MATHS

BOOKS - VIKAS GUPTA MATHS (HINGLISH)

SOLUTION OF TRIANGLES

Exercise 1 Single Choice Problems

1. In a ΔABC if $9(a^2+b^2)=17c^2$ then the value of the $\dfrac{\cot A+\cot B}{\cot C}$ is

A. $\dfrac{13}{4}$

B. $\dfrac{7}{4}$

C. $\dfrac{5}{4}$

D. $\frac{9}{4}$

Answer: D

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2. Let H be the orthocenter of triangle ABC, then angle subtended by side BC at the centre of incircle of ΔCHB is :

A. $\frac{A}{2}+\frac{\pi}{2}$

B. $\frac{B+C}{2}+\frac{\pi}{2}$

C. $\frac{B-C}{2}+\frac{\pi}{2}$

D. $\frac{B+C}{2}+\frac{\pi}{4}$

Answer: B

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3. Circumradius of a ΔABC is 3 units, let O be the circumcentre and H be the orthocentre then the value of $\frac{1}{64}\left(AH^2+BC^2\right)\left(BH^2+AC^2\right)\left(CH^2+AB^2\right)$ equals :

A. 3^4

B. 9^3

C. 27^6

D. 81^4

Answer: B

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4. The angles A, B and C of a triangle ABC are in arithmetic progression. If $2b^2=3c^2$ then the angle A is :

A. 15°

B. 60°

C. 75°

D. 90°

Answer: C

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5. In a triangle ABC if $\tan.\frac{A}{2}\tan.\frac{B}{2}=\frac{1}{3}$ and ab = 4, then the value of c can be

A. 1

B. 2

C. 4

D. 6

Answer: B

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6. In a triangle ABC the expression $a\cos B\cos C + b\cos C\cos A + c\cos A\cos B$ equals to :

A. $\frac{rs}{R}$

B. $\frac{r}{sR}$

C. $\frac{R}{rs}$

D. $\frac{Rs}{r}$

Answer: A

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7. The set of all real numbers a such that $a^2 + 2a, 2a + 3, and a^2 + 3a + 8$ are the sides of a triangle is_____

A. $(0, \infty)$

B. $(5, 8)$

C. $\left(-\frac{11}{3}, \infty\right)$

D. $(5, \infty)$

Answer: D

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8. In a $\Delta ABC, \angle B = \frac{\pi}{3}$ and $\angle C = \frac{\pi}{4}$ let D divide BC internally in the ratio $1:3$, then $\frac{\sin(\angle BAD)}{\sin(\angle CAD)}$ is equal to :

A. $\frac{1}{\sqrt{6}}$

B. $\frac{1}{3}$

C. $\frac{1}{\sqrt{3}}$

D. $\frac{\sqrt{2}}{3}$

Answer: A

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9. Let AD, BE, CF be the lengths of internal bisectors of angles A, B, C respectively of triangle ABC. Then the harmonic mean of $AD\sec\frac{A}{2}, BE\sec\frac{B}{2}, CF\sec\frac{C}{2}$ is equal to :

A. Harmonic mean of sides of ΔABC

B. Geometric mean of sides of ΔABC

C. Arithmetic mean of sides of ΔABC

D. Sum of reciprocals of the sides of ΔABC

Answer: A

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10. In a triangle ABC, if $2b = a + c$ and $A - C = 90$, then $\sin B$ equals

A. $\frac{\sqrt{7}}{5}$

B. $\frac{\sqrt{5}}{8}$

C. $\frac{\sqrt{7}}{4}$

D. $\frac{\sqrt{5}}{3}$

Answer: C

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11. In a triangle ABC, if $2a\cos\left(\frac{B-C}{2}\right)=b+c$, then secA is equal to :

A. $\frac{2}{\sqrt{3}}$

B. $\sqrt{2}$

C. 2

D. 3

Answer: C

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12. In a triangle ABC if $BC=1 and AC=2$, then what is the maximum possible value of angle A?

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

Answer: A

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13. $\Delta I_1 I_2 I_3$ is an excentral triangle of an equilateral triangle ΔABC such that $I_1 I_2 = 4$ unit, if ΔDEF is pedal triangle of ΔABC, then $\frac{Ar(\Delta I_1 I_2 I_3)}{Ar(\Delta DEF)} =$

A. 16

B. 4

C. 2

D. 1

Answer: A

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14. Let ABC be a triangle with $\angle BAC = 2\pi/3$ and $AB = x$ such that (AB) (AC) = 1. If x varies, then find the longest possible length of the angle bisector AD

A. $\frac{1}{3}$

B. $\frac{1}{2}$

C. $\frac{2}{3}$

D. $\frac{\sqrt{2}}{3}$

Answer: B

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15. In a equilateral triangle r, R and r_1 form (where symbols used have usual meaning)

A. an A.P.

B. a G.P.

C. an H.P.

D. none of these

Answer: A

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16. In ΔABC if $\dfrac{\sin A}{\sin C} = \dfrac{\sin(A-B)}{\sin(B-C)}$, then a^2, b^2, c^2 are in :

A. A.P.

B. G.P.

C. H.P.

D. none of these

Answer: A

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17. In $\Delta ABC, \tan A = 2, \tan B = \frac{3}{2}$ and $c = \sqrt{65},$ then circumradius of the triangle is :

A. 65

B. $\frac{65}{7}$

C. $\frac{65}{14}$

D. none of these

Answer: C

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18. If the sides a, b, c of a triangle ABC are the roots of the equation $x^3 - 13x^2 + 54x - 72 = 0$, then the value of $\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c}$ is equal to :

A. $\frac{61}{144}$

B. $\frac{61}{72}$

C. $\frac{169}{144}$

D. $\frac{59}{144}$

Answer: A

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19. In $\Delta ABC, \quad \text{if} \quad \angle C = 90^\circ, \quad \text{then} \quad \frac{a+c}{b} + \frac{b+c}{a}$ is equal to :

A. $\frac{c}{r}$

B. $\frac{1}{2Rr}$

C. 2

D. $\frac{R}{r}$

Answer: A

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20. In a $\Delta ABC, \quad \text{if} \quad a^2 \sin B = b^2 + c^2$, then :

A. $\angle A$ is obtuse

B. $\angle A$ is acute

C. $\angle B$ is abtuse

D. $\angle A$ is right angle

Answer: A

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21. If R and R' are the circumradii of triangles ABC and OBC, where O is the orthocenter of triangle ABC, then :

A. $R' = \frac{R}{2}$

B. $R' = 2R$

C. $R' = R$

D. $R' = 3R$

Answer: C

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22. The acute angle of a rhombus whose side is a mean proportional between its diagonals is

A. 15°

B. 20°

C. 30°

D. 60°

Answer: C

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23. In a ΔABC right angled at A, a line is drawn through A to meet BC at D dividing BC in $2:1$. If $\tan(\angle ADC) = 3$ then $\angle BAD$ is :

A. 30°

B. 45°

C. 60°

D. 75°

Answer: B

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24. A circle is cirumscribed in an equilateral triangle of side $'l'$. The area of any square inscribed in the circle is :

A. $\frac{4}{3}l^2$

B. $\frac{2}{3}l^2$

C. $\frac{1}{3}l^2$

D. l^2

Answer: B

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25. if the sides of a triangle are in the ratio $2 : \sqrt{6} : \sqrt{3} + 1$, then the largest ange of the trangle will be (1) 60 (3) 90 (2) 75 (4) 120

A. 60°

B. 72°

C. 75°

D. 90°

Answer: C

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26. In a triangle ABC if a, b, c are in A.P. and $C - A = 120^\circ$, then $\frac{s}{r} =$ (where notations have their usual meaning)

A. $\sqrt{15}$

B. $2\sqrt{15}$

C. $3\sqrt{15}$

D. $6\sqrt{15}$

Answer: C

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27. If in $\Delta ABC, a=5, b=4$ and $\cos(A-B)=\frac{31}{32}$, then

A. $\sqrt{6}$

B. $6\sqrt{6}$

C. 6

D. $(216)^{1/4}$

Answer: C

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28. If semiperimeter of a triangle is 15, then the value of

$(b+c)\cos(B+C)+(c+a)\cos(C+A)+(a+b)\cos(A+B)$

is equal to :

(where symbols used have usual meanings)

A. -60

B. -15

C. -30

D. can not be determined

Answer: C

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29. Let triangle ABC be an isosceles with AB=AC. Suppose that the angle bisector of its angle B meets the side AC at a point D and that $BC = BD + AD$. Measure of the angle A in degrees, is :

A. 80

B. 100

C. 110

D. 130

Answer: B

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30. In triangle ABC if $A:B:C=1:2:4$, then $\left(a^2-b^2\right)\left(b^2-c^2\right)\left(c^2-a^2\right)=\lambda a^2 b^2 c^2$, where $\lambda=$

(where notations have their usual meaning)

A. 1

B. 2

C. 4

D. 9

Answer: A

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31. In a triangle ABC with altitude AD, $\angle BAC = 45^{\circ}, DB = 3 \text{ and } CD = 2$. The area of the triangle ABC is :

A. 6

B. 15

C. $15/4$

D. 12

Answer: B

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32. A triangle has base 10 cm long and the base angles of 50° and 70°. If the perimeter of the triangle is $x + y\cos z^{\circ}$ where $z \in (0, 90)$ then the value of $x + y + z$ equals :

A. 60

B. 55

C. 50

D. 40

Answer: D

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33. Let H be the orthocenter of triangle ABC, then angle subtended by side BC at the centre of incircle of ΔCHB is :

A. $\frac{A}{2} + \frac{\pi}{2}$

B. $\frac{B+C}{2} + \frac{\pi}{2}$

C. $\frac{B-C}{2} + \frac{\pi}{2}$

D. $\frac{B+C}{2} + \frac{\pi}{4}$

Answer: B

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34. Triangle ABC is right angle at A. The points P and Q are on hypotenuse BC such that $BP = PQ = QC$.if $AP = 3$ and $AQ = 4$, then length BC is equal to

A. $\sqrt{27}$

B. $\sqrt{36}$

C. $\sqrt{45}$

D. $\sqrt{54}$

Answer: C

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35. In a ΔABC if $b = a(\sqrt{3} - 1)$ and $\angle C = 30^\circ$ then the measure of the angle A is

A. 15°

B. 45°

C. 75°

D. 105°

Answer: D

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36. Through the centroid of an equilateral triangle, a line parallel to the base is drawn. On this line, an arbitrary point P is taken inside the triangle. Let h denote the perpendicular distance of P from the base of the triangle. Let h_1 and h_2 be the perpendicular distance of P from the other two sides of the triangle . Then :

A. $h = \frac{h_1 + h_2}{2}$

B. $h = \sqrt{h_1 h_2}$

C. $h = \frac{2h_1 h_2}{h_1 + h_2}$

D. $h = \frac{(h_1 + h_2)\sqrt{3}}{4}$

Answer: A

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37. The angles A, B and C of a triangle ABC are in arithmetic progression. AB=6 and BC=7. Then AC is :

A. $\sqrt{41}$

B. $\sqrt{39}$

C. $\sqrt{42}$

D. $\sqrt{43}$

Answer: D

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38. In ΔABC, if $A - B = 120^\circ$ and $R = 8r$, then the value of $\dfrac{1+\cos C}{1-\cos C}$ equals :

(All symbols used hav their usual meaning in a triangle)

A. 12

B. 15

C. 21

D. 31

Answer: B

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39. The lengths of the sides CB and CA of a triangle ABC are given by a and b and the angle C is $\frac{2\pi}{3}$. The line CD bisects the angle C and meets AB at D. Then the length of CD is :

A. $\frac{1}{a+b}$

B. $\frac{a^2+b^2}{a+b}$

C. $\frac{ab}{2(a+b)}$

D. $\frac{ab}{a+b}$

Answer: D

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40. In $\Delta ABC,$ angle A is 120°, $BC + CA = 20,$ and $AB + BC = 21$ Find the length of the side BC

A. 13

B. 15

C. 17

D. 19

Answer: A

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41. A triangle has sides 6,7, and 8. The line through its incenter parallel to the shortest side is drawn to meet the other two sides at P and Q. Then find the length of the segment PQ.

A. $\frac{12}{5}$

B. $\frac{15}{4}$

C. $\frac{30}{7}$

D. $\frac{33}{9}$

Answer: C

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42. The perimeter of a ΔABC is 48 cm and one side is 20 cm. Then remaining sides of ΔABC must be greater than :

A. 8 cm

B. 9 cm

C. 12 cm

D. 4 cm

Answer: D

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43. In a equilateral triangle r, R and r_1 form (where symbols used have usual meaning)

A. an A.P.

B. a G.P.

C. an H.P.

D. neither an A.P., G.P. nor H.P.

Answer: A

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44. The expression $\frac{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}{4b^2c^2}$ is equal to

A. $\cos^2 A$

B. $\sin^2 A$

C. $\cos A \cos B \cos C$

D. $\sin A \sin B \sin C$

Answer: B

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45. Circumradius of an isosceles ΔABC with $\angle A = \angle B$ is 4 times its in radius, then cosA is root of the equation :

A. $x^2 - x - 8 = 0$

B. $8x^2 - 8x + 1 = 0$

C. $x^2 - x - 4 = 0$

D. $4x^2 - 4x + 1 = 0$

Answer: B

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46. A is the orthocentre of ΔABC and D is reflection point of A w.r.t. perpendicualr bisector of BC, then orthocenter of ΔDBC is :

A. D

B. C

C. B

D. A

Answer: A

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47. If $a \neq b \neq c$ are all positive, then the value of the determinant $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is

A. ≥ 0

B. > 0

C. ≤ -1

D. < 0

Answer: D

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48. In triangle ABC if $A : B : C = 1 : 2 : 4$, then $\left(a^2 - b^2\right)\left(b^2 - c^2\right)\left(c^2 - a^2\right) = \lambda a^2 b^2 c^2$, where $\lambda =$

(where notations have their usual meaning)

A. 1

B. 2

C. 3

D. $\frac{1}{3}$

Answer: A

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49. In any triangle, the minimum value of $r_1 r_2 r_3 / r^3$ is equal to

A. 1

B. 3

C. 8

D. 27

Answer: D

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50. In $\Delta ABC, a=3, b=4$ and $c=5$, then value of $\sin A+\sin 2B+\sin 3C$ is

A. $\frac{24}{25}$

B. $\frac{14}{25}$

C. $\frac{64}{25}$

D. None

Answer: B

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51. In any triangle ABC, the value of $\frac{r_1+r_2}{1+\cos C}$ is equal to (where notation have their usual meaning) :

A. 2R

B. 2r

C. R

D. $\frac{2R^2}{r}$

Answer: A

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52. In a triangle ABC, medians AD and BE are deawn. IF $AD = 4, \angle DAB = \frac{\pi}{6}$ and $\angle ABE = \frac{\pi}{3}$, then the area of the trianlge ABC is-

A. $\frac{8}{3\sqrt{3}}$

B. $\frac{16}{3\sqrt{3}}$

C. $\frac{32}{3\sqrt{3}}$

D. $\frac{64}{3\sqrt{3}}$

Answer: C

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53. The sides of a triangle are $\sin\alpha, \cos\alpha$ and $\sqrt{1+\sin\alpha\cos\alpha}$ for some $\alpha, 0 < \alpha < \frac{\pi}{2}$.Then the greatest angle of the triangle is

A. $\frac{\pi}{3}$

B. $\frac{\pi}{2}$

C. $\frac{2\pi}{3}$

D. $\frac{5\pi}{6}$

Answer: C

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54. Let ABC be a right with $\angle BAC = \frac{\pi}{2}$, then $\left(\frac{r^2}{2R^2} + \frac{r}{R}\right)$ is equal to :

(where symbols used have usual meaning in a striangle)

A. sinB sinC

B. tanB tanC

C. secB secC

D. cotB cotC

Answer: A

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55. In a ΔABC, with usual notations, if $b > c$ then distance between foot of median and foot of altitude both drawn from vertex A on BC is :

A. $\dfrac{a^2 - b^2}{2c}$

B. $\dfrac{b^2 - c^2}{2a}$

C. $\dfrac{b^2 + c^2 - a^2}{2a}$

D. $\dfrac{b^2 + c^2 - a^2}{2c}$

Answer: B

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56. In a triangle ABC the expression $a\cos B\cos C + b\cos A\cos B$ equals to :

A. $\frac{rs}{R}$

B. $\frac{r}{sR}$

C. $\frac{R}{rs}$

D. $\frac{Rs}{r}$

Answer: A

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57. In a acute triangle ABC, altitudes from the vertices A, B and C meet the opposite sides at the points D, E and F respectively. If the radisu of the circumcircle of $\Delta AFE, \Delta BFD, \Delta CED, \Delta ABC$ be respectively R_1, R_2, R_3 and R. Then the maximum value of $R_1 + R_2 + R_3$ is :

A. $\frac{3R}{8}$

B. $\frac{2R}{3}$

C. $\frac{4R}{3}$

D. $\frac{3R}{2}$

Answer: D

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Exercise 2 One Or More Than One Answer Is Are Correct

1. If r_1, r_2, r_3 are radii of the escribed circles of a triangle ABC and r it the radius of its incircle, then the root(s) of the equation $x^2 - r(r_1r_2 + r_2r_3 + r_3r_1)x + (r_1r_2r_3 - 1) = 0$ is/are :

A. r_1

B. $r_2 + r_3$

C. 1

D. $r_1 r_2 r_3 - 1$

Answer: C::D

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2. In $\Delta ABC, \angle A = 60^\circ, \angle B = 90^\circ, \angle C = 30^\circ$. Let H be its orthocentre, then :

(where symbols used have usual meanings)

A. $AH = c$

B. $CH = a$

C. $AH = a$

D. $BH = 0$

Answer: A::B::D

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3. In an equilateral triangle, if inradius is a rational number then

A. circumradius is always rational

B. exradii are always rational

C. area is always ir-rational

D. perimeter is always rational

Answer: A::B::C

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4. Let A, B, C be angles of a triangle ABC and let $D=\frac{5\pi+A}{32}, E=\frac{5\pi+B}{32}, F=\frac{5\pi+C}{32}$, then :

(where $D, E, F \ne \frac{n\pi}{2}, n \in I, I$ denote set of integers)

A. $\cot D \cot E + \cot E \cot F + \cot D \cot F = 1$

B. $\cot D + \cot E + \cot F = \cot D \cot E \cot F$

C. $\tan D \tan E + \tan E \tan F + \tan F \tan D = 1$

D. $\tan D + \tan E + \tan F = \tan D \tan E \tan F$

Answer: B::C

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5. In a triangle ABC, if a=4, b=8 and $\angle C = 60^\circ$, then :

(where symbols used have usual meanings)

A. $c = 6$

B. $c = 4\sqrt{3}$

C. $\angle A = 30^\circ$

D. $\angle B = 90^\circ$

Answer: B::C::D

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6. In a ΔABC if $\frac{r}{r_1} = \frac{r_2}{r_3}$, then which of the following is/are true ?

(where symbols used have usual meanings)

A. $a^2 + b^2 + c^2 = 8R^2$

B. $\sin^2 A + \sin^2 B + \sin^2 C = 2$

C. $a^2 + b^2 = c^2$

D. $\Delta = s(s+c)$

Answer: A::B::C

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7. ABC is a triangle whose circumcentre, incentre and orthocentre are O, I and H respectively which lie inside the triangle, then :

A. $\angle BOC = A$

B. $\angle BIC = \frac{\pi}{2} + \frac{A}{2}$

C. $\angle BHC = \pi - A$

D. $\angle BHC = \pi - \frac{A}{2}$

Answer: B::C

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8. In a triangle ABC, tanA and tanB satisfy the inequality $\sqrt{3}x^2 - 4x + \sqrt{3} < 0$, then which of the following must be correct ?

(where symbols used have usual meanings)

A. $a^2 + b^2 - ab < c^2$

B. $a^2 + b^2 > c^2$

C. $a^2 + b^2 + ab > c^2$

D. $a^2 + b^2 < c^2$

Answer: A::C

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9. If in $\Delta ABC, \angle C = \frac{\pi}{8}, a = \sqrt{2} \text{ and } b = \sqrt{2+\sqrt{2}}$ then find the measure of angle A (in degree).

A. 45°

B. 135°

C. 30°

D. 150°

Answer: A

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10. In triangle ABC, $a = 3, b = 4, c = 2$. Point D and E trisect the side BC. If $\angle DAE = \theta$, then $\cot^2 \theta$ is divisible by :

A. 2

B. 3

C. 5

D. 7

Answer: B::C

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11. In a triangle ABC, $3\sin A + 4\cos B = 6$ and $4\sin B + 3\cos A = 1$.

Find the measure of angle C.

A. $\frac{\pi}{4}$

B. $\frac{\pi}{6}$

C. $\frac{\pi}{3}$

D. $\frac{5\pi}{6}$

Answer: B

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12. If the line joining the incentre to the centroid of a triangle ABC is parallel to the side BC. Which of the following are correct ?

A. $2b = a + c$

B. $2a = b + c$

C. $\cot\frac{A}{2}\cot\frac{C}{2} = 3$

D. $\cot\frac{B}{2}\cot\frac{C}{2} = 3$

Answer: B::D

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13. In a triangle the lengths of the two larger are 10 and 9 respectively.If the angles are in A.P., the , length of the third side can be (A) $5-\sqrt{6}$ (B) $3\sqrt{3}$ (C) 5 (D) $5+\sqrt{6}$

A. $5-\sqrt{6}$

B. $5+\sqrt{6}$

C. $6-\sqrt{5}$

D. $6+\sqrt{5}$

Answer: A::B

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14. If area of $\Delta ABC(\Delta)$ and angle C are given and if c opposite to given angle is minimum, then

A. $a = \sqrt{\frac{2\Delta}{\sin C}}$

B. $b = \left(\frac{2\Delta}{\sin C}\right)$

C. $a = \frac{4\Delta}{\sin C}$

D. $b = \frac{4\Delta}{\sin^2 C}$

Answer: A::B

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15. In a triangle ABC , if $\tan A = 2\sin 2C$ and $3\cos A = 2\sin B \sin C$, then C=

A. $\frac{\pi}{8}$

B. $\frac{\pi}{6}$

C. $\frac{\pi}{4}$

D. $\frac{\pi}{3}$

Answer: C::D

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Exercise 3 Comprehension Type Problems

1. Let $\angle A = 23^\circ, \angle B = 75^\circ$ and $\angle C = 82^\circ$ be the angles of ΔABC.

The incircle of ΔABC touches the sides BC, CA, AB at points D, E, F respectively. Let r', r_1' respectively be the inradius, exradius opposite to vertex D of ΔDEF and r be inradius of ΔABC, then

Q. $\frac{r'}{r} =$

A. $\sin\frac{A}{2} + \sin\frac{B}{2} + \sin\frac{C}{2} - 1$

B. $1-\sin\frac{A}{2}+\sin\frac{B}{2}+\sin\frac{C}{2}$

C. $\cos\frac{A}{2}+\cos\frac{B}{2}+\cos\frac{C}{2}-1$

D. $1-\cos\frac{A}{2}+\cos\frac{B}{2}+\cos\frac{C}{2}$

Answer: A

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2. Let $\angle A = 23^\circ$, $\angle B = 75^\circ$ and $\angle C = 82^\circ$ be the angles of ΔABC.

The incircle of ΔABC touches the sides BC, CA, AB at points D, E, F respectively. Let r', r_1' respectively be the inradius, exradius opposite to vertex D of ΔDEF and r be inradius of ΔABC, then

Q. $\frac{r_1'}{r} =$

A. $\sin\frac{A}{2}+\sin\frac{B}{2}+\sin\frac{C}{2}-1$

B. $1-\sin\frac{A}{2}+\sin\frac{B}{2}+\sin\frac{C}{2}$

C. $\cos\frac{A}{2}+\cos\frac{B}{2}+\cos\frac{C}{2}-1$

D. $1-\cos\frac{A}{2}+\cos\frac{B}{2}+\cos\frac{C}{2}$

Answer: B

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3. Internal bisectors of ΔABC meet the circumcircle at point D, E, and F

Area of ΔDEF is

A. $2R^2\cos^2\left(\frac{A}{2}\right)\cos^2\left(\frac{B}{2}\right)\cos^2\left(\frac{C}{2}\right)$

B. $2R^2\sin\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)\sin\left(\frac{C}{2}\right)$

C. $2R^2\sin^2\left(\frac{A}{2}\right)\sin^2\left(\frac{B}{2}\right)\sin^2\left(\frac{C}{2}\right)$

D. $2R^2 \cos\left(\frac{A}{2}\right)\cos\left(\frac{B}{2}\right)\cos\left(\frac{C}{2}\right)$

Answer: D

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4. Internal angle bisecotors of ΔABC meets its circum circle at D, E and F where symbols have usual meaning.

Q. The ratio of area of triangle ABC and triangle DEF is :

A. ≥ 1

B. ≤ 1

C. $\geq 1/2$

D. $\leq 1/2$

Answer: B

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5. Let triangle ABC is right triangle right angled at C such that $A < B$ and $r = 8, R = 41$.

Q. Area of ΔABC is :

A. 720

B. 1440

C. 360

D. 480

Answer: A

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6. Let triangle ABC is right triangle right angled at C such that $A < B$ and $r = 8, R = 41$.

Q. $\tan\frac{A}{2} =$

A. $\frac{1}{18}$

B. $\frac{1}{3}$

C. $\frac{1}{6}$

D. $\frac{1}{9}$

Answer: D

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7. Let the incircle of ΔABC touches the sides BC, CA, AB at A_1, B_1, C_1 respectively. The incircle of $\Delta A_1B_1C_1$ touches its sides of B_1C_1, C_1A_1 and A_1B_1 at A_2, B_2, C_2 respectively and so on.

Q. $\lim_{n \to \infty} \angle A_n =$

A. 0

B. $\frac{\pi}{6}$

C. $\frac{\pi}{4}$

D. $\frac{\pi}{3}$

Answer: D

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8. Let the incircle of ΔABC touches the sides BC, CA, AB at A_1, B_1, C_1 respectively. The incircle of $\Delta A_1B_1C_1$ touches its sides of B_1C_1, C_1A_1 and A_1B_1 at A_2, B_2, C_2 respectively and so on.

Q. In $\Delta A_4B_4C_4$, the value of $\angle A_4$ is:

A. $\dfrac{3\pi + A}{6}$

B. $\dfrac{3\pi - A}{8}$

C. $\dfrac{5\pi - A}{16}$

D. $\dfrac{5\pi + A}{16}$

Answer: D

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9. Let ABC be a given triangle. Points D and E are on sides AB and AC respectively and point F is on line segment DE. Let $\dfrac{AD}{AB} = x, \dfrac{AE}{AC} = y, \dfrac{DF}{DE} = z$. Let area of $\Delta BDF = \Delta_1$, Area of $\Delta CEF = \Delta_2$ and area of $\Delta ABC = \Delta$.

Q. $\dfrac{\Delta_1}{\Delta}$ is equal to :

A. xyz

B. $(1 - x)y(1 - z)$

C. $(1 - x)yz$

D. $x(1 - y)z$

Answer: C

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10. Let ABC be a given triangle. Points D and E are on sides AB and AC respectively and point F is on line segment DE. Let $\dfrac{AD}{AB} = x, \dfrac{AE}{AC} = y, \dfrac{DF}{DE} = z$. Let area of $\Delta BDF = \Delta_1$, Area of $\Delta CEF = \Delta_2$ and area of $\Delta ABC = \Delta$.

Q. $\frac{\Delta_2}{\Delta}$ is equal to :

A. $(1-x)y(1-z)$

B. $(1-x)(1-y)z$

C. $x(1-y)(1-z)$

D. $(1-x)yz$

Answer: C

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11. a, b, c ar the length of sides BC, CA, AB respectively of ΔABC satisfying $\log\left(1+\frac{c}{a}\right)+\log a-\log b=\log 2$. Also the quadratic equation $a(1-x^2)+2bx+c(1+x^2)=0$ has two equal roots. Q. a, b, c are in :

A. A.P.

B. G.P.

C. H.P.

D. None

Answer: A

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12. a, b, c ar the length of sides BC, CA, AB respectively of ΔABC satisfying $\log\left(a+\frac{c}{a}\right)+\log a-\log b=\log 2.$

Also the quadratic equation $a\left(1-x^2\right)+2bx+c\left(1+x^2\right)=0$ has two equal roots.

Q. Measure of angle C is :

A. 30°

B. 45°

C. 60°

D. 90°

Answer: D

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13. If a,b,c are the sides of triangle ABC satisfying $\log\left(1+\frac{c}{a}\right)+\log a-\log b=\log 2.$ Also $a(1-x^2)+2bx+c(1+x^2)=0$ has two equal roots. Find the value of $\sin A+\sin B+\sin C$.

A. $\frac{5}{2}$

B. $\frac{12}{5}$

C. $\frac{8}{3}$

D. 2

Answer: B

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14. Let ABC be a triangle inscribed in a circle and let $l_a = \frac{m_a}{M_a}, l_b = \frac{m_b}{M_b}, l_c = \frac{m_c}{M_c}$ where m_a, m_b, m_c are the lengths of the angle bisectors of angles A, B and C respectively , internal to the triangle and M_a, M_b and M_c are the lengths of these internal angle bisectors extended until they meet the circumcircle.

Q. l_a equals :

A. $\frac{\sin A}{\sin\left(B + \frac{A}{2}\right)}$

B. $\frac{\sin B \sin C}{\sin^2\left(\frac{B+C}{2}\right)}$

C. $\dfrac{\sin B \sin C}{\sin^2\left(B+\frac{A}{2}\right)}$

D. $\dfrac{\sin B + \sin C}{\sin^2\left(B+\frac{A}{2}\right)}$

Answer: C

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15. Let ABC be a triangle inscribed in a circle and let $l_a = \dfrac{m_a}{M_a}, l_b = \dfrac{m_b}{M_b}, l_c = \dfrac{m_c}{M_c}$ where m_a, m_b, m_c are the lengths of the angle bisectors of angles A, B and C respectively , internal to the triangle and M_a, M_b and M_c are the lengths of these internal angle bisectors extended until they meet the circumcircle.

Q. The maximum value of the product

$$(l_a l_b l_c) \times \cos^2\left(\frac{B-C}{2}\right) \times \frac{\cos^2(C-A)}{2}\Bigg) \times \cos^2\left(\frac{A-B}{2}\right)$$

is equal to :

A. $\frac{1}{8}$

B. $\frac{1}{64}$

C. $\frac{27}{64}$

D. $\frac{27}{32}$

Answer: C

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16. Let ABC be a triangle inscribed in a circle and let $l_a = \frac{m_a}{M_a}, l_b = \frac{m_b}{M_b}, l_c = \frac{m_c}{M_c}$ where m_a, m_b, m_c are the lengths of the angle bisectors of angles A, B and C respectively , internal to the triangle and M_a, M_b and M_c are the lengths of these internal angle bisectors extended until they meet the circumcircle.

Q. The minimum value of the expression $\frac{l_a}{\sin^2 A} + \frac{l_b}{\sin^2 B} + \frac{l_c}{\sin^2 C}$ is :

A. 2

B. 3

C. 4

D. none of these

Answer: B

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Exercise 5 Subjective Type Problems

1. If the median AD of triangle ABC makes an angle $\frac{\pi}{4}$ with the side BC, then find the value of $|\cot B - \cot C|$.

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2. In a $\Delta ABC, a = \sqrt{3}, b = 3$ and $\angle C = \frac{\pi}{3}$. Let internal angle bisectors of angle C intersects side AB at D and altitude from B meets the angle bisector CD at E. If Q_1 and Q_2 are incentres of ΔBEC and ΔBED. Find the distance between the vertex B and orthocentre of $\Delta O_1 E O_2$.

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3. In a ΔABC; inscribed circle with centre I touches sides AB, AC and $BCatD, E, F$ respectively.Let area of quadrilateral ADIE is 5 units and area of quadrilteral BFID is 10 units. Find the value of $\dfrac{\cos\left(\frac{C}{2}\right)}{\sin\left(\frac{A-B}{2}\right)}$.

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4. If Δ be area of incircle of a triangle ABC and $\Delta_1, \Delta_2, \Delta_3$ be the area of excircles then find the least value of $\dfrac{\Delta_1\Delta_2\Delta_3}{729\Delta^3}$

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5. In $\Delta ABC, b = c, \angle A = 106^\circ$, M is an interior point such that

$\angle MBA = 7^\circ, \angle MAB = 23^\circ$ and $\angle MCA = \theta^\circ$, then $\dfrac{\theta}{2}$

is equal to

(where notations have their usual meaning)

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6. In an acute angled triangle ABC, $\angle A = 20^\circ$, let DEF be the feet of altitudes through A, B, C respectively and H is the orthocentre of ΔABC. Find $\frac{AH}{AD} + \frac{BH}{BE} + \frac{CH}{CF}$.

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7. Let ΔABC be inscribed in a circle having radius unity. The three internal bisectors of the angles A, B and C are extended to intersect the circumcircle of ΔABC at A_1B_1 and C_1 respectively. Then $\frac{AA_1\cos\frac{A}{2}BB_1\cos\frac{B}{2} + CC_1\cos\frac{C}{2}}{\sin A + \sin B + \sin C} =$

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8. If the quadratic equation $ax^2 + bx + c = 0$ has equal roots where a, b, c denotes the lengths of the sides opposite to vertex A, B and C of the ΔABC respectively then find the number of integers in the range of $\frac{\sin A}{\sin C} + \frac{\sin C}{\sin A}$

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9. If in the triangle ABC, $\tan\frac{A}{2}, \tan\frac{B}{2}$ and $\tan\frac{C}{2}$ are in harmonic progression then the least value of $\cot^2\frac{B}{2}$ is equal to :

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10. In ΔABC, if circumradius 'R' and inradius 'r' are connected by relation $R^2 - 4Rr + 8r^2 - 12r + 9 = 0$, then the greatest integer which is less than the semiperimeter of ΔABC is :

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11. Let a, b, c be sides of a triangle ABC and Δ denotes its area .

If $a = 2, \Delta = \sqrt{3}$ and $a \cos C + \sqrt{3} a \sin C - b - c = 0$, then find the value of $(b + c)$.

(symbols used have usual meaning in ΔABC).

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12. Circumradius of ΔABC is 3 cm and its area is $6 cm^2$. If DEF is the triangle formed by feet of the perpendicular drawn from A,B and C on the sides BC, CA and AB, respectively, then the perimeter of ΔDEF (in cm) is _____

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