MATHS

BOOKS - VIKAS GUPTA MATHS (HINGLISH)

SEQUENCE AND SERIES

Exercise Single Choice Problems

1. If a, b, c are positive real numbers such that $a+b+c=1$, then the greatest value of `(1-a)(1-b)(1-c), is

A. 1

B. $\frac{2}{3}$

C. $\frac{8}{27}$

D. $\frac{4}{9}$

Answer: C

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2. If $xyz = (1-x)(1-y)(1-z)$ Where $0 \le x, y, z \le 1$, then the minimum value of $x(1-z) + y(1-x) + z(1-y)$ is

A. $\frac{3}{2}$

B. $\frac{1}{4}$

C. $\frac{3}{4}$

D. $\frac{1}{2}$

Answer: C

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3. If $\sec(\alpha - 2\beta), \sec\alpha, \sec(\alpha + 2\beta)$ are in arithmetical progressin then $\cos^2\alpha = \lambda\cos^2\beta(\beta \ne n\pi, n \in I)$, the value of λ is:

A. 1

B. 2

C. 3

D. $\frac{1}{2}$

Answer: B

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4. Let a,b,c,d, e ar non-zero and distinct positive real numbers. If a,b, c are

In a,b,c are in A.B, b,c, dare in G.P. and c,d e are in H.P, the a,c,e are in :

A. A.P.

B. G.P.

C. H.P.

D. Nothing can be said

Answer: B

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5. if $(m+1)th, (n+1)th$ and $(r+1)th$ term of an AP are in GP.and m, n and r in HP. . find the ratio of first term of A.P to its common difference

A. $-\frac{n}{2}$

B. $-n$

C. $-2n$

D. $+n$

Answer: A

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6. If the equation $x^4-4x^3+ax^2+bx+1=0$has four positive roots, then the value of (a+ b) is :

A. -4

B. 2

C. 6

D. can not be determined

Answer: B

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7. If $S_1, S_2 \text{ and } S_3$ are the sums of first n natureal numbers, their squares and their cubes respectively, then $\frac{S_1^4 S_2^2 - S_2^2 S_3^2}{S_1^2 + S_2^2} =$

A. 4

B. 2

C. 1

D. 0

Answer: D

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8. If $S_n = \frac{1.2}{3!} + \frac{2.2^2}{4!} + \frac{3.2^3}{5!} + \ldots$ upto n terms then the sum infinite terms is

A. 1

B. $\frac{2}{3}$

C. e

D. $\frac{\pi}{4}$

Answer: A

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9. If $\tan\left(\frac{\pi}{12} - x\right), \tan\left(\frac{\pi}{12}\right), \tan\left(\frac{\pi}{12} + x\right)$ in G.P. then sum of all the solutions in [0,314] is $k\pi$. Find k

A. 4950

B. 5050

C. 2525

D. 5010

Answer: A

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10. Let $S_n = 1 + 2 + 3 + \quad + n$ and $P_n = \frac{S_2}{S_2 - 1} \frac{\dot{S}_3}{S_3 - 1} \frac{\dot{S}_4}{S_4 - 1} \frac{S_n}{S_n - 1}$

Where $n \in N$, $(n \geq 2)\cdot$ Then $(\lim)_{x \overrightarrow{\infty}} P_n = \_ \quad \_\_ \; \_$

A. $\frac{1}{3}$

B. 1

C. 3

D. 0

Answer: C

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11. If $a > 0, b > 0, c > 0$ are respectively the $p^{\text{th}}, q^{\text{th}}, r^{\text{th}}$ terms of a G.P.,.

Then the value of the determinant $\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix}$ is

A. -1

B. 2

C. 1

D. 0

Answer: D

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12. The numbers of natural numbers < 300 that are divisible by 6 but not by 9 :

A. 49

B. 37

C. 33

D. 16

Answer: C

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13. If $x, y, x > 0$ and $x + y + z = 1$ then $\dfrac{xyz}{(1-x)(1-y)(1-z)}$ is necessarily.

A. ≥ 8

B. $\leq \dfrac{1}{8}$

C. 1

D. None of these

Answer: B

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14. If the roots of the equation $px^2 + qx + r = 0$, where $2p$, q, $2r$ are in G.P, are of the form α^2, $4\alpha - 4$. Then the value of $2p + 4q + 7r$ is :

A. 82

B. 10

C. 14

D. 18

Answer: A

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15. Let $x_1, x_2, x_3, \ldots, x_k$ be the divisors of positive integer n (including 1 and x). If $x_1 + x_2 + \ldots + x_k = 75$, then $\sum_{i=1}^{k} \frac{1}{x_i}$ is equal to

A. $\frac{75}{k}$

B. $\frac{75}{n}$

C. $\frac{1}{n}$

D. $\frac{1}{75}$

Answer: B

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16. If $a_a, a_2, a_3, \ldots., a_n$ are in H.P. and $f(k) = \sum_{r=1}^{n} a_r - a_k$, then $\frac{a_1}{f(1)}, \frac{a_2}{f(2)}, \ldots. \frac{a_3}{f(n)}$ are in :

A. A.P.

B. G.P.

C. H.P.

D. None of these

Answer: C

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17. if α, β be roots of equation $375x^2 - 25x - 2 = 0$ and $s_n = \alpha^n + \beta^n$

then $\lim_{n \to \infty} \left(\sum_{r=1}^{n} S_r \right) = \dots\dots\dots$

A. $\frac{1}{12}$

B. $\frac{1}{4}$

C. $\frac{1}{3}$

D. 1

Answer: A

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18. If $a_i, i = 1, 2, 3, 4$ be four real members of same sign, then the minimum value of $\sum \left(\frac{a_i}{a_j} \right), i, j \in \{1, 2, 3, 4\}, i \neq j$ is : (a) 6 (b) 8 (c) 12 (d) 24

A. 6

B. 8

C. 12

D. 24

Answer: C

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19. Given that x, y, z are positive reals such that $xyz = 32$. The minimum value of $x^2 + 4xy + 4y^2 + 2z^2$ is equal to :

A. 64

B. 256

C. 96

D. 216

Answer: C

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20. In an A.P. five times the fifth term is equal to eight times its eight term. Then the sum of the first twenty five terms is equal to :

A. 25

B. $\frac{25}{2}$

C. -25

D. 0

Answer: D

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21. Let α, β be two distinct values of x lying in $(0, \pi)$ for which $\sqrt{5}\sin x, 10\sin x, 10(4\sin^2 x + 1)$ are 3 consecutive terms of a G.P. Then minimum value of $|\alpha - \beta| =$

A. $\frac{\pi}{10}$

B. $\frac{\pi}{5}$

C. $\frac{2\pi}{5}$

D. $\frac{3\pi}{5}$

Answer: B

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22. In an infinite G.P. the sum of first three terms is 70. If the extreme terms are multiplied by 4 and the middle term is multiplied by 5, the resulting terms form an A.P. then the sum to infinite terms of G.P. is :

A. 120

B. 40

C. 160

D. 80

Answer: D

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23. Find the $\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}\frac{k}{2^{n+k}}$.

A. 5

B. 4

C. 3

D. 2

Answer: D

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24. Let $p, q, r \in R^{+}$ and $27pqr \geq (p+q+r)^{3}$ and $3p+4q+5r=12$ then $p^{3}+q^{4}+r^{5}$ is equal to

A. 3

B. 6

C. 2

D. 4

Answer: A

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25. Find the sum of the infinte series $\frac{1}{9}+\frac{1}{18}+\frac{1}{30}+\frac{1}{45}+\frac{1}{63}+\dots$

A. $\frac{1}{3}$

B. $\frac{1}{4}$

C. $\frac{1}{5}$

D. $\frac{2}{3}$

Answer: A

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26. If S_r denotes the sum of r terms of an A.P. and $\frac{S_a}{a^2} = \frac{S_b}{b^2} = c$. Then $S_c =$ (A) c^3 (B) $\frac{c}{a}b$ (C) abc (D) $a+b+c$

A. c^2

B. c^3

C. c^4

D. abc

Answer: B

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27. In an infinite $G.P.$ second term is x and its sum is 4, then complete set of values of x is in

A. $(-8, 0)$

B. $\left[-\frac{1}{8}, \frac{1}{8}\right) - \{0\}$

C. $\left[-1, -\frac{1}{8}\right) \cup \left(\frac{1}{8}, 1\right]$

D. $(-8, 1] - \{0\}$

Answer: D

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28. The number of terms of an A.P. is odd. The sum of the odd terms $\left(1^{st}, 3^{rd} etc, \right)$ is 248 and the sum of the even terms is 217. The last term exceeds the first by 56 then :

A. the number of terms is 17

B. the first term is 3

C. the number of terms is 13

D. the first term is 1

Answer: B

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29. Let $A_1, A_2, A_3, \ldots.., A_n$ be squares such that for each $n \geq 1$ the length of a side of A_n equals the length of a diagonal of A_{n+1}. If the side of A_1 be 20 units then the smallest value of 'n' for wheich area of A_n is less than 1.

A. 7

B. 8

C. 9

D. 10

Answer: D

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30. Let $S_k = \lim_{n \to \infty} \sum_{i=0}^{n} \frac{1}{(k+1)^i}$. Then $\sum_{k=1}^{n} kS_k$ equals

A. $\frac{n(n+1)}{2}$

B. $\frac{n(n-1)}{2}$

C. $\frac{n(n+2)}{2}$

D. $\frac{n(n+3)}{2}$

Answer: D

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31. The sum of the series $\frac{2}{1.2}+\frac{5}{2.3}2^1+\frac{10}{3.4}2^2+\frac{17}{4.5}2^3+\ldots.$ upto n terms is equal :

A. $\frac{n2^n}{n+1}$

B. $\left(\frac{n}{n+1}\right)2^n+1$

C. $\frac{n2^n}{n+1}-1$

D. $\frac{(n-1)2^2}{n+1}$

Answer: A

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32. If $(1.5)^{30} = k$, then the value of $\sum_{(n=2)}^{29} (1.5)^n$, is :

A. $2k - 3$

B. $k + 1$

C. $2k + 7$

D. $2k - \frac{9}{2}$

Answer: D

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33. Suppose that n arithmetic means are inserted between then numbers 7 and 49. If the sum of these means is 364 then the sum their squares is

A. 11

B. 12

C. 12

D. 14

Answer: C

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34. The third term of a G.P. is 2. Then the product of the first five terms, is :

A. 2^3

B. 2^4

C. 2^5

D. None of these

Answer: C

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35. The sum of first n terms of an A.P. is $5n^2 + 4n,$ its common difference is :

A. 9

B. 10

C. 3

D. -4

Answer: B

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36. If $x+y=a$ and $x^2+y^2=b$, then the value of $\left(x^3+y^3\right)$, is

A. ab

B. a^2+b

C. $a+b^2$

D. $\dfrac{3ab-a^3}{2}$

Answer: D

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37. If $S_1, S_2, S_3, \ldots\ldots, S_n$ are the sum of infinite geometric series whose first terms are $1, 3, 5\ldots, (2n-1)$ and whose common rations are $\frac{2}{3}, \frac{2}{5}, \ldots, \frac{2}{2n+1}$ respectively, then $\left\{\frac{1}{S_1 S_2 S_3} + \frac{1}{S_2 S_3 S_4} + \frac{1}{S_3 S_4 S_5} + \text{.........upon infinite terms}\right\} =$

A. $\frac{1}{15}$

B. $\frac{1}{60}$

C. $\frac{1}{12}$

D. $\frac{1}{3}$

Answer: B

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38. Sequence $\{t_n\}$ of positive terms is a G.P If $t_6 2, 5, t_{14}$ form another G.P in that order then the product $t_1 t_2 t_3 \ldots\ldots.t_{18} t_{19}$ is equal to

A. 10^9

B. 10^{10}

C. $10^{17/2}$

D. $10^{19/2}$

Answer: D

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39. The minimum value of $\dfrac{(A^2+A+1)(B^2+B+1)(C^2+C+1)}{ABCD}$ where $A, B, C, D > 0$ is :

A. $\dfrac{1}{3^4}$

B. $\dfrac{1}{2^4}$

C. 2^4

D. 3^4

Answer: D

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40. if $\sum_{1}^{20} r^3 = a, \sum_{1}^{20} r^2 = b$ then sum of products of 1,2,3,4,....20 taking two at a time is :

A. $\frac{a-b}{2}$

B. $\frac{a^2-b^2}{2}$

C. $a-b$

D. a^2-b^2

Answer: A

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41. The sum of first $2n$ terms of an AP is α. and the sum of next n terms is β, its common difference is

A. $\frac{\alpha-2\beta}{3n^2}$

B. $\dfrac{2y-x}{3n^2}$

C. $\dfrac{x-2y}{3n}$

D. $\dfrac{2y-x}{3n}$

Answer: B

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42. The number of non-negative integers 'n' satisfying $n^2 = p + q$ and $n^3 = p^2 + q^2$ where p and q are integers

A. 2

B. 3

C. 4

D. Infinite

Answer: B

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43. Concentric circles of radii $1, 2, 3, \ldots, 100cm$ are drawn. The interior of the smallest circle is colored red and the angular regions are colored alternately green and red, so that no two adjacent regions are of the same color. Then, the total area of the green regions in sq. cm is equal to 1000π b. 5050π c. 4950π d. 5151π

A. 1000π

B. 5050π

C. 4950π

D. 5151π

Answer: B

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44. If $\log_2(4)$, $\log_{\sqrt{2}}(8)$ and $\log_3(9k-1)$ are consecutive terms of a geometric sequence,then the number of integers that satisfy the system of inequalities $x^2 - x > 6$and $|x| < k^2$ is

A. 193

B. 194

C. 195

D. 196

Answer: A

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45. Let T_r be the rth term of an A.P. whose first term is -1/2 and common difference is 1, then $\sum_{r=1}^{n} \sqrt{1 + T_r T_{r+1} T_{r+2} T_{r+3}}$

A. $\dfrac{n(n+1)(2n+1)}{6} - \dfrac{5n}{4}$

B. $\dfrac{n(n+1)(2n+1)}{6} - \dfrac{5n}{4} + \dfrac{1}{4}$

C. $\dfrac{n(n+1)(2n+1)}{6} - \dfrac{5n}{4} + \dfrac{1}{2}$

D. $\dfrac{n(n+1)(2n+1)}{12} - \dfrac{5n}{8} + 1$

Answer: C

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46. If $\sum_{r-1}^{n} T_r = \frac{n(n+1)(n+2)}{3}, then \lim_{x \to \infty} \sum_{r=1}^{n} \frac{2008}{T_r} =$

A. 2008

B. 3012

C. 4016

D. 8032

Answer: A

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47. The sum of the infinite series, $1^2 - \frac{2^2}{5} + \frac{3^2}{5^3} + \frac{3^2}{5^3} + \frac{5^2}{5^4} - \frac{6^2}{5^5} + \dots.$ is:

A. $\frac{1}{2}$

B. $\frac{25}{24}$

C. $\frac{25}{54}$

D. $\frac{125}{252}$

Answer: C

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48. The absolute term in P(x) = $\sum_{r=1}^{n}\left(x-\frac{1}{r}\right)\left(x-\frac{1}{r+1}\right)\left(x-\frac{1}{r+2}\right)$ as n approches to infinity is :

A. $\frac{1}{2}$

B. $\frac{-1}{2}$

C. $\frac{1}{4}$

D. $\frac{-1}{4}$

Answer: D

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49. Suppose A, B, C are defined as $A = a^2b + ab^2 - a^2c - ac^2, B = b^2c + bc^2 - a^2b - ab^2,$ and $C = a^2c + ac^2 - b^2c - bc^2,$ where $a > b > c > 0$ and the equation $Ax^2 + Bx + C = 0$ has equal roots, then a, b, c are in

A. A.P.

B. G.P.

C. H.P.

D. None of these

Answer: C

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50. It T_k denotes the k^{th} term of an H.P. from the bgegining and $\dfrac{T_2}{T_6} = 9,$ then $\dfrac{T_{10}}{T_4}$ equals :

A. $\frac{17}{5}$

B. $\frac{5}{17}$

C. $\frac{7}{19}$

D. $\frac{19}{7}$

Answer: B

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51. Number of terms common to the two sequences $17, 21, 25, \ldots\ldots\ldots\ldots, 417$ and $16, 21, 26, \ldots\ldots\ldots., 466$ is

A. 19

B. 20

C. 21

D. 22

Answer: B

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52. The sum of the series $1+\frac{2}{3}+\frac{1}{3^2}+\frac{2}{3^3}+\frac{1}{3^4}+\frac{2}{3^5}+\frac{1}{3^6}+\frac{2}{3^7}+\ldots\ldots$ upto infinite terms is equal to :

A. $\frac{15}{8}$

B. $\frac{8}{15}$

C. $\frac{27}{8}$

D. $\frac{21}{8}$

Answer: A

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53. The coefficient of x^8 in the polynomial $(x-1)(x-2)(x-3)\ldots\ldots(x-10)$ is :

A. 2640

B. 1320

C. 1370

D. 2740

Answer: B

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54. Let $\alpha = \lim_{x \to \infty} \left(\left(1^3 - 1^2\right) + \left(2^3 - 2^2\right) + \ldots + \frac{n^3 n^2}{\pi^4}, \right.$ then α is equal is:

A. $\frac{1}{3}$

B. $\frac{1}{4}$

C. $\frac{1}{2}$

D. non-exisitent

Answer: B

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55. If $16x^4 - 32x^3 + ax^2 + bx + 1 = 0$, $a, b \in R$ has positive real roots only, then $a - b$ is equal to :

A. -32

B. 32

C. 49

D. -49

Answer: B

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56. if ABC is a triangle and $\tan\left(\frac{A}{2}\right), \tan\left(\frac{B}{2}\right), \tan\left(\frac{C}{2}\right)$ are in H.P. Then find the minimum value of $\cot\left(\frac{B}{2}\right)$

A. $\sqrt{3}$

B. 1

C. $\dfrac{1}{\sqrt{2}}$

D. $\dfrac{1}{\sqrt{3}}$

Answer: A

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57. If α and β are the roots of the quadratic equation $4x^2 + 2x - 1 = 0$ then the value of $\sum_{r=1}^{\infty} (a^r + \beta^r)$ is :

A. 2

B. 3

C. 6

D. 0

Answer: D

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58. The sum of the series $(2)^2 + 2(4)^2 + 3(6)^2 + \ldots.$ upto 10 terms is

A. 11300

B. 12100

C. 12300

D. 11200

Answer: B

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59. If a and b are positive real numbers such that $a + b = c$, then the minimum value of $\left(\frac{4}{a} + \frac{1}{b}\right)$ is equal to :

A. $\frac{2}{3}$

B. $\frac{1}{3}$

C. 1

D. $\frac{3}{2}$

Answer: D

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60. The first term of an infinite G.R is the value of satisfying the equation $\log_4(4^x - 15) + x - 2 = 0$ and the common ratio is $\cos\left(2011\frac{\pi}{3}\right)$ The sum of G.P is ?

A. 1

B. $\frac{4}{3}$

C. 4

D. 2

Answer: C

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61. Let a, b, c be positive numbers, then the minimum value of

$$\frac{a^4 + b^4 + c^2}{abc}$$

A. 4

B. $2^{3/4}$

C. $\sqrt{2}$

D. $2\sqrt{2}$

Answer: D

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62. If $xy = 1$, then minimum value of $x^2 + y^2$ is :

A. 1

B. 2

C. $\sqrt{2}$

D. 4

Answer: B

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63. Find the value of $\frac{2}{1^3}+\frac{6}{1^3+2^3}+\frac{12}{1^3+2^3+3^3}+\frac{20}{1^3+2^3+3^3+4^3}+\ldots\ldots\ldots$ upto 60 terms :

A. 2

B. $\frac{1}{2}$

C. 4

D. $\frac{1}{4}$

Answer: C

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64. $\sum_{n=1}^{\infty}\frac{1}{(n+1)(n+2)(n+3)\ldots\ldots\ldots(n+k)}$

A. $\dfrac{1}{(k-1)(k-1)!}$

B. $\dfrac{1}{k \cdot kl}$

C. $\dfrac{1}{(-1)kl}$

D. $\dfrac{1}{kl}$

Answer: C

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65. Consider two positive numbers a and b. If arithmetic mean of a and b exceeds their geometric mean by 3/2, and geometric mean of aand b exceeds their harmonic mean by 6/5 then the value of $a^2 + b^2$ will be

A. 150

B. 153

C. 156

D. 159

Answer: D

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66. Sum of first 10 terms of the series, $S = \frac{7}{2^2 \cdot 5^2} + \frac{13}{5^2 \cdot 7^2} + \frac{19}{8^2 \cdot 11^2} + \ldots\ldots$ is :

A. $\frac{255}{1024}$

B. $\frac{88}{1024}$

C. $\frac{264}{1024}$

D. $\frac{85}{1024}$

Answer: D

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67. $\sum_{r=1}^{10} \frac{r}{1 - 3r^2 + r^4}$

A. $-\dfrac{50}{109}$

B. $-\dfrac{54}{109}$

C. $-\dfrac{55}{111}$

D. $-\dfrac{55}{109}$

Answer: D

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68. The rth term of a series is given by $t_r = \dfrac{r}{1 + r^2 + r^4}$, then

$$\lim (n \to \infty) \sum_{r=1}^{n} (t_r)$$

A. $\dfrac{1}{2}$

B. 1

C. 2

D. $\dfrac{1}{4}$

Answer: A

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69. The sum to infinity of the series

$1+\frac{4}{5}+\frac{7}{5^2}+\frac{10}{5^3}+\ldots,$ is

A. $\frac{31}{12}$

B. $\frac{41}{16}$

C. $\frac{45}{16}$

D. $\frac{35}{16}$

Answer: D

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70. The third term of a G.P. is 2. Then product of the first five terms, is :

A. 2^3

B. 2^4

C. 2^5

D. None of these

Answer: C

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71. If $x_1, x_2, x_3, \ldots\ldots x2_n$ are in $A.P$, then $\sum_{r=1}^{2n}(-1)^{r+1}x_r^2$ is equal to

A. $\frac{n}{(2n-1)}\left(x_1^2 - x_{2n}^2\right)$

B. $\frac{2n}{(2n-1)}\left(x_1^2 - x_{2n}^2\right)$

C. $\frac{n}{(n-1)}\left(x_1^2 - x_{2n}^2\right)$

D. $\frac{n}{(2n+1)}\left(x_1^2 - x_{2n}^2\right)$

Answer: A

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72. Let two numbers have arithmatic mean 9 and geometric mean 4.Then these numbers are roots of the equation (a) $x^2 + 18x + 16 = 0$ (b) $x^2 - 18x - 16 = 0$ (c) $x^2 + 18x - 16 = 0$ (d) $x^2 - 18x + 16 = 0$

A. $x^2 + 18x + 16 = 0$

B. $x^2 - 18x - 16 = 0$

C. $x^2 + 18x - 16 = 0$

D. $x^2 - 18x + 16 = 0$

Answer: D

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73. If p and q are positive real numbers such that $p^2 + q^2 = 1$, then the maximum value of p+q is

A. 2

B. $\frac{1}{2}$

C. $\frac{1}{\sqrt{2}}$

D. $\sqrt{2}$

Answer: D

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74. A person is to count 4500 currency notes. Let an denote the number of notes he counts in the nth minute. If $a_1 = a_2 = \dots\dots = a_{10} = 150$ and $a_{10}, a_{11}, \dots\dots$ are in A.P. with common difference 2, then the time taken by him to count all notes is (1) 34 minutes (2) 125 minutes (3) 135 minutes (4) 24 minutes

A. 34 minutes

B. 24 minutes

C. 125 minutes

D. 35 minutes

Answer: A

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75. A non constant arithmatic progression has common difference d and first term is $(1-ad)$ If the sum of the first 20 term is 20, then the value of a is equal to :

A. $\frac{2}{19}$

B. $\frac{19}{2}$

C. $\frac{2}{9}$

D. $\frac{9}{2}$

Answer: B

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76. The value of $\sum_{n=3}^{\infty} \frac{1}{n^5 - 5n^3 + 4n}$ is equal to -

A. $\frac{1}{120}$

B. $\frac{1}{96}$

C. $\frac{1}{24}$

D. $\frac{1}{144}$

Answer: B

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77. Find the value of $\frac{2}{1^3} + \frac{6}{1^3 + 2^3} + \frac{12}{1^3 + 2^3 + 3^3} + \frac{20}{1^3 + 2^3 + 3^3 + 4^3} + \dots$ upto infinite terms

A. 2

B. $\frac{1}{2}$

C. 4

D. $\frac{1}{4}$

Answer: C

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78. The minimum value of the expression $2^{x}+2^{2x+1}+\frac{5}{2^{x}}, x \in R$ is :

A. 7

B. $(7.2)^{1/7}$

C. 8

D. $(3.10)^{1/3}$

Answer: C

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79. $\sum_{r=1}^{\infty} \frac{(4r+5)5^{-r}}{r(5r+5)}$

A. $\frac{1}{5}$

B. $\frac{2}{5}$

C. $\frac{1}{25}$

D. $\frac{2}{25}$

Answer: A

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Exercise One Or More Than One Answer Is Are Correct

1. If first and $(2n-1)^t h$ terms of an AP, GP. and HP. are equal and their nth terms are a, b, c respectively, then (a) a=b=c (b)a+c=b (c) a>b>c and $ac-b^2=0$ (d) none of these

A. $a+c=2b$

B. $a \geq b \geq c$

C. $\frac{2ac}{a+c}=b$

D. $ac=b^2$

Answer: B::D

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2. If a, b, c are distinct positive real numbers such that the quadratic expression $Q_1(x) = ax^2 + bx + c$,

$Q_2(x) = bx^2 + cx + a, Q_3(x) = cx^2 + x + b$ are always non-negative, then possible integer in the range of the expression $y = \dfrac{a^2b^2 + c^2}{ab + bc + ca}$ is

A. 1

B. 2

C. 3

D. 4

Answer: B::C

Watch Video Solution

3. If a,b,c are in H.P, where $a > c > 0$, then :

A. $b > \dfrac{a + c}{2}$

B. $\frac{1}{a-b} - \frac{1}{b-c} < 0$

C. $ac > b^2$

D. $bc(1-a), ac(1-b), ab(1-c)$ are in A.P.

Answer: B::C::D

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4. In an A.P. let T_r denote r^{th} term from beginning, $T_p - \frac{1}{q(p+q)}, T_q - \frac{1}{p(p+q)}$, then :

A. $T_1 =$ common difference

B. $T_{p+q} = \frac{1}{pq}$

C. $T_{pq} = \frac{1}{p+q}$

D. $T_{p+q} = \frac{1}{p^2q^2}$

Answer: A::B::C

Watch Video Solution

5. Which of the following statement (s) is (are) correct ?

A. Sum of the reciprocal of all the n harmonic means inserted between a and b is equal to n times the harmonic mean between two given numbers a and b.

B. Sum of the cubes of first n natural number is equal to square of the sum of the first a natural numbers.

C. If $a, A_1, A_2, A_3, \ldots\ldots, A_{2n}, b$ are in A.P. then $\sum_{I=1}^{2n} A_l = n(a+b)$.

D. If the first term of the geometric progression $g_1, g_2, g_3, \ldots\ldots, \infty$ is unity, then the value of the common ratio of the progression such that $(4g_2 + 5g_3)$ is minimum equals $\frac{2}{5}$.

Answer: B::C

Watch Video Solution

6. If a,b,c are in 3 distinct numbers in H.P. $a, b, c > 0$, then :

A. $\frac{b+c-a}{a}, \frac{a+b-c}{b}, \frac{a+b-c}{c}$ are in AP

B. $\frac{b+c}{a}, \frac{c+a}{b}, \frac{a+b}{c}$ ar in A.P.

C. $a^5 + c^5 \geq 2b^5$

D. $\frac{a-b}{b-c} = \frac{a}{c}$

Answer: A::B::C::D

Watch Video Solution

7. All roots of equation $x^5 - 40x^4 + \alpha x^3 + \beta x^2 + \gamma x + \delta = 0$ are in G.P. if the sum of their reciprocals is 10, then δ can be equal to :

A. 32

B. -32

C. $\frac{1}{32}$

D. $-\frac{1}{32}$

Answer: A::B

Watch Video Solution

8. Let $a_1, a_2, a_3 \ldots \ldots$ be a sequence of non-zero rela numbers with are in A.P. for $k \in N$. Let $f_k(x) = a_k x^2 + 2a_{k+1}x + a_{k+2}$

A. $f_k(x) = 0$ has real roots for each $k \in N$.

B. Each of $f_k(x) = 0$ has one root in common.

C. Non-common roots of $f_1(x) = 0, f_2(x) = 0, f_3(x) = 0, \ldots \ldots$ from an A.P.

D. None of these

Answer: A::B

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9. Given a,b,c are in A.P. b,c,d are in G.P. and c,d,e are in H.P. if $a=2$ and $e=18$, then the possible value of 'c' can be :

A. 9

B. -6

C. 6

D. -9

Answer: B::C

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10. The number a, b, c in that order form a term A.P and $a+b+c=60$. The number $(a-2), b, (c+3)$ in that order form a three G.P. All possible values of $\left(a^2+b^2+c^2\right)$ is/are

A. 1218

B. 1208

C. 1288

D. 1298

Answer: B::D

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11. If

$(x^2 + x + 1) + (x^2 + 2x + 3) + (x^2 + 3x + 5) + \ldots\ldots + (x^2 + 20x + 39$

then x is equal to :

A. 10

B. -10

C. 20.5

D. -20.5

Answer: A::D

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12. For ΔABC, if $81 + 144a^4 + 16b^4 + 9c^4 = 144$ abc, (where notations have their usual meaning), then :

A. $a > b > c$

B. $A < B < C$

C. Area of $\Delta ABC = \dfrac{3\sqrt{3}}{8}$

D. Triangle ABC is right angled

Answer: B::C::D

Watch Video Solution

13. Let $x, y, z \in \left(0, \dfrac{\pi}{2}\right)$ are first three consecutive terms of an arithmatic progression such that $\cos x + \cos y + \cos x = 1$ and $\sin x + \sin y + \sin x = \dfrac{1}{\sqrt{2}}$, then which of the following is/are correct ?

A. $\cot y = \sqrt{2}$

B. $\cos(x-y)=\dfrac{\sqrt{3}-\sqrt{2}}{2\sqrt{2}}$

C. $\tan 2y=\dfrac{2\sqrt{2}}{3}$

D. $\sin(x-y)+\sin(y-z)=0$

Answer: A::B

Watch Video Solution

14. If the number $16, 20, 16, d$ form a A.G.P. then d can be equal to :

A. 3

B. 11

C. -8

D. -16

Answer: D

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15. If $S_r = \sqrt{r + \sqrt{r + \sqrt{r + \sqrt{\ldots\ldots\infty}}}}, r > 0$ then which the following is\are correct.

A. S_2, S_6, S_{13}, S_{20} are in A.P.

B. S_4, S_9, S_{16} are irrational

C. $(2S_3 - 1)^2, (2S_4 - 1)^2, (2S_2 - 1)^2$ are in A.P.

D. S_2, S_{12}, S_{36} are in G.P.

Answer: A::B::C::D

Watch Video Solution

16. Consider the A.P. $50, 48, 46, 44.\ldots\ldots IfS_n$ denotes the sum to n terms of this A.P. then

A. S_n is maximum for $\pi = 25$

B. the first negative terms is 26^{th} term

C. the first negative term is 27^{th} term

D. the maximum value of S_n is 650

Answer: A::C::D

Watch Video Solution

17. Let S_n be the sum to n terms of the series $\frac{2}{1^2}+\frac{5}{1^2+2^2}+\frac{7}{1^2+2^2+3^2}+\frac{9}{1^2+2^2+3^2+6^2}+\ldots\ldots$ then

A. $S_5=5$

B. $S_{50}=\frac{100}{17}$

C. $\left(S_{1001}=\frac{1001}{97}\right.$

D. $S_\infty=6$

Answer: A::B::D

Watch Video Solution

18. For ΔABC, if $81 + 144a^4 + 16b^4 + 9c^4 = 144$ abc, (where notations have their usual meaning), then :

A. $a > b > c$

B. $A < B < C$

C. Area of $\Delta ABC = \frac{3\sqrt{3}}{8}$

D. Triangle ABC is right angled

Answer: B::C::D

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Exercise Comprehension Type Problems

1. The first four terms of a sequence are given by $T_1 = 0, T_2 = 1, T_3 = 1, T_4 = 2. The \geq \neq raltermsisgivenby$

T_(n)=Aalpha ^(n -1) +B beta ^(n-1)$whereA, B$ alpha, beta

$are \in dependentofa$ and $Aispositive. Thevalueof$(alpha ^(2) + beta ^(2)+ alpha beta)` is equal to :

A. 1

B. 2

C. 5

D. 4

Answer: B

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2. The first four terms of a sequence are given by $T_1 = 0, T_2 = 1, T_3 = 1, T_4 = 2. The \geq \neq raltermsisgivenby$ T_(n)=Aalpha ^(n -1) +B beta ^(n-1)$whereA, B$ alpha, beta $are \in dependentofa$ and $Aispositive. Thevalueof$5 (A^(2) + B ^(2)` is equal to :

A. 2

B. 4

C. 6

D. 8

Answer: A

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3. There are two sets A and B each of which consists of three numbers in A.P.whose sum is 15 and where D and d are the common differences such that $D - d = 1. If \frac{p}{q} = \frac{7}{8}$, where p and q are the product of the numbers ,respectively, and $d > 0$ in the two sets .

The sum of the product of the numbers in set B taken two at a time is

A. 51

B. 71

C. 74

D. 86

Answer: B

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4. There are two sets A and B each of which consists of three numbers in A.P.whose sum is 15 and where D and d are the common differences such that $D - d = 1. If \frac{p}{q} = \frac{7}{8}$, where p and q are the product of the numbers ,respectively, and $d > 0$ in the two sets .

The sum of the product of the numbers in set B taken two at a time is

A. 52

B. 54

C. 64

D. 74

Answer: D

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5. Let x, y, z are positive reals and $x + y + z = 60$ and $x > 3$.

Maximum value of $(x - 3)(y + 1)(x + 5)$ is :

A. $(17)(21)(25)$

B. $(20)(21)(23)$

C. $(21)(21)(21)$

D. $(23)(19)(15)$

Answer: C

Watch Video Solution

6. Let x, y, z are positive reals and $x + y + z = 60$ and $x > 3$.

Maximum value of $(x - 3)(2y + 1)(3z + 5)$ is:

A. $\frac{(355)^3}{3^3 . 6^2}$

B. $(355)^3$

C. $\frac{(355)^3}{3^2 \cdot 6^3}$

D. None of these

Answer: A

Watch Video Solution

7. Let x, y, z are positive reals and $x + y + z = 60 \text{ and } x > 3.$

Maximum value of xyz is :

A. 8×10^3

B. 27×10^3

C. 64×10^3

D. 125×10^3

Answer: A

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8. Two consecutive number from n natural numbers $1, 2, 3, \ldots\ldots, n$ are removed. Arithmetic mean of the remaining numbers is $\frac{105}{4}$.

The value of n is:

A. 48

B. 50

C. 52

D. 49

Answer: B

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9. Two consecutive number from n natural numbers $1, 2, 3, \ldots\ldots, n$ are removed. Arithmetic mean of the remaining numbers is $\frac{105}{4}$.

The G.M. of the removed numbers is :

A. $\sqrt{30}$

B. $\sqrt{42}$

C. $\sqrt{56}$

D. $\sqrt{72}$

Answer: C

Watch Video Solution

10. Two consecutive number from n natural numbers $1, 2, 3, \ldots\ldots,$ n are removed. Arithmetic mean of the remaining numbers is $\frac{105}{4}$.

Let removed numbers are x_1, x_2 then $x_1 + x_2 + n =$

A. 61

B. 63

C. 65

D. 69

Answer: C

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11. The sequence $\{a_n\}$ is defined by formula $a_0 = 4$ and $a_{m+1} = a_n^2 - 2a_n + 2$ for $n \geq 0$. Let the sequence $\{b_n\}$ is defined by formula $b_0 = \frac{1}{2}$ and $b_n = \frac{2a_0 a_1 a_2 \ldots \ldots . a_{n-1}}{\forall n \geq 1.}$

The value of a_{10} is equal to :

A. $1 + 2^{1024}$

B. 4^{1024}

C. $1 + 3^{1024}$

D. 6^{1024}

Answer: C

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12. The sequence $\{a_n\}$ is defined by formula $a_0 = 4$ and $a_{m+1} = a_n^2 - 2a_n + 2$ for $n \geq 0$. Let the sequence $\{b_n\}$ is

defined by formula $b_0 = \frac{1}{2}$ and $b_n = \frac{2a_0a_1a_2.....a_{n-1}}{\forall n \geq 1.}$

The value of n which $b_n = \frac{3280}{3281}$ is :

A. 2

B. 3

C. 4

D. 5

Answer: B

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13. The sequence $\{a_n\}$ is defined by formula $a_0 = 4$ and $a_{m+1} = a_n^2 - 2a_n + 2$ for $n \geq 0$. Let the sequence $\{b_n\}$ is defined by formula $b_0 = \frac{1}{2}$ and $b_n = \frac{2a_0a_1a_2.....a_{n-1}}{\forall n \geq 1.}$

The sequence $\{b_n\}$satisfies the recurrence formula :

A. $b_{n+1} = \frac{2b_n}{1 - b_n^2}$

B. $b_{n+1} = \frac{2b_n}{1 + b_n^2}$

C. $\dfrac{b_n}{1+b_n^2}$

D. $\dfrac{b_n}{1-b_n^2}$

Answer: B

View Text Solution

14. Let

$$f(n)=\sum_{r=2}^{n}\frac{r}{®C_2{}^{r+1}C_2},\ a=\lim_{x\to\infty} f(n)\ \text{and}\ x^2-\left(2n-\frac{1}{2}\right)x+t=0$$

has two positive roots α and β.

If value of $f(7)+f(8) is \dfrac{p}{q}$ where p and q are relatively prime, then $(p-q)$ is :

A. 53

B. 55

C. 57

D. 59

Answer: D

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15. Let

$$f(n) = \sum_{r=2}^{n} \frac{r}{{}^{r}C_2 {}^{r+1}C_2}, a = \lim_{x \to \infty} f(n) \text{ and } x^2 - \left(2n - \frac{1}{2}\right)x + t = 0$$

has two positive roots α and β.

minimum value of $\frac{4}{\alpha} + \frac{1}{\beta}$ is :

A. 2

B. 6

C. 3

D. 4

Answer: B

View Text Solution

16. Given the sequence of numbers $x_1, x_2, x_3, ...x_{1005}.$ which satisfy $\frac{x_1}{x_1+1} = \frac{x_2}{x_2+3} = \frac{x_3}{x_3+5} = ... = \frac{x_{1005}}{x_{1005}+2009.}$ Also, $x_1 + x_2 + ...x_{1005} = 2010.$ Nature of the sequence is

A. A.P.

B. G.P.

C. A.G.R

D. H.R.

Answer: A

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17. Given that sequence of number $a_1, a_2, a_3, \ldots\ldots, a_{1005}$ which satisfy $\frac{a_1}{a_1+1} = \frac{a_2}{a_2+3} + \frac{a_3}{a_3+5} = \ldots\ldots = \frac{a_{1005}}{a_{1005}+2009}$ $a_1 + a_2 + a_3 \ldots\ldots a_{1005} = 2010$ find the 21^{st} term of the sequence is equal to :

A. $\frac{86}{1065}$

B. $\frac{83}{1005}$

C. $\frac{82}{1005}$

D. $\frac{79}{1005}$

Answer: C

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Exercise Subjective Type Problems

1. Let a, b, c, d be four distinct real numbers in A.P. Then half of the smallest positive valueof k satisfying $a(a-b)+k(b-c)^2=(c-a)^3=2(a-x)+(b-d)^2+(c-d)^3$ is ________.

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2. The sum of all digits of n for which $\sum_{r=1}^{n} r2^r = 2 + 2^{n+10}$ is :

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3. If $\lim_{x \to \infty} \frac{r+2}{2^{r+1} r(r+1)} = \frac{1}{k}$, then k =

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4. The value of $\sum_{r=1}^{\infty} \frac{8r}{4r^4+1}$ is equal to :

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5. Three non-zero real numbers from an A.P. and the squares of these numbers taken in same order from a G.P. Then, the number of all possible value of common ratio of the G.P. is

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6. The sum of the fourth and twelfth term of an arithmetic progression is 20. What is the sum of the first 15 terms of the arithmetic progression?

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7. In an increasing sequence of four positive integers, the first 3 terms are in A.P., the last 3 terms are in G.P. and the fourth term exceed the first term by 30, then the common difference of A.P. lying in interval $[1, 9]$ is:

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8. The limit of $\frac{1}{n^4}\sum_{k=1}^{n} k(k+2)(k+4) as n \to \infty$ is equal to $\frac{1}{\lambda}$, then $\lambda =$

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9. Which is the last digit of $1+2+3+\ldots\ldots+$ n if the last digit of $1^3+2^3+\ldots\ldots+n^3$ is 1 ?

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10. There distinct positive numbers, a,b,c are in G.P. while $\log_c a, \log_b c, \log_a b$ are in A.P. with non-zero common difference d, then $2d=$

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11. The numbers $\frac{1}{3}, \frac{1}{3}\log_x y, \frac{1}{3}\log_y z, \frac{1}{7}\log_x x$ are in H.P. If $y=x^{\circledR}$ and $z=x^s$, then $4(r+s)=$

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12. If $\sum_{k=1}^{\infty} \frac{k^2}{3^k} = \frac{p}{q}$, where p and q are relatively prime positive integers. Find the value of $(p+q)$,

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13. The sum of the terms of an infinitely decreassing Geometric Progression (GP) is equal to the greatest value of the function $f(x) = x^3 + 3x - 9$ where $x \in [-4, 3]$ and the difference between the first and second term is $f'(0)$. The common ratio $r = \frac{p}{q}$ where p and q are relatively prime positive integers. Find $(p+q)$.

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14. A cricketer has to score 4500 runs. Let a_n denotes the number of runs he scores in the n^{th} match. If $a_1 = a_2 = \dots a_{10} = 150$ and $a_{10}, a_{11}, a_{12}\dots$ are in A.P. with common difference (-2). If N be the

total number of matches played by him to scoere 4500 runs. Find the sum of the digits of N.

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15. If $x = 10\sum_{r=3}^{100} \frac{1}{(r^2 - 4)}$, then $[x] =$

(where [.] denotes gratest integer function)

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16. Let $f(n) = \frac{4n + \sqrt{4n^2 + 1}}{\sqrt{2n + 1} + \sqrt{2n - 1}}, n \in N$ then the remainder when $f(1) + f(2) + f(3) + \ldots. + f(60)$ is divided by 9 is.

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17. Find the sum of series $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \frac{1}{9} + \frac{1}{12} + \ldots\ldots\infty,$ where the term are the reciprocals of the positive integers whose only prime factors are two's and three's :

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18. Let $a_1, a_2, a_3, \ldots\ldots, a_n$ be real numbers in arithmatic progressin such that $a_1 = 15$ and a_2 is an integer.Given $\sum_{r=1}^{10} (a_r)^2 = 1185. If S_n = \sum_{r=1}^{n} a_r$ and maximum value of n is N for which $S_n \geq S_{(n+1)}$, then find $N - 10$.

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19. Let the roots of the equation $24x^3 - 14x^2 + kx + 3 = 0$ form a geometric sequence of real numbers. If absolute value of k lies between the roots of the equation $x^2 + \alpha^2 x - 122 = 0$, then the largest integral value of α is :

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20. How many ordered pair (s) satisfy

$$\log\left(x^2+\frac{1}{3}y^3+\frac{1}{9}\right)=\log x+\log y$$

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21. The value of xyz is 55 or 343/55 according as the sequence a,x,y,z,b is an H.P or A.P Find the sum (a+b) given that a and b are positive integers

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