MATHS

BOOKS - VIKAS GUPTA MATHS (HINGLISH)

CONTINUITY, DIFFERENTIABILITY AND DIFFERENTIATION

Exercise Singe Choice Problems

1. Let 'f' be a fifferentiable real valued function satisfying $f(x+2y)=f(x)+f(2y)+6xy(x+2y)\,\forall x, y \in R.$ Then $f'(0), f(1), f'(2)\ldots..$ are in

A. AP

B. GP

C. HP

D. None of these

Answer: A

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2. Find the number of points of non-diferentiability for $f(x) = \max\{||x| - 1|, 1/2\}$.

A. 4

B. 3

C. 2

D. 5

Answer: D

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3. Number of points of discontinuity of $f(x) = \left\{\frac{x}{5}\right\} + \left\{\frac{x}{2}\right\}$ in $x \in [0, 100]$ is/are (where [-] denotes greatest integer function and {:}

denotes fractional part function)

A. 50

B. 51

C. 52

D. 61

Answer: A

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4. If $f(x)$ has isolated point of discontinuity at $x = a$ such that $|f(x)$ is continuous at $x = a$ then :

A. $\lim_{x \to a} f(x)$ doesn not exist

B. $\lim_{x \to a} f(x) + f(a) = 0$

C. $f(a) = 0$

D. None of these

Answer: B

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5. If $f(x)$ is a thrice differentiable function such that $\lim_{x \to 0} \frac{f(4x) - 3f(3x) + 3f(2x) - f(x)}{x^3} = 12$ then the vlaue of $f'''(0)$ equais to :

A. 0

B. 1

C. 12

D. None of these

Answer: C

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6.

$$y=\frac{1}{(1+\tan\theta)^{\sin\theta-\cos\theta}+(\cot\theta)^{\cos\theta-\cot\theta}}+\frac{1}{(1+\tan\theta)^{\cos\theta-\sin\theta}+(\cot\theta)$$

then $\frac{dy}{dx}$ at $\theta=\frac{\pi}{3}$ is

A. 0

B. 1

C. $\sqrt{3}$

D. None of these

Answer: A

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7. Let $f'(x)=\sin(x^2)$ and $y=f(x^2+1)$ then $\frac{dy}{dx}$ at $x=1$ is

A. $2\sin 2$

B. $2\cos 2$

C. $2\sin 4$

D. $\cos 2$

Answer: C

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8. If $f(x) = |\sin x - |\cos x||$, then the value of $f'(x)$ at $x = \frac{7\pi}{6}$ is

A. $\frac{\sqrt{3}+1}{2}$

B. $\frac{1-\sqrt{3}}{2}$

C. $\frac{\sqrt{3}-1}{2}$

D. $\frac{-1-\sqrt{3}}{2}$

Answer: C

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9. If $(\sin x)(\cos y) = \frac{1}{2}, then \frac{d^2y}{dx^2} at\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$ is -4 (b) -2 (c) -6 (d) 0

A. -4

B. -2

C. -6

D. 0

Answer: A

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10. f is a differentiable function such that $x = f(t^2), y = f(t^3)$ and $f'(1) \neq 0$ if $\left(\frac{d^2y}{dx^2}\right)_{t=1} =$

A. $\frac{3}{4}\left(\frac{f''(1) + f(1)}{(f'(1))^2}\right)$

B. $\frac{3}{4}\left(\frac{f(1).\, f'(1) - f'(1)}{f(f'(1))^2}\right)$

C. $\frac{4}{3}\frac{f'(1)}{(f'(1))^2}$

D. $\frac{4}{3}\left(\frac{f'(1)f''(1) - f'(1)}{(f'(1))^2}\right)$

Answer: A

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11. let $f(x)=\begin{cases} ax+1 & \text{if } x\leq 1 \\ 3 & \text{if } x=1 \\ bx^2+1 & \text{if } x>1 \end{cases}\Bigg)$ if $f(x)$ is continuous at $x=1$ then value of $a-b$ is (A) 0 (B) 1 (C) 2 (D) 4

A. 0

B. 1

C. 2

D. 4

Answer: A

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12. If

$$y=1+\frac{\alpha}{\left(\frac{a}{x}-\alpha\right)}+\frac{\beta/x}{\left(\frac{1}{x}-1\right)\left(\frac{1}{x}-\beta\right)}+\frac{\gamma/x^2}{\left(\frac{1}{x}-\alpha\right)\left(\frac{1}{x}-\beta\right)\left(\frac{1}{x}-\gamma\right)},$$

find $\frac{dy}{dx}$

A. $y\left(\frac{\alpha}{1/x-\alpha}+\frac{\beta}{1/x-\beta}+\frac{\gamma}{1/x-\gamma}\right)$

B. $\frac{y}{x}\left(\frac{\alpha}{1/x-\alpha}+\frac{\beta}{1/x-\beta}+\frac{\gamma}{1/x-\gamma}\right)$

C. $y\left(\frac{\alpha}{1/x-\alpha}+\frac{\beta}{1/x-\beta}+\frac{\gamma}{1/x-\gamma}\right)$

D. $y\left(\frac{\alpha/x}{1/x-\alpha}+\frac{\beta/x}{1/x-\beta}+\frac{\gamma/x}{1/x-\gamma}\right)$

Answer: B

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13. If $f(x)=\sqrt{\frac{1+\sin^{-1}x}{1-\tan^{-1}x}}$, then f (0) is equal to :

A. 4

B. 3

C. 2

D. 1

Answer: D

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14. Let $f(x) = \begin{cases} \sin^2 x, & \text{x is rational} \\ -\sin^2 x, & \text{x is irrational} \end{cases}$, then set of points, where f (x) is continuous, is:

A. $\left\{(2n+1)\frac{\pi}{2} \in I\right\}$

B. a null set

C. $\{n\pi, n \in I\}$

D. set of all rational numbers

Answer: C

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15. The number of values of x in $(0, 2\pi)$ where the function $f(x) = \frac{\tan x + \cot x}{2} - \left|\frac{\tan x - \cot x}{2}\right|$ continuous but non-derivable :

A. 3

B. 4

C. 0

D. 1

Answer: B

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16. If $f(x) = |x - 1|$ and $g(x) = f(f(f(x)))$, then for $x > 2, g'(x)$ is equal to

A. 1 for $x > 2$

B. 1 for $2 < x < 3$

C. -1 for $2 < x < 3$

D. -1 for $x > 3$

Answer: C

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17. If $f(x)$ is continuous function $\forall x \in R$ and the range of $f(x) is \left(2, \sqrt{26}\right) and g(x) = \left[\frac{f(x)}{c}\right]$ is continuous $\forall x \in R,$ then find the least positive integral value of $c,$ where $[.]$ denotes the greatest integer function.

A. 3

B. 5

C. 6

D. 7

Answer: C

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18. If $y = x + e^x,$ then $\left(\frac{d^2y}{dy^2}\right)_{x = \ln 2}$ is :

A. $-\frac{1}{9}$

B. $-\frac{2}{27}$

C. $\frac{2}{27}$

D. $\frac{1}{9}$

Answer: B

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19. Let $f(x) = x^3 + 4x^2 + 6x$ and $g(x)$ be inverse then the vlaue of $g'(-4)$:

A. -2

B. 2

C. $\frac{1}{2}$

D. None of these

Answer: C

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20. If $f(x)=2+|x|-|x-1|-|x+1|$, then $f'\left(\frac{1}{2}\right)+f'\left(\frac{3}{2}\right)+f'\left(\frac{5}{2}\right)$ is equal to:

A. 1

B. -1

C. 2

D. -2

Answer: D

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21. If $f(x)=\cos\left(x^2-4[x]\right), 0<x<1,$ (whre [.] denotes greatest integer function) then $f'\left(\frac{\sqrt{\pi}}{2}\right)$ is equal to:

A. $-\sqrt{\frac{\pi}{2}}$

B. $\sqrt{\frac{\pi}{2}}$

C. 0

D. $\sqrt{\frac{\pi}{4}}$

Answer: A

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22. Let g (x) be then inverse of f (x) such that $f'(x) = \frac{1}{1+x^5}$, then $\frac{d^2(g(x))}{dx^2}$ is equal to:

A. $\frac{1}{1+(g(x))^5}$

B. $\frac{g'(x)}{1+(g(x))^5}$

C. $f(g(x))^4(1+g(x))^5\big)$

D. $1+(g(x))^5$

Answer: C

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23. Let $f(x) = \begin{cases} \min\left(x, x^2\right) & x \geq 0 \\ \max\left(2x, x - 1\right) & x < 0 \end{cases}$, then which of the following is not true ?

A. f (x) is not differentiable at $x = 0$

B. f (x) is not differentiable are exactly two points

C. f (x) is continous everywhere

D. f (x) is strictly increasing $\forall x \in R$

Answer: B

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24. if $f(x) = \lim_{x \to \infty} \left(\prod_{i=l}^{n} \cos\left(\frac{x}{2^l}\right)\right)$ then f '(x) is equal to:

A. $\frac{\sin x}{x}$

B. $\frac{x}{\sin x}$

C. $\frac{x \cos - \sin x}{x^2}$

D. $\frac{\sin x - x \cos x}{\sin^2 x}$

Answer: C

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25. Let $f(x) = \begin{cases} \frac{1 - \tan x}{4x - \pi} & x \neq \frac{\pi}{4} \\ \lambda & x = \frac{\pi}{4} \end{cases}, x \in \left[0, \frac{\pi}{2}\right)$, If f (x) is continuous in $\left[0, \frac{\pi}{2}\right)$ then λ is equal to:

A. 1

B. $\frac{1}{2}$

C. $-\frac{1}{2}$

D. -1

Answer: C

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26. Let $f(x) = \begin{cases} e^{\frac{1}{x^2}} \sin\frac{1}{x} & x \neq 0 \\ \lambda & x = \frac{\pi}{4} \end{cases}, then f'(0) =$

A. 1

B. -1

C. 0

D. Does not exist

Answer: C

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27. Let f be a differentiable function satisfying $f'(x) = 2f(x) + 10 \forall x \in R$ and $f(0) = 0$, then the number of real roots of the equation $f(x) + 5\sec^2 x = 0$ In $(0, 2\pi)$ is:

A. 0

B. 1

C. 2

D. 3

Answer: A

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28. If $f(x)=\begin{cases}\frac{\sin\{\cos x\}}{x-\frac{\pi}{2}} & x\neq\frac{\pi}{2}\\ 1 & x=\frac{\pi}{2}\end{cases}$, where {k} represents the fractional park of k, then:

A. f (x) is continous at $x=\frac{\pi}{2}$

B. $\lim_{x\to\frac{\pi}{2}}$ f (x) does not exist

C. $\lim_{x\to\frac{\pi}{2}} f(x)$ exists, but f is not continuous at $x=\frac{\pi}{2}$

D. $\lim_{x\to\frac{\pi}{2}} f(x)=1$

Answer: B

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29. Let $f(x)$ be a polynomial in x . Then the second derivative of $f(e^x) w \dot{r} \dot{t} \xi s \quad f^{e^x} \dot{e}^x + f'(e^x) \quad f^{e^x} \dot{e}^{2x} + f'(e^x) \dot{e}^{2x} \quad f^{e^x} e^{2x}$ (d) $f^{e^x} \dot{e}^{2x} + f'(e^x) \dot{e}^x$

A. $f''(e^x) e^x + f(e^x)$

B. $f''(e^x) e^{2x} + f'(e^x) e^{2n}$

C. $f''(e^x) e^x + f'(e^x) e^{2x}$

D. $f''(e^x) e^{2x} + e^x f'(e^x)$

Answer: D

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30. If $e f(x) = \log x$ and g(x) is the inverse function of f(x), then $g'(x)$ is

A. $e^x + x$

B. $e^{e^{e^x}} e^{e^x}$

C. $e^{e^x + z}$

D. $e^{e^{x}}$

Answer: C

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31. If $y = f(x)$ is differentiable $\forall x \in R,$ then

A. $y = |f(x)|$ is differentiable $\forall x \in R$

B. $y = f^2(x)$ is not-differentiable for atleast one x

C. $y = f(x)|f(x)|$ is non-differentiable for atleast one x

D. $y = |f(x)|^3$ is differentiable $Ax \in R$

Answer: D

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32. If $f(x) = (x-1)^4(x-2)^3(x-3)^2$ then the value of $f'(1) + f''(2) + f''(3)$ is:

A. 0

B. 1

C. 2

D. 6

Answer: A

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33. Indicate all correct alternatives if, $f(x) = \frac{x}{2} - 1$, then on the interval $[0, \pi]$

A. $\tan(f(x))$ and $\frac{1}{f(x)}$ are both continous

B. $\tan(f(x))$ and $\frac{1}{f(x)}$ are both discontinous

C. $tna(f(x))$ and $f^{-1}(x)$ are both continous

D. $\tan f(x)$ is continous but $f^{-1}(x)$ is not

Answer: C

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34. Let f (x)= $\left\{\left(\frac{\left(\frac{1}{e^{x-2}}-3\right)}{\frac{1}{3^{x-2}}+1}, x>2\right), \left(\frac{b\sin\{-x\}}{\{-x\}}, x<2\right), (c, x=2),\right.$ whre {.} denotes fraction part function, is continous at $x=2$, then $b+c=$

A. 0

B. 1

C. 2

D. 4

Answer: A

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35. Let $f(x) = \frac{e^{\tan x} - e^{x} + \ln(\sec x + \tan x) - x}{\tan x - x}$ be a continuous function at x=0. The value f(0) equals

A. $\frac{1}{2}$

B. $\frac{2}{3}$

C. $\frac{3}{2}$

D. 2

Answer: C

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36. Let $f(x) = \begin{cases} (1+ax)^{1/x} & x < 0 \\ \frac{(x+c)^{1/3} - 1}{(x+1)^{1/2} - 1} & x > 0 \end{cases}$, is continous at $x = 0$, then $3(e^{a} + b + c)$ is equal to:

A. 3

B. 6

C. 7

D. 8

Answer: C

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37. $\sqrt{x+y}+\sqrt{y-x}=c, then \frac{d^2y}{dx^2}$ equals

A. $\frac{2}{5}$

B. $\frac{4}{25}$

C. $\frac{2}{25}$

D. $\frac{1}{25}$

Answer: C

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38. If $f(x) = x^2 + x^4 + \log x$ and g is the inverse of f, then $g'(2)$ is:

A. 8

B. $\frac{1}{8}$

C. 2

D. $\frac{1}{4}$

Answer: B

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39. The number of points at which the function,

$f(x) = \begin{cases} \min\{|x|, x^2\} & \text{if } x \in (-\infty, 1) \\ \min(2x-1, x^2\} & \end{cases}$ otherwise is not differentiable is:

A. 0

B. 1

C. 2

D. 3

Answer: B

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40. If $f(x)$ is a function such that $f(x) + f'(x) = 0$ and $g(x) = (f(x))^2 + (f'(x))^2$ and $g(3) = 8,$ rhen $g(8) =$

A. 0

B. 3

C. 5

D. 8

Answer: D

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41. Let f is twice differerntiable on R such that $f(0) = 1, f'(0) = 0$ and $f''(0) = -1$, then for $a \in R, \lim_{x \to \infty} \left(f\left(\frac{a}{\sqrt{x}}\right)\right)^x =$

A. e^{-e^2}

B. $e^{\frac{a^2}{4}}$

C. $e^{\frac{a^2}{2}}$

D. e^{-2a^2}

Answer: C

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42. Let $f_1(x) = e^x$ and $f_{n+1}(x) = e^{f_n(x)})$ for any $n \geq 1, n \in N$. Then for any fixed n, the vlaue of $\frac{d}{dx} f\big)(n)(x)$ equals:

A. $f_n(x)$

B. $f_n(x) f_{n-1}(x) \ldots . f_2(x) f_2(x)$

C. $f_n(x) f_{n-1}(x)$

D. $f_n(x) f_{n-1}(x) \ldots . f_2(x) f_1(x) e^x$

Answer: B

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43. If $y = \tan^{-1}\left(\frac{x^{1/3} - a^{1/3}}{1 + x^{1/3} a^{1/3}}\right), x > 0, a > 0, then \frac{dy}{dx}$ is:

A. $\frac{1}{x^{2/3}(1 + x^{2/3})}$

B. $\frac{3}{x^{2/3}(1 + x^{2/3})}$

C. $\frac{1}{3x^{2/3}(1 + x^{2/3})}$

D. $\frac{1}{3x^{1/3}(1 + x^{2/3})}$

Answer: C

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44. The value of $k+f(0)$ so that $f(x)=\begin{cases}\frac{\sin(4k-1)x}{3x}, & x<0\\ \frac{\tan(4k+1)x}{5x}, & 0<x<\frac{\pi}{2}\\ 1, & x=0\end{cases}$

can bemade continous at $x=0$ is:

A. 1

B. 2

C. $\frac{5}{4}$

D. 0

Answer: B

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45. If $y=\sin^{-1}\left(\frac{x}{1+\sqrt{1-x^2}}\right)$, $|x|\leq 1$, then $\frac{dy}{dx}at\left(\frac{1}{2}\right)$ is:

A. $\frac{1}{\sqrt{3}}$

B. 3

C. $\dfrac{\sqrt{3}}{2}$

D. $\dfrac{2}{\sqrt{3}}$

Answer: A

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46. Let $f(x) = \dfrac{e^x x \cos x - x \log_e(1+x) - x}{x^2}, x \neq 0.$ If $f(x)$ is continuous at $x = 0,$ then $f(0)$ is equal to

A. 0

B. 1

C. -1

D. 2

Answer: A

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47. A function $f(x) = \max(\sin x, \cos x, 1 - \cos x)$ is non-derivable for n values of $x \in [0, 2\pi]$. Then the vaue of n is:

A. 2

B. 1

C. 3

D. 4

Answer: C

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48. Let g be the inverse function of a differentiable function f and $G(x) = \frac{1}{g(x)}$. If $f(4) = 2$ and $f'(4) = \frac{1}{16}$, then the value of $(G'(2))^2$ equals to:

A. 1

B. 4

C. 16

D. 64

Answer: A

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49. If $f(x) = \max . \left(x^4, x^2 \frac{1}{81}\right) \forall x \in [0, \infty),$ then the sum of square of reciprocal of all the values of x where f (x) is non-differentiable, is equal to:

A. 1

B. 81

C. 82

D. $\frac{82}{81}$

Answer: C

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50. If $f(x)$ is derivable at $x = 2$ such that $f(2) = 2$ and $f'(2) = 4$, then the value of $\lim_{h \to 0} \frac{1}{h^2}\left(\ln f(2+h^2) - \ln f(2-h^2)\right)$ is equal to

A. 1

B. 2

C. 3

D. 4

Answer: D

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51. Let $f(x) = (x^2 - 3x + 2)\left|(x^3 - 6x^2 + 11x - 6)\right| + \left|\sin\left(x + \frac{\pi}{4}\right)\right|$

Number of points at which the function $f(x)$ is non-differentiable in $[0, 2\pi]$, is

A. 5

B. 4

C. 3

D. 2

Answer: C

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52. Let f and g be differrntiable functions on R (the set of all real numbers) such that $g(1) = 2 = g'(1)$ and $f'(0) = 4. If h(x) = f(2xg(x) + \cos \pi x - 3)$ then $h'(1)$ is equal to:

A. 28

B. 24

C. 32

D. 18

Answer: C

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53. If $f(x) = \frac{(x+1)^7\sqrt{1+x^2}}{(x^2-x+1)^6}$, then the value of $f'(0)$ is equal to:

A. 10

B. 11

C. 13

D. 15

Answer: C

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54. Statement.1 : The function $f(x) = \lim_{x \to \infty} \frac{\log_e(1+x) - x^{2n}\sin(2x)}{1+x^{2n}}$

is discontinuous at $x = 1$

Statement.2: $L.H.L. = R.H.L. \neq f(1)$.

A. Statent:1 is ture , Statement:2 is true and Statement:2 is correct explanation for Statement-1

B. Statement-1 is true, Statement-2 is true and Statement-2 is not the correct explanation for Statemetn-1

C. Statement-1 is true, Statement-2 is false

D. Statement-1 is false, Statement-2 is true

Answer: C

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55. If $f(x) = \begin{bmatrix} x & \text{if x is rational} \\ 1 - x & \text{if x is irrational} \end{bmatrix}$, then number of points for $x \in R$, where $y = f(f(x))$ discontinous is:

A. 0

B. 1

C. 2

D. Infinitely many

Answer: A

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56. Number of points where $f(x) = \begin{cases} \max\left(|x^2 - x - 2, x^2 - 3x\right) & x \geq 0 \\ \max\left(\ln(-x), e^x\right) & x < 0 \end{cases}$ is non-differentiable will be:

A. 1

B. 2

C. 3

D. None of these

Answer: C

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57. If the function $f(x) = -4e^{\frac{1-x}{2}} + 1 + x + \frac{x^2}{2} + \frac{x^3}{3}$ and $g(x) = f^{-1}(x)$, then the vlue of $g'\left(\frac{-7}{6}\right)$ equals to :

A. $\frac{1}{5}$

B. $-\frac{1}{5}$

C. $\frac{6}{7}$

D. $-\frac{6}{7}$

Answer: A

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58. Find k, if possible, so that

$$f(x) = \begin{cases} \frac{\ln(2-\cos 2x)}{\ln^2(1+\sin 3x)} & x < 0 \\ k & x = 0 \\ \frac{e^{\sin 2x}-1}{\ln(1+\tan 9x)} & x > 0 \end{cases}$$ is continious at $x = 0$.

A. $\frac{2}{3}$

B. $\frac{1}{9}$

C. $\frac{2}{9}$

D. not possible

Answer: C

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59. A function is reprersented parametrically by the equations $x = \frac{1+t}{t^3}; y = \frac{3}{2t^2} + \frac{2}{t} \dot{T}henthevalueof\left|\frac{dy}{dx} - x\left(\frac{dy}{dx}\right)^3\right|$ is________

A. 2

B. 0

C. -1

D. -2

Answer: C

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60. If $y^{-2} = 1 + 2\sqrt{2}\cos 2x,$ then :

$\frac{d^2y}{dx^2} = y\left(py^2 + 1\right)\left(qy^2 - 1\right)$ then the vlaue of $(p+q)$ equals to:

A. 7

B. 8

C. 9

D. 10

Answer: D

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61. If $y^{-2} = 1 + 2\sqrt{2}\cos 2x,$ then :

$\frac{d^2y}{dx^2} = y\left(py^2 + 1\right)\left(qy^2 - 1\right)$ then the vlaue of $(p+q)$ equals to:

A. 7

B. 8

C. 9

D. 15

Answer: B

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62. Let $f: R \rightarrow R$ is not identically zero, differentiable function and satisfy the equals $f(xy) = f(x)f(y)$ and $f(x+z) = f(x) + f(z)$, then $f(5) =$

A. 3

B. 5

C. 10

D. 15

Answer: B

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63. If $y=\left(x+\sqrt{1+x^2}\right)^n$ then $(1+x^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}$

A. n^2y

B. y^{-n^2}

C. $-y$

D. $2x^2y$

Answer: A

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64. Let $g(x)=f\left(x-\sqrt{1-x^2}\right)$ and $f'(x)=1-x^2$ then $g'(x)$ equal to:

A. $1-x^2$

B. $\sqrt{1-x^2}$

C. $2x\left(x+\sqrt{x-x^2}\right)$

D. $2x\left(x-\sqrt{1-x^2}\right)$

Answer: C

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65. Let $f(x) = \lim_{n \to \infty} \frac{\log(2+x) - x^{2n} \sin x}{1 + x^{2n}}$. then

A. f (x) is continuous at $x = 1$

B. $\lim_{x \to 1} f(x) = \log, 3$

C. $\lim_{x \to 1^+} f(x) = -\sin 1$

D. $\lim_{x \to 1^+} f(x)$ does not exist

Answer: C

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66. If $f(x+y) = f(x) \times f(y)$ for all $x, y \in R$ and $f(5) = 2, f'(0) = 3, then f'(5) =$

A. 3

B. 1

C. -6

D. 6

Answer: C

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67. Let $f(x) = \lim_{n \to \infty} \frac{\log(2+x) - x^{2n} \sin x}{1 + x^{2n}}$. then

A. f (x) continuous at $x = 1$

B. $\lim_{x \to 1^-} f(x) = \log_e 3$

C. $\lim_{x \to 1^+} f(x) = -\sin 1$

D. $\lim_{x \to 1^-} f(x)$ does not exist

Answer: C

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68. If $f(x) = \begin{cases} \frac{x - e^x + 1 - (1 - \cos 2x)}{x^2} & x \neq 0 \\ k & x = 0 \end{cases}$ is continous at $x = 0$ then which of the following statement is false ?

A. $k = \frac{-5}{2}$

B. $\{k\} = \frac{1}{2}$

C. $[k] = -2$

D. $[k]\{k\} = \frac{-3}{2}$

Answer: C

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69. Let $f(x) = \left|\left|x^2 - 10x + 21\right| - p\right|$, then the exhausive set of values of for which f (x) has exactly 6 points of non-derivability, is:

A. $(4, \infty)$

B. $(0, 4)$

C. $[0, 4]$

D. $(-4, 4)$

Answer: B

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70. If $f(x) = \sqrt{\frac{1 + \sin^{-1} x}{1 - \tan^{-1} x}}$, then f (0) is equal to :

A. 4

B. 3

C. 2

D. 1

Answer: D

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71. For $t \in (0, 1), \leq tx = \sqrt{2^{\sin^{-1}(t)}}$ and $y = \sqrt{2^{\cos^{-1} t}},$ then $1 + \left(\frac{dy}{dx}\right)^2$ equals :

A. $\frac{x^2}{y^2}$

B. $\frac{y^2}{x^2}$

C. $\frac{x^2 + y^2}{y^2}$

D. $\frac{x^2 + y^2}{x^2}$

Answer: D

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72. Let $f(x) = -1 + |x-2|$ and $g(x) = 1 - |x|$ then set of all possible value (s) of for which (fog) (x) is discontinuous is:

A. $\{0, 1, 2\}$

B. $\{0, 2\}$

C. $\{0\}$

D. an empty set

Answer: D

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73. If $f(x) = [x]\tan(\pi x)$ then $f'(k')$ is equal to $(k \in$ and $[.]$ denotes greatest integer function):

A. $(k-1)\pi(-1)^k$

B. $k\pi$

C. $k\pi(-1)^{k+1}$

D. $(k-1)\pi(-1)^{k-1}$

Answer: B

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74. If $f(x) = \begin{cases} \frac{ae^{\sin x} + be^{-\sin x} - c}{x^2} & x \neq 0 \\ 2 & x = 0 \end{cases}$ is continous at $x = 0$, then:

A. $a = b = c$

B. $a = 2b = 3c$

C. $a = b = 2c$

D. $2a = 2b = c$

Answer: D

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75. If tan x cot $y = \sec\alpha$ where α is constant and $\alpha \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) then \frac{d^2y}{dx^2} at \left(\frac{\pi}{4}, \frac{\pi}{4}\right)$ equal to:

A. 0

B. 1

C. 2

D. 3

Answer: A

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76. If $y = (x-3)(x-2)(x-1) \times (x+1)(x+2)(x+3)$, then $\frac{d^2y}{dx^2} at x = 1$ is:

A. -101

B. 48

C. 56

D. 190

Answer: C

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77. Let $f(x+y)=f(x)f(y)\,\forall x, y \in R, f(0) \neq 0$ If $f(x)$ is continous at $x=0,$ then $f(x)$ is continuous at :

A. all natural numbers only

B. all integers only

C. all rational numbers only

D. all real numbers

Answer: D

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78. If

$f(x) = 3x^9 - 2x^4 + 2x^3 - 3x^2 + x + \cos + 5$ and $g(x) = f^{-1}(x)$,

then the value of $g'(6)$ equals:

A. 1

B. $\frac{1}{2}$

C. 2

D. 3

Answer: A

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79. If $y = f(x)$ and $z = g(x)$ then $\frac{d^2y}{dx^2}$ equals

A. $\frac{d'f'' - f'g''}{(g')^2}$

B. $\frac{g'f'' - f'g''}{(g')^3}$

C. $\frac{fg'' - g'f''}{(g')^3}$

D. None of these

Answer: B

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80. Let

$$f(x)=\begin{bmatrix} x+1 & x<0 \\ (x-1) & x\geq 0 \end{bmatrix} \text{ and } g(x)=\begin{bmatrix} x+1 & x<0 \\ (x-1)^2 & x\geq 0 \end{bmatrix}$$ then

the number of points where $g(f(x))$ is not differentiable.

A. 0

B. 1

C. 2

D. None of these

Answer: C

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81. 5.f(x)=[sinx]+[cos] ,$x\varepsilon[0, 2\pi]$, where[.] denotes the greatest integer function. Total number of point where f (x) is non-differentiable is equal to (A) 2 (B) 3 (C) 5 (D) 4

A. 2

B. 3

C. 4

D. 5

Answer: D

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82. Let $f(x) = \cos x, g(x) = \begin{cases} \min\{f(t) : 0 \leq t \leq x\}, & x \in [0,\pi] \\ (\sin x) - 1, & x > \pi \end{cases}$

Then

A. g (x) is discontinuous at $x = \pi$

B. $g(x)$ is continous for $x \in [0, \infty)$

C. $g(x)$ is differentiable at $x = \pi$

D. $g(x)$ is differentiable for $x \in [0, \infty)$

Answer: B

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83. If $f(x) = (4 + x)^n, \text{n}\varepsilon N$ and $f^r(0)$ represents the r^{th} derivative of $f(x)$ at $x = 0$, then the value of $\sum_{r=0}^{\infty} \frac{(f^r(0))}{r!}$ is equal to

A. 2^n

B. 3^n

C. 5^n

D. 4^n

Answer: C

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84. Let $f(x) = \begin{cases} \frac{x}{1+|x|}, & |x| \geq 1 \\ \frac{x}{1-|x|}, & |x| < 1 \end{cases}$, then domain of $f'(x)$ is:

A. $(-\infty, \infty)$

B. $(-\infty, \infty) - \{-1, 0, 1\}$

C. $(-\infty, \infty) - \{-1, 1\}$

D. $(-\infty, \infty) - \{0\}$

Answer: C

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85. If the function $f(x) = -4e^{\frac{1-x}{2}} + 1 + x + \frac{x^2}{2} + \frac{x^3}{3}$ and $g(x) = f^{-1}(x)$, then the value of $g'\left(\frac{-7}{6}\right)$ equals :

A. $\frac{1}{5}$

B. $-\frac{1}{5}$

C. $\frac{6}{7}$

D. $-\frac{6}{7}$

Answer: A

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86. The number of geinis at which the function $f(x) = (x - |x|)2(1 - x + |x|)2$ is not disiferenitairies in the intensail (-3, 4) is

A. Zero

B. One

C. Two

D. Three

Answer: A

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87. If $f(x) = \sqrt{\frac{1 + \sin^{-1} x}{1 - \tan^{-1} x}}$, then $f'(0)$ is equal to:

A. 4

B. 3

C. 2

D. 1

Answer: D

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88. If

$f(x) = [(e^{x-1}, , , 0 \le x \le 1), (x + 1 - \{x\}, ', 1 < x < 3)$ and $g(x) = x$

such that $f(x)g(x)$ is continous $[0, 3]$ then the ordered pair (a,b) is

(where {.} denotes fractional part function):

A. $(2, 3)$

B. $(1, 2)$

C. $(3, 2)$

D. $(2, 2)$

Answer: C

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89. Use the following table and the fact that $f(x)$ is invertible and differentiable everywhere to find $\left(f^{-1}(3)\right)'$:

x	$f(x)$	$f'(x)$
3	1	7
6	2	10
9	3	5

A. 0

B. $\frac{1}{5}$

C. $\frac{1}{10}$

D. $\frac{1}{7}$

Answer: B

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90. Let $f(x) = \begin{cases} x^n\left(\sin\frac{1}{x},\right) & x \neq 0 \\ 0, & x = 0 \end{cases}$ Such that f (x) is continuous at $x = 0$, $f'(0)$ is real and finlte, and $\lim_{x \to 0^+} f'(x)$ does not exist. The holds true for which of the following values of n ?

A. 0

B. 1

C. 2

D. 3

Answer: C

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Exercise One Or More Than One Answer Is Are Correct

1. If $f(x) = \tan^{-1}(sgn(n^2 - \lambda x + 1))$ has exactly one point of discontinuity, then the value of λ can be:

A. 1

B. -1

C. 2

D. -2

Answer: C::D

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2. $f(x) = \begin{cases} 2(x+1) & x \leq -1 \\ \sqrt{1-x^2} & -1 < x < 1 \\ |||x|-1|-1| & x \geq 1 \end{cases}$, then:

A. f (x) is non-differentialbe at exactly three points

B. f (x) is continous in $(-\infty, 1]$

C. $f(x)$ is differentiable in $(-\infty, -1)$

D. f (x) is finite type of discontinuity at $x = 1$, but continous at $x = -1$

Answer: A::C::D

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3. Let $f(x) = \begin{cases} \frac{x(3e^{1/x}+4)}{2-e^{1/x}} & (x \neq 0) \\ 0 & x = 0 \end{cases}$ $x \neq \frac{1}{\ln 2}$ which of the following statement (s) is/are correct ?

A. f (x) is continous $at x = 0$

B. $f(x)$ is non-dervable at $x = 0$

C. $f'(0^+) = -3$

D. $f'(0^-)$ does not exist

Answer: A::B::C

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4. Let $|f(x)| \leq \sin^2 x, \ \forall x \in R,$ then

A. f (x) is continous at $x = 0$

B. f (x) is differentiable at $x = 0$

C. $f(x)$ is continous but not differentiable at $x = 0$

D. $f(0) = 0$

Answer: A::B::D

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5. Let $f(x) = \begin{cases} \dfrac{a(1 - x\sin x) + b\cos x + 5}{x^2} & x < 0 \\ \left(1 + \left(\dfrac{dx + dx^3}{dx^2}\right)\right)^{\frac{1}{x}} & x > 0 \end{cases}$

If f is continous at $x = 0$ then correct statement (s) is/are:

A. $a + c = -1$

B. $b + x = -4$

C. $a + b = -5$

D. $c + d = an$ irrational number

Answer: A::B::C::D

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6. If $f(x) = |||x| - 2| + p|$ have more than 3 points non-derivability then the value of p can be :

A. 0

B. -1

C. -2

D. 2

Answer: B::C

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7. Identify the options having correct statement:

A. $f(x) = \sqrt[3]{x^2|x|} - 1|x|$ is nowhere non-differentiable

B. $\lim_{x \to \infty} (x+5)\tan^{-1}(x+1)\Big) - \big((x+1)\tan^{-1}(x+1)\big) = 2\pi$

C. $f(x) = \sin\Big(\ln\Big(x + \sqrt{x^2+1}\Big)\Big)$ is an odd function

D. $f(x) = \dfrac{4-x^2}{4x-x^3}$ is discontinous at exactly one point

Answer: A::B::C

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8. A twice differentiable function f(x)is defined for all real numbers and satisfies the following conditions $f(0) = 2; f'(0) - -5$ and $f(0) = 3.$ The function $g(x)$ is defined by $g(x) = e^{ax} + f(x)\,\forall x \in R$, where 'a' is any constant If $g'(0) + g(0) = 0$. Find the value(s) of 'a'

A. 1

B. -1

C. 2

D. -2

Answer: A::D

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9. If $f(x) = |\sin x|$, then

A. differentiable everwhere

B. not differentiable at $x = n\pi n, \in I$

C. not diffentiable at $x = 0$

D. continous at $x = 0$

Answer: A::D

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10. Let $[.]$ represent the greatest integer function and $f(x) = [\tan^2 x]$ then :

A. $\lim_{x \to 0} f(x)$ does not exist

B. f (x) is continuous at $x = 0$

C. f (x) is not differentiable at $x = 0$

D. $f'(0) = 0$

Answer: B::D

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11. Let f be a differentiable function satisfying $f'(x) = f'(-x) \forall x \in R$. Then

A. If $f(1) = f(2)$, then $f(-1) = f(-2)$

B. $\frac{1}{2}f(x) + \frac{1}{2}f(y) = f\left(\frac{1}{2}(x+y)\right))$ for all ral values of x,y

C. Let $f(x)$ be an even function, then $f(x) = 0 \forall x \in R$

D. $f(x) + f(-x) = 2f(o) \forall x \in R$

Answer: A::D

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12. Let $f: R \to R$ be a function, such that $|f(x)| \leq x^{4n}, n \in N \forall n \in R$ then $f(x)$ is:

A. discontinous at $x = 0$

B. continous at $x = 0$

C. non-differentiable at $x = 0$

D. differentiable at $x = 0$

Answer: B::D

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13. Let $f(x) = [x]$ and $g(x) = 0$ when x is an integer and $g(x) = x^2$ when x is not an integer ([] is ghe greatest integer function) then:

A. $\lim_{x \to 1} g(x)$ exists, but g (x) is not continous at $x = 1$

B. $\lim_{x \to 1} f(x)$ does not exist

C. gog is continous for all x

D. fog is continous for all x

Answer: A::B

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14. Let $f : R \to R$ is given by

$f(x) = \begin{cases} p + qx + x^2 & x < 2 \\ 2px + 3qx^2 & x \geq 2 \end{cases}$ then:

A. f (x) is continous in R if $3p + 10q = 4$

B. f (x) is differentiable in R if $p = q = \frac{4}{13}$

C. If $p = -2, q = 1,$ then f(x) is continuos in R

D. f (x) is differentiable in R if $1p + 11q = 4$

Answer: A::B::C

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15. Let $f(x) = |2x - 9| + |2x + 9|$. Which of the following are true ?

A. f (x) is not differerntiable at $x = \frac{9}{2}$

B. f (x) is not differentiable at $x = \frac{-9}{2}$

C. f (x) is differentiable at $x = 0$

D. f (x) is differentiable at $x = \frac{-9}{2}, 0, \frac{9}{2}$

Answer: A::B::C

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16. Let $f(x) = \max\left(x, x^2 x^3\right) \in -2 \leq x \leq 2$. Then:

A. f (x) is continous in $-2 \le x \le 2$

B. $f(x)$ is not differentiable at $x = 1$

C. $f(-1) + f\left(\frac{3}{2}\right) = \frac{35}{8}$

D. $f(-1) f'\left(\frac{3}{2}\right) = \frac{-35}{4}$

Answer: A::B::C

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17. Let $f(x)$ be a differentiable function satisfying $f(y) f\left(\frac{x}{y}\right) = f(x)\ \forall$, x,y $\in$ R, $y \ne 0$ and $f(1) \ne 0$, $f'(1) = 3$ then

A. sgn $(f(x))$ is non-differentiable at exactly one point

B. $\lim_{x \to 0} \frac{x^2(\cos x - 1)}{f(x)} = 0$

C. $f(x) = x$ has 3 solutions

D. $f(f(x)) - f^3(x) = 0$ has infinitely many solutions

Answer: A::B::C::D

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18. Let $f(x) = (x^2 - 3x + 2)(x^2 + 3x + 2)$ and α, β, γ satisfy $\alpha < \beta\gamma$ are the roots of $f'(x) = 0$ then which of the following is/are correct ([.] denots greatest integer function) ?

A. $[\alpha] = -2$

B. $[\beta] = -1$

C. $[\beta] = 0$

D. $[\alpha] = 1$

Answer: A::C

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19. Let $f: R \to R$ is given by

$f(x) = \begin{cases} p + qx + x^2 & x < 2 \\ 2px + 3qx^2 & x \geq 2 \end{cases}$ then:

A. f (x) is continous in R if $3p + 10q = 4$

B. $f(x)$ is differentiable in R is $p = q = \frac{4}{13}$

C. If $p = -2, q = 1,$ then f(x) is continous in R

D. f (x) is differentiable in R is $2p + 11q = 4$

Answer: A::B::C

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20. If $y = e^{x \sin(x^3)} + (\tan x)^x$ then $\frac{dy}{dx}$ may be equal to:

A.

$$e^{x \sin(x^3)}\left[3x^3 \cos(x^3) + \sin(x^3)\right] + (\tan x)^x[\ln \tan x + 2x \cos ec2x]$$

B.

$$e^{x \sin(x^3)}\left[x^3 \cos(x^3) + \sin(x^3)\right] + (\tan x)^x[\ln \tan x + 2x \cos ec2x]$$

C. $e^{x \sin(x^3)}\left[x^3 \sin(x^3) + \sin(x^3)\right] + (\tan x)^x[\ln \tan x + 2 \cos ec2x]$

D. $e^{x \sin(x^3)}\left[3x^3 \cos(x^3) + \sin(x^3)\right] + (\tan x)^x\left[\ln \tan x + \frac{x \sec^2}{\tan x}\right]$

Answer: A::D

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21. Let

$f(x) = x + (1 - x)x^3 + (1 - x)(1 - x^2)x^3 + \dots\dots\dots + (1 - x)(1 - x^2)..$

then :

A. $f(x) = -\prod_{r=1}^{n}(1 - n^r)$

B. $f(x) = 1 - \prod_{r=1}^{n}(1 - x^r)$

C. $f(x) = (1 = f(x))\left(\prod_{r=1}^{n}\frac{rx^{r-1}}{(1 - x^r)}\right)$

D. $f'(x) = f(x)\left(\left(\prod_{r=1}^{n}\frac{rx^{r-4}}{(1 - x^r)}\right)\right.$

Answer: B::C

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22. Let $f(x)=\begin{bmatrix}x^2+a & 0\le x<1\\ 2x+b & 1\le x\le 2\end{bmatrix}$ and $g(x)=\begin{bmatrix}3x+b & 0\le x<1\\ x^3 & 1\le x\le 2\end{bmatrix}$

If derivative of $f(x)$ w.r.t. $g(x) at x=1$ exists and is equal to λ, then which of the followig is/are correct?

A. $a+b=-3$

B. $a-b=1$

C. $\frac{ab}{\lambda}=3$

D. $\frac{-b}{\lambda}=3$

Answer: A::B::C::D

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23. If $f(x)=\begin{bmatrix}\frac{\sin[x^2]\pi}{x^2-3x+8}+ax^3+b & 0\le x\le 1\\ 2\cos\pi x+\tan^{-1}x & 1<x\le 2\end{bmatrix}$ is differentiable in $[0,2]$ then: ([.] denotes greatest integer function)

A. $a=\frac{1}{3}$

B. $a = \frac{1}{6}$

C. $b = \frac{\pi}{4} - \frac{13}{6}$

D. $b = \frac{\pi}{4} - \frac{7}{3}$

Answer: B::C

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24. If $f(x) = \begin{cases} 1 + x & 0 \le x \le 2 \\ 3x - 2 & 2 < x \le 3 \end{cases}$, then $f(f(x))$ is not differentiable at:

A. $x = 1$

B. $x = 2$

C. $x = \frac{5}{2}$

D. $x = 3$

Answer: A::B

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25. Let

$f(x) = (x+1)(x+2)(x+3)\ldots..(x+100)$ and $g(x) = f(x)f''(x) - f$

Let n be the numbers of rreal roots of $g(x) = 0,$ then:

A. $n < 2$

B. $n > 2$

C. $n < 100$

D. $n > 100$

Answer: A::C

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26. Let $f(x) = \begin{cases} |x| - 3 & x < 1 \\ |x-2| + a & x \geq 1 \end{cases}$

$g(x) = \begin{cases} 2 - |x| & x < 1 \\ Sgn(x) - b & x \geq 1 \end{cases}$ If h(x)=f(x) is discontinuous at exactly one point, then which of the following are correct?

A. $a = -3, b = 0$

B. $a = -3, b = -1$

C. $a = 2, b = 1$

D. $a = 0, b = 1$

Answer: A::B::C::D

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27. Let f (x) be a continous function in $[-1, 1]$ such that

$$f(x) = \begin{cases} \frac{\ln(ax^2+bx+c)}{x^2} & -1 \le x < 0 \\ 1 & x = 0 \\ \frac{\sin\left(e^{x^2}-1\right)}{x^2} & 0 < x \le 1 \end{cases}$$

Then which of the following is/are corrent

A. $a + b + c = 0$

B. $b = a + c$

C. $c = 1 + b$

D. $b^2 + c^2 = 1$

Answer: C::D

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28. f (x) is differentiable function satisfying the relationship $f^2(x) + f^2(y) + 2(xy - 1) = f^2(x + y) \forall x, y \in R$

Also $f(x) > 0 \forall x \in R$ and $f(\sqrt{2}) = 2$. Then which of the following statement (s) is/are correct about $f(x)$?

A. $[f(3)] = 3([.]$ denotes greatest integer function)

B. $f(\sqrt{7}) = 3$

C. $f(x)$ is even

D. $f'(0) = 0$

Answer: A::B::C::D

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29. The function $f(x) = \left[\sqrt{1 - \sqrt{1 - x^2}}\right]$, (where [.] denotes greatest integer function):

A. has domain $[-1, 1]$

B. is discontinous at two points in its domain

C. is discontinous at $x = 0$

D. is discontinous at $x = 1$

Answer: A::B::D

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30. A function f(x) satisfies the relation $f(x+y) = f(x) + f(y) + xy(x+y), \forall x, y \in R$. If f'(0) = - 1, then

A. f (x) is a polynomial funciton

B. f (x) is an exponetial function

C. f (x) is twice differentiable for all $x \in R$

D. $f'(3) = 8$

Answer: A::C::D

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31. The points of discontinuities of $f(x) = \left[\frac{6x}{\pi}\right]\cos\left[\frac{3x}{\pi}\right]\text{in}\left[\frac{\pi}{6}, \pi\right]$ is/are:(where [.] denotes greattest integer function)

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{2}$

D. π

Answer: B::C::D

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32. If $f(x) = \begin{cases} \frac{x^2}{2} & 0 \le x < 1 \\ 2x^2 - 3x + \frac{3}{2} & 1 \le x \le 2 \end{cases}$ then,

A. $f(x), f'(x)$ are continous

B. $f'(x)$ is continous, $f''(x)$ is not continous

C. $f''(x)$ is continous

D. $f''(x)$ is non differentiable

Answer: A::B::D

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33. If $x = \phi(t), y = \Psi(t), \quad \text{then } \frac{d^2y}{dx^2}$ is equal to

A. $\frac{\phi'\psi'' - \psi'\phi''}{(\phi')^2}$

B. $\frac{\phi'\psi'' - \psi'\phi''}{(\phi')^3}$

C. $\frac{\psi''}{\phi'} - \frac{\psi'\phi''}{(\phi')^2}$

D. $\frac{\psi''}{(\phi')^2} - \frac{\psi'\phi''}{(\phi')^3}$

Answer: B::D

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34. $f(x) = [x]$ and $g(x) = \begin{cases} 0, & x \in I \\ x^2, & \notin I \end{cases}$ where [.] dentoes the greatest integer function. Then

A. gof is continous for all x

B. gof is not continous for all x

C. fog is continous everywhere

D. fog is not continous everywhere

Answer: A

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35. Let $f: R^+ \to R$ defined as $f(x) = e^x +$ ln x and $g = f^{-1}$ then correct statement (s) is/are:

A. $g''(e) = \frac{1-e}{(1+e)^3}$

B. $g''(e)\frac{e-1}{(1+e)^3}$

C. $g'(e) = e+1$

D. $g'(e) = \frac{1}{e+1}$

Answer: A::D

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36. Let $f(x) = \begin{cases} \frac{3x-x^2}{2} & x < 2 \\ [x-1] & 2 \le x < 3 \\ x^2 - 8x + 17 & x \ge 3 \end{cases}$: then which of the following hold(s) good ?

A. $\lim_{x \to 2} f(x) = 1$

B. $f(x)$ is differentiable at $x = 2$

C. $f(x)$ is continous at $x = 2$

D. $f(x)$ is discontinous at $x = 3$

Answer: A::C::D

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Exercise Comprehension Type Problems

1. Let $f(x) = \lim_{n \to \infty} n^2 \tan\left(\ln\left(\sec\frac{x}{n}\right)\right)$ and $g(x) = \min\left(f(x), \{x\}\right\}$

(where {.} denotes fractional part function)

Left derivative of $\phi(x) = e^{\sqrt{2f(x)}} atx = 0$ is:

A. 0

B. 1

C. -1

D. Does not exist

Answer: C

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2. Let $f(x) = \lim_{n \to \infty} n^2 \tan\left(\ln\left(\sec\frac{x}{n}\right)\right)$ and $g(x) = \min\left(f(x), \{x\}\right)$

(where {.} denotes fractional part function)

Number of points in $x \in [-1, 2]$ at which g (x) is discontinous :

A. 2

B. 1

C. 0

D. 3

Answer: A

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3. Let $f(x)$ and $g(x)$ be two differentiable functions, defined as:

$f(x) = x^2 + xg'(1) + g''(2)$ and $g(x) = f(1)x^2 + xf'(x) + f''(x)$.

The value of $f(1) + g(-1)$ is:

A. 0

B. 1

C. 2

D. 3

Answer: D

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4. Let $f(x)$ and $g(x)$ be two differentiable functions, defined as:

$f(x)^2 = x^2 + xg'(1) + g''(2)$ and $g(x) = f(1)x^2 + xf'(x) + f''(x)$.

The number of integers in the domain of the function

$F(x) = \sqrt{-\frac{f(x)}{g(x)}} + \sqrt{3-x}$ is:

A. 0

B. 1

C. 2

D. Infinite

Answer: C

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5. Define: $f(x) = |x^2 - 4x + 3| \ln x + 2(x-2)^{1/3}, x > 0$

$$h(x) = \begin{cases} x - 1, & x \in Q \\ x^2 - x - 2, & x \notin Q \end{cases}$$

$f(x)$ is non-differentiable at...... points and the sum of corresponding x value (s) is ......

A. 3, 6

B. 2, 3

C. 2, 4

D. 2, 5

Answer: D

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6. Define: $f(x) = \left|x^2 - 4x + 3\right| \ln x + 2(x-2)^{1/3}, x > 0$

$$h(x) = \begin{cases} x - 1, & x \in Q \\ x^2 - x - 2, & x \notin Q \end{cases}$$

h (x) is discontinous at $x = \ldots\ldots$

A. $1 + \sqrt{2}$

B. $\tan\frac{3\pi}{8}$

C. $\tan\frac{7\pi}{8}$

D. $\sqrt{2} - 1$

Answer: D

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7. Consider a function defined in $[-2, 2]$

$$f(x) = \begin{cases} \{x\} & -2l \leq x < -1 \\ |sgn x| & -1 \leq x \leq 1 \\ \{-x\} & 1 < x \leq 2 \end{cases}$$, where {.} denotes the fractional part function.

The total number of points of discontinuity of $f(x)$ for $x \in [-2, 2]$ is:

A. 0

B. 1

C. 2

D. 4

Answer: B

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8. Consider a function defined in $[-2, 2]$

$f(x) = \begin{cases} \{x\} & -2 \le x < -1 \\ |sgn x| & -1 \le x \le 1 \\ \{-x\} & 1 < x \le 2 \end{cases}$, where {.} denotes the fractional part function.

The number of points for $x \in [-2, 2]$ where $f(x)$ is non-differentiable is:

A. 0

B. 1

C. 2

D. 3

Answer: D

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9. Consider a function $f(x)$ in $[0, 2\pi]$ defined as :

$$f(x) = \begin{bmatrix} [\sin x] + [\cos x] & 0 \le x \le \pi \\ [\sin x] - [\cos x] & \pi < x \le 2\pi \end{bmatrix}$$

where {.} denotes greatest integer function then.

Number of points where $f(x)$ is non-derivable :

A. 2

B. 3

C. 4

D. 5

Answer: B

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10. Consider a function $f(x)$ in $[0, 2\pi]$ defined as :

$$f(x) = \begin{bmatrix} [\sin x] + [\cos x] & 0 \leq x \leq \pi \\ [\sin x] - [\cos x] & \pi < x \leq 2\pi \end{bmatrix}$$

where {.} denotes greatest integer function then.

$\lim_{x \to \left(\frac{3\pi}{2}\right)^+}$, $f(x)$ equals

A. 0

B. 1

C. -1

D. 2

Answer: C

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11. Let $f(x) = \begin{cases} x[x] & 0 \leq x < 2 \\ (x-1) & 2 \leq x \leq 3 \end{cases}$ where [x]= greatest integer less than or equal to x, then:

The number of values of x for $x \in [0, 3]$ where $f(x)$ is dicontnous is:

A. 0

B. 1

C. 2

D. 3

Answer: C

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12. Let $f(x) = \begin{cases} x[x] & 0 \le x < 2 \\ (x-1) & 2 \le x \le 3 \end{cases}$ where [x]= greatest integer less than or equal to x, then:

The number of values of x for $x \in [0, 3]$ where $f(x)$ is non-differentiable is :

A. 0

B. 1

C. 2

D. 3

Answer: D

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13. Let $f(x) = \begin{cases} x[x] & 0 \leq x < 2 \\ (x-1) & 2 \leq x \leq 3 \end{cases}$ where [x]= greatest integer less than or equal to x, then:

The number of integers in the range of $y = f(x)$ is:

A. 3

B. 4

C. 5

D. 6

Answer: A

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14. Let $f: R \rightarrow R$ be a continous and differentiable function such that $f(x+y)=f(x).F(y)\,\forall x, y, f(x) \neq 0$ and $f(0)=1$ and $f'(0)=2.$

Let $f(xy)=g(x).G(y)\,\forall x, y$ and $g'(1)=2.\ g(1) \neq\ =0$

Identify the correct option:

A. $f(2)=e^{4}$

B. $f(2)=2e^{2}$

C. $(1)<4$

D. $f(3)>729$

Answer: A

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15. Let $f: R \rightarrow R$ be a continous and differentiable function such that $f(x+y)=f(x).F(y)\,\forall x, y, f(x) \neq 0$ and $f(0)=1$ and $f'(0)=2.$

Let $f(xy) = g(x).\, G(y)\, \forall x, y$ and $g'(1) = 2.\, g(1) \neq\; = 0$

Identify the correct option:

A. $g(2) = 2$

B. $g(3) = 3$

C. $g(3) = 9$

D. $g(3) = 6$

Answer: C

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16. Let $f: R \to R$ be a continous and differentiable function such that $f(x + y) = f(x).\, F(y)\, \forall x, y,\, f(x) \neq 0$ and $f(0) = 1$ and $f'(0) = 2.$

Let $f(xy) = g(x).\, G(y)\, \forall x, y$ and $g'(1) = 2.\, g(1) \neq\; = 0$

The number of values of x, where $f(x)g(x)$:

A. 0

B. 1

C. 2

D. 3

Answer: B

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17. $f(x)=\dfrac{\cos^2 x}{1+\cos x+\cos^2 x}$ and $g(x)=k\tan x+(1-k)\sin x-x$, where $k\in R$, $g'(x)=$

A. $\dfrac{(1-\cos x)(f(x)-\lambda)}{\cos x}$

B. $\dfrac{(1-\cos x)(\lambda-f(x))}{\cos x}$

C. $\dfrac{(1-\cos x)(\lambda-f(x))}{f(x)}$

D. $\dfrac{(1-\cos x)(\lambda-f(x))}{(f(x))^2}$

Answer: C

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18. Let $f(x)\dfrac{\cos^2 x}{1+\cos+\cos^2 x}$ and $g(x)\lambda\tan x+1(1-\lambda)\sin x-x,$ where $\lambda \in R$ and $x \in [0, \pi/2]$.

The exhaustive seet of values of $'\lambda'$ such that $g'(x) \geq 0$ for any $x \in [0, \pi/2)$:

A. $[1, \infty)$

B. $[0, \infty)$

C. $\left[\frac{1}{2}, \infty\right)$

D. $\left[\frac{1}{3}, \infty\right)$

Answer: D

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19.

$f(x) = \lim_{x\to\infty} \dfrac{x^2+2(x+1)^{2n}}{(x+1)^{2n+1}+x^2+1}, n \in N$ and $g(x) = \tan\left(\frac{1}{2}\sin^{-1}\left(\right.\right.$

then

The number of points where $g(x)$ is non-differentiable $\forall x \in R$is:

A. 1

B. 2

C. 3

D. 4

Answer: D

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20.

$$f(x) = \lim_{x \to \infty} \frac{x^2 + 2(x+1)^{2n}}{(x+1)^{2n+1} + x^2 + 1}, n \in N \text{ and } g(x) = \tan\left(\frac{1}{2}\sin^{-1}\left(\right.\right.$$

then

$$\lim_{x \to -3} \frac{(x^2 + 4x + 3)}{\sin(x+3)g(x)}$$ is equal to:

A. 1

B. 2

C. 4

D. Non-existent

Answer: B

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21.

$$f(x) = \lim_{x \to \infty} \frac{x^2 + 2(x+1)^{2n}}{(x+1)^{2n+1} + x^2 + 1}, n \in N \text{ and } g(x) = \tan\left(\frac{1}{2}\sin^{-1}\left(\right.\right.$$

then

$\lim_{x \to 0^-} \left\{ \frac{f(x)}{tna^2 x} \right\} + \left| \lim_{x \to 2^-} f(x) \right| + \lim_{x \to 2^+} (5f(x))$ is equal to (where {.} denotes fraction part function)

A. 7

B. 8

C. 12

D. Non-existent

Answer: A

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22. Let f and g be two differentiable functins such that:

$f(x) = g'(1)\sin x + (g''(2) - 1)x$

$g(x) = x^2 - f'\left(\frac{\pi}{2}\right)x + f'\left(-\frac{\pi}{2}\right)$

The number of solution (s) of the equation $f(x) = g(x)$ is/are :

A. 1

B. 2

C. 3

D. infinite

Answer: B

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23. Let f and g be two differentiable functins such that:

$f(x) = g'(1)\sin x + (g''(2) - 1)x$

$g(x) = x^2 - f'\left(\frac{\pi}{2}\right)x + f'\left(-\frac{\pi}{2}\right)$

If $\int \frac{g(\cos x)}{f(x) - x} dx = \cos x + \ln(h(x)) + C$ where C is constant and $h\left(\frac{\pi}{2}\right) = 1$ then $\left|h\left(\frac{2\pi}{3}\right)\right|$ is:

A. $3\sqrt{2}$

B. $2\sqrt{3}$

C. $\sqrt{3}$

D. $\frac{1}{\sqrt{3}}$

Answer: B

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24. Let f and g be two differentiable functins such that:

$f(x) = g'(1)\sin x + (g''(2) - 1)x$

$g(x) = x^2 - f'\left(\frac{\pi}{2}\right)x + f'\left(-\frac{\pi}{2}\right)$

If $\phi(x) = f^{-1}(x)$ then $\phi'\left(\frac{\pi}{2} + 1\right)$ equals to :

A. $\frac{\pi}{2} + 1$

B. $\frac{\pi}{2}$

C. 1

D. 0

Answer: C

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25. Suppose a function f(x) satisfies the following conditions

$f(x+y) = \dfrac{f(x)+f(y)}{f+f(x)f(y)}, \forall x, y \in R \text{ and } f'(0) = 1$

Also $-1 < f(x) < 1, \forall x \in R$

f(x) increases in the complete interval :

A. $(-\infty, -1) \cup (-1, 0) \cup (0, 1) \cup (1, \infty)$

B. $(-\infty, \infty)$

C. $(-\infty, 1) \cup (-1, 0)$

D. $(0, 1) \cup (1, \infty)$

Answer: B

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26. Suppose a function f(x) satisfies the following conditions

$$f(x+y)=\frac{f(x)+f(y)}{f+f(x)f(y)}, \forall x, y \in R \text{ and } f'(0)=1$$

Also $-1 < f(x) < 1, \forall x \in R$

The value of the limit $lt_{x \to \infty} (f(x))^x$ is:

A. 0

B. 1

C. e

D. e^2

Answer: B

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27. Let $f(x)$ be a polynomial satisfying $\lim_{x \to \infty} \frac{x^4 f(x)}{x^8 + 1} = 3$

$f(2) = 5, f(3) = 10, f(-1) = 2, f(-6) = 37$

The value of $\lim_{x \to -6} \frac{f(x) - x^2 - 1}{3(x + 6)}$ equals to:

A. $- \, | \, \underline{6}$

B. $|\underline{6}$

C. $\frac{|\underline{6}}{2}$

D. $\frac{|\underline{-6}}{2}$

Answer: D

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28. Let $f(x)$ be a polynomial satisfying $\lim_{x \to \infty} \frac{x^4 f(x)}{x^8 + 1} = 3$

$f(2) = 5, f(3) = 10, f(-1) = 2, f(-6) = 37$

The number of points of discontinuity of discontinuity of

$f(x) = \frac{1}{x^2 + 1 - f(x)}$ in $\left[\frac{-15}{2}, \frac{5}{2}\right]$ equals:

A. 4

B. 3

C. 1

D. 0

Answer: B

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29. Consider $f(x) = x^{\ln x}$, and $g(x) = e^2 x$. Let α and β be two values of x satisfying $f(x) = g(x) (\alpha < \beta)$

$\lim_{x \to \beta} \frac{f(x) - c\beta}{g(x) - \beta^2} = l$ then the value of x -l equals to:

A. $4 - e^2$

B. $e \% (2) - 4$

C. $4 - e$

D. $e - 4$

Answer: B

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30. Consider $f(x) = x^{\ln x}$, and $g(x) = e^2 x$. Let α and β be two values of x satisfying $f(x) = g(x) (\alpha < \beta)$

If $h(x) = \frac{f(x)}{g(x)}$ then $h'(\alpha)$ equals to:

A. e

B. $-e$

C. $3e$

D. $-3e$

Answer: D

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31. Let $f_n x + f_n(y) = \frac{x^n + y^n}{x^n y^n} \forall x, y \in R - \{0\}.$ where $n \in N$ and $g(x) = \max\left\{f_2(x), f_3(x), \frac{1}{2}\right\} \forall x \in R - \{0\}$

The number of value of $\sum_{k=1}^{\infty} f_{2k}(\cos ec\theta) + \sum_{k=1}^{\infty} f_{2k}(\sec\theta), where$theta

ne (kpi)/(2), k in I is: `

A. 1

B. 2

C. $\sqrt{2}$

D. 4

Answer: B

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32. Let $f_n x + f_n(y) = \frac{x^n + y^n}{x^n y^n} \forall x, y \in R - \{0\}.$ where $n \in N$ and $g(x) = \max\left\{f_2(x), f_3(x), \frac{1}{2}\right\} \forall x \in R - \{0\}$

The number of values of x for which g(x) is non-differentiable $(x \in R - \{0\})$:

A. 3

B. 4

C. 5

D. 1

Answer: A

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Exercise Mathcing Type Problems

1. The function $\begin{aligned} f(x) &= ax(x-1)+b & x<1 \\ &= x-1 & 1 \le x \le 3 \\ &= px^2+qx+2 & x>3 \end{aligned}$

if (i) f (x) is continous for all x

(ii) $f'(1)$ does not exist

(iii) $f'(x)$ is continous at $x = 3,$ then

	Column-I		Column-II
(A)	a cannot has value	(P)	1/3
(B)	b has value	(Q)	0
(C)	p has value	(R)	-1
(D)	q has value	(S)	1

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Exercise Subjective Type Problems

1. Let $f(x) = \begin{cases} ac(x-1)+b & x<1 \\ x+2 & 1 \le x \le 3 \\ px^2+qx+2 & x>3 \end{cases}$ is continous $\forall x \in R$ except $x = 1$ but $|f(x)|$ is differentiable everywhere and $f'(x)$ is continous at $x = 3$ and $|a+p+q| = k,$ then k=

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2. If $y = \sin(8\sin^{-1}x)$ then $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = -ky,$ where k =

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3. If $y^2 = 4ax$, then $\frac{d^2y}{dx^2} = \frac{ka^2}{y^3}, wherek^2 =$

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4. The number of values of $x, xI[-2, 3]$ where $f(x) = [x^2]\sin(\pi x)$ is discontinous is (where [.] denotes greatest integer function)

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5. If f (x) is continous and fifferentiable in $[-3, 9]$ and $f'(x) \in [-2, 8] \forall x \in (-3, 9)$. Let N be the number of divisors of the greatest possible vaue of $f(9) - f(-3)$, then the sum of digits of N.

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6. In $f(x) = \left[\left(\cos x^3, , x < 0\right), \left(\sin x^3 - \left|x^3 - 1\right|, , x \geq 0\right)\right.$ then find the number of points where $g(x) = f(|x|)$ is non-differentiable.

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7. Let $f(x) = x^2 + ax + 3$ and $g(x) = x + b,$ where $F(x) = \lim_{x \to \infty} \frac{f(x) + \left(x^2\right)g(x)}{1 + \left(x^2\right)^n}.$ If F (x) is continous $atx = 1$ and $x = -1$ then find the value of $\left(a^2 + b^2\right)$

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8. Let $f(x) = \begin{cases} 2 - x, & -3 \leq x \leq 0 \\ x - 2, & 0 < x < 4 \end{cases}$ Then f^{-1} (x) is discontinous at x=

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9. If $f(x) + 2f(1 - x)\left(= x^2 + 2 \forall x \in R\right.$ and $f(x)$ is a differentiable function, then the vlaue of $f'(8)$ is

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10. Let $f(x) =$ signum (x) and $g(x) = x\left(x^2 - 10x + 21\right)$, then the number of points of discontinuity of $f[g(x)]$ is

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11. If $\frac{d^2}{dx^2}\left(\frac{\sin^4 x + \sin^2 x + 1}{\sin^2 x + sinx + 1}\right) = a\sin^2 x + b\sin x + c$ then the value of $b + c - a$ is

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12. $f(x) = a\cos(\pi y) + b, f'\left(\frac{1}{2}\right) = \pi$ and $\int_{1/2}^{3/2} f(x)dx = \frac{2}{\pi} + 1$, then find the value of $-\frac{12}{\pi}\left(\frac{\sin^{-1} a}{3} + \cos^{-1} b\right)$.

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13. Let

$\alpha(x) = f(x) - f(2x)$ and $\beta(x) = f(x) - f(4x)$ and $\alpha'(1) = 5\alpha'(2) =$

then 2nd the vlaue of $\beta'(1) - 10$

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14. Let $f(x) = -4 . e^{\frac{1-x}{2}} + \frac{x^3}{3} + \frac{x^2}{2} + x + 1$ and g be inverse function of f and $h(x) = \frac{a + bx^{3/2}}{x^{5/4}}, h'(5) = 0,$ then $\frac{a^2}{5b^2 g'\left(\frac{-7}{6}\right)} =$

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15. If $y = e^{2\sin^{-1}x}$ then $\left|\frac{(x^2 - 1)y'' + xy'}{y}\right|$ is equal to

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16. Let $f(x) = x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \frac{x^5}{5}$ and let $g(x) = f^{-1}(x)$. Find $g'''(o)$.

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17. In $f(x) = \left[\left(\cos x^2, , x < 0\right), \left(\sin x^3 - \left|x^3 - 1\right|, , x \geq 0\right)\right.$ then find the number of points where $g(x) = f(|x|)$ is non-differentiable.

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18. Let $f: R \to R$ be a differentiable function satisfying:

$f(xy) = \dfrac{f(x)}{x} \forall x, y \in R^+$ also $f(1) = 0, f'(1) = 1$

find $\lim_{x \to e} \left[\dfrac{1}{f(x)}\right]$ (where [.] denotes greatest integer function).

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19. For the curve $\sin x + \sin y = 1$ lying in first quadrant. If $\lim_{x \to 0} x^{\alpha} \dfrac{d^2y}{dx^2}$ exists and non-zero than $2\alpha =$

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20. Let $f(x) = x\tan^{-1}(x^2) + x^4$ Let $f^k(x)$ denotes k^{th} derivative of $f(x)$ w.r.t. x, $k \in N$. If

f^{2m} (0) != 0, m in N, then m =

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21. If $x = \cos\theta$ and $y = \sin^3\theta$, then $\left|\frac{yd^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right|$ at $\theta = \frac{\pi}{2}$ is:

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22. The value of $x, x \in (2, \infty)$ where $f(x) = \sqrt{x\sqrt{8x - 16}} + \sqrt{x - \sqrt{8x - 16}}$ is not differentiable is:

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23. The number of non differentiability of point of function $f(x) = \min\left([x], \{x\}, \left|x - \frac{3}{2}\right|\right)$ for $x \in (0, 2)$, where [.] and {.}

denote greatest integer function and fractional part function respectively.

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