



MATHS

BOOKS - CHHAYA PUBLICATION MATHS (BENGALI ENGLISH)

BINARY OPERATION



1. Let P(A) be the power set of a non-empty set A. Prove that union (\cup) and intersection (\cap) of two subsets X and Y of A are binary operations on P(A).



2. Let * be an operation defined on A, $= \{2, 4, 6, 8\}$ by a * b = k where k is the least non-negative remainder when the product ab is divided by 10 and $a, b \in A$. show that * is a binary operation on A.



3. Let S be a set of two elements. How many different

binary operaions can be defined on S?



4. The operation * is defined by $a * b = a^b$ on the set $Z = \{0, 1, 2, 3, \ldots\}$. Show that * is not a binary operation.

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5. Let $S = \sqrt{3}x + 2y$: $x, y \in Z$ }. Prove that the operation * on S defined by $(\sqrt{3}x_1 + 2y) * (\sqrt{3}x_2 + 2y_2) = \sqrt{3}(x_1 + x_2) + 2(y_1 + y_2)$ for all $x_1, x_2, y_1, y_2 \in Z$ is closed under * .

6. Let $A = \{0, 1, 2, 3, 4, 5\}$. If $a_1b \in A$, then an operation \circ on A is defined by $a \circ b = k$ where k is the least non-negative remainder when the sum (a + b) is divided by 6. Show that \circ is a binary operation on A.



7. Let $\mathbb R$ be the set of real numbers and $x, y \in R$. We define operaitons \wedge and \vee on $\mathbb R$ as

- $x \wedge y = \max$ maximum of x and y,
- $x \lor y =$ minimum of x and y.

Show the operation $~\wedge~$ and $~\vee~$ defined above are binary

operations on \mathbb{R} .



8. On the set C of all complex numbers an operation 'o' is defined by z_1 o $z_2 = \sqrt{z_1 z_2}$ for all $z_1, z_2 \in C$. Is o a binary operation on C ?

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9. Determine whether * on N defined by $a \cdot b = a^b$ for all $a, b \in N$ define a binary operation on the given set or not:

10. On Q , the set of all rational numbers a binary operation * is defined by $a\cdot b=rac{a+b}{2}$. Show that * is not associative on Q.

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11. Let \mathbb{R} be the set of real numbers. Show that the operation * defined on $\mathbb{R} - \{0\}$ by $a * b = |ab|, a, b \in \mathbb{R} - \{0\}$ is a binary operation on $\mathbb{R} - \{0\}$.

12. Let $M_2=egin{bmatrix} x & 0 \ 0 & y \end{bmatrix}, x,y\in \mathbb{R}-\{0\}$ be the set of 2 imes 2

matrices, prove that the operation * defined on M_2 by

 $A * B = AB, A, B \in M_2$ is a binary operation.

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13. Let S = (0, 1, 2, 3, 4,) and * be an operation on S defined by $a \cdot b = r$, where *r* is the least non-negative remainder when a + b is divided by 5. Prove that * is a binary operation on S.

14. Is
$$\circ$$
 defined on \mathbb{Q} the set of rational numbers, by
 $a \circ b = \frac{a-1}{b-1} (a, b \in \mathbb{Q})$, a binary operation?

15. Prove that an operation * on \mathbb{R} , the set of real numbers, defined by $x * y = 2xy + \sqrt{5}$, for all $x, y \in \mathbb{R}$, is a binary operation on \mathbb{R} .

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16. Let \circ be a binary operation on \mathbb{Q} , the set of rational numbers, defined by $a \circ b = \frac{1}{8}ab$ for all $a, b \in \mathbb{Q}$. Prove that \circ is commutive as well as associative.



17. let * be a binary operation on \mathbb{Z}^+ , the set of positive integers, defined by $a * b = a^b$ for all $a, b \in \mathbb{Z}^+$. Prove that * is neither commutative nor associative on \mathbb{Z}^+ .



18. Show that the binary operation * defined on \mathbb{R} by

a * b = ab + 2 is commutative but not associative.



19. let * be a binary operation on \mathbb{R} , the set of real numbers, defined by $a \circ b = \sqrt{a^2 + b^2}$ for all $a, b \in \mathbb{R}$. Prove that the binary operation \circ is commutative as well as associative.



20. Discuss the commutativity and associativity of binary operation * defined on \mathbb{Z} by the rule

a * b = |a|b for all $a, b \in \mathbb{Z}$.



21. Show that the operation * defined on $\mathbb{R} - \{0\}$ by a * b = |ab| is a binary operation. Show also that * is commutative and associative.

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22. Prove that the binary operation * on \mathbb{R} defined by

a*b=a+b+ab for all $a,b\in\mathbb{R}$

is commutative and associative.



23. Prove that the binary operation \circ defined on \mathbb{Q} by $a \circ b = a - b + ab$ for all a,b in \mathbb{Q} is neither commutative

nor associative.



24. Let S be a set of containing more than two elements and a binary operaton \circ on S be defined by $a \circ b = a$ for all $a, b \in S$.

Prove that \circ is associative but not commutative on .



25. let * be a binary on \mathbb{Q} , defined by

 $a st b = \left(a - b
ight)^2$ for all $a, b \in \mathbb{Q}.$ Show that the binary

operation * on \mathbb{Q} is commutative but not associative.

26. Let * and \circ be two binary operations on \mathbb{R} defined as,

 $a * b = |a - b| ext{ and } a \circ b = a ext{ for all } a, b \in \mathbb{R}.$

Examine the commutativity and associativity of * and \circ on \mathbb{R} . Show also that * is distributative over \circ but \circ is not distributive over *.



27. Let $S = \mathbb{N} imes \mathbb{N}$ and * is a binary operation on S defined by

(a,b)*(c,d)=(a+c,b+d) for all $a,b,c,d\in\mathbb{N}.$

Prove that * is a commutative and associative binary

operation on S.



28. Let $A = \mathbb{N} \times \mathbb{N}$ and \circ be a binary operation on A defined by

 $(a,b)\circ(c,d)=(ac,bd)$ for all $a,b,c,d\in\mathbb{N}.$

Discuss the commutativity and associativity of \circ on A.

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29. Show that the operation * on \mathbb{Z} , the set of integers, defined by.

a*b=a+b-2 for all $a,b\in\mathbb{Z}$

(i) is a binary operation:

(ii) satisfies commutaitve and associative laws:

(iii) Find the identity elemetn in \mathbb{Z} ,

(iv) Also find the inverse of an element $a\in\mathbb{Z}.$

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30. Prove that the operaton * on $\mathbb{Q}-\{1\}$ given by

$$a \cdot b = a + b - ab$$
 for all $a, b \in \mathbb{Q} - \{1\}$

(i) is closed:

(ii) satisfies the commutative and associative laws,

(iii) Find the identity element,

(iv) Find the inverse of any element $a \in \mathbb{Q} - \{1\}$.



31. An operation \circ on $\mathbb{Q} - \{-1\}$ is defined by $a \circ b = a + b + ab$ for $a, b \in \mathbb{Q} - \{-1\}$. Find the identity element $e \in \mathbb{Q} - \{-1\}$.

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32. On the set \mathbb{Q}^+ of all positive rational numbers if the binary operation * is defined by $a * b = \frac{1}{4}ab$ for all $a, b \in \mathbb{Q}^+$, find the identity element in \mathbb{Q}^+ . Also prove that any element in \mathbb{Q}^+ is invertible.

33. Let P(A) be the power set of a non-empty set A and a binary operation \circ on P(A) is defined by $X \circ Y = X \cup Y$ for all $Y \in P(A)$. Prove that, the binary operation \circ is commutative as well as associative on P(A). Find the identity element w.r.t. binary operation \circ on P(A). Also prove that $\Phi \in P(A)$ is the only invertible element in P(A).



34. Let * be a binary operation on $A = \mathbb{N} \times \mathbb{N}$, defined by, (a, b) * (c, d) = (ad + bc, bd) for all $(a, b)(c, d) \in A$. Prove that $A = \mathbb{N} \times \mathbb{N}$ has no identity element.

- **35.** A binary \circ on \mathbb{N} is defined by $a \circ b = L. \ C. \ M. \ (a, b)$ for all $a, b \in \mathbb{N}.$
- (i) Examine the commutativity and associativity of \circ on \mathbb{N} ,
- (ii) Find the identity element in \mathbb{N} ,
- (iii) Also find the invertible elements of \mathbb{N} .



36. If $a, b \in \mathbb{Z}$, find the values of

- (i) $3 +_4 1$
- (ii) $7 +_5 4$
- (iii) $5+_7 1$
- (iv) $4 imes_5 1$

(v) $6 imes_8 4$

(vi) $7 imes_5 4$

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37. Let $A = \{1, \omega, \omega^2\}$ be the set of cube roots of unity. Prepare the composition table for multiplication (\times) on A. Show that multiplication on A is a binary operation and it is commutative on A. Find the identity element for multiplication and show that every element of A is invertible.

38. Let $A = \{1, -1, I, -i\}$ be the set of fourth roots of unity. Prepare the composition table for multiplication (\times) on A . Show that multiplication on A. Find the identity element for multiplication and show that every element of A is invertible.



39. Complete the following multiplication table so as to

define a commutative binary operation * on

 $S = \{a, b, c, d\}$

6	С	b	а	*
6	b	d	b	a
		а		b
	d	а		с
1	С	С		d

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40. A binary operation * is defined on the set $S = \{0, 1, 2, 3, 4\}$ as follows: $a * b = a + b \pmod{5}$ Prove that $0 \in S$ is the identity element of the binary operation * and each element $a \in S$ is invertible with $5 - a \in S$ being the inverse of the element a. **41.** An operation * is defined on the set $S = \{1, 2, 3, 5, 6\}$ as follows: $a * b = ab \pmod{7}$ Construct the composition table for operation * on S and discuss its important properties.

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42. The binary operation * on the set $A = \{1, 2, 3, 4, 5\}$

is defined by a * b = maximum of a and b. Construct the

composition table of the binary operation * on A.



1. Let A be a set of 3 elements. The number of differentity binary operations can be defined A is...

A. 3⁹ B. 3³ C. 3²

D. 3^{6}

Answer: A



2. If $a * b = a^2$ then the value of (4 * 5) * 3 is...

A.
$$\left(4^2+5^2\right)+3^2$$

B. $(4+5)^2+3^2$
C. $\left(4^2+5^2\right)^2+3^2$
D. $4^2+5^2+3^2$

Answer: C



3. If the binary operation on $\mathbb Z$ is defined by $a*b=a^2-b^2+ab+4,$ then the value of (2*3)*4 is

A. 233

B. 33

C. 55

D. -55

Answer: B

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4. \mathbb{Q}^+ denote the set of all positive raional numbers. If the binary operation \circ on \mathbb{Q}^+ is defined as $a \circ b = \frac{ab}{2}$, then the inverse of 3 is---

A.
$$\frac{4}{3}$$

B. 2

C.
$$\frac{1}{3}$$

D. $\frac{2}{3}$

Answer: A



- 5. Subtraction of integer is--
 - A. commutative but not associative
 - B. comutative and associative
 - C. associtive but nor commutative
 - D. neither commutative nor associative

Answer: D



6. Which of the following statement is true?

A. * defined by $a * b = rac{a+b}{2}$ is a binary operation on $\mathbb Z$

B. * defined by $a * b = rac{a+b}{2}$ is a binary operation on $\mathbb Q$

C. all binary commutative operations are associative

D. Subtraction is a binary operation on $\mathbb N$

Answer: B





a * b = a + b + ab for all $a, b \in \mathbb{N}$ is--

A. commutaitive only

B. associative only

C. commutative and associative both

D. none of these

Answer: C

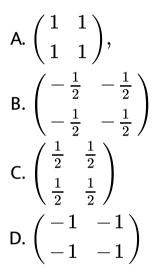


8. If the binary operation \circ is defined on the set \mathbb{Q}^+ of positive rational numbers by $a \circ b = rac{ab}{A}$. Then all $3 \circ \left(rac{1}{5} \circ rac{1}{2}
ight)$ is equal to--A. $\frac{3}{160}$ B. $\frac{5}{160}$ C. $\frac{3}{10}$ D. $\frac{3}{40}$

Answer: A



9. If M_2 be the set of all 2×2 matrices of the form $\begin{pmatrix} a & a \\ a & a \end{pmatrix}$, where $a \in R - \{0\}$, then the identity element with respect to the multiplication of matrices as binary operation, is--



Answer: C



1. Define a binary opeartion * on a non-empty set A.

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2. Define a commutative binary operation no a non-empty

set A.

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3. Define an associative binary operation on a non-empty

set S.



4. Let * and \circ be two binary operations on a non-empty setA. Then write the condition for which the binary operation * is distibutive over binary operation \circ

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5. Let P(A) be the power set of a non-empty set A. Prove that union (\cup) and intersection (\cap) of two subsets X and Y of A are binary operations on P(A).



6. Let * be an operation defined on \mathbb{N} , the set of natural numbers, by a * b = L. C. M. (a, b) for all $a, b \in \mathbb{N}$. Prove that * is a binary operation on \mathbb{N} .

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7. Let \circ be an operation defined on \mathbb{R} . The set of real numbers, by $a \circ b = \min(a, b)$ for all $a, b \in \mathbb{R}$. Show that \circ is a binary operation on \mathbb{R} .

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8. The operation \circ is defined by $a \circ b = b^a$ on the set $Z = \{0, 1, 2, 3, \ldots\}$. Prove that \circ is not a binary



9. Let
$$A = \left\{ 3x + \sqrt{5}y \colon x, y \in \mathbb{Z} \right\}$$
. Show that an operation $*$ on A defined by,

 $ig(3x_1+\sqrt{5}y_1ig)*ig(3x_2+\sqrt{5}y_2ig)=3(x_1+x_2)+\sqrt{5}(y_1+y_2)$

for all $x_1, x_2, y_1, y_2 \in \mathbb{Z}$ is binary operation on A.

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10. Prove that the operation 'addition' on the set of irrational numbers is not a binary operation.



11. Prove that the operation \circ on \mathbb{Q} , the set of rational numbers, defined by $a \circ b = ab + 1$ is binary operational on \mathbb{Q} .

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12. Let * be an opeartion defined on $S = \{1, 2, 3, 4\}$ by a * b = m where m is the least non-negative remainder when the product ab is divided by 5. Prove that * is a binary operation on S.



13. An operation * is defined on the set of real numbers \mathbb{R} by a * b = ab + 5 for all $a, b \in \mathbb{R}$. Is * a binary operation on \mathbb{R} ?.

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14. Let $S = \{0, 1, 2, 3, 4\}$, if $a, b \in S$, then an operation * on S is defined by, a * b = r where r is the nonnegative remaider when (a+b) is divided by 5. Prove that * is a binary operation on S.



15. Let M_2 be the set of all 2×2 singular matrices of the form $\begin{pmatrix} a & a \\ a & a \end{pmatrix}$ where $a \in \mathbb{R}$. On M_2 an operation \circ is defined as $A \circ B = AB$ for all $A, B \in M_2$. Show that \circ

is a binary operation on M_2 .



16. An operation * on the set of all complex numbers $\mathbb C$ is defined by $z_1*z_2=\sqrt{z_1z_2}$ for all $z_1,z_2\in\mathbb C$. Is * a

binary operation on \mathbb{C} ?



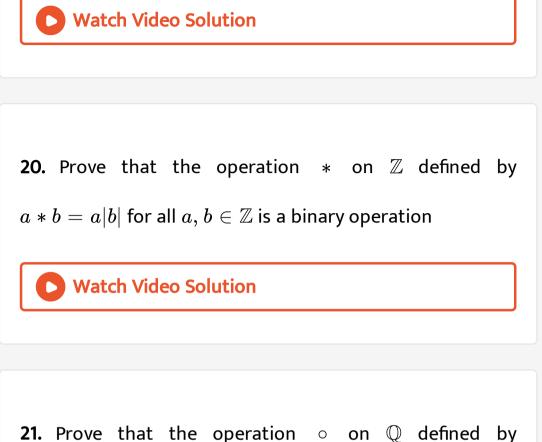
17. Show that an operation * on \mathbb{R} , the set of real numbers, defined by $a*b=3ab+\sqrt{2},\,$ for all $a,b\in\mathbb{R}.$ Is a binary operaion on $\mathbb{R}.$

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18. Examine whether the operation \circ on \mathbb{Z}^+ defined by $a \circ b = |a - b|$ for all $a, b \in \mathbb{Z}^+$, is a binary operation on \mathbb{Z}^+ or not.



19. Prove that the operation \land on $\mathbb R$ defined by $x \land y =$ min. of x and y for all $x, y \in \mathbb R$ is a binary operation on $\mathbb R$.



 $x\circ y=rac{x-2}{y-2}$ for all $x,y\in\mathbb{Q}$ does not represent a

binary operation on \mathbb{Q} .

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22. An operation * is defined on \mathbb{N} as a * b = HCF(a, b)for all $a, b \in \mathbb{N}$. Show that * is a binary operation on \mathbb{N} . Find the values of 25 * 15, 32 * 56, 9 * 11 and 34 * 38.

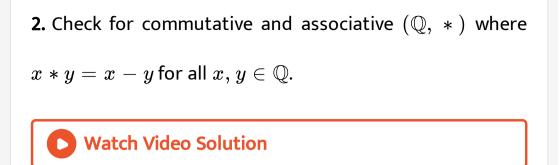


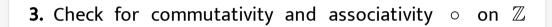
Exercise 3 Short Answer Type Questions

1. Discuss commutativity and associativity * on $\mathbb R$ defined

by $a * b = \min(a, b)$ for all $a, b \in \mathbb{R}$.







defined by $a \circ b = a|b|$ for all $a, b \in \mathbb{Z}$.

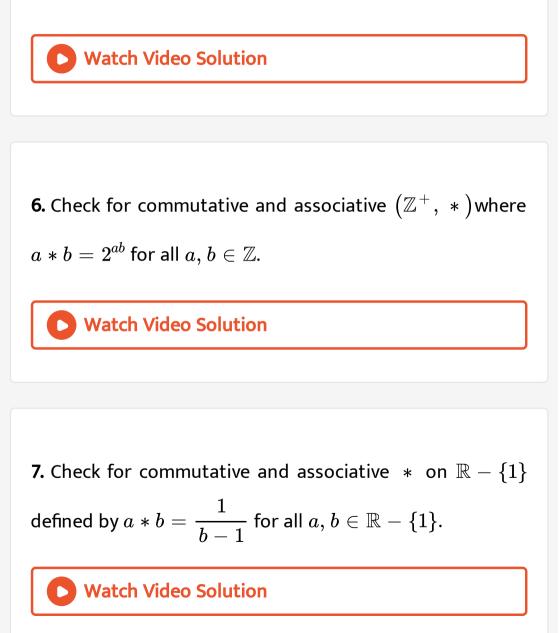


4. Check for commutative and associative (\mathbb{Z}, \circ) where $a \circ b = a + b + ab$ for all $a, b \in \mathbb{Z}$.



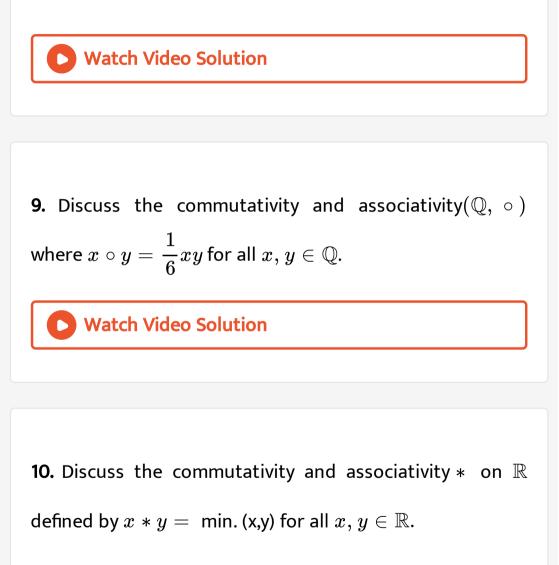
5. Discuss the commutativity and associativity * on $\mathbb R$

defined by a * b = |a + b| for all a,binRR`.



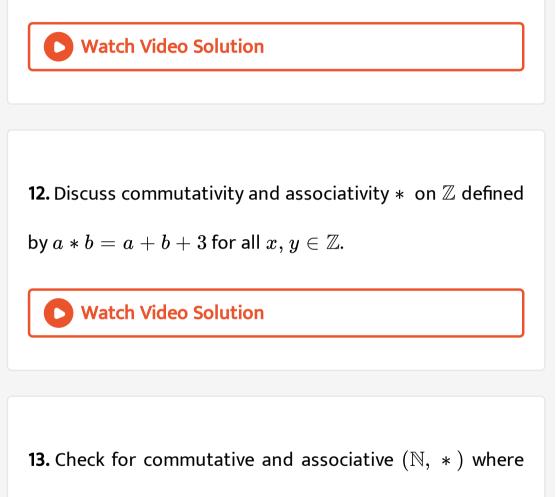
8. Check for commutative and associative \circ on $\mathbb R$ defined

by $x\circ y$ = max (x, y) for all $x,y\in\mathbb{R}.$





11. Discuss the commutativity and associativity * on $\mathbb Q$ defined by a * b = ab + 4 for all $a, b \in \mathbb Q.$

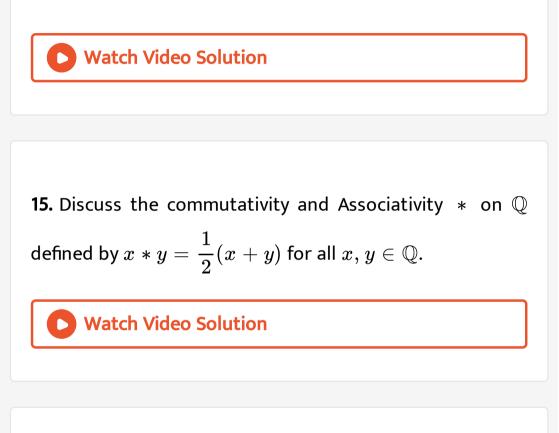


 $a*b=\gcd(a,b)$ for all $a,b\in\mathbb{N}.$



14. Check for commutativity and associativity * on $\mathbb Z$

defined by a * b = |a|b for all $a, b \in \mathbb{Z}$.



16. Discuss the commutativity and associativity * on $\mathbb R$

defined by a * b = |ab| for all $a, b \in \mathbb{R}$.



17. * on $\mathbb{Z} \times \mathbb{Z}$ defined by (a,b)*(c,d) = (a-c,b-d) for all $(a,b), (c,d) \in \mathbb{Z} \times \mathbb{Z}.$

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18. Check commutativity and associativity \circ on $M_2(\mathbb{R})$ defined by $A \circ B = \frac{1}{2}(AB - BA)$ for all $A, B \in M_2(\mathbb{R})$ where $M_2(\mathbb{R})$ is a 2×2 real matrix.

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19. An operation * on \mathbb{Z} , the set of integers, is defined as,

a * b = a - b + ab for all $a, b \in \mathbb{Z}$. Prove that * is a

binary operation on $\mathbb Z$ which is neither commutative nor

associative.



20. (I) Let * be a binary operation defined by a * b = 2a + b - 3. Find 3 * 4.

(ii) let * be a binary operation on $\mathbb{R} - \{-1\}$, defined by $a * b = \frac{1}{b+1}$ for all $a, b \in \mathbb{R} - \{-1\}$ Show that * is neither commutative nor associative. (iii) Let * be a binary operation on the set \mathbb{Q} of all raional numbers, defined as $a * b = (2a - b^2)$ for all $a, b \in \mathbb{Q}$.

Find 3 * 5 and 5 * 3. Is 3 * 5 = 5 * 3?

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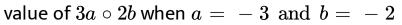
21. A binary operaiton \circ is defined on the set $\mathbb{R} - \{-1\}$ as $x \circ y = x + y + xy$ for all $x, y \in \mathbb{R} - \{-1\}$. Discuss the commutativity and associativity of \circ on $\mathbb{R} - \{-1\}$. If (3 * 2x) * 5 = 71, find x.

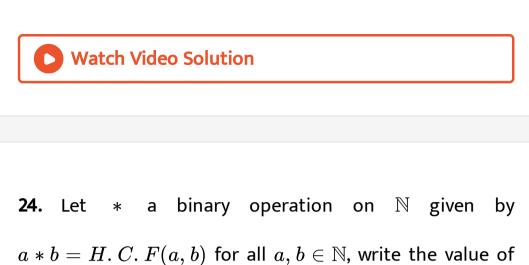


22. If * be the binary operation on the set \mathbb{Z} of all integers, defined by $a * b = a + 3b^2$, find 2 * 4.

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23. A binary operation \circ is defined on \mathbb{Z} , the set of integers, by $a \circ b = |a - b|$ for all $a, b \in \mathbb{Z}$. Find the





22 * 4

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25. If $+_6$ (addition modulo 6) is a binary operation on $A = \{0, 1, 2, 3, 4, 5\}$, find the value of $3 +_6 3^{-1} +_6 2^{-1}$. [note that the identity element is 0 and the inverse of the element 2 is 4 as $2 +_6 4 = 0$, the identity element.] **26.** A binary operation * is defined on the set \mathbb{R}_0 for all non-zero real numbers as $a * b = \frac{ab}{3}$ for all $a, b \in \mathbb{R}_0$, find the identity element in \mathbb{R}_0 .

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27. A binary operation * is defined on the set \mathbb{Z} of all integers by $a \circ b = a + b - 3$ for all $a, b \in \mathbb{Z}$. Determine the inverse of $5 \in \mathbb{Z}$.

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28. A binary operation * is defined on the set of real numbers $\mathbb R$ by a*b=2a+b-5 for all $a,b\in\mathbb R$. If 3*(x-2)=20 find x.

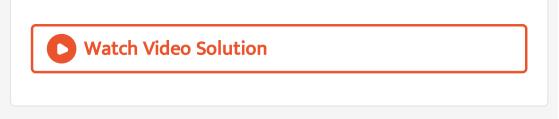
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29. For the binary operation multiplication modulo $5 \cdot (\times_5)$ defined on the set $A = \{1, 2, 3, 4\}$, find the value of $\{2 \times_5 3^{-1}$. [Note that the inverse of 3 is 2]

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30. A binary operation * on \mathbb{Q} the set of all rational numbers is defined as $a * b = \frac{1}{2}ab$ for all $a, b \in \mathbb{Q}$. Prove

that * is commutative as well as associative on \mathbb{Q} .



31. Prove that the identity element of the binary opeartion

* on $\mathbb R$ defined by a * b= min. (a,b) for all $a, b \in \mathbb R$, does

not exist.

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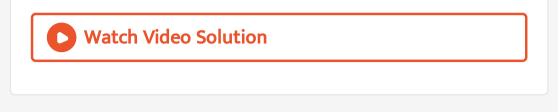
32. Find the identity element of the binary operation *

on $\mathbb Z$ defined by $a \ast b = a + b + 1$ for all $a, b \in \mathbb Z$.

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33. The binary operation * define on N by a*b = a+b+ab for

all $a,b\in N$ is



34. Prove that 0 is the identity element of the binary operation * on \mathbb{Z}^+ defined by x*y=x+y for all $x,y\in\mathbb{Z}^+.$

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35. A binary operation * on \mathbb{Q}_0 , the set of all non-zero rational numbers, is defined as $a * b = \frac{1}{3}ab$ for all $a, b \in \mathbb{Q}_0$ Prove that every element of \mathbb{Q}_0 , is invertible and find the inverse of the element $\frac{3}{5} \in \mathbb{Q}_0$.



36. A binary operation \circ on $\mathbb{Q} - \{1\}$ is defined by a * b = a + b - ab for all $a, b \in \mathbb{Q} - \{1\}$. Prove that every element of $\mathbb{Q} - \{1\}$ is invertible.



37. A binary operation * on \mathbb{Q} , the set of rational numbers, is defined by $a * b = \frac{a-b}{3}$ fo rall $a, b \in \mathbb{Q}$. Show that the binary opearation * is neither commutative nor associative on \mathbb{Q} .



38. Determine which of the following binary operations are associative and which are commutative:

(i) * on $\mathbb R$ defined by a*b=1 for all $a,b\in\mathbb R.$

(ii) * on $\mathbb R$ defined by $a*b=rac{a+b}{2}$ for all $a,b\in\mathbb R.$

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39. Let S be any set containing more than two elements. A binary operation \circ is defined on S by $a \circ b = b$ for all $a, b \in S$. Discuss the commutativity and associativity of \circ on S.



40. State whether the following statements are true or false with reasons.

(i) For any binary operation * on \mathbb{N} , $a*a = a \, \forall a \in \mathbb{N}$.

(ii) If *, a binary operation * on \mathbb{N} is commutative then,

 $a \ast (b \ast c) = (c \ast b) \ast a.$



Exercise 3 Long Answer Type Questions

1. A binary operation \circ is defined on $\mathbb{R} - \{-1\}$ by $a \circ b = a + b + ab$ for all $a, b \in \mathbb{R} - \{-1\}$. (i) Discuss the commutativity and associativity of \circ on $\mathbb{R} - \{-1\}$. Find the identity element, if exists.

(iii) Prove that every element of $\mathbb{R}-\{-1\}$ is invertible.

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2. The binary operation multiplication modulo $10(\times_{10})$ is defined on the set $A = \{0, 1, 3, 7, 9, \}$, find the inverse of the element 7.



3. Construct the composition table for the binary operation multiplication modulo $5(\times_5)$ on the set $A = \{0, 1, 2, 3, 4\}.$

4. A binary operation * on \mathbb{N} is defined by $a*b=L.\ C.\ M.\ (a,b)$ for all $a,b\in\mathbb{N}.$ (i) Find 15*18

(ii) Show that * is commutative as well as associative on \mathbb{N} .

(iii) Find the the identity element in \mathbb{N} .

(iv) Also find the invertible element in \mathbb{N} .

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5. Let P(A) b the power set of non-empty set A. A binary operation * is defined on P(A) as $X * Y = X \cap Y$ for all

 $X,Y\in P(A).$ Detemine the identity element In P(A).

Prove that A is the only invertible element in P(A).

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6. A binary operation * is defined in \mathbb{Q}_0 , the set of all nonzero rational numbers, by $a * b = \frac{ab}{3}$ for all $a, b \in \mathbb{Q}_0$. Find the identity element in \mathbb{Q}_0 . Also find the inverse of an element $x \in \mathbb{Q}_0$.

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7. Let P(C) be the power set of non-empty set C. A binary operation * is defined on P(C) as $A*B=(A-B)\cup(B-A)$ for all $A,B\in P(C)$. Prove that Φ is the identity element for * on P(C) and A is invertible for all $A\in \mathbb{Q}_0.$

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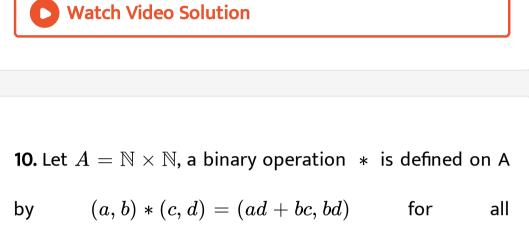
8. Let $A = N \cup \{0\} \times \mathbb{N} \cup \{0\}$, a binary operation * is defined on A by. (a, b) * (c, d) = (a + c, b + d) for all $(a, b), (c, d) \in A$. Prove that * is commutative as well as associative on A. Show also that (0,0) is the identity element In A.



9. Let $A = \{0, 1, 2, 3, 4, 5\}$ be a given set, a binary operation \circ is defined on A by $a \circ b = ab \pmod{6}$ for all

 $a,b\in A.$ Find the identity element for $\,\circ\,$ in A . Show that

1 and 5 are the only invetible elements in A.



 $(a,b), (c,d) \in A.$ Show that * possesses no identity element in A.

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11. Find the values of

(i) $4+_62$ (ii) $7+_57$ (iii) $5+_82$

(iv) $3 imes_7 2$ (v) $12 imes_{10} 5$ (vi) $6 imes_5 4$



12. Let $M_2(x)d = \left\{ \begin{pmatrix} x & x \\ x & x \end{pmatrix}, x \in \mathbb{R} \right\}$ be the set of 2×2 singular matrices. Considering multiplication of matrices as a binary operation, find the identity element in $M_2(x)$.

Also find the inverse of an element of M_2 .

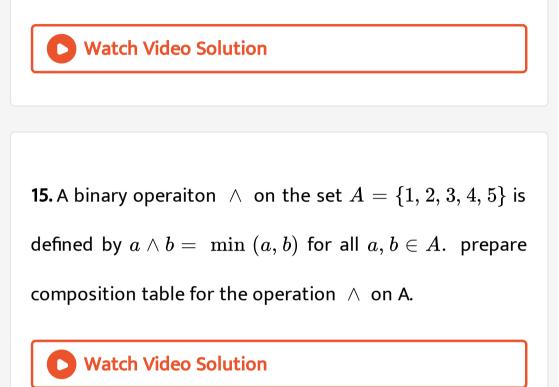
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13. Prepare the composition table for addition modulo

 $6(\ +_{6}\)$ on $A=\{0,1,2,3,4,5\}.$



14. Prepare the composition table for multiplication modulo 6 ($imes_6$) on $A=\{0,1,2,3,4,5\}.$



16. Let $S = \{1, \omega, \omega^2\}$ be the set of cube roots of unity. Prepare the composition table for multiplication (\times) on S, show that multiplication on S is a binary operation and it is commutative on S. Also, show that every element on S

is invertible.

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17. Prepare the composition table for multiplication (\times) on the set of fourth roots of unity and discuss its important properties.



18. Complete the following nultiplication table so as to

define a commutative binary operation * on

 $A = \{1, 2, 3, 4\}.$

*	1	2	3	4
1	2	4	2	1
2		1	estates a	
3		1	4	
4		3	3	2



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19. A binary operation \circ is defined on the set $A = \{0, 1, 2, 3, 4, 5\}$ as follows: $a \circ b = a + b \pmod{6}$ for all $a, b \in A$. Prove that $o \in A$ is the identity element In A is invertible with operation \circ and each element A is invertible with $6 - a \in A$ being the inverse of the element a.



20. An operation * is defined on the set $A = \{1, 2, 3, 4\}$ as follows: a * b, $ab \pmod{5}$ for all $a, b \in A$.Prepare the compositon table for * on A and from the table show that -

(i) multiplication $\mod{(5)}$ is a binary operation,

(ii) * is commutative on A,

(iii) is the identity element for multiplication $\mod{(5)}$ on A, and.

(iv) every element of A is invertible.



21. Let $A = \{1, -1\}$ be the set of square roots of unity. Considering multiplication (\times) as a binary operation on A, construct the composition table for multiplication on A. Determine the identity element for multiplication in A and the inverses of the elements.



22. Let * be the binary defined on the set $S = \{1, 2, 3, 4, 5, 6\}$ by a * b = r where r is the least nonnegative remainder when ab is divided by 7. Prepare the composition table * on S. Observing the composition table show that 1 is the identity element for * and every element of S is invertible.



Mcqs

1. Consider the binary operations $*: \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ and $\circ: \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ defined as a * b = |a - b| and $a \circ b = a$ for all $a, b \in \mathbb{R}$ then-

A. * is commutative but not associative on $\mathbb R$

- B. \circ is associative on \mathbb{R} .
- C. \circ is not distribution over *
- D. \circ is commutative on $\mathbb R$

Answer: A,B,C

2. If the binary oprations * on \mathbb{R} is defined by a * b = a + b + ab for all $a, b \in \mathbb{R}$ where on R.H.S. we have usual addition, subtraction and multiplication of real numbers. The relation * is---

A. not commutative

B. associative

C. commutative

D. not associative

Answer: B,C



3. Let * be a binary operation on \mathbb{N} , the set of natural numbers defined by $a * b = a^b$ for all $a, b \in \mathbb{N}$ is * associative or commutative on \mathbb{N} ?

A. not commutative

B. associative

C. commutative

D. not associative

Answer: A,D



4. Let * be a binary operation on set $\mathbb{Q} - \{1\}$ defined by $a * b = a + b - ab \in \mathbb{Q} - \{1\}$. e is the identity element with respect to * on \mathbb{Q} . Every element of $\mathbb{Q} - \{1\}$ is invertible, then value of e and inverse of an element a are--

A. 0

B. 1

C.
$$\frac{a}{a-1}$$

D. $\frac{a}{a+1}$

Answer: A,C



5. Consider the set $A = \{1, -1, i, -i\}$ of four roots of unity. Constructing the composition table for multiplication on S. which of the properties are true?

A. a binary operation on S

B. commutative on S

C. 1 is the identity element

D. i is the identity element

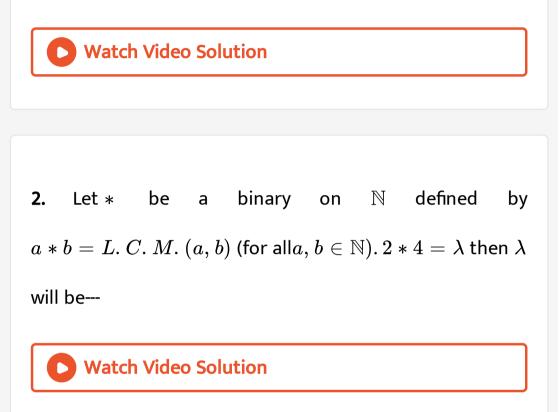
Answer: A,B,C



Integar Answer Type

1. Let $S = \{a, b, c\}$, the total number of binary operations

on S be K^9 . Find the value of K.



3. If the binary operation * on the set \mathbb{Z} is defined by a * b = a + b - 5, then the identity element with respect to * is K. Find the value of K.

4. Let * be a binary operation on \mathbb{Q}_0 (Set of all non-zero rational numbers) defined by $a * b = \frac{ab}{4}$, $a, b \in \mathbb{Q}_0$. The identity element in \mathbb{Q}_0 is e, then the value of e is--

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- 5. The total number of binary operations on the set
- $S = \{1, 2\}$ having 1 as the identity element is n . Find n.



Matrix Match Type

1. Match the following Column I and Column II

1.	Column I		Column II
۸	The brinary operation $*$ on \mathbb{Q} defined by $a * b = \frac{ab}{2}$ for all $a, b \in \mathbb{Q}$ is	(p)	neither commutative nor associative
₿	Let <i>A</i> be any set containing more than one element. The binary operation $*$ on <i>A</i> defined by $a * b = b$ for all <i>a</i> , $b \in A$ is	(q)	commutative but not associative
C	The binary operation $*$ on \mathbb{Q} defined by $a * b = ab^2$ for all $a, b \in \mathbb{Q}$ is		commutative and associative both
	The binary operation $*$ on $\mathbb Q$ defined by $a*b = ab+1$ for all $a, \ b \in \mathbb Q$ is		not commutative but associative

Match

2.

the

coloumn

2.	Column I		Column II
٨	Let * be a binary operation on \mathbb{N} given by, $a * b = \text{L.C.M.}(a, b)$ for all $a, b \in \mathbb{N}$, then the identity element in \mathbb{N} is	(p)	$\frac{25}{a}$
₿	On \mathbb{Q} , the set of all rational numbers, a binary operation $*$ is defined by $a * b = \frac{ab}{5}$ for all $a, b \in \mathbb{Q}$, then the inverse element in \mathbb{Q} is	(q)	<u>16</u> a

°C)	Let * be a binary operation on \mathbb{Q}_0 (Set of non-zero rational numbers) defined by $a * b = \frac{3ab}{5}$ for all $a, b \in \mathbb{Q}_0$ then the identity element in \mathbb{Q}_0 is	(r)	1 1
D	Let * be a binary operation on \mathbb{Q}_0 (Set of all non-zero rational numbers) de- fined by $a * b = \frac{ab}{4}$ for all $a, b \in \mathbb{Q}_0$ then the inverse element in \mathbb{Q}_0 is	(s)	<u>5</u> 3

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Comprehension Type

1. Let $A = \mathbb{R}_0 \times \mathbb{R}$ where \mathbb{R}_0 denote the set of all nonzero real numbers. A binary operation * is defined on A as follows: (a, b) * (c, d) = (ac, bc + d) for all $(a, b), (c, d) \in \mathbb{R}_0 \times \mathbb{R}$.

Binary operation * is--

A. commutative but not associative A

B. commutative and associative on A

C. associative but not commutative on A

D. none of these

Answer: B

2. Let $A = \mathbb{R}_0 \times \mathbb{R}$ where \mathbb{R}_0 denote the set of all nonzero real numbers. A binary operation * is defined on A as follows: (a, b) * (c, d) = (ac, bc + d) for all $(a, b), (c, d) \in \mathbb{R}_0 \times \mathbb{R}$.

Binary operation * is--Identity element in A is--

A. (0,1)

B. (0,0)

C. (1,0)

D. (1,1)

Answer: C

3. Let $A = \mathbb{R}_0 \times \mathbb{R}$ where \mathbb{R}_0 denote the set of all nonzero real numbers. A binary operation * is defined on A as follows: (a, b) * (c, d) = (ac, bc + d) for all $(a, b), (c, d) \in \mathbb{R}_0 \times \mathbb{R}$.

The inveritible elements in A is---

A.
$$\left(-\frac{1}{b}, -\frac{b}{a}\right)$$

B. $\left(-\frac{1}{b}, \frac{b}{a}\right)$
C. $\left(\frac{1}{b}, \frac{b}{a}\right)$
D. $\left(\frac{1}{a}, -\frac{b}{a}\right)$

Answer: D

4. Let the set $S = \{f_1, f_2, f_3, f_4\}$ of four functions from \mathbb{C} (the set of all complex numbers) to itself, defined by $f_1(z) = z, f_2(z) = -z, f_3(z) = \frac{1}{z}$ and $f_4(z) = -\frac{1}{z}$ for all $z \in \mathbb{C}$ Construct the composition table for the composition of functions (\circ) defined on the set S. Value of $f_4 \circ f_1(z)$ is --

A. f_1

 $\mathsf{B.}\,f_2$

C. *f*₃

D. f_4

Answer: D



5. Let the set $S = \{f_1, f_2, f_3, f_4\}$ of four functions from \mathbb{C} (the set of all complex numbers) to itself, defined by $f_1(z) = z, f_2(z) = -z, f_3(z) = \frac{1}{z}$ and $f_4(z) = -\frac{1}{z}$ for all $z \in \mathbb{C}$ Construct the composition table for the composition of functions (\circ) defined on the set S. value of $f_2 \circ f_1(z)$ is--

A. f_1

 $\mathsf{B.}\,f_2$

C. *f*₃

D. f_4

Answer: D



6. Let the set $S = \{f_1, f_2, f_3, f_4\}$ of four functions from \mathbb{C} (the set of all complex numbers) to itself, defined by $f_1(z) = z, f_2(z) = -z, f_3(z) = \frac{1}{z}$ and $f_4(z) = -\frac{1}{z}$ for all $z \in \mathbb{C}$ Construct the composition table for the composition of functions (\circ) defined on the set S. Value of $f_2 \circ f_4(z)$ is---

A. f_1

B. f_2

C. f_3

D. f_4

Answer: C



1. Let S be a non-empty set and P(S) be the power set of the Set S.

Statement -I: Φ is the identity element for union as a binary operation on P(S)

Statement -II: S is the identity element for intersection on P(S).

A. Statement -I is True Statement -II is True , Statement

-II is a correct explanation for Statement -I

B. Statement -I is True. Statement -II is True, Statement

-II is not a correct explanition for Statement -I

C. Statement -I is True, Statement -II is False.

D. Statement -I is False. Statement -II is True.

Answer: B

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2. On $\mathbb{R} - \{1\}$, a binary operation * is defined by a * b = a + b - ab

Statement - I: Every element of $\mathbb{R}-\{1\}$ is inveritble

Statement -II: o is the identity element for * on $\mathbb{R} - \{1\}$.

A. Statement -I is True Statement -II is True, Statement

-II is a correct explanation for Statement -I

B. Statement -I is True. Statement -II is True, Statement

-II is not a correct explanition for Statement -I

C. Statement -I is True, Statement -II is False.

D. Statement -I is False. Statement -II is True.

Answer: B