MATHS

BOOKS - CHHAYA PUBLICATION MATHS (BENGALI ENGLISH)

DEFINITE INTEGRAL AS AN AREA

Example

1. Find the area bounded by the curve $y^2 = 4x$the x-axis and the ordinate at x=4

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2. Find by integration the area of the triangle bounded by line $4y - 5x = 0$,the x axis and the ordinate x =4 Verify your result by using the definition of area of a triangle as half the product of the base and the altitude.

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3. Find by integration the area of the trapezoid bounded gy $y = 2x + 1, y = 0, x = 2$ and $x = 4$ Verify your result by finding the area of a trapezoid as the product of half the sum of the two parallel sides and the distance between them.

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4. Find the area bounded by the parabola $y = x^2$,the y-axis and the abscissa at y =4.

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5. Using integration find the area of the region bounded by the parabola $y^2 = 16x$ and the line x = 4

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6. Find the area in the second quadrant bounded by the curve $y = x^3 + 8$ and the coordinate axes.

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7. Find the area bounded by the parabola $y = x(4 - x)$ and the x-axis

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8. Find the area bounded by the parabola $y^2 = 4ax$ and its double ordinate x =b

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9. Using integration find the area of the triangle whose vertices are A (1,0) ,B(2,2) and C (3,1)

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10. Find the area in the first quadrant bounded by the circle $x^2 + y^2 = 16$,the x-axis and the ordinates x =1 and x=3

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11. Find the area of the region bounded by the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

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12. By finding the area of a regular polygon of n sides inscribed in a circle of radius r ,show that the area of the circle is πr^2

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13. Find the area included between $y^2 = 9x$ and $y = x$

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14. Find the area bounded by the parabola $y^2 = 4ax$ and $x^2 = 4ay$

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15. Find the area cut off from the parabola $4y = 3x^2$ by the straight line $3x - 2y + 12 = 0$

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16. On a diagram mark the area bounded by the parabola $y^2 = 4x$ and the circle x-4 $=4\cos\theta, y = 4\sin\theta$ above the x-axis and obtain the area by integration

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17. Find the area of the region $\{(x, y) : x^2 < y < x\}$

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18. Find the area of the smaller region bounded by the ellips $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the straight line $\frac{x}{a}+\frac{y}{b}=1$

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19. Find the area of the region $\{(x, y): x^2 < y < |x|\}$

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20. Find the area of the region enclosed between the two circles $x^2+y^2=1$ and $(x-1)^2+y^2=1$

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21. Find the area of the region bounded by the curve $y=\cos x, x$ axis and the ordinates x=0 and $x=2\pi$

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22. Draw a rough sketch of the curves $y = \sin x$ and $y = \cos x$ as x varies from 0 to $\frac{\pi}{2}$ and find the area of the region enclosed between them and x-axis

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23. Find the area in the first quadrant which is common to the circle $x^2 + y^2 = 4$ and the ellips $x^2 + 4y^2 = 9$

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24. From the point P(1.1) on the curve $y = x^4$ perpendicular Pm and PN are drawn upon the coordiante axes OX and OY

respectively.Show that the ratio of the smaller to the larger of the two areas in which the square OMPN is divided by the curve is 1: 4

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25. Using integration find the area of the triangle whose vertices are A(-4,3) ,B (3,4) and C(8,6) `

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26. By the method of integration find the area of the triangle formed by the straight lines $x + 3y - 8 = 0, 5x - y - 8 = 0$ and $x - y + 4 = 0$

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27. The curve y $ax^{\frac{1}{2}}$ +bx passes through the point (1,2) and the area enclosed by the curve the x-axis and the line x =4 is 8 square units. Determine a and b

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Exercise 17 M C Q

1. The area bounded by to the line $2x - 3y = 0$,x-axis and the ordinates x = 3 ,x=5 (in square units)-is

A. 16

B. 8

C. 4

D. $\frac{16}{3}$

Answer: C

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2. The area bounded by the curve $2y^2 - 3x = 0$ (in square units) y-axis and the two horizontals y=1 and y= 4 is-

A. 7

B. 14

C. $\frac{64}{9}$

D. $\frac{110}{9}$

Answer: B

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3. The area bounded by the curve y=$\cos x$,x-axis and the two ordinates $x = -\frac{\pi}{2}, x = \frac{\pi}{2}$ (in square units) is-

A. 2

B. -2

C. 1

D. -1

Answer: A

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4. The area (in square unit) bounded by the curve y= sin x, x-axis and the two ordinates x $= \pi, x = 2\pi$ is-

A. 1

B. -1

C. -2

D. 2

Answer: C

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Very Short Answer Type Questions

1. State the geometrival interpretation of $\int_a^b f(x)dx$

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2. State the geometrival interpretation of $\int_c^d \phi(y)dy$

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3. Using intregation find the area of the region bounded by the line $2y + x = 8$ and the lines x= 2 and x =4

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4. Draw a sketch graph showing the area of the region bounded by the parabola $y = x^2$,the x -axis and $x = 2$ Calculated its area.

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5. Draw sketch graph of $y^2 = x$ the y-axis and the straight line y=3 and shade the region bounded by them. Find the area of the shaded region

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6. Find the area bounded by the x-axis and one arc of the sine curve y= sin x between (0,0) and $(\pi, 0)$

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7. Draw the graph of y $= \cos x$between $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$ .Find the area between this curve and the x-axis

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8. Using integration,find the area of ΔPQR whose vertices are $P(2, 1), Q(3, 4)$ and $R(5, 2)$

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9. Find the area in the fourth quadrant bounded by the curve $y = x^3 - 8$ and the coordinate axes

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10. Determine the area bounded by the rectangular hyperbola $xy = c^2, thex - a\xi s$ and the two ordinates x =c,x=2c

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Short Answer Type Questions

1. Find by integration the area of the triangle bounded by the lines $3x - 2y = 6, y = 0$ and x=4 Verify your result by using the definition or area of a triangle as half the product of the base and altitude

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2. Find by integration the area of the trapezoid bounded by $y = 4x = -3, y = 0$ and x=3 .Verify your result by finding the area of a trapezoid as the product of half the sum of the two parallel sides and the distance between them.

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3. Find the area bounded by the parabola $x^2 = 12y$ and its latus rectum

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4. Find the area of the

(i) circle x =a cos θ , $y = a \sin \theta$

(ii) ellips $x = a \cos \theta, y = b \sin \theta$ by the method of integration

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5. Calculated the area enclosed by the ellipse $4x^2 + 9y^2 = 36$ and the x-axis

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6. Find the area of the plain region enclosed by the curve $y^2 = 2y - x$ and the y-axis

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7. Determine the area lying above the x-axis and under the parabola y=2x-x^2

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8. Find the area bounded by the parabola $y = 16(x-1)(4-x)$ and the x -axis

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9. Mark the area bounded by the curve $(y-1)(y-5) = 4x$ and the y-axis and obtain the area by integration

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10. Draw the graph of the curve $y = 3x^2 + 2x + 4$ shade the area enclosed by the curve ,the x-axis and the lines x =-1 and x =3 Find the area of the shaded region by the method of integration

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11. Using integration find the area of the region bounded by the lines $y = 1 + |x + 1|, x = 2, x = 3$ and $y = 0$

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12. Show that a triangle made by a tangent at any point on the curve xy $= c^2$ and the coordinates axes is of constant area.

A. s

B.

C.

D.

Answer:

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13. Find the common area between the parabolas $y^2 - ax = a^2$ and $y^2 + ax = a^2$

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Long Answer Type Questions

1. Find the area bounded by the curve $f(x) = 4 - |x|$ and the x axis

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2. (i) Using definite integral find the area of the triangle bounded by the straight lines x =0,y=4x and 2x+ y =6

(ii) Using definite integral find the area of the triangle bounded by the straight lines x =0 ,y=x and 2y+x=6

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3. Shade the area bounded by $y^2 = 8x$ and y=x above positive direction of x-axis and use integration to find the area of that part.

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4. Draw the sketch graph of the function $y = x^2$ and $y = x^3$ and shade the areas $\int_0^1 x^2 dx$ and $\int_0^1 x^3 dx$ what will be the value of the area enclosed by these two curves?

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5. Show that,the area bounded by the parabola y^2 =4ax and a double ordinate is two third of the rectangle formed by this ordinate and the abscissa.

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6. Using integration ,find the area of the region enclosed between the circles $x^2 + y^2 = 4$and $(x-2)^2 + y^2 = 4$

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7. Find the area of segment of the parabola $y = x^2 - 5x + 15$ cut off by the straight line $y = 3x + 3$

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8. Mark the area bounded by the curve $(y-1)(y-5) = 4x$ and the y-axis and obtain the area by integration

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9. shade the area enclosed by the parabolas y^2=x and $x^2 = y$ and use the method of integration to find the area so enclosed

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10. Using integration find the area of the regions common to the circle $x^2 + y^2 = 16$ and the parabola $y^2 = 6x$

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11. Draw rough sketch of the area bounded by the curves $x + 2y = 1$ find its area.

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12. Find the common area between the parabolas $y^2 = 4ax$ and $x^2 = 4by$

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13. Determine the area enclosed between the parabola $x^2 = 8y$ and the straight line $x - 2y + 8 = 0$

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14. Find the area of the region $\{(x, y) : x^2 + y^2 < 1 < x + y\}$

(ii) Find the area of the region $\{(x, y) : x^2 + y^2 < 2ax, y^2 > ax, x > 0, y > 0\}$

(iii) Using integration find the area of the region $\{x, y) : y^2 < 4x, 4x^2 + 4y^2 < 9\}$.

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15. Find the area included between the parabolas $y^2 = 16x$ and the line joining its vertes and an end of latus-rectum

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16. Show that,the area bounded between $y = x^3$ and $y = 4x$ in the first quadrant is 4 square units.

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17. Find the area in the first quadrant bounded by the circle $x^2 + y^2 = 16$ and the line y=x

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18. Find the area of the smaller part into which the circle $x^2 + y^2 = a^2$ is divided by the straight line $x = \frac{a}{\sqrt{2}}$

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19. The straight line $bx + ay = ab$ divides the ellipse $b^2x^2 + a^2y^2 = a^2b^2$ into two parts.prove that the area of the smallar part is $\frac{ab}{4}(\pi - 2)$ square units.

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20. If the area bounded by the parabola$y^2 = 4ax$ and its double ordinate $x = h$ is two times the area bounded by its latus rectum prove that $h : a, = 2^{\frac{3}{2}} : 1.$

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21. The area bounded in the first quadrant by the rectangular hyperbola $xy = k^2$ the x-axis and the ordinates $x = 3, x = cis2k^2 \log 5$, find c

(ii) If the area enclosed between the curves $y = kx^2$ and $x = ky^2 (k > 0)$ is 1 square units find k

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22. Prove that the curves $y^2 = 4ax$ and $x^2 = 4ay$ divide the square bounded by $x = 0, y = 0, x = 4a$and y=4a into three equal parts.

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23. Perpenicular PM and PN are drawn upon the coordinates axes OX and OY respectively from the point P(3,3) situated on the

parabola $y^2 = 3x$.Show that ,the ratio of the larger to the smallar of the two areas in which the area PNOM is divided by the are OP of the parabola is 2: 1

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24. IF A be the area bounded by the x-axis and one are of the curve $y = a\cos 3x$ between (0,0) and $\left(\frac{\pi}{6}, 0\right)$ and B be the area bounded by the x-axis and one are of the curve $y = a\cos\frac{x}{4}$ between (0,0) and $(2\pi, 0)$ show that ,A : B =1: 12

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25. Using integration find the area the area bounded by the lines $y = 4x + 5, y = 5 - x$ and $4y = x + 5$

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26. Show that ,the area of the coodinates axes is $\frac{a^2}{6}$ square units

(ii) Using the method of integration find the area bounded by the curve $|x| + |y| = 1$

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27. The area between $x = y^2$ and x = 4 is divided into two equal parts by the line x= a, find the value of a.

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28. Find the area of the region bounded by the parabola $y = x^2$ and $y = |x|$

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29. The area bounded by the curve $y = x^3$, x-axis and the ordinates: x = - 1 and x= 1 is given by

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30. A function $y = f(x)$ is defined as follow:

$$y = f(x) = \begin{cases} x^2 & when 0 < x < 1 \\ \sqrt{x} & when x > 1 \end{cases}$$

Find the area above the x-axis included between the curve $y = f(x)$ and the line x=4

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31. Find by integration the area of the triangle formed by the x-axis and the tangent and normal to the parabola $y = 6x - x^2$ at (5,5)

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Mcq

1. Area of the region bounded by the curve $y = e^x$ and lines x =0 and y=e is-

A. e-1

B. $\int_1^e (\log(e+1-y)dy$

C. $e - \int_0^1 e^x dx$

D. $\int_1^e \log y dy$

Answer: B::C::D

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2. The area enclosed between the curves $y = x$ and $x^2 = y$ is equal to-

A. $\frac{1}{6}$

B. $\int_0^1 (x - x^2) dx$

C. $\frac{1}{2}$

D. $\frac{3}{4}$

Answer: A::B::C

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3. Let S be the area of the region enclosed by $y = e^{-x^2}$,y=0 x =0 and x=1 ,Then -

A. $S > \frac{1}{e}$

B. $s \geq 1 - \frac{1}{e}$

C. $slq\frac{1}{4}\left(1 + \frac{1}{\sqrt{e}}\right)$

D. $S < \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{e}}\left(1 - \frac{1}{\sqrt{2}}\right)$

Answer: A::B::C

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4. The parabolas $y^2 = 4x$ and $x^2 = 4y$ divide the square region bounded by the lines x=4,y=4 and the coordinates axes. If S_1, S_2, S_3 are the areas of these parts numbered from the top to bottom respectively,then-

A. $S_1 : S_2 = 1 : 1$

B. $S_2 : S_3 = 1 : 2$

C. $S_1 : S_3 = 1 : 1$

D. $S_1 : (S_1 + S_2) = 1 : 2$

Answer: A::C::D

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5. Let the curve $y = ax^{\frac{1}{2}}$ +bx passes through the point (1,2) and lies above the axis for $0 < x < 9$ .If the area enclosed by the curve the x-axis and the line x =4 is 8 eq. units Then-

A. a=1

B. b=1

C. a=3

D. b=-1

Answer: C::D

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Integer Answer Type

1. The area of the figures bounded by the curves $y = |x - 1|$ and y=3 -|x| is-

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2. If S is the area bounded byb the curve y $= \sqrt{1 - x^2}$ and $y = x^3 - x$ then the value of $\frac{\pi}{S}$ is equal to-

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3. If the area inside the parabola $5x^2 - y = 0$ but outside the parabola $2x^2 - y + 9 = 0$ is $2K\sqrt{3}$ sq . Units then the value of K is equal to-

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4. If the area enclosed between the curves $|y| = 1 - x^2$ and $x^2 + y^2 = 1$ is $\frac{3\pi - K}{8}$ sq. unit ,then the value of K is equal to-

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5. The area enclosed between the curve $y = \log_e(x + e), x \log_e\left(\frac{1}{y}\right)$ and x-axis is K sq. units then the value of K is equal to-

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Matrix Match Type

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2.

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Comprehension Type

1. Consider the curves

$C_1 : x = 0, C_2 : y = 0, C_3 y = x^2 + 1, C_4 : y = 2, C_5 : x = 1$

the area enclosed between the curves C_1, C_2, C_3 and c_5 is (in square units) -

A. $\frac{5}{6}$

B. $\frac{4}{3}$

C. $\frac{2}{3}$

D. $\frac{7}{3}$

Answer: B

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2. Consider the curves

$C_1 : x = 0, C_2 : y = 0, C_3 y = x^2 + 1, C_4 : y = 2, C_5 : x = 1$

The area bounded by the curves C_3 and C_4 (in square units)

A. $\frac{20}{3}$

B. $\frac{2}{3}$

C. $\frac{5}{6}$

D. $\frac{7}{5}$

Answer: A

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3. Consider the curves

$C_1 : x = 0, C_2 : y = 0, C_3 y = x^2 + 1, C_4 : y = 2, C_5 : x = 1$

The area bounded by the curves C_1, C_3 and C_4 and which lies to the right of C_1 is (in square units)-

A. $\frac{4}{3}$

B. $\frac{5}{6}$

C. $\frac{7}{5}$

D. $\frac{2}{3}$

Answer: D

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4. The area of the region bounded by the curve and the line x=-1 is

A. $(\pi + 1)$ square units

B. $(\pi - 1)$ square units

C. $\left(\frac{\pi}{2} + 1\right)$ square units

D. $\left(\frac{\pi}{2} - 1\right)$ square units

Answer: A

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5. Line x =0 divides the region mentioned above in two parts,The ratio of the area left-hand side of the line to that of right-hand side of the line is-

A. $(2 + \pi) : \pi$

B. $(2 - \pi) : \pi$

C. $1 : 1$

D. $(\pi + 2) : \pi$

Answer: D

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6. The area of the region of curve and lines x =0 and $x = \frac{1}{2}$ is -

A. $\left(\frac{\sqrt{3}}{4} + \frac{\pi}{6}\right)$ square units

B. $\left(\frac{\sqrt{3}}{2} + \frac{\pi}{6}\right)$ square units

C. $\left(\frac{\sqrt{3}}{4} - \frac{\pi}{6}\right)$ square units

D. $\left(\frac{\sqrt{3}}{2} - \frac{\pi}{6}\right)$ square unit

Answer: A

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Assertion Reason Type

1. Statement-I : Area bounded by y =e^x ,y=0 and x=0 is 1 square units

Statement -II: Area bounded by y $= \log_e x, x = 0$ and $y = 0$ is 1 square units

A. Statement -I is True statement -II is a correct explanation for statement-I

B. Statement -I is true ,Statement-II is not True explanation for Statement-I

C. Statement -I is True Statement-II is False

D. Statement -I is False,Statement -II is True

Answer: A

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2. f(x) is a polynomial of degree 3 passing through the origin having local extrema at $x = \pm 2$

Statement-I: Ratio of the areas in which f(x) cuts the circle $x^2 + y^2 = 36$ is 1: 1

Statement-II Both $y = f(x)$and the circle are symmetric about the origin

A. Statement -I is True statement -II is a correct explanation for statement-I

B. Statement -I is true ,Statement-II is not True explanation for Statement-I

C. Statement -I is True Statement-II is False

D. Statement -I is False,Statement -II is True

Answer: A

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