MATHS

BOOKS - VIKAS GUPTA MATHS (HINGLISH)

INDEFINITE AND DEFINITE INTEGRATION

Exercise Single Choice Problems

1. $\int a^{x}\left(\ln x+\ln a.\ \ln\left(\frac{x}{e}\right)^{x}\right)dx=$

A. $a^{x}\ln\left(\frac{e}{x}\right)^{2x}+C$

B. $a^{x}\ln\left(\frac{x}{e}\right)^{x}+C$

C. $a^{x}+\ln\left(\frac{x}{e}\right)^{x}+C$

D. None of these

Answer: B

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2. The value of :

$$\lim_{n \to \infty}\left(\frac{1}{\sqrt{n}\sqrt{n+1}}+\frac{1}{\sqrt{n}\sqrt{n+2}}+\frac{1}{\sqrt{n}\sqrt{n+3}}+\ldots\ldots+\frac{1}{\sqrt{n}\sqrt{2n}}\right)$$

is:

A. $\sqrt{2}-1$

B. $2\left(\sqrt{2}-1\right)$

C. $\sqrt{2}+1$

D. $2\left(\sqrt{2}+1\right)$

Answer: B

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3. $\int \frac{\sin x}{\sin(x-\alpha)}dx = Ax + B\log(\sin(x-\alpha)) + C$, then find out $A\&B$

A. $(\sin\alpha, \cos\alpha)$

B. $(\cos\alpha, \sin\alpha)$

C. $(-\sin\alpha, \cos\alpha)$

D. $(-\cos\alpha, \sin\alpha)$

Answer: B

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4. The value of the integral $\int_0^2 \frac{\log(x^2+2)}{(x+2)^2}$, dx is

A. $\frac{\sqrt{2}}{3}\tan^{-1}\sqrt{2}+\frac{5}{12}\log 2-\frac{1}{4}\log 3$

B. $\frac{\sqrt{2}}{3}\tan^{-1}\sqrt{2}-\frac{5}{12}\log 2-\frac{1}{4}\log 3$

C. $\frac{\sqrt{2}}{3}\tan^{-1}\sqrt{2}+\frac{5}{12}\log 2+\frac{1}{4}\log 3$

D. $\frac{\sqrt{2}}{3}\tan^{-1}\sqrt{2}-\frac{5}{12}\log 2+\frac{1}{4}\log 3$

Answer: D

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5. If $I_1 = \int_0^1 \frac{1+x^8}{1+x^4} dxs$ and $I_2 = \int_0^1 \frac{1+x^9}{1+x^2} dx$, then:

A. $I_1 > 1, I_2 < 1$

B. $I_1 < 1, I_2 > 1$

C. $1 < I_1 < I_2$

D. $I_2 < I_1 < 1$

Answer: D

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6. Let $f : (0, 1) \in (0, 1)$ be a differenttiable function such that $f(x) \neq 0$ for all $x \in (0, 1)$ and $f\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2}$. Suppose for all x,

$$\lim_{x \to x} \frac{\int_0^1 \sqrt{1(f(s))^2} dx \int_0^x \sqrt{1(f(s))^2} ds}{f(t) - f(x)} = f(x)$$

Then, the value of $f\left(\frac{1}{4}\right)$ belongs to

A. $\left\{\frac{\sqrt{7}}{4}, \frac{\sqrt{15}}{4}\right\}$

B. $\left\{\frac{\sqrt{7}}{3}, \frac{\sqrt{15}}{3}\right\}$

C. $\left\{\frac{\sqrt{7}}{2}, \frac{\sqrt{15}}{2}\right\}$

D. $\{\sqrt{7}, \sqrt{15}\}$

Answer: A

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7. If $f(\theta)=\frac{4}{3}\left(1-\cos^6\theta-\sin^6\theta\right)$, then

$$\lim_{x\to o}\frac{1}{n}\left[\sqrt{f\left(\frac{1}{n}\right)}+\sqrt{f\left(\frac{2}{n}\right)}+\sqrt{f\left(\frac{n}{n}\right)}\right]=$$

A. $\frac{1-\cos 1}{2}$

B. $1-\cos 2$

C. $\frac{\sin 2}{2}$

D. $\frac{1-\cos 2}{2}$

Answer: D

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8. The value of $\int_0^1 \frac{(x^6 - x^3)}{(2x^3+1)^3}$ dx is equal to :

A. $-\frac{1}{6}$

B. $-\frac{1}{12}$

C. $-\frac{1}{18}$

D. $-\frac{1}{36}$

Answer: D

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9. $2\int_0^{\frac{1}{\sqrt{2}}} \frac{\sin^{-1}x}{x}dx - \int_0^1 \frac{\tan^{-1}x}{x}dx =$

A. $\frac{\pi}{8}\ln 2$

B. $\frac{\pi}{4}\ln 2$

C. $\frac{\pi}{2\sqrt{2}}\ln 2$

D. $\frac{\pi}{2}\ln 2$

Answer: B

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10. Let $f(x)$ be a differentiable function such that $f(x) = x^2 + \int_0^x e^{-t} f(x - t) dt$ then $\int_0^1 f(x) dx =$

A. $\frac{1}{3}$

B. $\frac{1}{4}$

C. $\frac{7}{12}$

D. $\frac{5}{12}$

Answer: D

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11. If $f'(x) = f(x) + \int_0^1 f(x)$ dx and given $f(0) = 1$, then $\int f(x)dx$ is equal to :

A. $\frac{2}{3-e}e^x + \left(\frac{3-e}{1-e}\right)x + C$

B. $\frac{2}{3-e}e^x + \left(\frac{1-e}{3-e}\right)x + C$

C. $\frac{3}{2-e}e^x + \left(\frac{1+e}{3+e}\right)x + C$

D. $\frac{2}{2-e}e^x + \left(\frac{1-e}{3+e}\right)x + C$

Answer: B

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12. For any $x \in R$, and $f$be a continuous function Let $I_1 = \int_{\sin^2 x}^{1-\cos^2 x} tft(2-t)dt$, $I_2 = \int_{\sin^2 x}^{1+\cos^2 x} f(t(2-t))dt$, then $I_1 =$

A. I_2

B. $\frac{1}{2}I_2$

C. $2I_2$

D. $3I_2$

Answer: A

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13. If the integral $\int \frac{5\tan x}{\tan x - 2} dx = x + a\text{I n}|\sin x - 2\cos x| + k$ then a is equal to (1) 1 (2) 2 (3) 1 (4) 2

A. 1

B. 2

C. -1

D. -2

Answer: B

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14. $\int \frac{(2+\sqrt{x})dx}{(x+1+\sqrt{x})^2}$ is equal to:

A. $\frac{x}{x+\sqrt{x}+1}+C$

B. $\frac{2x}{x+\sqrt{x}+1}+C$

C. $\frac{-2x}{x+\sqrt{x}+1}+C$

D. $\frac{-x}{x+\sqrt{x}+1}+C$

Answer: B

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15. Evaluate $\int \frac{\left(\sqrt[3]{x+\sqrt{2-x^2}}\right)\left(\sqrt[6]{1-x\sqrt{2-x^2}}\right)dx}{\sqrt[3]{1-x^2}}, x \in (0,1):$

A. $2^{\frac{1}{6}}x + C$

B. $2^{\frac{1}{12}}x + C$

C. $2^{\frac{1}{3}}x + C$

D. None of these

Answer: A

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16. $\int \frac{dx}{\sqrt{1-\tan^2 x}} = \frac{1}{\lambda}\sin^{-1}(\lambda \sin x) + C$,then $\lambda =$

A. $\sqrt{2}$

B. $\sqrt{3}$

C. 2

D. $\sqrt{5}$

Answer: A

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17. $\int \frac{dx}{\sqrt[3]{x^{5/2}(x+1)^{7/2}}}$ is equal to:

A. $-\left(\frac{x-1}{x}\right)^{1/6} + C$

B. $6\left(\frac{x+1}{x}\right)^{-1/6}+C$

C. $\left(\frac{x}{x+1}\right)^{5/6}+C$

D. $-\left(\frac{x}{x+1}\right)^{5/6}+C$

Answer: B

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18. If $I_n=\int(\sin x)^n dx, n\in N$, then $5I_4-6I_6$ is equal to

A. $\sin x.\ (\cos x)^5+C$

B. $\sin 2x\cos 2x+C$

C. $\frac{\sin 2x}{8}\left[1+\cos^2 2x-2\cos 2x\right]+C$

D. $\frac{\sin 2x}{8}\left[1+\cos^2 2x+2\cos 2x\right]+C$

Answer: C

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19. $\int \frac{x^2}{(a+bx)^2} dx$

A. $\frac{1}{b^3}\left(a+bx-a\ln|a+bx|-\frac{a^3}{a+bx}\right)+C$

B. $\frac{1}{b^3}\left(a+bx-2a\ln|a+bx|-\frac{a^3}{a+bx}\right)+C$

C. $\frac{1}{b^3}\left(a+bx+2n\ln|a+bx|\frac{a^2}{a+bx}\right)+C$

D. $\frac{1}{b^3}\left(a+bx+2n\ln|a-bx|\frac{a^2}{a+bx}\right)+C$

Answer: B

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20. $\int \frac{8x^{43}+13x^{38}}{(x^{13}+x^5+1)^4} dx =$

A. $\frac{x^{39}}{3(x^{13}-x^5+1)^3}+C$

B. $\frac{x^{39}}{(x^{13}-x^5+1)^3}+C$

C. $\frac{x^{39}}{5(x^{13}-x^5+1)^5}+C$

D. None of these

Answer: A

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21. $\int\left(\frac{\cos 6x+6\cos 4x+15\cos 2x+10}{10\cos^2 x+5\cos x\cos 3x+\cos x\cos 5x}\right)dx=f(x)+C,$ then $f(10)$ is equal to:

A. 20

B. 10

C. $2\sin 10$

D. $2\cos 10$

Answer: A

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22. $\int\left(1+x-x^{-1}\right)e^{x+x^{-1}}dx=$

A. $(x+1)e^{x+x^{-1}}+C$

B. $(x-1)e^{x+x^{-1}}+C$

C. $-xe^{x+x^{-1}}+C$

D. $xe^{x+x^{-1}}+C$

Answer: D

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23. $\int e^x\left(\frac{2\tan x}{1+\tan x}+\cot^2\left(x+\frac{\pi}{4}\right)\right)dxisequa < o$ $e^x\tan\left(\frac{\pi}{4}-x\right)+c\,e^x\tan\left(x-\frac{\pi}{4}\right)+c\,e^x\tan\left(\frac{3\pi}{4}-x\right)+c$ (d) none of these

A. 0

B. 1

C. -1

D. 2

Answer: B

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24. $\int e^{x\sin x+\cos x}\left(\frac{x^4\cos^3 x - x\sin x+\cos x}{x^2\cos^2 x}\right)dx =$

A. $e^{x\sin x+\cos x}\left(x-\frac{1}{\cos x}\right)C$

B. $e^{x\sin x+\cos x}\left(x-\frac{1}{x\cos x}\right)C$

C. $e^{x\sin x+\cos x}\left(1-\frac{1}{x\cos x}\right)C$

D. $e^{x\sin x+\cos x}\left(1-\frac{1}{\cos x}\right)C$

Answer: B

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25. $\int \frac{1+x+\sqrt{x+x^2}}{\sqrt{x}+\sqrt{1+x}}dx =$

A. $\frac{1}{3}\left(2^{1/2}-1\right)$

B. $\frac{2}{3}\left(2^{1/2}-1\right)$

C. $\frac{2}{3}\left(2^{2/3}-1\right)$

D. $\frac{1}{3}\left(2^{3/2}-1\right)$

Answer: C

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26. $\int x^{x^2+1}(2\ln x+1)dx$

A. $x^{2x}+C$

B. $x^2\ln x+C$

C. $x^{(x^x)}+C$

D. $\left(x^3\right)^x+C$

Answer: D

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27. If $\int \frac{\cos ec^2 x - 2010}{\cos^{2010} x} dx = = \frac{f(x)}{(g(x))^{2010}} + C$, where $f\left(\frac{\pi}{4}\right) = 1$, then the number of solution of the equation $\frac{f(x)}{g(x)} = \{x\}$ in $[0, 2\pi]$ is/are: (where {.} represents fractional part function)

A. 0

B. 1

C. 2

D. 3

Answer: A

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28. $\int x^x \left((\ln x)^2 - \frac{1}{x}\right)$ dx is equal to:

A. $x^2\left((\ln x)^2 - \frac{1}{x} + C\right.$

B. $x^x(\ln x - x) + C$

C. $x^x \frac{(\ln x)^2}{2} + C$

D. $x^x \ln x + C$

Answer: D

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29. $\int \frac{x^2 - 1}{x^3 \sqrt{2x^4 - 2x^2 + 1}} dxisequa < o \quad \frac{\sqrt{2x^4 - 2x^2 + 1}}{x^3} + C$ (b) $\frac{\sqrt{2x^4 - 2x^2 + 1}}{x} + C \ \frac{\sqrt{2x^4 - 2x^2 + 1}}{x^2} + C$ (d) $\frac{\sqrt{2x^4 - 2x^2 + 1}}{2x^2} + C$

A. $\frac{\sqrt{2x^4 - 2x^2 + 1}}{x^2} + C$

B. $\frac{\sqrt{2x^4 - 2x^2 + 1}}{x} + C$

C. $\frac{\sqrt{2x^4 - 2x^2 + 1}}{x} + C$

D. $\frac{\sqrt{2x^4 - 2x^2 + 1}}{2x^2} + C$

Answer: D

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30. $\int\left\{\left(\frac{\log x-1}{1+(\log x)^2}\right\}^2 dx$ is equal to

A. $\frac{x}{x^2+1}+C$

B. $\frac{\ln x}{(\ln x)^2+1}+C$

C. $\frac{x}{1+(\ln x)^2}+C$

D. $e^x\left(\frac{x}{x^2+1}\right)+C$

Answer: C

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31. $I=\int\frac{dx}{\sqrt[4]{(x-1)^3(x+2)^5}}=k\sqrt[4]{\frac{x-1}{x+2}}+C,$ then 'k is equal to:

A. $\frac{1}{3}$

B. $\frac{2}{3}$

C. $\frac{3}{4}$

D. $\frac{4}{3}$

Answer: D

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32. If : $\int \frac{1-x^7}{x(1+x^7)}dx = a.\log|x| + b.\log|x^7+1| + c,$ then : $(a, b) \equiv$

A. $2P - 7Q = 0$

B. $2P + 7Q = 0$

C. $7P + 2Q = 0$

D. $7P - 2Q = 1$

Answer: B

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33. $\int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cos^2 x}dx =$

A. $\sin 2x + C$

B. $\frac{\sin 2x}{2} + C$

C. $\frac{-\sin 2x}{2} + C$

D. $-2\sin 2x + C$

Answer: C

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34. $\int \frac{(\sin 2x)^{1/3} d\left(\sqrt[3]{\tan x}\right)}{\sin^{2/3} x + \cos^{2/3} x}$ is equal to

A. $\frac{1}{2^{2/3}} \ln\left(1 + \tan^{1/3} x\right) + C$

B. $\ln\left(1 + \tan^{2/3} x\right) + C$

C. $2^{1/3} \ln\left(1 + \tan^{2/3} x\right) + C$

D. $\frac{1}{2^{2/3}} \ln\left(1 + \tan^{2/3} x\right) + C$

Answer: D

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35. $\int \sqrt{\frac{(2012)^{2x}}{1-(2012)^{2x}}}(2012)^{\sin^{-1}(2012)^x}dx =$

A. $(\log_{2012} e)^2(2012)^{\sin^{-1}(2012)^x} + C$

B. $(\log_{2012} e)^2(2012)^{x-\sin^{-1}(2012)^x} + C$

C. $(\log_{2012} e)^2(2012)^{\sin^{-1}(2012)^x} + C$

D. $\frac{(2012)^{\sin^{-1}(2012)^x}}{(\log_{2012} e)^2} + C$

Answer: C

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36. $\int \frac{x+2}{(x^2+3x+3)\sqrt{x+1}}dx$ is equal to

A. $\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3(x+1)}}\right) + C$

B. $\frac{2}{\sqrt{3}}\tan^{-1}\left(\sqrt{\frac{x}{3(x+1)}}\right) + C$

C. $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x}{3(x+1)}\right) + C$

D. $\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}(x+1)}\right)+C$

Answer: A

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37. $\int \frac{f(x)\cdot g'(x)-f'(x)g(x)}{f(x)\cdot g(x)}\{\log g(x)-\log f(x)\}\,dx$

A. $\log\left(\frac{g(x)}{f(x)}\right)+C$

B. $\frac{1}{2}\left(\frac{g(x)}{f(x)}\right)^2+C$

C. $\frac{1}{2}\left(\log\left(\frac{g(x)}{f(x)}\right)\right)^2+C$

D. $\log\left(\left(\frac{g(x)}{f(x)}\right)^2\right)+C$

Answer: C

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38. $\int\left(\int e^x\left(\ln x+\frac{2}{x}-\frac{1}{x^2}\right)dx\right)dx=$

A. $e^x \ln x + C_1 x + C_2$

B. $e^x \ln x + \frac{1}{x} + C_1 x + C_2$

C. $\frac{\ln x}{x} + C_1 x + C_2$

D. None of these

Answer: A

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39. Maximum vlaue of the function $f(x) = \pi^2 \int_0^2 t \sin(x + \pi t) dt$ over all real number x:

A. $\sqrt{\pi^2 + 1}$

B. $\sqrt{\pi^2 + 2}$

C. $\sqrt{\pi^2 + 3}$

D. $\sqrt{\pi^2 + 4}$

Answer: D

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40. Let 'f is a function, continuous on $[0, 1]$ such that $f(x) \leq \sqrt{5} \forall x \in [0, 1]$ and $f(x) \leq \frac{2}{x} \forall x \in \left[\frac{1}{2}, 1\right]$ then smallest 'a' for which $\int_0^1 f(x)dx \leq a$ a holds for all 'f' is

A. $\sqrt{5}$

B. $\frac{\sqrt{5}}{2} + 2\ln 2$

C. $2 + \ln\left(\frac{\sqrt{5}}{2}\right)$

D. $2 + 2\ln\left(\frac{\sqrt{5}}{2}\right)$

Answer: D

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41. Let $I_n = \int_1^{e^2} (\ln x)^n dt(x^2)$, then the vlaue of $2I_n + nI_{n-1}$ equals to:

A. 0

B. $2e^2$

C. e^2

D. 1

Answer: B

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42. Let a function $f: R \rightarrow R$ be defined as $f(x) = x + \sin x$. The value of $\int_0^{2\pi} f^{-1}(x)dx$ will be:

A. $2\pi^2$

B. $2\pi^2 - 2$

C. $2x^2 + 2$

D. π^2

Answer: A

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43. The value of the definite integral $\int e^{-x^4}\left(2+\ln\left(x+\sqrt{x^2+1}\right)+5x^3-8x^4\right)dx$ is equal to

A. 4e

B. $\frac{4}{e}$

C. $2e$

D. $\frac{2}{e}$

Answer: B

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44. $\int_{-10}^{0}\left(\left|\frac{2[x]}{3x-[x]}\right|/\frac{2[x]}{3x-[x]}\right)dx$ is equal to (where [*] denotes greatest integer function.) is equal to (where [*] denotes greatest integer function.)

A. $\frac{28}{3}$

B. $\frac{1}{3}$

C. 0

D. None of these

Answer: A

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45. If $f(x) = \dfrac{x}{1 + (Inx)(\ln x)\ldots\infty} \forall x \in [1.\infty)$ then $\int_1^{2e} f(x)dx$ equals:

A. $\frac{e^2 - 1}{2}$

B. $\frac{e^2 + 1}{2}$

C. $\frac{e^2 - 2e}{2}$

D. None of these

Answer: A

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46. $\int_0^4 \frac{(y^2-4y+5)\sin(y-2)dy}{[2y^2-8y+1]}$ is equal to

A. 0

B. 2

C. -2

D. None of these

Answer: A

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47. Let $\frac{d}{dx}F(x)=\left(\frac{e^{\sin x}}{x}\right), x>0.$ If $\int_1^4 \frac{3}{x}e^s \in x^3 dx = F(k)-F(1),$ then one of the possible values of k, is: 15 (b) 16 (c) 63 (d) 64

A. 15

B. 16

C. 63

D. 64

Answer: D

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48. Value of $\lim_{h \to 0} \frac{\int_0^{x - he^{-1/h}} dx - \int_0^{\pi} x^2 e^{-x^2} dx}{he^{-1/h}}$ in equal to:

A. $\pi(1-\pi^2)e^{-\pi^2}$

B. $2\pi(1-\pi^2)e^{-\pi^2}$

C. $\pi(1-\pi)e^{-\pi}$

D. $\pi^2 e^{-\pi^2}$

Answer: D

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49. Let $f : R^+ \to R$ be a differentiable function with $f(1) = 3$ and satisfying :

$\int_1^{xy} f(t)dt = y\int_1^x f(t)dt + x\int_1^y f(t)dt \,\forall x, y \in R^+$, then $f(e) =$

A. 3

B. 4

C. 1

D. None of these

Answer: D

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50. If [.] denotes the greatest integer functin, then the integral $\int_0^{x/2} \frac{e^{\sin x - (\sin x)} d(\sin^2 x - [\sin^2 x])}{\sin x - [\sin x]}$ is λ, then $[\lambda - 1]$ is equal to:

A. 0

B. 1

C. 2

D. 3

Answer: C

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51. Calculate the reciprocal of the limit $\lim_{x \to \infty} \int_0^x xe^{t^2 - x^2} dt$

A. 0

B. 1

C. 2

D. 3

Answer: C

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52. Let

$$L = \lim_{n \to \infty} \left(\frac{(2.1+n)}{1^2+n.1+n^2} + \frac{(2.2+n)}{2^2+n.2+n^2} + \frac{(2.3+n)}{3^2+n.3+n^2} + \ldots\ldots +$$

then value of e^L is:

A. 2

B. 3

C. 4

D. $\frac{3}{2}$

Answer: B

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53. The value of the definite integral $\int_0^2 \left(\sqrt{1+x^3} + \sqrt[3]{x^2+2x}\right)$ dx is :

A. 4

B. 5

C. 6

D. 7

Answer: C

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54. The volue of the ddefinite integral $\int_0^\infty \frac{\ln x}{x^2+4} dx$ is:

A. $\frac{\pi \ln 3}{2}$

B. $\frac{\pi \ln 2}{3}$

C. $\frac{\pi \ln 2}{4}$

D. $\frac{\pi \ln 4}{3}$

Answer: C

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55. The value of the definite integral $\int_0^{10}\left((x-5)+(x-5)^2+(c-5)^3\right)$ dx is:

A. $\frac{125}{3}$

B. $\frac{250}{3}$

C. $\frac{125}{6}$

D. $\frac{250}{4}$

Answer: B

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56. The value of definite integral $\int_0^{\infty}\frac{dx}{(1+x^9)(1+x^2)}$ equal to:

A. $\frac{\pi}{16}$

B. $\frac{\pi}{8}$

C. $\frac{\pi}{4}$

D. $\frac{\pi}{2}$

Answer: C

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57. The value of the definite integral $\int_0^{\frac{\pi}{2}} \left(\frac{1+\sin 3x}{1+2\sin x}\right) dx$ equals to

A. $\frac{\pi}{2}$

B. 1

C. $\frac{1}{2}$

D. $\frac{\pi}{4}$

Answer: B

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58. The value of $\lim_{x \to \infty} \frac{\int_0^x \left(\tan^{-1} x\right)^2}{\sqrt{x^2+1}} dx$

A. $\frac{\pi^2}{16}$

B. $\frac{\pi^2}{4}$

C. $\frac{\pi^2}{2}$

D. None of these

Answer: B

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59. If

$$\int_0^1 \left(\sum_{r=1}^{2013} \frac{x}{x^2+r^2} \prod_{r=1}^{2013} (x^2+r^2)\right) dx = \frac{1}{2}\left[\left(\prod_{r=1}^{2013} (1+r^2)\right) - k^2\right]$$

then k=

A. 2013

B. 2013!

C. 2013^2

D. 2013^{2013}

Answer: B

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60. Let $f(x) = 2x - \tan^{-1} x - \ln\left(x + \sqrt{1+x^2}\right); x \in R,$ then

A. strictly increases $\forall x \in R$

B. strictly increases only in $(0, \infty)$

C. strictly decreases $\forall x \in R$

D. strictly decreases in (o, ∞) and strictly increases I $(-\infty, 0)$

Answer: A

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61. The value of the definite integral $\int_0^{\frac{\pi}{2}} \frac{dx}{\tan x + \cot x + \cos ecx + \sec x}$ is

A. $1-\frac{\pi}{4}$

B. $\frac{\pi}{4}+1$

C. $\pi+\frac{1}{4}$

D. None of these

Answer: A

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62. The value of the definite integral $\int_3^7 \frac{\cos x^2}{\left(\cos x^2+\cos(10)-x\right)^2}dx$ is:

A. 2

B. 1

C. $\frac{1}{2}$

D. None of these

Answer: A

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63. The value of the integral $\int_{e^{-1}}^{e^2}\left|\frac{\log_e x}{x}\right|dx$ is

A. $\frac{3}{2}$

B. $\frac{5}{2}$

C. $\frac{3}{2}$

D. 5

Answer: B

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64. The value of $\lim_{x\to\frac{\pi}{4}}\frac{\int_2^{\cos ec^2 x} tg(t)dt}{x^2-\frac{\pi^2}{16}}$ is:

A. $\frac{2}{\pi}g(2)$

B. $-\frac{4}{\pi}g(2)$

C. $-\frac{16}{\pi}g(2)$

D. $-\frac{16}{\pi}g(2)$

Answer: C

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65. The value of $\lim \sum_{k=1}^{n} \frac{n-k}{n} \cos\left(\frac{4k}{n}\right)$ equals to

A. $\frac{1}{4}\sin 4 + \frac{1}{16}\cos 4 - \frac{1}{16}$

B. $\frac{1}{4}\sin 4 - \frac{1}{16}\cos 4 + \frac{1}{16}$

C. $\frac{1}{16}(1 - \sin 4)$

D. $\frac{1}{16}(1 - \cos 4)$

Answer: D

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66. For each ositive integer n, define a function $f_n on [0, 1]$ as follows:

$$f_n(x) = \begin{cases} 0 & \text{if} \quad x = 0 \\ \sin\frac{\pi}{2n} & \text{if} \quad 0 < x \le \frac{1}{n} \\ \sin\frac{2\pi}{2n} & \text{if} \quad \frac{1}{n} < x \le \frac{2}{n} \\ \sin\frac{3\pi}{2\pi} & \text{if} \quad \frac{2}{n} < x \le \frac{3}{n} \\ \sin{'}\frac{n\pi}{2\pi} & \text{if} \quad \frac{n-1}{n} < x \le 1 \end{cases}$$

Then the value of $\lim_{x \to \infty} \int_0^1 f_n(x)dx$ is:

A. π

B. $\frac{\pi}{2}$

C. $\frac{1}{\pi}$

D. $\frac{2}{\pi}$

Answer: D

Watch Video Solution

67. $$\int_0^{n+1} \min\{(x-1|, |x-2|, |x-3|, \ldots\ldots\ldots\ldots, |x-n\ |\ )\}dx$$

equais to (where n be a positive integer)

A. $\frac{(n+1)}{4}$

B. $\frac{(n+2)}{4}$

C. $\frac{(n+3)}{4}$

D. $\frac{(n+4)}{4}$

Answer: A

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68. For positive integers k=1,2,3,....n,, let S_k denotes the area of $\triangle AOB_k$ such that $\angle AOB_k = \frac{k\pi}{2n}$, OA=1 and $OB_k = k$ The value of the $\lim_{n \to \infty} \frac{1}{n^2} \sum_{k-1}^{n} S_k$ is

A. $\frac{2}{\pi^2}$

B. $(4)/(\pi^2)$

C. $\frac{8}{\pi^2}$

D. $\frac{1}{2\pi^2}$

Answer: D

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69. If $A = \int_0^1 \prod_{r=1}^{2014} (r - x)dx$ and $B = \int_0^1 \prod_{r=0}^{2013} (r + x)dx$, then:

A. $A = 2B$

B. $2A = B$

C. $A + B = 0$

D. $A = B$

Answer: D

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70. If $f(x) = \left[\frac{x}{120} + \frac{x^3}{30}\right]$ defined in $[0, 3]$ then $\int_0^1 ((f(x)) + 2)dx =$

(where [.] dentoes greatest integer function)

A. 0

B. 1

C. 2

D. 4

Answer: C

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71. If $f(x)=\int_0^{g(x)}\frac{dt}{\sqrt{1+t^3}}, g(x)=\int_0^{\cos x}(1+\sin t)^2 dt,$ then the value of $f'\left(\frac{\pi}{2}\right)$ is equal to:

A. 1

B. -1

C. 0

D. $\frac{1}{2}$

Answer: D

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72. Let $f(x) = \frac{1}{x^2}\int_4^x (4t^2 - 2f'(t)dt,$ find $9f'(4)$

A. 16

B. 4

C. 8

D. 32

Answer: B

Watch Video Solution

73. Evaluate $\lim_{x \to \infty}\left(\frac{1^2}{n^3+1^3}+\frac{2^2}{n^3+2^3}+\frac{3^2}{n^3+3^3}+....+\frac{4}{9n}\right).$

A. $\frac{1}{3}\ln 3$

B. $\frac{\ln 9}{3}$

C. $\frac{\ln 4}{3}$

D. $\frac{\ln 6}{3}$

Answer: A

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74. The value of $\int_0^{2\pi} \cos^{-1}\left(\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)$ dx is:

A. π^2

B. $\frac{\pi^2}{2}$

C. $2\pi^2$

D. π^3

Answer: D

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75. Given a function 'g' continous everywhere such that $\int_0^1 g(t)dt = 2$ and $g(1) = 5$. If $f(x) = \frac{1}{2}\int_0^x (x-t)^2 g(t)dt$, then the vlaue of $f'(1) - f'(1)$ is:

A. 0

B. 1

C. 2

D. 3

Answer: B

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76. If $\int_0^{\pi} \frac{x^3 \cos^4 x \sin^2 x}{\pi^2 - 3\pi x + 3x^2} dx = \lambda \int_0^{\frac{\pi}{2}} \sin^2 x dx$, then the value of λ is:

A. $\frac{\pi}{12}$

B. $\frac{\pi}{(8)}$

C. $\frac{\pi}{4}$

D. $\frac{\pi}{3}$

Answer: A

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77. $\int_0^{\sqrt{3}}\left(\frac{1}{2}\frac{d}{dx}\left(\tan^{-1}\frac{2x}{1-x^2}\right)\right)dx$ equals to:

A. $\frac{\pi}{3}$

B. $-\frac{\pi}{6}$

C. $\frac{\pi}{2}$

D. None of these

Answer: B

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78. Let $y = \{x\}^{\{x\}}$ then the value of $\int_0^3 y dx$ equals to (where {.} and [.] denote fractional part and integerpart function respectively)

A. 1

B. $\frac{11}{6}$

C. 3

D. $\frac{5}{6}$

Answer: C

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79. $\int_0^1 \frac{\tan^{-1}}{x} dx =$

A. $\int_0^{\pi/4} \frac{\sin x}{x} dx$

B. $\int_0^{\pi/2} \frac{x}{\sin x} dx$

C. $\frac{1}{2}\int_0^{\pi/2} \frac{x}{\sin x} dx$

D. $\frac{1}{2}\int_0^{\pi/2}\frac{x}{\sin x}dx$

Answer: C

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80. The value of $\int_0^{4/\pi}\left(3x^2\sin\frac{1}{x}-x\cos\frac{1}{x}\right)dx$ is:

A. $\frac{8\sqrt{2}}{\pi^3}$

B. $\frac{24\sqrt{2}}{\pi^3}$

C. $\frac{32\sqrt{2}}{\pi^3}$

D. None of these

Answer: C

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81. Number of values of x satisfying the equation $\int_{-1}^{x}\left(8t^2 + \frac{28}{3}t + 4\right)dt = \frac{\left(\frac{3}{2}\right)x + 1}{\log_{x+1}\sqrt{x+1}}$, is

A. 0

B. 1

C. 2

D. 3

Answer: B

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82. Evaluate : $\lim_{n\to\infty}\frac{1^4 + 2^4 + 3^4 + \dots + n^4}{n^5} - \lim_{n\to\infty}\frac{1^3 + 2^3 + \dots + n^3}{n^5}$

A. $\frac{1}{30}$

B. zero

C. $\frac{1}{4}$

D. $\frac{1}{5}$

Answer: D

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83. The value of $\lim_{x \to 0^+} \frac{\int_1^{\cos x} (\cos^{-1} t)\, dt}{2x - \sin(2x)}$ is equal to

A. 0

B. -1

C. $\frac{2}{3}$

D. $-\frac{1}{4}$

Answer: D

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84. Consider a parabola $y = \frac{x^2}{4}$ and the point $F(0, 1)$.

Let $A_1(x_1, y_1), A_1(x_2, y_2), A_3(x_3, y_3), \ldots.., A_n(x_n, y_n)$ are 'n' points on the parabola such $x_k > -$ and $\angle OFA_k = \frac{k\pi}{2\pi}(k = 1, 2, 3, \ldots\ldots, n)$.

Then the value of $\lim_{n\to\infty} \frac{1}{n}\sum_{k=1}^{n} FA_k$, is equal to :

A. $\frac{2}{\pi}$

B. $\frac{4}{\pi}$

C. $\frac{8}{\pi}$

D. None of these

Answer: B

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85. The minimum value of $f(x) = \int_0^4 e^{|x-t|}dt$ where $x \in [0, 3]$ is : (A) $2e^2 - 1$ (B) $e^4 - 1$ (C) $2(e^2 - 1)$ (D) $e^2 - 1$

A. $2e^2 - 1$

B. $e^4 - 1$

C. $e(e^2 - 1)$

D. $e^2 - 1$

Answer: C

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86. If $\int_0^\infty \frac{\cos x}{x} dx = \frac{\pi}{2}$ then $\int_0^\infty \frac{\cos^3 x}{x} dx$ is equal to

A. $\frac{\pi}{2}$

B. $\frac{\pi}{4}$

C. π

D. $\frac{3\pi}{2}$

Answer: A

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87. $\int \sqrt{1+\sin x}\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)$ dx is:

A. $\frac{1+\sin x}{2}+C$

B. $(1+\sin x)^2$

C. $\frac{1}{\sqrt{1+\sin x}}+C$

D. $\sin x + C$

Answer: D

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88. If $I_n = \int_0^{\pi} \frac{\sin(2nx)}{\sin 2x} dx,$ then the value of $I_{n+\frac{1}{2}}$ is equal to $(n \in I)$:

A. $\frac{n\pi}{2()}$

B. π

C. $\frac{\pi}{2}$

D. 0

Answer: D

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89. Find the value of the function $f(x) = 1 + x + \int_1^x \left((\ln t)^2 + 2\ln t\right) dt$ where $f'(x)$ vanishes

A. $\frac{1}{e}$

B. 0

C. $\frac{2}{e}$

D. $1 + \frac{2}{e}$

Answer: D

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90. Let $f(x)$ be a differentiable function such that $f(x) = x^2 + \int_0^x e^{-t} f(x - t) dt$ then $\int_0^1 f(x) dx =$

A. $\frac{1}{3}$

B. $\frac{1}{4}$

C. $\frac{7}{12}$

D. $\frac{5}{12}$

Answer: D

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91. The value of the definite integral $\int_{-(\pi/2)}^{\pi/2} \frac{\cos^2 x}{1+5^x}$ equal to:

A. $\frac{3n}{4}$

B. π

C. $\frac{\pi}{2}$

D. $\frac{\pi}{4}$

Answer: D

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92. $\int\left(\frac{x^2-x+1}{x^2+1}\right)e^{\cot^{-1}(x)}\,dx = f(x).\,e^{\cot^{-1}(x)} + C$

where C is constant of integration. Then f (x) is equal to:

A. $-x$

B. $\sqrt{1-x}$

C. x

D. $\sqrt{1+x}$

Answer: C

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93. $\lim_{x\to\infty}\frac{1}{n^3}\left(\sqrt{n^2+1}+2\sqrt{n^2+2^2}+\ldots.+n\sqrt{(n^2+n^2)}\right)=:$

A. $\frac{3\sqrt{2}-1}{2}$

B. $\frac{2\sqrt{2}-1}{3}$

C. $\dfrac{3\sqrt{3}-1}{3}$

D. $\dfrac{4\sqrt{2}-1}{2}$

Answer: B

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94. $\displaystyle\int \frac{x^3-1}{(x^4+1)(x+1)}dx$ is

A. $\frac{1}{4}\ln(1+x^4)+\frac{1}{3}\ln(1+x^3)+c$

B. $\frac{1}{4}\ln(1+x^4)-\frac{1}{3}\ln(1+x^3)+c$

C. $\frac{1}{4}\ln(1+x^4)-\ln(1+x)+c$

D. $\frac{1}{4}\ln(1+x^4)+\ln(1+x)+c$

Answer: C

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95. The value of $\lim_{x \to 0^+} \frac{\int_1^{\cos x} \left(\cos^{-1} t\right) dt}{2x - \sin(2x)}$ is equal to

A. 0

B. -1

C. $\frac{2}{3}$

D. $\frac{-1}{4}$

Answer: D

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96. Let $f(x) = \lim_{n \to \infty} \frac{\cos x}{\left(1 + \tan^{-1} x\right)^n}$, then $\int_0^{\infty} f(x) dx =$

A. $\tan(\sin 1)$

B. $\sin(\tan 1)$

C. 0

D. $\sin\left(\frac{\tan 1}{2}\right)$

Answer: B

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97. The value of $\lim_{x \to \infty} \sum_{k=1}^{n} \left(\frac{k}{n^2 + n + 2k} \right) =$

A. $\frac{1}{4}$

B. $\frac{1}{3}$

C. $\frac{1}{2}$

D. 1

Answer: C

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98. The value of $\lim_{y \to 1^+} \frac{\int_1^y |t-1| dt}{\tan(y-1)}$ is:

A. 0

B. 1

C. 2

D. does not exist

Answer: A

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99. Given that $\int \frac{dx}{(1+x^2)} = \frac{x}{2(n-1)(1+x^2)^{n-1}} + \frac{(2n-3)}{2(n-1)} \int \frac{dx}{(1+x^2)^{n-1}}$. Find the vlaue of $\int_0^1 \frac{dx}{(1+x^2)^4}$, (you may or may not use reduction formula given)

A. $\frac{11}{48} + \frac{5\pi}{64}$

B. $\frac{11}{48} + \frac{5\pi}{32}$

C. $\frac{1}{24} + \frac{5\pi}{64}$

D. $\frac{1}{96} + \frac{5\pi}{32}$

Answer: A

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100. Find the value of $\int_0^{\pi/4} (\sin x)^4$ dx :

A. $\frac{3\pi}{16}$

B. $\frac{3\pi}{32} - \frac{1}{4}$

C. $\frac{3\pi}{32} - \frac{3}{4}$

D. $\frac{3\pi}{16} - \frac{7}{8}$

Answer: B

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101. $\int \frac{\cos 9x + \cos 6x}{2\cos 5x - 1} dx = A \sin 4x + B \sin x + C,$ then $A + B$ is equal to:

(Where C is constant of integration)

A. $\frac{1}{2}$

B. $\frac{3}{4}$

C. 2

D. $\frac{5}{4}$

Answer: D

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102. $\int \frac{dx}{x^{2014}+x} = \frac{1}{p}\ln\left(\frac{x^q}{1+x^r}\right)+C$ where $p, qr \in N$ then the value of $(p+q+r)$ equals (Where C is constant of integration)

A. 6039

B. 6048

C. 6047

D. 6021

Answer: A

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103. If $\int_0^1 e^{-x^2}dx = 0,$ then $\int_0^1 x^2e^{-x^2}$ dx is equal to

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104. $f(x)$ is a continuous function for all real values of x and satisfies $\int_n^{n+1} f(x)dx = \frac{n^2}{2}\forall n \in I\cdot$ Then $\int_{-3}^{5} f(|x|)dx$ is equal to $\frac{19}{2}$ (b) $\frac{35}{2}$ (c) $\frac{17}{2}$ (d) none of these

A. $\frac{19}{2}$

B. $\frac{35}{2}$

C. $\frac{17}{2}$

D. $\frac{37}{2}$

Answer: B

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105. If $\int \frac{dx}{x^4(1+x^2)} = a \ln\left|\frac{1+x^3}{x^3}\right| + \frac{b}{x^3} + \frac{c}{1+x^2} + d.$ then (where d is arbitrary constant)

A. $a = \frac{1}{3}, b = \frac{1}{3}, x = \frac{1}{3}$

B. $a = \frac{2}{3}, b = -\frac{1}{3}, c = \frac{1}{3}$

C. $a = \frac{2}{3}, b = -\frac{1}{3}, c = -\frac{1}{3}$

D. $a = \frac{2}{3} b = \frac{1}{3}, c = -\frac{1}{3}$

Answer: C

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106. $\lim_{x \to \infty} \frac{1}{\sqrt{n}\sqrt{n+1}} + \frac{1}{\sqrt{n}\sqrt{n+2}}$ is equal to :

A. 2

B. 4

C. $2(\sqrt{2} - 1)$

D. $2\sqrt{2}-1$

Answer: A

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107. Let $f(x)=\int_z^2 \frac{dy}{\sqrt{1+y^3}}$. The value of the integral $\int_0^2 x f(x)dx$ is equal to:

A. 1

B. $\frac{1}{3}$

C. $\frac{4}{3}$

D. $\frac{2}{3}$

Answer: D

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108. The value of the definite integral $\int_0^{\pi/3} \ln\left(1+\sqrt{3}\tan x\right)dx$ equals

A. $\frac{\pi}{23}\ln 2$

B. $\frac{\pi}{3}$

C. $\frac{\pi^2}{6}\ln 2$

D. $\frac{\pi}{2}\ln 2$

Answer: A

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109. Q. if $\int_0^{100} (f(x)dx = a$, then $\sum_{r=1}^{100}\left(\left(\int_0^1 (f(r-1+x)dx)\right)\right.$ =

A. 100 a

B. a

C. 0

D. 0.4

Answer: B

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110. The value of $\int_0^1 \lim_{n \to \infty} \sum_{k=0}^{n} \frac{x^{k+2}2^k}{k!} dx$ is:

A. $e^2 - 1$

B. 2

C. $\frac{e^2 - 1}{2()}$

D. $\frac{e^2 - 1}{4}$

Answer: D

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111. Evaluate: $\int x^5 \sqrt{1 + x^3} dx$.

A. $\frac{1}{15}\left(1 + x^2\right)^{5/2} - \frac{1}{9}\left(1 + x\right)^3\Big)^{3/2} + xc$

B. $\frac{2}{15}(1+x^3)^{5/2} - \frac{1}{9}(1+x^3)^{3/2} + c$

C. $\frac{2}{15}(1+x^2)^{5/2} - \frac{2}{9}(1+x^3)^{3/2} + c$

D. $\frac{1}{15}(1+x^3)^{5/2} - \frac{2}{9}(1+x^3)^{3/2} + c$

Answer: C

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112. If $f(x)$=int_o^x sint/t dt` which of the following is true?

A. $f(0) > f(1.1)$

B. $f(0) < f(1.1) > f(2.1)$

C. $f(0) < f(1.1) < f(2.1) > f(3.1)$

D. $f(0) < f(1.1) < f(2.1) < f(3.1) > f(4.1)$

Answer: D

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113. Evalutate : $\int \frac{x^3 + 3x^2 + x + 9}{(x^2+1)(x^2+3)} dx.$

A. $\ln|x^2 + 3| + 3\tan^{-1} x + c$

B. $\frac{1}{2}\ln|x^2 + 3| + \tan^{-1} x + c$

C. $\frac{1}{2}\ln|x^2 + 3| + 3\tan^{-1} x + c$

D. $\ln|x^2 + 3| - \tan^{-1} x + c$

Answer: C

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114. $\int \frac{\sqrt{\sec^5 x}}{\sqrt{\sin^3 x}}$ dx equals to:

A. $(\tan x)^{3/2} - \sqrt{\tan x} + C$

B. $2\left(\frac{1}{3}(\tan x)^{3/2} - \frac{1}{\sqrt{\tan x}}\right) + C$

C. $\frac{1}{3}(\tan x)^{3/2} - \sqrt{\tan x} + C$

D. $\sqrt{\sin x} + \sqrt{\cos x} + C$

Answer: B

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115. $\lim_{x \to 0} \int_0^x \frac{e^{\sin(tx)}}{x} dt$ equals to :

A. 1

B. 2

C. e

D. does not exist

Answer: A

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116. If $A = \int_0^{\pi} \frac{\sin x}{x^2} dx$, then $\int_0^{\pi/2} \frac{\cos 2x}{x} dt$ is equal to:

A. $1 - A$

B. $\frac{3}{2} - A$

C. $A - 1$

D. $1 + A$

Answer: C

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Exercise One Or More Than One Answer Is Are Correct

1. $\int \frac{dx}{(1+\sqrt{x})^8} = -\frac{1}{3(1+\sqrt{x})^{k_2}} + \frac{2}{7(1+\sqrt{x})^{k_2}} + C,$ then:

A. $K_1 = 5$

B. $k_1 = 6$

C. $k_2 = 7$

D. $k_2 = 8$

Answer: B::C

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2. If $\int_{-\alpha}^{\alpha}\left(e^{x}+\cos x \ln\left(x+\sqrt{1+x^{2}}\right)\right)dx > \frac{3}{2}$, then posible value of α can be:

A. 1

B. 2

C. 3

D. 4

Answer: A::B::C::D

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3. If $\int\sqrt{\frac{x}{a^{3}-x^{3}}}dx=\frac{d}{b}\sin^{-1}\left(\frac{x^{\frac{3}{2}}}{a^{\frac{3}{2}}}\right)+C$ (where $b\&d$ are coprime integer) then $b+d$ equals to.

A. $A=\frac{2}{3}$

B. $B = a^{3/2}$

C. $A = \frac{1}{3}$

D. $B = a^{1/2}$

Answer: A::B

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4. Let $\int x \sin x \sec^3 x dx = \frac{1}{2}(x. f(x) - g(x)) + k$, then

A. $f(x) \notin (-1, 1)$

B. $g(x) = \sin x$ has 6 solution for $x \in [-\pi, 2\pi]$

C. $g'(x) = f(x), \forall x \in R$

D. $f(x) g g(x)$ has no solution

Answer: A::C::D

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5. If $\int(\sin 3\theta + \sin\theta)\cos\theta e^{\sin\theta} dth\eta =$ $\left(A\sin^3\theta + B\cos^2\theta + C\sin\theta + D\cos\theta + E\right)e^{\sin\theta} + F$, then

A. $A = -4$

B. $B = -12$

C. $C = -20$

D. None of these

Answer: A::B::C

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6. For $a > 0$, if $I = \int\sqrt{\frac{x}{a^3 - x^3}}dx = A\sin^{-1}\left(\frac{x^{3/2}}{B}\right) + C$, where C is any arbitary constant, then:

A. $A = \frac{2}{3}$

B. $B = a^{3/2}$

C. $A = \frac{1}{3}$

D. $B = a^{1/2}$

Answer: A::B

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7. If $f(\theta) = \lim_{x \to \infty} \sum_{r=o}^{n\theta} \frac{2r}{n\sqrt{(3\theta n - 2r)(n\theta + 2r)}}$ then:

A. $f(1) = \frac{\pi}{6}$

B. $f(\theta) = \frac{\theta}{2}\int_0^{\theta} \frac{dx}{\sqrt{\theta^2 - \left(x - \frac{\theta}{2}\right)^2}}$

C. $f(\theta)$ is constnat function

D. $y = f(\theta)$ is invertible

Answer: A::B::D

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8. If $f(x+y)=f(x)f(y)$ for all x,y and $f(0)\neq 0, \text{ and } F(x)=\dfrac{f(x)}{1+(f(x))^2}$ then:

A. $\int_{-2010}^{2011} F(x)dx = \int_0^{2011} F(x)dx$

B. $\int_{-2010}^{2011} F(x)dx = \int_0^{2011} F(x)dx = \int_0^{2011} F(x)dx$

C. $\int_{-2010}^{2011} F(x)dx = 0$

D. $\int_{-2010}^{2010} (2F(-x)-F(x))dx = e\int_0^{2010} F(x)dx$

Answer: B::D

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9. Let $J=\int_{-1}^{2}\left(\cot^{-1}\dfrac{1}{x}+\cot^{-1}x\right)dx,\ K=\int_{-2\pi}^{7\pi}\dfrac{\sin x}{|\sin x|}dx.$ Then which of the following alternative (s) is/are correct ?

A. $2J+3K=8\pi$

B. $4J^2+K^2=26\pi^2$

C. $2J = K = 3\pi$

D. $\dfrac{J}{K} = \dfrac{2}{5}$

Answer: A::B

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10. Which of the following function (s) is/are even ?

A. $f(x) = \int_0^x \ln\left(t + \sqrt{1+t^2}\right) dt$

B. $g(x) = \int_0^x \dfrac{(2^t + 1)t}{2^t - 1} dt$

C. $h(x) = \int_0^x \left(\sqrt{1+t+t^2} - \sqrt{1-t+t^2}\right) dt$

D. $l(x) = \int_0^x \ln\left(\dfrac{1-t}{1+t}\right) dt$

Answer: A::B::C::D

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11. Let $\lim_{x \to \infty} \sqrt{\frac{x - \cos^2 x}{x + \sin x}}$ and $I_2 = \lim_{h \to 0^+} \int_{-1}^{1} h \frac{dx}{h^2 + x^2}$. Then

A. Both l_1 and l_2 are less than 22/7

B. One of the two limits is rational and other irrational

C. $l_2 > l_1$

D. l_2 is greater than 3 times of l_1

Answer: A::B::C::D

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12. For $a > 0$, if $I = \int \sqrt{\frac{x}{a^3 - x^3}} dx = A \sin^{-1}\left(\frac{x^{3/2}}{B}\right) + C,$

where C is any arbitary constant, then :

A. $A = \frac{2}{3}$

B. $B = a^{3/2}$

C. $A = \frac{1}{3}$

D. $B = a^{1/2}$

Answer: A::B

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13. If $\int \frac{dx}{1-\sin^4 x} = a \tan x + b \tan^{-1}(c \tan x) + D,$ then:

A. $a = \frac{1}{2}$

B. $b = \sqrt{2}$

C. $c = \sqrt{2}$

D. $b = \frac{1}{2\sqrt{2}}$

Answer: A::C

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14. The value of definite integral: $\int_{-2014}^{2014} \frac{dx}{1+\sin^{2015} x+\sqrt{1+\sin^{4030} x}}$ equals :

A. 0

B. 2014

C. (2014)^(2)`

D. 4028

Answer: B

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15. Let $L = \lim_{n \to \infty} \int_a^{\infty} \frac{ndx}{1+n^2x^2}$, where $a \in R$, then L can be

A. π

B. $\frac{\pi}{2}$

C. 0

D. $\frac{\pi}{3}$

Answer: A::B::C

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16. Let $I=\int_0^1 \sqrt{\frac{1+\sqrt{x}}{1-\sqrt{x}}}dx$ and $J=\int_0^1 \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}dx$then correct statement (s) is/are:

A. $I+J=2$

B. $I-J=\pi$

C. $I=\frac{2+\pi}{2}$

D. $J=\frac{4-\pi}{2}$

Answer: B::C

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Exercise Comprehension Type Problems

1. Let $f(x) = \int x^2 \cos^2 x \left(2x + 6\tan x - 2x\tan^2 x\right) dx$ and $f(x)$ passes through the point $(\pi, 0)$

If $f: R - (2n+1)\frac{\pi}{2} \to R$ then $f(x)$ be a :

A. even function

B. odd function

C. neither even nor odd

D. even as well as odd both

Answer: A

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2. Let $f(x) = \int x^2 \cos^2 x \left(2x + 6\tan x - 2x\tan^2 x\right) dx$ and $f(x)$ passes through the point $(\pi, 0)$

The number of solution (s) of the equation $f(x) = x^3$ in $[0, 2\pi]$ be:

A. 0

B. 3

C. 2

D. None of these

Answer: B

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3. Let f (x) be a twice differentiable function defined on $(-\infty, \infty)$ such that $f(x) = f(2-x)$ and $f'\left(\frac{1}{2}\right) = f'\left(\frac{1}{4}\right) = 0$. Then The minimum number of values where $f''(x)$ vanishes on $[0, 2]$ is :

A. 2

B. 3

C. 4

D. 5

Answer: C

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4. Let f (x) be a twice differentiable function defined on $(-\infty, \infty)$ such that $f(x) = f(2-x)$ and $f'\left(\frac{1}{2}\right) = f'\left(\frac{1}{4}\right) = 0.$ Then $\int_{-1}^{1} f'(1+x)x^2 e^{x^2} dx$ is equal to :

A. 1

B. π

C. 2

D. 0

Answer: D

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5. Let f (x) be a twice differentiable function defined on $(-\infty, \infty)$ such that $f(x) = f(2-x)$ and $f'\left(\frac{1}{2}\right) = f'\left(\frac{1}{4}\right) = 0.$ Then $\int_0^1 f(1-t)e^{-\cos \pi t}dt - \int_1^2 f(2-t)e^{\cos \pi t}dt$ is equal to :

A. $\int_0^2 f'(t)e^{\cos \pi t}dt$

B. 1

C. 2

D. π

Answer: A

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6. Consider the function $f(x)$ and $g(x)$, both defined from $R \rightarrow R$

$f(x) = \frac{x^3}{2} + 1 - x\int_0^x g(t)dt$ and $g(x) = x - \int_0^1 f(t)dt,$ then minimum value of $f(x)$ is:

A. 0

B. 1

C. $\frac{3}{2}$

D. does not exist

Answer: B

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7. Consider the function $f(x)$ and $g(x)$, both defined from $R \rightarrow R$

$$f(x) = \frac{x^3}{2} + 1 - x\int_0^x g(t)dt \text{ and } g(x) = x - \int_0^1 f(t)dt, \text{ then}$$

The number of points of intersection of $f(x)$ and $g(x)$ is/are:

A. 0

B. 1

C. 2

D. 3

Answer: A

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8. Consider the function $f(x)$ and $g(x)$, both defined from $R \to R$

$f(x) = \frac{x^3}{2} + 1 - x\int_0^x g(t)dt$ and $g(x) = x - \int_0^1 f(t)dt$, then

The area borunded by $g(x)$ with co-ordinate axes is (in square units):

A. $\frac{9}{4}$

B. $\frac{9}{2}$

C. $\frac{9}{8}$

D. None of these

Answer: C

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9. Let $f(x)$ be function defined on $[0, 1]$ such that $f(1) = 0$ and for any $a \in (0, 1]$, $\int_0^a f(x)dx - \int_a^1 f(x)dx = 2f(a) + 3a + b$ where b is constant.

b=

A. $\frac{3}{2e}-3$

B. $\frac{3}{2e}-\frac{3}{2}$

C. $\frac{3}{2e}+3$

D. $\frac{3}{2e}+\frac{3}{2}$

Answer: A

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10. Let $f(x)$ be function defined on $[0, 1]$ such that $f(1)=0$ and for any $a \in (0, 1]$, $\int_0^a f(x)dx - \int_a^1 f(x)dx = 2f(a)+3a+b$ where b is constant.

The length of the subtangent of the curve $y=f(x)atx=1/2$ is:

A. $\sqrt{e}-1$

B. $\frac{\sqrt{e}-1}{2}$

C. $\sqrt{e}+1$

D. $\frac{\sqrt{e}+1}{2}$

Answer: A

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11. Let $f(x)$ be function defined on $[0, 1]$ such that $f(1)=0$ and for any $a \in (0, 1], \int_0^a f(x)dx - \int_a^1 f(x)dx = 2f(a) + 3a + b$ where b is constant.

$\int_0^1 f(x)dx =$

A. $\frac{1}{e}$

B. $\frac{1}{2e}$

C. $\frac{3}{2e}$

D. $\frac{2}{e}$

Answer: C

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12. Let $f_a(x) = Inx$ and for $n \geq 0$ and $x > 0$

Let $f_a(x) = \int_0^x f_a(t)dt$ then:

$f_a(x)$ equals :

A. $\frac{x^3}{3}\left(\ln x - \frac{5}{6}\right)$

B. $\frac{x^3}{3}\left(\ln x - \frac{11}{6}\right)$

C. $\frac{x^3}{\lfloor 3}\left(\ln x - \frac{11}{6}\right)$

D. $\frac{x^3}{\lfloor 3}\left(\ln x - \frac{5}{6}\right)$

Answer: C

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13. Let $f_a(x) = Inx$ and for $n \geq 0$ and $x > 0$

Let $f_a(x) = \int_0^x f_a(t)dt$ then:

Value of $\lim_{x \to \infty} \frac{(\lfloor n) f_n(1)}{\ln(n)}$:

A. 0

B. 1

C. -1

D. $-e$

Answer: C

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14. Let $f: R \rightarrow \left[\frac{3}{4}, \infty\right)$ be a surjective quadratic function with line of symmetry $2x - 1 = 0$ and $f(1) = 1$

If $g(x) = \frac{f(x) + f(-x)}{2} then \int \frac{dx}{\sqrt{g(e^x) - 2}}$ is equal to:

A. $\sec^{-1}(e^{-x}) + C$

B. $\sec^{-1}(e^x) + C$

C. $\sin^{-1}(e^{-x}) + C$

D. $\sin^{-1}(e^x) + C$

Answer: B

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15. Let $f: R \rightarrow \left[\frac{3}{4}, \infty\right)$ be a surjective quadratic function with line of symmetry $2x - 1 = 0$ and $f(1) = 1$

$$\int \frac{e^x}{f(e^x)} dx$$

A. $\cot^{-1}\left(\frac{2x^2 - 1}{sqet3}\right) + C$

B. $\frac{2}{\sqrt{3}} \cot^{-1}\left(\frac{2x^2 - 1}{sqet3}\right) + C$

C. $\tan^{-1}\left(\frac{2x^2 - 1}{sqet3}\right) + C$

D. $\frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2x^2 - 1}{sqet3}\right) + C$

Answer: D

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16. let $g(x) = x^c e^{cx}$ and $f(x) = \int_0^x te^{2t}(1+3t^2)^{\frac{1}{2}}dt.$ if $L = \lim_{x \to \infty} \frac{f'(x)}{g'(x)}$ is non-zero finit number then :

A. 7

B. $\frac{3}{2}$

C. 2

D. 3

Answer: C

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17. let $g(x) = x^c e^{cx}$ and $f(x) = \int_0^x te^{2t}(1+3t^2)^{\frac{1}{2}}dt.$ if $L = \lim_{x \to \infty} \frac{f'(x)}{g'(x)}$ is non-zero finit number then :

A. $\frac{2}{7}$

B. $\frac{1}{2}$

C. $\frac{\sqrt{3}}{4}$

D. $\frac{\sqrt{3}}{2}$

Answer: D

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Exercise Subjective Type Problems

1. $\int \frac{x+(\cos^{-1}3x)^2}{\sqrt{1-9x^2}}dx = \frac{1}{k_1}\left(\sqrt{1-9x^2}+(\cos^{-1}3x)^{k_2}\right)+c,$ then $k_1^2+k_2^2=$ (where C is an arbitrary constnat.)

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2. If $\int_0^\infty \frac{x^3}{(a^2+x^2)}dx = \frac{1}{ka^6},$ then find the value of $\frac{k}{8}$.

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3. Let $f(x) = x\cos x, x \in \left[\frac{3\pi}{2}, 2\pi\right]$ and $g(x)$ be its inverse. If $\int_0^{2\pi} g(x)dx = \alpha\pi^2 + \beta\pi + \gamma$, where α, β and $\gamma \in R$, then find the value of $2(\alpha + \beta + \gamma)$.

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4. If

$$\int \left(x^6 + x^4 + x^2\right)\sqrt{2x^4 + 3x^2 + 6dx} = \frac{\left(\alpha x^6 + \beta x^4 + \gamma x^2\right)^{3/2}}{18} + C$$

where C is constnat, then find the value of $(\beta + \gamma - \alpha)$.

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5. If the value of the definite integral $\int^{-1} \hat{} (1)\cos^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right).\left(\cot^{-1}\frac{x}{\sqrt{1-(x^2)^{|x|}}}\right)dx = \frac{\pi^2\left(\sqrt{a} - \sqrt{\right.}}{\sqrt{c}}$

where $a, b, c \in N$ in their lowest from, then find the value of $(a + b + c)$.

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6. The value of $\int \frac{\tan x}{\tan^2 x + \tan x + 1} dx = x - \frac{2}{\sqrt{A}} \tan^{-1}\left(\frac{2\tan x + 1}{\sqrt{A}}\right) + C.$ Then the value of A is:

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7. Let $\int_0^1 \frac{4x^3\left(1 + \left(x^4\right)^{2010}\right)}{\left(1 + x^4\right)^{2012}} dx = \frac{\lambda}{\mu}$

where λ and μ are relatively prime positive integers. Find unit digit of

`mu.

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8. Let $\int_1^{\sqrt{5}} \left(x^{2x^2+1} + \ln\left(x^{2x^{2x^2+1}}\right)\right) dx = N.$ Find the valur of $(N - 6)$.

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9. If $\int \frac{dx}{\cos^3 x - \sin^3} = A \tan^{-1}(f(x)) + b \ln\left|\frac{\sqrt{2} + f(x)}{\sqrt{2} - f(x)}\right| + C$ where $f(x) = \sin x + \cos x$ find the value of $\left(12A + 9\sqrt{2}V\right) - 3.$

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10. Find the value of |a| for which the area of triangle included between the coordinate axes and any tangent to the curve $xy^a = \lambda^{a+1}$ is constant (where λ is constant.)

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11. Let $I = \int_0^\pi x^6 (\pi - x)^8 dx,$ then $\frac{\pi^{15}}{({}^{15}C_9)I} =$

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12. If maximum value of $\int_0^1 (f(x))^2$ dx under the condition $-1 \leq f(x) \leq 1, \int_0^1 f(x) dx - 0$ is $\frac{p}{q}$ (where p and q are relatively prime positive integers). Find $p+q$.

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13. Let a differentiable function f (x) satisfies $f(x).\,F'(-x).\,F'(x)$ and $f(0)-1.$ Find the value of $\int^{-2}\frac{dx}{1+f(x)}.$

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14. If {x} denotes the fractional part of x, then $I=\int_0^{100}(\sqrt{x})dx,$ then the value of $\frac{9f}{155}$ is:

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15. Let $I_n=\int_0^{\pi}\frac{\sin\left(n+\frac{1}{2}\right)x}{\sin\left(\frac{x}{2}\right)}dx$ where $n \in W.\,If l_1^2+l_2^2+l_3^2+\,\ldots\ldots\,+l_{20}^2=m\pi^2,$ then find the largest prime factor of m.

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16. IF M be the maximum valur of $72\int_0^y \sqrt{x^4 + (y - y^2)^2}\,dx$ for $y \in [0, 1]$, then find `M/6.

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17. Find the number points where $f(\theta) = \int_{-1}^{1} \frac{\sin\theta dx}{2x\cos\theta + x^2}$ is discontinous where $\theta \in [0, 2\pi]$.

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18. $\lim_{n \to \infty}\left[\frac{1}{\sqrt{n}} + \frac{1}{\sqrt{2n}} + \frac{1}{\sqrt{3n}} + \dots + \frac{1}{n}\right]$

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19. The maximum value of $\int_{-\pi/2}^{2\pi/2} \sin x.\ f(x)dx,$ subject to the condition $|f(x) \leq 5$ is M, then `M/10is equal to :

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20. Given a funtion g, continous everywhere such that $g(1) = 5$ and $\int_0^1 g(t)dt = 2.$ If $f(x) = \frac{1}{2}\int_0^x (x-t)^2 g(t)dt,$ then find the value of $f'''(1) + f''(1).$

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21. If $f(n) = \frac{1}{\pi}\int_0^{\pi/2} \frac{\sin^2(n\theta)d\theta}{\sin^2\theta}, n \in N,$ then evulate $\frac{f(15) + f(3)}{f(12) - f(10)}.$

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22. Let $f(2-x) = f(2+x$ and $f(4-x) = f(4+x)$. Function f (x) satisfies $\int_0^2 f(x)dx = 5$. if $\int_0^{50} f(x)dx = l$. Find $\left[\sqrt{l} - 2\right]$. (where [.] denotes greatest integer function.)

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23. Let $l_n = \int_{-1}^{1} |x| \left(1 + x + \frac{x^2}{2} + \frac{x^2}{3} + \dots . + \frac{x^{2n}}{2n}\right) dx$ if $\lim_{x \to \infty} l_n$ can be expressed as rational $\frac{p}{q}$ in this lowest form, then find the value of $\frac{pq(p+q)}{10}$

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24. Let $\lim_{x \to \infty} n^{\frac{1}{2}\left(1+\frac{1}{n}\right)} . \left(1^1 . 2^2 . 3^3 \dots . n^n\right)^{\frac{1}{n^2}} = e^{\frac{-p}{q}}$

where p and q are relative prime positive integers. Find the vlaur of $|p+q|$.

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25. If $\int_a^b |\sin x| dx = 8$ and $\int_0^{a+b} |\cos x| dx = 9$ then the value of $\frac{1}{\sqrt{2}\pi}\left|\int_a^b x \sin x dx\right|$ is:

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26. If $f(x), g(x), h(x)$ and $\phi(x)$ are polynomial in x, $\left(\int_1^x f(x)h(x)dx\right)\left(\int_1^x g(x)\phi(x)dx\right) - \left(\int_1^x g(x)h(x)dx\right)$ is divisible by $(x-1)^2$. Find maximum value of λ.

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27. If $\int_0^2 (3x^2 - 3x + 1)\cos\left(x^2 - 3x^2 + 4x - 2\right)dx = a\sin(b)$, where a and b are positive integers. Find the value of $\frac{a+b}{2}$.

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28. let $f(x) = \int_0^x e^{x-y} f'(y) dy - (x^2 - x + 1) e^x$

Find the number of roots of the equation $f(x) = 0$.

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29. For a positive integer n, let $I_n = \int_{-\pi}^{\pi} \left(\frac{\pi}{2} - |x|\right) \cos nx dx$

Find the value of $[I_1 + I_3 + I_4]$ (where [.] denotes greatest integer function) .

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