# MATHS

## BOOKS - CHHAYA PUBLICATION MATHS (BENGALI ENGLISH)

## ARCHIVE

2012

**1.** Which statement is correct ?

A. $\{1\} \in \{1, 2, 3\}$

B. $\{1\} \notin \{1, 2, 3\}$

C. $\{1\} \subseteq \{1, 2, 3\}$

D. $\{1\} \subset \{1, 2, 3\}$

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2. For any three sets A, B, C, prove that

$A - (B \cup C) = (A - B) \cap (A - C)$

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3. Find the domain of the function $f(x) = \sqrt{2x + 1} + \sqrt{5 - x}$ .

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4. There are 100 students in a class. In an examination, 50 of them failed in Mathematics, 45 failed in Physics, 40 failed in Biology and 32 failed in exactly two of the three subjects. Only one student passed in all the subjects. Then the number of students failing in all the three subjects____

A. is 12

B. is 4

C. is 2

D. cannot be determined from given information.

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**5.** Let $X = \{1, 2, 3, 4, 5\}$ . The number of different ordered pairs (Y, Z) that can be formed such that $Y \subseteq X, Z \subseteq X$ and $Y \cap Z$ is empty is___

A. $2^5$

B. $5^3$

C. $5^2$

D. $3^5$

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## 2013

**1.** Which of the following sets is the null set $\phi$ ?

A. $A = \{\text{x : x is a prime number and } 31 < x < 37\}$

B. $B = \{\text{x : x is an integer and } 0 < x \leq 1\}$

C. $C = \{\phi\}$

D. $D = \{x : x + 1 = 1\}$

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**2.** State whether the statement is true or false :

" All relations are mapping but the converse is not true ".

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**3.** State whether the following statements is true or false :

" For two non-empty sets A and B if $A \subset B$, then $A \cap B = A$.''

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**4.** Find the domain of definition of the function

$$f(x) = \frac{1}{\sqrt{x^2 - 4x}}$$

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**5.** For three sets A, B and C if $A \cap B = A \cap C$ and $A \cup B = A \cup C$, then prove that B = C .

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**6.** Let A, B, C are three sets then prove that

$(A \cup B) \times C = (A \times C) \cup (B \times C)$.

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**7.** Draw the rough sketch of the graph and discuss the continuity of the function $f(x) = |x - 1| + |x - 2|$.

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**8.** Let $f(x) = x\left(\frac{1}{x-1} + \frac{1}{x} + \frac{1}{x+1}\right), x > 1$. Then _____

A. $f(x) \leq 1$

B. $1 < f(x) \leq 2$

C. $2 < f(x) \leq 3$

D. $f(x) > 3$

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**9.** Let A and B be two sets containing 2 elements and 4 elements respectively. The number of subsets of $A \times B$ having 3 or more elements is____

A. 256

B. 220

C. 219

D. 211

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2014

**1.** If $A = \{x : 0 < x < 4\}$ and $B = \{x : 3 \leq x \leq 6\}$, where x is an integer, then which of the following is $A \cap B$ ?

A. {2}

B. {3}

C. {4}

D. $\phi$

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**2.** If $A = \{x : x = 3n, n \in \mathbb{Z}\}$ and

$B = \{x : x = 6n, n \in \mathbb{Z}\}$, then find $A \cap B$ and $A \cup B$.

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**3.** $A = \{1, 2, 3, 5\}$ and $B = \{4, 6, 9\}$. A relation R is defined from A to B by $R = \{(x, y)$: the difference between x and y is odd}. Write R in Roster form.

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**4.** For any three sets A, B, C prove that,

$A - (B \cup C) = (A - B) \cap (A - C)$

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**5.** Let the number of elements of set A and B be p and q respectively. Then the number of relations from the set A to the set B is___

A. $2^{p+q}$

B. $2^{pq}$

C. p + q

D. pq

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**6.** There is a group of 265 persons who like either singing or dancing or painting. In this group 200 like singing, 110 like dancing and 55 painting. If

60 persons like both singing and dancing, 30 like both singing and painting and 10 like all these activities, then the number of persons who like only dancing and painting is___

A. 10

B. 20

C. 30

D. 40

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**7.** Let $X_n = \left\{z = x + iy : |z|^2 \leq \frac{1}{n}\right\}$ for all integers $n \geq 1$.

Then $\cap_{n=1}^{\infty} X_n$ is___

A. a singleton set

B. not a finite set

C. an empty set

D. a finite set with more than one elements.

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**8.** The range of the function $y = 3\sin\left(\sqrt{\frac{\pi^2}{16} - x^2}\right)$ is___

A. $\left[0, \sqrt{\frac{3}{2}}\right]$

B. $[0, 1]$

C. $\left[0, \frac{3}{\sqrt{2}}\right]$

D. $[0, \infty)$

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**9.** If $a \in \mathbb{R}$ and the equation

$-3(x - [x])^2 + 2(x - [x]) + a^2 = 0$ (where [x] denotes the greatest integer $\leq x$) has no integral solution, then all possible values of a lie in the interval____

A. $(-1, 0) \cup (0, 1)$

B. (1, 2)

C. (-2, -1)

D. $(-\infty, -2) \cup (2, \infty)$

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**10.** If $X = \{4^n - 3n - 1 : n \in \mathbb{N}\}$ and $Y = \{9(n - 1) : n \in \mathbb{N}\}$ where $\mathbb{N}$ is the set of natural numbers, then $X \cup Y$ is equal to__

A. Y - X

B. X

C. Y

D. $\mathbb{N}$

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## 2015

**1.** Let R be the relation defined on the set $\mathbb{N}$ of natural number as $R = \{(x, y) \mid 4x + 5y = 50, x, y \in \mathbb{N}\}$. Express $R$ and $R^{-1}$ as set of ordered pairs.

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**2.** If $2f(x) + 3f(-x) = 2x + 1$, then find f(x).

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**3.** Prove that $A \times (B \cup C) = (A \times B) \cup (A \times C)$

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**4.** Find the domain and range of the function $f(x) = \dfrac{x}{x^2 - 5x + 4}$

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**5.** Let $f: \mathbb{R} \to \mathbb{R}$ be defined as $f(x) = \dfrac{x^2 - x + 4}{x^2 + x + 4}$. Then the range of the function f(x) is____

A. $\left[\frac{3}{5}, \frac{5}{3}\right]$

B. $\left(\frac{3}{5}, \frac{5}{3}\right)$

C. $\left(-\infty, \frac{3}{5}\right) \cup \left(\frac{5}{3}, \infty\right)$

D. $\left[-\frac{5}{3}, -\frac{3}{5}\right]$

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**6.** $\{x \in \mathbb{R} : |\cos x| \geq \sin x\} \cap \left[0, \frac{3\pi}{2}\right] =$

A. $\left[0, \frac{\pi}{4}\right] \cup \left[\frac{3\pi}{4}, \frac{3\pi}{2}\right]$

B. $\left[0, \frac{\pi}{4}\right] \cup \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$

C. $\left[0, \frac{\pi}{4}\right] \cup \left[\frac{5\pi}{4}, \frac{3\pi}{2}\right]$

D. $\left[0, \frac{3\pi}{2}\right]$

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**7.** For the function $f(x) = \left[\frac{1}{[x]}\right]$, where [x] denotes the greatest integer less than or equal to x, which of the following statements are true?

A. The domain is $(-\infty, \infty)$

B. The range is $\{0\} \cup \{-1\} \cup \{1\}$

C. The domain is $(-\infty, 0) \cup [1, \infty)$

D. The range is $\{0\} \cup \{1\}$

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**8.** The number of real solutions of the equation

$(\sin x - x)\left(\cos x - x^2\right) = 0$ is__

A. 1

B. 2

C. 3

D. 4

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**9.** The number of real roots of equation $\log_e x + ex = 0$ is__

A. 0 (zero)

B. 1

C. 2

D. 3

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**10.** Let $S = \{(a, b, c) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N}$ : a + b + c = 21,

$a \leq b \leq c\}$ and $T = \{(a, b, c) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N}$ : a, b, c

are in A. P.}, where $\mathbb{N}$ is the set of all natural numbers.

Then the number of elements in the set $S \cap T$ is___

A. 6

B. 7

C. 13

D. 14

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**11.** Given that x is real number satisfying $\frac{5x^2 - 26x + 5}{3x^2 - 10x + 3} < 0$, then___

A. $x < \frac{1}{5}$

B. $\frac{1}{5} < x < 3$

C. $x > 5$

D. $\frac{1}{5} < x < \frac{1}{3}$ or $3 < x < 5$

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## 2016

**1.** If $f(x) = \log_3 x$ and $\psi(x) = x^2$, then the value of $f\{\psi(3)\}$ will be

A. 0

B. 1

C. 2

D. 3

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**2.** Find the domain for which the functions $f(x) = 3x^2 - 2x$ and $g(x) = 9x - 6$ are equal.

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**3.** If $f(x) = e^{px+q}$, then show that

$f(a). f(b). f(c) = f(a + b + c). e^{2q}$

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**4.** For any three sets A, B and C, prove that

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

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**5.** Find the domain of definition and range of the function

$f(x) = \dfrac{x}{1 + x^2}$

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**6.** If $A = \left\{5^n - 4n - 1 : n \in \mathbb{N}\right\}$ and

$B = \{16(n - 1) : n \in \mathbb{N}\}$, then

A. A = B

B. $A \cap B = \phi$

C. $A \subseteq B$

D. $B \subseteq A$

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**7.** If $f(x) + 2f\left(\frac{1}{x}\right) = 3x, x \neq 0$ and

$S = \{x \in \mathbb{R} : f(x) = f(-x)\},$ then S

A. is an empty set

B. contains exactly one element

C. contains exactly two elements

D. contains more than two elements

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WBHS Archive (2012)

**1.** Two unbaised coins are tossed. The probability that the first coin shows 'head' and the second coin shows 'tall' is-

A. $\frac{1}{2}$

B. $\frac{1}{4}$

C. $\frac{1}{8}$

D. $\frac{1}{16}$

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**2.** Two dice are thrown simultaneously. Find the probability that the sum of the numbers obtained will be 10.

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**3.** A bag contains 12 red and 10 white balls. 5 balls are taken from the bag at random. Find the probability that 3 balls are red and 2 balls are white.

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## WBHS Archive (2013)

**1.** An unbiased coin is tossed three times in succession. What is the probability of obtaining 'Tail' at least once ?

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## WBHS Archive (2014)

**1.** A and B be two mutually exclusive events such that $P(A) = \frac{3}{8}, P(B) = \frac{1}{3}$ then $P[A \cup B]$ is given by-

A. $\frac{17}{24}$

B. $\frac{2}{9}$

C. $\frac{7}{24}$

D. $\frac{13}{24}$

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**2.** The range of 62,72,44,25,54,9,56,71,27,-13,-3 is-

A. 82

B. 75

C. 85

D. 81

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**3.** If $P(A) = \frac{2}{3}, P(B) = \frac{1}{2}, P(A \cap B) = \frac{1}{6},$ then find the value of $P(A \cap B)$ and $P(A \cup B)$

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**4.** Find the mean deviation about the mean for the following data: 39,51,59,62,74

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**5.** What is the probability that a year, selected at random, in between 2001 and 2010 (both inclusive) will contain 53 Mondays ?

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**6.** For a group of 200 students the mean and S.D. of marks obtained by them were found to be 40 and 15 respectively. Later on, it was found that the score 23 was misread as 32. Find the correct mean and correct S.D.

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**1.** If y=2x+3 and variance of y is 4, then the standard deviation of x is-

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**2.** If $P(A) = 0.54, P(B) = 0.69$ and $P(A \cap B) = 0.35$, then the value of $P(A' \cup B')$ is-

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**3.** The standard deviation of 32 numbers is 5. If the sum of the numbers is 80, determine the sum of the squares of the numbers.

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**4.** Three dies are thrown simultaneously. Find the probability of obtaining a total score of 4.

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5. Two roots of the equation $ax^2 + bx + c = 0$ is $\alpha, \beta$ then find the value of $\alpha^2 + \beta^2$

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6. Find the probability that three numbers chosen at random from first nine natural numbers from an A.P.

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## WBHS Archive (2016)

1. For two mutually exclusive events A and B, $P(A) = \frac{1}{2}$ and $P(A \cup B) = \frac{2}{3}$, then the value of P(B) will be

A. $\frac{1}{4}$

B. $\frac{1}{6}$

C. $\frac{1}{3}$

D. $\frac{1}{5}$

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**2.** Find the probability of obtaining total 7 points with the rolling of two dice simultaneously.

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**3.** Two variables x and y are related by y=10-3x. The standard deviation of x be 4, find the standard deviation of y.

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**4.** Find the probability that a leap year, selected at random will contain 53 Sundays.

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**5.** Two roots of the equation $ax^2 + bx + c = 0$ is $\alpha, \beta$ then find the value of $\frac{1}{\alpha^3} + \frac{1}{\beta^3}$

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## WBJEE Archive (2012)

**1.** An urn contains 8 red and 5 white balls. Three balls are drawn at random. Then the probability that balls of both colours are drawn is-

A. $\frac{40}{143}$

B. $\frac{70}{143}$

C. $\frac{3}{13}$

D. $\frac{10}{13}$

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**2.** Two decks of playing cards are well shuffled and 26 cards are randomly distributed to a player. Then the probability that the player gets all distinct cards is-

A. $\frac{{}^{52}C_{26}}{{}^{104}C_{26}}$

B. $\frac{2 \times {}^{52}C_{26}}{{}^{104}C_{26}}$

C. $\frac{2^{13 \times 52}C_{26}}{{}^{104}C_{26}}$

D. $\frac{2^{26 \times {}^{52}C_{26}}}{{}^{104}C_{26}}$

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## WBJEE Archive (2013)

**1.** Each of a and b can take values 1 or 2 with equal probability. The probability that the equation $ax^2 + bx + 1 = 0$ has real roots, is equal to-

A. $\frac{1}{2}$

B. $\frac{1}{4}$

C. $\frac{1}{8}$

D. $\frac{1}{16}$

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## WBJEE Archive (2014)

**1.** A fair six faced die is rolled 12 times. The probability that each face turns up twice is equal to-

A. $\frac{12!}{6!6!6^{12}}$

B. $\frac{2^{12}}{2^{6}6^{12}}$

C. $\frac{12!}{2^{6}6^{12}}$

D. $\frac{12!}{6^{2}6^{12}}$

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**2.** A poker hand consists of 5 cards drawn at random from a well-shuffled pack of 52 cards. Then the probability that a poker hand consists of a pair and triple of equal face values (for example 2 sevens and 3 kings or 2 aces and 3 queens etc.) is-

A. $\frac{6}{4165}$

B. $\frac{23}{4165}$

C. $\frac{1797}{4165}$

D. $\frac{1}{4165}$

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**3.** For two events A and B, Let P(A)=0.7 and P(B)=0.6. The necessarily false statements (s) is/are-

A. $P(A \cap B) = 0.35$

B. $P(A \cap B) = 0.45$

C. $P(A \cap B) = 0.65$

D. $P(A \cap B) = 0.28$

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## WBJEE Archive (2015)

**1.** In a certain town, 60% of the families own a car, 30% own a house and 20% own both a car and a house. If a family is randomly chosen, what is the probability that this family owns a car or a house but not both ?

A. 0.5

B. 0.7

C. 0.1

D. 0.9

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**2.** The variance of first 20 natural numbers is-

A. $\frac{133}{4}$

B. $\frac{279}{12}$

C. $\frac{133}{2}$

D. $\frac{399}{4}$

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**3.** If the letters of the word PROBABILITY are written down at random in a row, the probability that two B-s are together is-

A. $\frac{2}{11}$

B. $\frac{10}{11}$

C. $\frac{3}{11}$

D. $\frac{6}{11}$

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**4.** If 5 distinct balls are placed at random into 5 cells, then the probability that exactly once cell remains empty is

A. $\frac{48}{125}$

B. $\frac{12}{125}$

C. $\frac{8}{125}$

D. $\frac{1}{125}$

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## WBJEE Archive (2016)

**1.** Standard deviation of n observations $a_1, a_2, a_3, \ldots, a_n$ is $\sigma$. Then the standard deviation of the observations $\lambda a_1, \lambda a_2, \ldots, \lambda a_n$ is

A. $\lambda\sigma$

B. $-\lambda\sigma$

C. $|\lambda|\sigma$

D. $\lambda^n\sigma$

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## JEE Main (AIEEE) Archive (2012)

**1.** Let $x_1, x_2, \ldots, x_n$ be n observations, and let $\bar{x}$ be their arithmatic mean and $\sigma^2$ be their variance.

Statement 1 : Variance of $2x_1, 2x_2, \ldots, 2x_n$ is $4\sigma^2$

Statement 2 : Arithmatic mean of $2x_1, 2x_2, \ldots, 2x_n$ is $4\bar{x}$.

A. Statement-1 is true, Statement-2 is true.

Statement-2 is not a correct explnation for Statement-1.

B. Statement-1 is true, Statement-2 is false.

C. Statement-1 is false, Statement-2 is true.

D. Statement-1 is true, Statement-2 is ture, Statement-2 is a correct explanation for Statement-1

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## JEE Main (AIEEE) Archive (2013)

**1.** All the students of a class performed poorly in Mathematics. The teacher decided to give grace marks of 10 to each of the students. Which of the following statistical measures will not change even after the grace marks were given ?

A. mean

B. median

C. mode

D. variance

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## JEE Main (AIEEE) Archive (2014)

**1.** The variance of first 50 even natural numbers is-

A. $\frac{833}{4}$

B. 833

C. 437

D. $\frac{437}{4}$

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## JEE Main (AIEEE) Archive (2015)

**1.** The mean of the data set comprising of 16 observations is 16. If one of the observation valued 16 is deleted and three new observations valued 3, 4 and 5 are added to the data, then the mean of the resultant data, is-

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## JEE Main (AIEEE) Archive (2016)

**1.** If the standard deviation of the numbers 2, 3, a and 11 is 3.5. Then which of the following is true ?

A. $3a^2 - 26a + 55 = 0$

B. $3a^2 - 32a + 84 = 0$

C. $3a^2 - 34a + 91 = 0$

D. $3a^2 - 23a + 44 = 0$

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## WBHS Archive 2012

**1.** The term independent of x in the expansion of $\left(x + \frac{1}{x}\right)^{10}$ is -

A. $.^{10}C_5$

B. . ${}^{10}C_6$

C. . ${}^{10}C_7$

D. none of these

**Answer: A**

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**2.** If $x, x^2$ and 8 are three consecutive terms of G.P. then which of the following is the value of x ?

A. -2

B. 2

C. 3

D. 4

**Answer: B**

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**3.** Prove by mathematical induction that for any positive integer n , $3^{2n} - 1$ is always divisible by 8 .

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**4.** For $|a| < 1$ , the sum of the infinite series $a + 3a^2 + 5a^3 + 7a^4 + ....$ is 1 .Find the value of a .

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**5.** In the expansion of $\left(\sqrt{x} - \frac{k}{x^2}\right)^{10}$ the term independent of x is 405 . Find the value of k .

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**6.** Express $\frac{i}{1+i} + \frac{1-i}{2i}$ in the form A + iB. .

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**7.** Find value of x where $.^{4-x}P_2 = 6$

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**8.** There number are in A .P .whose sum is 6 and product of first and third number is 3 . Find the three numbers .

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**9.** If the equation $a(b-c)x^2+b(c-a)x+c(a-b)=0$ has two equal roots , then show that $\frac{1}{a}, \frac{1}{b}$ and $\frac{1}{c}$ are in A. P .

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**10.** Find the sum of 10th term of the series

$(1) + (5 + 5^2) + (5^3 + 5^4 + 5^5) + (5 + 5^6 + 5^7 + 5^8 + 5^9) + \ldots$

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**11.** How many triangles can be formed by joining vertices of decagon ?

Find number of diagonals of a decagon .

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**12.** If $x = a + b, y = a\omega + b\omega^2, z = a\omega^2 + b\omega$then show that $xyz = a^3 + b^3$where $\omega$ is a complex cube root of 1 . Find square root of - 2i.

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**13.** In an A .P ., p -th term be q and q -th term be p , then show that (p + q ) -th term = 0 .

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**14.** First term n - th term and product of first n term be a , b and p respectively of a G .P . Then show that $p^2 = (ab)^n$ .

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**15.** How many even number between 3000 and 4000 can be formed by using the digits 1 ,2 , 3 ,4 ,5 , 6 ?)no repetitions being allowed ).

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**16.** If x is real , then $\sqrt[3]{x + iy} = a + ib(a \neq 0, b \neq 0)$,then pove that

$$\frac{x}{a} + \frac{y}{b} = 4\left(a^2 - b^2\right)$$

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**17.** Which of the following integrals can be represented by

$\lim_{h \to 0} h \sum_{r=1}^{n} r^2 h^2$ where nh = 2 ?

A. $\int_0^{-1} x^2 dx$

B. $\int_0^x x dx$

C. $\int_0^2 x^2 dx$

D. $\int_0^1 x dx$

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**18.** The order and degree of the differential equation

$\left(\frac{d^3y}{dx^3}\right) + 6\left(\frac{d^3y}{dx^3}\right) = 2\left(\frac{d^2y}{dx^2}\right)^3 + 5y\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^2$ are respectively -

A. 2, 3

B. 3, 2

C. 3, 1

D. none of these

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19. The function $f:[0, 2] \rightarrow$ R defined by $f(x) = x^3 - 3x$ is increasing in which of the following intervals ?

A. $(1, 2)$

B. $(0, 1)$

C. $(0, 2)$

D. $\left(\frac{1}{2}, \frac{3}{2}\right)$

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**20.** f has a local maximum at x = a and local minimum at x = b. Then -

A. $f(a) > f(b)$

B. $f(a) \geq f(b)$

C. $f(a) < f(b)$

D. no definite conclusion can be made

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**21.** If $f'(x) = \frac{\sin 2x}{\cos^2 2x}$, then $f(x) =$

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**22.** Fill in the blank :

The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} |\sin x| dx$ is

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**23.** State whether the following statement is true or false : If x, y both are functions of t, then $\frac{d^2y}{dx^2} = \frac{\frac{d^2y}{dt^2}}{\frac{d^2x}{dt^2}}$.

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**24.** Without taking the help of a graph paper, define the derivative of a function at a point in its domian of definition. Use this definition to show that the derivative of a differentiable even function is an odd function.

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**25.** Define the continuity of a function at a point within its domain of definition.

The definition of a function is given below:

$$f(x) = \begin{cases} \frac{1 - \cos 5x}{x^2} & \text{when } (x \neq 0) \\ k & \text{when } x = 0 \end{cases}$$

Find the value of k for which f(x) is continuous at x = 0.

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**26.** Diffferentiate $\sin^2 x$ w.r.t. $\tan x$.

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**27.** If $y = \tan^{-1}\frac{\cos x + \sin x}{\cos x - \sin x}$, then find $\frac{dy}{dx}$.

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**28.** Find $\int \frac{2}{e^x + e^{-x}} dx$.

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29. Evaluate : $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(a\cos^2 x + b\sin^3 x\right)dx$

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30. Solve : $\frac{dy}{dx} = e^{y-x} + 2$

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31. Find the differential equation of all straight lines passing through the point (0,3).

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32. Find the point in the first quadrant and lying on the hyperbola $5x^2 - 3y^2 = -6$ at which the tangent to the hyperbola makes an angle $45^\circ$ with the positive direction of the x-axis.

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**33.** Find the maximum value of the function $2x^3 + 3x^2 - 36x + 10$.

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**34.** Find the area bounded by the parabola $y^2 = 4ax$, y-axis and the line y = 2a.

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**35.** The radius of a circle is increasing at a rate of 2.5 cm/sec. Find the rate of increase of the area of the circle when its radius is 5cm.

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**36.** Evaluate : $\int_a^b \frac{x}{\sqrt{(x-a)+(b-x)}}dx$

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**37.** With the help of definite integral find the value of

$$\lim_{n \to \infty}\left\{\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\ldots\left(1+\frac{n}{n}\right)\right\}^{\frac{1}{n}}$$

Watch Video Solution

**38.** Evaluate : $\int_0^2 \left|x^2 - 1\right| dx$

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**39.** Show that the function $f(x) = \frac{x+2}{x+1}(x > 0)$ decreasing.

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**40.** Solve : $\frac{dy}{dx} = \frac{x - 3y}{3x + y}$

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41. Solve : $\sqrt{a - x}dx + \sqrt{a + x}dy = 0$

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42. Volume of a right circular cone is $100\text{cm}^3$. The radius of the base of the cone increases at the constant rate of 1 cm/sec keeping volume of the cone unchanged. Find the rate of change of the height of he cone when radius is 10cm.

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43. Prove that the area of the triangle formed by any tangent to the hyperbola $xy = c^2$ and the coordinate axes is constant.

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**44.** The sum of the total surface areas of a sphere and a cube is constant. Find the ratio of the radius of the sphere and the length of the edge of the cube so that the sum of their volumes is minimum.

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**45.** If $x = a\cos^3\theta$ and $y = b\sin^3\theta$, then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$.

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**46.** Draw the rough sketch of the smaller region enclosed by the curve $x^2 + y^2 = a^2$ and the line y = x and find the area of the enclosed region.

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**47.** If $y = (x + \sqrt{x^2 - 1})^m$, then prove that, $\left(1 - x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + m^2y = 0$

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**48.** If $x^2 + y^2 = t + \frac{1}{t}$ and $x^4 + y^4 = t^2 + \frac{1}{t^2}$ then show that $\frac{dy}{dx} = -\frac{y}{x}$.

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**49.** If $x^y + y = x, \quad \text{find } \frac{dy}{dx}$.

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**50.** Find $\int \frac{x + \sin x}{1 + \cos x} dx$.

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**51.** Find $\int \frac{2\sqrt{\tan x}}{\sin 2x} dx$.

Watch Video Solution

**52.** Find $\int \frac{\cos x - \cos 2x}{1 - \cos x} dx.$

Watch Video Solution

**53.** Find the value of $\int_1^2 (x+1)^2 \log x dx.$

Watch Video Solution

**54.** Find $\int \frac{x^2}{x^4 + x^2 + 9} dx.$

Watch Video Solution

**55.** Find the value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$ f(x) dx, where

$$f(x) = \begin{cases} \sin^2 x & \text{when } x > 0 \\ 1 - \cos x & \text{when } x \le 0 \end{cases}$$

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**56.** Find $\int \frac{dx}{(1 + x)\sqrt{x^2 + x - 1}}$.

Watch Video Solution

**57.** Find $\int \tan^{-1}\sqrt{\frac{1 - x}{1 + x}}dx$.

Watch Video Solution

**58.** Differentiate $7^x x^7$ w.r.t. x.

Watch Video Solution

**59.** Without taking the help of a graph paper, define the derivative of a function at a point in its domain of definition. Use this definition to show that the derivative of a differentiable even function is an odd function.

Watch Video Solution

**1.** If positive integers $a_1, a_2, a_3....$ are in A .P .such that $a_8 + a_{10} = 24$, then the value of $a_9$ is -

A. 10

B. 11

C. 12

D. 9

**Answer: C**

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**2.** If x , y are real and $x + iy = -i(-2 + 3i)$ , then (x , y) is -

A. $(2, -3)$

B. $(3, 2)$

C. $(-2, 3)$

D. $(-3, -2)$

**Answer: B**

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**3.** $(n)p_r = k^nC_{n-r}, k =$

Watch Video Solution

**4.** Find the argument of the complex number $-3-3i$.

Watch Video Solution

**5.** Which term of the G.P $\left(\sqrt{2}, \sqrt{6}, 3\sqrt{2}, 3\sqrt{6}, \ldots\right)$ is $243\sqrt{2}$?

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**6.** If a , b , c , are three unequal number such that a , b ,c are in A.P and $(b - a), (c - b)$, a are in G.P then find a: b: c .

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**7.** In how many ways can 4 subjects be chosen by two students so that each student should take at least one subject ?

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**8.** If $z_1, z_2$ are two complex numbers , prove that ,

$\left|z_1 + z_2\right| \leq \left|z_1\right| + \left|z_2\right|$

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**9.** How many even number of three digits and greater than 300 can be formed with the digits 1 , 2 ,3 ,4 ,5 ? Repetition of the digits is allowed .

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**10.** Express $\sqrt{i} + \sqrt{-i}$ in the form of A + iB.

Watch Video Solution

**11.** If $a_1, a_2, a_3, \ldots a_n$ are in A.P , prove that,

$$\frac{1}{\sqrt{a_1} + \sqrt{a_1}} + \frac{1}{\sqrt{a_2} + \sqrt{a_3}} + \ldots. + \frac{1}{\sqrt{a_{n-1}} + \sqrt{a_n}} = \frac{n-1}{\sqrt{a_1} + \sqrt{a_n}}$$

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**12.** x is a real number and $x^2 + 5 < 6x$ . Then prove that , x must be between 1 to 5 .

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**13.** Prove that , $.^{2n}C_n = 2^n \frac{1.3.5\ldots(2n-1)}{\lfloor n}$

Watch Video Solution

**14.** Find the middle term of the expansion of $\left(2x^3 - \frac{1}{x^2}\right)^6$

Watch Video Solution

**15.** Show that for any positive integer $3^{2n+2} - 8n - 9$ is divisible by 64 .

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**16.** If $f(x) = \log(\cos x)$, then which of the following is f''(x) ?

A. $\sec^2 x$

B. $-\sec^2 x$

C. $\operatorname{cosec}^2 x$

D. $-\operatorname{cosec}^2 x$

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17. The value of $\int_{-1}^{1} x|x|dx$ is -

A. 2

B. $-1$

C. 0

D. 1

Watch Video Solution

18. The gradient of normal at t = 2 of the curve $x = t^2 - 3, y = 2t + 1$ is -

A. $-\frac{1}{2}$

B. $\frac{1}{2}$

C. 2

D. $-2$

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19. The maximum value of $y = 8 - x^2$ is -

A. $-8$

B. 0

C. 8

D. none of these

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**20.** The differential equation for the straight lines $y = mx + c$ (m and c are parameters) is -

A. $\frac{d^2x}{dy^2} = 0$

B. $y = x\frac{dy}{dx} + c$

C. $\frac{d^2y}{dx^2} = 0$

D. $\frac{dy}{dx} = m$

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**21.** $\lim_{x \to 0}\frac{\sin^{-1}x}{x}$ is equal to -

A. 0

B. 1

C. $-1$

D. does not exist

Watch Video Solution

**22.** State whether the following statement is true or false :

$\frac{d}{dx}\left(\cos^{-1}x - \sin^{-1}x\right) = \frac{2}{\sqrt{1 - x^2}}$, when $|x| < 1$.

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**23.** Fill in the blank :

If $n \neq -1$, then $\int \{f(x)\}^n . f'(x)dx =$ _ _ _ _ _ _ _ _ _ $+ c$, $c$ is a constant.

Watch Video Solution

**24.** Fill in the blank :

If the rate of change of volume of a sphere with respect to its radius is k times the surface area of the sphere, then the value of k is __________ .

Watch Video Solution

**25.** If u and v are two differentiable functions of x show that,

$$\frac{d}{dx}\left(\tan^{-1}\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{u^2 + v^2}.$$

Watch Video Solution

**26.** If $x = a(\theta + \sin\theta)$ and $y = a(1 + \cos\theta)$, then find the simplified value of $\frac{dy}{dx}$ and show that, $\frac{dy}{dx} = -1$, when $\theta = \frac{\pi}{2}$.

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**27.** Find $\int\left(e^{a\log x} + e^{x\log a}\right)dx, x, a > 0.$

Watch Video Solution

**28.** Using the definition of definite integral as the limit of a sum evaluate $\int_a^b cdx$ where a, b, c, are three constants and $b > a$.

Watch Video Solution

**29.** The gradient of the tangent to the curve at a point is k times to the gradient of the straight line joining this pint and origin. Find the equation of the curve, where $k(\neq 0)$ is constant.

Watch Video Solution

**30.** Solve : $dx - dy + ydx + xdy = 0$.

Watch Video Solution

**31.** The length of each side of an equilateral triangle increases at the rate of 2cm/sec. Find the rate of increase of the area of the triangle, when the length of each sides is 10cm.

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**32.** If $x \geq 0$ then show that $x \geq \sin x$.

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**33.** At which point of the curve $y = 1 + x - x^2$ the tangent is parallel to x-axis ?

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**34.** Find the area in the 1st quadrant, bounded by the parabola $y = x^2$, the straight line $y = 4$ and the y-axis.

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35. $\int \frac{xe^{x}dx}{(x+1)^2}$

Watch Video Solution

36. If $f(x) = \begin{cases} \sin x \text{ when } -\frac{\pi}{2} < x < \frac{\pi}{2} \\ |\cos x - 2| \text{ otherwise} \end{cases}$ then find the value of $\int_0^{\pi} f(x)dx$.

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37. Find the value of

$$\lim_{n \to \infty} \left[\frac{1^2}{1^3+n^3} + \frac{2^2}{2^3+n^3} + \frac{3^2}{3^3+n^3} + \dots + \frac{n^2}{n^3+n^3}\right]$$

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38. Solve : $\frac{dy}{dx} = \frac{3x+2y}{2x-3y}$, given y = 0 when x = 1.

Watch Video Solution

**39.** Solve : $\left(x+xy^2\right)dx+\left(y+x^2y\right)dy=0$

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**40.** Using calculus find the coordinates of the point on the parabola $y=x^2$, which is at the least distance from the straight line $y=2x-4$.

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**41.** The tangent at any point P on the circle $x^2+y^2=2$, cuts the axes at L and M. Find the equation of the locus of the midpoint L.M.

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**42.** Using calculus find the area enclosed by $y=1-x$, $y=1-2x$ and y = 0 (a rough sketch in necessary).

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**43.** In a certain culture of bacteria, the rate of increase of its number is equal to half of the number present at that moment. When the number of bacteria will be double of its initial number? (Given $\log_e 2 = 0.693$)

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**44.** The value of a quadratic function of x is 19 when x = 1 and its maximum value is 20 when x = 2. Find the function.

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**45.** If $\sqrt{1 - x^2} + \sqrt{1 - y^2} = x - y$, then prove that, $\frac{dy}{dx} = \frac{\sqrt{1 - y^2}}{\sqrt{1 - x^2}}, x \neq y$

Watch Video Solution

**46.** Differentiate $x^{\sin x}$ w.r.t. $\sin x$

Watch Video Solution

**47.** Find $\frac{dy}{dx}$, when $e^{xy} - 4xy = 2$

Watch Video Solution

**48.** If $y = \cos\left(2\sin^{-1}x\right)$, then show that

$\left(1 - x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = 0$

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**49.** Find $\int \sin^{-1}\sqrt{\frac{x}{a + x}}dx.$

Watch Video Solution

**50.** If f(x) is an odd function, then prove that $\int_{-a}^{a} f(x)dx = 0.$

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**51.** Evaluate : $\int_0^{\pi} \frac{x\sin x}{1 + \cos^2 x} dx$

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**52.** Find $\int \frac{dx}{1 - 2\cos x}.$

Watch Video Solution

**53.** $\int \frac{dx}{(x - 1)(x + 2)^2}$

Watch Video Solution

**54.** If $f(x) = |x - 1| + |x + 1|$, then evaluate $\int_{-2}^{2} f(x)dx$.

Watch Video Solution

**55.** Show that : $\int_2^3 \frac{dx}{(x - 1)\sqrt{x^2 - 2x}} = \frac{\pi}{3}$

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**56.** Find : $\int \frac{\sin x + \cos x}{\sqrt{\sin 2x}} dx$

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**57.** A continuous function f(x) is defined as follows :

$$f(x) = \begin{cases} x & \text{when } x < 1 \\ ax + bx^2 & \text{when } 1 \le x \le 2 \\ 2x^2 & \text{when } x > 2 \end{cases}$$

Find the values of a and b.

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**58.** Find the derivative w.r.t. x of $\tan^{-1}\frac{a\cos x - b\sin x}{b\cos x + a\sin x}$.

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## WBHS Archive 2014

**1.** If n - th term of an A.P be $2n - 4$ , then which one of the following is common difference ?

A. $-2$

B. $-\frac{1}{2}$

C. $\frac{1}{2}$

D. 2

**Answer: B**

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**2.** Value of $i^n + i^{n+1} + i^{n+2} + i^{n+3}\left(\text{when} i = \sqrt{-1}\right)$ -

A. 1

B. $-1$

C. 0

D. none of these

**Answer: C**

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**3.** In a G. P . $t_5 : t_3 := 7 : 9$ then value $t_9 : t_5$ -

A. $7 : 9$

B. $9 : 7$

C. $81 : 49$

D. $49:81$

**Answer: A::D**

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**4.** If $\omega$ be imaginary cube root of 1 , then prove that

$$\frac{x\omega^2 + y\omega + z}{x\omega + y + z\omega^2} = \left(\frac{x\omega + y + z\omega^2}{x\omega^2 + y\omega + z}\right)^2$$

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**5.** If joining vertices of a polygon of a polygon with n sides makes 12n traingles , then find the value of n .

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**6.** Find the term independent of x in the expansion of $\left(x - \frac{2}{x^2}\right)^{15}$

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**7.** Find the sum of the following G .P series , if exist $\frac{1}{3} - \frac{2}{9} + \frac{4}{27} - \frac{8}{81} + ....$

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**8.** Prove by mathematical induction $1^2 + 2^2 + 3^2 + ..... + (n+1)^2 = \frac{(n+1)(n+2)(n+3)}{6}$

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**9.** If z = x + iy and $|2z + 1| = |z - 2i|$,then show that $3\left(x^2 + y^2\right) + 4(x + y) = 3$

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**10.** 10 articles out of 14 articles are of same kind and each of remaining are of different kind . Find the number of combination if 10 articles are taken at a time .

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**11.** Ratio of seventh term from beginning and seventh term from end in the expansion of $\left(\sqrt[3]{2}+\frac{1}{3_{\sqrt{3}}}\right)^{n}$ is $1:6$.Find value of n .

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**12.** If in a G .P sum of first n terms be p, sum of first 2n terms 3p , then prove that , sum of first 3n tems is 7p .

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**13.** Draw the graph of the inequations and solve them :

$3x + 4y \leq 12, x \geq -2, y \leq 3$ .(use graph)

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**14.** Solve $3x^2 - 5ix + 3 = 0\left(i = \sqrt{-1}\right)$.

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**15.** How many numbers are between 100 and 1000 formed by 0 , 3 , 4 , 6 , 8 , 9 ?

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**16.** For any positive integer $n, 3^{2n} + 7$ is divisible by 8 . Prove by mathematical induction .

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**17.** Which of the following is the value of $\frac{d}{dx}\{|x - 1| + |x - 5|\}$ at the point x = 3 ?

A. -2

B. 0

C. 2

D. 4

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**18.** Which of the following relations is satisfied by the function $f(x) = \int_1^x \frac{dt}{t}$ ?

A. $f(x + y) = f(x) + f(y)$

B. $f\left(\frac{x}{y}\right) = f(x) - f(y)$

C. $f(xy) = f(x)f(y)$

D. $f\left(\frac{x}{y}\right) = \frac{f(x)}{f(y)}$

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**19.** Order and degree of the differential equation

$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{\frac{1}{3}} + x = 0$ are respectively-

A. 2 and 3

B. 2 and 12

C. 2 and 6

D. 2 and 4

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**20.** A particle moves according to the law $s = t^3 - 9t^2 + 24t$. The distance covered by the particle before it first comes to rest is -

A. 10 units

B. 16 units

C. 20 units

D. 24 units

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**21.** Fill in the blank :

If $f(x) = e^x$, $g(x) = 2\log_e x$ and $F(x) = f\{g(x)\}$, then $\frac{dF}{dx} =$ ________ .

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**22.** Fill in the blank:

The value of $\int_{-1}^{1}\left(1 + x + 3x^3 + 5x^5 + \ldots + 99x^{99}\right)dx$ is __________.

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**23.** Fill in the blank :

If $\int f(x)dx = \frac{e^x}{2}(\sin x - \cos x)$ then f(x) will be ____________.

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**24.** State whether the following statement is true or false :

$f(x) = |x|$ has no minimum value.

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**25.** If $y = x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots\infty$, show that $\frac{dy}{dx} = \frac{1}{1 - x^2}$.

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**26.** If y=acos(logx)+bsin(logx), show, that,

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0.$$

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**27.** If f(x)is integrable function in the interval $[-a, a]$ then show that

$$\int_{-a}^{a}f(x)dx=\int_0^a[f(x)+f(-x)]dx.$$

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**28.** Using the definition of definite integral as the limit of a sum evaluate

$\int_0^2 axdx$ where a is a constant.

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**29.** Find the differential equation of all circles passing through the origin and having their centres on the x - axis.

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**30.** Find the equation of the curve which passes through the point (4, 3) and the gradient of the tengent to the curve at any point on it is equal to the reciprocal of the ordinate of the point.

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**31.** If $y = a\log|x| + bx^2 + x$ has extreme values at x = -1 and x = 2, find the values of a and b.

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**32.** The tengent at a point (a, b) to the curve $y = \sin x$ is parallel to the line $2y = x$. If $a > 0$ find b.

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**33.** Find the area of the region bounded by the curve $y_2$ = x and the lines x = 1, x = 4 and the x-axis in the first quadrant.

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**34.** Find the values of x for which the function $f(x) = x^3 - 7x^2 + 8x - 10$ is monotonic increasing.

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**35.** A particle moves in a straight line and its velocity v at time t seconds is given by $v = \left(6t^2 - 2t + 3\right)$ cm/sec. Find the distance travelled by the particle during the first 5 second after the start.

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**36.** Find : $\int \left\{ \log(\log x) + \frac{1}{(\log x)^2} \right\} dx.$

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**37.** Find the value of

$$\lim n \to \infty \left[ \frac{1}{\sqrt{n^2 - 1^2}} + \frac{1}{\sqrt{n^2 - 2^2}} + \frac{1}{\sqrt{n^2 - 3^2}} + \dots + \frac{1}{\sqrt{n^2 - (n-1)^2}} \right]$$

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**38.** If $f(x) = (|x - 2| + |x - 4|)$, then evaluate $\int_1^4 f(x)dx.$

Watch Video Solution

**39.** Solve : $\left(e^y + 1\right)\cos x dx + \sin x dy = 0$

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**40.** Solve : $\frac{dy}{dx} = \frac{y\left(x\cos\frac{y}{x} + y\sin\frac{y}{x}\right)}{x\left(y\sin\frac{y}{x} - x\cos\frac{y}{x}\right)}$

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**41.** If $y = mx + c$ is the tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at any point on it, show that $c^2 = a^2m^2 + b^2$. Find the coordinates of the point of contact.

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**42.** Using calculus, find the area of the region bounded by $y^2 = 8x$ and y = x (a rough sketch is necessary).

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**43.** A spherical ice ball melts in such a way that the rate of melting is proportional to its volume of ice at that instant. If half the quantity of ice melts in 30 minutes, show that after 90 minutes from the start of melting, the volume of ice that remains is $\frac{1}{8}$ time of the original volume of the ice ball.

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**44.** Using calculus show that the portion of the normal to the curve $y = \frac{a}{2}\left(e^{\frac{x}{a}} + e^{\frac{-x}{a}}\right)$ at $(x_1, y_1)$ intercepted between the curve and the x-axis is $\frac{y_1^2}{a}$.

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**45.** If $f(x) = \begin{cases} x + \sin x & \text{when } x < 0 \\ 0 & \text{when } x \geq 0 \end{cases}$

examine whether f(x) is continuous at x = 0.

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**46.** If $y = \left(\frac{1 + x}{1 - x}\right)^n$, prove that $\left(1 - x^2\right)\frac{d^2y}{dx^2} = 2(n + x)\frac{dy}{dx}$.

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**47.** If $x = \log\left(1 + t^2\right), y = t - \tan^{-1}t$, find $\frac{dy}{dx}$.

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**48.** If $x = \sin\theta$ and $y = \cos p\theta$, p is constant, prove that $\left(1 - x^2\right)y_2 - xy_1 + p^2y = 0$.

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**49.** Find : $\int \frac{dx}{3\cos x - 4\sin x + 5}$.

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**50.** Evaluate : $\int_1^5 \frac{\sqrt{x+1}}{\sqrt{x+1} + \sqrt{7-x}} dx$.

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**51.** Find : $\int e^x[\log(\sec x + \tan x) + \sec x]dx$.

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**52.** Show that : $\int_1^2 \sqrt{(x-1)(2-x)}dx = \frac{\pi}{8}$.

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53. Find : $\int \frac{3x+1}{\sqrt{2-3x-2x^2}}dx.$

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54. Prove that $\int_a^b f(x)dx = \int_{a+c}^{b+c} f(x-c)dx$ , and when $f(x)$ is odd function, $\int_{-a}^{a} f(x)dx = 0$

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55. Find : $\int \frac{\log x}{(1+\log x)^2}dx.$

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56. Show that $\int_0^1 \left(\cos^{-1}x\right)^2 dx = \pi - 2.$

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**1.** $i^2 = -1$, then $\sum_{n=0}^{225} i^n$ is -

A. 0

B. $1 + i$

C. $-1$

D. i

**Answer: B**

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**2.** If $.^{15}C_r = ^{15}C_{r+1}$ then the value ofr is -

A. 6

B. 7

C. 4

D. 3

**Answer: B**

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**3.** The cofficient of $x^{17}$ in the expansion of $(x - 1)(x - 2)(x - 3)\ldots.(x - 18)$ is -

A. $-171$

B. 171

C. 153

D. $-153$

**Answer: D**

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**4.** Using principle of mathematical induction prove that $2^{3n} - 1$ is divisible by 7 .

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**5.** Find the principal amplitude of $-3 - \sqrt{3i}$

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**6.** If $y = x + x^2 + x^3 + \dots \infty \text{where} |x| < 1$ prove that $x = \frac{y}{1 + y}$.

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**7.** Prove that the middle term in the expansion of $(1 + x)^{2n}$ is $\frac{1.3.5....(2n - 1)}{n!}.2^n x^n$

Watch Video Solution

**8.** Show that 1 is a root of $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$. Hence show that if roots of this equation are equal then $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P

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**9.** the sum of first n terms of $1 + 3 + 7 + 15 + 31 + \ldots.$

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**10.** How many numbers can be formed which are divisible by 5 and less than one thousand ? (Repetition is not allowed )

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**11.** Determine the solution set of the given inequation :

$$\left|\frac{2x - 3}{x - 1}\right| > 3$$

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12. If G.M of two unequal positive numbers a and b is $\frac{a^{p+1}+b^{p+1}}{a^p+b^p}$, then find the the value of p .

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13. Find the number of permutations and the number of combinations in the letters of the word 'EXPRESSIOn' taken four at a time.

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14. $\text{amp}(z) - \text{amp}(-z) = \pm\pi$ according as amp (z) is positive or negative . (z is a complex number . )

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15. If f$f(x) = \mu x - \sin x, x > 0$ is a monotonic increasing function then -

A. $\mu > -1$

B. $\mu < 1$

C. $\mu > 1$

D. $\mu < -1$

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16. The value of $\int_{-2}^{2} x|x|dx$.

A. 2

B. $-\frac{1}{2}$

C. $\frac{1}{2}$

D. $-2$

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**17.** If $f(2) = 4, f'(2) = 4$, then value of $\lim_{x \to 2} \frac{xf(2) - 2f(x)}{2(x - 2)}$ is -

A. -4

B. -2

C. 2

D. 0

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**18.** Verify Rolle's theorem for the function $f(x) = 4^{\sin x}$ in the interval $[0, \pi]$.

Watch Video Solution

**19.** Evaluate the limit: $\lim_{x \to 0} \frac{e^{2x} + e^{-x} - 2}{x}$

Watch Video Solution

**20.** If the function $f(x) = \begin{cases} \frac{\sin x}{kx} + k, \text{when } x \neq 0 \\ 2 \text{ when } x = 0 \end{cases}$ is continuous at x = 0, find k.

Watch Video Solution

**21.** If $\sin^{-1}\left(\frac{x^2 - y^2}{x^2 + y^2}\right) = k$, k is a constant, then prove that $\frac{dy}{dx} = \frac{y}{x}$.

Watch Video Solution

**22.** Evaluate: $\int \sin\sqrt{x}dx$.

Watch Video Solution

**23.** Find the differential equation of the curves given by $y = Ae^{2x} + Be^{-2x}$, where A and B are parameters.

Watch Video Solution

24. Find $\frac{dy}{dx}$ where $x = \cos^{-1}\left(8t^4 - 8t^2 + 1\right)$ and $y = \sin^{-1}\left(3t - 4t^3\right), \left[0 < t < \frac{1}{2}\right]$.

Watch Video Solution

25. If $x = 2\cos\theta - \cos 2\theta$ and $y = 2\sin\theta - \sin 2\theta$, then find the value of $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{2}$.

Watch Video Solution

26. Evaluate : $\int \frac{2^x dx}{\sqrt{4^x - 2^{x+2} + 5}}$.

Watch Video Solution

**27.** Evaluate : $\int \frac{3x + 5}{x^3 - x^2 - x + 1} dx.$

Watch Video Solution

**28.** Solve : $\left(1 + y^2\right)dx = \left(\tan^{-1}y - x\right)dy$, given that $y = 0$ when x = -1.

Watch Video Solution

**29.** If the rate of increase of population is $5\%$ per year, then in how many years the population will be doubled?

Watch Video Solution

**30.** Evaluate (with the help of definite integral) :

$$\lim_{n \to \infty} \left[\frac{1}{\sqrt{n}} + \frac{1}{\sqrt{2n}} + \frac{1}{\sqrt{3n}} + \ldots + \frac{1}{n}\right]$$

Watch Video Solution

**31.** Find the value of $\int_0^{\frac{\pi}{2}} \frac{\sin x \cos x}{\left(a \sin^2 x + b \cos^2 x\right)^2} dx$

Watch Video Solution

**32.** A lamp is on the top of the lamp post of height 2a metres situated on a straight road. A boy of height a metre walks towards the post of the speed of c metre/minute. Find the rate of decrease of the shadow.

Watch Video Solution

**33.** Find the area of region

$\left\{(x, y): 0 \leq y \leq x^2 + 1, 0 \leq y \leq x + 1, 0 \leq x \leq 2\right\}$

Watch Video Solution

**34.** If the line $x\cos\alpha + y\sin\alpha =$ touches the curve

$x^m y^n = a^{m+n}$, prove that

$p^{m+n}. m^m. n^n = a^{m+n}(m+n)^{m+n}. \cos^m\alpha. \sin^n\alpha$

Watch Video Solution

**35.** The total surface area of a right circular cone is given. Show that the volume of that cone will be maximum if the semi vertical angle is $\sin^{-1}\frac{1}{3}$.

Watch Video Solution

## WBHS Archive 2016

**1.** If the ratio of the sum of first three terms to the sum of next three terms of a geometric series be $125:27$ ,then the common ratio of the series be

A. $\frac{5}{3}$

B. $\frac{1}{4}$

C. $\frac{3}{5}$

D. $\frac{1}{2}$

**Answer: C**

Watch Video Solution

**2.** If $i^2 = -1$ ,then value of modulus of $(3i - 1)^2$ will be

A. 9

B. 10

C. 8

D. 6

**Answer: A**

Watch Video Solution

**3.** Find the square root of the complex number ( 7 - 24i) .

Watch Video Solution

**4.** Find the (r+1) -th term form the end in the expansion of $(1 - 3x)^n$

Watch Video Solution

**5.** If $n \in \mathbb{N}$ ,the prove by mathematical that $1 + 3 + 5 + \ldots . + (2n - 1) = n^2$

Watch Video Solution

**6.** If the sum pf first n , 2n , 3n , terms of an arithmetic progression be $S_1 S_2$ and $S_1$ respectively , the prove that

$S_3 = 3\left(S_2 - S_1\right)$

Watch Video Solution

**7.** Prove be mathematical induction :

$$\frac{1}{3.6} + \frac{1}{6.9} + \frac{1}{9.12} + \ldots + \frac{1}{3n(3n+3)} = \frac{n}{9(n+1)}$$

Watch Video Solution

**8.** If the p -th and q-th terms of an AP are a and b resepectively , then show that the sum of first (p + q) terms of that AP is

$$\frac{1}{2}(q+p)\left(a+b+\frac{a-b}{p-q}\right)$$

Watch Video Solution

**9.** Find the probbiliity of drawing 4 cards from a pack of 52 cards such that at least two cards will be aces .

Watch Video Solution

**10.** Find the sum to n terms of the following series:

$$\left(x+\frac{1}{x}\right)^2+\left(x^2+\frac{1}{x^2}\right)^2+\left(x^3+\frac{1}{x^3}\right)^2+\left(x^4+\frac{1}{x^4}\right)^2+\ldots$$

Watch Video Solution

**11.** If $z = x + iy$ and $|z - 1| + |z + 1| = 4$, show that

$3x^2 + 4y^2 = 12\left(i = \sqrt{-1}\right)$.

Watch Video Solution

**12.** In how many ways can 10 boys and 5 girls be seated in a round table so that two girls never be seated together ?

Watch Video Solution

13. If $a_1 + a_2, a_3, \ldots a_4$ are in AP then show that

$$\frac{1}{a_1 a_2} + \frac{1}{a_2 a_3} + \frac{1}{a_3 a_4} + \ldots + \frac{1}{a_{n-1} a_n} = \frac{n-1}{a_1 a_n}$$

Watch Video Solution

14. $\frac{|x| - 2}{|x| - 3} \geq 0$, where $x \in \mathbb{R}$ and $x \neq \pm 3$

Watch Video Solution

15. The solution of the differential equation $\frac{dy}{dx} = e^{x+y}$ is

A. $e^x + e^y = c$

B. $e^x - e^{-y} = c$

C. $e^x + e^{-y} = c$

D. None of these

Watch Video Solution

**16.** If $f(x) = \log_x(\log_e x)$, then the value of $f'(e)$ is -

A. e

B. $\frac{2}{e}$

C. $\frac{1}{e}$

D. 0

Watch Video Solution

**17.** If $f(x) = \begin{cases} \frac{|\sin x|}{x} & \text{when } x \neq 0 \\ 1 & \text{when } x = 0 \end{cases}$ then examine the continuity of the function at x = 0.

Watch Video Solution

**18.** Find the differential coefficient of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ with respect to $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$

Watch Video Solution

**19.** Evaluate: $\int\left\{\frac{1}{\log_e x} - \frac{1}{\left(\log_e x\right)^2}\right\}dx$

Watch Video Solution

**20.** Verify Lagrange's Mean Value Theorem for the function

$f(x) = 4(6 - x)^2$ in $5 \le x \le 7$

Watch Video Solution

**21.** Find the differential equation of all circles which touch the x -axis at the origin.

Watch Video Solution

**22.** Evaluate: $\lim_{x \to 0} \frac{\log_e(1 + \alpha x)}{e^{2x} - 1}$

Watch Video Solution

**23.** If $\sin y = x\sin(a + y)$, prove that, $\frac{dy}{dx} = \frac{\sin a}{1 - 2x\cos a + x^2}$.

Watch Video Solution

**24.** If $pv^a = c$ (a and c are costants) then show that $v^2 \frac{d^2 p}{dv^2} = a(a + 1)p$

Watch Video Solution

**25.** Evaluate: $\int e^x \frac{x - 4}{(x - 2)^3} dx$

Watch Video Solution

**26.** Evaluate: $\int \sqrt{2 + 2\operatorname{cosec} x} dx$

Watch Video Solution

**27.** If the area of a circle increases uniformly, then show that the rate of increment of its circumference is inversely proportional to its radius.

Watch Video Solution

**28.** Solve: $\left(1 + 3e^{\frac{y}{x}}\right)dx + \left(1 - \frac{y}{x}\right)dx = 0$

Watch Video Solution

**29.** Evaluate (with the help of definite integral) :

$$\lim_{n \to \infty} \left\{ \left(1 + \frac{1}{n}\right)\left(1 + \frac{2}{n}\right)\ldots\left(1 + \frac{n}{n}\right) \right\}^{\frac{1}{n}}$$

Watch Video Solution

**30.** Evaluate: $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{\cos\theta}{1 + \sin\theta} d\theta$

Watch Video Solution

**31.** If the normal at any point to the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ makes an angle $\phi$ with the x-axis, then prove that the equation of the normal is $y\cos\phi - x\sin\phi = a\cos 2\phi$.

Watch Video Solution

**32.** Find the area of the region cut off by the straight line $3x - 2y + 12 = 0$ from the parabola $3x^2 = 4y$.

Watch Video Solution

**33.** Find the volume of the largest cylinder inscribed in the sphere of radius r cm.

Watch Video Solution

**34.** Solve: $x^2dy + y(x + y)dx = 0$

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## WBJEE Archive 2012

**1.** If a ,b ,c , are in A.P then the roots of the equation $ax^2 - 2bx + c = 0$

A. $1$ and $\frac{c}{a}$

B. $-\frac{1}{a}$ and $-c$

C. $-1$ and $\frac{-c}{a}$

D. $-2$ and $\frac{-c}{2a}$

**Answer: A**

Watch Video Solution

**2.** The remainder obtained when $1! + 2! + \ldots. + 95!$ is divided by 15 is -

A. 14

B. 3

C. 1

D. 0

**Answer: B**

Watch Video Solution

**3.** Let

$(1+x)^{10} = \sum_{r=0}^{10} c_r x^r$ and $(1+x)^7 = \sum_{r=0}^{7} d_r x^r$. if $p = \sum_{r=0}^{5}$ and $Q = \sum_{r=}^{3}$

is equal to -

A. 4

B. 8

C. 16

D. 32

**Answer: B**

Watch Video Solution

**4.** The maximum value of |z| when the complex number z satisfies the condition $\left|z + \frac{2}{z}\right| = 2$ is -

A. $\sqrt{3}$

B. $\sqrt{3} + \sqrt{2}$

C. $\sqrt{3} + 1$

D. $\sqrt{3} - 1$

**Answer: C**

Watch Video Solution

**5.** If $\left(\frac{3}{2} + i\frac{\sqrt{3}}{2}\right)^{50} = 3^{25}(x + iy)$ ,where x and y are real , then the ordered pair (x,y) is -

A. $(-3, 0)$

B. $(0, 3)$

C. $(0, -3)$

D. $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

**Answer: D**

Watch Video Solution

**6.** If $\frac{z - 1}{z + 1}$ is purely imaginary ,then-

A. $|z| = \frac{1}{2}$

B. $|z| = 1$

C. $|z| = 2$

D. $|z| = 3$

**Answer: B**

Watch Video Solution

**7.** A vehicle registration number consists of 2 letters of english alphbets followed by 4 digits ,where the first digit is not zero . Then the total number of vehicles with distinct registration number is -

A. $26^2 \times 10^4$

B. $.^{26}p_2 \times ^{10}p_4$

C. $.^{26}p_2 \times 9 \times ^{10}p_3$

D. $26^2 \times 9 \times 10^3$

**Answer: D**

Watch Video Solution

**8.** The number of words that can be written using all the letters of the word IRRATIONAL is -

A. $\dfrac{10!}{(2!)^3}$

B. $\dfrac{10!}{(2!)^2}$

C. $\dfrac{10!}{2!}$

D. $10!$

**Answer: A**

Watch Video Solution

**9.** Four speakers will address a meeting where speaker Q will always speak after speaker p .Then the number of ways in whcich the prder of speakers can be prepared is -

A. 256

B. 128

C. 24

D. 12

**Answer: D**

Watch Video Solution

**10.** The number of diagonals in a regular polygon of 100 sides is -

A. 4950

B. 4850

C. 4750

D. 4650

**Answer: B**

Watch Video Solution

**11.** Let the coefficients of powers of x in the 2nd ,3rd and 4th terms in the expansion of $(1+x)^n$ ,where n is a positive integer ,be in arithmetic progression .Then the sum of the cofficients of odd powers of x in the expansion is-

A. 32

B. 64

C. 128

D. 256

**Answer: B**

Watch Video Solution

**12.** The sum $1 \times 1! + 2 \times 2! + .... + 50 \times 50!$

A. $51!$

B. $51! - 1$

C. $51! + 1$

D. $2 \times 51!$

**Answer: B**

Watch Video Solution

**13.** Six number are in A.P such that their sum is 3 ,The first term is 4 times the third term .Then the fifth term is -

A. $-15$

B. $-3$

C. 9

D. $-4$

**Answer: D**

Watch Video Solution

**14.** The equations $x^2 + x + a = 0$ and $x^2 + ax + 1 = 0$ have a common root-

A. For no value of a

B. for a single value of a

C. for two values of a

D. for exactly three values of a

**Answer: B**

Watch Video Solution

**15.** If 64 , 27 , 36 are the p-th ,Q -th and R-th terms of , a G.P ,then p+2Q is equal to -

A. R

B. 2R

C. 3R

D. 4R

**Answer: C**

Watch Video Solution

**16.** The coefficient of $x^{10}$ in the expansion of $1 + (1 + x) + \ldots. + (1 + x)^{20}$ is -

A. $.^{19}C_9$

B. $.^{20}C_{10}$

C. .${}^{21}C_{11}$

D. .${}^{22}C_{12}$

**Answer: C**

Watch Video Solution

**17.** The points representing the complex number z for which arg $\left(\frac{z-2}{z+2}\right)=\frac{\pi}{3}$ lie on -

A. a circle

B. a straight line

C. an ellipse

D. a parabola

**Answer: A**

Watch Video Solution

**18.** Let , a , b ,c ,d , p ,q ,r be positive real number such that a ,b , c are in G.P and $a^p = b^q = c^r$ Then-

A. p ,q ,r ,are in G.P

B. p , q ,r are in A.P

C. p , q , r are in H.P

D. $p^2, q^2, r^2$ are in A.P

**Answer: C**

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**19.** $\alpha + \sqrt{\beta}$ and $\alpha - \sqrt{\beta}$ are two the two roots of the equation $x^2 + px + q = 0$ (where , $\alpha, \beta, p$ and $q$ are real numbers). Therefore , the roots of the equation $\left(p^2 - 4q\right)\left(p^2x^2 + 4px\right) - 16q = 0$ are -

A. $\frac{1}{\alpha} + \frac{1}{\sqrt{\beta}}$ and $\frac{1}{\alpha} - \frac{1}{\sqrt{\beta}}$

B. $\frac{1}{\sqrt{\alpha}} + \frac{1}{\beta}$ and $\frac{1}{\sqrt{\alpha}} - \frac{1}{\beta}$

C. $\frac{1}{\sqrt{\alpha}} + \frac{1}{\sqrt{\beta}}$ and $\frac{1}{\sqrt{\alpha}} - \frac{1}{\sqrt{\beta}}$

D. $\sqrt{\alpha} + \sqrt{\beta}$ and $\sqrt{\alpha} - \sqrt{\beta}$

**Answer: A**

Watch Video Solution

**20.** The number of solutions of the equation $\log_2\left(x^2 - 2x - 1\right) = 1$ is -

A. 0

B. 1

C. 2

D. 3

**Answer: C**

Watch Video Solution

**21.** The quadratic equation $2x^2 - \left(a^3 + 8a - 1\right)x + a^2 - 4a = 0$ possesse roots of opposite sign . Then -

A. $a \le 0$

B. $0 < a < 4$

C. $4 \le a < 8$

D. $a \ge 8$

**Answer: B**

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**22.** If $\log_e\left(x^2 - 16\right) \le \log_e(4x - 11)$ ,then-

A. $-1 \le x \le 5$

B. $x < -4$ or $x > 4$

C. $4 \le x \le 5$

D. $x < -1$ or $x > 5$

**Answer: A**

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## WBJEE Archive 2013

**1.** The number of solutions of the equation x + y + z = 10 in positive integers x , y , z, is equal to -

A. 36

B. 55

C. 72

D. 45

**Answer: A**

Watch Video Solution

**2.** If $\alpha$ and $\beta$ are the roots of $x^2 - x + 1 = 0$ ,then the value of $\alpha^{2013} + \beta^{2013}$ is equal to -

A. 2

B. $-2$

C. $-1$

D. 1

**Answer: B**

Watch Video Solution

**3.** The value of $1000\left[\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \dots . + \frac{1}{999 \times 1000}\right]$ is equal to -

A. 1000

B. 999

C. 1001

D. $\frac{1}{999}$

**Answer: B**

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**4.** $\alpha$ and $\beta$ are the roots of the equation Then $\left(\alpha - \frac{1}{\beta}\right)$ and $\left(\beta - \frac{1}{\alpha}\right)$ are the roots of the equation -

A. $ax^2 + a(b - 1)^x + (a - 1)^2 = 0$

B. $bx^2 + a(b - 1)^x + (a - 1)^2 = 0$

C. $x^2 + ax + b = 0$

D. $abx^2 + bx + a = 0$

**Answer: B**

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**5.** Let $z_1 = 2 + 3i$ and $z_2 = 3 + 4i$ be two points on the complex plane . Then the set of complex number z satisfying $|z - z_1|^2 + |z - z_2|^2 = |z_1 - z_2|^2$ represents -

A. a straight line

B. a point

C. a circle

D. a pair of staight lines

**Answer: C**

Watch Video Solution

**6.** Five number are in H.P The middle term is 1 and the ratio of the second and the fourth term is $2:1$ .Then the sum of the first three terms is -

A. $\frac{11}{2}$

B. 5

C. 2

D. $\frac{14}{3}$

**Answer: A**

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**7.** Suppose $z = x + iy$ where x and y are real numbers and $i = \sqrt{-1}$ . The points (x ,y) for which $\frac{z - 1}{z - i}$ is real . Lie om -

A. an ellipse

B. a circle

C. a parabola

D. a straight line

**Answer: D**

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**8.** Six positive numbers are in G.P ,such that their product is 1000 . If the fourth is 1 ,then the last term is -

A. 1000

B. 100

C. $\frac{1}{100}$

D. $\frac{1}{1000}$

**Answer: C**

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**9.** Let n be a positive even integer. The ratio of the largest coefficinet and the 2nd largest coefficient in the expansion of $(1+x)^n$ is 11 : 10 . The number of terms in the expansion of $(1+x)^n$ is -

A. 20

B. 21

C. 10

D. 11

**Answer: B**

Watch Video Solution

**10.** Five number are A.P with common differnce $\neq 0$ If the 1st , 3rd and 4th terms are in G.P ., then-

A. the 5th term is always 0

B. the 1st term is always 0

C. the middle term is always 0

D. the middle term is always $-2$

**Answer: A**

Watch Video Solution

**11.** Number of solutions of the equation $\frac{1}{2}\log_{\sqrt{3}}\left(\frac{x+1}{x+5}\right) + \log_9(x+5)^2 = 1$ is -

A. 0

B. 1

C. 2

D. infinite

**Answer: B**

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**12.** Let p(x) be a quadratic polynomial with constant term 1 . Suppose p(x) when divided by $x$ - $1$ leaves remiander 2 and when divided by x+1 leaves remainder 4 . Then the sum of the roots of p (x) = 0 is -

A. - $1$

B. 1

C. $-\frac{1}{2}$

D. $\frac{1}{2}$

**Answer: D**

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**13.** If $\alpha, \beta$ are the roots of the quadratic equation $ax^2 + bx + c = 0$ and $3b^2 = 16ac$ then-

A. $\alpha = 4\beta$ or $\beta = 4a$

B. $\alpha = -4\beta$ or $\beta = -4a$

C. $\alpha = 3\beta$ or $\beta = 3a$

D. $\alpha = -3\beta$ or $\beta = -3a$

**Answer: C**

Watch Video Solution

**14.** If P , Q ,R are angles of an isosceles traingle and $\angle P = \frac{\pi}{2}$ then the value of

$$\left(\cos\frac{p}{3} - i\sin\frac{p}{3}\right)^3 + (\cos Q + i\sin Q)(\cos R - i\sin R) + (\cos P - i\sin P)(\cos Q - i\sin Q)(\cos$$

then the value of

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**15.** The sum of the series

$$\frac{1}{1 \times 2}.{}^{25}C_0 + \frac{1}{2 \times 3}.{}^{25}C_1 + \frac{1}{3 \times 4}.{}^{25}C_2 + .... + \frac{1}{26 \times 27}.{}^{25}C_{25}$$

A. $\frac{2^{27} - 1}{26 \times 27}$

B. $\frac{2^{27} - 28}{26 \times 27}$

C. $\frac{1}{2}\left(\frac{2^{26} + 1}{26 \times 27}\right)$

D. $\frac{2^{26} - 1}{52}$

**Answer: B**

Watch Video Solution

**16.** Let $\sin\alpha, \cos\alpha$ be the roots of the equation $x^2 - bx + c = 0$ .Then which of the following statements is /are correct ?

A. $c \leq \frac{1}{2}$

B. $b \leq \frac{1}{2}$

C. $c > \frac{1}{2}$

D. $b > \sqrt{2}$

**Answer: A**

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## WBJEE Archive 2014

**1.** Let $z_1, z_2$ be two fixed complex numbers in the Argand plane and z be an arbitrary point satisfying $|z - z_1| + |z - z_2| = 2|z_1 - z_2|$ . Then the locus of z will be

A. an ellipse

B. a straight line joining $z_1$ and $z_2$

C. a parabola

D. a bisector of the line segment joining $z_1$ and $z_2$

**Answer: A**

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**2.** Out of 7 consonants and 4 vowels ,the number of words (not necssarily meaninguful ) that can be made ,each consisting of 3 consonants and 2 vowles ,is -

A. 2480

B. 2510

C. 2520

D. 2540

**Answer: C**

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**3.** The remainder obtained when $1! + 2! + 3! + .... + 11!$ is divided by 12 is -

A. 9

B. 8

C. 7

D. 6

**Answer: A**

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**4.** If $\alpha, \beta$ are the roots of the quadratic equation $x^2 + px + q = 0$ ,then the values of $\alpha^3 + \beta^3$ and $\alpha^4 + \alpha^2\beta^2 + \beta^4$ are respectively .

A. $3qp - p^3$ and $p^4 - 3p^2q + 3q^2$

B. $-p\left(3q - p^2\right)$ and $\left(p^2 - q\right)\left(p^2 + 3q\right)$

C. $pq - 4$ and $p^4 - q^4$

D. $3pq - p^3$ and $\left(p^2 - q\right)\left(p^2 - 3q\right)$

**Answer: D**

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**5.** Let p ,q be real numbers . If $\alpha$ is a root of $x^2 + 3p^2x + 5q^2 = 0, \beta$ is a root of $x^2 + 9p^2 + 15q^2 = 0$ and $0 < \alpha < \beta$ then the e

has a root $\gamma$ that always satisfies -

A. $\gamma = \frac{\alpha}{4} + \beta$

B. $\beta < \gamma$

C. $\gamma = \frac{\alpha}{2} + \beta$

D. $\alpha < \gamma < \beta$

**Answer: D**

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**6.** The value of the sum

$\left(.^{n}C_{1}\right)^{2}+\left(.^{n}C_{2}\right)^{2}+\left(.^{n}C_{3}\right)^{2}+\ldots.+\left(.^{n}C_{n}\right)^{2}$ is -

A. $\left(.^{2n}C_{n}\right)^{2}$

B. $.^{2n}C_{n}$

C. $.^{2n}C_{n}+1$

D. $.^{2n}C_{n}-1$

**Answer: D**

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**7.** Let $\alpha, \beta$ be the roots of $x^{2}-x-1=0$ and $s_{n}=\alpha^{n}+\beta^{n}$ for all integers $n \geq 1$. Then for every integer $n \geq 2$ -

A. $S_n + S_{n-1} = S_{n+1}$

B. $S_n + S_{n-1} = S_n$

C. $S_{n-1} = S_{n+1}$

D. $S_n + S_{n-1} = 2S_{n+1}$

**Answer: A**

Watch Video Solution

**8.** In the Argand plane ,the distinct roots of $1 + z + z^3 + z^4 = 0$ (z is a complex number ) represent vertices of -

A. a square

B. an equilateral

C. a rhombus

D. a rectangle

**Answer: B**

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**9.** The number of digits in $20^{301}$ (given `log_(10)2=0.3010) is

A. 602

B. 301

C. 392

D. 391

**Answer: C**

Watch Video Solution

**10.** The solution of the equation

$\log_{101}\log_7\left(\sqrt{x+7}+\sqrt{x}\right)=0$ is -

A. 3

B. 7

C. 9

D. 49

**Answer: C**

Watch Video Solution

**11.** If $\alpha, \beta$ are the roots of $ax^2 + bx + c = 0 (a \neq 0)$ and $\alpha + h, \beta + h$ are the roots of $px^2 + qx + r = 0 (p \neq \; =$

then the ration of the squares of their discriminants is -

A. $a^2 : p^2$

B. $a : p^2$

C. $a^2 : p$

D. $a : 2p$

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**12.** Suppose that $z_1, z_2, z_3$ are three vertices of an equilateral triangle in the Angand plane . Ley $\alpha = \frac{1}{2}\left(\sqrt{3} + i\right)$ and $\beta$ be a non-zero complex number . The points $\alpha z_1 + \beta, \alpha z_2 + \beta, \alpha z_3 + \beta$will be -

A. the vertices of an equilateral triangle

B. the vertices of an isosceles triangle

C. collinear

D. the vertices of an scalene triangle

**Answer: A**

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**13.** The value of $|z|^2 + |z - 3|^2 + |z - i|^2$ is minimum when z equals -

A. $2 - \frac{2}{3}i$

B. $45 + 3i$

C. $1 + \frac{i}{3}$

D. $1 - \frac{i}{3}$

**Answer: C**

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**14.** If the coefficient of $x^8 \text{in}\left(ax^2 + \frac{1}{bx}\right)^{13}$ is equal to the coefficeinet of $x^{-8} \text{ in}\left(ax - \frac{1}{bx^2}\right)^{13}$, then a and b will satisfy the relation -

A. ab + 1 =0

B. ab = 1

C. $a = 1 - b$

D. $a + b = -1$

**Answer: A**

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**15.** The number of solutions (s) of the equation $\sqrt{x+1} - \sqrt{x+1} = \sqrt{4x+1}$

is /are -

A. 2

B. 0

C. 3

D. 1

**Answer: B**

Watch Video Solution

**16.** If a , b , c , are positive numbers in G.P . ,then the roots of the quadratic

equation

$\left(\log_e a\right)x^2 - \left(2\log_e b\right)x + \left(\log_e c\right) = 0$

A. $-1 \text{ and } \dfrac{\log_e c}{\log_e a}$

B. $1 \text{ and } \dfrac{\log_e c}{\log_e a}$

C. $1$ and $\log_a c$

D. $-1$ and $\log_c a$

**Answer: C**

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**17.** Let $z_1$ be a fixed point on the circle of radius 1 centered at the origin in the Argand plane and $z_1 \neq \pm 1$. Consider an equilateral traingle inscribed in the circle with $z_1, z_2, z_3$ as the vertices taken in the counter clockwise direction .Then $z_1z_2z_3$ is equal to -

A. $z_1^2$

B. $z_1^3$

C. $z_1^4$

D. $z_1$

**Answer: B**

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Watch Video Solution

18. Let $\alpha, \beta$ denote the cube roots of unity other then 1 and let $s = \sum_{n=0}^{302} (-1)^n \left(\frac{\alpha}{\beta}\right)^n$. Then the value of s is -

A. either $-2\omega$ or $-2\omega^2$

B. either $-2\omega$ or $2\omega^2$

C. either $2\omega$ or $-2\omega^2$

D. either $2\omega$ or $2\omega^2$

**Answer: A**

Watch Video Solution

19. The minimum value of $2^{\sin x} + 2^{\cos x}$ is -

A. $2^{1 - \frac{1}{\sqrt{2}}}$

B. $2^{1 + \frac{1}{\sqrt{2}}}$

C. $2\sqrt{2}$

D. 2

**Answer: A**

Watch Video Solution

**20.** Let $S = \frac{2n}{1}C_0 + \frac{2^2 n}{2}C_1 + \frac{2^3 n}{3}C_2 + \dots . + \frac{2^{n+1} n}{n+1}C_n.$. Then S equals-

A. $\frac{2^{n+1} - 1}{1}$

B. $\frac{3^{n+1} - 1}{1}$

C. $\frac{3^n - 1}{1}$

D. $\frac{2^n - 1}{1}$

**Answer: B**

Watch Video Solution

**21.** The sum of the series $\sum^{\infty} n+1 \sin\left(\frac{n!\pi}{720}\right)$ is -

A. $\sin\left(\frac{\pi}{180}\right) + \sin\left(\frac{\pi}{360}\right) + \sin\left(\frac{\pi}{540}\right)$

B. $\sin\left(\frac{\pi}{6}\right) + \sin\left(\frac{\pi}{30}\right) + \sin\left(\frac{\pi}{120}\right) + \sin\left(\frac{\pi}{360}\right)$

C. $\sin\left(\frac{\pi}{6}\right) + \sin\left(\frac{\pi}{30}\right) + \sin\left(\frac{\pi}{120}\right) + \sin\left(\frac{\pi}{360}\right) + \sin\left(\frac{\pi}{720}\right)$

D. $\sin\left(\frac{\pi}{180}\right) + \sin\left(\frac{\pi}{360}\right)$

**Answer: C**

Watch Video Solution

## WBJEE Archive 2015

**1.** The letters of the word COCHIN are permuted and all the permutations are arranged in alphabetical order as in before the word COCHIN is -

A. 360

B. 192

C. 96

D. 48

**Answer: C**

Watch Video Solution

**2.** If $\alpha, \beta$ are the roots $x^2 - px + 1 = 0$ and $\gamma$ is a root of $x^2 + px + 1 = 0$ ,then $(\alpha + \gamma)(\beta + \gamma)$ is -

A. 0 (zero)

B. 1

C. $-1$

D. p

**Answer: A**

Watch Video Solution

**3.** Number of irrational terms on the binomial expansion of $\left(3^{\frac{1}{5}} + 7^{\frac{1}{3}}\right)^{100}$

is -

A. 90

B. 88

C. 93

D. 95

Watch Video Solution

**4.** The quadratic expression $(2x + 1)^2 - px + q \neq 0$ for any real x if

A. $p^2 - 16p - 8q < 0$

B. $p^2 - 8p + 16q < 0$

C. $p^2 - 8p - 16q < 0$

D. $p^2 - 16p + 8q < 0$

**Answer: C**

Watch Video Solution

**5.** The value of $\left(\frac{1+\sqrt{3i}}{1-\sqrt{3i}}\right)^{64}+\left(\frac{1-\sqrt{3i}}{1+\sqrt{3i}}\right)^{64}$ is -

A. 0 (zero)

B. $-1$

C. 1

D. i

**Answer: B**

Watch Video Solution

**6.** Let d (n) denote the number of divisors of n including 1 and itself . Then d (225), d(1125) and d (640) are -

A. in A.P

B. in H.P

C. in G.P

D. consecutive integers

**Answer: C**

Watch Video Solution

**7.** If $2+i$ and $\sqrt{5}-2i$ are the roots of the equation $\left(x^2+ax+b\right)\left(x^2+cx+d\right)=0$ where a , b , c , d are real constants then product of all roots of the equation is -

A. 40

B. $9\left(\sqrt{5}\right)$

C. 45

D. 35

**Answer: C**

Watch Video Solution

**8.** If x and y are digits such that $\left(x^2 + ax + b\right)\left(x^2 + cx + d\right) = 0$ then x + y equals -

A. 15

B. 6

C. 12

D. 13

**Answer: A**

View Text Solution

**9.** Which of the following is /are always false ?

A. A quadratic equation with rational coefficients has zero or two irrational roots.

B. A quadratic equation with real coefficients has zero or two non - real roots .

C. A quadratic equation with irrational coefficients has zero or two rational roots .

D. A quadratic equation with integer coefficients has zero or two irrational roots.

**Answer: C**

Watch Video Solution

**10.** Find the maximum value of $|z|$when $\left|z - \frac{3}{z}\right| = 2, z$ being a complex number .

A. $1 + \sqrt{3}$

B. 3

C. $1 + \sqrt{2}$

D. 1

**Answer: B**

Watch Video Solution

**11.** Let a , b , c ,d ,be any four real number . Then $a^n + b^n = c^n + d^n$ holds for any natural no . If -

A. a + b = c + d

B. $a - b = c - d$

C. $a + b = c + d, a^2 + b^2 = c^2 + d^2$

D. $a - b = c - d, a^2 - b^2 = c^2 - d^2$

**Answer: D**

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WBJEE Archive 2016

**1.** If x is a positive real number different from 1 such that $\log_a^x, \log_b^x, \log_c^x$ are in A.P , then

A. $b = \frac{a+c}{2}$

B. $b = \sqrt{ac}$

C. $c^2 = (ac)^{\log_a b}$

D. none of A ,B ,C are correct

**Answer: C**

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**2.** If a ,x are real number $|a| < 1, |x| < 1$ then $1 + (1+a)x + \left(1 + a + a^2\right)x^2 + .... \infty$ is equal to

A. $\dfrac{1}{(1-a)(1-ax)}$

B. $\dfrac{1}{(1-a)(1-x)}$

C. $\dfrac{1}{(1-x)(1-ax)}$

D. $\dfrac{1}{(1-ax)(1-a)}$

**Answer: C**

Watch Video Solution

**3.** If $\log_{0.3}(x-1) < \log_{0.09}(x-1)$ ,then x lies in the interval

A. $(2, \infty)$

B. $(1, 2)$

C. $(-2, -1)$

D. None of these

**Answer: A**

Watch Video Solution

**4.** The value of $\sum_{n=1}^{13}\left(i^n + i^{n-1}\right), \; = i = \sqrt{-1}$ is -

A. i

B. $i - 1$

C. 1

D. 0

Watch Video Solution

**5.** If

$$\left|z_1\right| = \left|z_2\right| = \left|z_3\right| = \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = 1 \text{ and } z_1, z_2, z_3 \text{ are imaginary numbers ,th}$$

is

A. equal to 1

B. less than 1

C. greater than 1

D. equal to 3

**Answer: A**

Watch Video Solution

6. If , p , q are the roots of the equation $x^2 + px + q = 0$ , then

A. p=1,q=-2

B. p= 0 ,q = 1

C. $p = -2, q = 0$

D. $p = -2, q = 1$

**Answer: A**

Watch Video Solution

**7.** The number of values of k for which the equation $x^2 - 3x + k = 0$ has two distinct roots lying in the interval ( 0 , 1 ) are

A. three

B. two

C. infinitely many

D. no value of k satisfies the requirment

Watch Video Solution

**8.** The number of ways in which the letters of the word ARRANGE can be permuted such that the R's occur together is

A. $\frac{\lfloor 7}{\lfloor 2 \lfloor 2}$

B. $\frac{\lfloor 7}{\lfloor 2}$

C. $\frac{\lfloor 6}{\lfloor 2}$

D. $\lfloor 5 \times \lfloor 2$

**Answer: C**

Watch Video Solution

**9.** If $\frac{1}{.^5C_r} + \frac{1}{.^6C_r} = \frac{1}{.^4C_r}$, then the value of r equals to

A. 4

B. 2

C. 5

D. 3

**Answer: B**

Watch Video Solution

**10.** For + ve integer $n, n^3 + 2n$ is always divisible by

A. 3

B. 7

C. 5

D. 6

**Answer: A**

Watch Video Solution

**11.** In the expansion of $(x - 1)(x - 2)...(x - 18)$ the coefficient of $x^{17}$ is

A. 684

B. $-171$

C. 171

D. $-342$

**Answer: B**

Watch Video Solution

**12.** $1 + .^{n}C_{1}\cos\theta + .^{n}C_{2}\cos 2\theta + \ldots + .^{n}C_{n}\cos n\theta$ equals

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**13.** The letters of the word COCHIN are permuted and all the permutations are arranged in alphabetical order as in an English dictionary .The number of word that appear before the word COCHIN is

A. 96

B. 48

C. 183

D. 267

**Answer: A**

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**14.** The sum of n terms of the following series , $1^3 + 3^3 + 5^3 + 7^3 + \ldots\ldots.$ is

A. $n^2\left(2n^2 - 1\right)$

B. $n^3(n - 1)$

C. $n^3 + 8n + 4$

D. $2n^4 + 3n^2$

**Answer: A**

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**15.** If $\alpha$ and $\beta$ are roots of $ax^2 + bx + c = 0$ then the equationnwhose roots are $\alpha^2$ and $\beta^2$ is

A. $a^2x^2 - \left(b^2 - 2ac\right)x + c^2 = 0$

B. $a^2x^2 + \left(b^2 - 2ac\right)x + c^2 = 0$

C. $a^2x^2 + \left(b^2 + ac\right)x + c^2 = 0$

D. $a^2x^2 + \left(b^2 + 2ac\right)x + c^2 = 0$

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16. If $\omega$ is an imaginary cube root of unity then the value of

$(2 - \omega), \left(2 - \omega^2\right) + 2(2 - \omega)\left(3 - \omega^2\right) + .... + (n - 1)(n - \omega)\left(n - \omega^2\right)$ is

A. $\frac{n^2}{4}(n + 1)^2 - n$

B. $\frac{n^2}{4}(n + 1)^2 + n$

C. $\frac{n^2}{4}(n + 1)^2$

D. $\frac{n^2}{4}(n + 1)^2 - n$

**Answer: D**

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**17.** If $.^{n}C_{r-1} = 36, .^{n}C_{r} = 84,$ and $^{n}C_{r+1} = 126,$ " then the value of "
.^(n)C_(8)` is

A. 10

B. 7

C. 9

D. 8

**Answer: C**

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**18.** If the first and the (2n+1) -th terms pf AP, GP and HP are equal and their n-th terms are respectively a , b , c then always

A. $a = b = c$

B. $a \geq b \geq c$

C. $a + c = b$

D. $ac - b^2 = 0$

**Answer: B::D**

View Text Solution

19. If the equation $x^2 + y^2 - 10x + 21 = 0$ has real roots $x = \alpha$ and $y = \beta$ then

A. $3 \le x \le 7$

B. $3 \le y \le 7$

C. $-2 \le y \le 2$

D. $-2 \le x \le 2$

**Answer: A::C**

Watch Video Solution

**20.** If $z = \sin\theta - i\cos\theta$ then for any integer n

A. $z^n + \frac{1}{z^n} = 2\cos\left(\frac{n\pi}{2} - n\theta\right)$

B. $z^n + \frac{1}{z^n} = 2\sin\left(\frac{n\pi}{2} - n\theta\right)$

C. $z^n - \frac{1}{z^n} = 2i\sin\left(n\theta - \frac{n\pi}{2}\right)$

D. $z^n - \frac{1}{z^n} = 2i\cos\left(\frac{n\pi}{2} - n\theta\right)$

**Answer: A::C**

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## JEE Main (ALEEE) Archive 2012

**1.** If n is a positive integer , then $\left(\sqrt{3} + 1\right)^{2n} - \left(\sqrt{3} - 1\right)^{2n}$ is -

A. an even positive integer

B. a rational number other than positive integers

C. an irrational number

D. an odd positive integer.

**Answer: C**

Watch Video Solution

**2.** Assuming the balls to be identical except for difference in colours , the number of ways in which one or more balls can be selected from 10 white , 9 green and 7 black balls is -

A. 630

B. 879

C. 880

D. 629

**Answer: B**

Watch Video Solution

**3.** If $z \neq 1$ and $\frac{z^2}{z - 1}$ is real , then the point represented by the complex number z lies -

A. either on the real axis or on a circle not passing through the origin.

B. on the imaginary axis

C. either on the real axis or on a circle passing through the origin

D. on a circle with centre at the origin.

**Answer: C**

Watch Video Solution

**4.** If 100 times the 100 -th term of an A.P . With non-zero common difference equals the 50 times its 50-th terms, thenthe 150-th term , of this A.P is -

A. 150

B. zero

C. $-150$

D. 150 tem its 50 -th term

**Answer: B**

Watch Video Solution

**5.** The equation $e^{\sin x} - e^{-\sin}04 = 0$ has

A. no real root

B. exactly one real root

C. exactly four real roots

D. infinite number of real roots

**Answer: A**

View Text Solution

**6.** Statement -1 : The sum of the series

$(1 + (1 + 2 + 3 + 4) + (4 + 5 + 9) + (9 + 12 + 16) + (361 + 380 + 400) \text{is} 8000$

Statement -2 : $\sum_{k=1}^{n} \left[k^3 - (k - 1)^3\right] = n^3$ for any natural number n .

A. Statement -1 is true ,statement -2 is true . Statement-2 is not a correct explantion for Statement -1.

B. Statement -1 is true ,Statement -2 is false .

C. Statement -1 is false ,Statement -2 is true .

D. Statement -1 is true , Statement -2 is true ,Statement -2 is a correct explanation for Statement -1.

**Answer: D**

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## JEE Main (ALEEE) Archive 2013

**1.** The sum of first 20 term of the squence $.7, 0.77, 0.777, \ldots$ is -

A. $\frac{7}{81}\left(179 - 10^{-20}\right)$

B. $\frac{7}{9}\left(99 - 10^{-20}\right)$

C. $\frac{7}{81}\left(179 + 10^{-20}\right)$

D. $\frac{7}{9}\left(99 + 10^{-20}\right)$

**Answer: C**

Watch Video Solution

**2.** If the equation $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0, a, b, c \in \mathbb{R}$, have a cmmon root , then a : b : c is -

A. $1:2:3$

B. $3:2:1$

C. $1:3:2$

D. $3:1:2$

**Answer: A**

Watch Video Solution

**3.** The term independent of x in the expansion of $\left(\frac{x+1}{x^{\frac{2}{3}}-x^{\frac{1}{3}}+1}-\frac{x-1}{x-x^{\frac{1}{2}}}\right)^{10}$ is -

A. 4

B. 120

C. 210

D. 310

**Answer: C**

Watch Video Solution

**4.** Let $T_n$ be the number of all possible triangles formed by joining vertices of a n-side regular polygon . If $T_{n+1}$ - $T_n = 10$ ,then the value of n is -

A. 7

B. 5

C. 10

D. 8

**Answer: B**

Watch Video Solution

**5.** If z is complex number of unit modulus and argument $\theta$, then arg $\frac{1+z}{1+z}$ equals-

A. - $\theta$

B. $\frac{\pi}{2}$ - $\theta$

C. $\theta$

D. $\pi - \theta$

**Answer: C**

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## JEE Main (ALEEE) Archive 2014

**1.** If the cofficients of $x^3$ and $x^4$ in the expansion of $\left(1 + ax + bx^2\right)(1 - 2x)^{18}$ in powers pf x are both zero , the (a , b) is equal to

-

A. $\left(16, \dfrac{251}{3}\right)$

B. $\left(14, \dfrac{251}{3}\right)$

C. $\left(16, \dfrac{272}{3}\right)$

D. $\left(16, \dfrac{272}{3}\right)$

**Answer: D**

Watch Video Solution

**2.** If z is complex number such that $|z| \geq 2$, minimum value of $\left|z + \frac{1}{2}\right|$ -

A. is equal to $\frac{5}{2}$

B. lies in the interval (1,2)

C. in stictly greater than $\frac{5}{2}$

D. is strictly than $\frac{3}{2}$ but less than $\frac{5}{2}$

**Answer: B**

Watch Video Solution

**3.** Let $\alpha$ and $\beta$ be the roots of equation $px^2 + qx + r = 0$ $p \neq 0$. If $p, q, r$ are in A.P and $\frac{1}{\alpha} + \frac{1}{\beta} = 4$, then the value of $|\alpha -$

is -

A. $\frac{\sqrt{61}}{9}$

B. $2\frac{\sqrt{17}}{9}$

C. $\frac{\sqrt{34}}{9}$

D. $2\frac{\sqrt{13}}{9}$

**Answer: D**

Watch Video Solution

**4.** Three positive numbers form an increasing G.P .If the middle term in this G.P is doubled , the new numbers are in A.P .Then the common ration of the G.P is -

A. $\sqrt{2} + \sqrt{3}$

B. $3 + \sqrt{2}$

C. $2 - \sqrt{3}$

D. $2+\sqrt{3}$

**Answer: D**

Watch Video Solution

**5.** If $10^9+2(11)^1(10)^8+3(11)^2(10)^7+\ldots+10(11)^9=k(10)^9$ , then k is equal to -

A. $\dfrac{121}{10}$

B. $\dfrac{441}{100}$

C. 100

D. 110

**Answer: C**

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JEE Main (ALEEE) Archive 2015

**1.** The sum pf coefficients of integral powers of x is the binomial expansion $\left(1 - 2\sqrt{x}\right)^{50}$ is -

A. $\frac{1}{2}\left(3^{50} - 1\right)$

B. $\frac{1}{2}\left(2^{50} + 1\right)$

C. $\frac{1}{2}\left(3^{50} + 1\right)$

D. $\frac{1}{2}\left(3^{50}\right)$

**Answer: C**

Watch Video Solution

**2.** The sum pf first 9 terms of the series

$\frac{1^3}{1} + \frac{1^3 + 2^3}{1 + 3} + \frac{1^3 + 2^3 + 3^3}{1 + 3 + 5} + \ldots\ldots$ is -

A. 142

B. 192

C. 71

D. 96

**Answer: D**

Watch Video Solution

**3.** A complex number z is said to be unimodular if $|z| = 1$ Suppose $z_1$ and $z_2$ are complex number such that $\frac{z_1 - 2z_2}{2 - z_1 z_2}$ is unimodular and $z_2$ is not unimodular .Then the point $z_1$ lies on a -

A. circle of radius 2

B. circle pf radius $\sqrt{2}$

C. stright line parallel to x - axis

D. straight line parallel to y - aixs

**Answer: A**

Watch Video Solution

**4.** If m is the A.M of two distinct real number l and n $(l, n > 1)$ and $G_1, G_2$ and $G_3$ are three geometric means between l and n , then $G_1^4 + 2G_2^4 + C_3^4$ equals-

A. $4lmn^2$

B. $4l^2m^2n^2$

C. $4l^2mn$

D. $4lm^2n$

**Answer: D**

Watch Video Solution

**5.** Let $\alpha$ and $\beta$ be the roots of equation $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$, for $n \geq 1$, then the value of $\frac{(a_{10}) - (2a_8)}{2a_9}$ is equal to -

A. 3

B. $-3$

C. 6

D. $-6$

**Answer: A**

Watch Video Solution

**6.** Let A and B be two sets containing four and two elements repectively . Then the number of subsets of the set $A \times B$ , each having at least elements is -

A. 275

B. 510

C. 219

D. 256

**Answer: C**

Watch Video Solution

7. The number of integers greater than 6000 that can be formed ,using the digits 3, 5 , 6 , 7 and 8 , without repetition is -

A. 120

B. 72

C. 216

D. 192

**Answer: D**

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## JEE Main (ALEEE) Archive 2016

1. A value of $\theta$ for which $\dfrac{2+3i\sin\theta}{1-2i\sin\theta}$ is purely imaginary is

A. $\frac{\pi}{3}$

B. $\frac{\pi}{6}$

C. $\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

D. $\sin^{-1}\left(\frac{3}{\sqrt{4}}\right)$

**Answer: D**

Watch Video Solution

**2.** The sum pf all real values of x satisfying the equation $\left(x^2 - 5x + 5\right)^{x^2 - 4x - 60} = 1$ is

A. 3

B. -4

C. 6

D. 5

**Answer: A**

Watch Video Solution

**3.** If all the words (with or without meaning ) having five letters ,formed using the letters of the word SMALL and arranged as in a dictionary ,then the position of the word SMALL as in a dictionary , then the position of the word SMALL is

A. 46th

B. 59th

C. 52th

D. 58th

**Answer: D**

Watch Video Solution

**4.** IF the number of terms in the expansion of $\left(1 - \frac{2}{x} + \frac{4}{x^2}\right)^n, x \neq 0$ is 28 ,then the sum of the coefficients of all the terms in this expansion is

A. 74

B. 2187

C. 243

D. 729

**Answer: D**

Watch Video Solution

**5.** If the 2nd ,5ht and 9th terms of a non-constant AP are in GP , then the common ratio of this GP is

A. $\frac{8}{5}$

B. $\frac{4}{3}$

C. 1

D. $\frac{7}{4}$

**Answer: B**

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**6.** If the sum of the first ten terms pf the series

$\left(1\frac{3}{5}\right)^2+\left(2\frac{2}{5}\right)^2+\left(3\frac{1}{5}\right)^2+4^2+\left(4\frac{4}{5}\right)^2 \dots..,$ is $\frac{16}{5}m$ ,then m is equal to

A. 102

B. 101

C. 100

D. 99

**Answer: B**

Watch Video Solution

**1.** Let complex numbers $\alpha$ and $\frac{1}{\alpha}$ lie on circles $(x - x_0)^2 + (y - y_0)^2 = r^2$ and $(x - x_0)^2 + (y - y_0)^2 = 4r^2$ respectively. If $z_0 = x_0 + iy_0$ satisfies the equation $2|z_0|^2 = r^2 + 2$ then $|\alpha|$=

A. $\frac{1}{\sqrt{2}}$

B. $\frac{1}{2}$

C. $\frac{1}{\sqrt{7}}$

D. $\frac{1}{3}$

**Answer: C**

Watch Video Solution

**2.** Let

$w = \frac{\sqrt{3} + i}{2}$ and $p = \{w^n : n = 1, 2, 3, \ldots.\}$. Further $H_1 = \left\{z \in \mathbb{C} : Rez > \frac{1}{2}\right\}$ a

, where $\mathbb{C}$ is the set of all complex numbers , If $z_1 \in p \cap H_1, z_2 \in p \cap H_2$ and $O$ represents the orgin,then $\angle z_1 O z_2$ =

A. $\frac{\pi}{2}$

B. $\frac{\pi}{6}$

C. $\frac{2\pi}{3}$

D. $\frac{5\pi}{6}$

**Answer: C::D**

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**3.** Let

$S = S_1 \cap S_2 \cap S_3$, where $S_1 = \{z \in \mathbb{C} : |z| < 4\}, S_2 = \left\{z \in \mathbb{C} : \text{Im}\left[\frac{(z - 1) + \sqrt{3i}}{1 - \sqrt{3i}}\right.\right.$

Area of s =

A. $\frac{10\pi}{3}$

B. $\frac{20\pi}{3}$

C. $\frac{16\pi}{3}$

D. $\frac{32\pi}{3}$

**Answer: B**

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**4.** Let

$S = S_1 \cap S_2 \cap S_3$, where $S_1 = \{z \in \mathbb{C} : |z| < 4\}, S_2 = \left\{z \in \mathbb{C} : \mathrm{Im}\left[\frac{(z-1)+\sqrt{3}i}{1-\sqrt{3}i}\right.\right.$

$\min_{z \in S}|1 - 3i - z| =$

A. $\frac{2-\sqrt{3}}{2}$

B. $\frac{2+\sqrt{3}}{2}$

C. $\frac{3-\sqrt{3}}{2}$

D. $\frac{3-\sqrt{3}}{2}$

**Answer: C**

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5. The coefficients of three consecutive terms of $(1+x)^{n+5}$ are in the ration 5 : 10 : 14 . Then n =

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6. The number of points in $(-\infty, \infty)$, for which $x^2 - x\sin x - \cos x = 0$ is

A. 6

B. 4

C. 2

D. 0

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**7.** Let, $f(x) = x\sin\pi x, x > 0$. Then for all natural numbers n, f'(x) vanishes at

A. a unique point in the interval $\left(n, n + \frac{1}{2}\right)$

B. a unique point in the interval $\left(n + \frac{1}{2}, n + 1\right)$

C. a unique point in the interval $(n, n + 1)$

D. two points in the interval $(n, n + 1)$

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**8.** Let, $f: [0, 1] \rightarrow \mathbb{R}$ (the set of all real numbers) be a function. Suppose the function f is twice differentiable, $f(0) = f(1) = 0$ and satisfies

$f''(x) - 2f'(x) + f(x) \geq e^x, x \in [0, 1]$

Which of the following is true for $0 < x < 1$ ?

A. $0 < f(x) < \infty$

B. $-\frac{1}{2} < f(x) < \frac{1}{2}$

C. $-\frac{1}{4} < f(x) < 1$

D. $-\infty < f(x) < 0$

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**9.** Let, $f:[0, 1] \to \mathbb{R}$ (the set of all real numbers) be a function. Suppose the function f is twice differentiable, $f(0) = f(1) = 0$ and satisfies

$f''(x) - 2f'(x) + f(x) \geq e^x, x \in [0, 1]$

If the function $e^{-x}f(x)$ assumes its minimum in the interval $[0, 1]$ at $x = \frac{1}{4}$, which of the following is true?

A. $f'(x) < f(x), \frac{1}{4} < x < \frac{3}{4}$

B. $f'(x) > f(x), 0 < x < \frac{1}{4}$

C. $f'(x) < f(x), 0 < x < \frac{1}{4}$

D. $f'(x) < f(x), \frac{3}{4} < x < 1$

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**10.** A rectangular sheet of fixed perimeter with sides having their lengths in the ratio $8:15$ is converted into an open rectangular box by folding after removing squares of equal area from all four corners. If the total area of the removed squares is 100, the resulting box has maximum volume. The lengths of the side of the rectangular sheet are

A. 24

B. 32

C. 45

D. 60

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**11.** Find derivative of $\cos^{-1}x$ with respect to $\cos x$

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**12.** The function $f(x) = 2|x| + |x + 2| - ||x + 2| - 2|x||$ has a local minimum or a local maximum at x =

A. -2

B. $-\frac{2}{3}$

C. 2

D. $\frac{2}{3}$

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**13.** Let $f: \left[\frac{1}{2}, 1\right] \rightarrow \mathbb{R}$ (the set of all real numbers) be a positive, non-constant and differentiable function such that

$f'(x) < 2f(x)$ and $f\left(\frac{1}{2}\right) = 1$

Then the value of $\int_{\frac{1}{2}}^{1} f(x)dx$ lies in the interval

A. $(2e - 1, 2e)$

B. $(e - 1, 2e - 1)$

C. $\left(\frac{e - 1}{2}, e - 1\right)$

D. $\left(0, \frac{e - 1}{2}\right)$

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**14.** For $a \in \mathbb{R}$ (the set of all real numbers), $a \neq -1$, $\lim_{n \to \infty} \frac{1^a + 2^a + \dots + n^a}{(n + 1)^{a - 1}[(na + 1) + (na + 2) + \dots + (na + n)]} = \frac{1}{60}$

Then a =

A. 5

B. 7

C. $-\frac{15}{2}$

D. $-\frac{17}{2}$

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**15.** The area exclosed by the curves $y = \sin x + \cos x$ and $y = |\cos x - \sin x|$ over the interval $\left[0, \frac{\pi}{2}\right]$ is

A. $4\left(\sqrt{2} - 1\right)$

B. $2\sqrt{2}\left(\sqrt{2} - 1\right)$

C. $2\left(\sqrt{2} + 1\right)$

D. $2\sqrt{2}\left(\sqrt{2} + 1\right)$

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**16.** A curve passes through the point $\left(1, \frac{\pi}{6}\right)$. Let, the slope of the curve at each point $(x, y)$ be $\frac{y}{x} + \sec\left(\frac{y}{x}\right), x > 0$. Then the equation of the curve is

A. $\sin\left(\frac{y}{x}\right) = \ln x + \frac{1}{2}$

B. $\operatorname{cosec}\left(\frac{y}{x}\right) = \ln x + 2$

C. $\sec\left(\frac{2y}{x}\right) = \ln x + 2$

D. $\cos\left(\frac{2y}{x}\right) = \ln x + \frac{1}{2}$

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## JEE Advanced Archive 2014

**1.** A pack contains n cards m=numbered form 1 to n . Two consecutive numbered cards are removed from the pack and the sum of the numbers of the remaining cards is 1224. .If thesmaller of the numberse on the removed cards is k , then $k - 20 =$

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**2.** The quadratic equation $p(x) = 0$ with real coefficients has purely imaginary roots. Then the equation $p(p(x)) = 0$ has

A. only imaginary roots

B. all real roots

C. two real and two purely imaginary roots

D. nither real nor purely imaginary roots

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**3.** Let $n \geq 2$ be an integer .Take n distinct points ona circle and join each pair of points by a line segment .Colour the line segement joining every pair of adjacent points by blue and the rest by red . If the number of red and blue line segments are equal, then the value of n is

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**4.** Let $n_1 < n_2 < n_3 < n_4 < n_5$ be positive integers such that $n_1 + n_2 + n_3 + n_4 + n_5 = 20$. Then the number of such distinct arrangements $(n_1, n_2, n_3, n_4, n_5)$ is

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**5.** Six cards and six envelopes are numbered 1 ,2 , 3, 4 , 5,6 and cards are to be placed in envelopes so that each envelope contains exaxtly one card and no card is placed in the envelope bearing the same number and moreover the card numbered 1 is always placed in envelope number 2 . Then the number of ways it cann be done is -

A. 264

B. 265

C. 53

D. 67

**Answer: C**

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**6.** Coefficient of $x^{11}$ in the expansion of $\left(1+x^2\right)^4\left(1+x^3\right)^7\left(1+x^4\right)^{12}$ is -

A. 1051

B. 1106

C. 1113

D. 1120

**Answer: C**

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**7.** If $S_n=\sum_{k=1}^{4n}(-1)^{\frac{k(k+1)}{2}}.k^2$ then the possible values of $s_n$ are-

A. 1056

B. 1088

C. 1120

D. 1332

**Answer: A::D**

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**8.** Let a ,b , c be positive integers such that $\frac{b}{a}$ is an integer . If a , b , c are in geometric progression and the arithmetic mean of a , b , c , is b + 2 , then value of $\frac{a^2 + a - 14}{a + 1}$

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**9.** Let, $f:[a, b] \rightarrow [1, \infty)$ be a continuous function and let $g:\mathbb{R} \rightarrow \mathbb{R}$ be defined as

$$g(x) = \begin{cases} 0 & \text{if } x < a, \\ \int_a^x f(t)dt & \text{if } a \leq x \leq b \\ \int_a^b f(t)dt & \text{if } x > b \end{cases}$$ Then

A. g(x) is continuous but not differentiable at a

B. g(x) is differentiable on $\mathbb{R}$

C. g(x) is continuous but not differentiable at b

D. g(x) is continuous and differentiable at either a or b but not both

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**10.** Let, $f:[0, 4\pi] \rightarrow [0, \pi]$ be defined by $f(x) = \cos^{-1}(\cos x)$. The number of points $x \in [0, 4\pi]$ satisfying the equation

$$f(x) = \frac{10 - x}{10}$$

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**11.** Let, $a \in \mathbb{R}$ and let $f: \mathbb{R} \rightarrow \mathbb{R}$ be given by $f(x) = x^5 - 5x - a$. Then

A. f(x) has three real roots if $a > 4$

B. f(x) has only one real root if $a > 4$

C. f(x) has three real roots if $a < -4$

D. f(x) has three real roots if $-4 < a < 4$

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12. The common tangents to the circle $x^2 + y^2 = 2$ and the parabola $y^2 = 8x$ touch the circle at the points P, Q and the parabola at the points R, S. Then the area of the quadrilateral PQSR is

A. 3

B. 6

C. 9

D. 15

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**13.** The slope of the tangent to the curve $\left(y - x^5\right)^2 = x\left(1 + x^2\right)^2$ at the point (1, 3) is

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**14.** Let, $f:(0, \infty) \rightarrow R$ be given by

$$f(x) = \int_{\frac{1}{x}}^{x} e^{-\left(t + \frac{1}{t}\right)} \frac{dt}{t}$$

Then

A. f(x) is monotonically increasing on $[1, \infty)$

B. f(x) is monotonically decreasing on (0, 1)

C. $fx + f\left(\frac{1}{x}\right) = 0, \quad \text{for all } x \in (0, \infty)$

D. $f\left(2^x\right)$ is an odd function of x on R

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**15.** The value of

$$\int_0^1 4x^3\left\{\frac{d^2}{dx^2}\left(1-x^2\right)^5\right\}dx$$

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**16.** The following integral $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}(2\ \text{cosec}\ x)^{17}dx$ is equal to

A. $\int_0^{\log\left(1+\sqrt{2}\right)} 2\left(e^u+e^{-u}\right)^{16}\text{du}$

B. $\int_0^{\log\left(1+\sqrt{2}\right)}\left(e^u-e^{-u}\right)^{17}du$

C. $\int_0^{\log\left(1+\sqrt{2}\right)}\left(e^u-e^{-u}\right)^{17}du$

D. $\int_0^{\log\left(1+\sqrt{2}\right)} 2\left(e^u-e^{-u}\right)^{17}du$

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**17.** $\int\frac{1}{1-\tan^2 x}dx$

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**18.** Given that for each $a \in (0, 1)$

$$\lim_{h \to 0^+} \int_h^{1-h} t^{-a}(1-t)^{a-1}dt$$

exists. Let, this limit be g(a). In addition, it is given that the function g(a) is differentiable on (0, 1).

The value of $g\left(\frac{1}{2}\right)$ is

A. $\pi$

B. $2\pi$

C. $\frac{\pi}{2}$

D. $\frac{\pi}{4}$

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**19.** Given that for each $a \in (0, 1)$

$$\lim_{h \to 0^+} \int_h^{1-h} t^{-a}(1-t)^{a-1}dt$$

exists. Let, this limit be g(a). In addition, it is given that the function g(a) is differentiable on (0, 1).

The value of $g'\left(\frac{1}{2}\right)$ is

A. $\frac{\pi}{2}$

B. $\pi$

C. $-\frac{\pi}{2}$

D. 0

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**20.** For a point P in the plane, let $d_1(P)$ and $d_2(P)$ be the distances of the point P from the lines $x - y = 0$ and $x + y = 0$ respectively. The area of the region R consisting of all points P lying in the first quadrant of the plane and satisfying $2 \leq d_1(P) + d_2(P) \leq 4$, is

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**21.** Let $f:[0, 2] \rightarrow \mathbb{R}$ be a function which is continuous on $[0, 2]$ and is differentiable on (0, 2) with $f(0) = 1$.

Let $F(x) = \int_0^{x^2} f(\sqrt{t})dt$ for $x \in [0, 2]$. If $F'(x) = f'(x)$ for all $x \in (0, 2)$, the F(2) equals

A. $e^2 - 1$

B. $e^4 - 1$

C. $e - 1$

D. $e^4$

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**22.** The function $y = f(x)$ is the solution of the differential equation

$$\frac{dy}{dx} + \frac{xy}{x^2 - 1} = \frac{x^4 + 2x}{\sqrt{1 - x^2}}$$

in (-1, 1) satisfying $f(0) = 0$.

Then $\int_{\frac{-\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} f(x)dx$ is

A. $\frac{\pi}{3} - \frac{\sqrt{3}}{2}$

B. $\frac{\pi}{3} - \frac{\sqrt{3}}{4}$

C. $\frac{\pi}{6} - \frac{\sqrt{3}}{4}$

D. $\frac{\pi}{6} - \frac{\sqrt{3}}{2}$

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## JEE Advanced Archive 2015

**1.** Let S be the set of the non zero real numbers $\alpha$ such that the quadratic equation $\alpha x^2 - x + \alpha = 0$ has two distinct real roots $x_1$ and $x_2$ satisfying the inequlity $|x_1 - x_2| < 1$. Which of the following intervals is (are) a subset (s) of s?

A. $\left(-\frac{1}{2}, -\frac{1}{\sqrt{5}}\right)$

B. $\left(-\frac{1}{\sqrt{5}}, 0\right)$

C. $\left(0, -\frac{1}{\sqrt{5}}\right)$

D. $\left(\frac{1}{\sqrt{5}}, \frac{1}{2}\right)$

**Answer: A::D**

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**2.** For any integer K , let $\alpha_k = \cos\left(\frac{k\pi}{7}\right) + i\sin\left(\frac{k\pi}{7}\right)$, where $i = \sqrt{-1}$ The value of the expression

$$\frac{\sum_{k=1}^{12}\left|\alpha_{k+1} - \alpha_k\right|}{\sum_{k=1}^{3}\left|\alpha_{4k-1} - \alpha_{4k-2}\right|}$$

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**3.** Suppose that all the terms of an arithmetic progression (A.P) are natural number .If the ratio of the sum of the first seven terms to the sum of the first eleven terms is 6 : 11 and the seventh term lies in between 130 and 140 , then the common differnce of A.P . is -

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**4.** The coefficient of $x^9$ in the expansion of $(1+x)\left(1+x^2\right)\left(1+x^3\right)\ldots.\left(1+x^{100}\right)$ is -

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**5.** Let , n be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that all the girls stand consecutively in the queue .Ley m be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that exaxtly four girls stand consescutively in the queue. Then the value of $\frac{m}{n}$ is -

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**6.** Suppose that the foci of the ellipse $\frac{x^2}{9} + \frac{y^2}{5} = -1$ are $(f_1, 0)$ and $(f_2, 0)$ where $f_1 > 0$ and $f_2 < 0$. Let $P_1$ and $P_2$ be two parabolas with a common vertex at (0, 0) and with foci at $(f_1, 0)$ and $(2f_2, 0)$ respectively. Let $T_1$ be a tangent to $P_1$ which passes through $(2f_2, 0)$ and $T_2$ be a tangent to $P_2$ which passes Through $(f_1, 0)$. If $m_1$ is the slope of $T_1$ and $m_2$ is the slope of $T_2$, then the value of $\left(\frac{1}{m_1^2} + m_2^2\right)$ is -

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**7.** Let m and n be two positive integers greater than 1. If

$$\lim_{\alpha \to 0}\left(\frac{e^{\cos\left(\alpha^n\right)} - e}{\alpha^m}\right) = -\left(\frac{e}{2}\right) \text{ then the value of } \frac{m}{n} \text{ is-}$$

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**8.** If $\alpha = \int_0^1 \left( e^{9x + 3\tan^{-1}x} \left( \frac{12 + 9x^2}{1 + x^2} \right) dx \right.$ where $\tan^{-1}x$ takes only principal values, then the value of $\left( \log_e|1 + \alpha| - \frac{3\pi}{4} \right)$ is-

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**9.** Let $f: \mathbb{R} \to \mathbb{R}$ be a continuous odd function, which vanishes exactly at one point and $f(1) = \frac{1}{2}$. Suppose that $F(x) = \int (-1)^x f(t) dt$ for all $x \in [-1, 2]$ and

$G(x) = \int_{-1}^{x} t|f(ft)|dt$ for all $x \in [-1, 2]$. If $\lim_{x \to 1} \frac{F(x)}{G(x)} = \frac{1}{14}$ then the value of $f\left(\frac{1}{2}\right)$ is-

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**10.** Consider the hyperbola $H: x^2 - y^2 = 1$ and a circle S with centre $N(x_2, 0)$. Suppose that H ans S touch each other at a point $P(x_1, y_1)$ with $x_1 > 1$ and $y_1 > 0$. The common tangent to H ans S at P intersects the x-axis at point M. If (l, m) is the centroid of the triangle PMN, then the correct expression(s) is (are)-

A. $\frac{dl}{dx_1} = 1 - \frac{1}{3x_1^2}$ for $x_1 > 1$

B. $\frac{dm}{dx_1} = \frac{x_1}{3\left(\sqrt{x_1^2 - 1}\right)}$ for $x_1 > 1$

C. $\frac{dl}{dx_1} = 1 + \frac{1}{3x_1^2}$ for $x_1 > 1$

D. $\frac{dm}{dy_1} = \frac{1}{3}$ for $y_1 > 0$

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**11.** Let $E_1$ and $E_2$ be two ellipses whose centres are at the origin. The major axes of $E_1$ and $E_2$ lie along the x-axis and the y-axis respectively. Let S be the circle $x^2 + (y - 1)^2 = 2$. The straight line $x + y = 3$ touches the curves $S, E_1$ and $E_2$ at P, Q and R respectively. Suppose that $PQ = PR = \frac{2\sqrt{2}}{3}$. If $e_1$ and $e_2$ are the eccentricities of $E_1$ and $E_2$ respectively, then the correct expression (s) is (are)-

A. $e_1^2 + e_2^2 = \frac{43}{40}$

B. $e_1 e_2 = \frac{\sqrt{7}}{2\sqrt{10}}$

C. $\left|e_1^2 - e_2^2\right| = \frac{5}{8}$

D. $e_1 e_2 = \frac{\sqrt{3}}{4}$

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**12.** The options(s) with the values of a and L that satisfy the following equation is(are)

$$\frac{\int_0^{4\pi} e^t\left(\sin^6 at + \cos^4 at\right)dt}{\int_0^{\pi} e^t\left(\sin^6 at + \cos^4 at\right)dt} = L?$$

A. $a = 2, L = \frac{e^{4\pi} - 1}{e^{\pi} - 1}$

B. $a = 2, L = \frac{e^{4\pi} + 1}{e^{\pi} + 1}$

C. $a = 4, L = \frac{e^{4\pi} - 1}{e^{\pi} - 1}$

D. $a = 4, L = \frac{e^{4\pi} + 1}{e^{\pi} + 1}$

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13. If the normals of the parabola $y^2 = 4x$ drawn at the end points of its latus rectum are tangents to the circle $(x - 3)^2 + (y + 2)^2 = r^2$, then the value of $r^2$ is -

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14. A cylindrical container is to be made from certain solid material with the following constraints :

It has a fixed inner volume of V $\text{mm}^3$, has a 2 mm thick solid wall andis open at the top. The bottom of the container is a solid circular disc of thickness 2 mm and is of radius equal to the outer radius of the container. If the volume of the material used to make the container is minimum when the inner radius of the container is 10 mm, then the value of $\dfrac{V}{250\pi}$ is-

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**15.** Let $F(x) = \int_x^{x^2 + \frac{\pi}{6}} 2\cos^2 t dt$ for all $x \in R$ and $f: \left[0, \frac{1}{2}\right] \to [0, \infty)$ be continuous function. For $a \in \left[0, \frac{1}{2}\right]$, if $F'(a) + 2$ is the area of the region bounded by $x = 0, y = 0, y = f(x)$ and x = a, then f(0) is-

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**16.** Let $f: \mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) = \begin{cases} [x] & x \leq 2 \\ 0 & x > 2 \end{cases}$, where [x] is the greatest integer less than or equal to x.

If $I = \int_{-1}^{2} \frac{xf(x^2)}{2 + f(x + 1)} dx$, then the value of $(4I - 1)$ is-

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**17.** Let $g: \mathbb{R} \to \mathbb{R}$ be a differentiable function with $g(0) = 0, g'(0) = 0$ and $g'(1) \neq 0$.

Let $f(x) = \begin{cases} \frac{x}{|x|}g(x), & x \neq 0 \\ 0, & x = 0 \end{cases}$ and $h(x) = e^{|x|}$ for all $x \in \mathbb{R}$.

Let $(\text{foh})(x)$ denote $f(h(x))$ and $(\text{hof})(x)$ denote $h(f(x))$. Then which of the following is (are) true?

A. f is differentiable at x = 0

B. h is differentiable at x = 0

C. foh is differentiable at x = 0

D. hof is differentiable at x = 0

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**18.** Let y(x) be a solution of the differential equation $\left(1 + e^x\right)y' + ye^x = 1$.

If $y(0) = 2$, then which of the following statements is (are) true?

A. $y(-4) = 0$

B. $y(-2) = 0$

C. y(x) has a critical point in the interval $(-1, 0)$

D. y(x) has no cirtical point in the interval $(-1, 0)$

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**19.** Consider the family of all circles whose centres lie on the straight line y = x. If this family of circles is represented by the differential equation $Py'' + Qy' + 1 = 0$, where P, Q are functions of x, y and y' (here $y' = \frac{dy}{dx}, y'' = \frac{d^2y}{dx^2}$), then which of the following statements is (are) true?

A. $P = y + x$

B. $P = y - x$

C. $P + Q = 1 - x + y + y' + (y')^2$

D. $P - Q = x + y - y' - (y')^2$

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**20.** The order and degree of $\left(\frac{d^2y}{dx^2}\right)^{\frac{1}{3}} = 10 + 9x\frac{dy}{dx}$ is:

A. 2,3

B. 2,1

C. 1,3

D. 1,1

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**21.** Let $f(x) = 7\tan^8 x + 7\tan^6 x - 3\tan^4 x - 3\tan^2 x$ for all $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Then the correct expression(s) is (are)-

A. $\int_0^{\frac{\pi}{4}} xf(x)dx = \frac{1}{12}$

B. $\int_0^{\frac{\pi}{4}} f(x)dx = 0$

C. $\int_0^{\frac{\pi}{4}} xf(x)dx = \frac{1}{6}$

D. $\int_0^{\frac{\pi}{4}} f(x)dx = 1$

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**22.** Let $f'(x) = \dfrac{192x^3}{2 + \sin^4 \pi x}$ for all $x \in R$ with $f\left(\dfrac{1}{2}\right) = 0$. If $m \leq \int_{\frac{1}{2}}^{1} f(x)dx \leq M$, then the possible values of m and M are-

A. m = 13, M = 24

B. $m = \dfrac{1}{4}, M = \dfrac{1}{2}$

C. $m = -11, M = 0$

D. m = 1, M = 12

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**23.** Let $F : \mathbb{R} \rightarrow \mathbb{R}$ be a thrice differentiable function. Suppose that $F(1) = 0, F(3) = -4$ and $F'(x) < 0$ for all $x \in \left(\frac{1}{2}, 3\right)$. Let $f(x) = xF(x)$ for all $x \in \mathbb{R}$.

The correct statement(s) is (are)-

A. $f'(1) < 0$

B. $f(2) < 0$

C. $f'(x) \neq 0$ for any $x \in (1, 3)$

D. $f'(x) = 0$ for some $x \in (1, 3)$

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**24.** Let $F : \mathbb{R} \rightarrow \mathbb{R}$ be a thrice differentiable function. Suppose that $F(1) = 0, F(3) = -4$ and $F'(x) < 0$ for all $x \in (1, 3)$. Let $f(x) = xF(x)$ for all $x \in \mathbb{R}$.

If $\int_1^3 x^2 F'(x)dx = -12$ and $\int_1^3 x^3 F''(x)dx = 40$, then the correct expression(s) is(are)-

A. $9f'(3) + f'(1) - 32 = 0$

B. $\int_1^3 f(x)dx = 12$

C. $9f'(3) - f'(1) + 32 = 0$

D. $\int_1^3 f(x)dx = -12$

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## JEE Advanced Archive 2016

**1.** A debate club consists of 6 girls and 4 boys A.term of 4 members is to be selected from this club including the selection of a caption (from among these 4 members ) for the term .If the term has to include at most one boy , then the number of ways of selecting the term is

A. 380

B. 320

C. 260

D. 95

**Answer: A**

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**2.** Let $-\frac{\pi}{6} < \theta < -\frac{\pi}{12}$ Suppose $\alpha_1$ and $\beta_1$ are the roots of the equation $x^2 - 2x\sec\theta + 1 = 0$ and $\alpha_2$ and $\beta_2$ are the roots of the equation $x^2 + 2x\tan\theta - 1$. if $\alpha_1 > \beta_1$ and $\alpha_2 > \beta_2$, then $\alpha_1 + \beta_2$ equals

A. $2(\sec\theta - \tan\theta)$

B. $2\sec\theta$

C. $-2\tan\theta$

D. 0

**Answer: C**

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**3.** The least value of $a \in \mathbb{R}$ for which $4ax^2 + \frac{1}{x} \geq 1$ for all $x > 0$, is

A. $\frac{1}{64}$

B. $\frac{1}{32}$

C. $\frac{1}{27}$

D. $\frac{1}{25}$

**Answer: C**

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**4.** Let , m be the the smallest positive interger such that the coefficient of $x^2$ in the expansion of $(1+x)^2 + (1+x)^3 + .... + (1+x)^{49} + (1+mx)^{50}$ is $(3n+1)^{51}C_3$ for some positive integer n . Then the value of n is ,

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**5.** Let $b_i > 1$ for $i = 1, 2, ...., 101$. Suppose $\log_e b_1, \log_e b_2, ...., \log_e b_{101}$ are in arithmetic progression (AP), with thecommon differnce $\log_e 2$ .Suppose $a_1, a_2 ....., a_{101}$ are in AP such that $a_1 = b_1$ and $a_{51} = b_{51}$. if $t = b_1 + b_2 + .... + b_{51}$ and $s = a_1 + a_2 + .... + a_{51}$, then

A. $s > t \text{and} a_{101} > b_{101}$

B. $s > t \text{and} a_{101} < b_{101}$

C. $s < t \text{and} a_{101} > b_{101}$

D. $s < t \text{and} a_{101} < b_{101}$

**Answer: B**

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**6.** Let $f: (0, \infty) \rightarrow \mathbb{R}$ be a differentiable function such that $f'(x) = 2 - \frac{f(x)}{x}$ for all $x \in (0, \infty)$ and $f(1) \neq 1$. Then-

A. $\lim_{x \to 0^+} f'\left(\frac{1}{x}\right) = 1$

B. $\lim_{x \to 0^+} xf\left(\frac{1}{x}\right) = 2$

C. $\lim_{x \to 0^+} x^2 f'(x) = 0$

D. $|f(x)| \leq 2$ for all $x \in (0, 2)$

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7. A solution curve of the differential equation $\left(x^2 + xy + 4x + 2y + 4\right)\frac{dy}{dx} - y^2 = 0, x > 0$, passes through the point (1, 3). Then the solution curve-

A. intersects $y = x + 2$ exactly at one point

B. intersects $y = x + 2$ exactly at two points

C. intersects $y = (x + 2)^2$

D. does not intersect $y = (x + 3)^2$

Watch Video Solution

**8.** The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x^2 \cos x}{1 + e^x} dx$ is

A. $\frac{\pi^2}{4} - 2$

B. $\frac{\pi^2}{4} + 2$

C. $\pi^2 - e^{-\frac{\pi}{2}}$

D. $\pi^2 + e^{\frac{\pi}{2}}$

Watch Video Solution

**9.** Area of the region

$\left\{(x, y) \in \mathbb{R} : y \geq \sqrt{|x + 3|}, 5y \leq (x + 9) \leq 15\right\}$ is equal to

A. $\frac{1}{6}$

B. $\frac{4}{3}$

C. $\frac{3}{2}$

D. $\frac{5}{3}$

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10. Let $f(x) = \lim_{n \to \infty} \left[ \frac{n^n (x+n)\left(x + \frac{n}{2}\right) \dots \left(x + \frac{n}{n}\right)}{n!\left(x^2 + n^2\right)\left(x^2 + \frac{n^2}{4}\right) \dots \left(x^2 + \frac{n^2}{n^2}\right)} \right]^{\frac{x}{n}}$ for all $x > 0$

. Then,

A. $f\left(\frac{1}{2}\right) \geq f(1)$

B. $f\left(\frac{1}{3}\right) \leq f\left(\frac{2}{3}\right)$

C. $f'(2) \leq 0$

D. $\frac{f'(3)}{f(3)} \geq \frac{f'(2)}{f(2)}$

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**11.** Let $f : \mathbb{R} \to (0, \infty)$ and $g : \mathbb{R} \to \mathbb{R}$ be twice differentiable functions such that f'' and g'' are continuous functions on $\mathbb{R}$. Suppose $f'(2) = g(2) = 0, f''(2) \neq 0$ and $g'(2) \neq 0$. If $\lim_{x \to 2} \frac{f(x)g(x)}{f'(x)g'(x)} = 1$. Then

A. f has a local minimum at x = 2

B. f has a local maximum at x = 2

C. $f''(2) > f(2)$

D. $f(x) - f''(x) = 0$ for at least one $x \in \mathbb{R}$

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**12.** Let P be the point on the parabola $y^2 = 4x$ which is at the shortest distance from the centre S of the circle $x^2 + y^2 - 4x - 16y + 64 = 0$. Let Q be the point on the circle dividing the line segment SP internally. Then

A. $SP = 2\sqrt{5}$

B. $SQ : QP = (\sqrt{5} + 1) : 2$

C. The x-intercept of the normal to the parabola at P is 6.

D. The slpe of the tangent to the circle at Q is $\frac{1}{2}$.

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**13.** Let $F_1(x_1, 0)$ and $F_2(x_2, 0)$, for $x_1 < 0$ and $x_2 > 0$, be the foci of the ellipse $\frac{x^2}{9} + \frac{y^2}{8} = 1$. Suppose a parabola having vertex at the orgin and focus at $F_2$ intersects the ellipse at point M in the first quadrant and at point N in the fourth quadrant.

The orthocentre of the triangle $F_1MN$ is

A. $\left(-\frac{9}{10}, 0\right)$

B. $\left(\frac{2}{3}, 0\right)$

C. $\left(\frac{9}{10}, 0\right)$

D. $\left(\frac{2}{3}, \sqrt{6}\right)$

Watch Video Solution

**14.** Let $F_1(x_1, 0)$ and $F_2(x_2, 0)$, for $x_1 < 0$ and $x_2 > 0$, be the foci of the ellipse $\frac{x^2}{9} + \frac{y^2}{8} = 1$. Suppose a parabola having vertex at the orgin and focus at $F_2$ intersects the ellipse at point M in the first quadrant and at point N in the fourth quadrant.

If the tangents to the ellipse at M and N meet at R and the normal to the parabola at M meets the x-axis at Q, then the ratio of area of the triangle MQR to area of the quadrilateral $MF_1NF_2$ is

A. $3:4$

B. $4:5$

C. $5:8$

D. $2:3$

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**15.** Let $f:\mathbb{R} \to \mathbb{R}, g:\mathbb{R} \to \mathbb{R}$ and $h:\mathbb{R} \to \mathbb{R}$ be differentiable functions such that $f(x) = x^3 + 3x + 2, g(f(x)) = x$ and $h(g(g(x))) = x$ for all $x \in \mathbb{R}$. Then,

A. $g'(2) = \frac{1}{15}$

B. $h'(1) = 666$

C. $h(0) = 16$

D. $h(g(3)) = 36$

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**16.** Let $a, b \in \mathbb{R}$ and $f:\mathbb{R} \to \mathbb{R}$ be defined $f(x) = a\cos\left(\left|x^3 - x\right|\right) + b|x|\sin\left(\left|x^3 + x\right|\right)$. Then f is

A. differentiable at x = 0, if a = 0 and b = 1

B. differentiable at x = 1, if a = 1 and b = 0

C. not differentiable at x = 0 if a = 1 and b = 0

D. not differentiable at x = 1, if a = 1 and b = 1

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**17.** Let $f_i\left[-\frac{1}{2}, 2\right] \rightarrow \mathbb{R}$ and $g:\left[-\frac{1}{2}, 2\right] \rightarrow \mathbb{R}$ be functions defined by $f(x) = \left[x^2 - 3\right]$ and $g(x) = |x|f(x) + |4x - 7|f(x)$ where [y] denotes the greatest integer less then or equal to y for $y \in \mathbb{R}$. Then,

A. f is discontinuous exactly at three points in $\left[-\frac{1}{2}, 2\right]$

B. f is discontinuous exactly at four points in $\left[-\frac{1}{2}, 2\right]$

C. g is not differentiable exactly at four points in $\left(-\frac{1}{2}, 2\right)$

D. g is not differentiable exactly at five points in $\left(-\frac{1}{2}, 2\right)$

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## WBJEE Archive 2012

**1.** The general solution of the differential equation

$\frac{dy}{dx} = \frac{x + y + 1}{2x + 2y + 1}$ is -

A. $\log_e|3x + 3y + 2| + 3x + 6y = c$

B. $\log_e|3x + 3y + 2| - 3x + 6y = c$

C. $\log_e|3x + 3y + 2| - 3x - 6y = c$

D. $\log_e|3x + 3y + 2| + 3x - 6y = c$

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**2.** The value of the integral $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left(\frac{1 + \sin 2x + \cos 2x}{\sin x + \cos x}\right) dx$ is equal to-

A. 16

B. 8

C. 4

D. 1

Watch Video Solution

**3.** The vaue of the integral $\int_0^{\frac{\pi}{2}} \frac{1}{1+(\tan x)^{101}} dx$ is equal to -

A. 1

B. $\frac{\pi}{6}$

C. $\frac{\pi}{8}$

D. $\frac{\pi}{4}$

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4. The integrating factor of the differential equation $3x\log_e x\frac{dy}{dx} + y = 2\log_e x$ is given by -

A. $\left(\log_e x\right)^3$

B. $\log_e\left(\log_e x\right)$

C. $\log_e x$

D. $\left(\log_e x\right)^{\frac{1}{3}}$

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5. The value of the integral $\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{3 + \sin 2x} dx$ is equal to-

A. $\log_e 2$

B. $\log_e 3$

C. $\frac{1}{4}\log_e 2$

D. $\frac{1}{4}\log_e 3$

Watch Video Solution

**6.** Let $y = \left(\frac{3^x - 1}{3^x + 1}\right)\sin x + \log_e(1 + x), x > -1$, then at x = 0, $\frac{dy}{dx}$ equals-

A. 1

B. 0

C. $-1$

D. $-2$

Watch Video Solution

**7.** Maximum value of function $f(x) = \frac{x}{8} + \frac{2}{x}$ on the interval [1, 6] is-

A. 1

B. $\frac{9}{8}$

C. $\frac{13}{12}$

D. $\frac{17}{8}$

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**8.** For $-\frac{\pi}{2} < x < \frac{3\pi}{2}$, the value of $\frac{d}{dx}\left\{\tan^{-1}\frac{\cos x}{1+\sin x}\right\}$ is equal to-

A. $\frac{1}{2}$

B. $-\frac{1}{2}$

C. 1

D. $\frac{\sin x}{(1+\sin x)^2}$

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**9.** The value of the integral $\int_{-}(1+2\sin x)e^x dx$ is equal to-

A. 0

B. $e^2 - 1$

C. $2\left(e^2 - 1\right)$

D. 1

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**10.** The sum of the series

$1 + \frac{1}{2}{}^nC_1 + \frac{1}{3}{}^nC_2 + \ldots + \frac{1}{n+1}{}^nC_n$ is equal to-

A. $\frac{2^{n+1} - 1}{n+1}$

B. $\frac{3\left(2^n - 1\right)}{2n}$

C. $\frac{2^n + 1}{n+1}$

D. $\frac{2^n + 1}{2n}$

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**11.** If f is a real-valued differentiable function such that $f(x)f'(x) < 0$ for all real x, then -

A. f(x) must be an increasing function

B. f(x) must be a decreasing function

C. $|f(x)|$ must be an increasing function

D. $|f(x)|$ must be a decreasing function

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**12.** Rolle's theorem is applicable in the interval $[-2, 2]$ for the function-

A. $f(x) = x^3$

B. $f(x) = 4x^4$

C. $f(x) = 2x^3 + 3$

D. $f(x) = \pi|x|$

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13. The value of $\lim_{n \to \infty} \frac{(n!)^{\frac{1}{n}}}{n}$ is -

A. 1

B. $\frac{1}{e^2}$

C. $\frac{1}{2e}$

D. $\frac{1}{e}$

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14. The area of the region bounded by the curves $y = x^3, y = \frac{1}{x}$ between $x = 1$ to $x = 2$ is -

A. $4 - \log_e 2$

B. $\frac{1}{4} + \log_e 2$

C. $-\log_e 2$

D. $\frac{15}{4} - \log_e 2$

**Answer: D**

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**15.** Let y be the solution of the differential equation

$x\frac{dy}{dx} = \frac{y^2}{1 - y\log x}$ satisfying $y(1) = 1$. Then y satisfies -

A. $y = x^{y-1}$

B. $y = x^y$

C. $y = x^{y+1}$

D. $y = x^{y+2}$

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**16.** The area of the region, bounded by the curves $y = \sin^{-1}x + x(1 - x)$ and $y = \sin^{-1}x - x(1 - x)$ in the first quadrant is-

A. 1

B. $\frac{1}{2}$

C. $\frac{1}{3}$

D. $\frac{1}{4}$

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**17.** The value of the integral $\int_1^5 [|x - 3| + |1 - x|]dx$ is equal to-

A. 4

B. 8

C. 12

D. 16

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**18.** If f(x) and g(x) are twice differentiable functions on (0, 3) satisfying $f''(x) = g''(x), f'(1) = 4, g'(1) = 6, f(2) = 3, g(2) = 9,$ then $f(1) - g(1)$ is -

A. 4

B. -4

C. 0

D. -2

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**19.** Let [x] denote the greatest integer less than or equal to x, then the value of the integral $\int_{-1}^{1}(|x| - 2[x])dx$ is equal to-

A. 3

B. 2

C. $-2$

D. $-3$

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**20.** $\lim_{x \to 0} \frac{\pi^x - 1}{\sqrt{1 + x} - 1}$

A. does not exist

B. equals $\log_e(\pi^2)$

C. equals 1

D. lies between 10 and 11

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**1.** The value of the integral $\int_{-1}^{+1}\left\{\frac{x^{2013}}{e^{|x|}\left(x^2+\cos x\right)}+\frac{1}{e^{|x|}}\right\}dx$

A. 0

B. $1-e^{-1}$

C. $2e^{-1}$

D. $2\left(1-e^{-1}\right)$

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**2.** For the curve $x^2+4xy+8y^2=64$, the tangents are parallel to the x-axis only at the points-

A. $\left(0, 2\sqrt{2}\right)$ and $\left(0, -2\sqrt{2}\right)$

B. $(8, -4)$ and $(-8, 4)$

C. $\left(8\sqrt{2},\ -2\sqrt{2}\right)$ and $\left(-8\sqrt{2}, 2\sqrt{2}\right)$

D. $(8, 0)$ and $(-8, 0)$

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**3.** The value of $I = \int_0^{\frac{\pi}{4}}\left(\tan^{n+1}x\right)dx + \frac{1}{2}\int_0^{\frac{\pi}{2}}\tan^{n-1}\left(\frac{x}{2}\right)dx$ is equal to-

A. $\frac{1}{n}$

B. $\frac{n+2}{2n+1}$

C. $\frac{2n-1}{n}$

D. $\frac{2n-3}{3n-2}$

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**4.** Let $f(x) = \begin{cases} x^3 - 3x + 2 & \text{where } x < 2 \\ x^3 - 6x^2 + 9x + 2 & \text{where } x \geq 2 \end{cases}$. Then-

A. $f(x)_{x \to 2}$ does not exist

B. f is not continuous at x = 2

C. f is continuous but not differentiable at x = 2

D. f is continous and differentiable at x = 2

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**5.** The limit of $\sum^{1000} n = 1( - 1)^n x^n$ as $x \to \infty$

A. does not exist

B. exists and equals to 0

C. exists and approaches to $+\infty$

D. exists and approaches to $-\infty$

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**6.** If $f(x) = e^x(x - 2)^2$ then-

A. f is increasing in $( - \infty, 0)$ and $(2, \infty)$ and decreasing in (0, 2)

B. f is increasing in $( - \infty, 0)$ and decreasing in $(0, \infty)$

C. f is increasing in $(2, \infty)$ and decreasing in $( - \infty, 0)$

D. f is increasing in $(0, 2)$ and decreasing in $( - \infty, 0)$ and $(2, \infty)$

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**7.** The area of the region bounded by the parabola $y = x^2 - 4x + 5$ and the straight line $y = x + 1$ is-

A. $\frac{1}{2}$

B. 2

C. 3

D. $\frac{9}{2}$

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**8.** The value of the integral $\int_1^2 e^x \left(\log_e x + \frac{x+1}{x}\right)dx$ is-

A. $e^2\left(1+\log_e 2\right)$

B. $e^2 - e$

C. $e^2\left(1+\log_e 2\right) - e$

D. $e^2 - e\left(1+\log_e 2\right)$

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**9.** Let $f(x) = \sin x + 2\cos^2 x, \frac{\pi}{4} \le x \le \frac{3\pi}{4}$. Then f attains its-

A. minimum at $x = \frac{\pi}{4}$

B. maximum at $x = \frac{\pi}{2}$

C. minimum at $x = \frac{\pi}{2}$

D. maximum at $x = \sin^{-1}\left(\frac{1}{4}\right)$

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**10.** Let exp(x) denotes the exponential function $e^x$. If $f(x) = \exp\left(x^{\frac{1}{x}}\right), x > 0$,

then the minimum value of f in the interval [2, 5] is-

A. $\exp\left(e^{\frac{1}{e}}\right)$

B. $\exp\left(2^{\frac{1}{2}}\right)$

C. $\exp\left(5^{\frac{1}{5}}\right)$

D. $\exp\left(3^{\frac{1}{3}}\right)$

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**11.** The minimum value of the function $f(x) = 2|x - 1| + |x - 2|$ is -

A. 0

B. 1

C. 2

D. 3

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**12.** Let [a] denote the greatest integer which is less than of equal to a.

Then the value of the integral

$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} [\sin x \cos x] dx$ is-

A. $\frac{\pi}{2}$

B. $\pi$

C. $-\pi$

D. $-\frac{\pi}{2}$

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**13.** The value of the integral $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{(\sin x - x\cos x)}{x(x + \sin x)} dx$ is equal to-

A. $\log_e\left(\frac{2(\pi + 3)}{2\pi + 3\sqrt{3}}\right)$

B. $\log_e\left(\frac{\pi + 3}{2\left(2\pi + 3\sqrt{3}\right)}\right)$

C. $\log_e\left(\frac{2\pi + 3\sqrt{3}}{2(\pi + 3)}\right)$

D. $\log_e\left(\frac{2\left(2\pi + 3\sqrt{3}\right)}{\pi + 3}\right)$

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**14.** Let $F(x) = \int_0^x \frac{\cos t}{\left(1 + t^2\right)} dt, 0 \leq x \leq 2\pi$. Then -

A. F is increasing in $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ and decreasing in $\left(0, \frac{\pi}{2}\right)$ and $\left(\frac{3\pi}{2}, 2\pi\right)$

B. F is increasing in $(0, \pi)$ and decreasing in $(\pi, 2\pi)$

C. F is increasing in $(\pi, 2\pi)$ and decreasing in $(0, \pi)$

D. F is increasing in $\left(0, \frac{\pi}{2}\right)$ and $\left(\frac{3\pi}{2}, 2\pi\right)$ and decreasing in $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

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**15.** A family of curves is such that the length intercepted on the y-axis between the origin and the tangent at a point is three times the ordinate of the point of contact. The family of curves is-

A. xy = c, c is a constant

B. $xy^2 = c$, c is a constant

C. $x^2y = c$, $c$ is a constant

D. $x^2y^2 = c$, $c$ is a constant

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**16.** The solution of the diferential equation $\left(y^2 + 2x\right)\frac{dy}{dx} = y$ satisfies $x = 1, y = 1$. Then the solution is -

A. $x = y^2\left(1 + \log_e y\right)$

B. $y = x^2\left(1 + \log_e x\right)$

C. $x = y^2\left(1 - \log_e y\right)$

D. $y = x^2\left(1 - \log_e x\right)$

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**17.** The solution of the differential equation $y\sin\left(\frac{x}{y}\right)dx = \left(x\sin\left(\frac{x}{y}\right) - y\right)dy$ satisfying $y\left(\frac{\pi}{4}\right) = 1$ is-

A. $\cos\frac{x}{y} = -\log_e y + \frac{1}{\sqrt{2}}$

B. $\sin\frac{x}{y} = \log_e y + \frac{1}{\sqrt{2}}$

C. $\sin\frac{x}{y} = \log_e x - \frac{1}{\sqrt{2}}$

D. $\cos\frac{x}{y} = -\log_e x - \frac{1}{\sqrt{2}}$

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**18.** The area of the region enclosed between the parabola $y^2 = x$ and line $y = mx$ is $\frac{1}{48}$. Then the value of m is -

A. - 2

B. - 1

C. 1

D. 2

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**19.** $\frac{1}{1 \times 2} {}^{25}C_0 + \frac{1}{2 \times 3} {}^{25}C_1 + \frac{1}{3 \times 4} {}^{25}C_2 + \ldots + \frac{1}{26 \times 27} {}^{25}C_{25}$ is -

A. $\frac{2^{27} - 1}{26 \times 27}$

B. $\frac{2^{27} - 28}{26 \times 27}$

C. $\frac{1}{2}\left(\frac{2^{26} + 1}{26 \times 27}\right)$

D. $\frac{2^{26} - 1}{52}$

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**20.** The limit of $\left[\dfrac{1}{x^2} + \dfrac{(2013)^x}{e^x - 1} - \dfrac{1}{e^x - 1}\right]$ as $x \rightarrow 0$

A. approaches $+\infty$

B. approaches $-\infty$

C. is equal to $\log_e(2013)$

D. does not exist

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## WBJEE Archive 2014

**1.** The function $f(x) = \dfrac{\tan\left\{\pi\left[x - \frac{\pi}{2}\right]\right\}}{2 + [x]^2}$ where [x] denotes the greatest integer $< x$, is-

A. continuous for all values of x

B. discontinuous at $x = \dfrac{\pi}{2}$

C. not differentiable for some values of x

D. discontinuous at x = -2

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**2.** The area of the region bounded by the curves $y = x^2$ and $x = y^2$ is-

A. $\dfrac{1}{3}$

B. $\dfrac{1}{2}$

C. $\dfrac{1}{4}$

D. 3

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**3.** Let f(x) be a differentiable function in $[2, 7]$. If $f(2) = 3$ and $f'(x) \leq 5$ for all x in (2, 7), then the maximum possible value of f(x) at x = 7 is-

A. 7

B. 15

C. 28

D. 14

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**4.** Let f(x) be a differentiable function and $f'(4) = 5$. Then

$$\lim_{x \to 2} \frac{f(4) - f(x^2)}{2(x - 2)}$$ equals-

A. 0

B. 5

C. 20

D. -20

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**5.** The value of $\lim_{x \to 0} \frac{\int_0^{x^2} \cos(t^2) dt}{x \sin 2x}$

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**6.** If $f(x) = \begin{cases} 2x^2 + 1 & \text{where } x < 1 \\ 4x^3 + 1 & \text{where } x > 1 \end{cases}$, then $\int_0^2 f(x)dx$ is-

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**7.** If $I = \int_0^2 e^{x^4}(x - \alpha)dx = 0$, then $\alpha$ lies in the interval-

A. $(0, 2)$

B. $(-1, 0)$

C. $(2, 3)$

D. $(-2, -1)$

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**8.** Suppose that f(x) is a differentiable function such that f'(x) is continuous, $f'(0) = 1$ and $f''(0)$ does not exist. Let $g(x) = xf'(x)$. Then-

A. g'(0) does not exist

B. $g'(0) = 0$

C. $g'(0) = 1$

D. $g'(0) = 2$

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**9.** Let [x] denote the greatest integer less than or equal to x for any real number x. Then $\lim_{n \to \infty} \frac{[n\sqrt{2}]}{n}$ is equal to-

A. 0

B. 2

C. $\sqrt{2}$

D. 1

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**10.** If $\sqrt{y} = \cos^{-1}x$, then it satisfies the differential equation $\left(1 - x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = c$, where c is equal to-

A. 0

B. 3

C. 1

D. 2

Watch Video Solution

**11.** The integrating factor of the differential equaion $\left(1 + x^2\right)\frac{dy}{dx} + y = e^{\tan^{-1}x}$ is-

A. $\tan^{-1}x$

B. $1 + x^2$

C. $e^{\tan^{-1}x}$

D. $\log_e\left(1+x^2\right)$

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**12.** The solution of the differential equation $y\dfrac{dy}{dx}=x\left[\dfrac{y^2}{x^2}+\dfrac{\phi\left(\dfrac{y^2}{x^2}\right)}{\phi'\left(\dfrac{y^2}{x^2}\right)}\right]$ is

(where c is a constant)-

A. $\phi\left(\dfrac{y^2}{x^2}\right)=cx$

B. $x\phi\left(\dfrac{y^2}{x^2}\right)=c$

C. $\phi\left(\dfrac{y^2}{x^2}\right)=cx^2$

D. $x^2\phi\left(\dfrac{y^2}{x^2}\right)=c$

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**13.** A particle starting from a point A and moving with a positive constant acceleration along a straight line reaches another point B is time T. Suppose that the initial velocity of the particle is $u > 0$ and P is the midpoint of the line AB. If the velocity of the particle at point P is $v_1$ and if the velocity at time $\frac{T}{2}$ is $v_2$, then -

A. $v_1 = v_2$

B. $v_1 > v_2$

C. $v_1 < v_2$

D. $v_1 = \frac{1}{2}v_2$

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**14.** The curve $y = (\cos x + y)^{\frac{1}{2}}$ satisfies the differential equation-

A. $(2y - 1)\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 + \cos x = 0$

B. $\frac{d^2y}{dx^2} - 2y\left(\frac{dy}{dx}\right)^2 + \cos x = 0$

C. $(2y - 1)\frac{d^2y}{dx^2} - 2\left(\frac{dy}{dx}\right)^2 + \cos x = 0$

D. $(2y - 1)\frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^2 + \cos x = 0$

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**15.** Let R be the set of all real numbers and $f:[-1, 1] \rightarrow R$ be defined by

$f(x) = \begin{cases} x\sin\frac{1}{x} & \text{where } x \neq 0 \\ 0 & \text{where } x = 0 \end{cases}$, then-

A. f satisfies the conditions of Rolle's theorem of $[-1, 1]$

B. f satisfies the conditions of Lagrange's mean value theorem on $[-1, 1]$

C. f satisfies the conditions of Rolle's theorem on $[0, 1]$

D. f satisfies the conditions of Lagrange's mean value theorem on [0, 1]

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**16.** Suppose $M = \int_0^{\frac{\pi}{2}} \frac{\cos x}{x+2} dx, N = \int_0^{\frac{\pi}{4}} \frac{\sin x \cos x}{(x+1)^2} dx$. Then the value of (M - N) equals-

A. $\frac{3}{\pi + 2}$

B. $\frac{2}{\pi - 4}$

C. $\frac{4}{\pi - 2}$

D. $\frac{2}{\pi + 4}$

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**17.** The solution of the differention $\frac{dy}{dx} + \frac{y}{x\log_e x} = \frac{1}{x}$ under the condition y = 1 when x = e is-

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**18.** Let $f(x) = \max\{x + |x|, x - [x]\}$, where [x] denotes the greatest integer $\leq x$. Then the value of $\int_{-3}^{3} f(x)dx$ is-

A. 0

B. $\frac{51}{2}$

C. $\frac{21}{2}$

D. 1

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**19.** Applying Lagrange's mean value theorem for a suitable function f(x) in [0, h], we have $f(h) = f(0) + hf'(\theta h), 0 < \theta < 1$. Then for $f(x) = \cos x$, the value of $\lim_{h \to 0+} \theta$ is-

A. 1

B. 0

C. $\frac{1}{2}$

D. $\frac{1}{3}$

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**20.** Let $f(x) = \begin{cases} \int_0^x |1 - t| dt & \text{where } x > 1 \\ x - \frac{1}{2} & \text{where } x \le 1 \end{cases}$, then

A. f(x) is continuous at x = 1

B. f(x) is not continuous at x = 1

C. f(x) is differentiable at x = 1

D. f(x) is not differentiable at x = 1

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**21.** Suppose that the equation $f(x) = x^2 + bx + c = 0$ has two distinct real roots $\alpha$ and $\beta$. The angle between the tangent to the curve $y = f(x)$ at the point $\left(\frac{\alpha+\beta}{2}, f\left(\frac{\alpha+\beta}{2}\right)\right)$ and the positive direction of the x -axis is-

A. $0^{\circ}$

B. $30^{\circ}$

C. $60^{\circ}$

D. $90^{\circ}$

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**22.** The equation of the common tangent with positive slope to the parabola $y^2 = 8\sqrt{3}x$ and the hyperbola $4x^2 - y^2 = 4$ is-

A. $y = \sqrt{6}x + \sqrt{2}$

B. $y = \sqrt{6}x - \sqrt{2}$

C. $y = \sqrt{3}x + \sqrt{2}$

D. $y = \sqrt{3}x - \sqrt{2}$

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**23.** The point on the parabola $y^2 = 64x$ which is nearest to the line $4x + 3y + 35 = 0$ has coordinates-

A. $(9, -24)$

B. $(1, 81)$

C. $(4, -16)$

D. $(-9, -24)$

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**24.** The angle of intersection between the curves $y = [|\sin x| + |\cos x|]$ and $x^2 + y^2 = 10$, where [x] denotes the greatest integer $< x$, is-

A. $\tan^{-1}3$

B. $\tan^{-1}(-3)$

C. $\tan^{-1}\sqrt{3}$

D. $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

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**25.** If $y = 4x + 3$ is parallel to a tangent to the parabola $y^2 = 12x$, then its distance from the normal parallel to the given ine is-

A. $\frac{213}{\sqrt{17}}$

B. $\frac{219}{\sqrt{17}}$

C. $\frac{211}{\sqrt{17}}$

D. $\frac{210}{\sqrt{17}}$

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**26.** If u(x) and v(x) are two independent solutions of the differential equation $\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$, then additional solution(s) of the given differential equation is (are)-

A. $y = 5u(x) + 8v(x)$

B. $y = c_1[u(x) - v(x)\} + c_2v(x), c_1$ and $c_2$ are arbitrary constants

C. $y = c_1u(x)v(x) + c_2\frac{u(x)}{v(x)}, c_1$ and $c_2$ are arbitrary constants

D. $y = u(x)v(x)$

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**27.** Let $S = \frac{2}{1}{}^nC_0 + \frac{2^2}{2}{}^nC_1 + \frac{2}{3^3}{}^nC_2 + \ldots + \frac{2^{n+1}}{n+1}{}^nC_n$. Then S equals-

A. $\frac{2^{n+1} - 1}{n+1}$

B. $\frac{3^{n+1} - 1}{n+1}$

C. $\frac{3^n - 1}{n}$

D. $\frac{2^{n-1}}{n}$

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**28.** The function $f(x) = a\sin|x| + be^{|x|}$ is differentiable at x = 0 when -

A. $3a + b = 0$

B. $3a - b = 0$

C. $a + b = 0$

D. $a - b = 0$

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## WBJEE Archive 2015

**1.** Value of $\lim_{x \to 2} \int_2^x \frac{3t^2}{x - 2} dt$-

A. 10

B. 12

C. 8

D. 16

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**2.** If $\log_{0.2}(x - 1) > \log_{0.04}(x + 5)$, then-

A. $-1 < x < 4$

B. $2 < x < 3$

C. $1 < x < 4$

D. $1 < x < 3$

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**3.** Let $f: R \rightarrow R$ be defined as

$$f(x) = \begin{cases} 0, \text{ x is irrational} \\ \sin|x|, \text{ x is rational} \end{cases}$$

Then which of the following is true?

A. f is discontinuous for all x

B. f is continuous for all x

C. f is discontinuous at $x = k\pi$, where k is an integer

D. f is continuous at $x = k\pi$, where k is an integer.

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**4.** Let $f:[-2, 2] \rightarrow R$ be a continuous function such that f(x) assumes only irrational values. If $f\left(\sqrt{2}\right) = \sqrt{2}$ then-

A. $f(0) = 0$

B. $f\left(\sqrt{2} - 1\right) = \sqrt{2} - 1$

C. $f\left(\sqrt{2} - 1\right) = \sqrt{2} + 1$

D. $f\left(\sqrt{2} - 1\right) = \sqrt{2}$

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**5.** $\lim_{n \rightarrow \infty} \dfrac{\sqrt{1} + \sqrt{2} + \ldots + \sqrt{n - 1}}{n\sqrt{n}} =$

A. $\frac{1}{2}$

B. $\frac{1}{3}$

C. $\frac{2}{3}$

D. 0 (zero)

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**6.** If $\lim_{x \to 0} \frac{axe^x - b\log(1 + x)}{x^2} = 3$ then values of a and b respectively-

A. 2, 2

B. 1, 2

C. 2, 1

D. 2, 0

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**7.** The area of the region bounded by $y = |x|$ and $y = -|x| + 2$ is-

A. 4 sq. units

B. 3 sq. units

C. 2 sq. units

D. 1 sq. units

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**8.** Let P(x) be a polynomial, which when divided by $x - 3$ and $x - 5$ leaves remainders 10 and 6 respectively. If the polynomial is divided by $(x - 3)(x - 5)$ then the remainder is-

A. $-2x + 16$

B. 16

C. $2x - 16$

D. 60

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9. $\frac{dy}{dx} + \left(3x^2\tan^{-1}y - x^3\right)\left(1 + y^2\right) = 0$

The differential equation has integrating factor-

A. $e^{x^2}$

B. $e^{x^3}$

C. $e^{3x^2}$

D. $3^{3x^3}$

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**10.** Let f(x) denote the fractional part of a real number x. Then the value of $\int_0^{\sqrt{3}} f\left(x^2\right)dx$ is-

A. $2\sqrt{3} - \sqrt{2} - 1$

B. 0 (zero)

C. $\sqrt{2} - \sqrt{3} + 1$

D. $\sqrt{3} - \sqrt{2} + 1$

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**11.** Let $y = e^{x^2}$ and $y = e^{x^2}\sin x$ be two given curves. Then the angle between the tangents to the curves at any point of their intersection is-

A. 0 (zero)

B. $\pi$

C. $\frac{\pi}{2}$

D. $\frac{\pi}{4}$

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12. The value of $\int \frac{(x-2)dx}{\left\{(x-2)^2.(x+3)^7\right\}^{\frac{1}{3}}}$ is -

A. $\frac{3}{20}\left(\frac{x-2}{x+3}\right)^{\frac{4}{3}}+c$

B. $\frac{3}{20}\left(\frac{x-2}{x+3}\right)^{\frac{3}{4}}+c$

C. $\frac{5}{12}\left(\frac{x-2}{x+3}\right)^{\frac{4}{3}}+c$

D. $\frac{3}{20}\left(\frac{x-2}{x+3}\right)^{\frac{5}{3}}+c$

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**13.** If $\cos x$ and $\sin x$ are solutions of the differential equation

$$a_0\frac{d^2y}{dx^2} + a_1\frac{dy}{dx} + a_2y = 0$$

where, $a_0, a_1, a_2$ are real constants then which of the followings is/are always true?

A. $A\cos x + B\sin x$ is a solution, where A and B are real constants.

B. $A\cos\left(x + \frac{\pi}{4}\right)$ is a solution, where A is real constant.

C. $A\cos x\sin x$ is a solution, where A is real constant.

D. $A\cos\left(x + \frac{\pi}{4}\right) + B\sin\left(x - \frac{\pi}{4}\right)$ is a solution, where A and B are real constant.

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**14.** If the straight line $(a - 1)x - by + 4 = 0$ is normal to the hyperbola $xy = 1$ then which of the followings does not hold?

A. $a > 1, b > 0$

B. $a > 1, b < 0$

C. $a < 1, b < 0$

D. $a < 1, b > 0$

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15. Let f be any continuously differentiable function on [a, b] and twice differentiable on (a, b) such that $f(a) = f'(a) = 0$ and $f(b) = 0$. Then-

A. $f''(a) = 0$

B. $f'(x) = 0$ for some $x \in (a, b)$

C. $f''(x) = 0$ for some $x \in (a, b)$

D. $f'''(x) = 0$ for some $x \in (a, b)$

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**16.** Let f : $R \rightarrow R$ be such that, $f(2x - 1) = f(x)$ for all $x \in R$. If f is continuous at x = 1 and $f(1) = 1$ then-

A. $f(2) = 1$

B. $f(2) = 2$

C. f is continuous only at x = 1

D. f is continuous at all points

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**17.** For all real values $a_0, a_1, a_2, a_3$ of satisfying $a_0 + \frac{a_1}{2} + \frac{a_2}{3} + \frac{a_3}{4} = 0$, the equation

$a_0 + a_1x + a_2x^2 + a_3x^3 = 0$ has a real root in the interval

A. $[0, 1]$

B. $[-1, 0]$

C. $[1, 2]$

D. $[-2, -1]$

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**18.** The least value of $2x^2 + y^2 + 2xy + 2x - 3y + 8$ for real numbers x and y is -

A. 2

B. 8

C. 3

D. -1/2

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**19.** f: $R \rightarrow R$ is a continuous and $f(x) = \int_0^x f(t)dt$ then $f\left(\log_e 5\right) = ?$

A. 0

B. 2

C. 5

D. 3

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## WBJEE Archive 2016

**1.** If $y = (1 + x)\left(1 + x^2\right)\left(1 + x^4\right)\ldots\left(1 + x^{2n}\right)$ then the value of $\left(\frac{dy}{dx}\right)$ at x = 0 is

A. 0

B. $-1$

C. 1

D. 2

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**2.** If f(x) is an odd differentiable function defined on $(-\infty, \infty)$ such that $f'(3) = 2$, then $f'(-3)$ equal to

A. 0

B. 1

C. 2

D. 4

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**3.** $\lim_{x \to 1} \left(\frac{1+x}{2+x}\right)^{\frac{1-\sqrt{x}}{1-x}}$

A. is 1

B. does not exist

C. is $\sqrt{\frac{2}{3}}$

D. is In 2

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**4.** If $f(x) = \tan^{-1}\left[\frac{\log\left(\frac{e}{x^2}\right)}{\log\left(ex^2\right)}\right] + \tan^{-1}\left[\frac{3+2\log x}{1-6\log x}\right]$ then the value of $f''(x)$ is

A. $x^2$

B. x

C. 1

D. 0

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5. $\int \frac{\log\sqrt{x}}{3x}dx$ is equal to

A. $\frac{1}{3}\left(\log\sqrt{x}\right)^2 + c$

B. $\frac{2}{3}\left(\log\sqrt{x}\right)^2 + c$

C. $\frac{2}{3}(\log x)^2 + c$

D. $\frac{1}{3}(\log x)^2 + c$

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6. $\int 2^x(f'(x) + f(x)\log 2)dx$ is equal to

A. $2^x f'(x) + c$

B. $2^x \log 2 + c$

C. $2^x f(x) + c$

D. $2^x + c$

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**7.** $\int_0^1 \log\left(\frac{1}{x} - 1\right) dx$

A. 1

B. 0

C. 2

D. none of these

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**8.** The value of $\lim_{n \to \infty}\left[\frac{\sqrt{n+1}+\sqrt{n+2}+\ldots+\sqrt{2n-1}}{n^{\frac{3}{2}}}\right]$

A. $\frac{2}{3}\left(2\sqrt{2}-1\right)$

B. $\frac{2}{3}\left(\sqrt{2}-1\right)$

C. $\frac{2}{3}\left(\sqrt{2}+1\right)$

D. $\frac{2}{3}\left(2\sqrt{2}+1\right)$

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**9.** If the solution of the differential equation $x\frac{dy}{dx}+y=xe^{x}$ be, $xy=e^{x}\phi(x)+c$, then $\phi(x)$ is equal to

A. $x+1$

B. $x-1$

C. $1-x$

D. x

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**10.** The order of the differential equation of all paraboles whose axis of symmetry along x-axis is

A. 2

B. 3

C. 1

D. none of these

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**11.** The area enclosed by $y = \sqrt{5 - x^2}$ and $y = |x - 1|$ is

A. $\left(\frac{5\pi}{4} - 2\right)$ sq units

B. $\left(\frac{5\pi - 2}{2}\right)$ sq units

C. $\left(\frac{5\pi}{4} - \frac{1}{2}\right)$ sq units

D. $\left(\frac{\pi}{2} - 5\right)$ sq units

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**12.** Time period T of a simple pendulum of length l is given by $T = 2\pi\sqrt{\frac{l}{g}}$.

If the length is increased by $2\%$ then an approximate change in the time period is

A. $2\%$

B. $1\%$

C. $\frac{1}{2}\%$

D. none of these

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**13.** [x] denotes the greatest integer, less than or equal to x, then the value of the integral $\int_0^2 x^2[x]dx$ equals

A. $\frac{5}{3}$

B. $\frac{7}{3}$

C. $\frac{8}{3}$

D. $\frac{4}{3}$

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**14.** The number of points at which the function

$f(x) = \max\{a - x, a + x, b\}, \ -\infty < x < \infty, 0 < a < b$ cannot be differentiable

A. 0

B. 1

C. 2

D. 3

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15. General solution of $y\frac{dy}{dx} + by^2 = a\cos x, a < x < 1$ is

A. $y^2 = 2a(2b\sin x + \cos x) + ce^{-2bx}$

B. $\left(4b^2 + 1\right)y^2 = 2a(\sin x + 2b\cos x) + ce^{-2bx}$

C. $\left(4b^2 + 1\right)y^2 = 2a(\sin x + 2b\cos x) + ce^{2bx}$

D. $y^2 = 2a(2b\sin x + \cos x) + ce^{-2bx}$

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**16.** The points of the ellipse $16x^2 + 9y^2 = 400$ at which the ordinate decrease at the same rate at which the abscissa increases is/are given by

A. $\left(3, \frac{16}{3}\right)$ and $\left(-3, -\frac{16}{3}\right)$

B. $\left(3, \frac{-16}{3}\right)$ and $\left(-3, \frac{16}{3}\right)$

C. $\left(\frac{1}{16}, \frac{1}{9}\right)$ and $\left(-\frac{1}{16}, -\frac{1}{9}\right)$

D. $\left(\frac{1}{16}, -\frac{1}{9}\right)$ and $\left(-\frac{1}{16}, \frac{1}{9}\right)$

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**17.** If f(x) is a function such that $f'(x) = (x-1)^2(4-x)$ then,

A. $f(0) = 0$

B. $f(x)$ is increasing in (0, 3)

C. x = 4 is a critical point of f(x)

D. f(x) is decreasing in (3, 5)

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18. If $\phi(t)=\begin{cases}1 & \text{for } 0\le t<1\\ 0 & \text{otherwise}\end{cases}$ then,

$$\int_{-3000}^{3000}\left(\sum_{r'=2012}^{2016}\phi(t-r')\phi(t-2016)\right)dt=$$

A. a real number

B. 1

C. 0

D. does not exist

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**19.** The line $y = x + \lambda$ is tangent to the ellipse $2x^2 + 3y^2 = 1$, then $\lambda$ is-

A. -2

B. 1

C. $\sqrt{\frac{5}{6}}$

D. $\sqrt{\frac{2}{3}}$

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**20.** On the ellipse $4x^2 + 9y^2 = 1$, the points at which the tangents are parallel to the line $8x = 9y$ are-

A. $\left(\frac{2}{5}, \frac{1}{5}\right)$

B. $\left(-\frac{2}{5}, \frac{1}{5}\right)$

C. $\left(-\frac{2}{5}, -\frac{1}{5}\right)$

D. $\left(\frac{2}{5}, -\frac{1}{5}\right)$

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## JEE Main (AIEEE) Archive 2012

**1.** The population p(t) at time t of a certain mouse species satisfies the differential equation $\frac{dp(t)}{dt} = 0.5p(t) - 450$. If $p(0) = 850$, then the time at which the population becomes zero is-

A. $\frac{1}{2}\ln 18$

B. $\ln 18$

C. 2 ln 18

D. ln 9

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**2.** If $f: \mathbb{R} \to \mathbb{R}$ is a function defined by $f(x) = [x]\cos\left(\frac{2x - 1}{2}\right)\pi$, where [x] denotes the greatest integer function, then f is-

A. discontinuous only at non-zero integer values of x

B. continuous only at x = 0

C. continuous for every real x

D. discontinuous only at x = 0

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**3.** If the integral $\int \frac{5\tan x}{\tan x - 2} dx = x + a \ln|\sin x - 2\cos x| + k$, then a is equal to-

A. 1

B. 2

C. $-1$

D. $-2$

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**4.** If $g(x) = \int_0^x \cos^4 t dt,$ then $g(x + \pi)$ equals $g(x) + g(\pi)$ (b) $g(x)$ - $g(\pi)$ $g(x)g(\pi)$ (d) $\frac{g(x)}{g(\pi)}$

A. $g(x)$ - $g(\pi)$

B. $g(x) \cdot g(\pi)$

C. $\frac{g(x)}{g(\pi)}$

D. $g(x) + g(\pi)$

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**5.** A spherical balloon is filled with $4500\pi$ cubic metres of helium gas. If a leak in the balloon causes the gas to escape at the rate of $72\pi$ cubic metres per minute, the rate (in metres per minute) at which the radius of the ballon decreases 49 minutes after the leakage began is-

A. $\frac{2}{9}$

B. $\frac{9}{2}$

C. $\frac{9}{7}$

D. $\frac{7}{9}$

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6. A line is drawn through the point (1, 2) to meet the coordinate axes at P and Q such that it forms a triangle OPQ, where O is the origin. If the area of the triangle OPQ is least, then the slope of the line PQ is-

A. -2

B. $-\frac{1}{2}$

C. $-\frac{1}{4}$

D. -4

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7. The area bounded between the parabolas $x^2 = \frac{y}{4}$ and $x^2 = 9y$ and the straight line $y = 2$ is-

A. $\frac{20\sqrt{2}}{3}$

B. $10\sqrt{2}$

C. $20\sqrt{2}$

D. $\frac{10\sqrt{2}}{3}$

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**8.** $a, b \in \mathbb{R}$ be such that the function f given by $f(x) = \ln|x| + bx^2 + ax, x \neq 0$ has extreme values at $x = -1$ and x = 2.

Statement-I : f has local maximum at x = -1 and at x = 2.

Statement-II : $a = \frac{1}{2}$ and $b = -\frac{1}{4}$.

A. Statement-I is true, Statement-II is true, Statement-II is not a correct explanation for statement-I.

B. Statement-I is true, Statement-II is false.

C. Statement-I is false, Statement-II is true.

D. Statement-I is true, Statement-II is true, Statement-II is a correct explanation for Statement-I.

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**9.** Consider the function $f(x) = |x - 2| + |x - 5|, x \in \mathbb{R}$.

Statement-I : $f'(4) = 0$.

Statement-II : f is continuous is [2, 5], differentiable in (2, 5) and $f(2) = f(5)$.

A. Statement-I is true, Statement-II is true, Statement-II is true, Statement-II is not a correct explanation for Statement-I.

B. Statement-I is true, Statement-II is false.

C. Statement-I is false, Statement-II is true.

D. Statement-I is true, Statement-II is true , Statement-II is a correct explanation for Statement-I.

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**10.** Statement-I : An equation of a common tangent to the parabola $y^2 = 16\sqrt{3}x$ and the ellipse $2x^2 + y^2 = 4$ is $y = 2x + 2\sqrt{3}$.

Statement-II : If the line $y = mx + \dfrac{4\sqrt{3}}{m}, (m \neq 0)$ is a common tangent to the parabola $y^2 = 16\sqrt{3}x$ and the ellipse $2x^2 + y^2 = 4$, then m satisfies $m^4 + 2m^2 = 24$.

A. Statement-I is true, Statement-II is true, Statement-II is a correct explanation for Statement-I.

B. Statement-I is true, Statement-II is true, Statement-II is not a correct explanation for Statement-I.

C. Statement-I is true, Statement-II is false.

D. Statement-I is false, Statement-II is true.

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## JEE Main (AIEEE) Archive 2013

**1.** At present a firm is manufacturing 2000 items. It is estimated that the rate of change of production p w.r.t. additional number of workers x is given by $\frac{dp}{dx} = 100 - 12\sqrt{x}$. If the firm employs 25 more workers, then the new level of production of items is-

A. 2500

B. 3000

C. 3500

D. 4500

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2. The real number k for which the equation $2x^3 + 3x + k = 0$ has two distinct real roots in [0, 1]

A. lies between 1 and 2

B. lies between 2 and 3

C. lies between -1 and 0

D. does not exist-

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**3.** If $\int f(x)dx = \Psi(x)$, then $\int x^5 f\left(x^3\right)dx$ is equal to -

A. $\frac{1}{3}\left[x^3\Psi\left(x^3\right) - \int x^2\Psi\left(x^3\right)dx\right] + c$

B. $\frac{1}{3}x^3\Psi\left(x^3\right) - 3\int x^3\Psi\left(x^3\right)dx + c$

C. $\frac{1}{3}x^3\Psi\left(x^3\right) - \int x^2\Psi\left(x^3\right)dx + c$

D. $\frac{1}{3}\left[x^3\Psi\left(x^3\right) - \int x^3\Psi\left(x^3\right)dx\right] + c \quad \left[\because dt = 3x^2dx\right]$

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**4.** The area (in square units) bounded by the curves $y = \sqrt{x}, 2y - x + 3 = 0,$ x -axis and lying in the first quadrant is -

A. 9

B. 36

C. 18

D. $\frac{27}{4}$

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5. The intercepts on x-axis made by tangents to the curve $y = \int_0^x |t| dt, x \in \mathbb{R}$, which are parallel to the line $y = 2x$, are equal to-

A. $\pm 1$

B. $\pm 2$

C. $\pm 3$

D. $\pm 4$

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6. If $y = \sec\left(\tan^{-1}x\right)$, then $\frac{dy}{dx}$ at $x = 1$ is equal to-

A. $\frac{1}{\sqrt{2}}$

B. $\frac{1}{2}$

C. 1

D. $\sqrt{2}$

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**7.** Statement-I : The value of the integral $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\tan x}}$ is equal to $\frac{\pi}{6}$.

Statement-II : $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

A. Statement-I is true, Statement-II is true, Statement-II is a correct explanation for Statement-I.

B. Statement-I is true, Statement-II is true, Statement-II is not a correct explanation for Statement-I.

C. Statement-I is true, Statement-II is false.

D. Statement-I is false, Statement-II is true.

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**8.** Given : a circle $2x^2 + 2y^2 = 5$ and a parabola $y^2 = 4\sqrt{5}x$.

Statement-I : An equation of a common tangent to these curves is $y = x + \sqrt{5}$.

Statement-II : If the line $y = mx + \frac{\sqrt{5}}{m}(m \neq 0)$ is their common tangent, then m satisfies $m^4 - 3m^2 + 2 = 0$.

A. Statement-I is true, Statement-II is true, Statement-II is not a correct explanation for Statement-I.

B. Statement-I is true, Statement-II is true, Statement-II is a correct explanation for Statement-I.

C. Statement-I is true, Statement-II is false.

D. Statement-I is false, Statement-II is true.

**Answer: B**

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## JEE Main (AIEEE) Archive 2014

**1.** The integral $\int_0^x \sqrt{1 + 4\sin^2\frac{x}{2} - 4\sin\frac{x}{2}}dx$ equals-

A. $\pi - 4$

B. $\frac{2\pi}{3} - 4 - 4\sqrt{3}$

C. $4\sqrt{3} - 4$

D. $4\sqrt{3} - 4 - \frac{\pi}{3}$

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**2.** The integral $\int\left(1 + x - \frac{1}{x}\right)e^{x + \frac{1}{x}}$ is equal to-

A. $(x - 1)e^{x + \frac{1}{x}}$

B. $xe^{x+\frac{1}{x}}+c$

C. $(x+1)e^{x+\frac{1}{x}}+c$

D. $-xe^{x+\frac{1}{x}}+c$

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**3.** If g is the inverse of a function f and $f'(x)=\frac{1}{1+x^5}$, then g'(x) is equal to-

A. $1+x^5$

B. $5x^4$

C. $\frac{1}{1+\{g(x)\}^5}$

D. $1+\{g(x)\}^5$

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**4.** If f and g are differentiable functions in [0, 1] satisfying $f(0) = 2 = g(1), g(0) = 0$ and $f(1) = 6$, then for some $c \in [0, 1]$-

A. $2f'(c) = g'(c)$

B. $2f'(c) = 3g'(c)$

C. $f'(c) = g'(c)$

D. $f'(c) = 2g'(c)$

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**5.** Let the population of rabbits surviving at a time t be governed by the differential equation $\frac{dp(t)}{dt} = \frac{1}{2}p(t) - 200$. If $p(0) = 100$, then p(t) equals -

A. $400 - 300e^{\frac{t}{2}}$

B. $300 - 200e^{-\frac{t}{2}}$

C. $600 - 500e^{\frac{t}{2}}$

D. $400 - 300e^{-\frac{t}{2}}$

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**6.** The area of the region described by $A = \left\{(x, y): x^2 + y^2 \leq 1 \text{ and } y^2 \leq 1 - x\right\}$ is-

A. $\frac{\pi}{2} + \frac{4}{3}$

B. $\frac{\pi}{2} - \frac{4}{3}$

C. $\frac{\pi}{2} - \frac{2}{3}$

D. $\frac{\pi}{2} + \frac{2}{3}$

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**7.** The locus of the foot of perpendicular drawn from the centre of the ellipse $x^2 + 3y^2 = 6$ on any tangent to it is-

A. $\left(x^2 - y^2\right)^2 = 6x^2 + 2y^2$

B. $\left(x^2 - y^2\right)^2 = 6x^2 - 2y^2$

C. $\left(x^2 + y^2\right)^2 = 6x^2 + 2y^2$

D. $\left(x^2 + y^2\right)^2 = 6x^2 - 2y^2$

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**8.** The slope of the line touching both the parabolas $y^2 = 4x$ and $x^2 = -32y$ is -

A. $\frac{1}{2}$

B. $\frac{3}{2}$

C. $\frac{1}{8}$

D. $\frac{2}{3}$

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**9.** If $x = -1$ and x = 2 are extreme points of $f(x) = \alpha\log|x| + \beta x^2 + x$ then-

A. $\alpha = -6, \beta = \frac{1}{2}$

B. $\alpha = -6, \beta = -\frac{1}{2}$

C. $\alpha = 2, \beta = -\frac{1}{2}$

D. $\alpha = 2, \beta = \frac{1}{2}$

**Answer: C**

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## JEE Main (AIEEE) Archive 2015

**1.** The area (in sq. units) of the region described by $\left\{(x, y): y^2 \leq 2x \text{ and } y \geq 4x - 1\right\}$ is-

A. $\frac{15}{64}$

B. $\frac{9}{32}$

C. $\frac{7}{32}$

D. $\frac{5}{64}$

**Answer: B**

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**2.** The number of common tangents to the circles $x^2 + y^2 - 4x - 6y - 12 = 0$

and

$x^2 + y^2 + 6x + 18y + 26 = 0$, is -

A. 3

B. 4

C. 1

D. 2

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**3.** Let y(x) be the solution of the differential equation

$$(x\log x)\frac{dy}{dx} + y = 2x\log x, \ (x \geq 1)$$

Then y(e) is equal to-

A. 2

B. 2e

C. e

D. 0

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**4.** The integral $\int \frac{dx}{x^2\left(x^4+1\right)^{\frac{3}{4}}}$ equals-

A. $-\left(x^4+1\right)^{\frac{1}{4}}+c$

B. $-\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}}+c$

C. $\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}}+c$

D. $\left(x^4+1\right)^{\frac{1}{4}}+c$

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**5.** The normal to the curve, $x^2+2xy-3y^2=0$ at (1, 1)-

A. meets the curve again in the third quadrant

B. meets the curve again in the fourth quadrant

C. does not meet the curve again

D. meets the curve again in the second quadrant

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**6.** The integral $\int_2^4 \frac{\log x^2}{\log x^2 + \log\left(36 - 12x + x^2\right)} dx$ is equal to-

A. 1

B. 6

C. 2

D. 4

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**7.** The area (in sq. units) of the quadrilateral formed by the tangents at the end points of the letera recta to the ellipse

$\frac{x^2}{9} + \frac{y^2}{5} = 1$ is-

A. $\frac{27}{2}$

B. 27

C. $\frac{27}{4}$

D. 18

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**8.** Let f(x) be a polynomial of degree four having extreme values at x = 1 and x = 2.

If $\lim_{x \to 0}\left[1 + \frac{f(x)}{x^2}\right] = 3$, then $f(2)$ is equal to-

A. 0

B. 4

C. -8

D. -4

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**9.** $\lim_{x \to 0} \frac{(1 - \cos 2x)(3 + \cos x)}{x \tan 4x}$ is equal to-

A. 2

B. $\frac{1}{2}$

C. 4

D. 3

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## JEE Main (AIEEE) Archive 2016

**1.** The integral $\int \frac{2x^{12} + 5x^9}{\left(x^5 + x^3 + 1\right)^3} dx$ is equal to

A. $\frac{-x^5}{\left(x^5 + x^3 + 1\right)^2} + C$

B. $\frac{x^5}{2\left(x^5 + x^3 + 1\right)^2} + C$

C. $\dfrac{x^{10}}{2\left(x^5+x^3+1\right)^2}+C$

D. $\dfrac{-x^{10}}{2\left(x^5+x^3+1\right)^2}+C$

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**2.** $\lim_{n\to\infty}\left(\dfrac{(n+1)(n+2)\dots 3n}{n^{2n}}\right)^{\frac{1}{n}}$ is equal to

A. $\dfrac{18}{e^4}$

B. $\dfrac{27}{e^2}$

C. $\dfrac{9}{e^2}$

D. $3\log 3-2$

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**3.** If a curve y = f(x) passes through the point (1, -1) and satisfies the differential equation,

$y(1 + xy)dx = xdy$, then $f\left(-\frac{1}{2}\right)$ is equal to

A. $-\frac{2}{5}$

B. $-\frac{4}{5}$

C. $\frac{2}{5}$

D. $\frac{4}{5}$

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**4.** If the tangent at a point P, with parameter t, on the curve $x = 4t^2 + 3, y = 8t^3 - 1, t \in \mathbb{R}$, meets the curve again at a point Q, then the coordinates of Q are

A. $\left(t^2 + 3, -t^3 - 1\right)$

B. $\left(4t^2 + 3, \ - 8t^3 - 1\right)$

C. $\left(t^2 + 3, t^3 - 1\right)$

D. $\left(16t^2 + 3, \ - 64t^3 - 1\right)$

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**5.** The minimum distance of a point on the curve $y = x^2 - 4$ from the origin is

A. $\frac{\sqrt{19}}{2}$

B. $\sqrt{\frac{15}{2}}$

C. $\frac{\sqrt{15}}{2}$

D. $\sqrt{\frac{19}{2}}$

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6. If $\int \frac{dx}{\cos^3 x\sqrt{2\sin 2x}} = (\tan x)^A + C(\tan x)^B + K$ where K is a constant of integration, then $A + B + C$ is equal to

A. $\frac{21}{5}$

B. $\frac{16}{5}$

C. $\frac{7}{10}$

D. $\frac{27}{10}$

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7. If $2\int_0^1 \tan^{-1} x dx = \int_0^1 \cot^{-1}\left(1 - x + x^2\right) dx$ then $\int_0^1 \tan^{-1}\left(1 - x - x^2\right) dx$ is equal to

A. $\log 4$

B. $\frac{\pi}{2} + \log 2$

C. $\log 2$

D. $\frac{\pi}{2}$ - log4

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**8.** Two roots of the equation $ax^2 + bx + c = 0$ is $\alpha, \beta$ then find the value of $\alpha^3 + \beta^3$

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**9.** If f(x) is a differentiable function in the interval $(0, \infty)$ such that $f(1) = 1$ and $\lim_{t \to x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1$, for each $x > 0$ then $f\left(\frac{3}{2}\right)$ is equal to

A. $\frac{13}{6}$

B. $\frac{23}{18}$

C. $\frac{25}{9}$

D. $\frac{31}{18}$

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**10.** If m and M are the minimum and the maximum values of $4+\frac{1}{2}\sin^2 2x - 2\cos^4 x, x \in \mathbb{R}$ then M-m is equal to

A. $\frac{15}{4}$

B. $\frac{9}{4}$

C. $\frac{7}{4}$

D. $\frac{1}{4}$

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**11.** For $x \in \mathbb{R}$, $f(x) = |\log 2 - \sin x|$ and $g(x) = f(f(x))$, then

A. g is not differentiable at x = 0

B. $g'(0) = \cos(\log 2)$

C. $g'(0) = -\cos(\log 2)$

D. g is differentiable at x = 0 and $g'(0) = -\sin(\log 2)$

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12. Let $p = \lim_{x \to 0^+} \left(1 + \tan^2 \sqrt{x}\right)^{\frac{1}{2x}}$ then log p is equal to

A. 2

B. 1

C. $\frac{1}{2}$

D. $\frac{1}{4}$

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**1.** Determine the value of $\lambda$ for which the vectors $3i + \lambda\hat{j}$ and $\lambda\hat{i} + 12\hat{j}$ are collinear

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**2.** Find the unit vector in the direction of the vector whose initial point is P(4,5) and terminal point is $Q(-2, 13)$

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**3.** Prove that, in a parallelogram $PQRS$, $\overrightarrow{PR} - \overrightarrow{QS} = 2\overrightarrow{PQ}$

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**4.** ABCDEF is a regular hexagon. If $\overrightarrow{AB} = \vec{a}$ and $\overrightarrow{BC} = \vec{b}$, find $\overrightarrow{CD}, \overrightarrow{DE}, \overrightarrow{EF}$ and $\overrightarrow{FA}$ in terms of $\vec{a}$ and $\vec{b}$

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**5.** Find the area of the triangle whose positive vectors of the vertices are $6\hat{i}, 3\hat{j}$ and $-2\hat{i}$

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**6.** Find the median $\overrightarrow{AD}$ of the triangle ABC where positive vectors of the vertices A, B and C are respectively $\hat{i} + \hat{j}, 4\hat{i} + 6\hat{j}, \hat{i} - \hat{j}$

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**7.** If $\vec{x}, \vec{y}, \vec{z}$ are three vectors, show that the points having positive vectors $7\vec{x} - \vec{z}, \vec{x} + 2\vec{y} + 3\vec{z}$ and $-2\vec{x} + 3\vec{y} + 5\vec{z}$ are collinear

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8. If the area of the triangle the positive vectors of whose vertices are $a\hat{j}, 4\hat{j}$ and $\hat{i} + \hat{j}$ is 6 square units, find a.

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9. The value of m for which the planes $2x + 3y - z = 5$ and $3x - my + 3z = 6$ are perpendicular to each other is

A. $-1$

B. $\frac{1}{2}$

C. 1

D. $-\frac{1}{2}$

**Answer: C**

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**10.** The value of $\lambda$ for which the vectors $\vec{a} = \hat{i} + 3\hat{j} - \hat{k}$ and $\hat{b} = 2\hat{i} + 6\hat{j} + \lambda\hat{k}$ are parallel is-

A. 2

B. -2

C. $\frac{1}{2}$

D. $-\frac{1}{2}$

**Answer: B**

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**11.** Find the vector equation of the line whose cartesian equation is given by $x + y + z = 0$

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**12.** If $\vec{a}$ and $\vec{b}$ are two vector such that $\left|\vec{a}\right| = 2, \left|\vec{b}\right| = 3$ and $\vec{a}.\vec{b} = 4$, then find the value of $\left|\vec{a} - \vec{b}\right|$

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**13.** Find $\vec{c}$, when $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a}.\vec{c} = 3$ where $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$

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**14.** Find the value of $\lambda$ if three vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{c} = 3\hat{i} + \lambda\hat{j} + 5\hat{k}$ are coplanar

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**15.** Find the image of the point (1, 6,3) with respect to the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ and also find the equation of the line passing through the point and its image

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16. Find the equation of a plane through the intersection of the planes $\vec{r}.\left(\hat{i}+3\hat{j}-\hat{k}\right)=5$ and $\vec{r}.\left(2\hat{i}-\hat{j}+\hat{k}\right)$ and passing through the point $(2, 1, -2)$

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17. If $\left|\vec{a}\right|=4, \left|\vec{b}\right|=2\sqrt{3}$ and $\left|\vec{a}\times\vec{b}\right|=12$, then the angle between the vectors $\vec{a}$ and $\vec{b}$ is

A. $\frac{\pi}{3}$

B. $\frac{\pi}{6}$

C. $\frac{\pi}{4}$

D. $\frac{\pi}{2}$

**Answer: A**

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18. The line $\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z+5}{4}$ meets the plane $2x + 4y - z = 3$ at the point whose coordinates are

A. $(3, 1, -1)$

B. $(3, -1, 1)$

C. $(3, -1, -1)$

D. none of these

**Answer: C**

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19. If the projection of $\vec{a} = \lambda\hat{i} + \hat{j} + 4\hat{k}$ on $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$ is 4 units, then find $\lambda$

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**20.** Find the angle between the planes $x - y + 2z = 9$ and $2x + y + z = 7$

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**21.** The vectors $\vec{a}, \vec{b}, \vec{c}$ are such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. If $|\vec{a}| = 3, |\vec{b}| = 4$ and $|\vec{c}| = 5$, then show that $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = -25$

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**22.** If $\vec{\alpha} = \lambda\hat{i} + \hat{j} + 3\hat{k}, \vec{\beta} = -\hat{i} + 2\hat{j} + \hat{k}, \vec{\gamma} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\left[\vec{\alpha}\vec{\beta}\vec{\gamma}\right] = -10$, find the value of $\lambda$

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**23.** Find the vector equation of a straight line passing through the point $(2, -1, 3)$ and is perpendicular to each of the straight lines $\vec{r} = \left(\hat{i} + \hat{j} + \hat{k}\right) + \lambda\left(2\hat{i} - 2\hat{j} + \hat{k}\right)$ and $\vec{r} = \left(2\hat{i} - \hat{j} - 3\hat{k}\right) + \mu\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$

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24. Find the equation of the plane passing through the points $(-1, 1, 1)$ and $(1, -1, 1)$ and is perpendicular to the plane $x + 2y + 2z = 5$

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25. The function $y = x^2$ is

A. one-one

B. many-one

C. one-many

D. None of these

**Answer: A**

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**26.** If $a, b \in \{-2, -1, 0, 1, 2\}$ then find the probalility that the matrix

$\begin{pmatrix} \frac{a}{b} & \frac{b}{a} \end{pmatrix}$ is singular.

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**27.** Two dice are thrown simultaneously. The probability of sum 4 of two numbers so obtained is-

A. $\frac{1}{12}$

B. $\frac{1}{36}$

C. $\frac{1}{18}$

D. $\frac{1}{9}$

**Answer: A::B::C::D**

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**28.** Two dice thrown simultaneously. What is the probability that the first dice give even number and sum of two numbers is 8?

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**29.** A box contains 3 black and 5 white ball and an another box contains 5 black and 3 white ball. Two same colour of ball is transferred from first box to second box and a ball drawn at random from second box. What is the probability that ball is black?

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**30.** $IfP(A \cap B) = \frac{5}{13}$, then the value of $p\left(A^c \cup B^c\right) is$ -

A. $\frac{4}{13}$

B. $\frac{5}{13}$

C. $\frac{7}{13}$

D. $\frac{8}{13}$

**Answer: A::B::C**

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**31.** Find out the mean and variance of the following probabulity distribution:

| | | |
|---|---|---|
| $X = x_i$ | 0 | 1 |
| $p_i$ | $\frac{1}{2}$ | $\frac{1}{2}$ |

where $p_i = p(X = x_i)$

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**32.** Find the probaility that birthdays of any two members out of the five members in a family fall on Sunday.

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**33.** Sum and product of mean and variance of the binomial distribution are 24 and 128 respectively. Find the distribution.

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**34.** The probability distribution of random variable X is given as follows :

| | | | |
|---|---|---|---|
| $X = x_i$ | 0 | 1 | 2 |
| $p_i$ | $3k^3$ | $4k - 10k^2$ | $5k - 1$ |

Find the value of k.

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**35.** If the probability of success of a binomial distribution is $\frac{1}{4}$ and the standard deviation is 3, then the value of its mean is-

A. 6

B. 8

C. 12

D. 15

**Answer: A::B**

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**36.** $If P(A)=\frac{3}{7}, P(B)=\frac{4}{7}$, and$P(A\cap B)=\frac{2}{9}$, then the value of$P(A/B) is$ -

A. $\frac{7}{18}$

B. $\frac{14}{27}$

C. $\frac{5}{18}$

D. $\frac{4}{9}$

**Answer: A**

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**37.**

$If P(A/B)=0.75, P(B/A)=0.6$and$P(A)=0.4$, then find the value of$P\left(\bar{A}/\bar{B}\right)$.

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**38.** If the number of heads obtained is denoted by x when two unbliased coins are tossed then find the mean value of X .

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**39.** IF a and b are any two constants , then prove that var $(ax + b) = a^2 var(x)$.

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**40.** Two urns contain respectively 2 red , 3 white and 3 red 5 while balls . One ball is drawn at random from the first urn and transferred into the second .A ball is now drawn from the second second urn and it turns out to be red . find the probability that the transferred ball from first urn was white .

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WBJEE Archive

**1.** Which of the following is not always true?

A. $\left|\vec{a}+\vec{b}\right|^2=\left|\vec{a}\right|^2+\left|\vec{b}\right|^2$ if $\vec{a}$ and $\vec{b}$ are perpendicular to each other

B. $\left|\vec{a}+\lambda\vec{b}\right|\ge\left|\vec{a}\right|$ for all $\lambda\in R$ if $\vec{a}$ and $\vec{b}$ are perpendicular to each other

C. $\left|\vec{a}+\vec{b}\right|^2+\left|\vec{a}-\vec{b}\right|^2=2\left(\left|\vec{a}\right|^2+\left|\vec{b}\right|^2\right)$

D. $\left|\vec{a}+\lambda\vec{b}\right|\ge\left|\vec{a}\right|$ for all $\lambda\in R$ if $\vec{a}$ is parallel to $\vec{b}$

**Answer: D**

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**2.** If four points $2\hat{i}+\hat{j}+\hat{k}, \hat{i}+\hat{j}-\hat{k}, \hat{j}-\hat{k}$ and $\lambda\hat{j}+\hat{k}$ are coplanar then $\lambda$ =

A. 1

B. 2

C. $-1$

D. 0

**Answer: C**

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**3.** The value of $\lambda$ for which the straight line $\frac{x-\lambda}{3}=\frac{y-1}{2+\lambda}=\frac{z-3}{-1}$ may lie on the plane $x-2y=0$ is

A. 2

B. 0

C. $-\frac{1}{2}$

D. there is no such $\lambda$

**Answer: D**

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**4.** A straight line joining the point (1, 1, 1) and (0, 0, 0) intersects the plane $2x + 2y + z = 10$ at-

A. $(1, 2, 5)$

B. $(2, 2, 2)$

C. $(2, 1, 5)$

D. $(1, 1, 6)$

**Answer: B**

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**5.** Angle between the planes $x + y + 2z = 6$ and $2x - y + z = 9$ is

A. $\frac{\pi}{4}$

B. $\frac{\pi}{6}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

**Answer: C**

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**6.** The cosine of the angle between any two diagonals of a cube is-

A. $\frac{1}{3}$

B. $\frac{1}{2}$

C. $\frac{2}{3}$

D. $\frac{1}{\sqrt{3}}$

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**7.** For non-zero vectors $\vec{a}$ and $\vec{b}$, if $\left|\vec{a} + \vec{b}\right| < \left|\vec{a} - \vec{b}\right|$, then $\vec{a}$ and $\vec{b}$ are-

A. collinear

B. perpendicular to each other

C. inclined at an acute angle

D. inclined at an obtuse angle

**Answer: D**

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**8.** Two coins are available, one fair and the other two-headed. Choose a coin and toss it once, assume that the unbiased coin is chosen with probability $\frac{3}{4}$ Given that the outcome is head, the probability that the two-headed coin was chosen is-

A. $\frac{3}{5}$

B. $\frac{2}{5}$

C. $\frac{1}{5}$

D. $\frac{2}{7}$

**Answer: B**

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**9.** Let A and B be two events with $P\left(A^c\right) = 0.3, P(B) = 0.4 \text{and} P\left(A \cap B^c\right) \text{is equal to}$

A. $\frac{1}{4}$

B. $\frac{1}{3}$

C. $\frac{1}{2}$

D. $\frac{2}{3}$

**Answer: A::D**

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**10.** There are two coins, one unbiased with probability $\frac{1}{2}$ of getting heads and the other one is biased with probability $\frac{3}{4}$ of gatting heads. A coin is selected at random and tossed.It shows heads up. Then the probability that the unbiased coin was selected is-

A. $\frac{2}{3}$

B. $\frac{3}{5}$

C. $\frac{1}{2}$

D. $\frac{2}{5}$

**Answer: D**

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**11.** Cards are drawn one -by -one without replacement from a well shuffled pack of 52 cards. Then the probability that a face card (jack,Queen or King) will appear for the first time on the third turn is equal to -

A. $\frac{300}{2197}$

B. $\frac{36}{85}$

C. $\frac{12}{85}$

D. $\frac{4}{51}$

**Answer: A::B**

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**12.** An objective type test paper has 5 questions. Out of these 5 questions, 3 questions have four options eah (A,B,C,D) with one option being the correct answer. The other 2 questions have two options each, namely True and False. A candidate randomly ticks the options. Then the probability that he/she will tick the correct option in at least four question is-

A. $\frac{5}{32}$

B. $\frac{3}{128}$

C. $\frac{3}{256}$

D. $\frac{3}{64}$

**Answer: C::D**

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**13.** Ram is visiting a friend. Ram knows that his friend has 2 children and 1 of them is a boy. Assuming that a child is equally likely to be a boy or a girl,then the probability that the other child is a girl, is-

A. $\frac{1}{2}$

B. $\frac{1}{3}$

C. $\frac{2}{3}$

D. $\frac{7}{10}$

**Answer: B::C**

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**14.** A student answers a multiple choice question with 5 altenatives, of which exactly one is correct. The probability that he knows the correct answer is p, 0 < p < 1. If he does not know the correct answer, he randomly ticks one answer. Given that he has answered tha question correctly, the probabilty that he did not tick the answer randomly, is-

A. $\dfrac{3p}{4p+3}$

B. $\dfrac{5p}{3p+2}$

C. $\dfrac{5p}{4p+1}$

D. $\dfrac{4p}{3p+1}$

**Answer: A::D**

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**15.** A survey of people in a given region showed that 20% were smokers. The probablity of death due to lung cancer, given that a person smoked, was 10 times of probability of death due to lung cancer, given that a person did not smoke. If the probability of death due to lung cancer in the ragion is 0.006, what is the probability of death due to lung cancer given that a person is a smoker?

A. $\frac{1}{140}$

B. $\frac{1}{70}$

C. $\frac{3}{140}$

D. $\frac{1}{10}$

**Answer: A::C::D**

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**16.** Suppose a machine produces metal parts that contain some defective parts with probability. How many parts should be produced in order that the probability of at least one part defective is 1/2 or more? $\left(\text{Given } \log_{10}95 = 1.977 \text{and} \log_{10}2 = 0.3\right)$

A. 11

B. 12

C. 15

D. 14

**Answer: A::C::D**

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**17.** A fair coin is tossed a fixed number of times. If the probability of getting exactly 3 heads equals the probablity of getting exactly 5 heads, then the probability of getting exactly one head is-

A. $\frac{1}{64}$

B. $\frac{1}{32}$

C. $\frac{1}{16}$

D. $\frac{1}{8}$

**Answer: A::B::C**

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**18.** Let A and B be two events such that

$P(A \cap B) = \frac{1}{6}, P(A \cup B) = \frac{31}{45} \text{and} P(\bar{B}) = \frac{7}{10}$-

A. A and B are independent

B. A and B are mutually exclusive

C. $P\left(\frac{A}{B}\right) < \frac{1}{6}$

D. $P\left(\frac{B}{A}\right) < \frac{1}{6}$

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**19.** In a group of 14 males and 6 females, 8 and 3 of the males and the females respectively are aged above 40 years. The probability that a person selected at random from the group is aged above 40 years, given that the selected person is a female is-

A. $\frac{2}{7}$

B. $\frac{1}{2}$

C. $\frac{1}{4}$

D. $\frac{5}{6}$

**Answer: A::B**

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**20.** If A, B are two events such that $P(A \cup B) \geq \frac{3}{4}$ and $\frac{1}{8} \leq P(A \cap B) \leq \frac{3}{8}$

A. $P(A) + P(B) \leq \frac{11}{8}$

B. $P(A)P(B) \leq \frac{3}{8}$

C. $P(A) + P(B) \geq \frac{7}{8}$

D. none of these

**Answer: A::B**

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**1.** An equation of a plane parallel to the plane $x - 2y + 2z - 5 = 0$ and at a unit distance from the origin is

A. $x - 2y + 2z - 1 = 0$

B. $x - 2y + 2z + 5 = 0$

C. $x - 2y + 2z - 3 = 0$

D. $x - 2y + 2z + 1 = 0$

**Answer: C**

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**2.** Let $\hat{a}$ and $\hat{b}$ be two unit vectors. If the vectors $\hat{c} = \hat{a} + 2\hat{b}$ and $\hat{d} = 5\hat{a} - 4\hat{b}$ are perpendicular to each other, than the angle between $\hat{a}$ and $\hat{b}$ is

A. $\frac{\pi}{3}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{6}$

D. $\frac{\pi}{2}$

**Answer: D**

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**3.** Let ABCD be a parallelogram such that $\overrightarrow{AB} = \vec{q}, \overrightarrow{AD} = \vec{P}$ and $\angle BAD$ be an acute angle. If $\vec{r}$ is the vector that coincides with the altitude directed from the vertex B to the side AD, then $\vec{r}$ is given by-

A. $\vec{r} = \vec{q} - \left(\frac{\vec{p}.\vec{q}}{\vec{p}.\vec{p}}\right)\vec{p}$

B. $\vec{r} = -3\vec{q} + \frac{3(\vec{p}.\vec{q})}{(\vec{p}.\vec{p})}\vec{p}$

C. $\vec{r} = 3\vec{q} - \frac{3(\vec{p}.\vec{q})}{(\vec{p}.\vec{p})}\vec{p}$

D. $\vec{r} = -\vec{q} + \left(\frac{\vec{p}.\vec{q}}{\vec{p}.\vec{p}}\right)\vec{p}$

**Answer: D**

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**4.** If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then k is equal to-

A. $\frac{9}{2}$

B. 0

C. $-1$

D. $\frac{2}{9}$

**Answer: A**

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**5.** Distance between two parallel planes $2x + y + 2z = 8$ and $4x + 2y + 4z + 5 = 0$ is-

A. $\frac{3}{2}$

B. $\frac{5}{2}$

C. $\frac{7}{2}$

D. $\frac{9}{2}$

**Answer: C**

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**6.** If the lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{-k}$ and $\frac{x-1}{k}=\frac{y-4}{2}=\frac{z-5}{1}$ are coplanar, then k can have

A. any value

B. exactly one value

C. exactly two value

D. exactly three value

**Answer: C**

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7. If the vector $\overrightarrow{AB} = 3\hat{i} + 4\hat{k}$ and $\overrightarrow{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a triangle ABC, then the length of the median through A is-

A. $\sqrt{18}$

B. $\sqrt{72}$

C. $\sqrt{33}$

D. $\sqrt{45}$

**Answer: C**

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8. The image of the line $\frac{x-1}{3} = \frac{y-3}{1} = \frac{z-4}{-5}$ in the plane $2x - y + z + 3 = 0$ is the line-

A. $\frac{x+3}{3} = \frac{y-5}{1} = \frac{z-2}{-5}$

B. $\frac{x+3}{-3}=\frac{y-5}{-1}=\frac{z+2}{5}$

C. $\frac{x-3}{3}=\frac{y+5}{1}=\frac{z-2}{-5}$

D. $\frac{x-3}{-3}=\frac{y+5}{-1}=\frac{z-2}{5}$

**Answer: A**

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**9.** The angle between the lines whose direction cosines satisfy the equations $l+m+n=0$ and $l^2=m^2+n^2$ is

A. $\frac{\pi}{3}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{6}$

D. $\frac{\pi}{2}$

**Answer: A**

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**10.** If $\left[\vec{a}\times\vec{b}\quad\vec{b}\times\vec{c}\quad\vec{c}\times\vec{a}\right]=\lambda\left[\vec{a}\vec{b}\vec{c}\right]^2$ then $\lambda$ is equal to

A. 2

B. 3

C. 0

D. 1

**Answer: D**

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**11.** The equation of the plane containing the line $2x - 5y + z = 3, x + y + 4z = 5$ and parallel to the plane, $x + 3y + 6z = 1$ is

A. $x + 3y + 6z = 7$

B. $2x + 6y + 12z = -13$

C. $2x + 6y + 12z = 13$

D. $x + 3y + 6z = -7$

**Answer: A**

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**12.** The distance of the point $(1, 0, 2)$ from the point of intersection of the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane $x - y + z = 16$ is

A. $3\sqrt{12}$

B. 13

C. $2\sqrt{14}$

D. 8

**Answer: B**

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**13.** Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of them are collinear and $\left(\vec{a} \times \vec{b}\right) \times \vec{c} = \frac{1}{3}\left|\vec{b}\right|\left|\vec{c}\right|\vec{a}$. If $\theta$ is the angle between vector $\vec{b}$ and $\vec{c}$, then a value of $\sin\theta$ is

A. $\frac{2}{3}$

B. $\frac{-2\sqrt{3}}{3}$

C. $\frac{2\sqrt{2}}{3}$

D. $\frac{-\sqrt{2}}{3}$

**Answer: C**

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**14.** The distance of the point $(1, -5, 9)$ from the plane $x - y + z = 5$ measured along the line $x = y = z$ is

A. $3\sqrt{10}$

B. $10\sqrt{3}$

C. $\frac{10}{\sqrt{3}}$

D. $\frac{20}{3}$

**Answer: B**

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**15.** If the line $\frac{x-3}{2}=\frac{y+2}{-1}=\frac{z+4}{3}$ lies in the plane $lx+my-z=9$, then $l^2+m^2$ is equal to

A. 26

B. 18

C. 5

D. 2

**Answer: D**

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16. Let $\vec{a}.\vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\sqrt{3}}{2}(\vec{b} + \vec{c})$. If $\vec{b}$ is not parallel to $\vec{c}$, then the angle between $\vec{a}$ and $\vec{b}$ is

A. $\frac{3\pi}{4}$

B. $\frac{\pi}{2}$

C. $\frac{2\pi}{3}$

D. $\frac{5\pi}{6}$

**Answer: D**

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17. The shortest distance between the lines $\frac{x}{2} = \frac{y}{2} = \frac{z}{1}$ and $\frac{x+2}{-1} = \frac{y-4}{8} = \frac{z-5}{4}$ lies in the interval

A. $[0, 1)$

B. $[1, 2)$

C. $(2, 3]$

D. $(3, 4]$

**Answer: C**

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**18.** The distance of the point $(1, -2, 4)$ from the plane passing through the point (1, 2, 2) and perpendicular to the planes $x - y + 2z = 3$ and $2x - 2y + z + 12 = 0$, is-

A. $2\sqrt{2}$

B. 2

C. $\sqrt{2}$

D. $\frac{1}{\sqrt{3}}$

**Answer: A**

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**19.** In a $\Delta ABC$, right angled at the vertex A, If the positive vectors of A, B and C are respectively $3\hat{i} + \hat{j} - \hat{k}$, $-\hat{i} + 3\hat{j} + p\hat{k}$ and $5\hat{i} + q\hat{j} - 4\hat{k}$, then the point (p,q) lies on a line-

A. parallel to x-axis

B. parallel to y-axis

C. making an acute angle with the positive direction of x-axis

D. making an obtuse angle with the positive direction of x-axis

**Answer: C**

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JEE Advanced Archive

**1.** A line l passing through the origin is perpendicular to the lines

$l_1 : (3 + t)\hat{i} + (-1 + 2t)\hat{j} + (4 + 2t)\hat{k}, \; -\infty < t < \infty$

$l_2 : (3 + 2s)\hat{i} + (3 + 2s)\hat{j} + (2 + s)\hat{k}, \; -\infty < s < \infty$

Then the coordinate (s) of the point (s) on $l_2$ at a distance of $\sqrt{17}$ from the point of intersection of l and $l_1$ is (are)

A. $\left(\frac{7}{3}, \frac{7}{3}, \frac{5}{3}\right)$

B. $(-1, -1, 0)$

C. $(1, 1, 1)$

D. $\left(\frac{7}{9}, \frac{7}{9}, \frac{8}{9}\right)$

**Answer: A::B::D**

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**2.** Two lines $L_1 : x = 5, \frac{y}{3 - \alpha} = \frac{z}{-2}$ and $L_2 : x = \alpha, \frac{y}{-1} = \frac{z}{2 - \alpha}$ are coplanar.

Then $\alpha$ can take value (s)

A. 1

B. 2

C. 3

D. 4

**Answer: A::D**

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**3.** Solve: $\begin{vmatrix} 49 & 1 & 6 \\ 39 & 7 & 4 \\ 26 & 2 & 3 \end{vmatrix}$

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**4.** Perpendiculars are drawn from points on the line $\frac{x+2}{2} = \frac{y+1}{-1} = \frac{z}{3}$ to the plane $x + y + z = 3$. The feet of perpendiculars lie on the line

A. $\frac{x}{5} = \frac{y-1}{8} = \frac{z-2}{-13}$

B. $\frac{x}{2} = \frac{y-1}{3} = \frac{z-2}{-5}$

C. $\frac{x}{4} = \frac{y-1}{3} = \frac{z-2}{-7}$

D. $\frac{x}{2} = \frac{y-1}{-7} = \frac{z-2}{5}$

**Answer: D**

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**5.** Consider th set of eight vectors $V = \left\{a\hat{i} + b\hat{j} + c\hat{k} : a, b, c \in \{-1, 1\}\right\}$.

Three non-coplanar vectors can be chosen from V in 2P ways Then p is

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**6.** Let $\vec{PR} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{SQ} = \hat{i} - 3\hat{j} - 4\hat{k}$ determine diagonals of aparallelogram PQRS and $\vec{PT} = \hat{i} + 2\hat{j} + 3\hat{k}$ be another vector. Then the volume of the parallelopiped determined by the vectors $\vec{PT}, \vec{PQ}$ and $\vec{PS}$ is

A. 5

B. 20

C. 10

D. 30

**Answer: C**

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**7.** Find the area enclosed by $x^{\frac{1}{3}} + y^{\frac{1}{3}} = 1$ and the co-ordinate axis.

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**8.** From a point $P(\lambda, \lambda, \lambda)$ perpendiculars PQ and PR are drawn respectively on the lines $y = x, z = 1$ and $y = -x, z = -1$. If P is such that $\angle QPR$ is a right angle, then the possible value (s) of $\lambda$ is (are)

A. $\sqrt{2}$

B. 1

C. -1

D. $-\sqrt{2}$

**Answer: C**

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**9.** Let $\vec{x}, \vec{y}$ and $\vec{z}$ be three vectors each of magnitude $\sqrt{2}$ and the angle between each pair of them is $\frac{\pi}{3}$. If $\vec{a}$ is a non-zero vector perpendicular to $\vec{x}$ and $\vec{y} \times \vec{z}$ and $\vec{b}$ is a non-zero vector perpendicular to $\vec{y}$ and $\vec{z} \times \vec{x}$, then

A. $\vec{b} = \left(\vec{b}.\vec{z}\right)\left(\vec{z} - \vec{x}\right)$

B. $\vec{a} = \left(\vec{a}.\vec{y}\right)\left(\vec{y} - \vec{z}\right)$

C. $\vec{a}.\vec{b} = -\left(\vec{a}.\vec{y}\right)\left(\vec{b}.\vec{z}\right)$

D. $\vec{a} = \left(\vec{a}.\vec{y}\right)\left(\vec{z} - \vec{y}\right)$

**Answer: A::B::C**

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**10.** Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-coplanar unit vectors such that the angle between every pair of them is $\frac{\pi}{3}$. If $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} = p\vec{a} + q\vec{b} + r\vec{c}$, where p, q and r are scalars, then the value of $\frac{p^2 + 2q^2 + r^2}{q^2}$ is

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**11.** Differentiate $\tan^{-1}\left(\frac{1 + ax}{1 - ax}\right)$ with respect to $\sqrt{1 + a^2x^2}$

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**12.** If matrix $A$ is given by $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, then find $k$ so that $A^2 = 5A + kI$

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**13.** Find the domain of $\frac{1}{1 + \ln(3x)}$

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**14.** Suppose that $\vec{p}, \vec{q}$ and $\vec{r}$ are three non-coplanar vectors in R. Let the components of a vector $\vec{s}$ along $\vec{p}, \vec{q}$ and $\vec{r}$ be 4, 3 and 5 respectively. If the components of this vector $\vec{s}$ along $\left(-\vec{p} + \vec{q} + \vec{r}\right), \left(\vec{p} - \vec{q} + \vec{r}\right)$ and $\left(-\vec{p} - \vec{q} + \vec{r}\right)$ are x,y and z respectively, then the value of $2x + y + z$ is

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**15.** In R, consider the planes $P_1 : y = 0$ and $P_2 : x + z = 1$. Let $P_3$ be a plane, different from $P_1$ and $P_2$, which passes through the intersection of $P_1$ and $P_2$. If the distance of the point (0, 1,1) from $P_3$ is 1 and the distance of a point $(\alpha, \beta, \gamma)$ from $P_3$ is 2, then which of the following relations is (are) true?

A. $2\alpha + \beta + 2\gamma + 2 = 0$

B. $2\alpha + \beta - 2\gamma - 10 = 0$

C. $2\alpha - \beta + 2\gamma - 8 = 0$

D. $2\alpha - \beta + 2\gamma + 4 = 0$

**Answer: A::B::D**

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**16.** In $R^3$, let L be a straight line passing through the origin. Suppose that all the points on L are at a constant distance from the two planes $P_1 : x + 2y - z + 1 = 0$ and $P_2 : 2x - y + z - 1 = 0$. Let M be the locus of the feet of the perpendiculars drawn from the points on L to the plane $P_1$. Which of the following points lie (s) on M ?

A. $\left(0, -\frac{5}{6}, \frac{2}{3}\right)$

B. $\left(-\frac{1}{6}, \frac{1}{3}, \frac{1}{6}\right)$

C. $\left(-\frac{5}{6}, 0, \frac{1}{6}\right)$

D. $\left(-\frac{1}{3}, 0, \frac{2}{3}\right)$

**Answer: A::B::D**

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**17.** Let $\Delta PQR$ be a triangle. Let $\vec{a} = \overrightarrow{QR}, \vec{b} = \overrightarrow{RP}$ and $\vec{c} = \overrightarrow{PQ}$. If $|\vec{a}| = 12, |\vec{b}| = 4\sqrt{3}$ and $\vec{b}.\vec{c} = 24$, then which of the following is (are) true?

A. $\frac{|\vec{c}|^2}{2} - |\vec{a}| = 12$

B. $\frac{|\vec{c}|^2}{2} + |\vec{a}| = 30$

C. $|\vec{a} \times \vec{b} + \vec{c} \times \vec{a}| = 48\sqrt{3}$

D. $\vec{a}.\vec{b} = -72$

**Answer: A::C::D**

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18. Consider a pyramid OPQRS located in the first octant $(x \geq 0, y \geq 0, z \geq 0)$ with O as origin and OP and OR along the X-axis and the Y-axis, respectively. The base OPQRS of the pyramid is a square with OP = 3. The point S is directly above the midpoint T of diagonal OQ such that TS = 3. Then,

A. the acute angle between OQ and OS is $\frac{\pi}{3}$

B. the equation of the plane containing the $\Delta OQS$ is $x - y = 0$

C. the length of the perpendicular from P to the plane containing the $\Delta OQS$ is $\frac{3}{\sqrt{2}}$

D. the perpendicular distance from O to the straight line containing RS is $\sqrt{\frac{15}{2}}$

**Answer: B::C::D**

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**19.** Let P be the image of the point (3, 1, 7) with respect to the plane $x - y + z = 3$. Then, the equation of the plane passing through P and containing the straight line $\frac{x}{1} = \frac{y}{2} = \frac{z}{1}$ is

A. $x + y - 3z = 0$

B. $3x + z = 0$

C. $x - 4y + 7z = 0$

D. $2x - y = 0$

**Answer: C**

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**20.** Let $\hat{u} = u_1\hat{i} + u_2\hat{j} + u_3\hat{k}$ be a unit vector in $R^3$ and $\hat{w} = \frac{1}{\sqrt{6}}\left(\hat{i} + \hat{j} + 2\hat{k}\right)$. Given that there exists a vector $\vec{v}$ in $R^3$, such that $\left|\hat{u} + \vec{v}\right| = 1$ and $\hat{w}.\left(\hat{u} + \vec{v}\right) = 1$

Which of the following statement(s) is/are correct ?

A. There is exactly one choice for such $\vec{v}$

B. There are infinitely many choices from such $\vec{v}$

C. If $\hat{u}$ lies in the XY plane, then $|u_1| = |u_2|$

D. If $\hat{u}$ lies in the XZ plane then $2|u_1| = |u_3|$

**Answer: A::B::C::D**

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**21.** Four persons independently solve a certain problem correctly with probabilities, $\frac{1}{2}, \frac{3}{4}, \frac{1}{4}, \frac{1}{8}$ Then the probability that problem is solved correctly by at least one of them is-

A. $\frac{235}{256}$

B. $\frac{21}{256}$

C. $\frac{3}{256}$

D. $\frac{253}{256}$

**Answer: A**

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**22.** Of the three independent events $E_1, E_2$ and $E_3$ the probability that only $E_1$ occurs is $\beta$ and only $E_3$ occurs is $\gamma$. Let the probability $p$ that none of the events $E_1, E_2, \text{or} E_3$ occurs satisfy the equations $(\alpha - 2\beta)p = \alpha\beta$ and $(\beta - 3\gamma)p = 2\beta\gamma$. All the given probabilities are assumed to lie in the interval (0,1).Then $\frac{\text{Probability of occurrence of} E_1}{\text{Probability of occurrence of} E_3}$

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**23.** A box $B_1$ contains 1 white ball 3 red balls and 2 black balls. Another box $B_2$ contains 2 white balls, 3 red balls and 4 black balls. A third box $B_3$ contains 3 white balls, 4 red balls and 5 black balls.

(i) If 1 ball is drawn from each of the boxes $B_1, B_2$ and $B_3$ the probability that all 3 drawn balls are of the same colour is-

A. $\frac{82}{648}$

B. $\frac{90}{648}$

C. $\frac{558}{648}$

D. $\frac{566}{648}$

**Answer: B::D**

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**24.** A box $B_1$ contains 1 white ball 3 red balls and 2 black balls. Another box $B_2$ contains 2 white balls, 3 red balls and 4 black balls. A third box $B_3$ contains 3 white balls, 4 red balls and 5 black balls.

If 2 balls are drawn (without replacement) from a randomly selected box and one of the ball is white and the other ball is red the probability that these 2 balls are drawn from box $B_2$ is-

A. $\frac{116}{181}$

B. $\frac{126}{181}$

C. $\frac{65}{181}$

D. $\frac{55}{181}$

**Answer: A**

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**25.** Box 1 contains three cards bearing numbers 1, 2,3, box 2 contains five cards bearing numbers 1,2,3,4,5, and box 3 contains seven card bearing numbers 1,2,3,4,5,6,7. Acard is drawn from each of the boxes. Let,$x_i$ be the number on the card drawn from the $i^{th}$ box i = 1,2,3.

The probability that $x_1 + x_2, + x_3$ is odd,is-

A. $\frac{29}{105}$

B. $\frac{53}{105}$

C. $\frac{57}{105}$

D. $\frac{1}{2}$

**Answer: A::C**

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**26.** Box 1 contains three cards bearing numbers 1, 2,3, box 2 contains five cards bearing numbers 1,2,3,4,5, and box 3 contains seven card bearing numbers 1,2,3,4,5,6,7. Acard is drawn from each of the boxes. Let,$x_i$ be the number on the card drawn from the $i^{th}$ box i = 1,2,3.

The probability that $x_1, x_2, x_3$ are in an arithmetic progression, is-

A. $\frac{9}{105}$

B. $\frac{10}{105}$

C. $\frac{11}{105}$

D. $\frac{7}{105}$

**Answer: A**

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**27.** Three boys and two girls stand in a queue. The probability, that the number of boys ahead of every girls is at least one more then the number of girls ahead of her, is-

A. $\frac{1}{2}$

B. $\frac{1}{3}$

C. $\frac{2}{3}$

D. $\frac{3}{4}$

**Answer: A::B**

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**28.** Let $n_1 \text{and} n_2$ be the number of red and black balls, respectively, in box I. Let $n_3 \text{and} n_4$ be the number of red black balls, respectively in box II.

One of the two boxes, box 1 and box II was selected at random and a ball was found to be rad. if the probability that this red ball was drawn from box II is $\frac{1}{3}$ then the correct option (s) with the possible values of correct option (s) with the possible values of correct option (s) with the possible values of $n_1, n_2, n_3 \text{and} n_4 \text{is (are)}$

A. $n_1 = 3, n_2 = 3, n_3 = 5, n_4 = 15$

B. $n_1 = 3, n_2 = 6, n_3 = 10, n_4 = 50$

C. $n_1 = 8, n_2 = 6, n_3 = 5, n_4 = 20$

D. $n_1 = 6, n_2 = 12, n_3 = 5, n_4 = 20$

**Answer: A::B::C::D**

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**29.** Let $n_1 \text{and} n_2$ be the number of red and black balls, respectively, in box I. Let $n_3 \text{and} n_4$ be the number of red black balls, respectively in box II.

A ball is drawn at random from box 1 and transferred to box II. If the probability of drawing a red ball from box 1, after this transfer, is $\frac{1}{3}$ then the correct option (s) with the possible values of $n_1 \text{and} n_2$ is (are)-

A. $n_1 = 4 \text{and} n_2 = 6$

B. $n_1 = 2 \text{ and } n_2 = 3$

C. $n_1 = 10 \text{ and } n_2 = 20$

D. $n_1 = 3 \text{ and } n_2 = 6$

**Answer: A::B::C**

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**30.** The minimum number of times a fair coin needs to be tossed, so that the probability of getting at least two head is at least 0.96, is-

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**31.** A computer producing factory has only two plants $T_1 \text{ and } T_2 \text{ Plant } T_1$ produces 20% and plant $T_2$ produced 80% of the total computers produced. 7% of computers produced in the factory turn out to be defective. It is known that P (computer turns out to be defective, given that it is produced in plant $T_1 = 10$ P (computer turns out to be defective, given that it is produced in plant$T_2$)where P(E) denptes the probability of an event E. A computer produced in the factory is randomly selected and it does not turn out to be defective. Then the probability that it is produced in plant $T_2$is

A. $\frac{36}{73}$

B. $\frac{47}{79}$

C. $\frac{78}{93}$

D. $\frac{75}{83}$

**Answer: A::D**

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**32.** Football teams $T_1$and$T_2$ have to play two games against each other. It is assumed that the outcomes of the two games are independent. The probabillities of $T_1$ winning, drawing and losing a game against $T_2$are$\frac{1}{2}$, $\frac{1}{6}$and$\frac{1}{3}$ respectively. Each team gets 3 points for a win 1 point for a draw and 0 point for a loss in a game. Let X and Y denote the total points scored by teams $T_1 \text{and} T_2$ respectively after two games.

P(X>Y) is equal to

A. $\frac{1}{4}$

B. $\frac{5}{12}$

C. $\frac{1}{2}$

D. $\frac{7}{12}$

**Answer: A::B**

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**33.** Football teams $T_1 \text{and} T_2$ have to play two games against each other. It is assumed that the outcomes of the two games are independent. The probabillities of $T_1$ winning, drawing and losing a game against $T_2 \text{are} \frac{1}{2}, \frac{1}{6} \text{and} \frac{1}{3}$ respectively. Each team gets 3 points for a win 1 point for a draw and 0 point for a loss in a game. Let X and Y denote the total points scored by teams $T_1 \text{and} T_2$ respectively after two games.

P(X = Y) is equal to

A. $\frac{11}{36}$

B. $\frac{1}{3}$

C. $\frac{13}{36}$

D. $\frac{1}{2}$

**Answer: A::C**

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## JEE Main (AIEEE) Archive

**1.** Three numbers are chosen at random without replacement from {1,2,3, ...8}. The probability that their minimum is 3, given that their maximum is 6, is-

A. $\frac{1}{4}$

B. $\frac{2}{5}$

C. $\frac{3}{8}$

D. $\frac{1}{5}$

**Answer: A**

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**2.** A multiple choice examination has 5 question. Each question has three alternative answers of which exactly one is correct.The probability that a student will get 4 or more correct answers just by guessing is-

A. $\frac{17}{3^5}$

B. $\frac{13}{3^5}$

C. $\frac{11}{3^5}$

D. $\frac{10}{3^5}$

**Answer: A::C**

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3. Let A and B be two events such that

$P(\overline{A \cup B}) = \frac{1}{6}, P(A \cap B)$ and $P(\bar{A}) = \frac{1}{4}$, where $\bar{A}$ stands for the complement of the event A. Then the events A and B are-

A. mutually exclusive and independent

B. equally likely but not independent

C. independent but not equally likely

D. independent and equally likely

**Answer: A::B::C**

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4. If 12 identical balls are to be placed in 3 identical boxes,then the probability that one of the boxes contains exactly 3 balls is-

A. $220\left(\frac{1}{3}\right)^{12}$

B. $22\left(\frac{1}{3}\right)^{11}$

C. $\frac{55}{3}\left(\frac{2}{3}\right)^{11}$

D. $55\left(\frac{2}{3}\right)^{10}$

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**5.** Let two fair six faced dice A and B be thrown simultaneously. If $E_1$ is the event that die A shows up four, $E_2$ is the event that die B shows up two and $E_3$ is the event that the sum of numbers on both dice is odd, then which of the following statements is NOT true?

A. $E_1 \text{and} E_2$ are independent

B. $E_2 \text{ and} E_1$ are independent

C. $E_1 \text{ and} E_3$ are independent

D. $E_1, E_2 \text{ and} E_3$ are independent

**Answer: A::B::C::D**

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6. If A and B are any two events such that $P(A) = \frac{2}{5}$ and $P(A \cap B) = \frac{3}{20}$ then the conditional probability $P(A/A' \cup B')$ where A denotes the complement of A is equal to

A. $\frac{1}{4}$

B. $\frac{5}{17}$

C. $\frac{8}{17}$

D. $\frac{11}{20}$

**Answer: A**

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Others

**1.** If x is real , then find the maximum value of $3 - 20x - 25x^2$ and find for which value of x the expression maximum .

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