MATHS

BOOKS - VIKAS GUPTA MATHS (HINGLISH)

INVERSE TRIGONOMETRIC FUNTIONS

Exercise 1 Single Choice Problems

1. If $\sin^{-1}x \in \left(0, \frac{\pi}{2}\right)$, then the value of

$\tan\left(\frac{\cos^{-1}\left(\sin\left(\cos^{-1}x\right)\right) + \sin^{-1}\left(\cos\left(\sin^{-1}x\right)\right)}{2}\right)$ is :

A. 1

B. 2

C. 3

D. 4

Answer: A

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2. The solution set of inequality $\left(\cot^{-1}x\right)\left(\tan^{-1}x\right)+\left(2-\frac{\pi}{2}\right)\cot^{-1}x-3\tan^{-1}x-3\left(2-\frac{\pi}{2}\right)>0,$ is

A. $x\in(\tan 2, \tan 3)$

B. $x\in(\cot 3, \cot 2)$

C. $x\in(-\infty, \tan 2)\cup(\tan 3, \infty)$

D. $x\in(-\infty, \cot 3)\cup(\cot 2, \infty)$

Answer: B

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3. $\sec^2\left(\tan^{-1}2\right)+\cos ec^2\left(\cot^{-1}3\right)$ is equal to

A. 14

B. 15

C. 16

D. 17

Answer: B

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4. Sum the series : $\tan^{-1}\left(\frac{4}{1+3.4}\right)+\tan^{-1}\left(\frac{6}{1+8.9}\right)+\tan^{-1}\left(\frac{8}{1+15.16}\right)+\ldots\ldots\ldots\ldots\infty$ is :

A. $\cot^{-1}(2)$

B. $\tan^{-1}(2)$

C. $\frac{\pi}{2}$

D. $\frac{\pi}{4}$

Answer: A

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5. If $\cot^{-1}(\sqrt{\cos\alpha}) - \tan^{-1}(\sqrt{\cos\alpha}) = x$, then $\sin x$ is $\frac{\tan^2\alpha}{2}$ (b) $\frac{\cot^2\alpha}{2}$ (c) $\tan^2\alpha$ (d) $\frac{\cot\alpha}{2}$

A. $\tan^2\left(\frac{\alpha}{2}\right)$

B. $\cot^2\left(\frac{\alpha}{2}\right)$

C. $\tan\alpha$

D. $\cot\left(\frac{\alpha}{2}\right)$

Answer: A

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6. The sum of the infinite series $\cot^{-1}\left(\frac{7}{4}\right) + \cot^{-1}\left(\frac{19}{4}\right) + \cot^{-1}\left(\frac{39}{4}\right) \dots . \infty$

A. $\frac{\pi}{4}-\cot^{-1}(3)$

B. $\frac{\pi}{4}-\tan^{-1}(3)$

C. $\frac{\pi}{4}+\cot^{-1}(3)$

D. $\frac{\pi}{4}+\tan^{-1}(3)$

Answer: C

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7. the number of solutions of $\cos^{-1}(1-x)+m\cos^{-1}x=\frac{n\pi}{2}$ where $m>0, n\leq 0$

A. 0

B. 1

C. 2

D. none of these

Answer: A

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8. Number of solution(s) of the equation $2\tan^{-1}(2x - 1) = \cos^{-1}(x)$ is

:

A. 1

B. 2

C. 3

D. infinitely many

Answer: A

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9. $\sin^{-1}\left(\frac{x^2}{4} + \frac{y^2}{9}\right) + \cos^{-1}\left(\frac{x}{2\sqrt{2}} + \frac{y}{3\sqrt{2}} - 2\right)$

A. $\frac{\pi}{2}$

B. π

C. $\dfrac{\pi}{\sqrt{2}}$

D. $\dfrac{3\pi}{2}$

Answer: D

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10. Solution set of inequation

$\left(\cos^{-1}x\right)^2 - \left(\sin^{-1}x\right)^2 > 0$ is

A. $\left[0, \dfrac{1}{\sqrt{2}}\right)$

B. $\left[-1, \dfrac{1}{\sqrt{2}}\right)$

C. $(-1, 1)$

D. $\left[-1, \dfrac{1}{2}\right)$

Answer: B

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11. Let α, β are the roots of the equation $x^2 + 7x + k(k-3) = 0$, where $k \in (0, 3)$ and k is a constant. Then the value of $\tan^{-1}\alpha + \tan^{-1}\beta + \tan^{-1}\frac{1}{\alpha} + \tan^{-1}\frac{1}{\beta}$ is :

A. π

B. $\frac{\pi}{2}$

C. 0

D. $-\frac{\pi}{2}$

Answer: C

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12. Let $f(x) = a + 2b\cos^{-1}x, b > 0$. If domain and range of f(x) are the same set, then $(b - a)$ is equal to :

A. $1 - \frac{1}{\pi}$

B. $\frac{2}{\pi}$

C. $\frac{2}{\pi}+1$

D. $1+\frac{1}{\pi}$

Answer: D

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13. If $\left(\tan^{-1}x\right)^2+\left(\cot^{-1}x\right)^2=\frac{5\pi^2}{8}$ then x equals

A. -1

B. 4

C. 5

D. 6

Answer: A

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14. The total number of ordered pairs (x, y) satisfying $|y| = \cos x$ and $y = \sin^{-1}(\sin x)$, where $x \in [-2\pi, 3\pi]$ is equal to :

A. 2

B. 4

C. 5

D. 6

Answer: C

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15. If $\left[\sin^{-1}\left(\cos^{-1}\left(\sin^{-1}\left(\tan^{-1} x\right)\right)\right)\right] = 1$ where [.] denotes integer function, then complete set of values of x is :

A. $[\tan(\sin(\cos 1)), \tan(\cos(\sin 1))]$

B. $[\tan(\sin(\cos 1)), \tan(\sin(\cos(\sin 1)))]$

C. $[\tan(\cos(\sin 1)), \tan(\sin(\cos(\sin 1)))]$

D. $[\tan(\sin(\cos 1)), 1]$

Answer: B

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16. The number of ordered pair(s) (x, y) of real numbers satisfying the equation $1 + x^2 + 2x\sin(\cos^{-1} y) = 0$, is :

A. 0

B. 1

C. 2

D. 3

Answer: B

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17. The value of $\tan^{-1}1 + \tan^{-1}2 + \tan^{-1}3$ is

A. $\frac{\pi}{2}$

B. π

C. $\frac{3\pi}{4}$

D. $\frac{5\pi}{8}$

Answer: B

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18. The complete set of values of x for which $2\tan^{-1}x + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ is independent of x is :

A. $(-\infty, 0]$

B. $[0, \infty)$

C. $(-\infty, -1]$

D. $[1, \infty)$

Answer: A

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19. The number of ordered pair(s) (x,y) which satisfy $y = \tan^{-1} \tan x$ and $16(x^2 + y^2) - 48\pi x + 16\pi y + 31\pi^2 = 0$ is

A. 0

B. 1

C. 2

D. 3

Answer: D

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20. Domain (D) and range (R) of $f(x) = \sin^{-1}(\cos^{-1}[x])$ where [ ] denotes the greatest integer function is

A. $D \equiv [1, 2), R \equiv \{0\}$

B. $D \equiv [0, 1), R \equiv \{-1, 0, 1\}$

C. $D \equiv [-1, 1), R \equiv \left\{0, \frac{\pi}{2}, \pi\right\}$

D. $D \equiv [-1, 1], R \equiv \left\{-\frac{\pi}{2}, 0, \frac{\pi}{2}\right\}$

Answer: A

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21. If $2\sin^{-1}x + \{\cos^{-1}x\} > \frac{\pi}{2} + \{\sin^{-1}x\}$, then $x \in$: (where $\{\cdot\}$ denotes fractional part function)

A. (cos 1, 1]

B. [sin 1, 1]

C. (sin 1, 1]

D. none of these

Answer: B

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22. If $f(x) = x^{11} + x^9 - x^7 + x^3 + 1$ and $f\left(\sin^{-1}(\sin 8)\right) = \alpha, \alpha$ is constant, then $f\left(\tan^{-1}(\tan 8)\right)$ is equal to α (b) $2 - \alpha$ (c) $\alpha - 2$ (d) $\alpha + 2$

A. 2

B. 3

C. 4

D. 1

Answer: A

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23. The number of real values of x satisfying the equation $3\sin^{-1}x + \pi x - \pi = 0$ is/are :

A. 0

B. 1

C. 2

D. -3

Answer: B

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24. Range of $f(x) = \sin^{-1}x + x^2 + 4x + 1$ is :

A. $\left[-\frac{\pi}{2} - 2, \frac{\pi}{2} + 6\right]$

B. $\left[0, \frac{\pi}{2} + 6\right]$

C. $\left[-\frac{\pi}{2} - 2, \infty\right)$

D. $(-3, \infty)$

Answer: A

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25. The solution set of the ineuality $\left(cosec^{-1}x\right)^2 - 2cosec^{-1}x \geq \frac{\pi}{6}\left(cosec^{-1}x - 2\right)$ is $(-\infty, a] \cup [b, \infty)$, then $(a+b)$ equals

A. 0

B. 1

C. 2

D. -3

Answer: B

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26. Number of solution of the equation $2\sin^{-1}(x+2)=\cos^{-1}(x+3)$ is :

A. 0

B. 1

C. 2

D. None of these

Answer: B

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27. Find the sum series:

$$\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{7}\right)+\tan^{-1}\left(\frac{1}{13}\right)+\ldots\to\infty$$

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{6}$

Answer: A

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28. If $\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{2}{9}=\frac{1}{2}\cos^{-1}x$ then x is equal to

A. $\frac{1}{2}$

B. $\frac{2}{5}$

C. $\frac{3}{5}$

D. none of these

Answer: C

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29. The set of value of x, satisfying the equation $\tan^2(\sin^{-1}x)>1$ is :

A. $(-1, 1)$

B. $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

C. $[-1, 1] - \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

D. $(-1, 1) - \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$

Answer: D

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30. The sum of the series $\cot^{-1}\left(\frac{9}{2}\right) + \cot^{-1}\left(\frac{33}{4}\right) + \cot^{-1}\left(\frac{129}{8}\right) + \ldots\ldots\ldots \infty$ is equal to :

A. $\cot^{-1}(2)$

B. $\cot^{-1}(3)$

C. $\cot^{-1}(-1)$

D. $\cot^{-1}(1)$

Answer: A

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31. If $\int \frac{\text{In}(\cot x)}{\sin x \cos x} dx = \frac{-1}{k} \text{In}^2(\cot x) + C$

(where C is a constant), then the value of k is :

A. 1

B. 2

C. 3

D. $\frac{1}{2}$

Answer: B

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32. The number of solutions of $\sin^{-1} x + \sin^{-1}(1 + x) = \cos^{-1} x$ is/are

:

A. 0

B. 1

C. 2

D. infinite

Answer: B

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33. The value of x satisfying the equation

$$\left(\sin^{-1}x\right)^3-\left(\cos^{-1}x\right)^3+\left(\sin^{-1}x\right)\left(\cos^{-1}x\right)\left(\sin^{-1}x-\cos^{-1}x\right)=\frac{\pi^3}{16}$$

is :

A. $\cos\frac{\pi}{5}$

B. $\cos\frac{\pi}{4}$

C. $\cos\frac{\pi}{8}$

D. $\cos\frac{\pi}{12}$

Answer: C

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34. The complete solution set of the equation

$$\sin^{-1}\sqrt{\frac{1+x}{2}} - \sqrt{2-x} = \cot^{-1}\left(\tan\sqrt{2-x}\right) - \sin^{-1}\sqrt{\frac{1-x}{2}}$$ is :

A. $\left[2 - \frac{\pi^2}{4}, 1\right]$

B. $\left[1 - \frac{\pi^2}{4}, 1\right]$

C. $\left[2 - \frac{\pi^2}{4}, 0\right]$

D. $[-1, 1]$

Answer: A

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35. Let $f(x) = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ then which of the following is correct :

A. f(x) has only one integer in its range

B. Range of f(x) is $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right) - \{0\}$

C. Range of f(x) is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) - \{0\}$

D. Range of f(x) is $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right] - \{0\}$

Answer: B

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36. If $\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9} = \frac{1}{2}\cos^{-1}x$ then x is equal to

A. $\frac{1}{2}$

B. $\frac{2}{5}$

C. $\frac{3}{5}$

D. None of these

Answer: C

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37. The set of value of x, satisfying the equation $\tan^2(\sin^{-1}x) > 1$ is :

A. (-1, 1)

B. $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

C. $[-1, 1] - \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

D. $(-1, 1) - \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$

Answer: D

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38. The sum of the series $\cot^{-1}\left(\frac{9}{2}\right) + \cot^{-1}\left(\frac{33}{4}\right) + \cot^{-1}\left(\frac{129}{8}\right) + \ldots\ldots\infty$ is equal to :

A. $\cot^{-1}(2)$

B. $\cot^{-1}(3)$

C. $\cot^{-1}(-1)$

D. $\cot^{-1}(1)$

Answer: A

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39. The number of real values of x satisfying $\tan^{-1}\left(\frac{x}{1-x^2}\right)+\tan^{-1}\left(\frac{1}{x^3}\right)=\frac{3\pi}{4}$ is :

A. 0

B. 1

C. 2

D. infinitely many

Answer: B

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40. Number of integral values of λ such that the equation $\cos^{-1}x + \cot^{-1}x = \lambda$ possesses solution is :

A. 2

B. 8

C. 5

D. 10

Answer: C

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41. If the equation `x^3+b x^2+c x+1=0,(b

A. $-\frac{\pi}{2}$

B. $-\pi$

C. $\frac{\pi}{2}$

D. π

Answer: B

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42. Range of the function $f(x) = \cot^{-1}\{-x\} + \sin^{-1}\{x\} + \cos^{-1}\{x\}$, where $\{\cdot\}$ denotes fractional part function

A. $\left(\frac{3\pi}{4}, \pi\right)$

B. $\left[\frac{3\pi}{4}, \pi\right)$

C. $\left[\frac{3\pi}{4}, \pi\right]$

D. $\left(\frac{3\pi}{4}, \pi\right]$

Answer: D

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43. If $3 \leq a < 4$ then the value of $\sin^{-1}(\sin[a]) + \tan^{-1}(\tan[a]) + \sec^{-1}(\sec[a])$, where [x] denotes greatest integer function less than or equal to x, is equal to :

A. 3

B. $2\pi - 9$

C. $2\pi - 3$

D. $9 - 2\pi$

Answer: A

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44. The number of real solutions of $y + y^2 = \sin x$ and $y + y^3 = \cos^{-1}(\cos x)$ is/are

A. 0

B. 1

C. 3

D. Infinite

Answer: D

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45. Range of $f(x) = \sin^{-1}[x-1] + 2\cos^{-1}[x-2]$ ([.] denotes greatest integer function)

A. $\left\{-\frac{\pi}{2}, 0\right\}$

B. $\left\{\frac{\pi}{2}, 2\pi\right\}$

C. $\left\{\frac{\pi}{4}, \frac{\pi}{2}\right\}$

D. $\left\{\frac{3\pi}{2}, 2\pi\right\}$

Answer: D

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Exercise 2 One Or More Than One Answer Is Are Correct

1. $f(x) = \sin^{-1}(\sin x), g(x) = \cos^{-1}(\cos x)$, then :

A. $f(x) = g(x) \text{ if } x \in \left(0, \frac{\pi}{4}\right)$

B. $f(x) < g(x) \text{ if } x \in \left(\frac{\pi}{2}, \frac{3\pi}{4}\right)$

C. $f(x) < g(x) \text{ if } \left(\pi, \frac{5\pi}{4}\right)$

D. $f(x) > g(x) \text{ if } x \in \left(\pi, \frac{5\pi}{4}\right)$

Answer: A::B::C

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2. The solution(s) of the equation $\cos^{-1} x = \tan^{-1} x$ satisfy

A. $x^2 = \frac{\sqrt{5} - 1}{2}$

B. $x^2 = \frac{\sqrt{5} + 1}{2}$

C. $\sin(\cos^{-1} x) = \frac{\sqrt{5} - 1}{2}$

D. $\tan(\cos^{-1} x) = \frac{\sqrt{5} - 1}{2}$

Answer: A::C

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3. If the numerical value of $\tan\left(\cos^{-1}\left(\frac{4}{5}\right)+\tan^{-1}\left(\frac{2}{3}\right)\right)$ is $\left(\frac{a}{b}\right)$, where a, b are two positive integers and their H.C.F. is 1

A. $a+b=23$

B. $a-b=11$

C. $3b=a+1$

D. $2a=3b$

Answer: A::B::C

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4. A solution of the equation $\cot^{-1}2=\cot^{-1}x+\cot^{-1}(10-x)$ where $1 < x < 9$ is :

A. 7

B. 3

C. 2

D. 5

Answer: A::B

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5. Consider the equation $\sin^{-1}\left(x^2 - 6x + \frac{17}{2}\right) + \cos^{-1} k = \frac{\pi}{2}$, then :

A. the largest value of k for which equation has 2 distinct solution is 1

B. the equation must have real root if $k \in \left(-\frac{1}{2}, 1\right)$

C. the equation must have real root if $k \in \left(-1, \frac{1}{2}\right)$

D. the equation has unique solution if $k = -\frac{1}{2}$

Answer: A::B::D

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6. The value of x satisfying the equation

$$\left(\sin^{-1}x\right)^3 - \left(\cos^{-1}x\right)^3 + \left(\sin^{-1}x\right)\left(\cos^{-1}x\right)\left(\sin^{-1}x - \cos^{-1}x\right) = \frac{\pi^3}{16}$$

can not be equal to :

A. $\cos\frac{\pi}{5}$

B. $\cos\frac{\pi}{4}$

C. $\cos\frac{\pi}{8}$

D. $\cos\frac{\pi}{12}$

Answer: A::B::D

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Exercise 3 Comprehension Type Problems

1. Let $\cos^{-1}(4x^3 - 3x) = a + b\cos^{-1}x$

Q. If $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$, then $\sin^{-1}\left(\sin\frac{a}{b}\right)$ is :

A. $-\frac{\pi}{3}$

B. $\frac{\pi}{3}$

C. $-\frac{\pi}{6}$

D. $\frac{\pi}{6}$

Answer: A

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2. Let $\cos^{-1}(4x^3 - 3x) = a + b\cos^{-1}x$

If $x \in \left(\frac{1}{2}, 1\right]$, then the value of $\lim_{y \to a} b\cos(y)$ is

A. $-\frac{1}{3}$

B. -3

C. $\frac{1}{3}$

D. 3

Answer: D

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Exercise 5 Subjective Type Problems

1. The complete set of values of x satisfying the inequality $\sin^{-1}(\sin 5) > x^2 - 4x$ is $\left(2 - \sqrt{\lambda - 2\pi}, 2 + \sqrt{\lambda - 2\pi}\right)$, then $\lambda =$

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2. In a ΔABC, if $(II_1)^2 + (I_2I_3)^2 = \lambda R^2$, where I denotes incentre, I_1, I_2 and I_3 denote centres of the circles escribed to the sides BC, CA and AB respectively and R be the radius of the circum circle of ΔABC. Find λ.

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3. If $2\tan^{-1}\frac{1}{5} - \sin^{-1}\frac{3}{5} = -\cos^{-1}\frac{63}{\lambda}$, then $\lambda =$

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4. If $2\tan^{-1}\frac{1}{5} - \sin^{-1}\frac{3}{5} = -\cos^{-1}\frac{9\lambda}{65}$, then $\lambda =$

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5. If $\sum_{n=0}^{\infty} 2\cot^{-1}\left(\frac{n^2+n+4}{2}\right) = k\pi$ then find the value of k

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6. Find number of solutions of the equation $\sin^{-1}\left(\left|\log_6^2(\cos x) - 1\right|\right) + \cos^{-1}\left(\left|3\log_6^2(\cos x) - 7\right|\right) = \frac{\pi}{2}$, if $x \in [0, 4\pi]$.

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