MATHS

BOOKS - CHHAYA PUBLICATION MATHS (BENGALI ENGLISH)

BINOMIAL THEOREM

Iiiustrative Examples

1. Expand the following binomial expressions :

$(2+5x)^7$

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2. Expand the following binomial expressions :

$\left(x-\frac{1}{2x}\right)^8$

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3. Expand the following binomial expressions :

$\left(x^2+x-1\right)^4$

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4. Simplify :

$\left(a+\sqrt{a^2-1}\right)^8-\left(a-\sqrt{a^2-1}\right)^8$

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5. Simplify :

$(2x+y)^5-5y(2x+y)^4+10y^2(2x+y)^3-10y^3(2x+y)^2+5y^4(2x+y)$

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6. Determine :

the 8th term in the expansion of $(2a+b)^{15}$

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7. Determine :

the 11th term in the expansion of $\left(x-\frac{1}{x}\right)^{20}$

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8. Determine :

the 11th term in the expansion of $\left(\frac{x}{2}-3y\right)^{n}$.

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9. Determine the cofficients of

x^4 in the expansion of $\left(x^4+\frac{1}{x^3}\right)^{15}$

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10. Determine the cofficients of a^{-11} in the expansion of $\left(5a^3 - \frac{2}{a^2}\right)^{13}$.

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11. Find the term independent of x in the expansion of $\left(x - \frac{2}{x^2}\right)^{15}$

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12. Find the value of the term free from x in the expansion of $(1-x)^2\left(x + \frac{1}{x}\right)^7$

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13. Find the cofficient of x^{10} in the expansion of $\left(1-2x+3x^2\right)(1-x)^{15}$

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14. Find the middle term (or terms):

$$\left(a-\frac{1}{a}\right)^{12}$$

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15. Find the middle term (or terms):

$$\left(3x-\frac{1}{2x}\right)^{9}$$

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16. Show that the value of the middle term in the expansion of $\left(x+\frac{1}{2x}\right)^{2n}$ is $\frac{1.3.5.\ldots..(2n-1)}{n!}$

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17. Prove that the coefficient of the middle term in the expansion of $(1+x)^{12}$ is equal to the sum of the coeffcients of the two middle terms in the expansion of $(1+x)^{11}$

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18. If the coffcients of the (4r+5) ans (2r+1) th terms in the expansion of $(1+x)^{10}$ are equal , find the value of r .

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19. If the value of the 5th term be 24 times the value of the 3rd term in the expansion of $(1+x)^{11}$, find the value of x .

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20. In the expansion by binomial theorem of $\left(x^2+\frac{1}{x}\right)^m$ [m is a positive integer] , the sum of the cofficients of the first , second and third terms is 46 . Find the term independent of x in the expansion.

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21. If the term independent of x in the expansion of $\left(\frac{k}{3}x^2-\frac{3}{2x}\right)^9$ be 2268 , find the value of k .

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22. Prove that , $2^{2n}-3n-1$ is divisible by 9 for all positive intergral values of n greater than 1 .

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23. If n is a positive integer (>1) ,show that , $3^{2n+2}-8n-9$ is always divisible by 64.

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24. Find the term independent of x in the expansion of $(1+x)^p\left(1+\frac{1}{x}\right)^q$, p, q being positive integers.

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25. If the coefficients of the pth , $(p+1)$ and $(p+2)$ terms in the expansions of $(1+x)^n$ are in A.P , show that , $n^2-n(4p+1)+4p^2-2=0$

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26. Find the coefficient of x^r in the expansion of

$$(x+4)^n + (x+4)^{n-1}(x+3) + (x+4)^{n-2}(x+3)^2 + \ldots . + (x+3)^n$$

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27. If n be a positive interger and p_n denotes the product of the binomial coefficients in the expansion of $(1+x)^n$, Prove that, $\frac{P_{n+1}}{P_n} = \frac{(n+1)^n}{n!}$.

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28. Prove that,

$$C_0 + 2C_1 + 3C_2 + \ldots\ldots + (n+1)C_n = (n+2)2^{n-1}$$

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29. Prove that,

$$C_0^2 + C_1^2 + C_2^2 + \ldots\ldots + C_n^2 = \frac{(2n)!}{(n!)^2}$$

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30. find the middle term in the expansion of $\left(\frac{x}{2} + \frac{1}{x}\right)^7$

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31. If $(1+x)^n$, Prove that, $\frac{P_{n+1}}{P_n} = \frac{(n+1)^n}{n!}$

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32. If $(1+x)^n$, Prove that, $\frac{P_{n+1}}{P_n} = \frac{(n+1)^n}{n!}$

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33. If $(1+x)^n$, Prove that, $\frac{P_{n+1}}{P_n} = \frac{(n+1)^n}{n!}$

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34. If the third , fourth and fifth terms in the expansion of $(x+a)^n$ be 84 , 280 and 560 repectively , find x , a and n .

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35. If the coeffcients of four successive terms in the espansion of $(1+x)^n$be a_1, a_2, a_3 and a_4 respectively , show that ,

$$\frac{a_1}{a_1+a_2} + \frac{a_3}{a_3+a_4} = 2.\frac{a_2}{a_2+a_3}$$

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36. If n is a positive integer , find the cofficient of x^{-1} in the expansion of $(1+x)^n.\left(1+\frac{1}{x}\right)^n$

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37. using binomial theorem find the values of

$(99)^4$

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38. Using binomial theorem find the values of

$(1.1)^{10}$ correct to six decimal places .

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39. Find the power of x in that term of the expansion of $\left(2+\frac{5x}{2}\right)^{12}$ which has the greatest numerical cofficient. Also find the value of the coefficient .

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40. Show that the integral part of $\left(5+2\sqrt{6}\right)^n$ is odd n is a positive integer .

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41. find the number of terms in the expansion of $(a+b+c)^6$ where $n \in N$

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42. If a_r be the coefficient of x^r in the expression $\left(1+bx^2+cx^3\right)^n$,Prove that $2na_4=(n-1)a_2^2$

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Exercise 8 Multiple Choice Type Questions

1. The number of terms in the expansion of $\left(x - \frac{2}{3x}\right)^{11}$ is -

A. 10

B. 11

C. 12

D. 13

Answer: C

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2. The index of x in the 10th term in the expansion of $(a + x)^{19}$ is -

A. 9

B. 10

C. 19

D. 20

Answer: A

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3. If the number of terms in the expansion of $(a+x)^n$ is finite then n is a -

A. real number

B. positive integer

C. negative integer

D. positive fraction

Answer: B

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4. The index of a of 12th term in the expansion of $(a+2b)^{20}$ is -

A. 9

B. 10

C. 19

D. 20

Answer: A

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5. The general term in the expansion of $(a+x)^{32}$ is -

A. $.^{32}C_r ax$

B. $.^{32}C_r a^{32}x$

C. $.^{32}C_r ax^{32}$

D. $.^{32}C_r a^{32-r}x^r$

Answer: D

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6. The coefficient of x^m in the expansion of $(1+x)^{m+n}$ is -

A. $\frac{m!n!}{(m+n)}$

B. $(m+n)!$

C. $\frac{(m+n)!}{m!n!}$

D. none of these

Answer: C

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7. State which of the following is true ?

A. the expansion of $(a+x)^n$, alaways positive

B. If $x>0, y>0$, then the 7th term in the expansion of $(x-2y)^{15}$ in negative .

C. If n is positive integer , then $.^{n}C_{1}+^{n}C_{2}+\ldots\ldots+^{n}C_{n}=2^{n}$

D. In the expansion of $(1+x)^{n}$, the sum of the coefficients of the terms in even positions is 2^{n-1} .

Answer: D

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8. State which of the following is true ?

A. The general term in the expansion of $(a-x)^{n}$ is $\left(-^{n}C_{r}a^{n-r}x^{r}\right)$

B. There are two middle terms in the expansion of $(x-y)^{n}$ when n is an even positive integer .

C. In the expansion of $(1+x)^{n}$, the sum of the coefficients of the terms in even positions is equal to the sum of the coefficients of the terms in odd positions .

D. $\sqrt{2}$ is a rational number .

Answer: C

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9. The middle term in the expansion of $(2x - 3y)^{12}$ is -

A. 6th term

B. 7th term

C. 5th term

D. 8th term.

Answer: B

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10. Two middle terms in the expansion of $(3a - 4b)^{15}$ is -

A. 6th and 7th terms

B. 5th and 6th terms

C. 7th and 8th terms

D. 8th and 9th terms

Answer: D

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11. Total number of terms in the expansion of $\left(1-x-x^2\right)^5$ is -

A. 9

B. 10

C. 21

D. 12

Answer: C

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12. In the expansion of $(2x+y)^{15}$,the indices of x and y in the 8th term are respectively -

A. 8 and 7

B. 6 and 9

C. 9 and 6

D. 7 and 8

Answer: A

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13. The coefficient of the 10th term in ,the expansion of $\left(x-\frac{1}{x}\right)^{20}$ is -

A. $.^{20}C_9$

B. $.^{20}C_9$

C. $. \ -^{20}C_{10}$

D. . $^{20}C_{10}$

Answer: A

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14. In the expansion of $(a+x)^n$, find the coefficent of rth term from end.

A. . $-^nC_r$

B. . $-^nC_{n-r}$

C. . $-^nC_{r+1}$

D. . $-(n)C_{n-r+1}$

Answer: D

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Exercise Very Short Answer Type Questions

1. State the binomial theorem for a positive integral index.

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2. If n is a positive integer, show that, in the expansion of $(1+x)^n$ the sum of the coefficents of terms in the odd positions is equal to the sum of the cofficients of term in the even positions and each sum is equal to 2^{n-1}.

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3. Write down the binominal expansion of ,

$(a+2b)^5$

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4. Write down the binominal expansion of ,

$\left(a-\frac{3}{b}\right)^7$

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5. Write down the binominal expansion of ,

$$\left(\frac{x}{3}+\frac{2}{t}\right)^4$$

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6. Write down the dimension of entropy.

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7. Write down the binominal expansion of ,

$$\left(x^3-3x^2y+3xy^2-y^3\right)^3$$

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8. Write down the binominal expansion of ,

$(x^2 - x - 2)^3$

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9. Write down the binominal expansion of ,

$(1 - x + x^2)^4$.

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10. Simplify :

$(\sqrt{3} + 1)^5 - (\sqrt{3} - 1)^5$

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11. Simplify :

$(2 + \sqrt{3})^6 + (2 - \sqrt{3})^6$

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12. Simplify :

$(x+y)^4 - 4y(x+y)^3 + 6y^2(x+y)^2 - 4y^3(x+y) + y^4$

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13. Simplify :

$(a-3)^3 + 6(a-3)^2 + 12(a-3) + 8$

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14. Simplify :

$\left(x+\sqrt{x^2-1}\right)^7 + \left(x-\sqrt{x^2-1}\right)^7$

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15. Expand $\left(3x - \frac{5}{x^3}\right)^8$ up to the term independent of x.

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16. Determine:

10th term in the expansion of $\left(3a + \frac{2}{a}\right)^{12}$

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17. Determine:

4th term in the expansion of $\left(\frac{a}{b} - \frac{b}{a}\right)^{10}$

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18. Determine:

nth term in the expansion of $\left(x + \frac{1}{x}\right)^{2n}$

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19. Determine:

the general term in the expansion of $\left(a-\frac{1}{3a}\right)^m$

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20. 5th term from the end in the expansion of $\left(\left(\frac{x^3}{2}\right)-\left(\frac{2}{x^2}\right)\right)^{12}$ is

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21. Determine:

$(r+1)th$ term from the end in the expansion of $(1-3x)^n$.

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22. Show that the coefficients of x^m and x^n are equal to the expansion of $(1+x)^{m+n}$.

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23. Prove that the coefficents of x^n in the expansion of $(1+x)^{2n}$ is twice the coefficent of x^n in the expansion of $(1+x)^{2n-1}$.

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24. Prove that in the expansion of $(1+x)^{57}$, the coefficents of x^{14} and x^{43} are equal.

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25. Find the middle term (or terms) in the following expansions:

(i) $\left(x+\frac{1}{x}\right)^8$

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26. Find the middle term (or terms) in the following expansions:

(ii) $\left(\frac{x^2}{3}+\frac{3}{x^2}\right)^8$

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27. Find the middle term (or terms) in the following expansions:

(iii) $\left(x^2-\frac{1}{x}\right)^9$

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28. Find the middle term (or terms) in the following expansions:

(iv) $\left(2x-\frac{1}{3x}\right)^{2n}$

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29. Using binomial therom find the value:

$(999)^3$

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30. Using binomial therom find the value:

$(101)^4$

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Exercise Short Answer Type Questions

1. If n be a positive integer , then the terms in the expansion of $(1+x)^n$, having equal numerical coefficients shall be equidistant from the beginning and the end . - Prove this.

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2. Find the coefficients of

x^{15} in the expansion of $\left(x^3+\frac{2}{x^2}\right)^{10}$

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3. Find the coefficients of

x^{-2} in the expansion of $\left(2x^3-\frac{1}{x^2}\right)^6$

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4. Find the coefficients of

x^{-11} in the expansion of $\left(x^2-\frac{1}{x^3}\right)^{12}$

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5. Find the coefficients of

x^{-2} in the expansion of $\left(3x-\frac{7}{x}\right)^8$

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6. Find the coefficients of

y^{2r+1} in the expansion of $\left(y-\frac{1}{y}\right)^{2n+1}$

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7. Find the coefficients of

x^{16} in the expansion of $x^{10}(x-2)^{10}$

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8. Find the coefficients of

x in the expansion of $\left(1-x^2+2x^4\right)\left(1-\frac{1}{x}\right)^6$

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9. Find the coefficients of

x^{10} in the expansion of $\left(1+x+x^2\right)(1-x)^8$

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10. Find the term independent of x in each of the following expansions :

$$\left(2x+\frac{1}{3x^2}\right)^9$$

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11. Find the term independent of x in each of the following expansions :

$$\left(x^2+\frac{1}{x}\right)^{12}$$

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12. Find the term independent of x in each of the following expansions :

$$\left(\sqrt{x}-\frac{\sqrt{c}}{\sqrt{x}}\right)^{10}$$

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13. Find the term independent of x in each of the following expansions :

$$\left(\frac{3}{2}x^2-\frac{1}{3x}\right)^{12}$$

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14. Find the term independent of x in each of the following expansions :

$$\left(2x+\frac{1}{\sqrt{x}}\right)^{15}$$

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15. Find the term independent of x in each of the following expansions :

$$\left(9x^2 - \frac{1}{3x}\right)^{12}$$

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16. Find the term independent of x in each of the following expansions :

$$(1 - x^3)\left(x - \frac{1}{x}\right)^7$$

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17. Find the term independent of x in each of the following expansions :

$$(1 - x^3)\left(x - \frac{1}{x}\right)^6$$

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18. Which term in the expansion of $(1 + x)^p . \left(1 + \frac{1}{x}\right)^q$ is independent of x where p , q are positive integers ? What is the value of that term ?

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19. Examine whether or not there is any term independent of x in the expansion of $\left(x-\frac{3}{x}\right)^7$

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20. Examine whether or not there is any term containing x^{10} in the expnsion of $\left(2x^2-\frac{1}{x}\right)^{20}$

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21. If the coefficient of x^3 in the expansion of $\left(x^2+\frac{k}{x}\right)^6$ be 160, then the value of k is-

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22. If the term free from x in the expansion of $\left(\sqrt{x}-\frac{\sqrt{m}}{x^2}\right)^{10}$ be 405 , find the value of m .

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23. If the 21st and 22nd terms in the expansion of $(1+x)^{44}$ are equal , find the value of x .

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24. If the coefficient of x^r and x^{r+1} are equal in the expansion of $(1+x)^{2n+1}$, then find the value of r .

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25. In the expansion of $(1+x)^{20}$, the coefficient of the rth term is to that of the $(r+1)$ th term is in ratio 1 : 2 . Find the value of r .

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26. If in the expansion of $(1+x)^{43}$,the coefficient of (3r+1)th term be equal to the coefficient of (3r+2)th term . Find show these that , r = 7 .

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27. prove that the coefficient of the (r +1)th term in the expansion of $(1+x)^n$ is equal to the sum of the coefficients of the rth and (r+1)th terms in the expansion of $(1+x)^{n-1}$

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28. Show that the middle term in the expansion of $(x+1)^{2n}$ is $\frac{1.3.5. \ldots\ldots (2n-1)}{n!}.2^n.\, x^n.$

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29. Show that the coefficient of the middle term of $(1+x)^{2m}$ is equal to the sum of the coefficents of the two middle terms of $(1+x)^{2m-1}$.

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30. Show that the coefficient of the middle term in the expansion of $(1+x)^{40}$ is equal to the sum of the coefficients of the two middle terms of $(1+x)^{39}$.

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31. If the coefficients of (p +1) th and (p+3)th terms in the expansion of $(1+x)^{2n}$ be equal show that , $p = n-1$.

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32. Let $n \geq 5b \neq 0$, if in the binomial of $(a-b)^n$, the sum of the 5th and 6th terms is zero, then the value of $\frac{a}{b}$ is-

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33. If the coefficients of x^2 and x^{11} in the expansion of $\left(1+ax+2x^2\right)^6$ be 27 and (-192) respectively , show that , $a=-1$

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34. If in the expansion of $(1+x)^m(1-x)^n$ the coefficients of x and x^2 are 3 (-6) respectively then , find the value of m .

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35. Use binomial theorem to show that if $n \geq 1$ is an integer then ,

(a) $11^n-10n-1$ is divisible by 100

(b)$3^{2n}-8n-1$ is divisible by 64.

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36. $n(>1)$ is a positive integer . Show that $14^n - 13n - 1$ is divisible by 13 for all n .

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37. For every natural number $n(>1)$, show that , $4^n + 15n - 1$ is divisible by 9 .

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38. If n is a positive integer (>1) , show that, $\left(4^{2n+2} - 15n - 16\right)$ is always divisible by 225.

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39. Using binomial theorem find the value :

$(0.999)^4$ [correct to three places of decimals]

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40. Using binomial theorem find the value :

$(1.02)^4$ [correct to two places of decimals]

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41. Write down the expansion of $\left(4x - \frac{y}{4}\right)^4$. By giving suitable values to x and y , obtain the value of $(39.75)^4$ correct to four places of decimals.

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42. Prove that ,

$$\frac{x^n}{n!} + \frac{x^{n-1}.a}{(n-1)!1!} + \frac{x^{n-2}.a^2}{(n-2)!2!} + \frac{x^{n-3}.a^3}{(n-3)!3!} + \ldots\ldots\ldots \frac{a^n}{n!} = \frac{(x+a)^n}{n!}$$

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43. Two successive terms in the expansion of $\left(2+\frac{1}{2}\right)^9$ are equal .

Find these two terms and their equal value .

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44. If n be a positive integer and the sums of the odd terms ans even terms in the expansion of $(a+x)^n$ be A and B repectively prove that ,

$A^2-B^2=\left(a^2-x^2\right)^n$

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45. If n be a positive integer and the sums of the odd terms ans even terms in the expansion of $(a+x)^n$ be A and B repectively prove that ,

$4AB=(a+x)^{2n}-(a-x)^{2n}$

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46. Find the value of

$(1+x)^n + {}^nC_1(1+x)^{n-1}.(1-x) + {}^nC_2(1+x)^{n-2}(1-x)^2 + \ldots . + (1$

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47. If n be positive integer , show that the algebraic sum of the numerical coefficients in the expansion of the $(3x-2y)^n$ is 1 .

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48. Show that the sum of the coefficients of the odd terms in the expansion of $(1+x)^{2n}$ is 2^{2n-1}

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49. If

$\left(1+x+x^2\right)^n = a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots . + a_{2n}x^{2n}$ prove that, a_0

.

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50. If a_r be the coefficient of x^r in the expression of $\left(1+bx^2+cx^3\right)^n$, then prove that, $a_3 = nc$

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51. In a binomial expansion if the coefficients of two successive terms are equal , show that the coefficients of terms just preceding and succeding these terms are also equal .

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52. Coefficient of x^n in the expansion of $(a+x)^{2n}$

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53. If the coefficients of three successive terms in the expansion of $(1+x)^n$ be a ,b and c respectively , then show that ,

$n = \frac{2ac + b(a+c)}{b^2 - ac}$ and $\frac{a(b+c)}{b^2 - ac}$ = no. of the terms has coefficients a .

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Exercise Long Answer Type Questions

1. The coefficient of x^m in the extension $(1+x)^{m+n}$

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2. The coefficients of 5th ,6th and 7th terms in the expansion of $(1+x)^n$ are A.P . ,find n .

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3. If the coefficients if three successive terms in the expansion of $(1+x)^n$ are 120 ,210 and 252 respectively . Find n .

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4. If the 2nd ,3rd and 4th terms in the expansion of $(x+a)^n$ are 240 ,720and 1080 respectively , find x a and n .

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5. The three successive of the consecutive three terms in the expansion of $(1+x)^n$ are in the ratio 1 : 2 : 3 . Find n .

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6. If the coefficients of the consicutive four terms in the expansion of $(1+x)^n$ be a_1, a_2, a_3 and a_4 respectively , show that ,

$$\frac{a_1}{a_1+a_2}+\frac{a_3}{a_3+a_4}=2.\frac{a_2}{a_2+a_3}.$$

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7. If n be a positive integer and if the 3rd , 4th , 5th and 6th terms in the expansion of $(x+A)^n$,when expanded in ascending powers of x , be , a , b ,c and d respectively , show that ,

$$\frac{b_2-ac}{c^2-bd}=\frac{5a}{3c}.$$

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8. Find the coefficient of x^r in the following expression :

$$(x+n)^n+(x+2)^{n-1}(x+1)+(x+2)^{n-2}(x+1)^2+\ldots..+(x+1)^n$$

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9. If the first three terms in the expansion of $(1+x)^n$ are in A .P . , prove that,

$$n(n-1)x^2-4nx+2=0.$$

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10. If $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \ldots . + C_n x^n$ then show :

$C_1 + 2.\, C_2 + 3.\, C_3 + \ldots . + n.\, C_n = n.2^{n-1}$

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11. If $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \ldots . + C_n x^n$ then show :

$C_1 - 2C_2 + 3C_3 - \ldots . + (-1)^{n-1} n.\, C_n = 0$

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12. If $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \ldots . + C_n x^n$ then show :

$$\frac{C_0}{1} + \frac{C_1}{2} + \frac{C_2}{3} + \ldots\ldots + \frac{C_n}{n+1} = \frac{2^{n+1}-1}{n+1}$$

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13. If $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \ldots\ldots + C_n x^n$ then show :

$$\frac{C_0}{1} - \frac{C_1}{2} + \frac{C_2}{3} - \ldots\ldots + (-1)^n \cdot \frac{C_n}{n+1} = \frac{1}{n+1}$$

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14. If $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \ldots\ldots + C_n x^n$ then show :

$$C_0 . C_n + C_1 . C_{n-1} + C_2 . C_{n-2} + \ldots\ldots + C_n . C_0 = \frac{(2n)!}{(n!)^2}$$

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15. If $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \ldots\ldots + C_n x^n$ then show :

$$C_0 + 4C_1 + 8C_2 + 12C_3 + \ldots\ldots + 4nC_n = 1 + n.2^{n+1}$$

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16. If $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \ldots\ldots + C_n x^n$ then show :

$$\frac{C_1}{C_0} + \frac{2C_2}{C_1} + \frac{3C_3}{C_2} + \ldots\ldots + \frac{nC_n}{C_{n-1}} = \frac{n(n-1)}{2}$$

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17. If $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \ldots\ldots + C_n x^n$ then show :

$$\frac{C_0}{1} + \frac{C_2}{3} + \frac{C_4}{5} + \ldots\ldots = \frac{2^n}{n+1}$$

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18. If $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \ldots\ldots + C_n x^n$ then show :

$$\frac{C_1}{2} + \frac{C_3}{4} + \frac{C_5}{6} + \ldots\ldots = \frac{2^n - 1}{n+1}$$

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19. If $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \ldots\ldots + C_n x^n$ then show :

$$C_0 C_1 + C_1 C_2 + C_2 C_3 + \ldots\ldots + C_{\_}(n-1) C_n = \frac{(2n)!}{(n+1)!(n-1)!}$$

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20. If $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \ldots . + C_n x^n$ then show :

$$C_0^2 + \frac{C_1^2}{2} + \frac{C_2^2}{3} + \ldots . + \frac{C_n^2}{n+1} = \frac{(2n+1)!}{\{(n+1)!\}^2}$$

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21. If $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \ldots . + C_n x^n$ then show :

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22. If $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \ldots . + C_n x^n$ then show :

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23. If $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \ldots . + C_n x^n$ then show :

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24. If $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \ldots . + C_n x^n$ then show :

$3.^n C_0 - 8.^n C_1 + 13.^n C_2 - 18.^n C_3 + \ldots . \text{ up to} (n+1) \text{terms} = 0$

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25. The power of a of $10th$term in the extension $(a+2b)^{20}$-

A. 9

B. 10

C. 11

D. 20

Answer:

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26.

$(1-x)^n = C_0 - C_1 x + C_2 x^2 - C_3 x^3 + \ldots + C_r (-1)^r x^r + \ldots . + (-1)$

Show that , $C_1 + 2C_2 + 3C_3 + \ldots . + n.\, C_n = n.2^{n-1}$

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27. Find the sum of the series

$.^n C_0 + 2.^n C_1 x + 3.^n C_2 x^2 + \ldots . + (n+1).^n C_n x^n$

and hence show that ,

$.^n C_0 + 2.^n C_1 x + 3.^n C_2 x^2 + \ldots . + (n+1)^n C_n = (n+2)2^{n-1}$

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28. Show :

$x - ^n C_1 (x+y) + ^n C_2 (x+2y) - ^n C_3 (x+3y) + \ldots . = 0$

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29. Show :

$2^n - \frac{n}{1!}.2^{n-1} + \frac{n(n-1)}{2!}.2^{n-2} - \ldots . + (-1)^n = 1$

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30. If $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \ldots + C_n x^n$ show that ,

$$\frac{1}{1!(n-1)!} + \frac{1}{3!(n-3)!} + \frac{1}{5!(n-5)!} + \ldots. = \frac{2^{n-1}}{n!}$$

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31. Find numerically the greatest terms in the following expansion :

$$\left(1+\frac{2}{3}\right)^9$$

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32. Find numerically the greatest terms in the following expansion :

$$\left(1-\frac{1}{15}\right)^{13}$$

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33. Find numerically the greatest terms in the following expansion :

$(2a - 3b)^9 \text{when } a = \frac{5}{3}, b = \frac{3}{5}$

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34. Find numerically the greatest terms in the following expansion :

$\left[1 + \frac{x}{8}\right]^{12} \text{when} x = 5$

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35. Find numerically the greatest terms in the following expansion :

$\left\{\frac{2x}{y} + \frac{3y}{x}\right\}^{10} \text{when} x = \frac{1}{2}, y = \frac{1}{3}$

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36. Find numerically the greatest terms in the following expansion

$(ax - by)^{10} \text{when } a = 2, b = 5, x = 3, y = \frac{1}{2}$

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37. Find numerically the greatest cofficients in the following expansions :

$(2a + 3x)^{11}$

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38. Find numerically the greatest cofficients in the following expansions :

$(2 - 3x)^{15}$

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39. Which terms in the expansion of $\left(x + \frac{1}{2x}\right)^{3n}$ has the greatest coefficient ?

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40. In the expansion of the expression $(a+x)^{15}$,if the eleventh term is the geometric of the eighth and twelfth terms , which terms in the expansion is the greatest ?

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Sample Questions For Competitive Exams Multiple Correct Answers Type

1. If the coefficients of $r^{\text{th}}, (r+1)^{\text{th}}$ and $(r+2)^{\text{th}}$ terms in the expansion of $(1+x)^{14}$ are in A.P ., then r is /are -

A. 5

B. 12

C. 10

D. 9

Answer: A::D

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2. If the middle term in the expansion $\left(\frac{x}{2}+2\right)^8$ is 1120 , then $x \in \mathbb{R}$ is equal to -

A. -2

B. 3

C. -3

D. 2

Answer: A::D

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3. For natural numbers m , n if $(1-y)^m(1+y)^n = 1 + a_1 y + a_2 y^2 + a_3 y^3 + \ldots\ldots,$ and $a_1 = a_2 = 10$, then-

A. $m < n$

B. $m > n$

C. $m + n = 20$

D. $m - n = 20$

Answer: A::C

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4. Number of terms in the extension $\left(9x - \frac{2}{3x}\right)^{11}$ is-

A. 10

B. 11

C. 12

D. 13

Answer:

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5. If n $\in$ N then $\hat{}\ (n)C_0 +^{n+1} C_1 +^{n+2} C_2 + ... +^{n+r} C_r$ is equal to

A. $.^{m+n} C_n$

B. $.^{m+n} C_{n-1}$

C. $.^{m} C_1 +^{m+1} C_2 +^{m+2} C_3 + ... +^{m+n-1} C_n$

D. $.^{m+n} C_m$

Answer: A::B::D

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Sample Questions For Competitive Exams Integer Answer Type

1. Number of terms of the expansion $(4x + 7y)^{10} + (4x - 7y)^{10}$ will be -

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2. The power of x of the expansion

$\left[x+\left(x^3-1\right)^{\frac{1}{2}}\right]^5+\left[x-\left(x^3-1\right)^{\frac{1}{2}}\right]^5$ will be -

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3. In the binomial expansion $(1+ax)^n$, the first three terms are $1, 12x \text{ and } 64x^2$,then the value of n will be -

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4. If $\frac{T_2}{T_3}$ is the expansion of $(a+b)^n$ and $\frac{T_3}{T_4}$ is the expansion of $(a+b)^{n+3}$ are equal , then n is equal to -

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5. If the sum of the coefficients of the expansion $\left(a^2x^2-2ax+1\right)^{51}$ be zero , than the value of a will be -

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Sample Questions For Competitive Exams Matrix Match Type

1. Find the 2th term of the extension $\left(x-\frac{1}{x}\right)^{10}$

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Sample Questions For Competitive Exams

1. Find the 3th term of the extension $(2a+b)^8$

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Sample Questions For Competitive Exams Comprehension Type

1. The 2^{nd}, 3^{rd} and 4^{th} terms in the expansion of $(x+a)^n$ are 240 ,720 and 1080 repectively .

The value of $(x-a)^n$ can be -

A. 64

B. -1

C. -32

D. 81

Answer: B

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2. The value of least term in the expansion is -

A. 16

B. 160

C. 32

D. 81

Answer: C

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3. Find the 5th term of the extension $(2a+b)^8$

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4. An equation $a_0 + a_1x + a_2x^2 + ... + a_{99}x^{99} + x^{100} = 0$ has roots ${}^{99}C_0, {}^{99}C_1, {}^{99}C_2, ..$

The value of a_{99} is equal to -

A. 2^{98}

B. 2^{99}

C. -2^{99}

D. 2^{100}

Answer: C

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5. An equation $a_0 + a_1 x + a_2 x^2 + ... + a_{99} x^{99} + x^{100} = 0$ has roots ${}^{99}C_0, {}^{99}C_1, {}^{99}C_2, ..$

The value of a_{98} is -

A. $\frac{2^{198} - {}^{198}C_{99}}{2}$

B. $\frac{2^{198} + {}^{198}C_{99}}{2}$

C. $2^{99} - {}^{99}C_{49}$

D. none of these

Answer: A

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6. Find the 5th term of the extension $\left(x - \frac{1}{x}\right)^{10}$

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Sample Questions For Competitive Exams Assertion Reason Type

1. Statement -I , $3^{2n+2} - 8n - 9$ is divisible be 64 , $(\forall n \in \mathbb{N})$

Syatement - II : $(1+x)^n - nx - 1$ is divisible by x^2, $(\forall n \in \mathbb{N})$

A. Statement - I is true , Statement -II is true and Statement -II is a correct explanation for Statement -I.

B. Statement -I is true , Statement -II is true but Statement -II is not a correct explanetion of Statement -I .

C. Statement -I is true , Statement -II is false .

D. Statement -I is false , Statement -II is true .

Answer: A

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2. Find the 4th term of the extension $\left(x - \frac{1}{x}\right)^{10}$

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