MATHS

BOOKS - CHHAYA PUBLICATION MATHS (BENGALI ENGLISH)

INCREASING AND DECREASING FUNCTION

Example

1. Show that the function $f(x) = \dfrac{3x+5}{x+2}$ is strictly monotonic increasing in $[0, \infty)$.

Watch Video Solution

2. Prove that $\phi(x) = \cos x$ is a strictly monotonic decreasing function in $0 \leq x \leq \pi$.

Watch Video Solution

Illustrative Examples

1. Find the intervals in which the function $f(x) = \frac{x}{x^2+1}$ is increasing

Watch Video Solution

2. Find the intervals in which the function $f(x) = \frac{x}{x^2+1}$ is decreasing

Watch Video Solution

3. Show that the function $\left(x^3 - 3x^2 + 4x\right)$ increases with x.

Watch Video Solution

4. Find the intervals in which the function $f(x) = \frac{4x^2 + 1}{x}$ is increasing

Watch Video Solution

5. Find the intervals in which the function $f(x) = \frac{4x^2 + 1}{x}$ is decreasing

Watch Video Solution

6. Find the range of values of x for which the function $f(x) = 2x^3 - 9x^2 - 24x + 5$ increases with x

Watch Video Solution

7. Find the range of values of x for which the function $f(x) = 2x^3 - 9x^2 - 24x + 5$ decreases with x.

Watch Video Solution

8. If $f(x) = 10 - 9x + 6x^2 - x^3$, examine whether $f(x)$ increases or decreases for values of x, for which

$x > 3$

Watch Video Solution

9. If $f(x) = 10 - 9x + 6x^2 - x^3$, examine whether $f(x)$ increases or decreases for values of x, for which

$x < 1$

Watch Video Solution

10. If $f(x) = 10 - 9x + 6x^2 - x^3$, examine whether $f(x)$ increases or decreases for values of x, for which

$1 < x < 3.$

Watch Video Solution

11. Find the intervals in which the function f given by $f(x) = x^3 + \frac{1}{x^3}(x \neq 0)$ is increasing

Watch Video Solution

12. Find the intervals in which the function f given by $f(x) = x^3 + \frac{1}{x^3}(x \neq 0)$ is decreasing

Watch Video Solution

13. Show that, for $0 < \theta < \frac{\pi}{2}, \frac{\sin\theta}{\theta}$ steadily decreases as θ continually increases.

Watch Video Solution

14. Find the intervals in which the function f given by $f(x) = \sin x + \cos x (0 \leq x \leq 2\pi)$ is increasing

Watch Video Solution

15. Find the intervals in which the function f given by $f(x) = \sin x + \cos x (0 \leq x \leq 2\pi)$ is decreasing

Watch Video Solution

16. Find the intervals in which the function $f(x) = x^x (x > 0)$ is increasing

Watch Video Solution

17. Find the intervals in which the function $f(x) = x^x (x > 0)$ is decreasing.

Watch Video Solution

18. Show that $f(x) = \tan^{-1}(\cos x + \sin x)$ is a stictly increasing function in the interval $\left(0, \frac{\pi}{4}\right)$.

Watch Video Solution

19. Prove that the function $f(x) = x^3 - x^2$ is neither increasing nor decreasing in $-\frac{2}{3} < x < \frac{2}{3}$.

Watch Video Solution

20. Find the intervals in which the function $f(x) = (x+1)^3(x-3)^3$ is increasing

Watch Video Solution

21. Find the intervals in which the function $f(x) = (x+1)^3(x-3)^3$ is decreasing.

Watch Video Solution

22. Sparate the interval $\left[0, \frac{\pi}{2}\right]$ into sub-intervals in which $f(x) = \sin^4 x + \cos^4 x$ is increasing

Watch Video Solution

23. Sparate the interval $\left[0, \frac{\pi}{2}\right]$ into sub-intervals in which $f(x) = \sin^4 x + \cos^4 x$ is decreasing

Watch Video Solution

24. Find the intervals in which the function $f(x) = \log(1+x) - \frac{x}{1+x}$ is increasing

Watch Video Solution

25. Find the intervals in which the function $f(x) = \log(1+x) - \frac{x}{1+x}$ is decreasing

Watch Video Solution

26. If $x > 0$, show that, $x > \log(1+x) > x - \frac{x^2}{2}$.

Watch Video Solution

27. Let $y = 3x.\dfrac{x+a}{x+b} + 5$ [a and b are positive constants and $a > b$] be the total cost for x unit of output of a commodity. Show that the marginal cost falls continuosly as the output increases.

Watch Video Solution

28. Use the function $f(x) = x^{\frac{1}{x}}, x > 0,$ to determine the bigger of the two numbers e^{π} and π^{e}.

Watch Video Solution

29. let $h(x) = f(x) - [f(x)]^2 + [f(x)]^3$ for every real number x. Prove that $h(x)$ is increasing or decreasing according as $f(x)$ is increasing or decreasing.

Watch Video Solution

30. The function $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in

Watch Video Solution

Exercise 15 Mcq

1. If $0 < \pi < \frac{\pi}{2}$ then,

(i) $\sin x$ is an increasing function,

(ii) $\cos x$ is an increasing function

(iii) $\tan x$ is an increasing function, then -

A. (i) and (ii) are true

B. (ii) and (iii) are true

C. (i) and (iii) are true

D. only (i) is true

Answer: c

Watch Video Solution

2. If the function $f(x)$ is differentiable at $x = a$, then it is increasing at $x = a$ when -

A. $f'(a) > 0$

B. $f'(a) < 0$

C. $f'(a) \geq 0$

D. $f'(a) \leq 0$

Answer: a

Watch Video Solution

3. (i) The function $f(x) = x^3$ is decreasing in $(-\infty, \infty)$

(ii) The function $f(x) = x^4$ is increasing in $(-\infty, 0)$, then-

A. only (i) is true

B. only (ii) is true

C. both (i) and (ii) are true

D. both (i) and (ii) are false

Answer: d

Watch Video Solution

4. A function $f(x)$ is defined in $a < x < b$ and $a < x_1 < x_2 < b$, then $f(x)$ is strictly monotonic decreasing in $a \leq x \leq b$ when-

A. $f(x_2) > f(x_1)$ when $x_2 > x_1$

B. $f(x_2) < f(x_1)$ when $x_2 > x_1$

C. $f(x_2) > f(x_1)$ when $x_2 < x_1$

D. $f(x_2) < f(x_1)$ when $x_2 < x_1$

Answer: b

Watch Video Solution

Exercise 15 Very Short Answer Type Questions

1. State when the function f(x) is said to be increasing in a finite interval $a \leq x \leq b$.

Watch Video Solution

2. State when the function f(x) is said to be decreasing in a finite interval $a \leq x \leq b$.

Watch Video Solution

3. Show that each of the following functions is strictly increasing

$$f(x) = \frac{2x-3}{4x+5}(x > 0)$$

Watch Video Solution

4. Show that each of the following functions is strictly increasing

$$\phi(x) = \sin x\left(0 \leq x \leq \frac{\pi}{2}\right)$$

Watch Video Solution

5. Show that each of the following functions is strictly increasing

$f(x) = \log_e x (x > 0)$

Watch Video Solution

6. Show that each of the following functions is strictly increasing

$\phi(x) = e^x$

Watch Video Solution

7. Prove that each of the following functions is strictly decreasing

$$f(x) = \frac{x+2}{x+1}(x > 0)$$

Watch Video Solution

8. Prove that each of the following functions is strictly decreasing

$$\phi(x) = \sin x\left(\frac{\pi}{2} \leq x \leq \pi\right)$$

Watch Video Solution

9. Prove that each of the following functions is strictly decreasing

$$f(x) = \frac{1}{x+1} + \frac{1}{x+2} + \frac{1}{x+3} \quad (x > 0)$$

Watch Video Solution

10. Show that, the function $4x^2 - 6x - 11$ is increasing at $x = 4$ and the function $\frac{x^2}{x^2+16}$ is decreasing at $x = -2$.

Watch Video Solution

11. Prove that the function $f(x) = x^3 - 3x^2 + 3x - 100$ is increasing on R.

Watch Video Solution

12. If $x > \frac{1}{2}$, show that the function $f(x) = x\left(4x^2 - 3\right)$ is steadily increasing.

Watch Video Solution

13. Show that, the function $f(x) = \cos 2x$ is increasing at $x = \frac{3\pi}{4}$.

Watch Video Solution

Exercise 15 Short Answer Type Questions

1. Find the intervals in which the function $f(x) = \frac{3}{x} + \frac{x}{3}$ is increasing

Watch Video Solution

2. Find the intervals in which the function $f(x) = \frac{3}{x} + \frac{x}{3}$ is decreasing

Watch Video Solution

3. Prove that the function $f(x) = x + \frac{1}{x}(x \neq 0)$ is decreasing for all x in $(-1, 0) \cup (0, 1)$.

Watch Video Solution

4. Find the intervals in which the following functions are (a) increasing (b) decreasing :

$f(x) = 2x^3 - 15x^2 + 36x + 1$

Watch Video Solution

5. Find the intervals in which the following functions are (a) increasing (b) decreasing :

$f(x) = x^3 - 12x^2 + 36x + 17$

Watch Video Solution

6. Find the intervals in which the following functions are (a) increasing (b) decreasing :

$f(x) = x^3 - 6x^2 + 9x + 15$

Watch Video Solution

7. 24. Find the intervals in which the following function is (a) increasing and (b) decreasing

$f(x) = 2x^3 + 9x^2 + 12x - 1$

Watch Video Solution

8. 24. Find the intervals in which the following function is (a) increasing and (b) decreasing

$f(x) = 2x^3 + 9x^2 + 12x - 1$

Watch Video Solution

9. Find the intervals in which the following functions are (a) increasing (b) decreasing :

$f(x) = 4 - 9x + 6x^2 - x^3$

Watch Video Solution

10. Find the intervals in which the function $f(x) = \sin x - \cos x$ where $0 < x < 2\pi$ is increasing.

Watch Video Solution

11. Find the intervals in which the function $f(x) = \sin x - \cos x$ where $0 < x < 2\pi$ is decreasing.

Watch Video Solution

12. Prove that the function $f(x) = x^3 - 6x^2 + 12x - 18$ is increasing for all $x \in \mathbb{R}$.

Watch Video Solution

13. Prove that the function $f(x) = x^2 - x + 1$ is neither increasing nor decreasing on $(-1, 1)$.

Watch Video Solution

14. Find the intervals in which $f(x) = 2x^3 - 24x + 7$ is increasing or decreasing.

Watch Video Solution

15. If $f(x) = (x - 1)e^x + 1$, show that f(x) is positive for all positive values of x.

Watch Video Solution

16. Prove that, the function $5 + 24x + 3x^2 - x^3$ increases in the interval $-2 < x < 4$.

Watch Video Solution

17. If $f(\theta) = \theta \sin\theta + \cos\theta$, show that, $f(\theta)$ steadily increases as θ continually increases in $0 < \theta < \frac{\pi}{2}$.

Watch Video Solution

18. Find the intervals in which $f(x) = (x+2)e^{-x}$ is increasing

Watch Video Solution

19. Find the intervals in which $f(x) = (x+2)e^{-x}$ is decreasing

Watch Video Solution

20. Find the intervals in which $f(x) = (x-1)^3(x-2)^2$ is increasing or decreasing.

Watch Video Solution

21. Find the intervals in which $f(x) = (x-1)^3(x-2)^2$ is increasing or decreasing.

Watch Video Solution

22. Prove that the function $f(x) = x^3 - 6x^2 + 12x - 18$ is increasing on R.

Watch Video Solution

Exercise 15 Long Answer Type Questions

1. Let $y = 3x.\ \dfrac{x+7}{x+5} + 4$ be the total cost for x units of output of a product. Show that the marginal cost falls continuously as the output increases. [Note that the marginal cost of a

product is the rate of change in cost for unit change in the output.]

Watch Video Solution

2. If $x > 0$, show that,

$x > \log(x^2 + 1)$

Watch Video Solution

3. If $x > 0$, show that,

$\log(1 + x) < x$

Watch Video Solution

4. If $x > 0$, show that,

$x > \sin x$

Watch Video Solution

5. If $x > 0$, show that,

$x^2 > (1+x)[\log(1+x)]^2$

Watch Video Solution

6. If $x > 0$, show that,

$$1 + x \log\left(x + \sqrt{x^2 + 1}\right) > \sqrt{1 + x^2}$$

Watch Video Solution

7. If $0 < x < \frac{\pi}{2}$, show that,

$$\sin x < x < \tan x$$

Watch Video Solution

8. If $0 < x < \frac{\pi}{2}$, show that,

$$\sin x > x - \frac{x^3}{3!}$$

Watch Video Solution

9. If $0 < x < \frac{\pi}{2}$, show that,

$$\cos x > 1 - \frac{x^2}{2!}$$

Watch Video Solution

10. If $x > 0$, prove that,

$$x > \log(1+x) > \frac{x}{1+x}$$

Watch Video Solution

11. If $x > 0$, prove that,

$$x - \frac{x^2}{2} < \log(1+x) < x - \frac{x^2}{2(1+x)}$$

Watch Video Solution

12. If $x > 0$, prove that,

$$\log\left(x + \sqrt{x^2 + 1}\right) < x$$

Watch Video Solution

13. If $x > 0$, prove that,

$$\log\left(x + \sqrt{1 + x^2}\right) > \tan^{-1} x$$

Watch Video Solution

14. If

$f(x) = 2x - \tan^{-1} x - \log\left(x + \sqrt{x^2 + 1}\right)$,

then show that $f(x)$ steadily increases as x increases from zero to positive infinity and hence deduce that,

$2x > \tan^{-1} x + \log\left(x + \sqrt{x^2 + 1}\right)$

Watch Video Solution

15. Find the values of parameter a for which the function.

$y = ax^3 + 3x^2 + (2a + 1)x + 1000$

is strictly decreasing for all real values of x.

Watch Video Solution

16. Show that, for all real values of θ the function $\frac{2\sin\theta + \cos\theta}{3\sin\theta + 4\cos\theta}$ is increasing.

Watch Video Solution

Mcq

1. Find the intervals of decrease and increase for the function $f(x) = \cos\frac{\pi}{x}$-

(k being a non-negative interger)

A. $\left(\frac{1}{2k+1}, \frac{1}{2k}\right)$

B. $\left(\frac{1}{2k+2}, \frac{1}{2k}\right)$

C. $\left(\frac{1}{2k+2}, \frac{1}{2k+1}\right)$

D. $\left(\frac{1}{2k}, \frac{1}{2k+1}\right)$

Answer: A::C

Watch Video Solution

2. Let $g(x) = f(x) + f(1-x)$ and $f''(x) > 0 \forall x \in (0,1)$. Find the intervals of increase and decrease of $g(x)$-

A. $\left(-\frac{1}{2}, 1\right)$

B. $\left(\frac{1}{2}, 1\right)$

C. $\left(1, \frac{1}{2}\right)$

D. $\left(0, \frac{1}{2}\right)$

Answer: B::D

Watch Video Solution

3. Find the points of inflection for $f(x) = 3x^4 - 4x^3$.

A. 0

B. 1

C. $\frac{2}{3}$

D. -2

Answer: A::C

Watch Video Solution

4. If

$$f(x) = 2x + \cos^{-1} x + \log\left(\sqrt{1 + x^2} - x\right),$$

then $f(x)$ is-

A. increases in $[0, \infty)$

B. decreases in $[0, \infty]$

C. neither increases nor decreases in $[0, \infty)$

D. increases in $(-\infty, \infty)$

Answer: A::D

Watch Video Solution

5. Let $f(x) = \sin x + ax + b$, then which of the following is/are true-

A. $f(x) = 0$ has only one root which is positive if $a > 1, b < 0$

B. $f(x) = 0$ has only one real root which is negative if $a > 1, b > 0$

C. $f(x) = 0$ has only one real root which is negative if $a < -1, b < 0$

D. none of these

Answer: A::B::C

Watch Video Solution

Integer Answer Type

1. If $f(x) = ax^3 - 9x^2 + 9x + 3$ is monotonically increasing in each interval, then $a \geq k$, find k.

Watch Video Solution

2. If the function $f(x) = \dfrac{a\sin x + 2\cos x}{\sin x + \cos x}$ is strictly increasing for all values of x, then $a > k$, find k.

Watch Video Solution

3. The function $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{k}\right)$, then find k.

Watch Video Solution

4. If $f'(x) = |x| - \{x\}$ where $\{x\}$ denotes the fractional part of x, then $f(x)$ is decreasing in $\left(-\frac{1}{2}, k\right)$, find k.

Watch Video Solution

5. Let $f(x)$ be a function such that $f'(x) = \log_{\frac{1}{3}}[\log_3(\sin x + a)]$. If $f(x)$ is decreasing for all real values of x then $a > k$, find k.

Watch Video Solution

Comprehension Type

1. Let $f(x) = \dfrac{x}{1 + |x|}$

Domain of $f(x)$ is-

A. $\mathbb{R} - \{-1\}$

B. $\mathbb{R} - \{0\}$

C. $\mathbb{R}$

D. none of these

Answer: c

Watch Video Solution

2. Let $f(x) = \frac{x}{1 + |x|}$

Interval of increase of $f(x)$ is -

A. $(-\infty, 0)$

B. $(0, \infty)$

C. $(-1, 1)$

D. $\mathbb{R}$

Answer: d

Watch Video Solution

3. Let $f(x) = \frac{x}{1 + |x|}$

Range of $f(x)$ is -

A. $(-1, 1)$

B. $[-1, 1]$

C. $[-1, 0) \cup (0, 1]$

D. $\mathbb{R}$

Answer: a

Watch Video Solution

4. Let $f(x) = (1-x)^2 \sin^2 x + x^2$ for all $x \in \mathbb{R}$, and let $g(x) = \int_1^x \left(\frac{2(t-1)}{t+1} - \log t \right) f(t)dt$ for all $x \in (1, \infty)$.

Consider the statements :

P : there exists some $x \in \mathbb{R}$ such that

$f(x) + 2x = 2\left(1 + x^2\right)$

Q : There exists some $x \in \mathbb{R}$ such that

$2f(x) + 1 = 2x(1 + x)$ then-

A. both P and Q are true

B. P is true and Q is false

C. P is false and Q is true

D. both P and Q are false

Answer: c

Watch Video Solution

5. Let $f(x) = (1-x)^2 \sin^2 x + x^2$ for all $x \in \mathbb{R}$, and let $g(x) = \int_1^x \left(\frac{2(t-1)}{t+1} - \log t \right) f(t) dt$ for all

$x \in (1, \infty)$.

Which of the following is true?

A. g is increasing on $(1, \infty)$

B. g is decreasing on $(1, \infty)$

C. g is increasing on (1, 2) and decreasing on $(2, \infty)$

D. g is decreasing on (1, 2) and increasing on $(2, \infty)$

Answer: b, d

Watch Video Solution

6. Let $f(x) = (1-x)^2 \sin^2 x + x^2$ for all $x \in \mathbb{R}$, and let $g(x) = \int_1^x \left(\frac{2(t-1)}{t+1} - \log t \right) f(t)dt$ for all $x \in (1, \infty)$.

The number of real roots of the equation $f(x) = 1$ in the interval [0, 1] is -

A. 0

B. 1

C. 2

D. 4

Answer: b

Watch Video Solution

Assertion Reason Type

1. Let $f(x) = 2\sqrt{x}$ and $g(x) = 3 - \frac{1}{x}, x > 1.$

Statement - I : $f(x) > g(x)(x > 1).$

Statement - II : $f(x) - g(x)$ increases on $(1, \infty).$

A. Satement -I is True, Statement -II is True, Statement -II is a correct explanation for Statement -I

B. Satement -I is True, Statement -II is True, Statement -II is not a correct explanation for Statement -I

C. Stament -I is True, Statement -II is False.

D. Statement -I is False, Statement -II is True.

Answer: a

Watch Video Solution

2. Let $f(x) = 2x^3 + 3x^2 - 12x + 1$.

Statement - I : f decreases on (-2, 1).

Statement - II : The solution set of $x^2 + x - 2 < 0$ is (-2, 1).

A. Satement -I is True, Statement -II is True, Statement -II is a correct explanation for Statement -I

B. Satement -I is True, Statement -II is True, Statement -II is not a correct explanation for Statement -I

C. Stament -I is True, Statement -II is False.

D. Statement -I is False, Statement -II is True.

Answer: a

 Watch Video Solution