



# MATHS

# BOOKS - CHHAYA PUBLICATION MATHS (BENGALI ENGLISH)

# DIRECTION COSINES AND DIRECTION RATIOS



1. Find the direction cosines of the straight line

joining the points (4, -3, -1) and (1, -1, 5).





3. Prove that the straight line joining the points A(4,

-3, 2) and B(2, -1, -1) is parallel to the straight line

joining the point C(7, -3, 5) and D(1, 3, -4).





respectively. Find :  $\overrightarrow{AB}$ 





8. Points A and B have position vector  

$$\overrightarrow{a} = -3\hat{i} + 2\hat{j} + 7\hat{k}$$
 and  $\overrightarrow{b} = 3\hat{i} + 4\hat{j} - 5\hat{k}$   
respectively. Find : The direction ratios of  $\overrightarrow{AB}$ 

9. Points A and B have position vector  $\overrightarrow{a} = -3\hat{i} + 2\hat{j} + 7\hat{k}$  and  $\overrightarrow{b} = 3\hat{i} + 4\hat{j} - 5\hat{k}$ respectively. Find : The direction cosines l, m, n of  $\overrightarrow{AB}$ 



10. Points A and B have position vector  $\overrightarrow{a} = -3\hat{i} + 2\hat{j} + 7\hat{k}$  and  $\overrightarrow{b} = 3\hat{i} + 4\hat{j} - 5\hat{k}$ respectively. Show that  $l^2 + m^2 + n^2 = 1$  where I,m,n are the direction cosine of  $\overrightarrow{AB}$ 

11. Find the direction cosines of the straight line which makes equal angles with the coordinate axes.Watch Video Solution

**12.** A straight line in the yz-plane makes an angle  $60^{\circ}$  with the positive z-axis. Find the direction cosines of the line.



**13.** A straight line makes angles  $45^{\circ}$  and  $60^{\circ}$  with the positive directions of y-axis and z-axis respectively. Find the acute angle made by the line with the x-axis.



**14.** The acute angle between the z-axis and the straight line joining the points (3, 2, 3) and (-3, -1, 5)

is -



**15.** The acute angle between the straight lines whose direction numbers are 1, 1, 2 and  $(\sqrt{3}-1), (-\sqrt{3}, -1), 4$  is -

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16. Determine the values of a and b for which the straight line joining the points A(3, 4, -1) and B(4, a, 2) is parallel to the straight line joining the points P(2, 1, b) and Q(4, -3, 1).

17. Using direction ratio show that the points (2, 6,

3), (1, 2, 7) and (3, 10, -1) are collinear.



**18.** If A, B, C, D are the points (2, 3, -1), (5, 2, 3), (4, 3,

-5) and (-2, 1, -8) respectively, then find the projection

of AB on CD.



**19.** The direction cosines of the line which is perpendicular to the lines with direction ratios 1, -2, -2 and 0, 2, 1 are -

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**20.** The coordinates of the foot of the perpendicular drawn from the point P(1, 2, 1) to the straight line joining the points Q(1, 4, 6) and R(5, 4, 4) are -



**21.** If A(1, 4, 2), B(-2, 1, 2) and C(2, -3, 4) are the vertices of the triangle ABC, then find the angles of the triangle ABC.

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**22.** A(2, 3, 1), B(-2, 2, 0) and C(0, 1, -1) are the vertices of the triangle ABC. Show that the triangle ABC is right -angled.

**23.** If  $l_1$ ,  $m_1$ ,  $n_1$  and  $l_2$ ,  $m_2$ ,  $n_2$  are direction cosines of two mutually perpendicular straight lines, then prove that the direction cosines of the straight line which is perpendicular to both the given lines are

 $\pm (m_1n_2-m_2n_1),\ \pm (n_1l_2-n_2l_1),\ \pm (l_1m_2-l_2m_1)$ 

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24. Show that the pair of straight lines whose direction cosines are given by the equations 2l - m + 2n = 0 and mn + nl + lm = 0 are at right angles.

25. Find the acute angle vetween two straight lines whose direction cosines are given by the equations l - 5m + 3n = 0 and  $7l^2 + 5m^2 - 3n^2 = 0$ 

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26. Prove that the straight lines whose direction cosines are given by the equations al + bm + cn = 0 and fmn + gnl + hlm = 0 are at right angles if  $\frac{f}{a} + \frac{g}{b} + \frac{h}{c} = 0$ 

27. Prove that the straight lines whose direction cosines are given by the equations al + bm + cn = 0 and fmn + gnl + hlm = 0 are parallel if

 $a^2f^2+b^2g^2+c^2h^2-2(abfg+bcgh+cahf)=0$ 

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28. Prove that the acute angle between two

diagonals of a cube is  $\cos^{-1}\left(\frac{1}{3}\right)$ .

**29.** A straight line L makes angles  $\alpha, \beta, \gamma$  and  $\delta$  with

the four diagonals of a cube, prove that,

$$\sin^2lpha+\sin^2eta+\sin^2\gamma+\sin^2\delta=rac{4}{3}$$

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**30.** A variable line in two adjacent positions has direction cosines l, m, n and  $l + \delta l, m + \delta m, n + \delta n$ . Prove that the small angle  $\delta \theta$  between the two positions of the variable line is given by,  $(\delta \theta)^2 = (\delta l)^2 + (\delta m)^2 + (\delta n)^2$ 

**31.** Let  $\overrightarrow{P} = 2\hat{i} + \hat{j} - 2\hat{k}$  and  $\overrightarrow{Q} = 4\hat{i} - 3\hat{k}$ . Find the acute angle between  $\overrightarrow{P}$  and  $\overrightarrow{Q}$ .



1. The direction cosines of x-axis are-

A.0, 1, 0

B. 0, 0, 1

C. 1, 0, 0

D. 1, 1, 1



- 2. The direction cosines of y-axis are-
  - A. 1, 1, 1
  - B.1, 0, 0
  - C.0, 0, 1
  - D.0, 1, 0

#### Answer: D



- 3. The direction cosines of z-axis are-
  - A.0, 0, 1
  - B.0, 1, 0
  - C. 1, 1, 1
  - D.1, 0, 0

#### Answer: A



**4.** If a line has the direction ratios -18, 12, -4 then its

direction cosines will be -

A. 
$$\frac{9}{11}$$
,  $\frac{6}{11}$ ,  $\frac{2}{11}$   
B.  $-\frac{9}{11}$ ,  $\frac{6}{11}$ ,  $-\frac{2}{11}$   
C.  $\frac{9}{11}$ ,  $-\frac{6}{11}$ ,  $-\frac{2}{11}$ 

D. none of these

#### Answer: B



5. State which of the following statement is true?

A. If the direction angles of a line be  $lpha, eta, \gamma$  then

$$lpha+eta+\gamma
eq 2\pi.$$

B. If the direction angles of a line be  $\alpha, \beta, \gamma$  then

$$lpha+eta+\gamma=2\pi.$$

C. If the direction cosines of a line be l, m, n

then 
$$l^2+m^2+n^2
eq 1.$$

D. The direction cosine of the angle between

vectors is nothing but the cross product of their direction cosines.

Answer: A

**6.** The angle between the lines whose direction ratios are proportional to 1, -2, 1 and 4, 3, 2 is -

A. 
$$\frac{3\pi}{4}$$
  
B.  $\frac{\pi}{2}$   
C.  $\frac{\pi}{3}$   
D.  $\frac{\pi}{4}$ 

#### Answer: B

7. If O is the origin, OP = 3 with direction ratios proportional to -1, 2, -2 then the coordinates of P are-

A. (-1, 2, -2) B. (1, 2, 2) C.  $\left(-\frac{1}{9}, \frac{2}{9}, -\frac{2}{9}\right)$ D. (3, 6, -9)

#### **Answer: A**



8. The angle between the two diagonals of a cube is-

A. 
$$\frac{\pi}{6}$$
  
B.  $\frac{\pi}{4}$   
C.  $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$   
D.  $\cos^{-1}\left(\frac{1}{3}\right)$ 

#### Answer: D



**9.** The direction cosines of the line joining the points (1, 2, -3) and (-2, 3, 1) are-

$$A. -3, 1, 4$$

$$\begin{array}{l} \text{B.} -1, \, 5, \, -2 \\ \text{C.} -\frac{3}{\sqrt{26}}, \, \frac{1}{\sqrt{26}}, \, \frac{4}{\sqrt{26}} \\ \text{D.} -\frac{1}{\sqrt{30}}, \, \frac{5}{\sqrt{30}}, \, -\frac{2}{\sqrt{30}} \end{array}$$

#### Answer: C



**10.** If a line whose direction ratios are proportional to 0, 1, -1 then the inclination of the line with z-axis is-

A. 
$$\frac{\pi}{2}$$

B.  $\pi$ 

C. 
$$\frac{3\pi}{2}$$
  
D.  $\frac{3\pi}{4}$ 

#### Answer: D



**11.** If P(x, y, z) is a point in the space at a distance r from the origin O, then the direction cosines of the line OP are-

A. 
$$\frac{r}{x}, \frac{r}{y}, \frac{r}{z}$$
  
B.  $rx, ry, rz$   
C.  $\frac{x}{r}, \frac{y}{r}, \frac{z}{r}$ 

D. none of these

#### Answer: C



# Exercise 3 Very Short Type Questions

### 1. Define :

Direction angles of a straight line,



4. Can the numbers 1, 2, 3 be the direction cosines of

a straight line?

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5. Can the numbers 1, 2, 3 be the direction ratios of a

straight line?

. . . . .

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6. Find the direction cosines of the straight line

joining the points : (2, -1, 4) and (0, 1, 5)



**8.** The direction angles of a straight of a straight line are  $120^{\circ}$ ,  $45^{\circ}$ ,  $30^{\circ}$ . Is the statement true? Give reasons for your answer.

9. Find the acute angle between the pair of straight



**10.** Find the cosine of the angle between the lines BA and BC where A, B, C are the points (1, 2, 3), (2, 5, -1) and (-1, 1, 2) respectively.



11. Find the angle between the lines whose direction

ratios are

2, 3, 6 and 1, 2, 2

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12. Find the angle between the lines whose direction

ratios are

5, -12, 13 and -3, 4, 5

13. Find the angle between the lines whose direction

ratios are

p, q, r and q-r, r-p, p-q

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14. Find the angle between the lines whose direction

ratios are

2, 1, -2 and 3, -4, 5.

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**Exercise 3 Short Answer Type Questions** 

**1.** Find the direction ratios of a straight line which makes equal angles with the coordinate axes. How many such straight line are there?

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2. A straight line in the zx-plane makes an angle of  $\frac{\pi}{3}$  with the z-axis, find the direction cosines of the

line.

3. If  $lpha, eta, \gamma$  be the direction angles of a straight line,

then prove that  $\sin^2lpha + \sin^2eta + \sin^2\gamma = 2$ 



**4.** A line makes an acute angle with the positive direction of x-axis and passes through the points (6, -7, -1) and (2, -3, 1), find the direction cosines of the line.

5. A straight line in the xy-plane makes an angle  $\frac{\pi}{4}$  with y-axis. Find the direction cosines of the straight line.



**6.** Calculate the direction cosines of the sides of the triangle whose vertices are A(4, 5, 0), B(2, 6, 2) and C(2, 3, -1).

7. Using direction ratios show that the points P(2, 3,

4) Q(-1, -2, 1) and R(5, 8, 7) are collinear.



**8.** A staright line makes angles  $\frac{\pi}{4}$  and  $\frac{\pi}{3}$  with the positive directions of y-axis and z-axis respectively. Find the acute angle made by the line with x-axis.



**9.** If O is the origin and A(2, 3, 1), B(1, 1, -5) are two given points then show that the straight line OA is perpendicular to the straight line OB.

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**10.** Show that the staright line joining the points (-1, 0, -2) and (1, 3, -1) is perpendicular to the straight line joining the points (9, 1, -6) and (7, 2, -5)



**11.** Prove that the straight line joining the points (4, 5, 0) and (5, 3, 3) is parallel to the straight line joining the point (4, 3, -3) and (6, -1, 3).

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**12.** Calculating the angle of the triangle, prove that the points A(3, 4, -1) , B(1, 5, 1) and C(1, 2, -2) are the vertices of an isosceles triangle.

**13.** Find the acute angle between x-axis and the straight line joining the points (1, 1, 3) and (3, 2, 1).



14. Calculating direction ratios show that the triangle formed by joining the points (2, 3, 1), (-2, 2, 0) and (0, 1, -1) in right-angled.



**15.** Straight lines OA and OB are drawn from the origin O, if the direction ratios of the lines OA and OB are 1, -1, -1 and 2, -1, 1 respectively then find the direction cosines of the normal to the plane AOB.



**16.** Find the coordinates of the foot of the perpendicular drawn from the point A(1, 8, 4) on the straight line joining the points B(0, -11, 4) and C(2, -3, 1).



**17.** Determine the values of a and b for which the straight line joining the points (7, 4, 2) and (3, -2, 5) may be parallel to the line joining the points (2, a, 5) and (b, -15, 11).



18. If the straight line joining the points (4, -3, 2) and

(3, -1, 5) is perpendicular to the straight line joining the points (K, -2, 1) and (7, 3, -2) then find the value of K.



19. Find the acute angle between two straight lines

whose direction number are -4, 3, 5 and 3, 4, 5.



**20.** Let A(2, -3, -1), B(4, 5, 2), C(-3, 4, 1) and D(2, 3, 5)

are four given points. Find direction cosines of a straight line which is perpendicular to both the straight line AB and CD.

**21.** Using direction ratios prove that the points (4, 2, -6), (5, -3, 1), (12, 4, 5) and (11, 9, -2) taken in order are the vertices of a rectangle.

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**22.** Using direction numbers show that points P(4, 7, 8) and Q(2, 3, 4), R(-1, -2, 1) and S(1, 2, 5) taken in order, are the vertices of a parallelogram. Is the parallelogram a rectangle?

**23.** Let  $l_1, m_1, n_1$  and  $l_2, m_2, n_2$  be the direction ratios of two given straight lines. Find the direction ratios of a straight line which is perpendicular to both the given straight lines.



**24.** Let P(-9, 4, 5) and Q(11, 0, -1) be two given points. If O be the origin and ON be perpendicular to PQ then find the coordinates of N.



**25.** Let A(-2, 0, 3), B(0, 3, -3), C(3, 3, 5) and D(5, 4, 3) be four given points, find the acute angle between the lines AB and CD.

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**26.** Find the acute angle between the two straight lines whose direction cosines are given by

$$l+m+n=0, l^2+m^2-n^2=0$$

27. Find the angle between the two straight lines

whose direction cosines are given by

2l + 2m - n = 0, mn + nl + lm = 0

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**28.** Find the angle between the two straight lines whose direction cosines are given by

 $l+2m+3n=0, 3lm-4\ln+mn=0$ 

29. Find the acute angle between the two straight

lines whose direction cosines are given by

 $3l+m+5n=0,\,6mn-2nl+5lm=0$ 

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Exercise 3 Long Answer Type Question

1. Prove that the straight lines whose direction cosines are given by  $a^2l + b^2m + c^2n = 0$  and mn + nl + lm = 0 are parallel if a + b + c = 0.



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**3.** If  $l_1, m_1, n_1, l_2, m_2, n_2, l_3, m_3, n_3$  be the direction cosines of three mutually perpendicular straight lines, then show that the straight line whose direction numbers are  $(l_1 + l_2 + l_3), (m_1 + m_2 + m_3), (n_1 + n_2 + n_3)$  makes equal angles with the given lines.



**4.** A, B, C, D are consecutive vertices of a parallelogram. If the coordinates of A, B and C are (0, 0, 0), (3, -4, 4) and (7, 1, 4) respectively, find using direction ratios the coordinates of the vertex D.

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**5.** Find the projection of the line segment joining the points (-1, 0, 3) and (2, 5, 1) on the line whose direction ratios are 6, 2, 3.

6. Show that the pair of staright lines whose direction cosines are given by al + bm + cn = 0 and  $pl^2 + qm^2 + rn^2 = 0$  are perpendicular to each other if

$$a^2(q+r)+b^2(r+p)+c^2(p+q)=0$$

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## Mcqs

**1.** The direction cosines of a line equally inclined with the coordinates axes are-



#### Answer: A::B::C::D



**2.** A line segment has length 63 units and direction ratios are 3, -2, 6. The components of the line vector are-

A. - 27, 18, 54

- B. 27, -18, -54
- C. 27, -18, 54
- D. 27, 18, -54

### Answer: C::D

**3.** The midpoints of the sides of a triangle are (1, 5, -1), (0, 4, -2) and (2, 3, 4). The coordinates of its vertices are-

A. (1, 1, 1)

B. (1, 2, 3)

C. (3, 4, 5)

D. (-1, 6, -7)

Answer: B::C::D

**4.** The coordinates of the foot of perpendicular drawn from the point A (1, 2, 1) to the joining B(1, 4, 6) and C(5, 4, 4) is (x, y, z), then values of x, y, z will be-

A. 3

B.4

C. 5

D. none of these

Answer: A::B::C



5. Find the direction cosines of the two lines which are connected by the relations l-5m+3n=0 and  $7l^2+5m^2-3n^2=0$  –



Answer: A::B::C::D

1. The distance of the point (2, 3, 4) from the x-axis is

K units. Find the value of K.

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**2.** The inclination of a line with z-axis is  $\frac{K\pi}{4}$ , if its direction ratios are proportional to 0, 1,-1, then find the value of K.

3. The coordinates of the projection of the point P(2,

-3, 5) on y-axis is  $(0, -\beta, 0)$ . Find the value of  $\beta$ .



**4.** A parallelpiped is formed by planes drawn through the points (2, 3, 5) and (5, 9, 7) parallel to the coordinate planes. The length of diagonals of the parallelopiped is 'd' units, find 'd'.



5. If the line makes angles  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\delta$  with four diagonals of a cube, then the value of  $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma + \cos^2 \delta$  is equal to  $\frac{m}{3}$ . Find m.

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**Comprehension Type** 

**1.** A (-2, 2, 3) and B(13, -3, 13) are the points. L is a line through A.

Coordinates of the point P which divides the join of

A and B in the ratio 2:3 internally are-

A. 
$$\left(\frac{33}{5}, \frac{-2}{5}, 9\right)$$
  
B.  $(4, 0, 7)$   
C.  $\left(\frac{33}{5}, \frac{-12}{5}, \frac{17}{5}\right)$   
D.  $(20, 0, 35)$ 

#### **Answer: B**



**2.** A (-2, 2, 3) and B(13, -3, 13) are the points. L is a line through A.

Direction ratios of the normal to the plane passing through the origin and the points A and B areA. 15, -5, 10

B. 11, -1, 16

C. 3, 13, 2

D. 7, 13, -4

#### **Answer: D**



**3.** A (-2, 2, 3) and B(13, -3, 13) are the points. L is a line through A.

A point P moves in the space such that 3PA=2PB, then the locus of P is-

A. 
$$x^2 + y^2 + z^2 + 28x - 12y + 10z - 247 = 0$$
  
B.  $x^2 + y + z^2 - 28x + 12y + 10z - 247 = 0$   
C.  $x^2 + y^2 + z^2 + 28x - 12y - 10z + 247 = 0$   
D.  $x^2 + y^2 + z^2 - 28x + 12y - 10z + 247 = 0$ 

#### Answer: A



**4.** Consider the points A(0, 0, 0), B(5, 0, 0), C(3, 0, 4)

and D(0, 4, 3).

The points A, B, C and D are-

A. coplanar

B. the vertices of a square

C. non-coplanar

D. none of these

Answer: C

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5. Consider the points A(0, 0, 0), B(5, 0, 0), C(3, 0, 4)

and D(0, 4, 3).

The line AC and BD are-

A. coplanar

B. skew

C. parallel

D. none of these

**Answer: B** 

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6. Consider the points A(0, 0, 0), B(5, 0, 0), C(3, 0, 4)

and D(0, 4, 3).

Area of triangle ABC (in square units) is -

A. 5

B.  $5\sqrt{2}$ 

C. 10

D. none of these

Answer: C

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Assertion Reason Type

**1.** Consider the points A(0, 0, 0), B(2, 0, 0),  
$$C(1, \sqrt{3}, 0)$$
 and  $D\left(1, \frac{1}{\sqrt{3}}, \frac{2\sqrt{2}}{\sqrt{3}}\right)$ 

Statement - I : ABCD is a square.

Statement - II : |AB| = |BC| = |CD| = |DA|.

A. Satement -I is True, Statement -II is True,
Statement -II is a correct explanation for
Statement -I
B. Satement -I is True, Statement -II is True,
Statement -II is not a correct explanation for

Statement -I

C. Stament -I is True, Statement -II is False.

D. Statement -I is False, Statement -II is True.

#### Answer: D

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2. Statement -I : A line makes the same angle  $\theta$  with each of the x and z-axis. If it makes an angle  $\alpha$  with the y-axis, such that  $\sin^2 \alpha = 3 \sin^2 \theta$ , then  $\cos^2 \theta = \frac{3}{5}$ .

Statement -II : If a line with direction ratios, l, m, nmakes angles  $\alpha, \beta, \gamma$  respectively with x, y amd zaxis then

$$\coslpha=rac{l}{\sqrt{l^2+m^2+n^2}}, \coseta=rac{m}{\sqrt{l^2+m^2+n^2}}$$
 and  $\cos\gamma=rac{n}{\sqrt{l^2+m^2+n^2}}.$ 

A. Satement -I is True, Statement -II is True, Statement -II is a correct explanation for Statement -I B. Satement -I is True, Statement -II is True, Statement -II is not a correct explanation for Statement -I C. Stament -I is True, Statement -II is False.

D. Statement -I is False, Statement -II is True.

Answer: A

