

MATHS

BOOKS - VIKAS GUPTA MATHS (HINGLISH)

SOLUTION OF TRIANGLES

Exercise 1 Single Choice Problems

1. In a ΔABC if $9(a^2 + b^2) = 17c^2$ then the value of the $\frac{\cot A + \cot B}{\cot C}$ is A. $\frac{13}{4}$ B. $\frac{7}{4}$ C. $\frac{5}{4}$

Answer: D

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2. Let H be the orthocenter of triangle ABC, then angle subtended by side BC at the centre of incircle of ΔCHB is :

A.
$$\frac{A}{2} + \frac{\pi}{2}$$

B.
$$\frac{B+C}{2} + \frac{\pi}{2}$$

C.
$$\frac{B-C}{2} + \frac{\pi}{2}$$

D.
$$\frac{B+C}{2} + \frac{\pi}{4}$$

Answer: B

3. Circumradius of a ΔABC is 3 units, let O be the circumcentre and H be the orthocentre then the value of $rac{1}{64} (AH^2 + BC^2) (BH^2 + AC^2) (CH^2 + AB^2)$ equals :

A. 3^4

 $\mathsf{B.}\,9^3$

 $C. 27^{6}$

D. 81^4

Answer: B

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4. The angles A, B and C of a triangle ABC are in arithmetic

progression. If $2b^2=3c^2$ then the angle A is :

A. 15°

 $\mathrm{B.\,60}^{\,\circ}$

C. 75°

D. $90\,^\circ$

Answer: C

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5. In a triangle ABC if $an. \frac{A}{2} an. \frac{B}{2} = \frac{1}{3}$ and ab = 4, then the

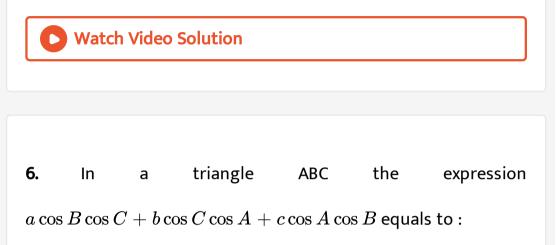
value of c can be

A. 1

B. 2

C. 4

Answer: B



A.
$$\frac{rs}{R}$$

B. $\frac{r}{sR}$
C. $\frac{R}{rs}$
D. $\frac{Rs}{r}$

Answer: A

7. The set of all real numbers a such that $a^2 + 2a, 2a + 3, anda^2 + 3a + 8$ are the sides of a triangle is____

A. $(0,\infty)$ B. (5,8)C. $\left(-\frac{11}{3},\infty\right)$ D. $(5,\infty)$

Answer: D

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8. In a $\triangle ABC$, $\angle B = \frac{\pi}{3}$ and $\angle C = \frac{\pi}{4}$ let D divide BC internally in the ratio 1:3, then $\frac{\sin(\angle BAD)}{\sin(\angle CAD)}$ is equal to :

A.
$$\frac{1}{\sqrt{6}}$$

B.
$$\frac{1}{3}$$

C.
$$\frac{1}{\sqrt{3}}$$

D.
$$\frac{\sqrt{2}}{3}$$

Answer: A



9. Let AD, BE, CF be the lengths of internal bisectors of angles A,

B, C respectively of triangle ABC. Then the harmonic mean of

$$AD \mathrm{sec} rac{A}{2}, BE \mathrm{sec} rac{B}{2}, CF \mathrm{sec} rac{C}{2}$$
 is equal to :

A. Harmonic mean of sides of ΔABC

B. Geometric mean of sides of ΔABC

C. Arithmetic mean of sides of ΔABC

D. Sum of reciprocals of the sides of ΔABC

Answer: A

10. In a triangle ABC, if 2b = a + c and A - C = 90, then $\sin B$

equals

A.
$$\frac{\sqrt{7}}{5}$$

B.
$$\frac{\sqrt{5}}{8}$$

C.
$$\frac{\sqrt{7}}{4}$$

D.
$$\frac{\sqrt{5}}{3}$$

Answer: C

11. In a triangle ABC, if
$$2a\cos\left(rac{B-C}{2}
ight)=b+c$$
, then secA is

equal to :

A.
$$\frac{2}{\sqrt{3}}$$

B. $\sqrt{2}$
C. 2

Answer: C

D. 3

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12. In a triangle ABC if BC = 1 and AC = 2, then what is

the maximum possible value of angle A?

A.
$$\frac{\pi}{6}$$

B. $\frac{\pi}{4}$
C. $\frac{\pi}{3}$
D. $\frac{\pi}{2}$

Answer: A



13. $\Delta I_1 I_2 I_3$ is an excentral triangle of an equilateral triangle ΔABC such that $I_1 I_2 = 4$ unit, if ΔDEF is pedal triangle of ΔABC , then $\frac{Ar(\Delta I_1 I_2 I_3)}{Ar(\Delta DEF)} =$

A. 16

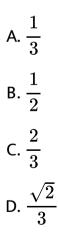
B.4

C. 2

Answer: A

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14. Let ABC be a triangle with $\angle BAC = 2\pi/3$ and AB = x such that (AB) (AC) = 1. If x varies, then find the longest possible length of the angle bisector AD



Answer: B

15. In a equilateral triangle r, R and r_1 form (where symbols used

have usual meaning)

A. an A.P.

B. a G.P.

C. an H.P.

D. none of these

Answer: A

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16. In ΔABC if $rac{\sin A}{\sin C} = rac{\sin(A-B)}{\sin(B-C)}$, then a^2, b^2, c^2 are in :

A. A.P.

B. G.P.

C. H.P.

D. none of these

Answer: A



17. In
$$\triangle ABC$$
, $\tan A = 2$, $\tan B = \frac{3}{2}$ and $c = \sqrt{65}$, then

circumradius of the triangle is :

A. 65

B.
$$\frac{65}{7}$$

C. $\frac{65}{14}$

D. none of these

Answer: C

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18. If the sides a, b, c of a triangle ABC are the roots of the equation $x^3 - 13x^2 + 54x - 72 = 0$, then the value of $rac{\cos A}{a} + rac{\cos B}{b} + rac{\cos C}{c}$ is equal to : A. $\frac{61}{144}$ B. $\frac{61}{72}$ C. $\frac{169}{144}$ D. $\frac{59}{144}$

Answer: A

19. In
$$\triangle ABC$$
, if $\angle C=90^\circ,$ then $\frac{a+c}{b}+\frac{b+c}{a}$ is equal to :

A.
$$\frac{c}{r}$$

B. $\frac{1}{2Rr}$
C. 2

D.
$$\frac{R}{r}$$

Answer: A



20. In a ΔABC , if $a^2 \sin B = b^2 + c^2$, then :

A. $\angle A$ is obtuse

B. $\angle A$ is acute

C. $\angle B$ is abtuse

D. $\angle A$ is right angle

Answer: A



21. If R and R' are the circumradii of triangles ABC and OBC, where O is the orthocenter of triangle ABC, then :

A.
$$R' = \frac{R}{2}$$

B. $R' = 2R$
C. $R' = R$

D. R'=3R

Answer: C



22. The acute angle of a rhombus whose side is a mean proportional between its diagonals is

A. $15^{\,\circ}$

B. 20°

C. 30°

D. $60^{\,\circ}$

Answer: C



23. In a $\triangle ABC$ right angled at A, a line is drawn through A to meet BC at D dividing BC in 2:1. If $\tan(\angle ADC) = 3$ then $\angle BAD$ is :

A. 30°

B. 45°

 $\mathrm{C.\,60}^\circ$

D. 75°

Answer: B



24. A circle is cirumscribed in an equilateral triangle of side 'l'.

The area of any square inscribed in the circle is :

A.
$$\frac{4}{3}l^2$$

B. $\frac{2}{3}l^2$
C. $\frac{1}{3}l^2$
D. l^2

Answer: B

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25. if the sides of a triangle are in the ratio $2: \sqrt{6}: \sqrt{3} + 1$, then the largest ange of the trangle will be (1) 60 (3) 90 (2) 75 (4) 120

A. $60^{\,\circ}$

B. 72°

C. 75°

D. 90°

Answer: C

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26. In a triangle ABC if a, b, c are in A.P. and $C-A=120^\circ$, then

 $\frac{s}{r}$ = (where notations have their usual meaning)

- A. $\sqrt{15}$
- B. $2\sqrt{15}$
- C. $3\sqrt{15}$
- D. $6\sqrt{15}$

Answer: C

27. If in $\triangle ABC$, a = 5, b = 4 and $\cos(A - B) = \frac{31}{32}$, then

A.
$$\sqrt{6}$$

B. $6\sqrt{6}$

C. 6

D. $(216)^{1/4}$

Answer: C



28. If semiperimeter of a triangle is 15, then the value of $(b+c)\cos(B+C) + (c+a)\cos(C+A) + (a+b)\cos(A+B)$ is equal to :

(where symbols used have usual meanings)

A. -60

B. -15

C. -30

D. can not be determined

Answer: C



29. Let triangle ABC be an isosceles with AB=AC. Suppose that the angle bisector of its angle B meets the side AC at a point D and that BC = BD + AD. Measure of the angle A in degrees, is :

A. 80

B. 100

C. 110

D. 130

Answer: B

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30. In triangle ABC if A:B:C=1:2:4, then $(a^2-b^2)(b^2-c^2)(c^2-a^2)=\lambda a^2b^2c^2$, where $\lambda=$

(where notations have their usual meaning)

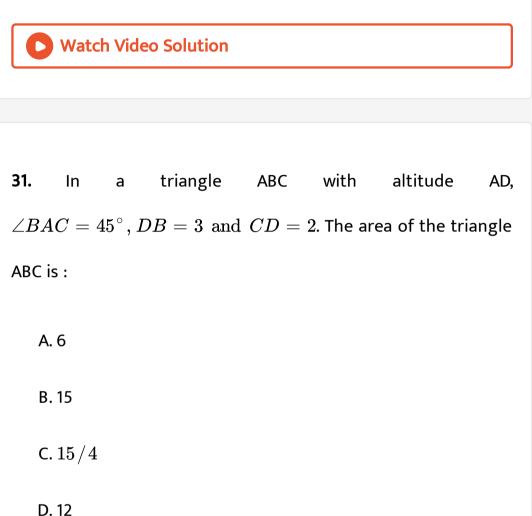
A. 1

B. 2

C. 4

D. 9

Answer: A



Answer: B

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32. A triangle has base 10 cm long and the base angles of 50° and 70° . If the perimeter of the triangle is $x + y \cos z^{\circ}$ where $z \in (0, 90)$ then the value of x + y + z equals :

A. 60

B. 55

C. 50

D. 40

Answer: D

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33. Let H be the orthocenter of triangle ABC, then angle subtended by side BC at the centre of incircle of ΔCHB is :

A.
$$\frac{A}{2} + \frac{\pi}{2}$$

B.
$$\frac{B+C}{2} + \frac{\pi}{2}$$

C.
$$\frac{B-C}{2} + \frac{\pi}{2}$$

D.
$$\frac{B+C}{2} + \frac{\pi}{4}$$

Answer: B

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34. Triangle ABC is right angle at A. The points P and Q are on hypotenuse BC such that BP = PQ = QC.if AP = 3 and AQ = 4, then length BC is equal to

A. $\sqrt{27}$

B. $\sqrt{36}$

C. $\sqrt{45}$

D.
$$\sqrt{54}$$

Answer: C

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35. In a $\triangle ABC$ if $b = a \left(\sqrt{3} - 1 \right)$ and $\angle C = 30^\circ$ then the measure of the angle A is

A. $15^{\,\circ}$

B. 45°

C. 75°

D. $105^{\,\circ}$

Answer: D

36. Through the centroid of an equilateral triangle, a line parallel to the base is drawn. On this line, an arbitrary point P is taken inside the triangle. Let h denote the perpendicular distance of P from the base of the triangle. Let h_1 and h_2 be the perpendicular distance of P from the other two sides of the triangle . Then :

A.
$$h = rac{h_1 + h_2}{2}$$

B. $h = \sqrt{h_1 h_2}$
C. $h = rac{2h_1 h_2}{h_1 + h_2}$
D. $h = rac{(h_1 + h_2)\sqrt{3}}{4}$

Answer: A



37. The angles A, B and C of a triangle ABC are in arithmetic progression. AB=6 and BC=7. Then AC is :

A. $\sqrt{41}$

B. $\sqrt{39}$

 $\mathsf{C}.\sqrt{42}$

D. $\sqrt{43}$

Answer: D



38. In $\triangle ABC$, if $A - B = 120^{\circ}$ and R = 8r, then the value of $\frac{1 + \cos C}{1 - \cos C}$ equals :

(All symbols used hav their usual meaning in a triangle)

A. 12

B. 15

C. 21

D. 31

Answer: B

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39. The lengths of the sides CB and CA of a triangle ABC are given by a and b and the angle C is $\frac{2\pi}{3}$. The line CD bisects the angle C and meets AB at D. Then the length of CD is :

A.
$$\frac{1}{a+b}$$

B. $\frac{a^2+b^2}{a+b}$
C. $\frac{ab}{2(a+b)}$

D.
$$rac{ab}{a+b}$$

Answer: D

View Text Solution 40. In $\triangle ABC$, angle A is $120^{\circ}, BC+CA=20, ~{
m and}~ AB+BC=21$ Find the length of the side BC A. 13 B. 15 C. 17 D. 19

Answer: A



41. A triangle has sides 6,7, and 8. The line through its incenter parallel to the shortest side is drawn to meet the other two sides at P and Q. Then find the length of the segment PQ.

A.
$$\frac{12}{5}$$

B. $\frac{15}{4}$
C. $\frac{30}{7}$
D. $\frac{33}{9}$

Answer: C



42. The perimeter of a ΔABC is 48 cm and one side is 20 cm.

Then remaining sides of ΔABC must be greater than :

A. 8 cm

B. 9 cm

C. 12 cm

D. 4 cm

Answer: D

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43. In a equilateral triangle r, R and r_1 form (where symbols used

have usual meaning)

A. an A.P.

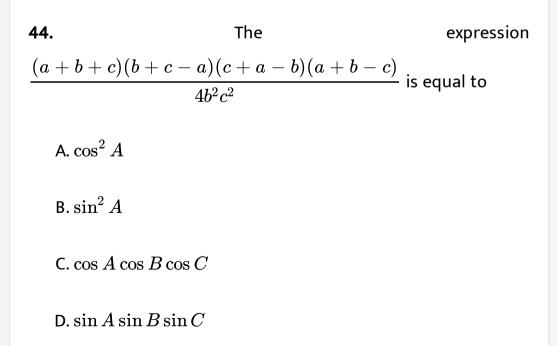
B. a G.P.

C. an H.P.

D. neither an A.P., G.P. nor H.P.

Answer: A

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Answer: B



45. Circumradius of an isosceles ΔABC with $\angle A = \angle B$ is 4 times its in radius, then cosA is root of the equation :

A.
$$x^2-x-8=0$$

B.
$$8x^2-8x+1=0$$

$$\mathsf{C.}\,x^2-x-4=0$$

D.
$$4x^2 - 4x + 1 = 0$$

Answer: B

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46. A is the orthocentre of ΔABC and D is reflection point of A w.r.t. perpendicualr bisector of BC, then orthocenter of ΔDBC is :

- A. D
- B.C
- С. В
- D. A

Answer: A

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47. If $a \neq b \neq c$ are all positive, then the value of the determinant $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is

A. ≥ 0

 $\mathsf{B.} > 0$

C. \leq -1

D. < 0

Answer: D

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48. In triangle ABC if A:B:C=1:2:4, then $(a^2-b^2)(b^2-c^2)(c^2-a^2)=\lambda a^2b^2c^2$, where $\lambda=$

(where notations have their usual meaning)

A. 1

B. 2

C. 3

 $\mathsf{D}.\,\frac{1}{3}$

Answer: A

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49. In any triangle, the minimum value of $r_1r_2r_3\,/\,r^3$ is equal to

A. 1

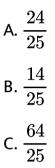
B. 3

C. 8

D. 27

Answer: D

50. In ΔABC , a=3, b=4 and c=5, then value of $\sin A + \sin 2B + \sin 3C$ is



D. None

Answer: B

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51. In any triangle ABC, the value of $rac{r_1+r_2}{1+\cos C}$ is equal to

(where notation have their usual meaning) :

A. 2R

B. 2r

C. R

D.
$$rac{2R^2}{r}$$

Answer: A



52. In a triangle ABC, medians AD and BE are deawn. IF $AD = 4, \angle DAB = \frac{\pi}{6}$ and $\angle ABE = \frac{\pi}{3}$, then the area of the triangle ABC is-

A.
$$\frac{8}{3\sqrt{3}}$$

B.
$$\frac{16}{3\sqrt{3}}$$

C.
$$\frac{32}{3\sqrt{3}}$$

D.
$$\frac{64}{3\sqrt{3}}$$

Answer: C

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53. The sides of a triangle are $\sin \alpha$, $\cos \alpha$ and $\sqrt{1 + \sin \alpha \cos \alpha}$ for some α , $0 < \alpha < \frac{\pi}{2}$. Then the greatest angle of the triangle .

is

A.
$$\frac{\pi}{3}$$

B. $\frac{\pi}{2}$
C. $\frac{2\pi}{3}$
D. $\frac{5\pi}{6}$

Answer: C

54. Let ABC be a right with $\angle BAC = rac{\pi}{2}$, then $\left(rac{r^2}{2R^2} + rac{r}{R}
ight)$ is

equal to :

(where symbols used have usual meaning in a striangle)

A. sinB sinC

B. tanB tanC

C. secB secC

D. cotB cotC

Answer: A

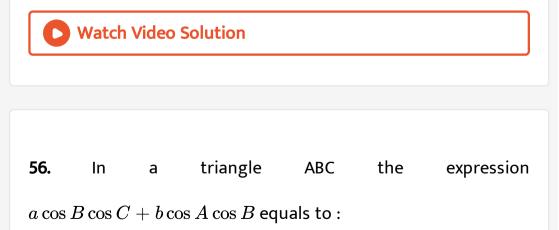


55. In a $\triangle ABC$, with usual notations, if b > c then distance between foot of median and foot of altitude both drawn from vertex A on BC is :

A.
$$\frac{a^2 - b^2}{2c}$$

B. $\frac{b^2 - c^2}{2a}$
C. $\frac{b^2 + c^2 - a^2}{2a}$
D. $\frac{b^2 + c^2 - a^2}{2c}$

Answer: B



A.
$$\frac{rs}{R}$$

B. $\frac{r}{sR}$
C. $\frac{R}{rs}$
D. $\frac{Rs}{r}$

Answer: A

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57. In a acute triangle ABC, altitudes from the vertices A, B and C meet the opposite sides at the points D, E and F respectively. If the radisu of the circumcircle of ΔAFE , ΔBFD , ΔCED , ΔABC be respectively R_1 , R_2 , R_3 and R. Then the maximum value of $R_1 + R_2 + R_3$ is :

A.
$$\frac{3R}{8}$$

B. $\frac{2R}{3}$
C. $\frac{4R}{3}$
D. $\frac{3R}{2}$

Answer: D

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Exercise 2 One Or More Than One Answer Is Are Correct

1. If r_1, r_2, r_3 are radii of the escribed circles of a triangle ABC and r it the radius of its incircle, then the root(s) of the equation $x^2 - r(r_1r_2 + r_2r_3 + r_3r_1)x + (r_1r_2r_3 - 1) = 0$ is/are : A. r_1

 $\mathsf{B.}\,r_2+r_3$

C. 1

D. $r_1 r_2 r_3 - 1$

Answer: C::D

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2. In $\Delta ABC, ar{A} = 60^\circ, ar{B} = 90^\circ, ar{C} = 30^\circ.$ Let H be its

orthocentre, then :

(where symbols used have usual meanings)

A. AH = c

 $\mathsf{B.}\, CH=a$

 $\mathsf{C}.\,AH=a$

 $\mathsf{D}.\,BH=0$

Answer: A::B::D



3. In an equilateral triangle, if inradius is a rational number then

A. circumradius is always rational

B. exradii are always rational

C. area is always ir-rational

D. perimeter is always rational

Answer: A::B::C



4. Let A, B, C be angles of a triangle ABC and let

$$D = \frac{5\pi + A}{32}, E = \frac{5\pi + B}{32}, F = \frac{5\pi + C}{32}$$
, then :
(where $D, E, F \neq \frac{n\pi}{2}, n \in I, I$ denote set of integers)
A. $\cot D \cot E + \cot E \cot F + \cot D \cot F = 1$
B. $\cot D + \cot E + \cot F = \cot D \cot E \cot F$
C. $\tan D \tan E + \tan E \tan F + \tan F \tan D = 1$
D. $\tan D + \tan E + \tan F = \tan D \tan E \tan F$

Answer: B::C

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5. In a triangle ABC, if a=4, b=8 and $\angle C=60^{\,\circ}$, then :

(where symbols used have usual meanings)

A. c=6B. $c=4\sqrt{3}$ C. $\angle A=30^{\circ}$ D. $\angle B=90^{\circ}$

Answer: B::C::D

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6. In a $\triangle ABC$ if $\frac{r}{r_1} = \frac{r_2}{r_3}$, then which of the following is/are true ?

(where symbols used have usual meanings)

A.
$$a^2+b^2+c^2=8R^2$$

B. $\sin^2 A+\sin^2 B+\sin^2 C=2$
C. $a^2+b^2=c^2$

D.
$$\Delta = s(s+c)$$

Answer: A::B::C



7. ABC is a triangle whose circumcentre, incentre and orthocentre are O, I and H respectively which lie inside the triangle, then :

A.
$$\angle BOC = A$$

B. $\angle BIC = \frac{\pi}{2} + \frac{A}{2}$
C. $\angle BHC = \pi - A$
D. $\angle BHC = \pi - \frac{A}{2}$

Answer: B::C



8. In a triangle ABC, tanA and tanB satisfy the inequality $\sqrt{3}x^2 - 4x + \sqrt{3} < 0$, then which of the following must be correct ?

(where symbols used have usual meanings)

A.
$$a^2+b^2-ab < c^2$$

B. $a^2+b^2 > c^2$
C. $a^2+b^2+ab > c^2$
D. $a^2+b^2 < c^2$

Answer: A::C

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9. If in $\triangle ABC, \angle C = \frac{\pi}{8}, a = \sqrt{2}$ and $b = \sqrt{2 + \sqrt{2}}$ then

find the measure of angle A (in degree).

A. $45^{\,\circ}$

B. 135°

C. 30°

D. 150°

Answer: A

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10. In triangle ABC, a=3, b=4, c=2. Point D and E trisect the

side BC. If $\angle DAE = heta$, then $\cot^2 heta$ is divisible by :

B. 3

C. 5

D. 7

Answer: B::C



11. In a triangle ABC, 3sinA + 4cosB = 6 and 4sinB + 3cosA = 1.

Find the measure of angle C.

A.
$$\frac{\pi}{4}$$

B. $\frac{\pi}{6}$
C. $\frac{\pi}{3}$
D. $\frac{5\pi}{6}$

Answer: B

?

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12. If the line joining the incentre to the centroid of a triangle ABC is parallel to the side BC. Which of the following are correct

A. 2b = a + cB. 2a = b + cC. $\cot \frac{A}{2} \cot \frac{C}{2} = 3$ D. $\cot \frac{B}{2} \cot \frac{C}{2} = 3$

Answer: B::D

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13. In a triangle the lengths of the two larger are 10 and 9 respectively. If the angles are in A.P., the , length of the third side can be (A) $5 - \sqrt{6}$ (B) $3\sqrt{3}$ (C) 5 (D) $5 + \sqrt{6}$

A. $5 - \sqrt{6}$ B. $5 + \sqrt{6}$ C. $6 - \sqrt{5}$ D. $6 + \sqrt{5}$

Answer: A::B



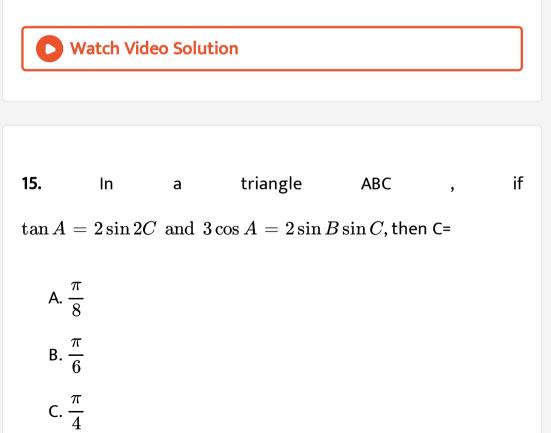
14. If area of $\Delta ABC(\Delta)$ and angle C are given and if c opposite

to given angle is minimum, then

A.
$$a = \sqrt{\frac{2\Delta}{\sin C}}$$

B. $b = \left(\frac{2\Delta}{\sin C}\right)$
C. $a = \frac{4\Delta}{\sin C}$
D. $b = \frac{4\Delta}{\sin^2 C}$

Answer: A::B



Answer: C::D



Exercise 3 Comprehension Type Problems

1. Let $\angle A=23^\circ, \angle B=75^\circ\,\, {
m and}\,\, \angle C=82^\circ\,\,$ be the angles of $\Delta ABC.$

The incircle of ΔABC touches the sides BC, CA, AB at points D, E, F respectively. Let r', r'_1 respectively be the inradius, exradius opposite to vertex D of ΔDEF and r be inradius of ΔABC , then

Q.
$$rac{r'}{r}=$$

A. ${
m sin}rac{A}{2}+{
m sin}rac{B}{2}+{
m sin}rac{C}{2}-1$

$$\begin{array}{l} \mathsf{B}.\,1-\sin\frac{A}{2}+\sin\frac{B}{2}+\sin\frac{C}{2}\\ \mathsf{C}.\,\cos\frac{A}{2}+\cos\frac{B}{2}+\cos\frac{C}{2}-1\\ \mathsf{D}.\,1-\cos\frac{A}{2}+\cos\frac{B}{2}+\cos\frac{C}{2}\end{array}$$

Answer: A



2. Let $\angle A=23^\circ, \angle B=75^\circ \,\, {
m and} \,\, \angle C=82^\circ \,\,$ be the angles of $\Delta ABC.$

The incircle of ΔABC touches the sides BC, CA, AB at points D, E, F respectively. Let r', r'_1 respectively be the inradius, exradius opposite to vertex D of ΔDEF and r be inradius of ΔABC , then

$$\mathtt{Q}.\,\frac{r_1^{\,\prime}}{r} =$$

A.
$$\sin \frac{A}{2} + \sin \frac{B}{2} + \sin \frac{C}{2} - 1$$

B. $1 - \sin \frac{A}{2} + \sin \frac{B}{2} + \sin \frac{C}{2}$
C. $\cos \frac{A}{2} + \cos \frac{B}{2} + \cos \frac{C}{2} - 1$
D. $1 - \cos \frac{A}{2} + \cos \frac{B}{2} + \cos \frac{C}{2}$

Answer: B

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3. Internal bisectors of ΔABC meet the circumcircle at point D,

E, and F

Area of ΔDEF is

A.
$$2R^2 \cos^2\left(\frac{A}{2}\right) \cos^2\left(\frac{B}{2}\right) \cos^2\left(\frac{C}{2}\right)$$

B. $2R^2 \sin\left(\frac{A}{2}\right) \sin\left(\frac{B}{2}\right) \sin\left(\frac{C}{2}\right)$
C. $2R^2 \sin^2\left(\frac{A}{2}\right) \sin^2\left(\frac{B}{2}\right) \sin^2\left(\frac{C}{2}\right)$

D.
$$2R^2 \cos\left(\frac{A}{2}\right) \cos\left(\frac{B}{2}\right) \cos\left(\frac{C}{2}\right)$$

Answer: D



- 4. Internal angle bisecotors of ΔABC meets its circum circle at
- D, E and F where symbols have usual meaning.
- Q. The ratio of area of triangle ABC and triangle DEF is :
 - A. ≥ 1
 - B. ≤ 1
 - $\mathsf{C.}\ \geq 1/2$
 - D. $\leq 1/2$

Answer: B



5. Let triangle ABC is right triangle right angled at C such that

$$A < B$$
 and $r = 8, R = 41$.

Q. Area of ΔABC is :

A. 720

B.1440

C. 360

D. 480

Answer: A



6. Let triangle ABC is right triangle right angled at C such that

$$A < B$$
 and $r = 8, R = 41$.
Q. $tan \frac{A}{2} =$
A. $\frac{1}{18}$
B. $\frac{1}{3}$
C. $\frac{1}{6}$
D. $\frac{1}{9}$

Answer: D



7. Let the incircle of ΔABC touches the sides BC, CA, AB at A_1, B_1, C_1 respectively. The incircle of $\Delta A_1B_1C_1$ touches its sides of B_1C_1, C_1A_1 and A_1B_1 at A_2, B_2, C_2 respectively

and so on.

- Q. $\lim_{n o \infty} \ \angle A_n =$
 - A. 0
 - B. $\frac{\pi}{6}$ C. $\frac{\pi}{4}$ D. $\frac{\pi}{3}$

Answer: D



8. Let the incircle of ΔABC touches the sides BC, CA, AB at A_1, B_1, C_1 respectively. The incircle of $\Delta A_1B_1C_1$ touches its sides of B_1C_1, C_1A_1 and A_1B_1 at A_2, B_2, C_2 respectively

and so on.

Q. In $\Delta A_4 B_4 C_4$, the value of $\angle A_4$ is:

A.
$$\frac{3\pi + A}{6}$$

B.
$$\frac{3\pi - A}{8}$$

C.
$$\frac{5\pi - A}{16}$$

D.
$$\frac{5\pi + A}{16}$$

Answer: D

9. Let ABC be a given triangle. Points D and E are on sides AB and AC respectively and point F is on line segment DE. Let $\frac{AD}{AB} = x, \frac{AE}{AC} = y, \frac{DF}{DE} = z$. Let area of $\Delta BDF = \Delta_1$, Area of $\Delta CEF = \Delta_2$ and area of $\Delta ABC = \Delta.$

Q.
$$rac{\Delta_1}{\Delta}$$
 is equal to :
A. xyz
B. $(1-x)y(1-z)$
C. $(1-x)yz$
D. $x(1-y)z$

Answer: C



10. Let ABC be a given triangle. Points D and E are on sides AB and AC respectively and point F is on line segment DE. Let $\frac{AD}{AB} = x, \frac{AE}{AC} = y, \frac{DF}{DE} = z$. Let area of $\Delta BDF = \Delta_1$, Area of $\Delta CEF = \Delta_2$ and area of $\Delta ABC = \Delta.$

Q.
$$rac{\Delta_2}{\Delta}$$
 is equal to :
A. $(1-x)y(1-z)$
B. $(1-x)(1-y)z$
C. $x(1-y)(1-z)$
D. $(1-x)yz$

Answer: C



11. a, b, c ar the length of sides BC, CA, AB respectively of ΔABC satisfying $\log(1 + \frac{c}{a}) + \log a - \log b = \log 2$. Also the quadratic equation $a(1 - x^2) + 2bx + c(1 + x^2) = 0$ has two equal roots. Q. a, b, c are in : A. A.P.

B. G.P.

C. H.P.

D. None

Answer: A

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12. a, b, c ar the length of sides BC, CA, AB respectively of ΔABC satisfying $\log\left(a+\frac{c}{a}\right) + \log a - \log b = \log 2$. Also the quadratic equation $a(1-x^2) + 2bx + c(1+x^2) = 0$ has two equal roots.

Q. Measure of angle C is :

A. 30°

B. 45°

C. 60°

D. 90°

Answer: D



13. If a,b,c are the sides of triangle ABC satisfying $\log(1+\frac{c}{a}) + \log a - \log b = \log 2$. Also $a(1-x^2) + 2bx + c(1+x^2) = 0$ has two equal roots. Find the value of $\sin A + \sin B + \sin C$.

A.
$$\frac{5}{2}$$

B. $\frac{12}{5}$
C. $\frac{8}{3}$

Answer: B

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14. Let ABC be a triangle inscribed in a circle and let $l_a = \frac{m_a}{M_a}, l_b = \frac{m_b}{M_b}, l_c = \frac{m_c}{M_c}$ where m_a, m_b, m_c are the lengths of the angle bisectors of angles A, B and C respectively, internal to the triangle and M_a, M_b and M_c are the lengths of these internal angle bisectors extended until they meet the circumcircle.

Q. l_a equals :

A.
$$rac{\sin A}{\sin \left(B+rac{A}{2}
ight)}$$

B. $rac{\sin B \sin C}{\sin^2 \left(rac{B+C}{2}
ight)}$

C.
$$\frac{\sin B \sin C}{\sin^2 \left(B + \frac{A}{2}\right)}$$

D.
$$\frac{\sin B + \sin C}{\sin^2 \left(B + \frac{A}{2}\right)}$$

Answer: C

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15. Let ABC be a triangle inscribed in a circle and let $l_a = \frac{m_a}{M_a}, l_b = \frac{m_b}{M_b}, l_c = \frac{m_c}{M_c}$ where m_a, m_b, m_c are the lengths of the angle bisectors of angles A, B and C respectively, internal to the triangle and M_a, M_b and M_c are the lengths of these internal angle bisectors extended until they meet the circumcircle.

Q. The maximum value of the product

$$(l_a l_b l_c) imes \cos^2 \! \left(rac{B-C}{2}
ight) imes rac{\cos^2 (C-A)}{2}
ight) imes \cos^2 \! \left(rac{A-B}{2}
ight)$$

is equal to :

A.
$$\frac{1}{8}$$

B. $\frac{1}{64}$
C. $\frac{27}{64}$
D. $\frac{27}{32}$

Answer: C

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16. Let ABC be a triangle inscribed in a circle and let $l_a = \frac{m_a}{M_a}, l_b = \frac{m_b}{M_b}, l_c = \frac{m_c}{M_c}$ where m_a, m_b, m_c are the lengths of the angle bisectors of angles A, B and C respectively, internal to the triangle and M_a, M_b and M_c are the lengths of

these internal angle bisectors extended until they meet the circumcircle.

Q. The minimum value of the expression $\frac{l_a}{\sin^2 A} + \frac{l_b}{\sin^2 B} + \frac{l_c}{\sin^2 C}$ is : A. 2 B. 3 C. 4

D. none of these

Answer: B



Exercise 5 Subjective Type Problems

1. If the median AD of triangle ABC makes an angle $\frac{\pi}{4}$ with the side BC, then find the value of $|\cot B - \cot C|$.

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2. In a ΔABC , $a = \sqrt{3}$, b = 3 and $\angle C = \frac{\pi}{3}$. Let internal angle bisectors of angle C intersects side AB at D and altitude from B meets the angle bisector CD at E. If Q_1 and Q_2 are incentres of ΔBEC and ΔBED . Find the distance between the vertex B and orthocentre of $\Delta O_1 EO_2$.

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3. In a $\triangle ABC$; inscribed circle with centre I touches sides AB, AC and BCatD, E, F respectively.Let area of

quadrilateral ADIE is 5 units and area of quadrilteral BFID is 10

units. Find the value of
$$\frac{\cos\left(\frac{C}{2}\right)}{\sin\left(\frac{A-B}{2}\right)}$$
.



4. If Δ be area of incircle of a triangle ABC and $\Delta_1, \Delta_2, \Delta_3$ be the area of excircles then find the least value of $\frac{\Delta_1 \Delta_2 \Delta_3}{729 \Delta^3}$

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5. In $\Delta ABC, b=c, \angle A=106^{\circ}$, M is an interior point such

that

Λ

is equal to

(where notations have their usual meaning)



6. In an acute angled triangle ABC, $\angle A = 20^{\circ}$, let DEF be the feet of altitudes through A, B, C respectively and H is the orthocentre of ΔABC . Find $\frac{AH}{AD} + \frac{BH}{BE} + \frac{CH}{CF}$.

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7. Let $\triangle ABC$ be inscribed in a circle having radius unity. The three internal bisectors of the angles A, B and C are extended to intersect the circumcircle of $\triangle ABC$ at A_1B_1 and C_1 respectively. Then $\frac{AA_1\cos\frac{A}{2}BB_1\cos\frac{B}{2} + CC_1\cos\frac{C}{2}}{\sin A + \sin B + \sin C} =$

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8. If the quadratic equation $ax^2 + bx + c = 0$ has equal roots where a, b, c denotes the lengths of the sides opposite to vertex A, B and C of the $\triangle ABC$ respectively then find the number of integers in the range of $\frac{\sin A}{\sin C} + \frac{\sin C}{\sin A}$

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9. If in the triangle ABC, $\tan\frac{A}{2}$, $\tan\frac{B}{2}$ and $\tan\frac{C}{2}$ are in harmonic progression then the least value of $\cot^2\frac{B}{2}$ is equal to

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10. In ΔABC , if circumradius 'R' and inradius 'r' are connected by relation $R^2 - 4Rr + 8r^2 - 12r + 9 = 0$, then the greatest integer which is less than the semiperimeter of ΔABC is :



11. Let a, b, c be sides of a triangle ABC and Δ denotes its area .

If $a=2, \Delta=\sqrt{3}$ and $a\cos C+\sqrt{3}a\sin C-b-c=0$, then

find the value of (b + c).

(symbols used have usual meaning in ΔABC).



12. Circumradius of ΔABC is 3 cm and its area is $6cm^2$. If DEF is the triangle formed by feet of the perpendicular drawn from A,B and C on the sides BC, CA and AB, respectively, then the perimeter of ΔDEF (in cm) is ____