MATHS

BOOKS - VIKAS GUPTA MATHS (HINGLISH)

LIMIT

Exercise Single Choice Problems

1. $\lim_{x \to 0} \frac{\cos(\tan x) - \cos x}{x^4} =$

A. $\frac{1}{6}$

B. $-\frac{1}{3}$

C. $-\frac{1}{6}$

D. $\frac{1}{3}$

Answer: B

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2. The value of $\lim_{x \to 0} \frac{(\sin x - \tan x)^2 - (1 - \cos 2x)^4 + x^5}{7(\tan^{-1} x)^7 + (\sin^{-1} x)^6 + 3 \sin^5 x}$ equal to :

A. 0

B. 1

C. 2

D. $\frac{1}{3}$

Answer: D

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3. Let $a = \lim_{x \to 0} \frac{\ln(\cos 2x)}{3x^2}, b = \lim_{x \to 0} \frac{\sin^2 2x}{x(1 - e^x)}, c = \lim_{x \to 1} \frac{\sqrt{x} - x}{\ln x}$

A. $a < b < c$

B. $b < c < a$

C. $a < c < b$

D. $b < a < c$

Answer: D

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4. If $f(x) = \cot^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)$ and $g(x) = \cos^{-1}\left(\frac{1 - x^2}{1 + x^2}\right)$, then $\lim_{x \to a} \frac{f(x) - f(a)}{g(x) - g(a)}, 0 < a < \frac{1}{2}$ is

A. $\frac{3}{2(1 + a^2)}$

B. $\frac{3}{2}$

C. $\frac{-3}{2(1 + a^2)}$

D. $-\frac{3}{2}$

Answer: D

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5. $\lim_{x \to 0} \left(\frac{(1+x)^{\frac{2}{x}}}{e^2} \right)^{\frac{4}{\sin x}}$ is :

A. e^4

B. e^{-4}

C. e^8

D. e^{-8}

Answer: B

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6. $\lim_{x \to \infty} \frac{3}{x}\left[\frac{x}{4}\right] = \frac{p}{q}$ where [.] denotes greatest integer function), then $p+q$ (where p,q are relative prime) is:

A. 2

B. 7

C. 5

D. 6

Answer: B

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7. f(x) = $\dfrac{x^n + \left(\frac{\pi}{3}\right)^n}{x^{n-1} + \left(\frac{\pi}{3}\right)^{n-1}}$,(n is an even number, then which of the following is correct

A. If $f, \left[\frac{\pi}{3}, \infty\right) \rightarrow \left[\frac{\pi}{3}, \infty\right),$ then function is inbvertible

B. $f(x) = f(-x)|$ has infinite number of solutions

C. $f(x) = |f(x)|$ has infinite nouber of solution

D. $f(x)$ is one-one function for all $x \in R$

Answer: D

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8. $\lim_{x \to 0} \frac{\sin(\pi \cos^2(\tan(\sin x)))}{x^2} =$

A. π

B. $\frac{\pi}{4}$

C. $\frac{\pi}{2}$

D. none of these

Answer: A

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9. if $f(x) = \frac{\left(e^{(x+3)\ln 27}\right)^{\frac{x}{27}} - 9}{3^x - 27}, x < 3$ and $f(x) = \lambda \frac{1 - \cos(x-3)}{(x-3)\tan(x-3)}$ if $\lim_{x \to 3} f(x)$ exist then $lmbda$ is

A. $\frac{9}{2}$

B. $\frac{2}{9}$

C. $\frac{2}{3}$

D. none of these

Answer: C

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10. $\lim_{x \to \frac{\pi}{3}} \frac{\sin\left(\frac{\pi}{3} - x\right)}{2\cos x - 1}$ is equal to:

A. $\frac{2}{\sqrt{3}}$

B. $\frac{1}{\sqrt{3}}$

C. $\sqrt{3}$

D. $\frac{1}{2}$

Answer: B

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11. $\lim_{x \to \frac{\pi}{2}} \frac{\sin x}{co^{-1}\left[\frac{1}{4}(3\sin x - \sin 3x)\right]}$ where [] denotes greatest integer function id:

A. $\frac{2}{\pi}$

B. 1

C. $\frac{4}{\pi}$

D. does not exist

Answer: A

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12. Let f be a continuous function on R such that $f\left(\frac{1}{4n}\right) = \frac{\sin e^n}{e^{n^2}} + \frac{n^2}{n^2+1}$ Then the value of $f(0)$ is

A. 1

B. 0

C. -1

D. $\frac{1}{6}$

Answer: A

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13. $\lim_{x \to 1^-} \frac{e^{\{x\}} - \{x\} - 1}{\{x\}^2}$ equal, where {.} is fractional part function and I is aan integer, to :

A. $\frac{I}{2}$

B. $e - 2$

C. I

D. does not exist

Answer: B

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14. $\lim_{x \to \infty} \left(e^{11x} - 7x\right)^{\frac{1}{3x}}$

A. $\frac{11}{3}$

B. $\frac{3}{11}$

C. $e^{\frac{3}{11}}$

D. $e^{\frac{11}{3}}$

Answer: D

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15. The value of $\lim_{x \to 0}\left[(1-2x)^{n}\sum_{r=0}^{n}{}^{-}r\left(\frac{x+x^{2}}{1-2x}\right)^{r}\right]^{\frac{1}{x}}$ is :

A. e^{n}

B. e^{-n}

C. e^{3n}

D. e^{-3n}

Answer: B

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16. For a certain value of 'c' $\lim_{x \to \infty} \left[\left(x^5 + 7x^4 + 2 \right)^c - x \right]$ is finite and non-zero. Then the value of limit is :

A. $\frac{7}{5}$

B. 1

C. $\frac{2}{5}$

D. None of these

Answer: A

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17. The integer n for which $\lim_{x \to 0} \frac{(\cos x - 1)(\cos x - e^x)}{x^n}$ is a finite non-zero number is :

A. 1

B. 2

C. 3

D. 4

Answer: C

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18. The vlaue of $\lim_{x \to 0} \left(\frac{\sin x}{x}\right)^{\frac{1}{1 - \cos x}}$:

A. $e^{-1/3}$

B. $e^{1/3}$

C. $e^{-1/=6}$

D. $e^{1/6}$

Answer: A

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19. If $\lim_{x \to \infty} \left(\sqrt{x^2 - x + 1} - ax - b\right) = 0$, then for $k \geq 2, (k \in N) \lim_{x \to \infty} \sec^{2n}(k!\pi b) =$

A. a

B. $-a$

C. 2a

D. b

Answer: A

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20. If f is a positive function such that $f(x+T) = f(x)(T > 0), \forall x \in R,$ then $\lim_{n \to \infty} n\left(\frac{f(x+T) + 2f(x+2T) + \ldots\ldots + nf(x+nT)}{f(x+T) + 4f(x+4T) + \ldots\ldots + n^2 f(x+n^2 T)}\right) =$

A. 2

B. $\frac{2}{3}$

C. $\frac{3}{2}$

D. none of these

Answer: C

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21. Let $f(x)=3x^{10}-7x^8+5x^6-21x^3+3x^2-7$

$$265\left(\lim_{h\to o}\frac{h^4+3h^2}{(f(1-h)-f(1))\sin 5h}\right)=$$

A. 1

B. 2

C. 3

D. -3

Answer: C

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22. $\lim_{x \to 0}\left(\frac{\cos x - \sec x}{x^2(x+1)}\right) =$

A. 0

B. $-\frac{1}{2}$

C. -1

D. -2

Answer: C

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23. Let $f(x)$ be a continuous and differentiable function satisfying $f(x+y) = f(x)f(y)\,\forall x, y \in R$ if $f(x)$ an be expressed as $f(x) = 1 + xP(x) + x^2Q(x)$ where $\lim_{x \to 0} P(x) = a$ and $\lim_{x \to 0} Q(x) = b$, then $f'(x)$ is equal to :

A. $af(x)$

B. $bf(x)$

C. $(a+b)f(x)$

D. $(a+2b)f(x)$

Answer: A

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24. The value of $\lim_{x+\frac{\pi}{2}} \frac{\left\{1-\frac{\tan(x)}{2}\right\}\{1-\sin x\}}{\left\{1+\frac{\tan(X)}{2}\right\}(\pi-2x)^3}$ equals

A. not exist

B. $\frac{1}{8}$

C. $\frac{1}{16}$

D. $\frac{1}{32}$

Answer: D

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25. $\lim_{x \to \infty} \left(\frac{x-3}{x+2}\right)^x$ is equal to

A. e

B. e^{-1}

C. e^{-5}

D. e^5

Answer: C

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26. $\lim_{x \to \frac{\pi}{2}} (\cos x)^{\cos x}$ is :

A. 1

B. 0

C. $\frac{1}{e}$

D. $\frac{2}{e}$

Answer: A

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27. If $\lim_{x \to c^-} \{\ln x\}$ and $\lim_{x \to c^+} \{\ln x\}$ exist finitely but they are not equal (where {.} denotes fractional part function), then:

A. c' can take only rational values

B. c' can take only irrational values

C. c' can take infinite values in which only one is irrational

D. c' can take infinite values I which only one is rational

Answer: D

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28. $\lim_{x \to 0} \left(1 + \frac{a \sin bx}{\cos x}\right)^{\frac{1}{x}}$, where a, b are non zero constants is equal to :

A. $e^{a/b}$

B. ab

C. e^{ab}

D. $e^{b/e}$

Answer: C

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29. the value of $\lim_{x \to 0} \left\{ (\cos x)^{\frac{1}{\sin^2 x}} + \frac{\sin 2x + 2\tan^{-13} x + 3x^2}{\ln(1 + 3x + \sin^2 x) + xe^x} \right\}$

A. $\sqrt{e} + \frac{3}{2}$

B. $\frac{1}{\sqrt{e}} + \frac{3}{2}$

C. $\sqrt{e} + 2$

D. $\frac{1}{\sqrt{e}} + 2$

Answer: D

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30. let

$a = \lim_{x \to 1}\left(\frac{x}{\ln x} - \frac{1}{x \ln x}\right), b = \lim_{x \to 0}\left(\frac{x^3 - 16x}{4x + x^2}\right), c = \lim_{x \to 0} \frac{\ln(1 + \sin}{x}$

and $d = \lim_{x \to -1} \frac{(x+1)^3}{3[\sin(x+1) - (x+1)]}$ then the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

A. Idempotent

B. Involutary

C. Non-singular

D. Nipotent

Answer: D

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31. The integral value of n so that $\lim_{x \to 0} f(x)$ where

$f(x) = \frac{(\sin x - x)\left(2 \sin x - \ln\left(\frac{1+x}{1-x}\right)\right)}{x^n}$ is a finite non-zero number

A. 2

B. 4

C. 6

D. 8

Answer: C

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32. Consider the function $f(x) = \begin{cases} \max\left(x, \frac{1}{x}\right), & If x \neq 0 \\ \min\left(x, \frac{1}{x}\right) & \\ 1, & \text{if } x = 0 \end{cases}$, then

$\lim_{x \to 0^-} \{f(x)\} + \lim_{x \to 1^-} \{f(x)\} + \lim_{x \to 1^-} [f(x)] =$

(where {.} denotes fraction part function and [.] denotes greatest integer function)

A. 0

B. 1

C. 2

D. 3

Answer: A

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33. $\lim_{x \to \frac{1}{\sqrt{2}}^{+}} \frac{\cos^{-1}\left(2x\sqrt{1-x^2}\right)}{\left(x-\frac{1}{\sqrt{2}}\right)} - \lim_{x \to \frac{1}{\sqrt{2}}^{-}} \frac{\cos^{-1}\left(2x\sqrt{1-x^2}\right)}{\left(x-\frac{1}{\sqrt{2}}\right)}$

A. $\sqrt{2}$

B. $2\sqrt{2}$

C. $4\sqrt{2}$

D. 0

Answer: C

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34.

$$\lim_{n \to \infty} \sum_{k=1}^{n} \left((\sin) \frac{\pi}{2k} - (\cos) \frac{\pi}{2k} - (\sin) \left(\frac{\pi}{2(k+2)} + (\cos) \frac{\pi}{2(k+2)} \right) =$$

A. 0

B. 1

C. 2

D. 3

Answer: D

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35. $\lim_{x \to 0} [1 + [x]]^{\frac{2}{x}}$, where [:] is greatest integer function, is equal to

A. 0

B. 1

C. e^2

D. Does not exist

Answer: B

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36. If and n are positive integers, then $\lim_{x \to 0} \frac{(\cos x)^{\frac{1}{m}} - (\cos x)^{\frac{1}{n}}}{x^2}$ equal to :

A. $m - n$

B. $\frac{1}{n} - \frac{1}{m}$

C. $\frac{n - n}{2mn}$

D. none of these

Answer: C

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37. $\lim_{x \to 0} \frac{x(1 + a\cos x) - b\sin x}{x^3} = 1$ then

A. $\left(-\frac{5}{2}, -\frac{3}{2}\right)$

B. $\left(\frac{5}{2}, \frac{3}{2}\right)$

C. $\left(-\frac{5}{2}, \frac{3}{2}\right)$

D. $\left(\frac{5}{2}, -\frac{3}{2}\right)$

Answer: A

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38. What is the value of $a + b$, if $\lim_{x \to 0} \frac{\sin(ax) - \ln(e^x \cos x)}{x\sin(bx)} = \frac{1}{2}$?

A. 1

B. 2

C. 3

D. $-\frac{1}{2}$

Answer: B

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39. Let $\alpha = \lim_{n \to \infty} \frac{(1^3 - 1^2) + (2^3 - 2^2) + \ldots . + (n^3 - n^2)}{n^4}$, then α is equal to :

A. $\frac{1}{3}$

B. $\frac{1}{4}$

C. $\frac{1}{2}$

D. None existent

Answer: B

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40. The value of $\lim_{x \to 0} \frac{\cos(\sin x) - \cos x}{x^4}$ is equal to :

A. $\frac{1}{5}$

B. $\frac{1}{6}$

C. $\frac{1}{4}$

D. $\frac{1}{12}$

Answer: D

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41. $\lim_{x \to 0} \frac{x(1 + a\cos x) - b\sin x}{x^3} = 1$ then

A. $\left(-\frac{5}{2}, -\frac{3}{2}\right)$

B. $\left(\frac{5}{2}, \frac{3}{2}\right)$

C. $\left(-\frac{5}{2}, \frac{3}{2}\right)$

D. $\left(\frac{5}{2}, \frac{3}{2}\right)$

Answer: A

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42. Consider the sequence $u_n = \sum_{r=1}^{n} \frac{r}{2^r}, n \geq 1$ then the $\lim it_{n \to \infty} u_n$

A. 1

B. e

C. $\frac{1}{2}$

D. 2

Answer: D

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43. the value of $\lim_{x \to 0} \left\{ (\cos x)^{\frac{1}{\sin^2 x}} + \frac{\sin 2x + 2\tan^{-13} x + 3x^2}{\ln(1 + 3x + \sin^2 x) + xe^x} \right\}$

A. $\sqrt{e} + \frac{3}{2}$

B. $\frac{1}{\sqrt{e}} + \frac{3}{2}$

C. $\sqrt{e} + 2$

D. $\frac{1}{\sqrt{e}}+2$

Answer: D

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44. For $n \in N$, let $f_n(x)=\tan\frac{x}{2}(1+\sec x)(1+\sec 2x)(1+\sec 4x)......(1+\sec 2^n x)$, the $\lim_{x\to 0}\frac{f_n(x)}{2x}$ is equal to :

A. 0

B. 2^n

C. 2^{n-1}

D. 2^{n+1}

Answer: C

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45. The value of $\lim_{x \to \frac{\pi}{4}} (1+[x])^{1/\ln(\tan x)}$ (where[.] denote the greatest integer function) is equal to

A. 0

B. 1

C. e

D. $\frac{1}{e}$

Answer: B

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46. If $(\lim)_{x \to 0} \frac{\{(a-n)nx - \tan x\}\sin nx}{x^2} - 0$, where n is nonzero real number, the a is 0 (b) $\frac{n+1}{n}$ (c) n (d) $n + \frac{1}{n}$

A. 0

B. $1 + \frac{1}{n}$

C. π

D. $n+\frac{1}{n}$

Answer: D

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47. The value of $\lim_{x \to \infty}\left(\frac{n!}{n^n}\right)^{\frac{3n^3+4}{4n^4-1}}, n \in N$ is equal to:

A. $\left(\frac{1}{e}\right)^{3/4}$

B. $e^{3/4}$

C. e^{-1}

D. 0

Answer: A

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48. The value of $\lim_{x \to \infty} \frac{ax^2 + bx + c}{dx + e} (a, b, c, d, e \in R - \{0\})$ depends on the sign of :

A. a only

B. d only

C. a and d only

D. a,b and d only

Answer: C

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49. If $f(x) = \lim_{n \to \infty} \tan^{-1}\left(4n^2\left(1 - \cos\left(\frac{x}{n}\right)\right)\right)$ and g(x) = $\lim_{n \to \infty} \frac{n^2}{2} \ln \cos\left(2\frac{x}{n}\right)$ then $\lim_{x \to 0} \frac{e^{-2g(x)} - e^{f(x)}}{x^6}$ equals

A. $\frac{8}{3}$

B. $\frac{7}{3}$

C. $\frac{5}{3}$

D. $\frac{2}{3}$

Answer: A

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50. If $f(x)$ be a cubic polynomial and $\lim_{x \to 0} \frac{\sin^2 x}{f(x)} = \frac{1}{3}$ then $f(1)$ can not be equal to :

A. 0

B. -5

C. 3

D. -2

Answer: C

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51. $\lim_{x \to 0} \frac{2e^{\sin x} - e^{-\sin x} - 1}{x^2 + 2x}$

A. $\frac{3}{2}$

B. $e^{3/2}$

C. 2

D. e^2

Answer: A

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52. If $x_1, x_2, x_3, \ldots, x_n$ are the roots of the equation $x^n + ax + b = 0$, the value of

$(x_1 - x_2)(x_1 - x_3)(x_1 - x_4)\ldots\ldots(x_1 - x_n)$ is

A. $nx_1 + b$

B. $nx_1^{n-1} + a$

C. nx_1^{n-1}

D. nx_1^{n-1}

Answer: B

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53. $\lim_{x \to 0} \frac{\sqrt[3]{1+\sin^2 x} - \sqrt[4]{1-2\tan x}}{\sin x + \tan^2 x}$ is equal to:

A. -1

B. 1

C. $\frac{1}{2}$

D. $-\frac{1}{2}$

Answer: C

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54. if $f(x) = \begin{vmatrix} x\cos x & 2x\sin x & x\tan x \\ 1 & x & 1 \\ 1 & 2x & 1 \end{vmatrix}$ find $\lim_{x \to 0} \frac{f(x)}{x^2}$

A. 0

B. 1

C. -1

D. Does not exist

Answer: C

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Exercise One Or More Than One Answer Is Are Correct

1. if $\lim_{x \to 0} \left(p\tan qx^2 - 3\cos^2 x + 4\right)^{\frac{1}{3x^2}} = e^{\frac{5}{3}}$

A. $p = \sqrt{2}, q = \frac{1}{2\sqrt{2}}$

B. $= \frac{1}{\sqrt{2}}, q = 2\sqrt{2}$

C. $p = 1, q = 2$

D. $p = 2, q = 4$

Answer: B::C

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2. $\lim_{x \to \infty} 2\left(\sqrt{25x^2 + x} - 5x\right)$ is:

A. $\lim_{x \to 0} \frac{2x - \log_e (1 + x)^2}{5x^2}$

B. $\lim_{x \to 0} \frac{e^{-x} - 1 + x}{x^2}$

C. $\lim_{x \to 0} \frac{2(1 - \cos x^2)}{5x^4}$

D. $\lim (x \to 0) \frac{\sin \frac{\pi}{5}}{x}$

Answer: A::C::D

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3. Let $\lim_{x \to \infty} \left(2^x + a^x + e^x\right)^{1/x} = L$ which of the following statement (s) is (are) correct ?

A. if $L = a(a > 0)$, then the range of a is $[e, \infty)$

B. if $L = 2e(a > 0)$, then the range of a is $\{2e\}$

C. if $L = e(a > 0)$, then the range of a is $(0, e]$

D. if $L = 2a(a > 1)$, then the range of a is $\left(\frac{e}{2}, \infty\right)$

Answer: A::B::C

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4. Let $\tan\alpha. x + \sin\alpha \cdot y = \alpha$ and $\alpha. \cos ec\alpha \bullet x + \cos\alpha \cdot y = 1$ be two variable straight lines, α being the parameter. Let P be the point of intersection of the lines. In the limiting position when $a \to 0$, the point P lies on the line :

A. $x = 2$

B. $x = -1$

C. $y + 1 = 0$

D. $y = 2$

Answer: A::C::D

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5. Let $f: R \to [-1, 1]$ be defined as $f(x) = \cos(\sin x),$ then which of the following is(are) correct ?

A. f is periodic with fundamental period 2π

B. Range of $f = [\cos 1, 1]$

C. $\lim_{x \to \frac{\pi}{2}} \left(f\left(\frac{\pi}{2} - x\right) + f\left(\frac{\pi}{2} + x\right)\right) = 2$

D. f is neither even nor odd function

Answer: B::C

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6. Let $f(x) = x + \sqrt{x^2 + 2x}$ and $g(x) = \sqrt{x^2 + 2x} - x$, then:

A. $\lim_{x \to \infty} g(x) = 1$

B. $\lim_{x \to \infty} f(x) = 1$

C. $\lim_{x \to -\infty} f(x) = -1$

D. $\lim_{x \to \infty} g(x) = -1$

Answer: A::C::D

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7. Which of the following limits does not exist ?(a) $\lim_{x \to \infty} \cos ec^{-1}\left(\frac{x}{x+7}\right)$(B) $\lim_{x \to 1} \sec^{-1}(\sin^{-1} x)$(C) $\lim_{x \to 0^+} x^{\frac{1}{x}}$(D) $\lim_{x \to 0} \left(\tan\left(\frac{\pi}{8} + x\right)\right)^{\cot x}$

A. $\lim_{x \to \infty} \cos ec^{-1}\left(\frac{x}{x+7}\right)$

B. $\lim_{x \to 1} \sec^{-1}(\sin^{-1} x)$

C. $\lim_{x \to 0^+} x^{\frac{1}{x}}$

D. $\lim_{x \to 0} \left(\tan\left(\frac{\pi}{8} + x\right)\right)^{\cot x}$

Answer: A::D

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8. If $f(x) = (\lim)_{n \vec{\infty}} \left(\frac{3}{2} + [\cos x]\left(\sqrt{n^2 + 1} - \sqrt{n^2 - 3n + 1}\right)\right)$ where $[y]$ denotes largest integer $\leq$, then identify the correct statement(s). $(\lim)_{n \vec{\infty}} f(x) = 0$ $(\lim)_{n \vec{\frac{\pi}{2}}} f(x) = \frac{3\pi}{4}$ $f(x) = \frac{3x}{2} \forall x \in \left[0, \frac{\pi}{2}\right]$ $f(x) = 0 \forall x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

A. $\lim_{x \to 0} f(x) = 0$

B. $\lim_{x \frac{\pi}{2}} f(x) = \frac{3\pi}{4}$

C. $f(x) = \frac{3\pi}{2} \forall x \in \left[0, \frac{\pi}{2}\right]$

D. $f(x) = 0 \forall x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

Answer: A::C::D

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9. Let $f: R \to R; f(x) = \begin{cases} (-1)^n & \text{if } x = \frac{1}{2^{2^n}}, n = 1, 2, 3 \ldots\ldots\ldots \text{ and } 0 \end{cases}$

otherwisw then identity the correct statement (s).

A. $\lim_{x \to 0} f(x) f(2x) = 0$

B. $\lim_{x \to 0} f(x)$ does not exist

C. $\lim_{x \to 0} f(x) f(2x) = 0$

D. $\lim_{x \to 0} f(x)(2x)$ does not exist

Answer: B::C

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10. If $\lim_{x \to a} f(x) = \lim_{x \to a} [f(x)]$ ([.] denotes the greatest integer function) and $f(x)$ is non-constantcontinuous function, then :

A. $\lim_{x \to 0} f(x)$ is an integer

B. $\lim_{x \to 0} f(x)$ is non-integer

C. $f(x)$ has local maximum at $x = a$

D. $f(x)$ has local minimum at $x = a$

Answer: A::D

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11. let $f(x) = \dfrac{\cos^{-1}(1 - \{x\})\sin^{-1}(1 - \{x\})}{\sqrt{2\{x\}(1 - \{x\})}}$ where $\{x\}$ denotes the fractional part of x then

A. $\lim_{x \to 0^+} f(x) = \dfrac{\pi}{4}$

B. $\lim_{x \to 0^+} f(x) = \sqrt{2} \lim_{x \to 0^-} f(x)$

C. $\lim_{x \to 0^-} f(x) = \dfrac{\pi}{4\sqrt{2}}$

D. $\lim_{x \to 0^-} f(x) = \dfrac{\pi}{2\sqrt{2}}$

Answer: B::D

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12. $\lim_{x \to 0} \frac{\sin(\sin x) - \sin x}{ax^3 + bx^5 + c} = -\frac{1}{12}$ then

A. $a = 2$

B. $a = -2$

C. $c = 0$

D. $b \in R$

Answer: A::C

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13. If $f(x) = \lim_{n \to \infty} \left(n\left(x^{1/n} - 1\right)\right)$ for $x > 0$, then which of the following is/are true?

A. $f\left(\frac{1}{x}\right) = 0$

B. $f\left(\frac{1}{x}\right) = \frac{1}{f(x)}$

C. $f\left(\frac{1}{x}\right) = -f(x)$

D. $f(xy) = f(x) + f(y)$

Answer: C::D

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14. $\lim_{n \to \infty} \cos^2\left(\pi\left(3\sqrt{n^3 + n^2 + 2n} - n\right)\right)$ where n is an integer,equals

A. $\frac{1}{3}$

B. $\frac{1}{2}$

C. $\frac{1}{4}$

D. $\frac{1}{9}$

Answer: A::B::C

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15. If $\alpha, \beta \in \left(-\frac{\pi}{2}, 0\right)$ such that $(\sin\alpha + \sin\beta) + \frac{\sin\alpha}{\sin\beta} = 0$ and $(\sin\alpha + \sin\beta)\frac{\sin\alpha}{\sin\beta} = -1$ and $\lambda = \lim_{n}$

then :

A. $\alpha = \frac{\pi}{6}$

B. $\lambda = 2$

C. $\alpha = -\frac{\pi}{3}$

D. $\lambda = 1$

Answer: B

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16. Let $f(x) = \{|x-2| + a^2 - 6a + 9, x < 2$ and $5 - 2x, x \geq 2$ If $\lim_{x \to 2} [f(x)]$ existsn the possible values a can take is/are (where [.] represents the grestest integer function)

A. 2

B. $\frac{5}{2}$

C. 3

D. $\frac{7}{2}$

Answer: B

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17. A circular disk of unit radius is filled with a number of smaller circular disks arranged in the form of hexagon. Let A_n denotes a stack of disks arranged in the shape of a hexagon having 'n' disks on a side. The figure shows the configuration A_3. If A be the area of large disk, S_n be the number of disks in A_n configuration and r_n be the radius of each disk in A_n configuration, then $\lim_{n \to \infty} \frac{S_n}{n^2} \lim_{n \to \infty} nr_n$

A. $\frac{1}{3}$

B. $\frac{1}{2}$

C. $\frac{1}{4}$

D. $\frac{1}{11}$

Answer: B

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18. Let

$f(x) = [(x+3, , -2 < x < 0), (4, x = 0), (2x+5, , 0 < x < 1),$

then

$\lim_{x \to 0^-} f([x - \tan])$ is : [.] denotes greatest integer function)

A. 2

B. 4

C. 5

D. none of these

Answer: B

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19. Let

$f(x) = [(x+3, , \quad -2 < x < 0), (4, x = 0), (2x+5, , 0 < x < 1),$

then

$\lim_{x \to 0}, f\left(\left\{\frac{x}{\tan x}\right\}\right) is :$ ($\{.\}$ denotes fractional part of function)

A. 4

B. 5

C. 7

D. none of these

Answer: C

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20. A certain function f(x) has the property that $f(3x) = \alpha f(x)$ for all positive real values of x and $f(x) = 1 - |x-2|$ for $1 \leq x \leq 3$,

$\lim_{x \to 2} (f(x))^{\cos ec\left(\frac{\pi x}{2}\right)}$ is

A. $\frac{2}{\pi}$

B. $-\frac{2}{\pi}$

C. $e^{2/\pi}$

D. none of these

Answer: D

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21. A certain function $f(x)$ has the property that $f(3x) = \alpha f(x)$ for all positive real values of x and $f(x) = 1 - |x - 2|$ for $1 \leq x \leq 3$ If the total area bounded by $y = f(x)$ and x-axis in $[1, \infty)$ converges to a finite quantity, then the range of α is:

A. $(-1, 1)$

B. $\left(-\frac{1}{2}, \frac{1}{2}\right)$

C. $\left(-\frac{1}{3}, \frac{1}{3}\right)$

D. $\left(-\frac{1}{4}, \frac{1}{4}\right)$

Answer: C

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22. Consider the limit $\lim_{x \to 0} \frac{1}{x^3}\left(\frac{1}{\sqrt{1+x}} - \frac{(1+ax)}{(1+bx)}\right)$ exists, finite and has the value equal to l (where a,b are real constants), then :

a=

A. 1

B. $\frac{3}{4}$

C. $\frac{1}{2}$

D. $\frac{1}{4}$

Answer: D

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23. Consider the limit $\lim_{x \to 0} \frac{1}{x^3}\left(\frac{1}{\sqrt{1+x}} - \frac{(1+ax)}{(1+bx)}\right)$ exists, finite and has the value equal to l (where a,b are real constants), then:

$a + b =$

A. $\frac{3}{4}$

B. $\frac{1}{2}$

C. 1

D. 0

Answer: C

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24. Consider the limit $\lim_{x \to 0} \frac{1}{x^3}\left(\frac{1}{\sqrt{1+x}} - \frac{(1+ax)}{(1+bx)}\right)$ exists, finite and has the value equal to l (where a,b are real constants), then :

$\left|\frac{b}{a}\right| =$

A. 38

B. 16

C. 72

D. 3

Answer: D

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25. For the curve $\sin x + \sin y = 1$ lying in first quadrant. If $\lim_{x \to 0} x^{\alpha} \frac{d^2y}{dx^2}$ exists and non-zero than $2\alpha =$

A. $\frac{1}{2}$

B. $\frac{1}{\sqrt{2}}$

C. $\frac{3}{2}$

D. 2

Answer: C

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26. Ror the curve $\sin x+\sin y=1$ lying in the first quadrant there exists a constant α for which $\lim_{x\to 0} x^{\alpha}\frac{d^2y}{dx^2}=I,$ (not zero)

The volue of L:

A. $\frac{1}{2}$

B. 1

C. $\frac{1}{2\sqrt{2}}$

D. $\frac{1}{2\sqrt{3}}$

Answer: C

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Exercise Comprehension Type Problems

1. A circular disk of unit radius is filled with a number of smaller circular disks arranged in the form of hexagon. Let A_n denotes a stack of disks arranged in the shape of a hexagon having 'n' disks on a side. The figure shows the configuration A_3. If A be the area of large disk, S_n be the number of disks in A_n configuration and r_n be the radius of each disk in A_n configuration, then $\lim_{n \to \infty} \frac{S_n}{n^2} \lim_{n \to \infty} nr_n$

A. 3

B. 4

C. 1

D. 11

Answer: A

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Exercise Mathching Type Problems

1. Solve for x: $\lim_{x \to -4} \left[\frac{x^2 - x - 20}{x + 4}\right]$

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Exercise Subjective Type Problems

1. If $\lim_{x \to 0} \frac{\ln \cot\left(\frac{\pi}{4} - \beta x\right)}{\tan \alpha x} = 1$, thetn $\frac{\alpha}{\beta} = \ldots.$

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2. If $\lim_{x \to 0} \frac{f(x)}{\sin^2 x} = 8, \ \lim_{x \to 0} \frac{g(x)}{2\cos x - xe^x + x^3 + x - 2} = \lambda$ and $\lim_{x \to 0} (1 + 2f(x))^{\frac{1}{g(x)}} = \frac{1}{e}$, then

The value of λ is

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3. If α, β are two distinct real roots of the equation $ax^3 + x - 1 - a = 0 (a \neq -1, 0)$, none of which is equal to unity. If the value of $\lim_{x \to \frac{1}{\alpha}} \frac{(1 + a)x^3 - x^2 - a}{(e^{1 - \alpha x} - 1)(x - 1)}$ is $\frac{al(k\alpha - \beta)}{\alpha}$ the value of $k + l$

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4. The value of $\lim_{x \to 0} \frac{(140)^x - (35)^x - (28)^x - (20)^x + 7^x + 5^x + 4^x - 1}{x \sin^2 x}$

$= 2 \ln 2 \ln k \ln 7$, then $k =$

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5. If $\lim_{x \to 0} \frac{a \cos x}{x^2} + \frac{b}{x^2} = \frac{1}{3}$, then $b - a =$

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6. Find the value of $\lim_{x \to \infty} \left(x + \frac{1}{x}\right) e^{1/x} - x$.

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7. FInd $\lim_{x \to \alpha^+} \left[\frac{\min(\sin x, \{x\})}{x - 1}\right]$ where alpha is the root of the equation $\sin x + 1 = x$ Here [.] represents greatest integer function and

{.} represents fractional part function

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