MATHS

BOOKS - VIKAS GUPTA MATHS (HINGLISH)

VECTOR & 3DIMENSIONAL GEOMETRY

Exercise 1 Single Choice Problems

1. If $ax+by+cz=p$, then minimum value of $x^2+y^2+z^2$ is

A. $\left(\frac{p}{a+b+c}\right)^2$

B. $\frac{p^2}{a^2+b^2+c^2}$

C. $\frac{a^2+b^2+c^2}{p^2}$

D. 0

Answer: B

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2. If the angle between the vectors $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{3}$ and the area of the triangle with adjacemnt sides parallel to $\vec{a}$ and $\vec{b}$ is 3 is

A. $\sqrt{3}$

B. $2\sqrt{3}$

C. $4\sqrt{3}$

D. $\frac{\sqrt{3}}{2}$

Answer: B

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3. Let B_1, C_1 and D_1 are points on AB, AC and AD of the parallelogram $ABCD,$ such that $\overrightarrow{AB_1} = k_1\overrightarrow{AC}, \overrightarrow{AC_1} = k_2\overrightarrow{AC}$ and $\overrightarrow{AD_1} = k_2\overrightarrow{AD},$ where k_1, k_2 and k_3 are scalar.

A. λ_1, λ_3 and λ_2 are in AP

B. λ_1, λ_3 and λ_2 are in GP

C. λ_1, λ_3 and λ_2 are in HP

D. $\lambda_1 + \lambda_2 + \lambda_3 = 0$

Answer: C

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4. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. If $\vec{c}$ is a vector such that $\vec{a} \cdot \vec{c} = |\vec{c}|, |\vec{c} - \vec{a}| = 2\sqrt{2}$ and the angle between $\vec{a} \times \vec{b}$ and $\vec{c}$ is 30° then $\left|\left(\vec{a} \times \vec{b}\right) \times \vec{c}\right|$ is equal to :

A. $\frac{2}{3}$

B. $\frac{3}{2}$

C. 2

D. 3

Answer: B

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5. If acute angle between the line $\vec{r} = \hat{i} + 2\hat{j} + \lambda\left(4\hat{i} - 3\hat{k}\right)$ and xy-plane is θ_1 and acute angle between planes $x + 2y = 0$ and $2x + y = 0$ is θ_2, then $\left(\cos^2\theta_1 + \sin^2\theta_2\right)$ equals to :

A. 1

B. $\frac{1}{4}$

C. $\frac{2}{3}$

D. $\frac{3}{4}$

Answer: A

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6. If a, b, c, x, y, z are real and $a^2+b^2+c^2=25, x^2+y^2+z^2=36$ and $ax+by+cz=30,$ then $\frac{a+b+c}{x+y+z}$ is equal to :

A. 1

B. $\frac{6}{5}$

C. $\frac{5}{6}$

D. $\frac{3}{4}$

Answer: C

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7. If $\vec{a}$ and $\vec{b}$ are non-zero, non-collinear vectors such that $|\vec{a}|=2, \vec{a}\cdot\vec{b}=1$ and angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{3}$. If $\vec{r}$ is any vector such that $\vec{r}\cdot\vec{a}=2, \vec{r}\cdot\vec{b}=8, \left(\vec{r}+2\vec{a}-10\vec{b}\right)\cdot\left(\vec{a}\times\vec{b}\right)=4\sqrt{3}$ and satisfy to $\vec{r}+2\vec{a}-10\vec{b}=\lambda\left(\vec{a}\times\vec{b}\right)$, then λ is equal to :

A. $\frac{1}{2}$

B. 2

C. $\frac{1}{4}$

D. None of these

Answer: D

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8. Let $\vec{a} = 3\hat{i} + 2\hat{j} + 4\hat{k}$, $\vec{b} = 2\left(\hat{i} + \hat{k}\right)$ and $\vec{c} = 4\hat{i} + 2\hat{j} + 3\hat{k}$ .Sum of the values of α for which the equation $x\vec{a} + y\vec{b} + z\vec{c} = \alpha\left(x\hat{i} + y\hat{j} + z\hat{k}\right)$ has non-trivial solution is:

A. -1

B. 4

C. 7

D. 8

Answer: C

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9. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + \hat{k}, \vec{c} = \hat{i} + 2\hat{j} - \hat{k}$, then the value of

$$\begin{vmatrix} \vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} & \vec{a}\cdot\vec{c} \\ \vec{b}\cdot\vec{a} & \vec{b}\cdot\vec{b} & \vec{b}\cdot\vec{c} \\ \vec{c}\cdot\vec{a} & \vec{c}\cdot\vec{b} & \vec{c}\cdot\vec{c} \end{vmatrix}$$ is equal to :

A. 2

B. 4

C. 16

D. 64

Answer: C

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10. $\vec{a}$ and $\vec{b}$ are two vectors such that $\left|\vec{a}\right| = 1, \left|\vec{b}\right| = 4$ and $\vec{a}.Vecb = 2. If \vec{c} = \left(2\vec{a} \times \vec{b}\right) - 3\vec{b}$ then find angle between $\vec{b}$ and $\vec{c}$.

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. $\frac{2\pi}{3}$

D. $\frac{5\pi}{3}$

Answer: D

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11. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors, then the value of $\left|\vec{a} - 2\vec{b}\right|^2 + \left|\vec{b} - 2\vec{c}\right|^2 + \left|\vec{c} - 2\vec{a}\right|^2$ does not exceed to :

A. 9

B. 12

C. 18

D. 21

Answer: D

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12. Two adjacent sides OA and OB of a rectangle $OACB$ are represented by $\vec{a}$ and $\vec{b}$ respectively, where o is origin. If $16\left|\vec{a}\times\vec{b}\right|=3\left|\vec{a}+\vec{b}\right|^2$ and θ is the angle between the diagonals OC and AB, then the value(s) of $\tan\left(\frac{\theta}{2}\right)$

A. $\frac{1}{\sqrt{2}}$

B. $\frac{1}{2}$

C. $\frac{1}{\sqrt{3}}$

D. $\frac{1}{3}$

Answer: D

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13. The vectors $\overrightarrow{AB} = 3\hat{i} + 4\hat{k}$ and $\overrightarrow{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a triangle ABC. The length of the median through A is (A) $\sqrt{72}$ (B) $\sqrt{33}$ (C) $\sqrt{2880}$ (D) $\sqrt{18}$

A. $\sqrt{288}$

B. $\sqrt{72}$

C. $\sqrt{33}$

D. $\sqrt{18}$

Answer: C

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14. If $\overrightarrow{a} = 2\hat{i} + \lambda\hat{j} + 3\hat{k}$, $\overrightarrow{b} = 3\hat{i} + 3\hat{j} + 5\hat{k}$, $\overrightarrow{c} = \lambda\hat{i} + 2\hat{j} + 2\hat{k}$ are inearly dependent vectors, then the number of possible values of λ is :

A. 0

B. 1

C. 2

D. More than 2

Answer: C

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15. The scalar triple product $\left[\vec{a}+\vec{b}-\vec{c} \quad \vec{b}+\vec{c}-\vec{a} \quad \vec{c}+\vec{a}-\vec{b}\right]$ is equal to

A. 0

B. $\left[\vec{a}\vec{b}\vec{c}\right]$

C. $2\left[\vec{a}\vec{b}\vec{c}\right]$

D. $4\left[\vec{a}\vec{b}\vec{c}\right]$

Answer: D

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16. If $\widehat{a}$ and $\hat{b}$ are unit vectors then the vector defined as $\overrightarrow{V}=\left(\widehat{a}\times\hat{b}\right)\times\left(\widehat{a}+\hat{b}\right)$ is collinear to the vector :

A. $\widehat{a}+\hat{b}$

B. $\hat{b}-\widehat{a}$

C. $2\widehat{a}-\hat{b}$

D. $\widehat{a}+2\hat{b}$

Answer: B

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17. The sine of angle formed by the lateral face ADC and plane of the base ABC of the terahedron ABCD, where $A=(3,-2,1), B=(3,1,5), C=(4,0,3)$ and $D=(1,0,0)$, is :

A. $\dfrac{2}{\sqrt{29}}$

B. $\frac{5}{\sqrt{29}}$

C. $\frac{3\sqrt{3}}{\sqrt{29}}$

D. $\frac{-2}{\sqrt{29}}$

Answer: B

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18. Let $\vec{a}_r = x_r\hat{i} + y_r\hat{j} + z_r\hat{k}, r = 1, 2, 3$ three mutually prependicular unit vectors then the value of $\begin{vmatrix} x_1 & -x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{vmatrix}$ is equal to

A. 0

B. ± 1

C. ± 2

D. ± 4

Answer: B

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19. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three non coplanar vectors and $\vec{r}$ is any vector in space, then

$$\left(\overrightarrow{\times} \vec{b}\right), \left(\vec{r} \times \vec{c}\right) + \left(\vec{b} \times \vec{c}\right) \times \left(\vec{r} \times \vec{a}\right) + \left(\vec{c} \times \vec{a}\right) \times \left(\vec{r} \times\right.$$

(A) $\left[\vec{a}\,\vec{b}\,\vec{c}\right]$ (B) $2\left[\vec{a}\,\vec{b}\,\vec{c}\right]\vec{r}$ (C) $3\left[\vec{a}\,\vec{b}\,\vec{c}\right]\vec{r}$ (D) $4\left[\vec{a}\,\vec{b}\,\vec{c}\right]\vec{r}$

A. $\left[\vec{a}\,\vec{b}\,\vec{c}\right]\vec{r}$

B. $2\left[\vec{a}\,\vec{b}\,\vec{c}\right]\vec{r}$

C. $4\left[\vec{a}\,\vec{b}\,\vec{c}\right]\vec{r}$

D. $\vec{0}$

Answer: B

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20. E and F are the interior points on the sides BC and CD of a parallelogram ABCD. Let $\overrightarrow{BE} = 4\overrightarrow{EC}$ and $\overrightarrow{CF} = 4\overrightarrow{FD}$. If the line EF meets the diagonal AC in G, then $\overrightarrow{AG} = \lambda\overrightarrow{AC}$, where λ is equal to :

A. $\frac{1}{3}$

B. $\frac{21}{25}$

C. $\frac{7}{13}$

D. $\frac{21}{5}$

Answer: B

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21. If $\vec{a}$ and $\vec{b}$ are unit vectors and $\vec{c}$ is such that $\vec{c}$ is such that $\vec{c} = \vec{c} = \vec{a} \times \vec{c} + \vec{b}$ then maximum value of $\left[\vec{a}\vec{b}\vec{c}\right]$ is

A. 1

B. $\frac{1}{2}$

C. 2

D. $\frac{3}{2}$

Answer: B

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22. Conside the matrices $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 1 & 2 \\ 1 & -1 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 1 & -1 \\ 2 & 2 & 3 \end{bmatrix}$ $C = \begin{bmatrix} 14 \\ 12 \\ 2 \end{bmatrix}$ $D = \begin{bmatrix} 13 \\ 11 \\ 14 \end{bmatrix}$. Now $x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ is such that solutions of equation $AX = C$ and $BX = D$ represent two points L andM respectively in 3 dimensional space. If L' and M' are hre reflections of L and M in the plane x+y+z=9 then find coordinates of L,M,L',M'

A. (3, 4, 2)

B. (5, 3, 4)

C. (7, 2, 3)

D. (1, 5, 6)

Answer: A

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23. The value of α for which point $M\left(\alpha\hat{i}+2\hat{j}+\hat{k}\right)$, lie in the plane containing three points $A\left(\hat{i}+\hat{j}+\hat{k}\right)$ and $C\left(3\hat{i}-\hat{k}\right)$ is :

A. 1

B. 2

C. $\frac{1}{2}$

D. $-\frac{1}{2}$

Answer: B

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24. Q is the image of point P(1, -2, 3) with respect to the plane $x-y+z=7$. The distance of Q from the origin is.

A. $\sqrt{\frac{70}{3}}$

B. $\frac{1}{2}\sqrt{\frac{70}{3}}$

C. $\sqrt{\frac{35}{3}}$

D. $\sqrt{\frac{15}{2}}$

Answer: A

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25. $\widehat{a}, \hat{b}$ and $\widehat{a}-\hat{b}$ are unit vectors. The volume of the parallelopiped, formed with $\widehat{a}, \hat{b}$ and $\widehat{a} \times \hat{b}$ as coterminous edges is :

A. 1

B. $\frac{1}{4}$

C. $\frac{2}{3}$

D. $\frac{3}{4}$

Answer: D

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26. A line passing through P(3, 7, 1) and R(2, 5, 7) meet the plane $3x+2y+11z-9=0$ at Q. Then PQ is equal to :

A. $\frac{5\sqrt{41}}{59}$

B. $\frac{\sqrt{41}}{59}$

C. $\frac{50\sqrt{41}}{59}$

D. $\frac{25\sqrt{41}}{59}$

Answer: D

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27. Let $\vec{a}, \vec{b}, \vec{c}$ be three non-zero non coplanar vectors and $\vec{p}, \vec{q}$ and $\vec{r}$ be three vectors given by $\vec{p}=\vec{a}+\vec{b}-2\vec{c}, \vec{q}=3\vec{a}-2\vec{b}+\vec{c}$ and $\vec{r}=\vec{a}-4vcb+2\vec{c}$

If the volume of the parallelopiped determined by $\vec{a}, \vec{b}$ and $\vec{c}$ is V_1 and that of the parallelopiped determined by $\vec{a}, \vec{q}$ and $\vec{r}$ is V_2, then $V_2 : V_1 =$

A. 10

B. 15

C. 20

D. None of these

Answer: B

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28. If the two lines represented by $x + ay = b, z + cy = d$ and $x = a'y + b', z = c'y + d'$ be perpendicular to each other, then the value of $aa' + cc'$ is :

A. 1

B. 2

C. 3

D. 4

Answer: A

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29. The distance between the line $\overrightarrow{r} = 2\hat{i} - 2\hat{j} + 3\hat{k} + \lambda\left(\hat{i} - \hat{j} + 4\hat{k}\right)$ and the plane $\overrightarrow{r}.\left(\hat{i} + 5\hat{j} + \hat{k}\right) = 5$ is

A. $\frac{10}{9}$

B. $\frac{10}{3\sqrt{3}}$

C. $\frac{3}{10}$

D. $\frac{10}{3}$

Answer: B

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30. If $\left(\overrightarrow{a} \times \overrightarrow{b}\right) \times \overrightarrow{c} = \overrightarrow{a} \times \left(\overrightarrow{b} \times \overrightarrow{c}\right)$, where $\overrightarrow{a}, \overrightarrow{b}$ and $\overrightarrow{c}$ are any three vectors such that $\overrightarrow{a} \cdot \overrightarrow{b} \neq 0, \overrightarrow{b} \cdot \overrightarrow{c} \neq 0$, then $\overrightarrow{a}$ and $\overrightarrow{c}$ are :

A. Inclined at an angle of $\frac{\pi}{3}$

B. Inclined at an angle of $\frac{\pi}{6}$

C. Perpendicular

D. Parallel

Answer: D

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31. Let $\overrightarrow{r}$ be position vector of variable point in cartesian plane OXY such that $\overrightarrow{r} \cdot \left(\overrightarrow{r} + 6\hat{j}\right) = 7$ cuts the co-ordinate axes at four distinct points, then the area of the quadrilateral formed by joining these points is :

A. $4\sqrt{7}$

B. $6\sqrt{7}$

C. $7\sqrt{7}$

D. $8\sqrt{7}$

Answer: D

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32. If $\left|\vec{a}\right| = 2, \left|\vec{b}\right| = 5 \text{ and } \vec{a} \cdot \vec{b} = 0,$ then $\vec{a} \times \left(\vec{a} \times \left(\vec{a} \times \left(\vec{a} \times \left(\vec{a} \times \left(\vec{a} \times \vec{b}\right)\right)\right)\right)\right)$ is equal to :

A. $64\vec{a}$

B. $64\vec{b}$

C. $-64\vec{a}$

D. $-64\vec{b}$

Answer: D

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33. If O (origin) is a point inside the triangle PQR such that $\overrightarrow{OP} + k_1\overrightarrow{OQ} + k_2\overrightarrow{OR} = 0$, where k_1, k_2 are constants such that $\frac{\text{Area}(\Delta PQR)}{\text{Area}(\Delta OQR)} = 4$, then the value of $k_1 + k_2$ is :

A. 2

B. 3

C. 4

D. 5

Answer: B

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34. Let PQ and QR be diagonals of adjacent faces of a rectangular box, with its centre at O. If $\angle QOR, \angle ROP$ and $\angle POQ$ are θ, ϕ and Ψ respectively then the value of $'\cos\theta + \cos\phi + \cos\Psi'$ is :

A. -2

B. $-\sqrt{3}$

C. -1

D. 0

Answer: C

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35. The value of $\begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \\ \vec{a}\cdot\vec{p} & \vec{b}\cdot\vec{p} & \vec{c}\cdot\vec{p} \\ \vec{a}\cdot\vec{q} & \vec{b}\cdot\vec{q} & \vec{c}\cdot\vec{q} \end{vmatrix}$ is equal to :

A. $\left(\vec{p}\times\vec{q}\right)\left[\vec{a}\times\vec{b} \quad \vec{b}\times\vec{c} \quad \vec{c}\times\vec{a}\right]$

B. $2\left(\vec{p}\times\vec{q}\right)\left[\vec{a}\times\vec{b} \quad \vec{b}\times\vec{c} \quad \vec{c}\times\vec{a}\right]$

C. $4\left(\vec{p}\times\vec{q}\right)\left[\vec{a}\times\vec{b} \quad \vec{b}\times\vec{c} \quad \vec{c}\times\vec{a}\right]$

D. $\left(\vec{p}\times\vec{q}\right)\sqrt{\left[\vec{a}\times\vec{b} \quad \vec{b}\times\vec{c} \quad \vec{c}\times\vec{a}\right]}$

Answer: D

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36. If

$$\vec{r} = a\left(\vec{m} \times \vec{n}\right) + b\left(\vec{n} \times \vec{I}\right) + c\left(\vec{I} \times \vec{m}\right) \text{ and } \left[\vec{I}\,\vec{m}\,\vec{n}\right] = 4, \text{ find}$$

:

A. $\frac{1}{4}$

B. $\frac{1}{2}$

C. 1

D. 2

Answer: A

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37. The volume of tetrahedron, for which three co-terminus edges are $\vec{a} - \vec{b}, \vec{b} + 2\vec{c}$ and $3\vec{a} - \vec{c}$ is :

A. 6k

B. 7k

C. 30k

D. 42k

Answer: D

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38. The equation of a plane passing through the line of intersection of the planes :

$x + 2y + z - 10 = 0$ and $3x + y - z = 5$ and passing through the origin is :

A. $5x + 3z = 0$

B. $5x - 3z = 0$

C. $5x + 4y + 3z = 0$

D. $5x - 4y + 3z = 0$

Answer: B

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39. Find the locus of a point whose distance from x-axis is equal the distance from the point (1, -1, 2) :

A. $y^2 + 2x - 2y - 4z + 6 = 0$

B. $x^2 + 2x - 2y - 4z + 6 = 0$

C. $x^2 - 2x + 2y - 4z + 6 = 0$

D. $z^2 - 2x + 2y - 4z + 6 = 0$

Answer: C

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Exercise 2 One Or More Than One Answer Is Are Correct

1. If equation of three lines are :

$\frac{x}{1} = \frac{y}{2} = \frac{z}{3}, \frac{x}{2} = \frac{y}{1} = \frac{z}{3}$ and $\frac{x-1}{1} = \frac{2-y}{1} = \frac{z-3}{0}$, then which of the following statement(s) is/are correct ?

A. Triangle formed by the line is equilateral

B. Triangle formed by the lines is isosceles

C. Equation of the plane containing the lines is $x + y = z$

D. Area of the triangle formed by the lines is $\frac{3\sqrt{3}}{2}$

Answer: B::C::D

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2. If $\overrightarrow{a} = \hat{i} + 6\hat{j} + 3\hat{k}; \overrightarrow{b} = 3\hat{i} + 2\hat{j} + \hat{k}$ and $\overrightarrow{c} = (\alpha + 1)\hat{i} + (\beta - 1)\hat{j} + \hat{k}$ are linearly dependent vectors and $\left|\overrightarrow{c}\right| = \sqrt{6}$; then the possible value(s) of $(\alpha + \beta)$ can be:

A. 1

B. 2

C. 3

D. 4

Answer: A::C

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3. Consider the lines: $L_1 : \frac{x-2}{1} = \frac{y-1}{7} = \frac{z+2}{-5}, L_2 : x-4 = y+3 = -z$ Then which of the following is/are correct ? (A) Point of intersection of L_1 and $L_2 is (1, -6, 3)$

A. Point of intersection of L_1 and L_2 is (1, -6, 3)

B. Equation of plane containing L_1 and L_2 is $x+2y+3z+2=0$

C. Acute angle between L_1 and L_2 is $\cot^{-1}\left(\frac{13}{15}\right)$

D. Equation of plane containing L_1 and L_2 is $x+2y+2z+3=0$

Answer: A::B::C

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4. Let $\widehat{a}, \hat{b}$ and $\hat{c}$ be three unit vectors such that $\widehat{a} = \hat{b} + \left(\hat{b} \times \hat{c}\right)$, then the possible value(s) of $\left|\widehat{a} + \hat{b} + \hat{c}\right|^2$ can be :

A. 1

B. 4

C. 16

D. 9

Answer: A::D

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5. The value(s) of μ for which the straight lines $\overrightarrow{r} = 3\hat{i} - 2\hat{j} - 4\hat{k} + \lambda_1\left(\hat{i} - \hat{j} + \mu\hat{k}\right)$ and

$\vec{r} = 5\hat{i} - 2\hat{j} + \hat{k} + \lambda_2\left(\hat{i} + \mu\hat{j} + 2\hat{k}\right)$ are coplanar is/are :

A. $\dfrac{5+\sqrt{33}}{4}$

B. $\dfrac{-5+\sqrt{33}}{4}$

C. $\dfrac{5-\sqrt{33}}{4}$

D. $\dfrac{-5-\sqrt{33}}{4}$

Answer: A::C

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6. If $\hat{i} \times \left[\left(\vec{a} - \hat{j}\right) \times \hat{i}\right] \times \left[\left(\vec{a} - \hat{k}\right) \times \hat{j}\right] + \vec{k} \times \left[\left(\vec{a} - \vec{i}\right) \times \hat{k}\right] = 0$, then find vector $\vec{a}$.

A. $x + y = 1$

B. $y + z = \dfrac{1}{2}$

C. $x + z = 1$

D. None of these

Answer: A::C

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7. The value of expression $\left[\vec{a}\times\vec{b} \quad \vec{c}\times\vec{d} \quad \vec{e}\times\vec{f}\right]$ is equal to :

A. $\left[\vec{a}\vec{b}\vec{d}\right]\left[\vec{c}\vec{e}\vec{f}\right]-\left[\vec{a}\vec{b}\vec{c}\right]\left[\vec{d}\vec{e}\vec{f}\right]$

B. $\left[\vec{a}\vec{b}\vec{e}\right]\left[\vec{f}\vec{c}\vec{d}\right]-\left[\vec{a}\vec{b}\vec{f}\right]\left[\vec{e}\vec{c}\vec{d}\right]$

C. $\left[\vec{c}\vec{d}\vec{a}\right]\left[\vec{b}\vec{e}\vec{f}\right]-\left[\vec{c}\vec{d}\vec{b}\right]\left[\vec{a}\vec{e}\vec{f}\right]$

D. $\left[\vec{b}\vec{c}\vec{d}\right]\left[\vec{a}\vec{e}\vec{f}\right]-\left[\vec{b}\vec{c}\vec{f}\right]\left[\vec{a}\vec{e}\vec{d}\right]$

Answer: A::B::C

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8. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are the position vectors of the points A, B, C and D respectively in three dimensionalspace no three of A, B, C, D are collinear and satisfy the relation $3\vec{a} - 2\vec{b} + \vec{c} - 2\vec{d} = 0$, then

A. A, B, C and D are coplanar

B. The line joining the points B and D divides the line joining the point A and C in the ratio of $2:1$

C. The line joining the points A and C divides the line joining the points B and D in the ratio of $1:1$

D. The four vectors $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are linearly dependent .

Answer: A::C::D

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9. If OAB is a tetrahedron with edges and $\hat{p}, \hat{q}, \hat{r}$ are unit vectors along bisectors of

$\overrightarrow{OA}, \overrightarrow{OB} : \overrightarrow{OB}, \overrightarrow{OC} : \overrightarrow{OC}, \overrightarrow{OA}$ respectively and

$\widehat{a} = \frac{\overrightarrow{OA}}{\left|\overrightarrow{OA}\right|}, \overrightarrow{b} = \frac{\overrightarrow{OB}}{\left|\overrightarrow{OB}\right|}, \overrightarrow{c} = \frac{\overrightarrow{OC}}{\left|\overrightarrow{OC}\right|}$, then :

A. $\frac{\left[\widehat{a}\hat{b}\hat{c}\right]}{[\hat{p}\hat{q}\hat{r}]} = \frac{3\sqrt{3}}{2}$

B. $\frac{\left[\widehat{a}+\hat{b} \quad \hat{b}+\hat{c} \quad \hat{c}+\widehat{a}\right]}{[\hat{p}+\hat{q} \quad \hat{q}+\hat{r} \quad \hat{r}+\hat{p}]} = \frac{3\sqrt{3}}{4}$

C. $\frac{\left[\widehat{a}+\hat{b} \quad \hat{b}+\hat{c} \quad \hat{c}+\widehat{a}\right]}{[\hat{p}\hat{q}\hat{r}]} = \frac{3\sqrt{3}}{2}$

D. $\frac{\left[\widehat{a}\hat{b}\hat{c}\right]}{[\hat{p}+\hat{q} \quad \hat{q}+\hat{r} \quad \hat{r}+\hat{p}]} = \frac{3\sqrt{3}}{4}$

Answer: A::D

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10. Let $\widehat{a}$ and $\hat{c}$ are unit vectors and $\left|\overrightarrow{b}\right| = 4$. If the angle between $\widehat{a}$ and $\hat{c}$ is $\cos^{-1}\left(\frac{1}{4}\right)$, and $\hat{b} - 2\hat{c} = \lambda\widehat{a}$, then the value of λ can be :

A. 2

B. 3

C. -3

D. 4

Answer: C::D

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11. Consider the lines $x = y = z$ and line $2x + y + z - 1 = 0 = 3x + y + 2z - 2$, then

A. The shortest distance between the two lines is $\frac{1}{\sqrt{2}}$

B. The shortest distance between the two lines is $\sqrt{2}$

C. Plane containing the line L_2 and parallel to line L_1 is $z - x + 1 = 0$

D. Perpendicular distance of origin from plane containing line L_2 and parallel to line L_1 is $\frac{1}{\sqrt{2}}$

Answer: A::D

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12. Let $\vec{r} = \sin x\left(\vec{a} \times \vec{b}\right) + \cos y\left(\vec{b} \times \vec{c}\right) + 2\left(\vec{c} \times \vec{a}\right)$, where $\vec{a}, \vec{b}$ are non-collinear and $\vec{c}, \vec{d}$ are also non-collinear then :

A. π^2

B. $\frac{5\pi^2}{4}$

C. $\frac{35\pi^2}{4}$

D. $\frac{37\pi^2}{4}$

Answer: B::D

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13. If $\left(\vec{a} \times \vec{b}\right) \times \left(\vec{c} \times \vec{d}\right) = h\vec{a} + k\vec{b} = r\vec{c} + s\vec{d}$, where $\vec{a}, \vec{b}$ are non-collinear and $\vec{c}, \vec{d}$ are also non-collinear then :

A. $h = \left[\vec{b}\vec{c}\vec{d}\right]$

B. $k = \left[\vec{a}\ \vec{c}\ \vec{d}\right]$

C. $r = \left[\vec{a}\ \vec{b}\ \vec{d}\right]$

D. $s = -\left[\vec{a}\ \vec{b}\ \vec{c}\right]$

Answer: B::C::D

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14. Let a be a real number and $\vec{\alpha} = \hat{i} + 2\hat{j}, \vec{\beta} = 2\hat{i} + a\hat{j} + 10\hat{k}, \vec{\gamma} = 12\hat{i} + 20\hat{i} + a\hat{k}$ be three vectors, then $\vec{\alpha}, \vec{\beta}$ and $\vec{\gamma}$ are linearly independent for :

A. $a > 0$

B. $a < 0$

C. $a = 0$

D. No value of a

Answer: A::B::C

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15. The volume of a tetrahedron prism $ABCA_1B_1C_1$ is equal to 3. Find the coordinates of the vertex A_1, if the coordinate of the base vertices of the prism are $A(1, 0, 1)$, $B(2, 0, 0)$ and $C(0, 1, 0)$.

A. (2, 2, 2)

B. (0, 2, 0)

C. (0, -2, 2)

D. (0, -2, 0)

Answer: A::D

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16. If $\overrightarrow{a} = x\hat{i} + y\hat{j} + z\hat{k}$, $\overrightarrow{b} = y\hat{i} + z\hat{j} + x\hat{k}$, and $\overrightarrow{c} = z\hat{i} + x\hat{j} + y\hat{k}$, then $\overrightarrow{a} \times \left(\overrightarrow{b} \times \overrightarrow{c}\right)$ is :

A. Parallel to $(y - z)\hat{i} + (z - x)\hat{j} + (x - y)\hat{k}$

B. Orthogonal to $\hat{i} + \hat{j} + \hat{k}$

C. Orthogonal to $(y + z)\hat{i} + (z + x)\hat{j} + (x + y)\hat{k}$,

D. Orthogonal to $x\hat{i} + y\hat{j} + z\hat{k}$

Answer: A::B::C::D

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17. If a line has a vector equation, $\vec{r} = 2\hat{i} + 6\hat{j} + \lambda\left(\hat{i} - 3\hat{j}\right)$ then which of the following statements holds good ?

A. the line is parallel to $2\hat{i} + 6\hat{j}$

B. the line passes through the point $3\hat{i} + 3\hat{j}$

C. the line passes through the point $\hat{i} + 9\hat{j}$

D. the line is parallel to xy plane

Answer: B::C::D

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18. Let M,N, P and Q be the mid points of the edges AB, CD, AC and BD respectively of the tetrahedron ABCD. Further, MN is perpendicular to both AB and CD and PQ is perpendicular to both AC and BD. Then which of the following is/are correct:

A. AB = CD

B. BC = DA

C. AC = BD

D. AN = BN

Answer: A::B::C::D

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19. The solution vectors $\vec{r}$ of the equation $\vec{r} \times \hat{i} = \hat{j} + \hat{k}$ and $\vec{r} \times \hat{j} = \hat{k} + \hat{j}$ represent two straight lines which are :

A. Intersecting

B. Non coplanar

C. Coplanar

D. Non intersecting

Answer: B::D

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20. Which of the following statement(s) is/are incorrect ?

A. The lines $\frac{x-4}{-3}=\frac{y+6}{-1}=\frac{z+6}{-1}$ and $\frac{x-1}{-1}=\frac{y-2}{-2}=\frac{z-3}{2}$ are orthogonal

B. The planes $3x-2y-4z=3$ and the plane $x-y-z=3$ are orthogonal

C. The function $f(x) = \text{In}\left(e^{-2} + e^{x}\right)$ is monotonic increasing $\forall x \in R$

D. If g is the inverse of the function, $f(x) = \text{In}\left(e^{-2} + e^{x}\right)$ then $g(x) = \text{In}\left(e^{x} - e^{-2}\right)$

Answer: A::B

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21. The lines with vector equations are, $\vec{r}_1 = 3\hat{i} + 6\hat{j} + \lambda\left(-4\hat{i} + 3\hat{j} + 2\hat{k}\right)$ and $\vec{r}_2 = -2\hat{i} + 7\hat{j} + \mu\left(-4\hat{i} +\right.$

are such that :

A. they are coplanar

B. they do not intersect

C. they are skew

D. the angle between then is $\tan^{-1}(3/7)$

Answer: B::C::D

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Exercise 3 Comprehension Type Problems

1. The vertices of ΔABC are (2, 0, 0), B(0, 1, 0), C(0, 0, 2). Its orthocentre is H and circumcentre is S. P is a point equidistant from A, B, C and the origin O.

Q. The z-coordinate of H is :

A. 1

B. $1/2$

C. $1/6$

D. $1/3$

Answer: D

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2. The vertices of ΔABC are (2, 0, 0), B(0, 1, 0), C(0, 0, 2). Its orthocentre is H and circumcentre is S. P is a point equidistant from A, B, C and the origin O.

Q. The y-coordinate of S is :

A. $5/6$

B. $1/3$

C. $1/6$

D. $1/2$

Answer: C

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3. The vertices of ΔABC are (2, 0, 0), B(0, 1, 0), C(0, 0, 2). Its orthocentre is H and circumcentre is S. P is a point equidistant from A, B, C and the origin O.

Q. PA is equal to :

A. 1

B. $\sqrt{2}$

C. $\sqrt{\frac{3}{2}}$

D. $\frac{3}{2}$

Answer: D

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4. Consider a plane $\pi : \vec{r} \cdot \vec{n} = d$ (where $\vec{n}$ is not a unti vector). There are two points $A\left(\vec{a}\right)$ and $B\left(\vec{b}\right)$ lying on the same side of the plane.

Q. If foot of perpendicular from A and B to the plane π are P and Q respectively, then length of PQ be :

A. $\frac{\left|\left(\vec{b} - \vec{a}\right) \cdot \vec{n}\right|}{\left|\vec{n}\right|}$

B. $\left|\left(\vec{b}-\vec{a}\right)\cdot\vec{n}\right|$

C. $\dfrac{\left|\left(\vec{b}-\vec{a}\right)\times\vec{n}\right|}{\left|\vec{n}\right|}$

D. $\left|\left(\vec{b}-\vec{a}\right)\times\vec{n}\right|$

Answer: C

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5. Consider a plane $\pi: \vec{r}\cdot\vec{n}=d$ (where $\vec{n}$ is not a unti vector). There are two points $A\left(\vec{a}\right)$ and $B\left(\vec{b}\right)$ lying on the same side of the plane.

Q. Reflection of $A\left(\vec{a}\right)$ in the plane π has the position vector :

A. $\vec{a}+\dfrac{2}{\left(\vec{n}\right)^2}\left(d-\vec{a}\cdot\vec{n}\right)\vec{n}$

B. $\vec{a}-\dfrac{1}{\left(\vec{n}\right)^2}\left(d-\vec{a}\cdot\vec{n}\right)\vec{n}$

C. $\vec{a}+\dfrac{2}{\left(\vec{n}\right)^2}\left(d+\vec{a}\cdot\vec{n}\right)\vec{n}$

D. $\vec{a}+\dfrac{2}{\left(\vec{n}\right)^2}\vec{n}$

Answer: A

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6. Consider a plane $\pi : \vec{r} \cdot \vec{n} = d$ (where $\vec{n}$ is not a unti vector). There are two points $A(\vec{a})$ and $B(\vec{b})$ lying on the same side of the plane.

Q. If a plane π_1 is drawn from the point $A(\vec{a})$ and another plane π_2 is drawn point $B(\vec{b})$ parallel to π, then the distance between the planes π_1 and π_2 is :

A. $\dfrac{\left|\left(\vec{a} - \vec{b}\right) \cdot \vec{n}\right|}{\left|\vec{n}\right|}$

B. $\left|\left(\vec{a} - \vec{b}\right) \cdot \vec{n}\right|$

C. $\left|\left(\vec{a} - \vec{b}\right) \times \vec{n}\right|$

D. $\dfrac{\left|\left(\vec{a} - \vec{b}\right) \times \vec{n}\right|}{\left|\vec{n}\right|}$

Answer: A

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7. Consider a plane $\prod : \overrightarrow{r} \cdot \left(2\hat{i} + \hat{j} - \hat{k}\right) = 5$, a line $L_1 : \overrightarrow{r} = \left(3\hat{i} - \hat{j} + 2\hat{k}\right) + \lambda\left(2\hat{i} - 3\hat{j} - \hat{k}\right)$ and a point $a(3, -4, 1) \cdot L_2$ is a line passing through A intersecting L_1 and parallel to plane $\prod$.

Q. Equation of L_2 is :

A. $\overrightarrow{r} = (1 + \lambda)\hat{i} + (2 - 3\lambda)\hat{j} + (1 - \lambda)\hat{k} : \lambda \in R$

B. $\overrightarrow{r} = (3 + \lambda)\hat{i} - (4 - 2\lambda)\hat{j} + (1 + 3\lambda)\hat{k}, \lambda \in R$

C. $\overrightarrow{r} = (3 + \lambda)\hat{i} - (4 + 3\lambda)\hat{j} + (1 - \lambda)\hat{k}, \lambda \in R$

D. None of the above

Answer: C

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8. Consider a plane $\prod : \overrightarrow{r} \cdot \left(2\hat{i} + \hat{j} - \hat{k}\right) = 5$, a line $L_1 : \overrightarrow{r} = \left(3\hat{i} - \hat{j} + 2\hat{k}\right) + \lambda\left(2\hat{i} - 3\hat{j} - \hat{k}\right)$ and a point

$a(3, -4, 1) \cdot L_2$ is a line passing through A intersecting L_1 and parallel to plane $\prod$.

Q. Plane containing L_1 and L_2 is :

A. parallel to yz-plane

B. parallel to x-axis

C. parallel to y-axis

D. passing through origin

Answer: B

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9. Consider a plane $\prod : \vec{r} \cdot \left(2\hat{i} + \hat{j} - \hat{k}\right) = 5$, a line $L_1 : \vec{r} = \left(3\hat{i} - \hat{j} + 2\hat{k}\right) + \lambda\left(2\hat{i} - 3\hat{j} - \hat{k}\right)$ and a point $a(3, -4, 1) \cdot L_2$ is a line passing through A intersecting L_1 and parallel to plane $\prod$.

Q. Line L_1 intersects plane $\prod$ at Q and xy-plane at R the volume of tetrahedron OAQR is :

(where 'O' is origin)

A. 0

B. $\frac{14}{3}$

C. $\frac{3}{7}$

D. $\frac{7}{3}$

Answer: D

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10. Consider three planes :

$2x + py + 6z = 8, x + 2y + qz = 5$ and $x + y + 3z = 4$

Q. Three planes intersect at a point if :

A. $p = 2, q \neq 3$

B. $p \neq 2, q \neq 3$

C. $p \neq 2, q = 3$

D. $p=2, q=3$

Answer: B

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11. Consider three planes :

$2x+py+6z=8, x+2y+qz=5 \text{ and } x+y+3z=4$

Q. Three planes do not have any common point of intersection if :

A. $p=2, q\neq 3$

B. $p\neq 2, q\neq 3$

C. $p\neq 2, q=3$

D. $p=2, q=3$

Answer: C

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12. The points A, B and C with position vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively lie on a circle centered at origin O. Let G and E be the centroid of ΔABC and ΔACD respectively where D is mid point of AB.

Q. If OE and CD are mutually perpendicular, then which of the following will be necessarily true ?

A. $\left|\vec{b} - \vec{a}\right| = \left|\vec{c} - \vec{a}\right|$

B. $\left|\vec{b} - \vec{a}\right| = \left|\vec{b} - \vec{c}\right|$

C. $\left|\vec{c} - \vec{a}\right| = \left|\vec{c} - \vec{b}\right|$

D. $\left|\vec{b} - \vec{a}\right| = \left|\vec{c} - \vec{a}\right| = \left|\vec{b} - \vec{c}\right|$

Answer: A

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13. The points A, B and C with position vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively lie on a circle centered at origin O. Let G and E be the centroid of ΔABC and ΔACD respectively where D is mid point of AB.

Q. If GE and CD are mutually perpendicular, then orthocenter of ΔABC must lie on :

A. median through A

B. median through C

C. angle bisector through A

D. angle bisector through B

Answer: B

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14. The points A, B and C with position vectors $\overrightarrow{a}$, $\overrightarrow{b}$ and $\overrightarrow{c}$ respectively lie on a circle centered at origin O. Let G and E be the centroid of ΔABC and ΔACD respectively where D is mid point of AB.

Q. If $\left[\overrightarrow{AB}\overrightarrow{AC}\overrightarrow{AB}\times\overrightarrow{AC}\right]=\lambda\left[\overrightarrow{AE}\overrightarrow{AG}\overrightarrow{AE}\times\overrightarrow{AG}\right]$, then the value of λ is :

A. -18

B. 18

C. -324

D. 324

Answer: D

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15. Consider a tetrahedron $D - ABC$ with position vectors if its angular points as

A(1, 1, 1), B(1, 2, 3), C(1, 1, 2)

and centre of tetrahedron $\left(\frac{3}{2}, \frac{3}{4}, 2\right)$.

Q. Shortest distance between the skew lines AB and CD :

A. $\frac{1}{2}$

B. $\frac{1}{3}$

C. $\frac{1}{4}$

D. $\frac{1}{5}$

Answer: B

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16. Consider a tetrahedron $D - ABC$ with position vectors if its angular points as

A(1, 1, 1), B(1, 2, 3), C(1, 1, 2)

and centre of tetrahedron $\left(\frac{3}{2}, \frac{3}{4}, 2\right)$.

Q. If N be the foot of the perpendicular from point D on the plane face ABC then the position vector of N are :

A. (-1, 1, 2)

B. (1, -1, 2)

C. (1, 1, -2)

D. (-1, -1, 2)

Answer: B

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17. In a triangle AOB, R and Q are the points on the side OB and AB respectively such that 3OR = 2RB and 2AQ = 3QB. Let OQ and AR intersect at the point P (where O is origin).

Q. If the point P divides OQ in the ratio of $\mu : 1$, then μ is :

A. $\frac{2}{19}$

B. $\frac{2}{17}$

C. $\frac{2}{15}$

D. $\frac{10}{9}$

Answer: D

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18. In a triangle AOB, R and Q are the points on the side OB and AB respectively such that 3OR = 2RB and 2AQ = 3QB. Let OQ and AR intersect at the point P (where O is origin).

Q. If the ratio of area of quadrilateral PQBR and area of ΔOPA is $\frac{\alpha}{\beta}$ then $(\beta - \alpha)$ is (where α and β are coprime numbers) :

A. 1

B. 9

C. 7

D. 0

Answer: D

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Exercise 4 Matching Type Problems

1.

	Column-I		Column-II
(A)	If $\vec{a}, \vec{b}$ and $\vec{c}$ are three mutually perpendicular vectors where $\lvert\vec{a}\rvert = \lvert\vec{b}\rvert = 2$, $\lvert\vec{c}\rvert = 1$, then $[\vec{a}\times\vec{b} \ \ \vec{b}\times\vec{c} \ \ \vec{c}\times\vec{a}]$ is	(P)	-12
(B)	If $\vec{a}$ and $\vec{b}$ are two unit vectors inclined at $\frac{\pi}{3}$, then $16[\vec{a} \ \ \vec{b} + (\vec{a}\times\vec{b}) \ \ \vec{b}]$ is	(Q)	0
(C)	If $\vec{b}$ and $\vec{c}$ are orthogonal unit vectors and $\vec{b}\times\vec{c} = \vec{a}$ then $[\vec{a}+\vec{b}+\vec{c} \ \ \vec{a}+\vec{b} \ \ \vec{b}+\vec{c}]$ is	(R)	16
(D)	If $[\vec{x} \ \vec{y} \ \vec{a}] = [\vec{x} \ \vec{y} \ \vec{b}] = [\vec{a} \ \vec{b} \ \vec{c}] = 0$, each vector being a non-zero vector, then $[\vec{x} \ \vec{y} \ \vec{c}]$ is	(S)	1
		(T)	4

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Exercise 5 Subjective Type Problems

1. A straight line L intersects perpendicularly both the lines :

$$\frac{x+2}{2} = \frac{y+6}{3} = \frac{z-34}{-10} \text{ and } \frac{x+6}{4} = \frac{y-7}{-3} = \frac{z-7}{-2},$$

then the square of perpendicular distance of origin from L is

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2. If $\hat{a}, \hat{b}$ and $\hat{c}$ are non-coplanar unti vectors such that $\left[\hat{a}\hat{b}\hat{c}\right] = \left[\hat{b} \times \hat{c} \quad \hat{c} \times \hat{a} \quad \hat{a} \times \hat{b}\right]$, then find the projection of $\hat{b} + \hat{c}$ on $\hat{a} \times \hat{b}$.

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3. Let OA, OB, OC be coterminous edges of a cubboid. If l, m, n be the shortest distances between the sides OA, OB, OC and their respective skew body diagonals to them, respectively, then find $\dfrac{\left(\frac{1}{l^2} + \frac{1}{m^2} + \frac{1}{n^2}\right)}{\left(\frac{1}{OA^2} + \frac{1}{OB^2} + \frac{1}{OC^2}\right)}$.

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4. Let OABC be a tetrahedron whose edges are of unit length. If $\vec{O}A = \vec{a}$, $\vec{O}B = \vec{b}$, and $\vec{O}C$ = $\alpha\left(\vec{a} + \vec{b}\right) + \beta\left(\vec{a} \times \vec{b}\right)$, then $(\alpha\beta)^2 = \dfrac{p}{q}$(where p & q are relatively prime to each other). then the value of $\left[\dfrac{q}{2}p\right]$ is

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5. Let $\vec{v}_0$ be a fixed vector and $\vec{v}_0 = \begin{bmatrix} \frac{1}{0} \end{bmatrix}$. Then for $n \geq 0$ a sequence is defined $\vec{v}_{n+1} = \vec{v}_n + \left(\frac{1}{2}\right)^{n+1} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}^{n+1} \vec{v}_0$ then $\lim_{n \to \infty} \vec{v}_n = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$. Find $\frac{\alpha}{\beta}$.

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6. If a is the matrix $\begin{bmatrix} 1 & -3 \\ -1 & 1 \end{bmatrix}$, then $A - \frac{1}{3}A^2 + \frac{1}{9}A^3 \dots\dots\dots\dots + \left(-\frac{1}{3}\right)^n A^{n+1} + \dots\dots\dots\infty = \frac{3}{13}\begin{bmatrix} 1 & a \\ b & 1 \end{bmatrix}$. Find $\left|\frac{a}{b}\right|$.

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7. A sequence of 2×2 matrices $\{M_n\}$ is defined as follows

$$M_n = \begin{bmatrix} \frac{1}{(2n+1)!} & \frac{1}{(2n+2)!} \\ \sum_{k=0}^{n} \frac{(2n+2)!}{(2k+2)!} & \sum_{k=0}^{n} \frac{(2n+1)!}{(2k+1)!} \end{bmatrix}$$ then

$\lim_{n \to \infty} \det.(M_n) = \lambda - e^{-1}$. Find λ.

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8. Let $\left|\vec{a}\right| = 1, \left|\vec{b}\right| = 1$ and $\left|\vec{a} + \vec{b}\right| = \sqrt{3}$. If $\vec{c}$ be a vector such that $\vec{c} = \vec{a} + 2\vec{b} - 3\left(\vec{a} \times \vec{b}\right)$ and $p\left|\left(\vec{a} \times \vec{b}\right) \times \vec{c}\right|$, then find $\left[p^2\right]$. (where [ ] represents greatest integer function).

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9. Let $\vec{r} = \left(\vec{a} \times \vec{b}\right)\sin x + \left(\vec{b} \times \vec{c}\right)\cos y + 2\left(\vec{c} \times \vec{a}\right)$, where $\vec{a}, \vec{b}, \vec{c}$ are non-zero and non-coplanar vectors. If $\vec{r}$ is orthogonal to $\vec{a} + \vec{b} + \vec{c}$, then find the minimum value of $\frac{4}{\pi^2}\left(x^2 + y^2\right)$.

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10. The plane denoted by $P_1 : 4x + 7y + 4z + 81 = 0$ is rotated through a right angle about its line of intersection with the plane

$P_2 : 5x + 3y + 10z = 25$. If the plane in its new position is denoted by P, and the distance of this plane from the origin is d, then find the value of $[k/2]$ (where $[\cdot]$ represents greatest integer less than or equal to k).

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11. ABCD is a regular tetrahedron, A is the origin and B lies on x-axis. ABC lies in the xy-plane $\left|\overrightarrow{AB}\right| = 2$ Under these conditions, the number of possible tetrahedrons is :

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12. A, B, C, D are four points in the space and satisfy $\left|\overrightarrow{AB}\right| = 3, \left|\overrightarrow{BC}\right| = 7, \left|\overrightarrow{CD}\right| = 11$ and $\left|\overrightarrow{DA}\right| = 9$. Then find the value of $\overrightarrow{AC} \cdot \overrightarrow{BD}$.

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13. Let OABC be a regular tetrahedron of edge length unity. Its volume be V and $6V = \sqrt{\frac{p}{q}}$ where p and q are relatively prime. The find the value of (p+q)

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14. If $\vec{a}$ and $\vec{b}$ are non zero, non collinear vectors and $\vec{a}_1 = \lambda \vec{a} + 3\vec{b}$, $\vec{b}_1 = 2\vec{a} + \lambda \vec{b}$, $\vec{c}_1 = \vec{a} + \vec{b}$. Find the sum of all possible real values of λ so that points A_1, B_1, C_1 whose position vectors are $\vec{a}_1$, $\vec{b}_1$, $\vec{c}_1$ respectively are collinear is equal to.

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15. Let P and Q are two points on curve $y = \log_{\frac{1}{2}}\left(x - \frac{1}{2}\right) + \log_2 \sqrt{4x^2 - 4x + 1}$ and P is also on $x^2 + y^2 = 10$. Q lies inside the given circle such that its abscissa is integer. Find the smallest possible value of $\overrightarrow{OP} \cdot \overrightarrow{OQ}$ where 'O' being origin.

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16. Let P and Q are two points on curve $y = \log_{\frac{1}{2}}\left(x - \frac{1}{2}\right) + \log_2 \sqrt{4x^2 - 4x + 1}$ and P is also on $x^2 + y^2 = 10$. Q lies inside the given circle such that its abscissa is integer. find the largest possible value of $\left|\overrightarrow{PQ}\right|$.

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17. If $a, b, c, l, m, n \in R - \{0\}$ such that $al + bm + cn = 0, bl + cm + an = 0, cl + am + bn = 0$. If a, b, c are distinct and $f(x) = ax^3 + bx^2 + cx + 2$. Find f(1) :

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18. Let $\overrightarrow{u} \, and \, \overrightarrow{v}$ be unit vectors such that $\overrightarrow{u} \times \overrightarrow{v} + \overrightarrow{u} = \overrightarrow{w}$ and $\overrightarrow{w} \times \overrightarrow{u} = \overrightarrow{v}$. Find the value of $\left[\overrightarrow{u} \, \overrightarrow{v} \, \overrightarrow{w}\right]$.

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