



MATHS

BOOKS - CHHAYA PUBLICATION MATHS (BENGALI ENGLISH)

COMPLEX NUMBER

Example

1. Simplify : $2\sqrt{-18} + 3\sqrt{-50} - 6\sqrt{-8}$



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2. $(2\sqrt{-5} + 3\sqrt{-2})(-3\sqrt{-8} - \sqrt{-20})$



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3. Simplify : $(3\sqrt{-1} + \sqrt{-2}) \div (2 - \sqrt{-4})$



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4. Express $\left(\frac{1+i}{1-i}\right)^3$ in the form A + iB (A and B are real numbers).



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5. Find the modulus of each of the following complex quantities :

$$-2\sqrt{3} + 2\sqrt{2}i$$



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6. Find the modulus of each of the following complex quantitiesm :

$$(3 - 4i)(-2 + \sqrt{5}i)$$



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7. Find The modulus of each of the following complex quantities :

$$\frac{x - iy}{-a + ib}$$



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8. Find The modulus of each of the following complex quantities :

$$\frac{2}{4 + 3i} + \frac{1}{3 - 4i}$$



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9. If $z_1 = -3 + 4i$ and $z_2 = 12 - 5i$, show that,

$$\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$$



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10. If $z_1 = -3 + 4i$ and $z_2 = 12 - 5i$, show that,

$$|z_1 + z_2| < |z_1| + |z_2|$$



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11. If $z_1 = -3 + 4i$ and $z_2 = 12 - 5i$, show that, $|z_1 z_2| = |z_1| |z_2|$



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12. If $z_1 = -3 + 4i$ and $z_2 = 12 - 5i$, show that, $\left| \frac{z_1}{z_2} \right| = \frac{|z_1|}{|z_2|}$



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13. Find the amplitude and modulus of $\frac{i}{1-i}$



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14. Find the amplitude and modulus of $\sqrt{12} + 6\left(\frac{1-i}{1+i}\right)$



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15. Express $(\sqrt{-1})$ in modulus-amplitude form.

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16. If $z = x + iy$ and $|z - 1| + |z + 1| = 4$, show that

$$3x^2 + 4y^2 = 12 (i = \sqrt{-1}).$$

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17. Find the arguments of

$z_1 = \sqrt{3} + i$ and $z_2 = -1 - i\sqrt{3}$ and hence, calculate $\arg(z_1 z_2)$ and \arg

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18. Find the conjugate of $\frac{x + iy}{x - iy}$ (x, y are real).

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19. If a, b, x, y are real and $3\sqrt{x+iy} = a + ib$ show that ,

$$3\sqrt{x-iy} = a - ib$$



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20. If a, b, x, y are real and $3\sqrt{x+iy} = a + ib$ show that ,

$$3\sqrt{x-iy} = a - ib \frac{x}{a} + \frac{y}{b} = 4(a^2 - b^2).$$



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21. Find the square roots of $\frac{-1 + \sqrt{-3}}{2}$



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22. Find the square roots of $-i$



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23. Find the square roots of $-i$

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24. Find the square roots of $a + \sqrt{a^2 - 1}$ ($a^2 < 1$)

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25. If $x = -1 + i\sqrt{2}$ and the value of $x^4 + 4x^3 + 6x^2 + 4x + 9$.

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26. ω is an imagianry cube root of unity, show that,

$$(1 - \omega^2)(1 - \omega^4)(1 - \omega^8)(1 - \omega^{10}) = 9$$

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27. ω is an imaginary cube root of unity, show that,

$$(ab + bc\omega + ca\omega^2)^2 + (ab\omega + bc\omega^2 + ca)^2 + (ab\omega^2 + bc + ac\omega)^2 = 0$$



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28. Factorise : $(x^2 + y^2)$



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29. Factorise : $x^2 + xy + y^2$



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30. Factorise : $a^3 + b^3 + c^3 - 3abc$



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31. If ω is an imaginary cube root of unity, prove that,

$$(a + b\omega + c\omega^2)^3 + (a + b\omega^2 + c\omega)^3 = (2a - b - c)(2b - c - a)(2c - a - b)$$



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32. If a, b are real and $a^2 + b^2 = 1$, then show that, the equation

$$\frac{1 - ix}{1 + ix} = a - ib \text{ is satisfied by a real value of } x.$$



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33. Express $(x^2 + a^2)(y^2 + b^2)(z^2 + c^2)$ as the sum of two squares.



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34. If x, y are and $(3 + ix^2y)$ is conjugate of the complex quantity

$$(x^2 + y + i4)$$
, find x and y .



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35. If $x + iy = \frac{3}{2 + \cos \theta + \sin \theta}$, prove that, $x^2 + y^2 = 4x - 3$.



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36. If z_1 and z_2 are two complex quantities, show that,

$$|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2[|z_1|^2 + |z_2|^2].$$



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37. If $z=x+iy$ and $\frac{|z-3|}{|z+3|} = 2$, find the position of the point z in the

Argand diagram.



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38. Find the fourth root of $(-7+24i)$.



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39. The points P,Q and R represent the complex numbers $(\sqrt{2} + i\sqrt{2})$, $(\sqrt{3} + i)$ and $(1 + i\sqrt{3})$ respectively in the z-plane. Show that the triangle PQR is isosceles.



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40. If $|z + 2| \leq 2$, find the maximum and minimum values of $|z|$.



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41. For any complex number z, show that the minimum value of $|z| + |z - 1|$ is 1.



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42. If $p < 0$ and α, β, γ are cube roots of p then for any a, b and c show that $\frac{a\alpha + b\beta + c\gamma}{a\beta + b\gamma + c\alpha} = \omega^2$ where ω is an imaginary cube root of 1.



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43. If the vertices of an equilateral triangle be represented by the complex numbers z_1, z_2, z_3 on the Argand diagram, then prove that,

$$z_1^2 + z_2^2 + z_3^2 = z_1z_2 + z_2z_3 + z_3z_1.$$



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44. For every real value of $a > 0$, determine the complex number which will satisfy the equation $|z|^2 - 2iz + 2a(1 + i) = 0$



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45. If the complex number z satisfies the equations

$$\frac{|z - 12|}{|z - 8i|} = \frac{5}{3} \text{ and } \frac{|z - 4|}{|z - 8|} = 1, \text{ find } z.$$



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Exercise 4 Multiple Choice Type Questions

1. If $\bar{z} = -3 + 5i$ then $z =$

A. $-3 - 5i$

B. $3+5i$

C. $5+3i$

D. $5-3i$

Answer: A



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2. If $z = -2 - \sqrt{-5}$ then $\bar{z} =$

A. $-2 + \sqrt{-5}$

B. $2 - \sqrt{-5}$

C. $2 + \sqrt{-5}$

D. $-\sqrt{5} + 2i$

Answer: A



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3. Which one is true ?

A. $2 + 3i > 1 + 4i$

B. $3 + 3i > 6 + 2i$

C. $5 + 9i > 5 + 6i$

D. none of these

Answer: D



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4. The modulus of the complex number $(1 + \sqrt{-8})$ is

A. 2

B. -2

C. 3

D. -3

Answer: C



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5. Let z be a complex number , if $|z| = 3$ and $\arg z = \left(\frac{-\pi}{4}\right)$ then the modulus- amplitude form of z is $z =$

A. $4\left(\cos\frac{\pi}{4} - i \sin\frac{\pi}{4}\right)$

B. $4\left(\cos\frac{\pi}{4} + i \sin\frac{\pi}{4}\right)$

C. $3\left(\cos\frac{\pi}{4} - i \sin\frac{\pi}{4}\right)$

D. $3\left(\cos\frac{\pi}{4} + i \sin\frac{\pi}{4}\right)$

Answer: C



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6. The amplitude of the complex number $z = -2$ is

A. π

B. $\frac{\pi}{2}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{6}$

Answer: A



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7. The argument of the complex number $z = 2i$ is

A. π

B. $\frac{\pi}{2}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{6}$

Answer: B



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8. If x, y are real and $x + iy = 0$ then

A. $x=0, y=1$

B. $x=1, y=0$

C. $x=1, y=1$

D. $x=0, y=0$

Answer: D



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9. The condition satisfied by ω the imaginary ,cube root of unity is

A. $\omega + \omega^2 = 1$

B. $\omega^3 = 0$

C. $1 + \omega + \omega^2 = 0$

D. $\omega^2 = \omega$

Answer: C



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10. If ω is an imaginary cube root of unity and $\omega = \frac{-1 + \sqrt{3}i}{2}$ then $\omega^2 =$

A. $\frac{-1 + \sqrt{3}i}{2}$

B. $\frac{1 - \sqrt{3}i}{2}$

C. $\frac{\sqrt{3}i}{2}$

D. $\frac{-1 - \sqrt{3}i}{2}$

Answer: D



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11. If ω is an imaginary cube root of unity then which of the following is the value of ω^{242}

A. 0

B. 1

C. ω

D. ω^2

Answer: D



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12. State which of the following is the value of $(1 + i + i^2 + i^3 + i^4)$ [given, $i = \sqrt{-1}$].

A. 0

B. 1

C. i

D. 2

Answer: B



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13. Given $(a+ib)(c+id)$ is a purely real quantity, then which of the following conditions is true?

A. $ac-bd=0$

B. $bc=0$

C. $ad=0$

D. $bc+ad=0$

Answer: D



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14. Given $(a+ib)(x-iy)$ is a purely imaginary quantity, then which of the following conditions is true?

A. $ax+by=0$

B. $bx-ay=0$

C. $ax-by=0$

D. $bx+ay=0$

Answer: A



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15. Which of the following is the value of amplitude of the complex number $z=1$?

A. 0

B. $\frac{\pi}{2}$

C. π

D. $-\frac{\pi}{2}$

Answer: A



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16. Which of the following is the value of modulus of the complex quantity $\frac{1-i}{\sqrt{5}-i\sqrt{3}}$?

A. $\frac{1}{2\sqrt{2}}$

B. $-\frac{1}{2}$

C. $\frac{1}{2}$

D. $\frac{1}{\sqrt{2}}$

Answer: C



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17. Which of the following statement is true?

A. Which of the following statement is true?

B. Which of the following statement is true?

C. The real part of a complex number cannot exceed its modulus.

D. If the sum of two complex quantities z_1 and z_2 is real then

z_1 and z_2 are conjugate of each other.

Answer: C



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18. If z_1 and z_2 are conjugate complex number, then $z_1 + z_2$ will be

- A. real
- B. imaginary
- C. positive integer
- D. negative integer.

Answer: A



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Exercise 4 Very Short Answer Type Questions

1. Do you think 4 as a complex number ? If so why ?



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2. If x, y are real and $x + iy = -i(-2 + 3i)$ find x and y .



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3. If x, y are real and $x + iy = \frac{5}{-3 + 4i}$ find x and y .



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4. If $(1 + i)(2 + i)(3 + i)\dots(n + i) = a + ib$ show that,
 $2 \cdot 5 \cdot 10\dots(n^2 + 1) = a^2 + b^2$.



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5. Prove that, $\frac{|x - iy|}{|-x + iy|} = 1$



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6. If $z = \frac{2+i}{-2+i}$ find \bar{z} , the conjugate of z in the form $a + ib$.



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7. Express in the form A+iB (A,B are real) : $(1 - i)^3$



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8. Express in the form A+iB (A,B are real) : $\frac{i}{1+i} + \frac{1+i}{i}$



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9. Express in the form A+iB (A,B are real) : $\frac{x+iy}{y-ix}$ ($x^2 + y^2 \neq 0$)



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10. Express in the form A+iB (A,B are real) : $\frac{i}{2+i} + \frac{3}{1-i} = 4i$



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11. Express in the form $A+iB$ (A,B are real) : $\frac{\sqrt{3} - i\sqrt{2}}{2\sqrt{3} - i\sqrt{2}}$



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12. Express in the form $A + iB$ (A,B are real) : $\frac{1}{1 - \cos \theta - i \sin \theta}$



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13. Find the conjugate : $\sqrt{-5} - 2$



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14. Find the conjugate : $2 - i\sqrt{3} - \sqrt{2}$



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15. Find the conjugate : $\sqrt{-5} - 2$



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16. Simplify : $\frac{i + i^2 + i^3 + i^4}{1 + i}$



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17. Simplify : $(1 + i^3) \left(1 + \frac{1}{i}\right)^2 \left(i^4 + \frac{1}{i^4}\right)$



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18. Simplify : $(1 + i)^2 + (1 - i)^2$



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19. Simplify : $(1 + i)^{-2} - (1 - i)^{-2}$



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20. Simplify : $\left(\frac{1+i}{1+i}\right)^2 + \left(\frac{1-i}{1+i^2}\right)$



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21. If x, y real and $(x - 3i)$ and $(-2 + iy)$ are conjugate of each other, find x and y .



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22. Find the least positive integral value of n so that $\left(\frac{1+i}{1-i}\right)^n = 1$
[Hint : Note that $\left(\frac{1+i}{1-i}\right)$ and $i^4 = 1$]



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23. Find the modulus : $2\sqrt{2} - i\sqrt{6}$



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24. Find the modulus : $(\sqrt{5} + i\sqrt{3})(-2 + i)$



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25. Find the modulus : $\frac{1 - i}{3 - 4i}$



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26. Find the modulus : $\frac{2}{1 + \cos \theta + i \sin \theta}$



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27. Find the argument of each of the following complex numbers : 2-2i



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28. Find the argument of each of the following complex numbers : -3-3i



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29. Find the argument of each of the following complex numbers :

$$(1 + i)(\sqrt{3} + i)$$



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30. Find the argument of each of the following complex numbers :

$$\frac{\sqrt{3} + i}{-1 - i\sqrt{3}}$$



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31. If $a = \frac{1+i}{\sqrt{2}}$, show that $a^6 + a^4 + a^2 + 1 = 0$.



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32. Find the cube roots of (-1) and 27.



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33. Factorise : $a^2 + 1$.



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34. If , ω, ω^2 denote the cube roots of unity, find the roots of $(x + 5)^3 + 27 = 0$.



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Exercise 4 Short Answer Type Question

1. Find all the roots of $x^3 - 1 = 0$. Show that if ω is a complex root of this equation, the other complex root is ω^2 and $1 + \omega + \omega^2 = 0$.



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2. If $\sqrt{x - iy} = a - ib$ and a,b,x,y are real then show that,
 $\sqrt{x + iy} = a + ib$.



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3. If $|z + 5| \leq 6$, find the maximum and minimum values of $|z + 2|$, z being a complex number.



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4. If x,y are real and $(y^2x - 5i)$ and $\{4 + i(x + y^2)\}$ are conjugate to each other find x and y



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5. If $z_1 = 4 - 3i$ and $z_2 = -12 + 5i$, show that,

$$\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$$



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6. If $z_1 = 4 - 3i$ and $z_2 = -12 + 5i$, show that,

$$|z_1 + z_2| < |z_1| + |z_2|$$



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7. If $z_1 = 4 - 3i$ and $z_2 = -12 + 5i$, show that,

$$\overline{z_1 z_2} = \overline{z_1} \cdot \overline{z_2}$$



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8. If $z_1 = 4 - 3i$ and $z_2 = -12 + 5i$, show that,

$$|z_1 z_2| = |z_1| |z_2|$$



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9. If $z_1 = 4 - 3i$ and $z_2 = -12 + 5i$, show that,

$$\left| \frac{z_1}{z_2} \right| = \frac{|z_1|}{|z_2|}$$



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10. If $z=2-3i$, express $\frac{1-2z}{z-3}$ in the form $A+Bi$ where A and B are real.



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11. If $\frac{2+i}{2-3i} = A + Bi$ (A, B are real, $i = \sqrt{-1}$), find the value of $A^2 + B^2$



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12. If $\frac{2+i}{2-3i} = A + Bi$ (A, B are real, $i = \sqrt{-1}$), find the value of $A^2 + B^2$



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13. Express each of the following complex numbers in modulus-amplitude form: $\sqrt{3} + i$



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14. Express each of the following complex numbers in modulus-amplitude form: $5-5i$



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15. Express each of the following complex numbers in modulus-amplitude form: $4i$



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16. Express each of the following complex numbers in modulus-amplitude form: $(-3 + 3i)(1 - i)$



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17. Express each of the following complex numbers in modulus-amplitude form: $\frac{\sqrt{3} - 1}{1 - \sqrt{3}i}$



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18. If a, b, c, d, x, y are real and $(a + ib)(c + id) = x + iy$, show that ,
 $(a - ib)(c - id) = x - iy$



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19. If a, b, c, d, x, y are real and $(a + ib)(c + id) = x + iy$, show that ,
 $(ac - bd)^2 + (ad + bc)^2 = x^2 + y^2$



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20. If $\frac{a + ib}{c + id} = x + iy$ prove that, $\frac{a - ib}{c - id} = x - iy$



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21. If $\frac{a + ib}{c + id} = x + iy$ prove that, $\frac{a^2 + b^2}{c^2 + d^2} = x^2 + y^2$



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22. If $x + iy = \sqrt{\frac{a + ib}{c + id}}$ prove that, $(x^2 + y^2)^2 = \frac{a^2 + b^2}{c^2 + d^2}$



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23. If a,b,p,q are real and

$3\sqrt{a - ib} = p - iq$ prove that $3\sqrt{a + ib} = p + iq.$



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24. If $z=x+iy$ and $|z + 6| = |2z + 3|$, prove that $x^2 + y^2 = 9$.



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25. If $z = x + iy$ and $|2z - 1| = |z - 2|$, prove that $x^2 + y^2 = 1$.



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26.

If

$z = x + iy$ and $|2z + 1| = |z - 2i|$, prove that $3(x^2 + y^2) + 4(x + y) = [x, y \text{ are real and } I = \sqrt{-1}]$.



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27. If $x=2+3i$ and $y=2-3i$ then find the values of $\frac{x^3 - y^3}{x^3 + y^3}$



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28. If $x=2+3i$ and $y=2-3i$ then find the values of : $\frac{x^2 + xy + y^2}{x^2 - xy + y^2}$



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29. Find the square roots : $7-24i$



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30. Find the square roots : $16+30i$



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31. Find the square roots : i



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32. Find the square roots : $1-i$



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33. Find the square roots : $1 + 2\sqrt{-6}$



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34. Find the square roots : $\frac{1+i}{1-i}$



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35. Find the square roots : $\frac{7-24i}{3+4i}$



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36. Find the square roots : $x^2 - 1 + 2ix$



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37. Find the square roots : $1 + i\sqrt{a^4 - 1}$



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38. Find the square roots : $x - i\sqrt{x^4 + x^2 + 1}$



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39. Find the square roots : $y + \sqrt{y^2 - x^2} (x^6 > y^2)$



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40. Find the square roots : $a^2 + \frac{1}{a^2} + 4i\left(a + \frac{1}{a}\right) - 2$



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41. If $x\sqrt{2} = 1 + \sqrt{-1}$, find the value of $x^6 + x^4 + x^2 + 2$.



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42. Show that one value of $(\sqrt{i} + \sqrt{-i})is\sqrt{2}$.



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43. Prove that one value of $(1 + i)^{1/2} - (1 - i)^{1/2}is\sqrt{2}(\sqrt{2} - 1)$.



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44. Show that one value of $(4 + 3i)^{-1/2} + (4 - 3i)^{-1/2}is\frac{3\sqrt{2}}{5}$.



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45. If ω be an imaginary cube root of unity, show that,

$$(1 - \omega)(1 - \omega^2)(1 - \omega^4)(1 - \omega^5) = 9$$



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46. If ω be an imaginary cube root of unity, show that

$$(1 + \omega - \omega^2)(1 - \omega + \omega^2) = 4$$



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47. If ω be an imaginary cube root of unity, show that

$$(3 + 3\omega + 5\omega^2)^6 = (3 + 5\omega + 3\omega^2)^6 = 64$$



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48. If ω be an imaginary cube root of unity, show that $\frac{x\omega^2 + y\omega + z}{x\omega + y + zw^2} = \omega$



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49. If ω be an imaginary cube root of unity, show that

$$(x + y\omega + z\omega^2)^2 + (x\omega + y\omega^2 + z)^2 + (x\omega^2 + y + z\omega)^2 = 0$$



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50. If $\alpha = \frac{-1 - \sqrt{-3}}{2}$ and $\beta = \frac{-1 + \sqrt{-3}}{2}$, show that
 $\alpha^2 + \alpha\beta + \beta^2 = 0$.



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51. Show that, $\left(\frac{-1 + \sqrt{-3}}{2}\right)^{19} + \left(\frac{-1 - \sqrt{-3}}{2}\right)^{19} = -1$.



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52. If α and β are the complex cube roots of 1 show that,

$$\alpha^4 + \beta^4 + \alpha^{-1} \cdot \beta^{-1} = 0.$$



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53. Find the value of $\sqrt{[-3 + \sqrt{(-3 + \sqrt{-3 + \dots \text{to infinity}})}]}$



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54. If ω be a complex cube root of unity and $x = \alpha + \beta, y = \alpha + \beta\omega, z = \alpha + \beta\omega^2$, show that,
 $x^3 + y^3 + z^3 = 3(\alpha^3 + \beta^3)$.



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55. If $x = a + b, y = a\alpha + b\beta$ and $z = a\beta + b\alpha$ where α and β are complex cube roots of unity, show that,

$$xyz = a^3 + b^3.$$



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56. If $z = x + iy$ and $|z - 1|^2 + |z + 1|^2 = 4$, determine the position of the point z in the complex plane.

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57. Factorise : $a^2 - ab + b^2$

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58. Factorise : $x^3 + y^3$

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Exercise 4 Long Answer Type Question

1. If $z_1 = 1 + i\sqrt{3}$ and $z_2 = \sqrt{3} - i$, show that

$$\arg(z_1 z_2) = \arg z_1 + \arg z_2$$



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2. If $z_1 = 1 + i\sqrt{3}$ and $z_2 = \sqrt{3} - i$, show that

$$\arg \frac{z_1}{z_2} = \arg z_1 - \arg z_2.$$



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3. If $|z_1| = |z_2|$ and $\arg z_1 + \arg z_2 = 0$, then show that z_1 and z_2 are two complex conjugate numbers.'



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4. If $|z_1| = |z_2| = 1$ and $\arg z_1 + \arg z_2 = 0$ then show that $z_1 = 1$



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5. If $|z_1| = |z_2|$ and $\arg z_1 - \arg z_2 = \pi$, show that $z_1 + z_2 = 0$.



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6.

If

$z_1 = (\sqrt{3} - 1) + (\sqrt{3} + 1)i$ and $z_2 = -\sqrt{3} + i$ find $\arg z_1$ and $\arg z_2$ hence, calculate $\arg(z_1 z_2)$.

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7. Find the values of x and y (real) for which the following equation is satisfied

$$\frac{(1+i)x - 2i}{3+i} + \frac{(2-3i)y + i}{3-i} = i$$

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8. If x,y,b and real, $z=x+iy$ and $\frac{z-i}{z-1} = ib$, show that,
$$\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{1}{2}$$
.

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9. If $x + iy = \frac{2}{3 + \cos \theta + i \sin \theta}$, show that $2x^2 + 2y^2 = 3x - 1$.



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10. If $y = \sqrt{x^2 + 6x + 8}$, show that one value of $\sqrt{1+iy} + \sqrt{1-iy}$ [$i = \sqrt{-1}$] is $\sqrt{2x+8}$.



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11. Factorise : $x^2 + y^2 + z^2 - xy - yz - zx$



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12. prove that , $(a + b\omega + c\omega^2)^3 + (a + b\omega^2 + c\omega)^3 = 27abc$ if $a + b + c = 0$.



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13. Express each of the following expressions as the sum of two squares:

$$(1 + x^2)(1 + y^2)$$



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14. Express each of the following expressions as the sum of two squares:

$$(a^2 + b^2)(c^2 + d^2)$$



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15. Express each of the following expressions as the sum of two squares:

$$(1 + x^2)(1 + y^2)(1 + z^2)$$



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16. If a, b are real and $a^2 + b^2 = 1$ then show that the equation

$$\frac{\sqrt{1+x} - i\sqrt{1-x}}{\sqrt{1+x} + i\sqrt{1-x}} = a - ib \text{ is satisfied by a real value of } x.$$





17. If $z = x + iy$ and $|z| = 1$, show that $\frac{z - 1}{z + 1}$ ($z \neq -1$) is a purely imaginary quantity.



18. If z_1 and z_2 be any two non-zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$ then prove that, $\arg z_1 = \arg z_2$.



19. If α be the real positive cube root and β, γ be the complex cube roots of m , a real positive number, then for any x, y, z show that $\frac{x\beta + y\gamma + z\alpha}{x\gamma + y\alpha + z\beta} = \omega^2$ where ω is a complex cube root of unity.



20. If $\arg \frac{z-1}{z+1} = \frac{\pi}{4}$, show that the locus of z in the complex plane is a circle.



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21. In the complex plane, the vertices of an equilateral triangle are represented by the complex numbers z_1 , z_2 and z_3 prove that,

$$\frac{1}{z_1 - z_2} + \frac{1}{z_2 - z_3} + \frac{1}{z_3 - z_1} = 0$$



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22. Show that the area of the triangle on the Argand Diagram formed by the complex numbers z , iz and $z + iz$ is $\frac{1}{2}|z|^2$.



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23.

If

$$z_1^2 + z_2^2 + z_3^2 - z_1 z_3 - z_1 z_2 - z_2 z_3 = 0 \quad \text{prove that} \quad |z_2 - z_3| = |z_3 - z_1|$$

are complex numbers.



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24. Solve $\bar{z} = iz^2$ (z being a complex number)



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25. Solve : $|z| + z = 2 + i$ (z being a complex number)



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26. If $z=x+iy$ then show that $|x| + |y| \leq \sqrt{2}|x + iy|$.



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27. Let $z=x+iy$ and $\omega = \frac{1 - iz}{z - i}$. If $|\omega| = 1$ show in the complex plane the point z lies on the real axis.



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Sample Question For Competitive Exams Multiple Correct Answer Type

1. If $\arg(z_1 z_2) = 0$ and $|z_1| = |z_2| = 1$, then

A. $z_1 + z_2 = 0$

B. $z_1 + z_2 = 1$

C. $z_1 = \overline{z_2}$

D. $z_1 + \overline{z_2} = 0$

Answer: B::C



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2. The values of $(-1)^{\frac{1}{3}}$ are

A. $\frac{\sqrt{3} - i}{2}$

- B. $\frac{\sqrt{3} + i}{2}$
- C. $\frac{-\sqrt{3} - i}{2}$
- D. $\frac{-\sqrt{3} + i}{2}$

Answer: A::C



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3. If $z = (\sqrt{5 - 12i} + \sqrt{-5 - 12i})$, then the principal value of $\arg z$ will be

- A. $-\frac{\pi}{4}$
- B. $\frac{\pi}{4}$
- C. $\frac{3\pi}{4}$
- D. $-\frac{3\pi}{4}$

Answer: A::B::C::D



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4. What will be the least positive integral value of n such that $\left(\frac{2i}{1+i}\right)^n$ be a positive integral value



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5. The value of $\frac{i^{592} + i^{590} + i^{586} + i^{584}}{i^{582} + i^{580} + i^{578} + i^{576} + i^{574}}$ will be



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6. Consider the complex number $z = \frac{1 - i \sin \theta}{1 + i \sin \theta}$

The value of θ for which z is purely imaginary is

A. $n\pi - \frac{\pi}{4}, n \in I$

B. $n\pi + \frac{\pi}{4}, n \in I$

C. $n\pi, n \in I$

D. none of these

Answer: D



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7. Consider the complex number $z = \frac{1 - i \sin \theta}{1 + i \cos \theta}$

If argument of z is $\frac{\pi}{4}$, then

A. $\theta = n\pi, n \in I$ only

B. $\theta = (2n + 1)\frac{\pi}{2}, n \in I$ only

C. both $\theta = n\pi$ and $\theta = (2n + 1)\frac{\pi}{2}, n \in I$

D. none of these

Answer: C



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8. Consider the equation $az + b\bar{z} + c = 0$ where $a, b, c \in Z$

If $|a| = |b|$ and $\overline{ac} \neq \overline{bc}$, then z has

A. infinite solutions

B. no solutions

C. finite solutions

D. cannot say anything

Answer: B



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9. Consider the equation $az + \bar{b}z + c = 0$ where $a, b, c \in Z$

If $|a| = |b| \neq 0$ and $\bar{a}c = \bar{b}c$, then $az + b\bar{z} + c = 0$ represents

A. an ellipse

B. a circle

C. a point

D. a straight line

Answer: D



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10. Let fourth roots of unity be z_1, z_2, z_3 and z_4 respectively.

Statement - I: $z_1^2 + z_2^2 + z_3^2 + z_4^2 = 0$

Statement - II: $z_1 + z_2 + z_3 + z_4 = 0$

A. Statement -I is true, Statement-II is true and Statement-II is a

correct explanation for Statement-I.

B. Statement-I is true, Statement-II is true but Statement-II is not a

correct explanation of Statement-I.

C. Statement-I is true, Statement-II is false.

D. Statement-I is false, Statement-II is true.

Answer: B



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11. Statement-I: If n is an odd integer greater than 3 but not a multiple of 3, then $(x + 1)^n - x^n - 1$ is divisible by $x^3 + x^2 = x$.

Statement - II: if n is an odd integer greater than 3 but not a multiple of 3, we have $1 + \omega^n + \omega^{2n} = 0$.

- A. Statement -I is true, Statement-II is true and Statement-II is a correct explanation for Statement-I.
- B. Statement-I is true, Statement-II is true but Statement-II is not a correct explanation of Statement-I.
- C. Statement-I is true, Statement-II is false.
- D. Statement-I is false, Statement-II is true.

Answer: C



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1. If $|z - 3| = \min \{|z - 1|, |z - 5|\}$, then the values of $\operatorname{Re}(z)$ will be

A. 2

B. $\frac{5}{2}$

C. $\frac{7}{2}$

D. 4

Answer: A::D



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2. The solution of the equation $x^2 - (3\sqrt{2} - 2i)x - 6\sqrt{2}i = 0$ are

A. $3\sqrt{2}$

B. $-2\sqrt{2}$

C. $-2i$

D. $2i$

Answer: A::C



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3. If z , $z + iz$ and iz are three complex numbers on a plane which forms a triangle of area 18 sq. Units, then the value of



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4. If $1, \alpha$ and α^2 be three cube roots of 1 and $x = a + b, ya + b\alpha, z = a + b\alpha^2$ and $x^3 + y^3 + z^3 = K(a^3 + b^3)$, then the value of K is



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5. If $iz^2 - \bar{z} = 0$, then the value of $|z|$ will be



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6. Consider the complex number $z = \frac{1 - i \sin \theta}{1 + i \cos \theta}$

The value of θ for number z is purely real is

A. $n\pi - \frac{\pi}{4}, n \in I$

B. $n\pi + \frac{\pi}{4}, n \in I$

C. $n\pi, n \in I$

D. none of these

Answer: A



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7. Consider the equation $az + b\bar{z} + c = 0$ where $a, b, c \in Z$

If $|a| \neq |b|$, then z represents

A. a circle

B. straight line

C. one point

D. ellipse

Answer: C



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