: D

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6. If ω is a complex nth root of unity, then $\sum_{r=1}^{n}(a+b)\omega^{r-1}$ is equal to $\frac{n(n+1)a}{2}$ b. $\frac{nb}{1+n}$ c. $\frac{na}{\omega-1}$ d. none of these

A. $\frac{n(n+1)a}{2\omega}$

B. $\frac{nb}{1-n}$

C. $\frac{na}{\omega-1}$

D. None of these

Answer: C

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7. If α and β are complex numbers then the maximum value of $\frac{\alpha\bar{\beta}+\overline{\alpha}\beta}{|\alpha\beta|}=$ (A) 1 (B) 2 (C) gt2 (D) lt1

A. 1

B. 2

C. greater than 2

D. less than 1

Answer: B

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8. let z_1, z_2, z_3 and z_4 be the roots of the equation $z^4 + z^3 + 2 = 0$, then the value of $\prod_{r=1}^{4}(2z_r + 1)$ is equal to :

A. 28

B. 29

C. 30

D. 31

Answer: D

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9. If $\arg\left(\frac{z-6-3i}{z-3-6i}\right)=\frac{\pi}{4}$, then maximum value of |z| :

A. $6\sqrt{2}+3$

B. $6\sqrt{3}+3$

C. $\sqrt{2}+3$

D. 6

Answer: A

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10. If $z_1 \neq -z_2$ and $|z_1+z_2|=\left|\frac{1}{z_1}+\frac{1}{z_2}\right|$ then :

A. at least one of z_1, z_2 is unimodular

B. both z_1, z_2 are unimodular

C. $z_1 . z_2$ is unimodular

D. $z_1 - z_2$ is unimodular

Answer: C

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11. If |z-i| ≤ 2 and $z_0 = 5 + 3i$, the maximum value of $|iz + z_0|$ is

A. $5 + \sqrt{13}$

B. $5 + \sqrt{2}$

C. 7

D. 8

Answer: C

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12. If z_1, z_2, z_3 are vertices of a triangle such that $|z_1 - z_2| = |z_1 - z_3|$, then arg $\left(\frac{2z_1 - z_2 - z_3}{z_3 - z_2}\right)$ is :

A. $\pm\frac{\pi}{3}$

B. 0

C. $\pm\frac{\pi}{2}$

D. $\pm\frac{\pi}{6}$

Answer: C

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13. It is given that complex numbers z_1 and z_2 satisfy $|z_1|$ =2 and $|z_2|$ =3 . If the included angle of their corresponding vectors is 60° , then $\left|\frac{z_1+z_2}{z_1-z_2}\right|$ can be expressed as $\frac{\sqrt{n}}{\sqrt{7}}$, where 'n' is a natural number then n=

A. 126

B. 119

C. 133

D. 19

Answer: C

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14. If al the roots oif $z^3 + az^2 + bz + c = 0$ are of unit modulus, then (A) $|a| \leq 3$ (B) $|b| \leq 3$ (C) $|c| = 1$ (D) none of these

A. $|a| \leq 3$

B. $|b| \leq 3$

C. $|c| = 1$

D. All of the above

Answer: D

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15. Let z be a complex number satisfying $\frac{1}{2} \leq |z| \leq 4$, then sum of greatest and least values of $\left|z + \frac{1}{z}\right|$ is :

A. $\frac{65}{4}$

B. $\frac{65}{16}$

C. $\frac{17}{4}$

D. 17

Answer: C

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16. if $|z-2i| \leq \sqrt{2}$, then the maximum value of |3+i(z-1)| is :

A. $\sqrt{2}$

B. $2\sqrt{2}$

C. $2+\sqrt{2}$

D. $3+2\sqrt{2}$

Answer: B

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17. Let $x - \frac{1}{x} = (\sqrt{2})i$ where $i = \sqrt{-1}$. Then the value of $x^{2187} - \frac{1}{x^{2187}}$ is :

A. $i\sqrt{2}$

B. $-i\sqrt{2}$

C. -2

D. $\frac{i}{\sqrt{2}}$

Answer: A

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18. If $z = re^{i\theta}$ (r gt 0 & $0 \leq \theta < 2\pi$) is a root of the equation $z^8 - z^7 + z^6 - z^5 + z^4 - z^3 + z^2 - z + 1 = 0$ then number of value of $'\theta'$ is :

A. 6

B. 7

C. 8

D. 9

Answer: C

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19. Let P and Q be two points on the circle |w|=r represented by w_1 and w_2 respectively, then the complex number representing the point of intersection of the tangents of P and Q is :

A. $\frac{w_1 w_2}{2(w_1 + w_2)}$

B. $\frac{2w_1 \overline{w}_2}{w_1 + w_2}$

C. $\frac{2w_1 w_2}{w_1 + w_2}$

D. $\frac{2\overline{w}_1 w_2}{w_1 + w_2}$

Answer: C

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20. If z_1, z_2, z_3 are complex number , such that $|z_1| = 2, |z_2| = 3, |z_3| = 4$, the maximum value $|z_1 - z_2|^2 + |z_2 - z_3|^2 + |z_3 - z_1|^2$ is :

A. 58

B. 29

C. 87

D. None of these

Answer: C

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21. If $Z = \dfrac{7+i}{3+4i}$, then find Z^{14}.

A. 2^7

B. $(-2)^7$

C. $(2^7)i$

D. $(-2^7)i$

Answer: C

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22. If |Z-4| + |Z+4|=10 , then the difference between the maximum and the minimum values of |Z| is :

A. 2

B. 3

C. $\sqrt{41}-5$

D. 0

Answer: A

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Exercise 2 One Or More Than One Answer Is Are Correct

1. Let Z_1 and Z_2 are two non-zero complex number such that $|Z_1 + Z_2| = |Z_1| = |Z_2|$, then $\frac{Z_1}{Z_2}$ may be :

A. $1 + \omega$

B. $1 + \omega^2$

C. ω

D. ω^2

Answer: C::D

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2. Let z_1, z_2 and z_3 be three distinct complex numbers , satisfying $|z_1| = |z_2| = |z_3| = 1$. Which of the following is/are true :

A. If $\arg\left(\frac{z_1}{z_2}\right) = \frac{\pi}{2}$ then $\arg\left(\frac{z - z_1}{z - z_2}\right) > \frac{\pi}{4}$ where |z| gt 1

B. $|z_1z_2 + z_2z_3 + z_3z_1| = |z_1 + z_2 + z_3|$

C. $lm\left(\frac{(z_1 + z_2)(z_2 + z_3)(z_3 + z_1)}{z_1 . z_2 . z_3}\right) = 0$

D. If $|z_1 - z_2| = \sqrt{2}|z_1 - z_3| = \sqrt{2}|z_2 - z_3|$, then Re$\left(\frac{z_3 - z_1}{z_3 - z_2}\right) = 0$

Answer: B::C::D

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3. The triangle formed by complex numbers z, iz, i^2z is Equilateral (b) Isosceles Right angle (d) Scalene

A. equilateral

B. isosceles

C. right angled

D. isosceles but not right angled

Answer: B::C

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4. if $A(z_1), B(z_2), C(z_3), D(z_4)$ lies on |z|=4 (taken in order) , where $z_1+z_2+z_3+z_4=0$ then :

A. Max. area of quadrilateral ABCD=32

B. Max. area of quadrilateral ABCD=16

C. The triangle ΔABC is right angled

D. The quadrilateral ABCD is rectangle

Answer: A::C::D

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5. Let z_1, z_2 and z_3 be three distinct complex numbers satisfying $|z_1|=|z_2|=|z_3|=1$. Which of the following is/are true ?

A. If $\arg\left(\frac{z_1}{z_2}\right)=\frac{\pi}{2}$ then $\arg\left(\frac{z-z_1}{z-z_2}\right)>\frac{\pi}{4}$ where | z| gt 1

B. $|z_1 z_2 + z_2 z_3 + z_3 z_1| = |z_1 + z_2 + z_3|$

C. $\lim \left(\frac{(z_1 + z_2)(z_2 + z_3)(z_3 + z_1)}{z_1 . z_2 . z_3} \right) = 0$

D. If $|z_1 - z_2| = \sqrt{2}|z_1 - z_3| = \sqrt{2}|z_2 - z_3|$, then Re $\left(\frac{z_3 - z_1}{z_3 - z_2} \right)$ = 0

Answer: B::C::D

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6. If $z_1 = a + ib$ and $z_2 = c + id$ are two complex numbers where a,b,c,d $\in$ R and $|z_1| = |z_2| = 1$ and Im $(z_1 \bar{z}_2) = 0$. If $w_1 = a + ic$ and $w_2 = b + id$, then :

A. $lm(w_1 \overline{w}_2) = 0$

B. $lm(\overline{w}_1 w_2) = 0$

C. $lm \left(\frac{(z_1 + z_2)(z_2 + z_3)(z_3 + z_1)}{z_1 . z_2 . z_3} \right) = 0$

D. Re $\left(\frac{w_1}{w_2} \right) = 0$

Answer: A::B::C

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7. The solutions of the equation $z^4 + 4iz^3 - 6z^2 - 4iz - i = 0$ represent vertices of a convex polygon in the complex plane. The area of the polygon is :

A. $2^{1/2}$

B. $2^{3/2}$

C. $2^{5/2}$

D. $2^{5/4}$

Answer: D

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8. Least positive argument of the 4^{th} root of the complex number $2 - i\sqrt{12}$ is :

A. $\frac{\pi}{6}$

B. $\frac{\pi}{12}$

C. $\frac{5\pi}{12}$

D. $\frac{7\pi}{12}$

Answer: C

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9. Let ω be the imaginary cube root of unity and $(a + b\omega + c\omega^2)^{2015} = (a + b\omega^2 + c\omega)$ where a,b,c are unequal real numbers . Then the value of $a^2 + b^2 + c^2 - ab - bc - ca$ equals.

A. 0

B. 1

C. 2

D. 3

Answer: B

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10. Let n be a positive integer and a complex number with unit modulus is a solution of the equation $Z^n + Z + 1 = 0$, then the value of n can be

A. 62

B. 155

C. 221

D. 196

Answer: A::B::C

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Exercise 3 Comprehension Type Problems

1. Let f(z) is of the form $\alpha z+\beta$, where α, β, z are complex numbers such that $|\alpha| \neq |\beta|$.f(z) satisfies following properties :

(i)If imaginary part of z is non zero, then $f(z)+\overline{f(z)}=f(\bar{z})+\overline{f(z)}$

(ii)If real part of of z is zero , then $f(z)+\overline{f(z)}=0$

(iii)If z is real , then $\overline{f(z)}f(z)>(z+1)^2 \forall z \in R$

$\frac{4x^2}{(f(1)-f(-1))^2}+\frac{y^2}{(f(0))^2}=1, x, y \in R$, in (x,y) plane will represent :

A. hyperbola

B. circle

C. ellipse

D. pair of line

Answer: A

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2. Let f(x) is of the form αz_{β} , where α, β are constants and α, β, z are complex numbers such that $|\alpha| \neq |\beta|$.f(x) satisfies following properties :

(i)If imaginary part of z is non zero, then $f(x) + \overline{f(z)} = f(\bar{z}) + \overline{f(z)}$

(ii)If real part of of z is zero , then $f(z) + \overline{f(x)} = 0$

(iii)If z is real , then $\overline{f(x)}\, f(x) > (x+1)^2 \,\forall z \in R$

Consider ellipse S: $\dfrac{x^2}{(Re(\alpha))^2} + \dfrac{y^2}{(Im(\beta))^2}\Bigg)$ =1 , x, y $\in$ R in (x,y) plane , then point (1,1) will lie :

A. outside the ellipse S

B. inside the ellipse S

C. on the ellipse S

D. none of these

Answer: B

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3. Let z_1 and z_2 be complex numbers such that $z_1^2 - 4z_2 = 16 + 20i$ and the roots α and β of $x^2 + z_1 x + z_2 + m = 0$ for some complex number m satisfies $|\alpha - \beta| = 2\sqrt{7}$.

The locus of the complex number m is a curve

A. a square with side 7 and centre (4,5)

B. a circle with radius 7 and centre (4,5)

C. a circle with radius 7 and centre (-4,5)

D. a square with side 7 and centre (-4,5)

Answer: B

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4. Let z_1 and z_2 be complex numbers such that $z_1^2 - 4z_2 = 16 + 20i$ and the roots α and β of $x^2 + z_1 x + z_2 + m = 0$ for some complex number m satisfies $|\alpha - \beta| = 2\sqrt{7}$.

The maximum value of $|m|$ is

A. $5\sqrt{21}$

B. $5+\sqrt{23}$

C. $7+\sqrt{43}$

D. $7+\sqrt{41}$

Answer: D

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5. Let z_1 and z_2 be complex numbers such that $z_1^2-4z_2=16+20i$ and the roots α and β of $x^2+z_1x+z_2+m=0$ for some complex number m satisfies $|\alpha-\beta|=2\sqrt{7}$.

The locus of the complex number m is a curve

A. $7-\sqrt{41}$

B. $7-\sqrt{43}$

C. $5-\sqrt{23}$

D. $5+\sqrt{21}$

Answer: A

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6. Let $z_1 = 3$ and $z_2 = 7$ represent two points A and B respectively on complex plane . Let the curve C_1 be the locus of pint P(z) satisfying $|z - z_1|^2 + |z - z_2|^2 = 10$ and the curve C_2 be the locus of point P(z) satisfying $|z - z_1|^2 + |z - z_2|^2 = 16$

Least distance between curves C_1 and C_2 is :

A. 4

B. 3

C. 2

D. 1

Answer: D

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7. Let $z_1 = 3$ and $z_2 = 7$ represent two points A and B respectively on complex plane . Let the curve C_1 be the locus of pint P(z) satisfying $|z - z_1|^2 + |z - z_2|^2 = 10$ and the curve C_2 be the locus of point P(z) satisfying $|z - z_1|^2 + |z - z_2|^2 = 16$

The locus of point from which tangents drawn to C_1 and C_2 are perpendicular , is :

A. |z-5|=4

B. |z-3|=2

C. |z-5|=3

D. |z-5|=$\sqrt{5}$

Answer: D

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8. In an Agrad plane z_1, z_2 and z_3 are, respectively, the vertices of an isosceles trinagle ABC with AC= BC and $\angle CAB = \theta$. If z_4 is incentre of triangle, then

The value of $AB \times AC/(IA)^2$ is

A. $\left|\frac{(Z_2 - Z_1)(Z_1 - Z_3)}{(Z_4 - Z_1)}\right|$

B. $\left|\frac{(Z_2 - Z_1)(Z_3 - Z_1)}{(Z_4 - Z_1)}\right|$

C. $\left|\frac{(Z_2 - Z_1)(Z_3 - Z_1)}{(Z_4 - Z_1)^2}\right|$

D. $\left|\frac{(Z_2 - Z_1)(Z_3 - Z_1)}{(Z_4 + Z_1)}\right|$

Answer: C

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9. In an Agrad plane z_1, z_2 and z_3 are, respectively, the vertices of an isosceles trinagle ABC with AC= BC and $\angle CAB = \theta$. If z_4 is incentre of triangle, then

The value of $(z_4 - z_1)^2(\cos\theta + 1)\sec\theta$ is

A. $(Z_2 - Z_1)(Z_3 - Z_1)$

B. $\frac{(Z_2 - Z_1)(Z_3 - Z_1)}{Z_4 - Z_1}$

C. $\dfrac{(Z_2 - Z_1)(Z_3 - Z_1)}{(Z_4 - Z_1)^2}$

D. $(Z_2 - Z_1)(Z_3 - Z_1)^2$

Answer: A

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Exercise 5 Subjective Type Problems

1. Let complex number 'z' satisfy the inequality $2 \le |x| \le 4$. A point P is selected in this region at random. The probability that argument of P lies in the interval $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ is $\frac{1}{K}$, then K =

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2. Let Z be a complex number satisfying $|Z - 1| \le |Z - 3|, |Z - 3| \le |Z - 5|, |Z + i| \le |Z - i|, |Z - i| \le |Z - 5i$

. Then area of region in which Z lies is A square units, Where A is equal to

:

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3. Complex number z_1 and z_2 satisfy $z + \bar{z} = 2|z - 1|$ and arg $(z_1 - z_2) = \frac{\pi}{4}$. Then the value of Im$(z_1 + z_2)$ is

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4. If $|z_1 = 1, |z_2| = 2, |z_3| = 3$ and $|z_1 + z_2 + z_3| = 1, then|9z_1z_2 + 4z_3z_1 + z$ is equal to (A) 3 (B) 36 (C) 216 (D) 1296

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5. If $|z_1|$ and $|z_2|$ are the distance of points on the curve $5z\bar{z} - 2i(z^2 - \bar{z}^2) - 9 = 0$ which are at maximum and minimum distance from the origin, then the value of $|z_1| + |z_2|$ is equal to :

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6. Let $\frac{1}{a_1 + \omega} + \frac{1}{a_2 + \omega} + \frac{1}{a_3 + \omega} + \ldots. + \frac{1}{a_n + \omega} = i$

where a_1, a_2, a_3 .... $a_n \in R$ and ω is imaginary cube root of unity , then evaluate $\sum_{r=1}^{n} \frac{2a_r - 1}{a_r^2 - a_r + 1}$.

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7. If $|z_1| = 2, |z_2| = 3, |z_3| = 4$ and $|2z_1 + 3z_2 + 4z_3| = 9$, then value of $|8z_2z_3 + 27z_3z_1 + 64z_1z_2|^{1/3}$ is :

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8. The sum of maximum and minimum modulus of a complex number z satisfying $|z - 25i| \leq 15, i = \sqrt{-1}$ is S , then $\frac{S}{10}$ is :

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