MATHS

BOOKS - CHHAYA MATHS (BENGALI ENGLISH)

MATHEMATICAL INDUCTION

Illustrative Example

1. Prove by mathematical induction that the sum of the squares of first n natural numbers is $\frac{1}{6}n(n+1)(2n+1)$.

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2. By principle of mathematical induction,prove that $1^3+2^3+3^3+\ldots.+n^3=\left[\frac{n(n+1)}{2}\right]^2$ for all $n \in \mathbb{N}$

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3. Prove by induction that $n(n+1)(2n+1)$ is divisible by 6 for all $n \in \mathbb{N}$

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4. By mathematical induction prove that, $\left(2^{2n}-1\right)$ is divisible by 3 where $n \geq 1$ is an integer.

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5. Prove by induction , 7 divides $3^{2n+1}+2^{n+2}$ for all $n \in \mathbb{N}$.

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6. By principle of mathematical induction show that $a^n - b^n$ is divisible by a+b when n is a positive even integer.

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7. Prove by induction that

$$\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\ldots+\frac{1}{(2n-1)(2n+1)}=\frac{n}{2n+1}(n\in\mathbb{N}).$$

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8. If $n\geq 3$ is an integer, prove by mathmatical induction that, $2n+1<2^n$.

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9. If $n\geq 4$ is an integer,prove by induction that $n^2<n!$.

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10. Prove by mathematical induction that,

$$1^2 - 2^2 + 3^2 - 4^2 + \dots. + (-1)^{n-1} \cdot n^2 = (-1)^{n-1} \cdot \frac{n(n+1)}{2}, n \in \mathbb{N}$$

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11. By induction method, find the value of

$1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \dots + n \cdot n!.$

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12. If $n \geq 0$ is an integer, prove by induction that $3 \cdot 5^{2n+1} + 2^{3n+1}$ is divisible by 17.

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13. If p be a natural number, then prove that, $p^{n+1}+(p+1)^{2n-1}$ is divisible by (p^2+p+1) for every positive integer n.

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14. By mathematical induction prove that,

$1\cdot 2+2\cdot 2^2+3\cdot 2^3+\ldots+n\cdot 2^n=(n-1)\cdot 2^{n+1}+2, n\in\mathbb{N}.$

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15. By mathematical indcution prove that,

$4+44+444+\ldots \text{to n terms} \quad =\frac{4}{81}\left(10^{n+1}-9n-10\right).$

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16. If $n\geq 2$ is an integer, prove by induction that,

$1+\frac{1}{4}+\frac{1}{9}+\ldots+\frac{1}{n^2}<2-\frac{1}{n}.$

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17. If $n > 1$ is an integer, prove induction that,

$$\frac{1}{n+1} + \frac{1}{n+2} + \ldots + \frac{1}{2n} > \frac{13}{24}.$$

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18. Prove by mathematical induction that $\left(11^{n+2} + 12^{2n+1}\right)$ is divisible by 133 for all non-negative integers.

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19. Let P(n) be the statement : $2^n > 3n$. IF P(m) is true, show that, P(m+1) is true. Do you conclude that, P(n) is true for all $n \in \mathbb{N}$?

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20. Using mathematical induction prove that,

$$\cos\theta+\cos 2\theta+\ldots+\cos n\theta=\frac{\cos\frac{n+1}{2}\theta\sin\frac{n\theta}{2}}{\sin\frac{\theta}{2}}.$$

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21. Using induction, prove that,

$$\cos\theta\cos 2\theta\cos 2^2\theta\ldots\cos 2^n\theta=\frac{\sin 2^{n+1}\theta}{2^{n+1}\sin\theta}.$$

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22. By induction prove that, $7^{2n}+2^{3(n-1)}\cdot 3^{n-1}$ is a multiple of 25 for all $n\in\mathbb{N}$.

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23. By mathematical induction prove that, $\frac{n^7}{7}+\frac{n^5}{5}+\frac{2n^3}{3}-\frac{n}{105}$ is an integer for all $n\in\mathbb{N}$.

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24. By mathematical induction prove the inequality $(1+x)^n \geq 1+nx$ for all $n \in \mathbb{N}$ and $x > -1$.

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25. Prove by induction that the sum of the cubes of three successive positive integers is divisible by 9.

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26. Using mathematical induction prove that for every integer $n \geq 1$, $\left(3^{2^n}-1\right)$ is divisible by 2^{n+2} but not by 2^{n+3}.

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27. Prove by mathematical induction that, $2n + 7 < (n+3)^2$ for all $n \in \mathbb{N}$.

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28. Prove by the principle of mathematical induction that when every even power of every odd integer greater than 1 is divided by 8, the remainder is always 1 .

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Exercise

1. The sum of first n numbers is -

A. n

B. $\dfrac{n(n+1)}{2}$

C. $\frac{1}{6}n(n+1)(2n+1)$

D. $\left[\frac{n(n+1)}{2}\right]^2$

Answer: B

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2. The sum of the squares of first n natural numbers is -

A. n^2

B. $\frac{n(n+1)}{2}$

C. $\frac{1}{6}n(n+1)(2n+1)$

D. $\left[\frac{n(n+1)}{2}\right]^2$

Answer: C

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3. The sum of the cubes of first n natural numbers is -

A. n^3

B. $\frac{n(n+1)}{2}$

C. $\frac{1}{6}n(n+1)(2n+1)$

D. $\left[\frac{n(n+1)}{2}\right]^2$

Answer: D

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4. State which of the following statement is true ?

A. If a mathematical relation involving the natural number n is true for all $n \in \mathbb{N}$.

B. A mathematical statement may be true or false.

C. (n^2+n+41) is a prime number for all natural number n.

D. If $n \in \mathbb{N}$ and $n \geq 2$. Then n n(n+1) is always divisible by 3.

Answer: B

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5. If $\left(2^{2n} - 1\right)$ is divisible by 3, then show that, $\left[2^{2(n+1)} - 1\right]$ is also divisible by 3.

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6. If $\left(2^{3n} - 1\right)$ is divisible by 7, then prove that, $\left[2^{3(n+1)} - 1\right]$ is also divisible by 7.

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7. If $1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \ldots + n \cdot n! = (n+1)! - 1$ then show that,

$$1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \ldots + n \cdot n! + (n+1)(n+1)! = (n+2)! - 1$$

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8. If $\left(10^{2n-1}+1\right)$ is divisible by 11, then prove that $\left(10^{2n+1}+1\right)$ is also divisible by 11.

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9. If $\left(15^{2n-1}+1\right)$ is divisible by 16, then show that $\left(15^{2n+1}+1\right)$ is also divisible by 16.

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10. If $\left[12^{n}+25^{n-1}\right]$ is divisible by 13, then show that $\left[12^{n+1}+25^{n}\right]$ is also divisible by 13.

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11. If $\left[n^3+(n+1)^3+(n+2)^3\right]$ is also divisible by 9. then show that, $\left[(n+1)^3+(n+2)^3+(n+3)^3\right]$ is also divisible by 9.

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12. If $n \in N, x > -1$and $(1+x)^n \geq 1+nx$, then prove that $(1+x)^{n+1} \geq 1+(n+1)x.$

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13. Let $f(n)=n(n+1)(2n+1)$, if f(n) is always divisible ny 6 then prove that, f(n+1) is also divisible ny 6.

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14. For all $n \in \mathbb{N}$, prove by principle of mathematical induction that, $1+3+5+\ldots+(2n-1)=n^2.$

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15. For all $n \in \mathbb{N}$, prove by principle of mathematical induction that,

$a + (a + d) + (a + 2d) + \ldots$ to n terms $= \frac{n}{2}[2a + (n - 1)d]$.

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16. For all $n \in \mathbb{N}$, prove by principle of mathematical induction that,

$$\sin x + \sin 2x + \sin 3x + \ldots + \sin nx = \frac{\sin\frac{n+1}{2}x \sin\frac{nx}{2}}{\sin\frac{x}{2}}$$

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17. For all $n \in \mathbb{N}$, prove by principle of mathematical induction that,

$$\sin x + \sin 3x + \ldots + \sin(2n - 1)x = \frac{\sin^2 nx}{\sin x}$$

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18. For all $n \in \mathbb{N}$, prove by principle of mathematical induction that,

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta \quad \left[i = \sqrt{-1}\right].$$

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19. For all $n \in \mathbb{N}$, prove by principle of mathematical induction that,

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots + \frac{1}{2^n} = 1 - \frac{1}{2^n}.$$

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20. For all $n \in \mathbb{N}$, prove by principle of mathematical induction that,

$$a + ar + ar^2 + \ldots \text{ to n terms } = a \cdot \frac{r^n - 1}{r - 1}[r \neq 1].$$

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21. For all $n \in \mathbb{N}$, prove by principle of mathematical induction that,

$$1^2 + 3^2 + 5^2 + \ldots + (2n-1)^2 = \frac{n}{3}\left(4n^2 - 1\right).$$

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22. For all $n \in \mathbb{N}$, prove by principle of mathematical induction that,

$$1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \ldots + n(n+1) = \frac{1}{3}n(n+1)(n+2).$$

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23. For all $n \in \mathbb{N}$, prove by principle of mathematical induction that,

$$2^2 + 5^2 + 8^2 + \ldots \text{ to n terms } = \frac{n}{2}\left(6n^2 + 3n - 1\right).$$

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24. For all $n \in \mathbb{N}$, prove by principle of mathematical induction that,

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \ldots + \frac{1}{n(n+1)} = \frac{n}{n+1}.$$

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25. For all $n \in \mathbb{N}$, prove by principle of mathematical induction that,

$$\frac{1}{1 \cdot 4} + \frac{1}{4 \cdot 7} + \frac{1}{7 \cdot 10} + \ldots \text{ to terms } = \frac{n}{3n+1}.$$

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26. For all $n \in \mathbb{N}$, prove by principle of mathematical induction that,

$$\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right) \ldots \left(1 - \frac{1}{n+1}\right) = \frac{1}{n+1}.$$

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27. For all $n \in \mathbb{N}$, prove by principle of mathematical induction that,

$$1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \ldots + \frac{1}{1+2+3+\ldots+n} = \frac{2n}{n+1}.$$

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28. For all $n \in \mathbb{N}$, prove by principle of mathematical induction that,

$$1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \ldots + n \cdot n! = (n+1)! - 1.$$

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29. For all $n \in \mathbb{N}$, prove by principle of mathematical induction that,

$2^{3n} - 1$ is divisible by 7.

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30. For all $n \in \mathbb{N}$, prove by principle of mathematical induction that,

$4^n + 15n - 1$ is a multiple of 9.

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31. For all $n \in \mathbb{N}$, prove by principle of mathematical induction that,

$2 \cdot 7^n + 3 \cdot 5^n - 5$ is divisible by 24.

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32. For all $n \in \mathbb{N}$, prove by principle of mathematical induction that,

$15^{2n-1} + 1$ is divisible by 16.

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33. For all $n \in \mathbb{N}$, prove by principle of mathematical induction that,

$12^n + 25^{n-1}$ is divisible by 13.

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34. For what natural numbers n the inequality $2^n > 2n + 1$ is valid ?

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35. $3^{2n+2} - 8n - 9$ is divisible by

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36. If $n \in \mathbb{N}$, then by principle of mathematical induction prove that,

$10^n + 3 \cdot 4^{n+2} + 5 \quad [n \geq 0]$ is divisible by 9.

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37. If $n \in \mathbb{N}$, then by principle of mathematical induction prove that,

$3^{4n+1} + 2^{2n+2} \quad [n \geq 0]$ is a multiple of 7.

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38. If $n \in \mathbb{N}$, then by principle of mathematical induction prove that,

$3^{2n+2} - 8n - 9$ is divisible by 64.

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39. If $n \in \mathbb{N}$, then by principle of mathematical induction prove that,

$7 + 77 + 777 + \ldots$ to n terms $= \frac{7}{81}\left(10^{n+1} - 9n - 10\right)$.

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40. If $n \in \mathbb{N}$, then by principle of mathematical induction prove that,

$$1 + 2 + 3 + \ldots. + n < \frac{1}{8}(2n+1)^2.$$

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41. If $n \in \mathbb{N}$, then by principle of mathematical induction prove that,

$$1^2 + 2^2 + 3^2 + \ldots. + n^2 > \frac{n^3}{3}.$$

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42. If $n \in \mathbb{N}$, then by princuple of mathematical induction prove that,

$\frac{1}{5}n^5 + \frac{1}{3}n^3 + \frac{1}{15} \cdot 7n$ is an integer.

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43. If $n \in \mathbb{N}$, then by princuple of mathematical induction prove that,

$\frac{n^{11}}{11} + \frac{n^5}{5} + \frac{n^3}{3} + \frac{62n}{165}$ is an integer.

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44. If $n \in \mathbb{N}$, then by princuple of mathematical induction prove that,

$$n \cdot 1 + (n-1) \cdot 2 + (n-2) \cdot 3 + \ldots + 2 \cdot (n-1) + 1 \cdot n = \frac{1}{6}n(n+1)$$

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45. If $n \in \mathbb{N}$, then by principle of mathematical induction prove that,

Prove that $2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \ldots n \text{ times}}}} < 4$, for all integers $n \geq 1$.

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46. If $n \in \mathbb{N}$, then by principle of mathematical induction prove that,

Use mathematical induction to prove

$$\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\dots\left(1-\frac{1}{(n+1)^2}\right)=\frac{n+2}{2n+2}$$

for all positive integers n.

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47. If $n \in \mathbb{N}$, then by princuple of mathematical induction prove that,

$$2+222+22222+\ldots+22\ldots\{(2n-1)\text{digits}\}=\frac{20}{891}\left(10^{2n}-1\right)-\frac{2n}{9}$$

for all positive integers n.

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48. If $n \in \mathbb{N}$, then by principle of mathematical induction prove that,

$$.^{n}C_0+.^{n}C_1+.^{n}C_2+\ldots+.^{n}C_n=2^n(n \in \mathbb{N})$$

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49. If x and are two real numbers, then prove by mathematical induction that $(x^n - y^n)$ is divisible by $(x - y)$ for all $n \in \mathbb{N}$.

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50. By induction method prove that, $(a^n + b^n)$ is divisible by (a+b) when n is an odd positive integer .

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51. If $n \in \mathbb{N}$ and $(2 \cdot 1 + 1) + (2 \cdot 2 + 1) + (2 \cdot 3 + 1) + \ldots + (2 \cdot n + 1) = n^2 + 2n + 5$ is true for n=m, then prove that it is also true for n=m+1. Can we conclude that is is true for all $n \in \mathbb{N}$?

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52. Prove by induction method that $n(n^2-1)$ is divisible by 24 when n is an odd positive integer.

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53. Find the positive integer n for which the inequality $2^n > n^2$ is true.

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54. Prove by the method of mathematica induction that for all $n \in \mathbb{N}$, 3^{2n} when divided by 8, the remainder is always 1.

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55. Prove by induction that $5^{n+1} + 4 \cdot 6^n$ when divided by 20 leaves the same remainder 9 for all $n \in \mathbb{N}$.

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56. Prove by induction that $2^{2^n}+1$ has 7 in unit's place for all $n \geq 2$.

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57. Using mathematical induction prove that for every integer $n, |\sin nx| \leq n|\sin x|$.

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Sample Questions For Competitive Exams

1. If for all $n \in \mathbb{N}$ and $n \geq 1$, then $\left(3^{2^n}-1\right)$ is always divisible by

A. 2^{n+2}

B. 2^{n-2}

C. 8

D. 9

Answer: A::C

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2. If $n \in \mathbb{N}$, both expression $n(n+1)(n+2)$ and $n(n+1)(n+5)$ are multiple of -

A. 5

B. 2

C. 3

D. 6

Answer: B::C::D

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3. The value of n for which $n! > 2^n$ will true, are -

A. 6

B. 3

C. 4

D. 5

Answer: A::C::D

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4. If n is even then the expression $n(n^2 + 20)$ is divisible by the numbers, are -

A. $\frac{2^3}{6^{-1}} \times (7 \times 5)^\circ$

B. $\frac{3^2}{6^{-1}} - (5 + 5^\circ)$

C. 96

D. 48

Answer: A::B::D

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5. If $p \in \mathbb{N}$ then the expression $p^{n+1} + (p+1)^{2n-1}$ is divisible by the expressions are -

A. $p^2 + p + 1$

B. $p^2 + p$

C. $\dfrac{(p^4 + p^2 + 1)}{(p^2 - p + 1)}$

D. $p^2 + 1$

Answer: A::C

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6. $10^n + 3 \cdot 4^2 + 5$ is always divisible by the number -

A. 9

B. 13

C. 21

D. 11

Answer: A

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7. 3^{2n} when divided by 8 leves the remainder-

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8. If n be a positive integer, then the digit in the unit's place of $3^{2n-1}+2^{2n-1}$ is -

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9. For a positive integer n, $n(n+1)(2n+1)$ when divided by 6 leaves the remainder -

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10. $n \in \mathbb{N}$, $(n+1)^3 + (n+2)^3 + (n+3)^3$ when divided by 9, then the remainder will be -

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11. Match the entries given in left column with those given in right column.

	Column I		Column II
A	If $n \in \mathbb{N}$, then $4^n + 15n - 1$ is divisible by	(p)	8
B	If 1 is added with the sum of the squares of three consecutive natural numbers, then the expression is divisible by	(q)	9
C	$2 \cdot 7^n + 3 \cdot 5^n - 5 (n \geqslant 1)$ is divisible by	(r)	12
D	$3^{2n+2} - 8n - 9 (n \geqslant 1)$ is divisible by	(s)	25
E	When n is a natural number, then $7^{2n} + (2^{3n-3}) \cdot (3^{n-1})$ is always divisible by	(t)	24

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12. Match the entries given in left column with those given in right column.

2.	Column I		Column II
A	$\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\cdots+\frac{1}{(2n-1)(2n+1)}=$	(p)	$\frac{n}{3n+1}$
B	$1+\frac{1}{1+2}+\frac{1}{1+2+3}+\cdots+\frac{1}{1+2+3+\cdots+n}=$	(q)	$\frac{12}{5}(6^n-1)$
C	$\frac{1}{1\cdot 4}+\frac{1}{4\cdot 7}+\frac{1}{7\cdot 10}+\cdots+\frac{1}{(3n-2)(3n+1)}=$	(r)	$(n-1)2^{n+1}+2$
D	$1\cdot 2+2\cdot 2^2+3\cdot 2^3+\cdots+n\cdot 2^{n+1}=$	(s)	$\frac{n}{2n+1}$
E	$3\cdot 2^2+3^2\cdot 2^3+3^3\cdot 2^4+\cdots+3^n\cdot 2^{n+1}=$	(t)	$\frac{2n}{n+1}$

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13. $P(n): 11^{n+2} + 1^{2n+1}$ where n is a positive integer

p(n) is divisible by -

A. 2

B. 7

C. 23

D. 9

Answer: B

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14. $P(n): 11^{n+2} + 1^{2n+1}$ where n is a positive integer

If P(n) = 14642, then the value of n is -

A. 3

B. 4

C. 5

D. 2

Answer: D

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15. $P(n): 11^{n+2} + 1^{2n+1}$ where n is a positive integer

If $n = 3$, then the last digit of expression is -

A. 5

B. 2

C. 9

D. 6

Answer: C

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16. If n be a positive integer and $P(n) : 4^{5n} - 5^{4n}$

P(n) is divisible by -

A. 399

B. 401

C. 397

D. 430

Answer: A

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17. If n be a positive integer and $P(n) : 4^{5n} - 5^{4n}$

If P(n) be negative value, then the value of n is -

A. 4

B. 5

C. 7

D. none of these

Answer: D

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18. If n be a positive integer and $P(n): 4^{5n} - 5^{4n}$

When n=3 then the last digit of the expression will be -

A. 4

B. 9

C. 6

D. 1

Answer: B

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19. $\lim_{n \to \infty} \left[\frac{1}{\sqrt{n}} + \frac{1}{\sqrt{2n}} + \frac{1}{\sqrt{3n}} + \ldots + \frac{1}{n} \right]$

A. Statement-I is true, Statement -II is true and Statement -II is a correct explanation for Statement - I.

B. Statement -I is true, Statement-II is true but Statement -II is not a correct explanation of Statement - I.

C. Statement -I is true, Statement - II is false.

D. Statement -I is false, Statement - II is true.

Answer: B

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20. Statement - I : For every natural number $\frac{(n+4)!}{(n+1)!}$ is divisible by 6.

Statement - II : Product of three consective natural numbers is divisible by 3 !

A. Statement-I is true, Statement -II is true and Statement -II is a correct explanation for Statement - I.

B. Statement -I is true, Statement-II is true but Statement -II is not a correct explanation of Statement - I.

C. Statement -I is true, Statement - II is false.

D. Statement -I is false, Statement - II is true.

Answer: B

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