# © 'doubtnut 

## MATHS

# BOOKS - VIKAS GUPTA MATHS (HINGLISH) 

## FUNCTION

Single Choice Problems

1. Range of the function $f(x)=\log _{\sqrt{2}}\left(2-\log _{2} 16 \sin ^{2} x+1\right)$ is:
A. $[0,1]$
B. $(-\infty, 2]$
C. $[-1,1]$
D. $(-\infty, \infty)$

Answer: B
2. The values of $\alpha$ and $\beta$ for which $\left|e^{|x-\beta|}-\alpha\right|=2$ has four distinct solutions are
A. $a \in(-2, \infty), b=0$
B. $a \in(2, \infty), b=0$
C. $a \in(3, \infty), b \in R$
D. $a \in(2, \infty) b=0$

## Answer: C

## Watch Video Solution

3. The range of the function : $f(x)=\tan ^{-1} x+\frac{1}{2} \sin ^{-1} x$
A. $(-\pi / 2, \pi / 2)$
B. $[-\pi / 2, \pi / 2]-\{0\}$
C. $[-\pi / 2, \pi / 2]$
D. $(-3 \pi / 4,3 \pi / 4)$

## Answer: C

## - Watch Video Solution

4. Find the number of real ordered pair(s) ( $x, y$ ) for which: $16^{x^{2}+y}+16^{x+y^{2}}=1$
A. 0
B. 1
C. 2
D. 3

## Answer: B

5. The range of values of 'a' such that $\left(\frac{1}{2}\right)^{|x|}=x^{2}-a$ is satisfied for maximum number of values of ' $x$ '
A. $(-\infty,-1)$
B. $(-\infty, \infty)$
C. $(-1,1)$
D. $(-1, \infty)$

## Answer: D

## D Watch Video Solution

6. For a real number $x$, let $[x]$ denote the greatest integer less than or equal to $x$. Let $\mathrm{f}: \mathrm{R} \rightarrow \mathrm{R}$ be defined as $f(x)=2 x+[x]+\sin x \cos x$, then $f$ is
A. One-one but not onto
B. onto but not one-one
C. Both one-one and onto
D. Neither one-one nor onto

## Answer: A

## - Watch Video Solution

7. The maximum value of $\sec ^{-1}\left(\frac{7-5\left(x^{2}+3\right)}{2\left(x^{2}+2\right)}\right)$ is:
A. $\frac{5 \pi}{6}$
B. $\frac{5 \pi}{12}$
C. $\frac{7 \pi}{12}$
D. $\frac{2 \pi}{3}$

Answer: D
8. Number of ordered pair (a,b) the set $A=\{1,2,3,4,5\}$ so that the functon $f(x)=\frac{x^{3}}{3}+\frac{a}{2} x^{2}+b x+10$ is an injective mapping $\forall x \in R$ :
A. ' 13
B. 14
C. 15
D. 16

## Answer: C

## - Watch Video Solution

9. let A be the greatest value of the function $f(x)=\log _{x}[x]$, (where [. ] denotes gratest integer function) and $B$ be the least value of the function $g(x)=|\sin x|+|\cos x|$, then :
A. $A>B$
B. $A<B$
C. $A=B$
D. $2 A+B=4$

## Answer: C

## - Watch Video Solution

10. The function $f:(a, \infty) \rightarrow R$ where R denotes the range corresponding to the given domain, with rule $f(x)=2 x-3 x^{2}+6$, will have an inverse provided
A. $a=1, B=[5, \infty)$
B. $a=2, B=[10, \infty)$
C. $a, 0, B=[6, \infty)$
D. $a=-1, B=[1, \infty)$

## Answer: A

11. Solution of the inequation $\{x\}(\{x\}-1)(\{x\}+2) \geq 0$ where $\{$.$\} denots fractin part function) is :$
A. $x \in(-2,1)$
B. $x \in I$ (I denote set of integers)
C. $x \in[0,1)$
D. $x \in[-2,0)$

## Answer: B

## - Watch Video Solution

12. Let $f(x), g(x)$ be two real valued functions then the function $h(x)=2 \max \{f(x)-g(x), 0\}$ is equal to :
A. $f(x)-g(x)-|g(x)-f(x)|$
B. $f(x)+g(x)-|g(x)-f(x)|$
C. $f(x)-g(x)+|g(x)-f(x)|$
D. $f(x)+g(x)+|g(x)-f(x)|$

## Answer: C

## - Watch Video Solution

13. Let $R=\{(1,3),(4,2),(2,4),(2,3),(3,1)\}$ be a relation the set $A=\{1,2,3,4\}$. The relation R is
A. a function
B. reflexive
C. not symmetric
D. transitive

## Answer: C

14. The true set of valued of ' $K$ ' for which $\sin ^{-1}\left(\frac{1}{1+\sin ^{2} x}\right)=\frac{k \pi}{6}$ may have a solution is :
A. $\left[\frac{1}{4}, \frac{1}{2}\right]$
B. $[1,2]$
C. $\left[\frac{1}{6}, \frac{1}{2}\right]$
D. $[2,4]$

## Answer: B

## - Watch Video Solution

15. A ral valued functin $f(x)$ satisfies the functional equation $f(x-y)=f(x) f(y)-f(a-x) f(a+y)$ where 'a' is a given constant and $f(0)=1, f(2 a-x)$ is equal to :
A. $-f(x)$
B. $f(x)$
C. $f(a)+f(a-x)$
D. $f(-x)$

## Answer: A

## - Watch Video Solution

16. Let $g: R \rightarrow R$ be given by $g(x)=3+4 x$ if $g^{n}(x)=$ gogogo......og $(\mathrm{x})$ n times. Then inverse of $g^{n}(x)$ is equal to:
A. $\left(x+1-4^{n}\right) \cdot 4^{-n}$
B. $\left(x-1+4^{n}\right) 4^{-n}$
C. $\left(x+1+4^{n}\right) 4^{-n}$
D. None of these

## Answer: A

## - Watch Video Solution

17. Let $f: D \rightarrow R$ bge defined as : $f(x)=\frac{x^{2}+2 x+a}{x^{2}+4 x+3 a}$ where D and R denote the domain of $f$ and the set of all the real numbers respectively. If $f$ is surjective mapping. Then the complete range of $a$ is :
A. $0<a \leq 1$
B. $0<a \leq 1$
C. $0 \leq a<1$
D. $0<a<1$

## Answer: D

## - Watch Video Solution

18. Let $f:(-\infty, 2] \rightarrow(-\infty, 4]$ be a function defined by $f(x)=4 x-x^{2}$. Then, $f^{-1}(x)$ is
A. $2-\sqrt{4-x}$
B. $2+\sqrt{4-x}$
C. $-2+\sqrt{4-x}$
D. $-2-\sqrt{4-x}$

## Answer: A

## - Watch Video Solution

19. IF $[5 \sin x]+[\cos x]+6=0$, then range of $f(x)=\sqrt{3} \cos x+\sin \mathrm{x}$ corresponding to solution set of the given equation is: (where [.] denotes greatest integer function)
A. $[-2,-1]$
B. $\left(-\frac{3 \sqrt{3}+2}{5},-1\right)$
C. $[-2,-\sqrt{3})$
D. $\left(-\frac{3 \sqrt{3}+4}{5},-1\right)$

## Answer: D

20. If $f: R \rightarrow R$ where $f(x)=a x+\cos x$ is an invertible function, then
A. $(-2,-1] \cup[1,2)$
B. $[-1,1]$
C. $(-\infty,-1] \cup[1, \infty)$
D. $(-\infty,-2] \cup[2, \infty)$

## Answer: C

## - Watch Video Solution

21. The
range
$f(x)=[1+\sin x]+\left[2+s \in \frac{2}{x}\right]+\left[3+s \in \frac{x}{3}\right]++\left[n+s \in \frac{x}{n}\right] \forall x$
, where [.] denotes the greatest integer function, is,

$$
\begin{array}{ll}
\left\{\frac{n+n-2^{2}}{2}, \frac{n(n+1)}{2}\right\} & \left\{\frac{n(n+1)}{2}\right\} \\
\left\{\frac{n^{2}+n-2^{\square}}{2}, \frac{n(n+1)}{2} \frac{n^{2}+n+2}{2}\right\}\left[\frac{n(n+1)}{2},\right. & \left.\frac{n^{2}+n+2}{2}\right]
\end{array}
$$

A. $\left\{\frac{n^{2}+n-2}{2}, \frac{n(n+1)}{2}\right\}$
B. $\left\{\frac{n(n+1)}{2}\right\}$
C. $\left\{\frac{n(n+1)}{2}, \frac{n^{2}+n+2}{2}, \frac{n^{2}+n+4}{2}\right\}$
D. $\left\{\frac{n(n+1)}{2}, \frac{n^{2}+n-2}{2}\right\}$

## Answer: D

## - Watch Video Solution

22. $f: R \rightarrow R$, where $f(x)=\frac{x^{2}+a x+1}{x^{2}+x+1}$ Complete set of values of 'a' such that $f(x)$ is onto, is
A. $(-\infty, \infty)$
B. $(-\infty, 0)$
C. $(0, \infty)$
D. not possible

## - Watch Video Solution

23. If $f(x)$ and $g(x)$ are two function such that $f(x)=[x]+[-x]$ and $g(x)=\{x\} \forall x \in R$ and $h(x)=f(g(x)$, then which of the following is incorrect ? [.] denotes greatest integer function and $\{$.$\} denotes fractional part function)$
A. $f(x)$ and $h(x)$ are identical functions
B. $f(x)=g(x)$ has no solution
C. $f(x)+h(x)>0$ has no solution
D. $f(x)-h(x)$ is a periodic function

## Answer: B

## - Watch Video Solution

24. Find the number of values of $f(x)=\left[\frac{x}{15}\right]\left[-\frac{15}{x}\right]$ can take where $x \in(0,90)$ whete [.] =GIF
A. 5
B. 6
C. 7
D. Infinite

## Answer: B

## - Watch Video Solution

25. The graph of function $f(x)$ is shown below :


Then the graph of $g(x)=\frac{1}{f(|x|)}$ is:

A.

c.

D.

## Answer: C

26. Which of the following function is homogeneous ?
A. $f(x)=x \sin y+y \sin x$
B. $g(x)=x z \frac{y}{x}+y e \frac{x}{y}$
C. $h(x)=\frac{x y}{x+y^{2}}$
D. $\phi(x)=\frac{x-y \cos x}{y \sin x+y}$

## Answer: B

## - Watch Video Solution

27. Let $f(x)=\left[\begin{array}{ll}2 x+3 & x>1 \\ \alpha^{2} x+1 & x \leq 1\end{array}\right.$ If range of $f(x)=R$ (set of real numbers) then number orf integral value(s), which $\alpha$ any take :
A. 2
B. 3
C. 4
D. 5

## D Watch Video Solution

28. The maximum integral values of $x$ in the domain of $f(x)=\log _{10}\left(\log _{1 / 3}\left(\log _{4}(x-5)\right)\right.$ is :
A. 5
B. 7
C. 8
D. 9

## Answer: C

## - Watch Video Solution

29. Range of the function $f(x)=\log _{2}\left(\frac{4}{\sqrt{x+2}+\sqrt{2-x}}\right)$ is
A. $(0, \infty)$
B. $\left[\frac{1}{2}, 1\right]$
C. $[1,2]$
D. $\left[\frac{1}{4}, 1\right]$

## Answer: B

## - Watch Video Solution

30. Number of integers stastifying the equation $\left|x^{2}+5 x\right|+\left|x-x^{2}\right|=|6 x|$ is:
A. 3
B. 5
C. 7
D. 9

## Answer: C

31. Which of the following is not an odd function ?
A. $\ln \left(\frac{x^{4}+x^{2}+1}{\left(x^{2}+x+1\right)^{2}}\right)$
B. $\operatorname{sgn}(\operatorname{sgn}(x))$
C. $\sin (\tan x)$
D. $f(x)$,

$$
f(x)+f\left(\frac{1}{x}\right)=f(x) f\left(\frac{1}{x}\right) \forall x \in R-\{0\} \text { and } f(2)=33
$$

## Answer: D

## - Watch Video Solution

32. Which of the following function is periodic with fundamental period $\pi$
A. $f(x)=\cos x\left|\frac{\sin x}{2}\right|$, where [.] denotes greatest integer function
B. $g(x)=\frac{\sin x+\sin 7 x}{\cos x+\cos 7 x}+|\sin x|$
C. $h(x)=\{x\}+|\cos x|$, where $\{$.$\} denotes functional part function$
D. $\phi(x)=|\cos x|+\ln (\sin x)$

## Answer: B

## (D) Watch Video Solution

33. Let $f: N \rightarrow Z$ and $f(x)=\left[\begin{array}{ll}\frac{x-1}{2} & \text { when } \mathrm{x} \text { is odd } \\ -\frac{x}{2} & \text { when } \mathrm{x} \text { is even }\end{array}\right.$, then:
A. $f(x)$ is bijective
B. $f(x)$ is injective but not surjective
C. $f(x)$ is not injective but surjective
D. $\mathrm{f}(\mathrm{x})$ is neither injective nor subjective

## Answer: A

34. Let $g(x)$ be the inverse of $f(x)=\frac{2^{x+1}-2^{1-x}}{2^{x}+2^{-x}}$ then $g(x)$ be :
A. $\frac{1}{2} \log _{2}\left(\frac{2+x}{2-x}\right)$
B. $-\frac{1}{2} \log _{2}\left(\frac{2+x}{2-x}\right)$
C. $\log _{2}\left(\frac{2+x}{2-x}\right)$
D. $\log _{2}\left(\frac{2-x}{2+x}\right)$

## Answer: C

## D Watch Video Solution

35. Which of the following is the graph of the curve $\sqrt{|y|}=x$ is ?


B.

C.

D.

Answer: B
36. Range of $f(x)=\log _{(x)}\left(9-x^{2}\right)$, where [.] denotes G.I.E.' is :
A. $\{1,2\}$
B. $(-\infty, 2)$
C. $\left(-\infty, \log _{2} 5\right]$
D. $\left.[\log )_{2} 5,3\right]$

## Answer: C

## - Watch Video Solution

37. if $e^{x}+e^{f(x)}=e$ then for $f(x)$
A. Domain is $(-\infty, 1)$
B. Range is $(-\infty, 1]$
C. Domain is $(-\infty, 0]$
D. Range is $(-\infty, 0]$

## Answer: A

38. A lion moves in the region given by the graph $y-|y|-x+|x|=0$. curve a person can move so that he does not encounter lion -
A. $y=x^{2}$
B. $y=\operatorname{sgn}\left(-e^{2}\right)$
C. $y=\log _{1 / 3} x$
D. $y=m+|x|, m>3$

## Answer: D

## - Watch Video Solution

39. If $\left|f(x)+6-x^{2}\right|=|f(x)|+\left|4-x^{2}\right|+2$, then $\mathrm{f}(\mathrm{x})$ is necessarily non-negaive for
A. $x \in[-2,2]$
B. $x n(-\infty,-2) \cup(2, \infty)$
C. $x \in[-\sqrt{6}, \sqrt{6}]$
D. $x \in[-5,-2] \cup[2,5]$

## Answer: A

## - Watch Video Solution

40. Let $f(x)=\cos (p x)+\sin x$ be periodic, then p must be :
A. Positive real number
B. Negative real number
C. Rational
D. Prime

## Answer: C

## - Watch Video Solution

41. The domain of $f(x) i s(0,1)$. Then the domain of $\left(f\left(e^{x}\right)+f(1 n|x|)\right.$ is $(-1, e)(b)(1, e)(-e,-1)(d)(-e, 1)$
A. $\left(\frac{1}{e}, 1\right)$
B. $(-e, 1)$
C. $\left(-1,-\frac{1}{e}\right)$
D. $(-e,-1) \cup(1, e)$

## Answer: B

## - Watch Video Solution

42. Let $A=\{1,2,3,4\}$ and $f: A \rightarrow \quad$ A satisfy
$f(1)=2, f(2)=3, f(3)=4, f(4)=1$. Suppose $g: A \rightarrow A$ satisfies $g(1)=3$ and $f o g=g \circ f$, then $\mathrm{g}=$
A. $\{(1,3),(2,1),(3,2),(4,4)\}$
B. $\{(1,3),(2,4),(3,1),(4,2)\}$
C. $\{(1,3),(2,2),(3,4),(4,3)\}$
D. $\{(1,3),(2,4),(3,2),(4,1)\}$

## Answer: B

## - Watch Video Solution

43. Number of solutions of the equation, $[y+[y]]=2 \cos x$ is: (where $y=1 / 3)[\sin x+[\sin x+[\sin x]]]$ and []$=$ greatest integer function) 0
(b) 1 (c) 2 (d) $\infty$
A. 0
B. 1
C. 2
D. Infinite

## Answer: A

44. The function $f(x)=\left\{\frac{\left(x^{2 n}\right)}{\left(x^{2 n} \operatorname{sgn} x\right)^{2 n+1}}\left(\frac{e^{\frac{1}{x}}-e^{-\frac{1}{x}}}{e^{\frac{1}{x}}+e^{-\frac{1}{x}}}\right) x \neq 0 n \in N\right.$ is:
A. Odd function
B. Even function
C. Neither odd nor even function
D. Constant function

## Answer: B

## - Watch Video Solution

45. $f(1)=1$ and $f(n)=2 \sum_{r=1}^{n-1} f(r)$. Then $\sum_{n=1}^{m} f(n)$ is equal to (A) $3^{m}-1(\mathrm{~B}) 3^{m}(\mathrm{C}) 3^{m-1}$ (D)none of these
A. $\frac{3^{m}-1}{2}$
B. $3^{m}$
C. $3^{m-1}$
D. $\frac{3^{m-1}-1}{2}$

## Answer: C

## - Watch Video Solution

46. Let $f(x)=\frac{x}{\sqrt{1+x^{2}}}$ then n times(fo fo fo ......of)( x ) ${ }^{\text {' }}$ is :
A. $\frac{x}{\sqrt{1+\left(\sum_{r=1}^{n} r\right) x^{2}}}$
B. $\frac{x}{\sqrt{1+\left(\sum_{r=1}^{n} 1\right) x^{2}}}$
C. $\left(\frac{x}{\sqrt{1+x^{2}}}\right)^{n}$
D. $\frac{n \pi}{\sqrt{1+\pi x^{2}}}$

## Answer: B

47. Let $f: R \rightarrow R$, then $f(x)=2 x+|\cos x|$ is
A. One-one into
B. One-one and onto
C. May-one and into
D. Many-one and onto

## Answer: B

## - Watch Video Solution

48. Let $f: R \rightarrow R$ be a function defined by $f(x)=x^{3}+x^{2}+3 x+\sin x$.

Then $f$ is
A. One-one end into
B. One-one and onto
C. Many-one and into
D. many-one and onto

## Answer: B

## - Watch Video Solution

49. If $f(x)=\{x\}+\{x+1\}+\{x+2\} \ldots \ldots \ldots . .\{x+99)$, then the value of $[f(\sqrt{2})]$ is, where (.) denotes fractional part function \& $[$.$] denotes the$ greatest integer function
A. 5050
B. 4950
C. 41
D. 14

## Answer: C

50. If $|\cot x+\cos e c x|=|\cot x|+\cos e c x|, x \in|[0,2 \pi]$, then complete set of values of $x$ is :
A. $[0, \pi]$
B. $\left(0, \frac{\pi}{2}\right]$
C. $\left(0, \frac{\pi}{2}\right] \cup\left[\frac{3 \pi}{2}, 2 \pi\right)$
D. $\left(\pi, \frac{3 \pi}{2}\right] \cup\left[\frac{7 \pi}{4}, 2 \pi\right]$

## Answer: C

## - Watch Video Solution

51. The funcrtin $f(x)=0$ has eight distinvt real solution and f also satisfy $f(4+x)=f(4-x)$. The sum of all the eight solution of $f(x)=0$ is :
A. 12
B. 32
C. 16
D. 15

## Answer: B

## - Watch Video Solution

52. Let $f(x)$ polynomial of degree 5 with leading coefficient unity such that $f(1)=5, f(2)=4, f(3)=3, f(4)=2, f(5)=1$, then $f(6)$ is equal to
A. 0
B. 24
C. 120
D. 720

## Answer: C

53. Let $f: A \rightarrow B$ be a function such that $f(x)=\sqrt{x-2}+\sqrt{4-x}$, is invertible, then which of the following is not possible?
A. $A=[3,4]$
B. $A=[2,3]$
C. $A=[2,2 \sqrt{3}]$
D. $\{2,2 \sqrt{2}\}$

## Answer: C

## - Watch Video Solution

54. Find the number of positive integral values of $x$ satisfying $\left[\frac{x}{9}\right]=\left[\frac{x}{11}\right]$ is where [.] -=Gl.F)
A. 21
B. 22
C. 23
D. 24

## Answer: D

## - Watch Video Solution

55. The domain of function $f(x)=\log _{\left[x+\frac{1}{2}\right]}\left(2 x^{2}+x-1\right)$, where [.] denotes the greatest integer function is :
A. $\left[\frac{3}{2}, \infty\right)$
B. $(2, \infty)$
C. $\left(-\frac{1}{2}, \infty\right)-\left\{\frac{1}{2}\right\}$
D. $\left(\frac{1}{2}, 1\right) \cup(1, \infty)$

## Answer: A

## - Watch Video Solution

56. The solution set of the equation $[x]^{2}+[x+1]-3=0$, where [.] represents greatest integeral function is :
A. $[-1, o) \cup[1,2)$
B. $[-2,-1) \cup[1,2]$
C. $[1,2]$
D. $[-3,-2) \cup[2,3)$

## Answer: B

## - Watch Video Solution

57. Which among the following relations is a function ?
A. $x^{2}+y^{2}=r^{2}$
B. $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}=r^{2}$
C. $y^{2}=4 a x$
D. $x^{2}=d x y$

## - Watch Video Solution

58. A function $f: R \rightarrow R$ is defined as $f(x)=3 x^{2}+1$. then $f^{-1}(x)$ is:
A. $\frac{\sqrt{x-1}}{3}$
B. $\left(\frac{1}{2} \sqrt{x}-1\right.$
C. $f^{-1}$ does not exist
D. $\sqrt{\frac{x-1}{3}}$

## Answer: C

## - Watch Video Solution

59. If $f(x)=\left\{\begin{array}{ll}2+x, & x \geq 0 \\ 4-x, & x<0\end{array}\right.$, then $f(f(x))$ is given by :
A. $f(f(x))= \begin{cases}4+x, & x \geq 0 \\ 6-x, & x<0\end{cases}$
B. $f(f(x))= \begin{cases}4+x, & x \geq 0 \\ x, & x<0\end{cases}$
C. $f(f(x))= \begin{cases}4-x, & x \geq 0 \\ x, & x<0\end{cases}$
D. $f(f(x))= \begin{cases}4-x, & x \geq 0 \\ x+2 x, & x<0\end{cases}$

## Answer: A

## - Watch Video Solution

60. The function $f: R \rightarrow R$ defined as $f(x)=\frac{3 x^{2}+3 x-4}{3+3 x-4 x^{2}}$ is :
A. One ot one buty not onto
B. Onto but not one to one
C. Both one to one and onto
D. Neither one to one nor onto

## Answer: B

61. The number of solutions of the equation $e^{x}-\log |x|=0$ is:
A. 0
B. 1
C. 2
D. 5

## Answer: B

62. If complete solution set of $e^{-x} \leq 4-x$ is $[\alpha, \beta]$ then $[\alpha]+[\beta]$ is equal to : (where [.] denotes greatest integer function )
A. 0
B. 2
C. 1
D. 4

## Answer: C

## - Watch Video Solution

63. Range of $f(x)=\sqrt{\sin \left(\log _{7}(\cos (\sin x))\right)}$ is:
A. $[0,1]$
B. $\{0,1\}$
C. $\{0\}$
D. $[1,7]$

## Answer: C

## - Watch Video Solution

64. If the domain of $y=f(x) i s[-3,2]$, then find the domain of $g(x)=f(|[x]|)$, wher [] denotes the greatest integer function.
A. $[-3,2]$
B. $[-2,3]$
C. $[-3,3]$
D. $[-2,3]$

## Answer: B

## - Watch Video Solution

65. Range of the function
$f(x)=\cot ^{-1}\{-x\}+\sin ^{-1}\{x\}+\cos ^{-1}\{x\}, \quad$ where $\{$.$\} denotes$ fractional part function:
A. $\left(\frac{3 \pi}{4}, \pi\right)$
B. $\left[\frac{3 \pi}{4}, \pi\right)$
C. $\left[\frac{3 \pi}{4}, \pi\right]$
D. $\left(\frac{3 \pi}{4}, \pi\right]$

## - Watch Video Solution

66. 

$f: R-\left\{\frac{3}{2}\right\} \rightarrow R, f(x)=\frac{3 x+5}{2 x-3} . \operatorname{Let}_{1}(x)=f(x), f_{n}(x)=f\left(f_{n-1}(x)\right.$
for $\pi \geq 2, n \in N$, then $f_{2008}(x)+f_{2009}(x)=$
A. $\frac{2 x^{2}+5}{2 x-3}$
B. $\frac{x^{2}+5}{2 x-3}$
C. $\frac{2 x^{2}-5}{2 x-3}$
D. $\frac{x^{2}-5}{2 x-3}$

## Answer: A

## D Watch Video Solution

67. Find the range of the function $f(x)=\frac{\left(1+x+x^{2}\right)\left(1+x^{4}\right)}{x^{3}}$
A. $[0, \infty]$
B. $[2, \infty]$
C. $[4, \infty]$
D. $[6, \infty]$

## Answer: D

## - Watch Video Solution

68. The function $f:(-\infty, 3] \rightarrow\left(o, e^{7}\right]$ defined by $f(x)=e^{x^{3}-3 x^{2}-9 x+2}$ is
A. Many one and onto
B. Many one and into
C. One to one and onto
D. One to one and into
69. If $f(x)=\sin (\log )_{e}\left\{\frac{\sqrt{4-x^{2}}}{1-x}\right\}$, then the domain of $f(x)$ is and its range is $\qquad$ .
A. $[-1,1]$
B. $[0,1]$
C. $[-1,1)$
D. None of these

## Answer: A

## - Watch Video Solution

70. Set of values of 'a' for which the function $f: R \rightarrow R$, given by $f(x)=x^{3}+(a+2) x^{2}+3 a x+10$ is one-one is given by:

$$
\text { A. }(-\infty, 1] \cup[4, \infty)
$$

B. $[1,4]$
C. $[1, \infty]$
D. $[-\infty, 4]$

## Answer: B

## - Watch Video Solution

71. If the range of the function $F(x)=\tan ^{-1}\left(3 x^{2}+b x+c\right)$ is $\left[0, \frac{\pi}{2}\right)$; (domain in R) then :
A. $b^{2}=3 c$
B. $b^{2}=4 c$
C. $b^{2}=12 c$
D. $b^{2}=8 c$

## Answer: C

72. Let $f(x)=\sin ^{-1} x-\cos ^{-1}, \mathrm{x}$, then the set of values of k for which of $|f(x)|=k$ has exactly two distinct solutions is:
A. $\left(0, \frac{\pi}{2}\right]$
B. $\left(0, \frac{\pi}{2}\right]$
C. $\left[\frac{\pi}{2}, \frac{3 \pi}{2}\right)$
D. $\left[\pi, \frac{3 \pi}{2}\right]$

## Answer: A

## - Watch Video Solution

73. Let $\quad f: R \rightarrow R$ is
defined
by
$f(x)=\left\{\begin{array}{ll}(x+1)^{3} & x \leq 1 \\ \ln x+\left(b^{2}-3 b+10\right) & x>1\end{array}\right.$ If $\mathrm{f}(\mathrm{x})$ is invertible, then the
set of all values of ' $b$ ' is :
A. $\{1,2\}$
B. $\phi$
C. $\{2,5\}$
D. None of these

## Answer: A

## - Watch Video Solution

74. 

If
$f(x)$
is
continuous such
that
$|f(x)| \leq 1, \forall x \in R$ and $g(x)=\frac{e^{f(x)}-e^{-|f(x)|}}{e^{f(x)}+e^{-|f(x)|}}$, then range of $g(x)$ is
A. $[0,1]$
B. $\left[0, \frac{e^{2}+1}{e^{2}-1}\right]$
C. $\left[0, \frac{e^{2}-1}{e^{2}+1}\right]$
D. $\left[\frac{e^{2}+1}{e^{2}+1}, 0\right]$
75. Consider all function $f:\{1,2,3,4\} \rightarrow\{1,2,3,4\}$ which are one-one, onto and satisfy the following property :

If $f(k)$ is odd then $f(k+1)$ is even, $K=1,2,3$. The number of such function is :
A. 4
B. 8
C. 12
D. 16

## Answer: C

## - Watch Video Solution

76. Consider the function $f: R-\{1\} \rightarrow R-\{2\}$ given by $f\{x\}=\frac{2 x}{x-1}$. Then
A. $f$ is one-one but not onto
B. $f$ is onto but not one-one
C. f is neither one-one nor onto
D. $f$ is one-one and onto

## Answer: D

## D Watch Video Solution

77. If rang of fraction $f(x)$ whose domain is set of all real numbers is $[-2,4]$, then range of function $g(x)=\frac{1}{2} f(2 x+1)$ is equal to :
A. $[-2,4]$
B. $[-1,2]$
C. $[-3,9]$
D. $[-2,2]$
78. Let $f: R \rightarrow$ and $f(x)=\frac{x\left(x^{4}+1\right)(x+1)+x^{4}+2}{x^{2}+x+1}$, then $f(x)$ is
A. one-one, inot
B. Many -one onto
C. One-one, onto
D. Mny one, into

## Answer: D

## - Watch Video Solution

79. Let $\mathrm{f}(\mathrm{x})$ be defined as

$$
f(x)= \begin{cases}|x| & 0 \leq x<1 \\ |x-1|+|x-2| & 1 \leq x<2 \\ |x-3| & 2 \leq x<3\end{cases}
$$

The range of function $g(x)=\sin (7(f(x))$ is:
A. $[0,1]$
B. $[-1,0]$
C. $\left[-\frac{1}{2}, \frac{1}{2}\right]$
D. $[-1,1]$

## Answer: D

## - Watch Video Solution

80. If $[x]^{2}-7[x]+10<0$ and $4[y]^{2}-16[y]+7<0$, then $[x+y]$ cannot be ([.]` denotes greatest integer function):
A. 7
B. 8
C. 9
D. both (b) and (c )

## Answer: C

81. Let $f: R \rightarrow R$ be a function defined by, $f(x)=\frac{e^{|x|}-e^{-x}}{e^{x}+e^{-x}}$ then
A. $f(x)$ is many one, onto function
B. $f(x)$ is many one, into function
C. $\mathrm{f}(\mathrm{x})$ is decreasing function $\forall n \in R$
D. $f(x)$ is bijective function

## Answer: B

## - Watch Video Solution

82. The function $f(x)$ satisfy the equation
$f(1-x)+2 f(x)=3 x \forall x \in R$, then $f(0)=$
A. -2
B. -1
C. 0
D. 1

## Answer: B

## - Watch Video Solution

83. Let $f:[0,5] \rightarrow[0,5)$ be an invertible function defined by $f(x)=a x^{2}+b x+C$, where $a, b, c \in R, a b c \neq 0$, then one of the root of the equation $c x^{2}+b x+a=0$ is:
A. a
B. $b$
C. c
D. $a+b+c$

## Answer: A

84. Let $f(x)=x^{2}+\lambda x+\mu \cos x, \lambda$ being an integer and $\mu$ is a real number. The number of ordered pairs $(\lambda, \mu)$ for which the equation $f(x)=0$ and $f(f(x))=0$ have the same (non empty) set of real roots is:
A. 2
B. 3
C. 1
D. 6

## Answer: C

## - Watch Video Solution

85. Consider all function $f:\{1,2,3,4\} \rightarrow\{1,2,3,4\}$ which are one-one, onto and satisfy the following property :

If $f(k)$ is odd then $f(k+1)$ is even, $K=1,2,3$. The number of such function is :
A. 4
B. 8
C. 12
D. 16

## Answer: C

## - Watch Video Solution

86. Which of the following is closest to the graph of $y=\tan (\sin x), x>0 ?$

A.
B.

C.
D.


## Answer: B

## - Watch Video Solution

87. Consider the function $f: R-\{1\} \rightarrow R-\{2\}$ given by $f(x)=\frac{2 x}{x-1}$. Then
A. $f$ is one-one but not onto
B. $f$ is onto but not one-one
C. $f$ is one-one nor onto
D. f is both one-one and onto

Answer: D

## - Watch Video Solution

88. If rang of fraction $f(x)$ whose domain is set of all real numbers is $[-2,4]$, then range of function $g(x)=\frac{1}{2} f(2 x+1)$ is equal to :
A. $[-2,4]$
B. $[-1, .2]$
C. $[-3,9]$
D. $[-2,2]$

## Answer: B

## - Watch Video Solution

89. Let $f: R \rightarrow$ and $f(x)=\frac{x\left(x^{4}+1\right)(x+1)+x^{4}+2}{x^{2}+x+1}$, then $f(x)$ is
A. One-one, into
B. Many one, onto
C. One-one, onto
D. Many one, into

## Answer: D

## - Watch Video Solution

90. Let $f(x)$ be defined as
$f(x)= \begin{cases}|x| & 0 \leq x<1 \\ |x-1|+|x-2| & 1 \leq x<2 \\ |x-3| & 2 \leq x<3\end{cases}$
The range of function $g(x)=\sin (7(f(x))$ is :
A. $[0,1]$
B. $[-1,0]$
C. $\left[-\frac{1}{2}, \frac{1}{2}\right]$
D. $[-1,1]$

## Answer: D

## - Watch Video Solution

91. Number of integral values of $x$ in the domain of function $f(x)=\sqrt{\ln (|\ln x|)}+\sqrt{7|x|-(|x|)^{2}-10}$ is equal to
A. 5
B. 6
C. 7
D. 8

## Answer: B

92. The complete set of values of $x$ in the domain of function $f(x)=\sqrt{\log _{x+2\{x\}}\left([x]^{2}-5[x]+7\right)}$ where [.] denote greatest integer functioon and $\{$.$\} denote fraction pert function) is :$
A. $\left(-\frac{1}{3}, 0\right) \cup\left(\frac{1}{3}, 1\right) \cup(2, \infty)$
B. $(0,1) \cup(1, \infty)$
c. $\left(-\frac{2}{3}, 0\right) \cup\left(\frac{1}{3}, 1\right) \cup(1, \infty)$
D. $\left(-\frac{1}{3}, 0\right) \cup\left(\frac{1}{3}, 1\right) \cup(1, \infty)$

## Answer: D

## - Watch Video Solution

93. The number of integral ordered pair ( $x, y$ ) that satisfy the system of equatin $|x+y-4|=5$ and $|x-3|+|y-1|=5$ is/are:
A. 2
B. 4
C. 6
D. 12

## Answer: D

## - Watch Video Solution

94. $f: R \rightarrow R$, where $f(x)=\frac{x^{2}+a x+1}{x^{2}+x+1}$ Complete set of values of 'a' such that $f(x)$ is onto, is
A. $(-\infty, \infty)$
B. $(-\infty, 0)$
C. $(0, \infty)$
D. Empty set

## Answer: D

95. If $A=\{1,2,3,4\}$ and $f: A->A$, then total number of invertible functions,'f',such that $f(2) \neq 2, f(4) \neq 4, f(1)=1$ is equal to:
A. 1
B. 2
C. 3
D. 4

## Answer: C

96. The domian of definition of $f(x)=\log _{\left(x^{2}-x+1\right)}\left(2 x^{2}-7 x+9\right)$ is:
A. R
B. $R-\{0\}$
C. $R-\{0,1\}$
D. $R-\{1\}$

## Answer: C

## - Watch Video Solution

97. If $A=\{1,2,3,4\} B=\{1,2,3,4,5,6\}$ and $f: A \rightarrow \mathrm{~B}$ is an injective mapping satisfying $f(i) \neq I$, then number of such mappings are :
A. 182
B. 181
C. 183
D. None of these

## Answer: B

98. Let $f\left(x 0=x^{2}-2 x-3, x \geq 1\right.$ and $g(x)=1+\sqrt{x+4}, x \geq-4$ then the number of real solution os equation $f(x)=g(x)$ is/are
A. 0
B. 1
C. 2
D. 4

## Answer: B

## - Watch Video Solution

## One Or More Than One Answe Is Are Correct

1. $f(x)$ is an even periodic function with period 10 In
$[0,5], f(x)=\left\{\begin{array}{ll}2 x & 0 \leq x<2 \\ 3 x^{2}-8 & 2 \leq x<4 . \\ 10 x & 4 \leq x \leq 5\end{array}\right.$ Then:
A. $f(-4)=40$
B. $\frac{f(-13)-f(11)}{f(13)+f(-11)}=\frac{17}{21}$
C. $f(5)$ is not defined
D. Range of $f(x)$ is $[0,50]$

## Answer: A::B::D

## - Watch Video Solution

2. Let $f(x)=\left|\left|x^{2}-4 x+3\right|-2\right|$. Which of the following is/are correct ?
A. $f(x)=m$ has exactly two real solutios of different sign $\forall m>2$
B. $f(x)=m$ has exactly two real solution $\forall m \in(2, \infty) \cup\{0\}$
C. $f(x)=m$ has no solutions $\forall m<0$
D. $f(x)=m$ has four distinct real solution $\forall m \in(0,1)$
3. Let $f(x)=\cos ^{-1}\left(\frac{1-\tan ^{2}(x / 2)}{1+\tan ^{2}(x / 2)}\right)$ Which of the following statement (s) is/are correct about $f(x)$ ?
A. Domain is $R$
B. Range is $[0, \pi]$
C. $f(x)$ is even
D. $f(x)$ is derlvable in $(\pi, 2 \pi)$

## Answer: A::B::C::D

## - Watch Video Solution

4. $\left|\log _{e}\right| x||=|k-1|-3$ has four distict roots then k satisfies : (where $\left.|x|<d^{2}, x \neq 0\right)$

$$
\text { A. }(-4,-2)
$$

B. $(4,6)$
C. $\left(e^{-1}, e\right)$
D. $\left(d^{-2}, e^{-1}\right)$

## Answer: A: B

## - Watch Video Solution

5. Which of the following functions are defined for all $x \in R$ ? (Where[.] = denotes greatest integer function)
A. $f(x)=\sin [x]+\cos [x]$
B. $f(x)=\sec ^{-1}\left(1+\sin ^{2} x\right)$
C. $f(x)=\sqrt{\frac{9}{8}+\cos x+\cos 2 x}$
D. $f(x)=\tan (\ln (1+|x|))$

## Answer: A::B::C

6. Let $f(x)=\left\{\begin{array}{ll}x^{2} & 0<x<2 \\ 2 x-3 & 2 \leq x<3 \\ x+2 & x \geq 3\end{array}\right.$ then the tune equations:
A. $f\left(f\left(f\left(\frac{3}{2}\right)\right)=f\left(\frac{3}{2}\right)\right.$
B. $1+f\left(f\left(f\left(\frac{5}{2}\right)\right)\right)=f\left(\frac{5}{2}\right)$
C. $f(f(f(2)))=f(1)$
D. $\underbrace{f(f(f(\ldots \ldots f(4)) \ldots))}=2012$

## Answer: A::B::C

## - Watch Video Solution

7. Let $f:\left[\frac{2 \pi}{3}, \frac{5 \pi}{3}\right] \rightarrow[0,4]$ be a function defined as $f(x)=\sqrt{3} \sin x-\cos x+2$, then :
A. $f^{-1}(1)=\frac{4 \pi}{3}$
B. $f^{-1}(1)=\pi$
C. $f^{-1}(2)=\frac{5 \pi}{6}$
D. $f^{-1}(2)=\frac{7 \pi}{6}$

## Answer: A::D

## - Watch Video Solution

8. Let $f(x)$ be invertible function and let $f^{-1}(x)$ be is inverse. Let equation $f\left(f^{-1}(x)\right)=f^{-1}(x)$ has two real roots $\alpha$ and $\beta$ (with in domain of $f(x))$, then :
A. $f(x)=x$ also have same two rreal roots
B. $f^{-1}(x)=x$ also have same two real roots
C. $f(x)=f^{-1}(x)$ also have same two real roots
D. Area of triangle formed by $(0,0),(\alpha, f(\alpha))$, and $(\beta, f(\beta))$ is 1 unit
9. In function $f(x)=\cos ^{-1} x+\cos ^{-1}\left(\frac{x}{2}+\frac{\sqrt{3-3 x^{2}}}{2}\right)$, then Range of $f(x)$ is $\left.\left[\frac{\pi}{3}, \frac{10 \pi}{3}\right]\right]$ Range of $f(x)$ is $\left[\frac{\pi}{3}, 5 \pi\right] f(x)$ is one-one for $x \in\left[-1, \frac{1}{2}\right] f(x)$ is one-one for $x \in\left[\frac{1}{2}, 1\right]$
A. Range of $f(x) i s\left[\frac{\pi}{3}, \frac{10 \pi}{3}\right]$
B. Rang $f(x) i s\left[\frac{\pi}{3}, \frac{5 \pi}{3}\right]$
C. $\mathrm{f}(\mathrm{x})$ is one-one for $x \in\left[-1, \frac{1}{2}\right]$
D. $\mathrm{f}(\mathrm{x})$ is one-one for $x \in\left[\frac{1}{2}, 1\right]$

## Answer: B::C

## - Watch Video Solution

10. Let $f: R \rightarrow R$ defined by $f(x)=\cos ^{-1}(-\{-x\})$, where $\{x\}$ denotes fractional part of $x$. Then, which of the following is/are correct?
A. $f$ is many coe but not even function
B. Eange of $f$ contains two prime numbers
C. f is a periodic
D. Graph of $f$ does not lie below $x$-axis

## Answer: A::B::D

## - Watch Video Solution

11. Which option (s) is/are ture ?
A. $f: R \rightarrow R, f(x)=e^{|x|}-e^{-x}$ is many-one into function
B. $f: R \rightarrow R, f(x)=2 x+|\sin x|$ is one-one onto
C. $f: R \rightarrow R, f(x)=\frac{x^{2}+4 x+30}{x^{2}-8 x+18}$ is many-one onto
D. $f: R \rightarrow R, f(x)=\frac{2 x^{2}-x+5}{7 x^{2}+2 x+10}$ is many-one into

## Answer: A::B::D

12. If $f(x)=\left[\frac{\ln (x)}{e}+\left[\frac{\ln (e)}{x}\right]\right.$, where [.] denotes greatest interger function, the which of the following are ture?
A. range of $h(x) i s\{-1,0\}$
B. If $h(x)=-1$, then x can be rational as well as irractional
C. If $h(x)=-1$, then x can be rational as well as irrational
D. $h(x)$ is periodic function

## Answer: A:C

## - Watch Video Solution

13. If $f(x)=\left\{\begin{array}{ll}x^{3} & x \in Q \\ -x^{3} & x \notin Q\end{array}\right.$, then :
A. $f(x)$ is periodic
B. $f(x)$ is many-one
C. $f(x)$ is one-one
D. range of the function is $R$

## Answer: C::D

## - Watch Video Solution

14. Let $\mathrm{f}(\mathrm{x})$ be a real valued function such that $f(0)=\frac{1}{2}$ and $\mathrm{f}(\mathrm{x}+\mathrm{y})=\mathrm{f}(\mathrm{x}) \mathrm{f}(\mathrm{a}-\mathrm{y})+\mathrm{f}(\mathrm{y}) \mathrm{f}(\mathrm{a}-\mathrm{x}), \forall x, y \in R$, then for some real a,
A. $f(x)$ is perodic function
B. $f(x)$ is a constant function
C. $f(x)=\frac{1}{2}$
D. $f(x)=\frac{\cos x}{2}$

## Answer: A::B::C

15. $f(x)$ is an even periodic function with period 10 In
$[0,5], f(x)=\left\{\begin{array}{ll}2 x & 0 \leq x<2 \\ 3 x^{2}-8 & 2 \leq x<4 . \\ 10 x & 4 \leq x \leq 5\end{array}\right.$. Then:
A. $f(-4)=40$
B. $\frac{f(-13)-x(11)}{f(13)+f(-11)}=\frac{17}{21}$
C. $f(5)$ is not defined
D. Range of $f(x)$ is $[0,50]$

## Answer: A::B::D

## - Watch Video Solution

16. For the equation $\frac{e^{-x}}{x+1}$ which of the following statement(s) is/are correct ?
A. when $\lambda \in(0, \infty)$ equation has 2 real and distinct roots
B. when $\lambda \in\left(-\infty,-e^{2}\right)$ equation has 2 real anddistinct roots
C. when $\lambda \in(0, \infty)$ equatio has 1 real root
D. when $\lambda \in(-e, 0)$ equation has no real root

## Answer: B::C::D

## - Watch Video Solution

17. . For $x \in R^{+}, \quad$ if $x,[x],\{x\}$ are in harmonic progression then the value of $x$ can not be equal to (where [*] denotes greatest integer function, $\left\{{ }^{*}\right\}$ denotes fractional part function)
A. $\frac{1}{\sqrt{2}} \tan \frac{\pi}{8}$
B. $\frac{1}{\sqrt{2}} \cot \frac{\pi}{8}$
C. $\frac{1}{\sqrt{2}} \tan \frac{\pi}{12}$
D. $\frac{1}{\sqrt{2}} \cot \frac{\pi}{12}$

## Answer: A::C::D

18. The equation $||x-1|+a|=4, a \in R$, has:
A. 3 distinct real roots for unique value of a.
B. 4 distinct real roots for $a \in(-\infty,-4)$
C. 2 distinct real roots for $|a|<4$
D. no rela roots for $a>4$

## Answer: A::B::C::D

## - Watch Video Solution

19. Let $f_{n}(x)=(\sin x)^{1 / \pi}, x \in R$, then:
A. $f_{2}(x)>1$ for all $x \in\left(2 k \pi,(4 k+1) \frac{\pi}{2}\right), k \in I$
B. $f_{2}(x)=1$ for $x=2 k \pi, k \in I$
C. $f_{2}(x)>f_{3}(x)$ for all $x \in\left(2 k \pi,(4 k+1) \frac{\pi}{2}\right), k \in I$
D. $f_{3}(x) \geq f_{5}(x)$ for all $x \in\left(3 k \pi(4 k+1) \frac{\pi}{2}\right), k \in I$

## D View Text Solution

20. If the domain of $f(x)=\frac{1}{\pi} \cos ^{-1}\left[\log _{3}\left(\frac{x^{3}}{3}\right)\right]$ where, $x>0$ is [a,b] and the range of $f(x)$ is $[\mathrm{c}, \mathrm{d}]$, then :
A. $\mathrm{a}, \mathrm{b}$ are the roots of the equation $x^{4}-3 x^{4}-3 x c^{3}-x+3=0$
B. a,b are the roots of the equatin $x^{4}-x^{3}+x^{2}-2 x+1=0$
C. $a^{3}+d^{3}=1$
D. $a^{2}+b^{2}+c^{2}=11$

## Answer: A:D

## - Watch Video Solution

21. The number of real values of x satisfying the equation $;\left[\frac{2 x+1}{3}\right]+\left[\frac{4 x+5}{6}\right]=\frac{3 x-1}{2}$ are greater than or equal to $\left\{\left[{ }^{*}\right]\right.$
denotes greatest integer function):
A. 7
B. 8
C. 9
D. 10

## Answer: A::B::C

## - Watch Video Solution

22. Let $f\left(x=\sin ^{6}\left(\frac{x}{4}\right)+\cos ^{6}\left(\frac{x}{4}\right) . I f f^{n}(x)\right.$ denotes $n^{\text {th }}$ derivative of f evaluated at x . Then which of the following hold ?
A. $f^{2014}(0)=-\frac{3}{8}$
B. $f^{2015}(0)=\frac{3}{8}$
C. $f^{2010}\left(\frac{\pi}{2}\right)=0$
D. $f^{2011}\left(\frac{\pi}{2}\right)=\frac{3}{8}$

## - Watch Video Solution

23. Which of the following is (are) incorrect ?
A. If $f(x)=\sin x$ and $g(x)=$ in x then range of $g(f(x))$ is $[-1,1]$
B.
C. If $f(x)=\left(2011-x^{2012}\right)^{\frac{1}{2012}}$ then $f(f(2))=\frac{1}{2}$
D. The function $f: R \rightarrow R$ defined as $f(x)=\frac{x^{2}+4 x+30}{x^{2}-8 x+18}$ is not surjective.

## Answer: A: B

## - View Text Solution

24. If $[x]$ denotes the integral part of $x$ for real $x$, and $S=\left[\frac{1}{4}\right]+\left[\frac{1}{4}+\frac{1}{200}\right]+\left[\frac{1}{4}+\frac{1}{100}\right]+\left[\frac{1}{4}+\frac{3}{200}\right] \cdots .+\left[\frac{1}{4}+\frac{199}{200}\right.$
then S is

## - Watch Video Solution

25. Let $f(x)=\log _{\{x\}}[x]$
$g(x)=\log _{\{x\}}-\{x\}$
$h(x) \log _{\{x\}}\{x\}$
where [], $\}$ denotes the greatest integer function and fractional part function respectively.

For $x \in(1,5)$ the $f(x)$ is not defined at how many points :
A. 5
B. 4
C. 3
D. 2

## Answer: C

1. Let $f(x)=\log _{\{x\}}[x]$
$g(x)=\log _{\{x\}}-\{x\}$
$h(x) \log _{\{x\}}\{x\}$
where [], $\}$ denotes the greatest integer function and fractional part function respectively.

If $A=\{x: x \in$ domine of $f(x)))$ and $B\{x: x$ domine of $g(x)\}$ then $\forall x \in(1,5), A-B$ will be :
A. $(2,3)$
B. $(1,3)$
C. $(1,2)$
D. None of these

## Answer: D

2. Let $f(x)=\log _{\{x\}}[x]$
$g(x)=\log _{\{x\}}-\{x\}$
$h(x) \log _{\{x\}}\{x\}$
where [], $\}$ denotes the greatest integer function and fractional part function respectively.

Domine of $h(x)$ is :
A. $[2, \infty)$
B. $[1, \infty)$
C. $[2, \infty)-\{I\}$
D. $R^{+}-\{I\}$

## Answer: C

## - Watch Video Solution

3. $\theta$ is said to be well behaved if it lies in interval $\left[0, \frac{\pi}{2}\right]$. They are intelligent if they make domain of $f+g$ and $g$ equal. The vlaue of $\theta$ for
which $h(\theta)$ is defined are handosome. Let
$f(x)=\sqrt{\theta x^{2}-2\left(\theta^{2}-3\right) x-12 \theta}, g(x)=\ln \left(x^{2}-49\right)$,
$h(\theta) \ln \left[\int_{0}^{\theta} 4 \cos ^{2} t d t-\theta^{2}\right]$, where $\theta$ is in radians.
Complete set of vlaues of $\theta$ which are well behaved as well as intellignent is:
A. $\left[\frac{3}{4}, \frac{\pi}{2}\right]$
B. $\left[\frac{3}{5}, \frac{7}{8}\right]$
C. $\left[\frac{5}{6}, \frac{\pi}{2}\right]$
D. $\left[\frac{6}{7}, \frac{\pi}{2}\right]$

## Answer: D

## - View Text Solution

4. $\theta$ is said to be well behaved if it lies in interval $\left[0, \frac{\pi}{2}\right]$. They are intelligent if they make domain of $f+g$ and $g$ equal. The vlaue of $\theta$ for which $h(\theta)$ is defined are handosome. Let
$f(x)=\sqrt{\theta x^{2}-2\left(\theta^{2}-3\right) x-12 \theta,} g(x)=\ln \left(x^{2}-49\right)$,
$h(\theta) \ln \left[\int_{0}^{\theta} 4 \cos ^{2} t d t-\theta^{2}\right]$, where $\theta$ is in radians.
Complete set of alues of $\theta$ which are intelligent is :
A. $\left[\frac{6}{7}, \frac{7}{2}\right]$
B. $\left(0, \frac{\pi}{3}\right]$
C. $\left[\frac{1}{4}, \frac{6}{7}\right]$
D. $\left[\frac{1}{2}, \frac{\pi}{2}\right]$

## Answer: A

## - View Text Solution

5. $\theta$ is said to be well behaved if it lies in interval $\left[0, \frac{\pi}{2}\right]$. They are intelligent if they make domain of $f+g$ and $g$ equal. The vlaue of $\theta$ for which $h(\theta)$ is defined are handosome. Let

$$
f(x)=\sqrt{\theta x^{2}-2\left(\theta^{2}-3\right) x-12 \theta,} g(x)=\ln \left(x^{2}-49\right)
$$

$h(\theta) \ln \left[\int_{0}^{\theta} 4 \cos ^{2} t d t-\theta^{2}\right]$, where $\theta$ is in radians.

Complete set of values of $\theta$ which are well behaved, intelligent adn handsome is :
A. $\left(0, \frac{\pi}{2}\right]$
B. $\left[\frac{6}{7}, \frac{\pi}{2}\right]$
C. $\left[\frac{3}{4}, \frac{\pi}{2}\right]$
D. $\left[\frac{3}{5}, \frac{\pi}{2}\right]$

## Answer: B

## D View Text Solution

6. Let $f(x)=2-|x-3|, 1 \leq x \leq 5$ and for rest of the values $\mathrm{f}(\mathrm{x})$ can be obtained by using the relation $f(5 x)=\alpha f(x) \forall x \in R$ The maximum value of $f(x)$ in $\left[5^{4}, 5^{5}\right]$ for $\alpha=2$ is
A. 16
B. 32
C. 64
D. 8

## Answer: B

## - Watch Video Solution

7. Let $f(x)=2-|x-3|, 1 \leq x \leq 5$ and for rest of the values $\mathrm{f}(\mathrm{x})$ can be obtained by unsing the relation $f(5 x)=\alpha f(x) \forall x \in R$. The vlaue of f (2007), taking $\alpha=5$, is :
A. 1118
B. 2007
C. 1050
D. 132

## Answer: A

## - View Text Solution

8. An even periodic functin $f: R \rightarrow R$ with period 4 is such that
$f(x)=\left[\begin{array}{ll}\max .\left(|x|, x^{2}\right) & 0 \leq x<1 \\ x & 1 \leq x \leq 2\end{array}\right.$
The value of $\{f(5.12)\}$ (where $\{$.$\} denotes fractional part function), is:$
A. $\{f(3.26)\}$
B. $\{f(7.88)\}$
C. $\{f(2.12)\}$
D. $\{f(5.88)\}$

## Answer: B

## - Watch Video Solution

9. An even periodic functin $f: R \rightarrow R$ with period 4 is such that
$f(x)=\left[\begin{array}{ll}\max .\left(|x|, x^{2}\right) & 0 \leq x<1 \\ x & 1 \leq x \leq 2\end{array}\right.$
The number of solution of $f(x)|3 \sin x|$ for $x \in(-6,6)$ are :
A. 5
B. 3
C. 7
D. 9

## Answer: C

## D Watch Video Solution

10. Let $f(x)=\frac{2|x|-1}{x-3}$

Range of $f(x)$ :
A. $R-\{3\}$
B. $\left(-\infty, \frac{1}{3}\right] \cup(2, \infty)$
C. $\left(-2, \frac{1}{3}\right]$ uu $(2,00)^{\text {. }}$
D. $R$

## Answer: B

11. Let $f(x)=\frac{2|x|-1}{x-3}$

Range of the values of ' $k$ ' for which $f(x)=k$ has exactly two distinct solutions:
A. $\left(-2, \frac{1}{3}\right)$
B. $(-2,1]$
C. $\left(0, \frac{2}{3}\right]$
D. $(-\infty,-2)$

## Answer: A

## - Watch Video Solution

12. Let $f(x)$ be a continuous function (define for all $\mathbf{x}$ ) which satisfies
$f^{3}(x)-5 f^{2}(x)+10 f(x)-12 \geq 0, f^{2}(x)+3 \geq 0$ and $f^{2}(x)-5 f(x)+$ If distinct positive number $b_{1}, b_{2}$ and $b_{3}$ ar in G.P. then $\left.f(1)+\ln b_{91}\right), f(2)+\ln b_{2}, f(3)+\ln b_{3}$ are in :
A. A.P.
B. G.P.
C. H. P.
D. A. G. P.

## Answer: A

## D Watch Video Solution

13. Let $f(x)$ be a continuous function (define for all x ) which satisfies $f^{3}(x)-5 f^{2}(x)+10 f(x)-12 \geq 0, f^{2}(x)+3 \geq 0$ and $f^{2}(x)-5 f(x)+$ The equation of tangent that can be drawn from $(2,0)$ on the curve $y=x^{2} f(\sin x)$ is :
A. $y=24(x+2)$
B. $y=12(x+2)$
C. $y=24(x-2)$
D. $y=12(x-2)$

## - Watch Video Solution

14. Let $f:[2, \infty) \rightarrow\{1, \infty)$
defined
$f(x)=2^{x^{4}-4 x^{3}}$ and $g:\left[\frac{\pi}{2}, \pi\right] \rightarrow A$ defined by $g(x)=\frac{\sin x+4}{\sin x-2}$ be two invertible functions, then
$f^{-1}(x)$ is equal to
A. $\sqrt{2+\sqrt{4-\log _{2} x}}$
B. $\sqrt{2+\sqrt{4+\log _{2} x}}$
C. $\sqrt{4+\sqrt{4+\log _{2} x}}$
D. $\sqrt{4-\sqrt{2+\log _{2} x}}$

## Answer: B

## - Watch Video Solution

15. Let

$$
f:[2, \infty) \rightarrow\{1, \infty)
$$

defined
$f(x)=2^{x^{4}-4 x^{3}}$ and $g:\left[\frac{\pi}{2}, \pi\right] \rightarrow A$ defined by $g(x)=\frac{\sin x+4}{\sin x-2}$ be two invertible functions, then

The set "A" equals to
A. $[5,2]$
B. $[-2,5]$
C. $[-5,2]$
D. $\left[\begin{array}{ll}-5, & -2]\end{array}\right.$

## Answer: D

## - Watch Video Solution

## Matching Type Problems

1. Let $f(x)= \begin{cases}1+x, & 0 \leq x \leq 2 \\ 3-x, & 2<x \leq 3\end{cases}$
find (fof) ( $x$ ).

## (D) Watch Video Solution

## Subjective Type Problems

1. Let $f(x)$ be a polynomial of degree 6 with leading coefficient 2009. Suppose further that $f(1)=1, f(2)=3, f(3)=5, f(4)=7, f(5)=9, f^{\prime}(2)=2$. Then the sum of all the digits of $f(6)$ is

## - Watch Video Solution

2. If $f(x)=x^{3}-3 x+1$, then the number of distinct real roots of the equation $f(f(x))=0$ is

## - Watch Video Solution

3. If $f(x+y+1)=\{\sqrt{f(x)}+\sqrt{f(y)}\}^{2} \quad$ and $f(0)=1 \forall x, y \in R, \operatorname{determ} \in e f(n), n \in N$.
4. If the domain of $f(x)=\sqrt{12-3^{x}-3^{3-x}}+\sin ^{-1}\left(\frac{2 x}{3}\right) i s[a, b]$, then $a=\ldots \ldots$.

## - Watch Video Solution

5. The number of elements in the range of functions: $y=\sin ^{-1}\left[x^{2}+\frac{5}{9}\right]+\cos ^{-1}\left[x^{2}-\frac{4}{9}\right]$ where where [.] denotes the greatest integer function is:

## - Watch Video Solution

6. The number of integers in the range of function $f(x)=[\sin x]+[\cos x]+[\sin x+\cos x]$ is (where $\quad[]=$. denotes greatest integer function)
7. If $P(x)$ is polynomial of degree 4 such than $P(-1)=P(1)=5$ and $P(-2)=P(0)=P(2)=2 \quad$ find $\quad$ the maximum vaue of $P(x)$.

## - Watch Video Solution

8. The number of integral vlaue (s) of $k$ for which the curve $y=\sqrt{-x^{2}-2 x}$ and $x+y-k=0$ intersect at 2 distinct points is/are

## - Watch Video Solution

9. Let the solution set of the equation $\sqrt{\left[x+\left[\frac{x}{2}\right]\right]}+\left[\sqrt{\{x\}}+\left[\frac{x}{3}\right]\right]=3$ is $[a, b)$. Find the product ab . (where $[\cdot]$ and $\{\cdot\}$ denote greatest integer and fractional part function, respectively).

## - Watch Video Solution

10. For the real number x , let $f(x)=\frac{1}{2011 \sqrt{1-x^{2011}}}$. Find the number of real roots of the equation
$f(f(\ldots .(f(x)) \ldots)=.(\{-x\}$
where $f$ is applies 2013 times and \{.\} denotes fractional part function.

## - Watch Video Solution

11. Find the number of elements contained in the range of the function $\left.f(x)=\left[\frac{x}{6}\right]\left[\frac{-6}{x}\right] \forall x \in(0,30)\right]$ where [.] denotes greatest integer function)

## - Watch Video Solution

12. Let $f(x, y)=x^{2}-y^{2}$ and $g(x, y)=2 x y$. such that $(f(x, y))^{2}-(g(x, y))^{2}=\frac{1}{2}$ and $f(x, y) . G(x, y)=\frac{\sqrt{3}}{4} \quad$ Find $\quad$ the number of ordered pairs $(x, y)$ ?
13. Let $f(x)=\frac{x+5}{\sqrt{x^{2}+1}}$, then the smallest integral value of k for which $f(x) \leq k \forall x \in R$ is

## - Watch Video Solution

14. The number of integral values of $m$ for which $f: R \rightarrow R, f(x)=\frac{x^{2}}{3}+(m-1) x^{2}+(m+5) x+n$ is bijective is :

## - Watch Video Solution

15. The number of roots of equation $\left.\left.\frac{(x-1)(x-3)}{(x-2)(x-4)}-e^{x}\right) \frac{(x+1)(x+3) e^{x}}{(x+2)(x+4)}-1\right)\left(x^{3}-\cos x\right)=0:$

## ( Watch Video Solution

16. Let $f(x)=x^{2}-b x+c, b$ is an odd positive integer. Given that $\mathrm{f}(\mathrm{x})=0$ has two prime numbers as roots and $b+c=35$. If the least value of
$f(x) \forall x \in R$ is $\lambda$, then $\left[\left|\frac{\lambda}{3}\right|\right]$ is equal to (where [.] denotes greatest integer function)

## - Watch Video Solution

17. Let $f(x)$ be a continuous function such that $f(0)=1$ and $f(x)=f\left(\frac{x}{7}\right)=\frac{x}{7} \forall x \in R$, then $f(42)$ is

## - Watch Video Solution

## 18.

$f(x)=4 x^{3}-x^{2}-2 x+1$ and $g(x)= \begin{cases}\min \{f(t): 0 \leq t \leq x\} & 0 \leq: \\ 3-x & 1<:\end{cases}$ and if $\lambda=g\left(\frac{1}{4}\right)+g\left(\frac{3}{4}\right)+g\left(\frac{5}{4}\right)$, then $2 \lambda=$

## - Watch Video Solution

19. If $x=10 \sum_{r=3}^{100} \frac{1}{\left(r^{2}-4\right)}$, then $[x]=$
(where [.] denotes gratest integer function)

## - Watch Video Solution

20. Let $f x=\frac{a x+b}{x a+d}$, where $\mathrm{a}, \mathrm{b}, \mathrm{c} \mathrm{d}$ are non zero If $f(7)=7, f(11)=11$ and $f(f(x))=x$ for all x expect $-\frac{d}{c}$. The unique number which is not is the range of $f$ is

## - Watch Video Solution

21. 

Let

$$
A=\left\{x \mid x^{2}-4 x+3<0, x \in R\right\}
$$

$B=\left\{x \mid 2^{1-x}+p \leq 0 ; x^{2}-2(p+7) x+5 \leq 0\right\}$ If $A \subset B$, then the range of real number $p \in[a, b]$ where, $\mathrm{a}, \mathrm{b}$ are integers. Find the value of $(b-a)$.

## - Watch Video Solution

22. Let the maximum value of expression $y=\frac{x^{4}-x^{2}}{x^{6}+2 x^{3}-1}$ for $x>1 i s \frac{p}{q}$, where p and q are relatively prime natural numbers, then $p+q=$

## Watch Video Solution

23. If $f(x)$ is an even function then find the number of distinct real numbers x such that $f(x)=f\left(\frac{x+1}{x+2}\right)$.

## - Watch Video Solution

24. The least integral value of $m, m \in R$ for which the range of function $f(x)=\frac{x+m}{x^{2}+1}$ contains the interval $[0,1]$ is :

## ( Watch Video Solution

25. Let $x_{1}, x_{2}, x_{3}$ satisfying the equation $x^{3}-x^{2}+\beta x+\gamma=0$ are in GP where $\left(x_{1}, x_{2}, x_{3}>0\right)$, then the maximum value of $[\beta]+[\gamma]+2$ is, [.] is greatest integer function.

## - Watch Video Solution

26. The value of $\sum_{r=1}^{1024}\left[\log _{2} r\right]$ is equal to, ([.] denotes the greatest integer function)

## - Watch Video Solution

27. Let $f(x)=\frac{a x+b}{c x+d}$, where $\mathrm{a}, \mathrm{b}, \mathrm{c} \quad \mathrm{d}$ are non zero If $f(7)=7, f(11)=11$ and $f(f(x))=x$ for all x except $-\frac{d}{c}$. The unique number which is not is the range of $f$ is

## - Watch Video Solution

28. It is pouring down rain and the amount of rain hitting point $(x, y)$ is given by $f(x, y)=\left|x^{3}+2 x^{2} y-5 x y^{2}-6 y^{3}\right|$. If Mr. 'A' starts at $(0,0)$, find number of possible value (s) for ' $m$ ' such that $\mathrm{y}=\mathrm{mx}$ is a line along which Mr.' A could walk without any rain falling on him.

## - Watch Video Solution

29. Let $P(x)$ be a cubic polynomical with leading co-efficient unity. Let the remainder when $\mathrm{P}(\mathrm{x})$ is divided by $x^{2}-5 x+6$ equals 2 times the remainder when $\mathrm{P}(\mathrm{x})$ is divided by $x^{2}-5 x+4$. If $P(0)=100$, find thesum of the digits of $P(5)$,

## - Watch Video Solution

30. Let $f(x)=x^{2}+10 x+20$. Find the number of real solution of the equation $f(f(f(f(x))))=0$
31. If range of $f(x)=\frac{(\ln x)\left(\ln x^{2}\right)+\ln x^{3}+3}{\ln ^{2} x+\ln x^{2}+2}$ can be expressed as $\left[\frac{a}{b}, \frac{c}{d}\right]$ where a,b,c and d are prime numbers (not nacessarily distinct) then find the value of $\frac{(a+b+c+d)}{2}$.

## - Watch Video Solution

32. Polynomial $P(x)$ contains only terms of aodd degree. when $P(x)$ is divided by $(x-3)$, the ramainder is 6 . If $P(x)$ is divided by $\left(x^{2}-9\right)$ then remainder is $g(x)$. Then find the value of $g(2)$.

## - Watch Video Solution

33. The equation $2 x^{3}-3 x^{2}+p=0$ has three real roots. Then find the minimum value of $p$.

## - Watch Video Solution

## - Watch Video Solution

