MATHS

BOOKS - CHHAYA PUBLICATION MATHS (BENGALI ENGLISH)

BINARY OPERATION

Example

1. Let P(A) be the power set of a non-empty set A. Prove that union ($\cup$) and intersection ($\cap$) of two subsets X and Y of A are binary operations on P(A).

Watch Video Solution

2. Let $*$ be an operation defined on $A, = \{2, 4, 6, 8\}$ by $a * b = k$ where k is the least non-negative remainder when the product ab is divided by 10 and $a, b \in A$. show that $*$ is a binary operation on A.

Watch Video Solution

3. Let S be a set of two elements. How many different binary operaions can be defined on S?

Watch Video Solution

4. The operation $*$ is defined by $a * b = a^b$ on the set $Z = \{0, 1, 2, 3, \ldots\}$. Show that $*$ is not a binary operation.

Watch Video Solution

5. Let $S = \sqrt{3}x + 2y : x, y \in Z\}$. Prove that the operation $*$ on S defined by $\left(\sqrt{3}x_1 + 2y\right) * \left(\sqrt{3}x_2 + 2y_2\right) = \sqrt{3}(x_1 + x_2) + 2(y_1 + y_2)$ for all $x_1, x_2, y_1, y_2 \in Z$ is closed under $*$.

Watch Video Solution

6. Let $A = \{0, 1, 2, 3, 4, 5\}$. If $a_1 b \in A$, then an operation $\circ$ on A is defined by $a \circ b = k$ where k is the least non-negative remainder when the sum $(a + b)$ is divided by 6. Show that $\circ$ is a binary operation on A.

Watch Video Solution

7. Let $\mathbb{R}$ be the set of real numbers and $x, y \in R$. We define operaitons $\wedge$ and $\vee$ on $\mathbb{R}$ as

$x \wedge y =$ maximum of x and y,

$x \vee y =$ minimum of x and y.

Show the operation $\wedge$ and $\vee$ defined above are binary operations on $\mathbb{R}$.

Watch Video Solution

8. On the set C of all complex numbers an operation $'o'$ is defined by $z_1 \, o \, z_2 = \sqrt{z_1 z_2}$ for all $z_1, \ z_2 \in C$. Is o a binary operation on C ?

Watch Video Solution

9. Determine whether * on N defined by $a \cdot b = a^b$ for all $a, \ b \in N$ define a binary operation on the given set or not:

Watch Video Solution

10. On Q , the set of all rational numbers a binary operation * is defined by $a \cdot b = \frac{a+b}{2}$. Show that * is not associative on Q.

Watch Video Solution

11. Let $\mathbb{R}$ be the set of real numbers. Show that the operation $*$ defined on $\mathbb{R} - \{0]$ by $a * b = |ab|, a, b \in \mathbb{R} - \{0\}$ is a binary operation on $\mathbb{R} - \{0\}$.

Watch Video Solution

12. Let $M_2 = \begin{bmatrix} x & 0 \\ 0 & y \end{bmatrix}, x, y \in \mathbb{R} - \{0\}$ be the set of 2×2 matrices, prove that the operation $*$ defined on M_2 by $A * B = AB, A, B \in M_2$ is a binary operation.

Watch Video Solution

13. Let $S = (0, 1, 2, 3, 4,)$ and * be an operation on S defined by $a \cdot b = r$, where$r$ is the least non-negative remainder when $a + b$ is divided by 5. Prove that * is a binary operation on S.

Watch Video Solution

14. Is $\circ$ defined on $\mathbb{Q}$ the set of rational numbers, by $a \circ b = \frac{a-1}{b-1}(a, b \in \mathbb{Q})$, a binary operation?

Watch Video Solution

15. Prove that an operation $*$ on $\mathbb{R}$, the set of real numbers, defined by $x * y = 2xy + \sqrt{5}$, for all $x, y \in \mathbb{R}$, is a binary operation on $\mathbb{R}$.

Watch Video Solution

16. Let $\circ$ be a binary operation on $\mathbb{Q}$, the set of rational numbers, defined by $a \circ b = \frac{1}{8}ab$ for all $a, b \in \mathbb{Q}$. Prove that $\circ$ is commutive as well as associative.

Watch Video Solution

17. let $*$ be a binary operation on $\mathbb{Z}^+$, the set of positive integers, defined by $a * b = a^b$ for all $a, b \in \mathbb{Z}^+$. Prove that $*$ is neither commutative nor associative on $\mathbb{Z}^+$.

Watch Video Solution

18. Show that the binary operation $*$ defined on $\mathbb{R}$ by $a * b = ab + 2$ is commutative but not associative.

Watch Video Solution

19. let $*$ be a binary operation on $\mathbb{R}$, the set of real numbers, defined by $a \circ b = \sqrt{a^2 + b^2}$ for all $a, b \in \mathbb{R}$. Prove that the binary operation $\circ$ is commutative as well as associative.

Watch Video Solution

20. Discuss the commutativity and associativity of binary operation $*$ defined on $\mathbb{Z}$ by the rule

$a * b = |a|b$ for all $a, b \in \mathbb{Z}$.

Watch Video Solution

21. Show that the operation $*$ defined on $\mathbb{R}-\{0\}$ by $a*b=|ab|$ is a binary operation. Show also that $*$ is commutative and associative.

Watch Video Solution

22. Prove that the binary operation $*$ on $\mathbb{R}$ defined by $a*b=a+b+ab$ for all $a, b \in \mathbb{R}$

is commutative and associative.

Watch Video Solution

23. Prove that the binary operation $\circ$ defined on $\mathbb{Q}$ by $a \circ b=a-b+ab$ for all a,b in $\mathbb{Q}$ is neither commutative nor associative.

Watch Video Solution

24. Let S be a set of containing more than two elements and a binary operaton $\circ$ on S be defined by

$a \circ b = a$ for all $a, b \in S$.

Prove that $\circ$ is associative but not commutative on .

Watch Video Solution

25. let $*$ be a binary on $\mathbb{Q}$, defined by

$a * b = (a - b)^2$ for all $a, b \in \mathbb{Q}$. Show that the binary operation $*$ on $\mathbb{Q}$ is commutative but not associative.

Watch Video Solution

26. Let $*$ and $\circ$ be two binary operations on $\mathbb{R}$ defined as,

$a * b = |a - b|$ and $a \circ b = a$ for all $a, b \in \mathbb{R}$.

Examine the commutativity and associativity of $*$ and $\circ$ on $\mathbb{R}$. Show also that $*$ is distributative over $\circ$ but $\circ$ is not distributive over $*$.

Watch Video Solution

27. Let $S = \mathbb{N} \times \mathbb{N}$ and $*$ is a binary operation on S defined by

$(a, b) * (c, d) = (a + c, b + d)$ for all $a, b, c, d \in \mathbb{N}$.

Prove that $*$ is a commutative and associative binary operation on S.

Watch Video Solution

28. Let $A = \mathbb{N} \times \mathbb{N}$ and $\circ$ be a binary operation on A defined by

$(a, b) \circ (c, d) = (ac, bd)$ for all $a, b, c, d \in \mathbb{N}$.

Discuss the commutativity and associativity of $\circ$ on A.

Watch Video Solution

29. Show that the operation $*$ on $\mathbb{Z}$, the set of integers, defined by.

$a * b = a + b - 2$ for all $a, b \in \mathbb{Z}$

(i) is a binary operation:

(ii) satisfies commutaitve and associative laws:

(iii) Find the identity elemetn in $\mathbb{Z}$,

(iv) Also find the inverse of an element $a \in \mathbb{Z}$.

Watch Video Solution

30. Prove that the operaton $*$ on $\mathbb{Q} - \{1\}$ given by $a \cdot b = a + b - ab$ for all $a, b \in \mathbb{Q} - \{1\}$

(i) is closed:

(ii) satisfies the commutative and associative laws,

(iii) Find the identity element,

(iv) Find the inverse of any element $a \in \mathbb{Q} - \{1\}$.

Watch Video Solution

31. An operation $\circ$ on $\mathbb{Q}-\{-1\}$ is defined by $a \circ b = a + b + ab$ for $a, b \in \mathbb{Q}-\{-1\}$. Find the identity element $e \in \mathbb{Q}-\{-1\}$.

Watch Video Solution

32. On the set $\mathbb{Q}^{+}$ of all positive rational numbers if the binary operation $*$ is defined by $a * b = \frac{1}{4}ab$ for all $a, b \in \mathbb{Q}^{+}$, find the identity element in $\mathbb{Q}^{+}$. Also prove that any element in $\mathbb{Q}^{+}$ is invertible.

Watch Video Solution

33. Let P(A) be the power set of a non-empty set A and a binary operation $\circ$ on P(A) is defined by $X \circ Y = X \cup Y$ for all $Y \in P(A)$. Prove that, the binary operation $\circ$ is commutative as well as associative on P(A). Find the identity element w.r.t. binary operation $\circ$ on P(A). Also prove that $\Phi \in P(A)$ is the only invertible element in P(A).

Watch Video Solution

34. Let $*$ be a binary operation on $A = \mathbb{N} \times \mathbb{N}$, defined by, $(a, b) * (c, d) = (ad + bc, bd)$ for all $(a, b)(c, d) \in A$. Prove that $A = \mathbb{N} \times \mathbb{N}$ has no identity element.

Watch Video Solution

35. A binary $\circ$ on $\mathbb{N}$ is defined by $a \circ b = L.C.M.(a, b)$ for all $a, b \in \mathbb{N}$.

(i) Examine the commutativity and associativity of $\circ$ on $\mathbb{N}$,

(ii) Find the identity element in $\mathbb{N}$,

(iii) Also find the invertible elements of $\mathbb{N}$.

Watch Video Solution

36. If $a, b \in \mathbb{Z}$, find the values of

(i)$3 +_4 1$

(ii) $7 +_5 4$

(iii) $5 +_7 1$

(iv) $4 \times_5 1$

(v) $6 \times_8 4$

(vi) $7 \times_5 4$

Watch Video Solution

37. Let $A = \{1, \omega, \omega^2\}$ be the set of cube roots of unity. Prepare the composition table for multiplication $(\times)$ on A. Show that multiplication on A is a binary operation and it is commutative on A. Find the identity element for multiplication and show that every element of A is invertible.

Watch Video Solution

38. Let $A = \{1, -1, I, -i\}$ be the set of fourth roots of unity. Prepare the composition table for multiplication $(\times)$ on A . Show that multiplication on A. Find the identity element for multiplication and show that every element of A is invertible.

Watch Video Solution

39. Complete the following multiplication table so as to define a commutative binary operation $*$ on

$S = \{a, b, c, d\}$

*	a	b	c	d
a	b	d	b	a
b		a		
c		a	d	
d		c	c	b

Watch Video Solution

40. A binary operation $*$ is defined on the set $S = \{0, 1, 2, 3, 4\}$ as follows: $a * b = a + b(\mod 5)$ Prove that $0 \in S$ is the identity element of the binary operation $*$ and each element $a \in S$ is invertible with $5 - a \in S$ being the inverse of the element a.

Watch Video Solution

Watch Video Solution

41. An operation $*$ is defined on the set $S = \{1, 2, 3, 5, 6\}$ as follows: $a * b = ab(\bmod 7)$ Construct the composition table for operation $*$ on S and discuss its important properties.

Watch Video Solution

42. The binary operation $*$ on the set $A = \{1, 2, 3, 4, 5\}$ is defined by $a * b =$ maximum of a and b. Construct the composition table of the binary operation $*$ on A.

Watch Video Solution

Exercise 3 Mcqs

1. Let A be a set of 3 elements. The number of differentity binary operations can be defined A is...

A. 3^9

B. 3^3

C. 3^2

D. 3^6

Answer: A

Watch Video Solution

2. If $a * b = a^2$ then the value of $(4 * 5) * 3$ is...

A. $(4^2 + 5^2) + 3^2$

B. $(4 + 5)^2 + 3^2$

C. $(4^2 + 5^2)^2 + 3^2$

D. $4^2 + 5^2 + 3^2$

Answer: C

Watch Video Solution

3. If the binary operation on $\mathbb{Z}$ is defined by $a * b = a^2 - b^2 + ab + 4,$ then the value of $(2 * 3) * 4$ is

--

A. 233

B. 33

C. 55

D. -55

Answer: B

Watch Video Solution

4. $\mathbb{Q}^+$ denote the set of all positive raional numbers. If the binary operation $\circ$ on $\mathbb{Q}^+$ is defined as $a \circ b = \frac{ab}{2}$, then the inverse of 3 is---

A. $\frac{4}{3}$

B. 2

C. $\frac{1}{3}$

D. $\frac{2}{3}$

Answer: A

Watch Video Solution

5. Subtraction of integer is--

A. commutative but not associative

B. comutative and associative

C. associtive but nor commutative

D. neither commutative nor associative

Answer: D

Watch Video Solution

6. Which of the following statement is true?

A. $*$ defined by $a * b = \frac{a+b}{2}$ is a binary operation on $\mathbb{Z}$

B. $*$ defined by $a * b = \frac{a+b}{2}$ is a binary operation on $\mathbb{Q}$

C. all binary commutative operations are associative

D. Subtraction is a binary operation on $\mathbb{N}$

Answer: B

Watch Video Solution

7. The binary operation $*$ defined on $\mathbb{N}$ by $a * b = a + b + ab$ for all $a, b \in \mathbb{N}$ is--

A. commutaitive only

B. associative only

C. commutative and associative both

D. none of these

Answer: C

Watch Video Solution

8. If the binary operation $\circ$ is defined on the set $\mathbb{Q}^{+}$ of all positive rational numbers by $a \circ b = \frac{ab}{4}$. Then $3 \circ \left(\frac{1}{5} \circ \frac{1}{2}\right)$ is equal to--

A. $\frac{3}{160}$

B. $\frac{5}{160}$

C. $\frac{3}{10}$

D. $\frac{3}{40}$

Answer: A

Watch Video Solution

9. If M_2 be the set of all 2×2 matrices of the form $\begin{pmatrix} a & a \\ a & a \end{pmatrix}$, where $a \in R - \{0\}$, then the identity element with respect to the multiplication of matrices as binary operation, is--

A. $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$,

B. $\begin{pmatrix} -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} \end{pmatrix}$

C. $\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

D. $\begin{pmatrix} -1 & -1 \\ -1 & -1 \end{pmatrix}$

Answer: C

Watch Video Solution

Exercise 3 Very Short Answer Type Questions

1. Define a binary opeartion $*$ on a non-empty set A.

Watch Video Solution

2. Define a commutative binary operation no a non-empty set A.

Watch Video Solution

3. Define an associative binary operation on a non-empty set S.

Watch Video Solution

4. Let $*$ and $\circ$ be two binary operations on a non-empty setA. Then write the condition for which the binary operation $*$ is distibutive over binary operation $\circ$

Watch Video Solution

5. Let P(A) be the power set of a non-empty set A. Prove that union $(\cup)$ and intersection $(\cap)$ of two subsets X and Y of A are binary operations on P(A).

Watch Video Solution

6. Let $*$ be an operation defined on $\mathbb{N}$, the set of natural numbers, by $a * b = L.C.M.(a, b)$ for all $a, b \in \mathbb{N}$. Prove that $*$ is a binary operation on $\mathbb{N}$.

Watch Video Solution

7. Let $\circ$ be an operation defined on $\mathbb{R}$. The set of real numbers, by $a \circ b = \min(a, b)$ for all $a, b \in \mathbb{R}$. Show that $\circ$ is a binary operation on $\mathbb{R}$.

Watch Video Solution

8. The operation $\circ$ is defined by $a \circ b = b^a$ on the set $Z = \{0, 1, 2, 3, \ldots\}$. Prove that $\circ$ is not a binary opeartion on Z.

Watch Video Solution

9. Let $A = \{3x + \sqrt{5}y : x, y \in \mathbb{Z}\}$. Show that an operation * on A defined by, $(3x_1 + \sqrt{5}y_1) * (3x_2 + \sqrt{5}y_2) = 3(x_1 + x_2) + \sqrt{5}(y_1 + y_2)$ for all $x_1, x_2, y_1, y_2 \in \mathbb{Z}$ is binary operation on A.

Watch Video Solution

10. Prove that the operation 'addition' on the set of irrational numbers is not a binary operation.

Watch Video Solution

11. Prove that the operation $\circ$ on $\mathbb{Q}$, the set of rational numbers, defined by $a \circ b = ab + 1$ is binary operational on $\mathbb{Q}$.

Watch Video Solution

12. Let $*$ be an opeartion defined on $S = \{1, 2, 3, 4\}$ by $a * b = m$ where m is the least non-negative remainder when the product ab is divided by 5. Prove that $*$ is a binary operation on S.

Watch Video Solution

13. An operation $*$ is defined on the set of real numbers $\mathbb{R}$ by $a * b = ab + 5$ for all $a, b \in \mathbb{R}$. Is $*$ a binary operation on $\mathbb{R}$?.

Watch Video Solution

14. Let $S = \{0, 1, 2, 3, 4\}$, if $a, b \in S$, then an operation $*$ on S is defined by, $a * b = r$ where r is the non-negative remaider when (a+b) is divided by 5. Prove that $*$ is a binary operation on S.

Watch Video Solution

15. Let M_2 be the set of all 2×2 singular matrices of the form $\begin{pmatrix} a & a \\ a & a \end{pmatrix}$ where $a \in \mathbb{R}$. On M_2 an operation $\circ$ is defined as $A \circ B = AB$ for all $A, B \in M_2$. Show that $\circ$ is a binary operation on M_2.

Watch Video Solution

16. An operation $*$ on the set of all complex numbers $\mathbb{C}$ is defined by $z_1 * z_2 = \sqrt{z_1 z_2}$ for all $z_1, z_2 \in \mathbb{C}$. Is $*$ a binary operation on $\mathbb{C}$?

Watch Video Solution

17. Show that an operation $*$ on $\mathbb{R}$, the set of real numbers, defined by $a * b = 3ab + \sqrt{2}$, for all $a, b \in \mathbb{R}$. Is a binary operaion on $\mathbb{R}$.

Watch Video Solution

18. Examine whether the operation $\circ$ on $\mathbb{Z}^{+}$ defined by $a \circ b = |a - b|$ for all $a, b \in \mathbb{Z}^{+}$, is a binary operation on $\mathbb{Z}^{+}$ or not.

Watch Video Solution

19. Prove that the operation $\wedge$ on $\mathbb{R}$ defined by $x \wedge y =$ min. of x and y for all $x, y \in \mathbb{R}$ is a binary operation on $\mathbb{R}$.

Watch Video Solution

20. Prove that the operation $*$ on $\mathbb{Z}$ defined by $a * b = a|b|$ for all $a, b \in \mathbb{Z}$ is a binary operation

Watch Video Solution

21. Prove that the operation $\circ$ on $\mathbb{Q}$ defined by $x \circ y = \frac{x-2}{y-2}$ for all $x, y \in \mathbb{Q}$ does not represent a binary operation on $\mathbb{Q}$.

Watch Video Solution

22. An operation $*$ is defined on $\mathbb{N}$ as $a * b = HCF(a, b)$ for all $a, b \in \mathbb{N}$. Show that $*$ is a binary operation on $\mathbb{N}$. Find the values of $25 * 15, 32 * 56, 9 * 11$ and $34 * 38$.

Watch Video Solution

Exercise 3 Short Answer Type Questions

1. Discuss commutativity and associativity $*$ on $\mathbb{R}$ defined by $a * b =$ min. (a, b) for all $a, b \in \mathbb{R}$.

Watch Video Solution

2. Check for commutative and associative $(\mathbb{Q}, *)$ where $x * y = x - y$ for all $x, y \in \mathbb{Q}$.

Watch Video Solution

3. Check for commutativity and associativity $\circ$ on $\mathbb{Z}$ defined by $a \circ b = a|b|$ for all $a, b \in \mathbb{Z}$.

Watch Video Solution

4. Check for commutative and associative $(\mathbb{Z}, \circ)$ where $a \circ b = a + b + ab$ for all $a, b \in \mathbb{Z}$.

Watch Video Solution

5. Discuss the commutativity and associativity $*$ on $\mathbb{R}$ defined by $a * b = |a + b|$ for all a,binRR`.

Watch Video Solution

6. Check for commutative and associative $(\mathbb{Z}^+, *)$ where $a * b = 2^{ab}$ for all $a, b \in \mathbb{Z}$.

Watch Video Solution

7. Check for commutative and associative $*$ on $\mathbb{R} - \{1\}$ defined by $a * b = \dfrac{1}{b - 1}$ for all $a, b \in \mathbb{R} - \{1\}$.

Watch Video Solution

8. Check for commutative and associative $\circ$ on $\mathbb{R}$ defined by $x \circ y$ = max (x, y) for all $x, y \in \mathbb{R}$.

Watch Video Solution

9. Discuss the commutativity and associativity$(\mathbb{Q}, \circ)$ where $x \circ y = \frac{1}{6}xy$ for all $x, y \in \mathbb{Q}$.

Watch Video Solution

10. Discuss the commutativity and associativity $*$ on $\mathbb{R}$ defined by $x * y =$ min. (x,y) for all $x, y \in \mathbb{R}$.

Watch Video Solution

11. Discuss the commutativity and associativity $*$ on $\mathbb{Q}$ defined by$a * b = ab + 4$ for all $a, b \in \mathbb{Q}$.

Watch Video Solution

12. Discuss commutativity and associativity $*$ on $\mathbb{Z}$ defined by $a * b = a + b + 3$ for all $x, y \in \mathbb{Z}$.

Watch Video Solution

13. Check for commutative and associative $(\mathbb{N}, *)$ where $a * b = \gcd(a, b)$ for all $a, b \in \mathbb{N}$.

Watch Video Solution

14. Check for commutativity and associativity $*$ on $\mathbb{Z}$ defined by $a * b = |a|b$ for all $a, b \in \mathbb{Z}$.

Watch Video Solution

15. Discuss the commutativity and Associativity $*$ on $\mathbb{Q}$ defined by $x * y = \frac{1}{2}(x + y)$ for all $x, y \in \mathbb{Q}$.

Watch Video Solution

16. Discuss the commutativity and associativity $*$ on $\mathbb{R}$ defined by $a * b = |ab|$ for all $a, b \in \mathbb{R}$.

Watch Video Solution

17. $*$ on $\mathbb{Z} \times \mathbb{Z}$ defined by $(a, b) * (c, d) = (a - c, b - d)$ for all $(a, b), (c, d) \in \mathbb{Z} \times \mathbb{Z}$.

Watch Video Solution

18. Check commutativity and associativity $\circ$ on $M_2(\mathbb{R})$ defined by $A \circ B = \frac{1}{2}(AB - BA)$ for all $A, B \in M_2(\mathbb{R})$ where $M_2(\mathbb{R})$ is a 2×2 real matrix.

Watch Video Solution

19. An operation $*$ on $\mathbb{Z}$, the set of integers, is defined as, $a * b = a - b + ab$ for all $a, b \in \mathbb{Z}$. Prove that $*$ is a binary operation on $\mathbb{Z}$ which is neither commutative nor associative.

Watch Video Solution

20. (I) Let $*$ be a binary operation defined by $a * b = 2a + b - 3$. Find $3 * 4$.

(ii) let $*$ be a binary operation on $\mathbb{R} - \{-1\}$, defined by $a * b = \frac{1}{b+1}$ for all $a, b \in \mathbb{R} - \{-1\}$ Show that $*$ is neither commutative nor associative.

(iii) Let $*$ be a binary operation on the set $\mathbb{Q}$ of all raional numbers, defined as $a * b = (2a - b^2)$ for all $a, b \in \mathbb{Q}$. Find $3 * 5$ and $5 * 3$. Is $3 * 5 = 5 * 3$?

Watch Video Solution

21. A binary operaiton $\circ$ is defined on the set $\mathbb{R} - \{-1\}$ as $x \circ y = x + y + xy$ for all $x, y \in \mathbb{R} - \{-1\}$. Discuss the commutativity and associativity of $\circ$ on $\mathbb{R} - \{-1\}$. If $(3 * 2x) * 5 = 71,$, find x.

Watch Video Solution

22. If $*$ be the binary operation on the set $\mathbb{Z}$ of all integers, defined by $a * b = a + 3b^2$, find $2 * 4$.

Watch Video Solution

23. A binary operation $\circ$ is defined on $\mathbb{Z}$, the set of integers, by $a \circ b = |a - b|$ for all $a, b \in \mathbb{Z}$. Find the value of $3a \circ 2b$ when $a = -3$ and $b = -2$

Watch Video Solution

24. Let $*$ a binary operation on $\mathbb{N}$ given by $a * b = H.C.F(a, b)$ for all $a, b \in \mathbb{N}$, write the value of $22 * 4$

Watch Video Solution

25. If $+_6$ (addition modulo 6) is a binary operation on $A = \{0, 1, 2, 3, 4, 5\}$, find the value of $3 +_6 3^{-1} +_6 2^{-1}$. [note that the identity element is 0 and the inverse of the element 2 is 4 as $2 +_6 4 = 0$, the identity element.]

Watch Video Solution

26. A binary operation $*$ is defined on the set $\mathbb{R}_0$ for all non- zero real numbers as $a * b = \frac{ab}{3}$ for all $a, b \in \mathbb{R}_0$, find the identity element in $\mathbb{R}_0$.

Watch Video Solution

27. A binary operation $*$ is defined on the set $\mathbb{Z}$ of all integers by $a \circ b = a + b - 3$ for all $a, b \in \mathbb{Z}$. Determine the inverse of $5 \in \mathbb{Z}$.

Watch Video Solution

28. A binary operation $*$ is defined on the set of real numbers $\mathbb{R}$ by $a * b = 2a + b - 5$ for all $a, b \in \mathbb{R}$. If $3 * (x - 2) = 20$ find x.

Watch Video Solution

29. For the binary operation multiplication modulo $5 \cdot (\times_5)$ defined on the set $A = \{1, 2, 3, 4\}$, find the value of $\{2 \times_5 3^{-1}$. [Note that the inverse of 3 is 2]

Watch Video Solution

30. A binary operation $*$ on $\mathbb{Q}$ the set of all rational numbers is defined as $a * b = \frac{1}{2}ab$ for all $a, b \in \mathbb{Q}$. Prove that $*$ is commutative as well as associative on $\mathbb{Q}$.

Watch Video Solution

31. Prove that the identity element of the binary opeartion $*$ on $\mathbb{R}$ defined by $a * b$= min. (a,b) for all $a, b \in \mathbb{R}$, does not exist.

Watch Video Solution

32. Find the identity element of the binary operation $*$ on $\mathbb{Z}$ defined by $a * b = a + b + 1$ for all $a, b \in \mathbb{Z}$.

Watch Video Solution

33. The binary operation * define on N by a*b = a+b+ab for all $a, b \in N$ is

Watch Video Solution

34. Prove that 0 is the identity element of the binary operation $*$ on $\mathbb{Z}^+$ defined by $x * y = x + y$ for all $x, y \in \mathbb{Z}^+$.

Watch Video Solution

35. A binary operation $*$ on $\mathbb{Q}_0$, the set of all non-zero rational numbers, is defined as $a * b = \frac{1}{3}ab$ for all $a, b \in \mathbb{Q}_0$ Prove that every element of $\mathbb{Q}_0$, is invertible and find the inverse of the element $\frac{3}{5} \in \mathbb{Q}_0$.

Watch Video Solution

36. A binary operation $\circ$ on $\mathbb{Q} - \{1\}$ is defined by $a * b = a + b - ab$ for all $a, b \in \mathbb{Q} - \{1\}$. Prove that every element of $\mathbb{Q} - \{1\}$ is invertible.

Watch Video Solution

37. A binary operation $*$ on $\mathbb{Q}$, the set of rational numbers, is defined by $a * b = \dfrac{a - b}{3}$ fo rall $a, b \in \mathbb{Q}$. Show that the binary opearation $*$ is neither commutative nor associative on $\mathbb{Q}$.

Watch Video Solution

38. Determine which of the following binary operations are associative and which are commutative:

(i) $*$ on $\mathbb{R}$ defined by $a * b = 1$ for all $a, b \in \mathbb{R}$.

(ii) $*$ on $\mathbb{R}$ defined by $a * b = \frac{a+b}{2}$ for all $a, b \in \mathbb{R}$.

Watch Video Solution

39. Let S be any set containing more than two elements. A binary operation $\circ$ is defined on S by $a \circ b = b$ for all $a, b \in S$. Discuss the commutativity and associativity of $\circ$ on S.

Watch Video Solution

40. State whether the following statements are true or false with reasons.

(i) For any binary operation $*$ on $\mathbb{N}$, $a * a = a \forall a \in \mathbb{N}$.

(ii) If $*$, a binary operation $*$ on $\mathbb{N}$ is commutative then, $a * (b * c) = (c * b) * a$.

Watch Video Solution

Exercise 3 Long Answer Type Questions

1. A binary operation $\circ$ is defined on $\mathbb{R} - \{-1\}$ by $a \circ b = a + b + ab$ for all $a, b \in \mathbb{R} - \{-1\}$.

(i) Discuss the commutativity and associativity of $\circ$ on $\mathbb{R} - \{-1\}$.

Find the identity element, if exists.

(iii) Prove that every element of $\mathbb{R} - \{-1\}$ is invertible.

Watch Video Solution

2. The binary operation multiplication modulo $10(\times_{10})$ is defined on the set $A = \{0, 1, 3, 7, 9, \}$, find the inverse of the element 7.

Watch Video Solution

3. Construct the composition table for the binary operation multiplication modulo $5(\times_5)$ on the set $A = \{0, 1, 2, 3, 4\}$.

Watch Video Solution

4. A binary operation $*$ on $\mathbb{N}$ is defined by $a * b = L. C. M. (a, b)$ for all $a, b \in \mathbb{N}$.

(i) Find $15 * 18$

(ii) Show that $*$ is commutative as well as associative on $\mathbb{N}$.

(iii) Find the the identity element in $\mathbb{N}$.

(iv) Also find the invertible element in $\mathbb{N}$.

Watch Video Solution

5. Let P(A) b the power set of non-empty set A. A binary operation $*$ is defined on P(A) as $X * Y = X \cap Y$ for all

$X, Y \in P(A)$. Detemine the identity element In P(A).

Prove that A is the only invertible element in P(A).

Watch Video Solution

6. A binary operation $*$ is defined in $\mathbb{Q}_0$, the set of all non-zero rational numbers, by $a * b = \frac{ab}{3}$ for all $a, b \in \mathbb{Q}_0$. Find the identity element in $\mathbb{Q}_0$. Also find the inverse of an element $x \in \mathbb{Q}_0$.

Watch Video Solution

7. Let $P(C)$ be the power set of non-empty set C. A binary operation $*$ is defined on P(C) as $A * B = (A - B) \cup (B - A)$ for all $A, B \in P(C)$. Prove that Φ is the identity element for $*$ on P(C) and A is invertible for all $A \in \mathbb{Q}_0$.

Watch Video Solution

8. Let $A = N \cup \{0\} \times \mathbb{N} \cup \{0\}$, a binary operation $*$ is defined on A by. $(a, b) * (c, d) = (a + c, b + d)$ for all $(a, b), (c, d) \in A$. Prove that $*$ is commutative as well as associative on A. Show also that (0,0) is the identity element In A.

Watch Video Solution

9. Let $A = \{0, 1, 2, 3, 4, 5\}$ be a given set, a binary operation $\circ$ is defined on A by $a \circ b = ab(\mod 6)$ for all

$a, b \in A$. Find the identity element for $\circ$ in A . Show that 1 and 5 are the only invetible elements in A.

Watch Video Solution

10. Let $A = \mathbb{N} \times \mathbb{N}$, a binary operation $*$ is defined on A by $(a, b) * (c, d) = (ad + bc, bd)$ for all $(a, b), (c, d) \in A$. Show that $*$ possesses no identity element in A.

Watch Video Solution

11. Find the values of

(i) $4 +_6 2$ (ii) $7 +_5 7$ (iii) $5 +_8 2$

(iv) $3 \times_7 2$ (v) $12 \times_{10} 5$ (vi) $6 \times_5 4$

Watch Video Solution

12. Let $M_2(x)d = \left\{\begin{pmatrix} x & x \\ x & x \end{pmatrix}, x \in \mathbb{R}\right\}$ be the set of 2×2 singular matrices. Considering multiplication of matrices as a binary operation, find the identity element in $M_2(x)$. Also find the inverse of an element of M_2.

Watch Video Solution

13. Prepare the composition table for addition modulo $6(\ +_6\ )$ on $A = \{0, 1, 2, 3, 4, 5\}$.

Watch Video Solution

14. Prepare the composition table for multiplication modulo 6 ($\times_6$) on $A = \{0, 1, 2, 3, 4, 5\}$.

Watch Video Solution

15. A binary operaiton $\wedge$ on the set $A = \{1, 2, 3, 4, 5\}$ is defined by $a \wedge b = \min (a, b)$ for all $a, b \in A$. prepare composition table for the operation $\wedge$ on A.

Watch Video Solution

16. Let $S = \{1, \omega, \omega^2\}$ be the set of cube roots of unity. Prepare the composition table for multiplication ($\times$) on S, show that multiplication on S is a binary operation and it is commutative on S. Also, show that every element on S is invertible.

Watch Video Solution

17. Prepare the composition table for multiplication $(\times)$ on the set of fourth roots of unity and discuss its important properties.

Watch Video Solution

18. Complete the following nultiplication table so as to define a commutative binary operation $*$ on

$A = \{1, 2, 3, 4\}$.

*	1	2	3	4
1	2	4	2	1
2		1		
3		1	4	
4		3	3	2

Watch Video Solution

19. A binary operation $\circ$ is defined on the set $A = \{0, 1, 2, 3, 4, 5\}$ as follows: $a \circ b = a + b (\bmod 6)$ for all $a, b \in A$. Prove that $o \in A$ is the identity element In A is invertible with operation $\circ$ and each element A is invertible with $6 - a \in A$ being the inverse of the element a.

Watch Video Solution

20. An operation $*$ is defined on the set $A = \{1, 2, 3, 4\}$ as follows: $a * b, ab(\mod 5)$ for all $a, b \in A$.Prepare the compositon table for $*$ on A and from the table show that -

(i) multiplication $\mod (5)$ is a binary operation,

(ii) $*$ is commutative on A,

(iii) is the identity element for multiplication $\mod (5)$ on A, and.

(iv) every element of A is invertible.

21. Let $A = \{1, -1\}$ be the set of square roots of unity. Considering multiplication $(\times)$ as a binary operation on A, construct the composition table for multiplication on A. Determine the identity element for multiplication in A and the inverses of the elements.

Watch Video Solution

22. Let $*$ be the binary defined on the set $S = \{1, 2, 3, 4, 5, 6\}$ by $a * b = r$ where r is the least non-negative remainder when ab is divided by 7. Prepare the composition table $*$ on S. Observing the composition table show that 1 is the identity element for $*$ and every element of S is invertible.

Watch Video Solution

Mcqs

1. Consider the binary operations $* : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ and $\circ : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ defined as $a * b = |a - b|$ and $a \circ b = a$ for all $a, b \in \mathbb{R}$ then-

A. $*$ is commutative but not associative on $\mathbb{R}$

B. $\circ$ is associative on $\mathbb{R}$.

C. $\circ$ is not distribution over $*$

D. $\circ$ is commutative on $\mathbb{R}$

Answer: A,B,C

Watch Video Solution

Watch Video Solution

2. If the binary oprations $*$ on $\mathbb{R}$ is defined by $a * b = a + b + ab$ for all $a, b \in \mathbb{R}$ where on R.H.S. we have usual addition, subtraction and multiplication of real numbers. The relation $*$ is---

A. not commutative

B. associative

C. commutative

D. not associative

Answer: B,C

Watch Video Solution

3. Let $*$ be a binary operation on $\mathbb{N}$, the set of natural numbers defined by $a * b = a^b$ for all $a, b \in \mathbb{N}$ is $*$ associative or commutative on $\mathbb{N}$?

A. not commutative

B. associative

C. commutative

D. not associative

Answer: A,D

Watch Video Solution

4. Let $*$ be a binary operation on set $\mathbb{Q} - \{1\}$ defined by $a * b = a + b - ab \in \mathbb{Q} - \{1\}$. e is the identity element with respect to $*$ on $\mathbb{Q}$. Every element of $\mathbb{Q} - \{1\}$ is invertible, then value of e and inverse of an element a are---

A. 0

B. 1

C. $\dfrac{a}{a-1}$

D. $\dfrac{a}{a+1}$

Answer: A,C

Watch Video Solution

5. Consider the set $A = \{1, -1, i, -i\}$ of four roots of unity. Constructing the composition table for multiplication on S. which of the properties are true?

A. a binary operation on S

B. commutative on S

C. 1 is the identity element

D. i is the identity element

Answer: A,B,C

Watch Video Solution

Integar Answer Type

1. Let $S = \{a, b, c\}$, the total number of binary operations on S be K^9. Find the value of K.

Watch Video Solution

2. Let $*$ be a binary on $\mathbb{N}$ defined by $a * b = L.C.M.(a, b)$ (for all$a, b \in \mathbb{N}$). $2 * 4 = \lambda$ then λ will be---

Watch Video Solution

3. If the binary operation $*$ on the set $\mathbb{Z}$ is defined by $a * b = a + b - 5$, then the identity element with respect to $*$ is K. Find the value of K.

Watch Video Solution

4. Let $*$ be a binary operation on $\mathbb{Q}_0$ (Set of all non-zero rational numbers) defined by $a * b = \frac{ab}{4}, a, b \in \mathbb{Q}_0$. The identity element in $\mathbb{Q}_0$ is e, then the value of e is--

Watch Video Solution

5. The total number of binary operations on the set $S = \{1, 2\}$ having 1 as the identity element is n . Find n.

Watch Video Solution

Matrix Match Type

1. Match the following Column I and Column II

1.	Column I		Column II
(A)	The brinary operation $*$ on $\mathbb{Q}$ defined by $a * b = \frac{ab}{2}$ for all $a, b \in \mathbb{Q}$ is	(p)	neither commutative nor associative
(B)	Let A be any set containing more than one element. The binary operation $*$ on A defined by $a * b = b$ for all $a, b \in A$ is	(q)	commutative but not associative
(C)	The binary operation $*$ on $\mathbb{Q}$ defined by $a * b = ab^2$ for all $a, b \in \mathbb{Q}$ is	(r)	commutative and associative both
(D)	The binary operation $*$ on $\mathbb{Q}$ defined by $a * b = ab + 1$ for all $a, b \in \mathbb{Q}$ is	(s)	not commutative but associative

Watch Video Solution

2. Match the coloumn

2.	Column I		Column II
(A)	Let $*$ be a binary operation on $\mathbb{N}$ given by, $a * b =$ L.C.M. (a, b) for all $a, b \in \mathbb{N}$, then the identity element in $\mathbb{N}$ is	(p)	$\frac{25}{a}$
(B)	On $\mathbb{Q}$, the set of all rational numbers, a binary operation $*$ is defined by $a * b = \frac{ab}{5}$ for all $a, b \in \mathbb{Q}$, then the inverse element in $\mathbb{Q}$ is	(q)	$\frac{16}{a}$
(C)	Let $*$ be a binary operation on $\mathbf{Q}_0$ (Set of non-zero rational numbers) defined by $a * b = \frac{3ab}{5}$ for all $a, b \in \mathbf{Q}_0$ then the identity element in $\mathbf{Q}_0$ is	(r)	1
(D)	Let $*$ be a binary operation on $\mathbf{Q}_0$ (Set of all non-zero rational numbers) defined by $a * b = \frac{ab}{4}$ for all $a, b \in \mathbf{Q}_0$ then the inverse element in $\mathbf{Q}_0$ is	(s)	$\frac{5}{3}$

Watch Video Solution

Comprehension Type

1. Let $A = \mathbb{R}_0 \times \mathbb{R}$ where $\mathbb{R}_0$ denote the set of all non-zero real numbers. A binary operation $*$ is defined on A as follows: $(a, b) * (c, d) = (ac, bc + d)$ for all $(a, b), (c, d) \in \mathbb{R}_0 \times \mathbb{R}$.

Binary operation $*$ is--

A. commutative but not associative A

B. commutative and associative on A

C. associative but not commutative on A

D. none of these

Answer: B

Watch Video Solution

2. Let $A = \mathbb{R}_0 \times \mathbb{R}$ where $\mathbb{R}_0$ denote the set of all non-zero real numbers. A binary operation $*$ is defined on A as follows: $(a, b) * (c, d) = (ac, bc + d)$ for all $(a, b), (c, d) \in \mathbb{R}_0 \times \mathbb{R}$.

Binary operation $*$ is--Identity element in A is--

A. (0,1)

B. (0,0)

C. (1,0)

D. (1,1)

Answer: C

Watch Video Solution

3. Let $A = \mathbb{R}_0 \times \mathbb{R}$ where $\mathbb{R}_0$ denote the set of all non-zero real numbers. A binary operation $*$ is defined on A as follows: $(a, b) * (c, d) = (ac, bc + d)$ for all $(a, b), (c, d) \in \mathbb{R}_0 \times \mathbb{R}$.

The inveritible elements in A is---

A. $\left(-\frac{1}{b}, -\frac{b}{a}\right)$

B. $\left(-\frac{1}{b}, \frac{b}{a}\right)$

C. $\left(\frac{1}{b}, \frac{b}{a}\right)$

D. $\left(\frac{1}{a}, -\frac{b}{a}\right)$

Answer: D

Watch Video Solution

4. Let the set $S = \{f_1, f_2, f_3, f_4\}$ of four functions from $\mathbb{C}$ (the set of all complex numbers) to itself, defined by $f_1(z) = z,\ f_2(z) = -z,\ f_3(z) = \dfrac{1}{z}$ and $f_4(z) = -\dfrac{1}{z}$ for all $z \in \mathbb{C}$ Construct the composition table for the composition of functions ($\circ$) defined on the set S.

Value of $f_4 \circ f_1(z)$ is ---

A. f_1

B. f_2

C. f_3

D. f_4

Answer: D

Watch Video Solution

5. Let the set $S = \{f_1, f_2, f_3, f_4\}$ of four functions from $\mathbb{C}$ (the set of all complex numbers) to itself, defined by $f_1(z) = z,\ f_2(z) = -z,\ f_3(z) = \frac{1}{z}$ and $f_4(z) = -\frac{1}{z}$ for all $z \in \mathbb{C}$ Construct the composition table for the composition of functions ($\circ$) defined on the set S.

value of $f_2 \circ f_1(z)$ is--

A. f_1

B. f_2

C. f_3

D. f_4

Answer: D

Watch Video Solution

6. Let the set $S = \{f_1, f_2, f_3, f_4\}$ of four functions from $\mathbb{C}$ (the set of all complex numbers) to itself, defined by $f_1(z) = z$, $f_2(z) = -z$, $f_3(z) = \frac{1}{z}$ and $f_4(z) = -\frac{1}{z}$ for all $z \in \mathbb{C}$ Construct the composition table for the composition of functions ($\circ$) defined on the set S.

Value of $f_2 \circ f_4(z)$ is---

A. f_1

B. f_2

C. f_3

D. f_4

Answer: C

Watch Video Solution

Assertion Reason Type

1. Let S be a non-empty set and P(S) be the power set of the Set S.

Statement -I: Φ is the identity element for union as a binary operation on P(S)

Statement -II: S is the identity element for intersection on P(S).

A. Statement -I is True Statement -II is True , Statement -II is a correct explanation for Statement -I

B. Statement -I is True. Statement -II is True, Statement -II is not a correct explanition for Statement -I

C. Statement -I is True, Statement -II is False.

D. Statement -I is False. Statement -II is True.

Answer: B

Watch Video Solution

2. On $\mathbb{R} - \{1\}$, a binary operation $*$ is defined by $a * b = a + b - ab$

Statement - I: Every element of $\mathbb{R} - \{1\}$ is inveritble

Statement -II: o is the identity element for * on $\mathbb{R} - \{1\}$.

A. Statement -I is True Statement -II is True , Statement -II is a correct explanation for Statement -I

B. Statement -I is True. Statement -II is True, Statement -II is not a correct explanition for Statement -I

C. Statement -I is True, Statement -II is False.

D. Statement -I is False. Statement -II is True.

Answer: B

Watch Video Solution