MATHS

BOOKS - CHHAYA PUBLICATION MATHS (BENGALI ENGLISH)

LIMIT

Example

1. Prove that, $\lim_{x \to 0} \frac{(1+x)^{\frac{1}{2}} - 1}{(1+x)^{\frac{1}{3}} - 1} = \frac{3}{2}$

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2. Evaluate: $\lim_{h \to 0} \frac{e^{5h} - 1}{3h}$

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3. Evaluate: $\lim_{x \to 0} \frac{e^{px} - e^{-qx}}{x}$

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4. Evaluate: $\lim_{x \to 1} \frac{e^{\log x} - 1}{e^{x-1} - 1}$

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5. Evaluate: $\lim_{x \to 2} \frac{e^{x} - e^{2}}{x - 2}$

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6. Prove that, $\lim_{x \to 0} \frac{5^{x} - 4^{x}}{x} = \log_e \left(\frac{5}{4} \right)$

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7. Evaluate: $\lim_{x \to 0} \frac{\log(1 + \alpha x)}{e^{2x} - 1}$

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8. Evaluate: $\lim_{x \to 0} \frac{e^{ax} - 1}{\sin bx}$

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9. Evaluate: $\lim_{x \to 0} \frac{\sin x}{\log_e (1 + x)^{\frac{1}{4}}}$

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10. Evaluate: $\lim_{x \to 0} \frac{(e^x - 1)\log(1 + x)}{\sin^2 x}$

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11. Evaluate: $\lim_{x \to 0} \frac{3^x - 1}{(4 + x)^{\frac{1}{2}} - 2}$

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12. Show that , $\lim_{x \to 0} \frac{3^x + 3^{-x} - 2}{x^2}$

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13. Show that, $\lim_{x \to 0} \frac{\log(1 + x^3)}{\sin^3 x}$ = 1

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14. If $\lim_{x \to 0} (1 + x)^{\frac{1}{x}} = e$, prove that $\lim_{x \to 0} (1 + 3x)^{\frac{(x+2)}{x}} = e^6$.

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15. Evaluate the following limits: $\lim_{x \to 3} \frac{\sin(e^{x-3} - 1)}{\log(x - 2)}$

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16. Evaluate the following limits: $\lim_{x \to 0} \left[\frac{1}{x} - \frac{\log(1+x)}{x^2} \right]$

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17. Evaluate the following limits: $\lim_{x \to 0} \frac{xe^x - \log(1+x)}{x^2}$

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18. Find the value of : $\lim_{x \to \infty} \left(\frac{x+5}{x+1} \right)^5$

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19. Find the value of : $\lim_{x \to \infty} \left(\frac{x+5}{x-1} \right)^5$

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20. Does $\lim_{x \to 4} \sqrt{4 - x}$ exist ?

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21. Does $\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$ exist If so , find its value .

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22. A function f (x) is defined as follwos :

f(x) $= 1 \quad , \quad \text{when} \quad x \neq 0$
$= 0 \quad , \quad \text{when} \quad x = 0$

Does $\lim_{x \to 0} f(x)$ exist ?

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23. Draw the graph of $y = f(x) = \frac{x^2}{x}$ and from the graph examine whether $\lim_{x \to 0} f(x)$ exists or not .

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24. Draw the graph of the function $f(x) = \frac{|x|}{x}$ and from the graph discuss whether $\lim_{x \to 0} f(x)$ Exist or not

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25. Let $f(x) \begin{array}{ll} = x^2 & , \text{ when } x < 1 \\ = 1 & , \text{when } x \leq 1 \end{array}$

Draw the graph of the function f (x) and evaluate $\lim_{x \to 1} f(x)$.

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26. Show that $\lim_{x \to 1} \frac{1}{(x-1)^2} = +\infty$.

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27. Evaluate : $\lim_{x \to 0} \frac{1}{x}$.

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28. Evaluate : $\lim_{x \to -\infty} \left(1 + \frac{2}{x^2}\right)$.

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29. Prove that $\lim_{x \to \infty} \frac{1}{x} = 0$.

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30. Show that $\lim_{x \to 2} \left(x^2 - 3x + 5\right) = 3$.

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31. Prove that $\lim_{x \to 2} \left(x^2 - 3x + 5\right) = 3$. Where n is a positive interger .

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32. Evaluate : $\lim_{y \to -1} \frac{y^2 - 3y + 6}{2y^2 + 5y}$.

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33. If

$p(x) = a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \ldots . + a_{n-1} x + a_n,$ prove that $\lim_{x \to a}$

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34. Evaluate : $\lim_{x \to 1} \sin(2x^2 - x - 1)$.

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35. show that $\lim_{x \to -2} \sqrt{x^3 + 3x^2 - x + 3} = 3$

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36. Evaluate : $\lim_{h \to 0} e^2 h^{2-3h+2}$

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37. Prove that $\lim_{x \to 4} \log\left(2.\ x^{3/2} - 3.\ x^{1/2} - 1\right) = 2\log 3.$

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38. Show that $\lim_{\theta \to \alpha} \sin\theta = \sin\alpha.$

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Exercise Multiple Choice Type Questions

1. find the the value of $\lim_{x \to 0} \frac{e^{3x} - 1}{2x}$ and $\lim_{x \to 0} \frac{\log(1+4x)}{3x}$

A. $\frac{3}{2}$

B. $\frac{1}{6}$

C. $\frac{2}{3}$

D. $\frac{3}{4}$

Answer: A

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2. find the the value of $\lim_{x \to 0} \frac{e^{3x} - 1}{2x}$ and $\lim_{x \to 0} \frac{\log(1 + 4x)}{3x}$

A. 4

B. 3

C. $\frac{1}{2}$

D. $\frac{1}{12}$

Answer: C

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3. The value of $\lim_{x \to 0} \frac{a^{2x} - 1}{2x}$ is

A. $\frac{1}{4}\log_e a$

B. 1

C. $\frac{1}{2}$

D. $\log_e a$

Answer: D

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4. The value of $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x$ is

A. 1

B. e

C. e^{-1}

D. does not exist

Answer: B

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5. The value of $\lim_{x \to 0} \frac{e^{x2} - 1}{x}$ is

A. 1

B. 0

C. does not exist

D. nono of these

Answer: B

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6. The value of $\lim_{x \to 1} \frac{\log x}{x - 1}$ is

A. 1

B. 0

C. e

D. -1

Answer: A

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Exercise Very Short Answer Type Questions

1. Using $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$, deduce that, $\lim_{x \to 0} \frac{a^x - 1}{x} = \log_e a$ $[a > 0]$.

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2. Using $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$, show that, $\lim_{x \to 0} \frac{\log_e(1 + x)}{x} = 1$

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3. Show that: $\lim_{x \to 0} \frac{1}{x}\left[(1+x)^8 - 1\right] = 8$

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4. Show that $\lim_{x \to 0} \frac{(1+x)^9 - 1}{(1+x)^6 - 1} = \frac{3}{2}$

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5. Evaluate: $\lim_{x \to 0} \frac{e^{4x} - 1}{x}$

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6. Evaluate: $\lim_{x \to 0} \frac{e^{4x} - e^{-x}}{x}$

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7. Evaluate: $\lim_{x \to 0} \frac{e^{13x} - e^{7x}}{x}$

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8. Evaluate: $\lim_{x \to 0} \frac{e^{ax} + e^{Bx} - 2}{x}$

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9. Evaluate: $\lim_{x \to 0} \frac{e^{\sin x} - 1}{x}$

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10. Evaluate: $\lim_{h \to 0} \frac{e^{\tan h} - 1}{h}$

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Exercise Short Answer Type Questions

1. Evaluate: $\lim_{h \to 0} \frac{e^{(x+h)^2} - e^{x^2}}{h}$

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2. Evaluate: $\lim_{x \to 0} \frac{e^{2x} - 1}{\sin 3x}$

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3. Evaluate: $\lim_{x \to 0} \frac{4^x - 1}{\sin 2x}$

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4. Evaluate: $\lim_{h \to 0} \frac{e^{2h} - 1}{e^{3h} - 1}$

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5. Evaluate: $\lim_{x \to 0} \frac{a^{\alpha x} - b^{\beta x}}{x}$

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6. Evaluate: $\lim_{x \to 0} \frac{\log(1 + 3x)}{x}$

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7. Evaluate: $\lim_{x \to 0} \frac{2^x - 1}{\sqrt{1 + x} - 1}$

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8. Evaluate: $\lim_{x \to 0} \frac{e^x + e^{-x} - 2}{x^2}$

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9. Prove that: $\lim_{x \to 0} \frac{\log(1+2x)}{\sin 3x} = \frac{2}{3}$

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10. Prove that: $\lim_{x \to -2} \frac{\log(x+3)}{x+2} = 1$

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11. Prove that: $\lim_{x \to e} \frac{\log - 1}{x - e} = \frac{1}{e}$

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12. Prove that: $\lim_{x \to 0} \frac{p^x - q^x}{\tan x} = \log \frac{p}{q}$

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13. Prove that: $\lim_{x \to 0} \frac{\sin \log(1+x)}{x} = 1$

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14. Prove that: $\lim_{x \to 0} \frac{\log(1+x) + \sin x}{e^x - 1} = 2$

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15. Prove that: $\lim_{x \to 0} \frac{\log \cos x}{\sin^2 x} = -\frac{1}{2}$

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16. Prove that: $\lim_{h \to 0} \frac{\log(x+h) - \log x}{h} = \frac{1}{x}$

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17. Prove that: $\lim_{x \to 0} \frac{\log(1+\sin x)}{x} = 1$

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18. Prove that: $\lim_{x \to 0} \frac{\sqrt{1+x}-1}{\log(1+x)} = \frac{1}{2}$

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19. Evaluate: $\lim_{x \to 0} \frac{\sin x}{\log_e (1+x)^{\frac{1}{2}}}$

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20. Evaluate: $\lim_{x \to 0} \frac{(e^x - 1)\log(1+x)}{\sin^2 x}$

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21. Evaluate: $\lim_{x \to 0} \frac{(e^2 - 1)\log(1+x)}{\sin x}$

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22. Evaluate: $\lim_{x \to 4} \frac{x^{\frac{7}{2}} - 4^{\frac{7}{2}}}{\log_e(x-3)}$

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23. Evaluate: $\lim_{x \to 0} \frac{\log(1+\alpha x)}{\sin \beta x}$

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24. Evaluate: $\lim_{x \to 0} \frac{e^{3x} - 1}{\log(1+5x)}$

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25. Evaluate: $\lim_{x \to 0} \frac{2^{3x} - 1}{3^{2x} - 1}$

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Exercise Long Answer Type Questions

1. Prove that $\lim_{x \to \infty} \frac{ae^x + be^{-x}}{ce^x + de^{-x}} = \frac{a}{c}[2 < e < 3, c \neq 0].$

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2. Prove that $\lim_{x \to 00} \frac{ae^x + be^{-x}}{ce^x + de^{-x}} = \frac{b}{d}[2 < e < 3, d \neq 0].$

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3. Prove that $\lim_{x \to 1} \frac{4^x - 4}{x - 1} = 8 \log 2$

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4. Prove that $\lim_{x \to 2} \frac{\log(2x - 3)}{2(x - 2)} = 1$

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5. prove that $\lim_{x \to 0} (1+2x)^{\frac{1}{x}} = e^2$

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6. Prove that $\lim_{x \to 0} (1+3x)^{\frac{3}{x}} = e^9$

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7. If $\lim_{x \to 0} x^{\frac{1}{1-x}} = e^{-1}$

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8. $\lim_{x \to 0} (1+4x)^{\frac{x+2}{x}} = e^8$

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Sample Questions For Competitive Examination Multiple Correct Answer Tpye

1. If $f(x) = x\frac{e|x| + |x| - 2}{|x| + |x|}$(where |x| denotes the greaster integer less than or equal to x), then-

A. $\lim_{x \to 0+} f(x) = -1$

B. $\lim_{x \to 0-} f(x) = 0$

C. $\lim_{x \to 0+} f(x)$ does not exist.

D. underser (xrarr0)"lim"f(x) does not exist.

Answer: A,B,D

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2. The $\lim_{x \to 0} x^8 \left[\frac{1}{x^3}\right]$ (where [x] is the greatest integer function) is

A. a non-zero real number

B. a rational number

C. an integer

D. zero

Answer: B,C,D

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3. Let = $\lim_{x \to 0} a - \frac{\sqrt{a^2 - x^2} - \frac{x^2}{4}}{x^4}, a > 0.$ If is fintine, then

A. a = 2

B. a = 1

C. $L = \frac{1}{64}$

D. $L = \frac{1}{32}$

Answer: A,C

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4. If $\lim_{x \to 0} (\cos x + a \sin bx)^{\frac{1}{x}} = e^2$ then the values of a and b are

A. a = 1 ,b = 2

B. a = 2 , $b = \frac{1}{2}$

C. $a = 2\sqrt{2}, b = \frac{1}{\sqrt{2}}$

D. a = 4, b = 2

Answer: A,C

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5. If $\lim_{x \to -a} \frac{x^7 + a^7}{x + a} = 7$ then the value of a is

A. 1

B. -1

C. 7

D. -7

Answer: A,B,D

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Sample Questions For Competitive Examination Integer Answer Tpye

1. Find the value of $\lim_{x \to 3} \frac{\sqrt{16x^2 + 112} - 16}{3x^2 - 15x + 18}$

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2. Find the value of $\lim_{x \to 3} e(1 + \sin \pi x)^{\cot \pi x}$.

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3. Find the value of $\lim_{x \to 1} \frac{x^x - 1}{x \log x} - \lim_{x \to 0} \frac{\log(1 - 3x)}{x}$.

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4. The value of $\lim_{x \to \frac{\pi}{4}} \frac{8 - \sqrt{2}(\cos + \sin x)^5}{1 - \sin 2x} = 5k,$ then find the value of k.

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5. The value of $\left[\lim_{x \to 0} \left(\frac{\sin x}{x} \right)^{\frac{\sin x}{x - \sin x}} + \lim_{x \to 1} x^{\frac{1}{1-x}} \right]$ is

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6. $f(x) = \frac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1}$ find $f^{-1}(x)$

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Sample Questions For Competitive Examination Comprehension Types

1. Let

$$f(x)\frac{1-\cos 4x}{x^2}, g(x) = \frac{\sqrt{x}}{\left(\sqrt{16+\sqrt{x}}\right)-4} \text{ and } q(x) = \frac{e^{2x}-x^x+1}{x^{2x+e^x+1}}, (x$$

$$\lim_{x\to 0} f(x)$$

A. $\frac{1}{2}$

B. 2

C. $\frac{1}{8}$

D. 8

Answer: D

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2. Let

$$f(x)\frac{1-\cos 4x}{x^2}, g(x) = \frac{\sqrt{x}}{\left(\sqrt{16+\sqrt{x}}\right)-4} \text{ and } q(x) = \frac{e^{2x}-x^x+1}{x^{2x+e^x+1}}, (x$$

$$\lim_{x\to 0} g(x) =$$

A. $\frac{1}{8}$

B. 8

C. 2

D. $\frac{1}{2}$

Answer: B

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3. Let

$f(x)\frac{1-\cos 4x}{x^2}, g(x) = \frac{\sqrt{x}}{\left(\sqrt{16+\sqrt{x}}\right)-4}$ and $q(x) = \frac{e^{2x}-x^x+1}{x^{2x+e^x+1}}, (x$

$\lim_{x \to 0} q(x) =$

A. 0

B. 1

C. -1

D. 2

Answer: B

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4. If f , g , and h are functions having a cammon domain D and $h(x) \leq f(x) \leq g(x), x \in D$ and if $\lim_{x \to a} h(x) = \lim_{x \to a} g(x) = l \quad then \quad \lim_{x \to a} f(x) = l$

The value of $\lim_{x \to 0} \frac{|x|}{\sqrt{x^4 + 4x^2 + 7}} -$

A. 1

B. 0

C. $\frac{1}{2}$

D. $-\frac{1}{2}$

Answer: B

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5. If f , g , and h are functions having a cammon domain D and $h(x) \leq f(x) \leq g(x), x \in D$ and if $\lim_{x \to a} h(x) = \lim_{x \to a} g(x) = l \quad then$ $\lim_{x \to 0} x^4 \sin\left(\frac{1}{3\sqrt{x}}\right)$ is

A. 0

B. 1

C. $\frac{1}{3}$

D. $-\frac{1}{3}$

Answer: A

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Sample Questions For Competitive Examination Assertion Reason Type

1. Let $f(x) = x^2 \frac{e^{\frac{1}{x}} e^{-\frac{1}{x}}}{e^{\frac{1}{x}} + e^{-\frac{1}{x}}}, x \neq 0 \text{ and } f(0) = 1$ then-

A. $\lim_{x \to 0+} f(x)$ doesn't exist

B. $\lim_{x \to 0} f(x)$ doesn,t exist

C. $\lim_{x \to 0} f(x)$ exist

D. none of these

Answer: C

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2. Statement - I: if $\lim_{x \to 0} \left(\frac{\sin x}{x} + f(x) \right)$ does not exist, then $\lim_{x \to 0} f(x)$ does not exist.

Statement - II: $\lim_{x \to 0} \frac{\sin x}{x} = 1$

A. Statement -I is True , Statement -II is True , Statement -II is a correct explanation for Statement-I

B. Statement-I is True , Statement -II is True , Statement -II is no a correct explanation for Statement -I

C. Statement-I is True , Statement-II is false

D. Statement -I is False , Statement-II is true

Answer: C

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3. prove that $\lim_{x \to 0} \log_e \left(\frac{\sin x}{x} \right) = 0$

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Iiiustrative Examples

1. Show that $\lim_{x \to -2} \frac{x^3 - 5x + 3}{x^2 + 1} = 1$

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2. Evaluate : $\lim_{x \to 1} \frac{x^4 + 4x^3 - 5x^2}{x - 1}$

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3. Evaluate : $\lim_{x \to 1} \frac{x^2 - 3x + 2}{x^2 - 4x + 3}$

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4. Evaluate : $\lim_{x \to 2} \frac{x^2 - 3x + 2}{x^3 - 5x + 2}$.

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5. Evaluate :

$$\lim_{x \to 2} \frac{x - \sqrt{3x - 2}}{x^2 - 4}$$

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6. Evaluate :

$$\lim_{x \to 0} \frac{\sqrt{1+ax} - \sqrt{1-ax}}{x}$$

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7. Evaluate : $\lim_{h \to 0} \frac{f(1+h) - f(1)}{2h}, \quad \text{where} f(x) = \frac{1}{x}$.

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8. Evaluate :

$$\lim_{x \to 0} \frac{\sqrt{1-x^3} - \sqrt{1-x}}{\sqrt{1+x^2} - \sqrt{1+x}}$$

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9. Evaluate :

$$\lim_{x \to 3} \frac{\sqrt{x-3} + \sqrt{x} - \sqrt{3}}{\sqrt{x^2-9}}$$

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10. Prove that

$$\lim_{h \to -2} \frac{h^5 + 32}{h + 2} = 80$$

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11. Prove that

$$\lim_{x \to 0} \frac{(1+x)^{1/2} - 1}{(1+x)^{1/3} - 1} = \frac{3}{2}$$

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12. Prove that

$$\lim_{x \to y} \frac{x^{9/2} - y^{9/2}}{x^{5/2} - y^{5/2}} = \frac{9}{5} y^2$$

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13. Prove that

$$\lim_{h \to 0} \frac{\sqrt[3]{x+h} - \sqrt[3]{x}}{h} = \frac{1}{3} x^{-\frac{2}{3}}$$

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14. Prove that

$$\lim_{x \to a} \frac{(x+5)^{5/2} - (a+5)^{5/2}}{x-a} = \frac{5}{2}(a+5)^{3/2}$$

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15. Evaluate : $\lim_{x \to 0} \frac{\sqrt[3]{1+x} - \sqrt[3]{1-x}}{x}$

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16. Evaluate : $\lim_{x \to 2} \frac{2 - \sqrt{2+x}}{\sqrt[3]{2} - \sqrt[3]{4-x}}$

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17. Evaluate : $\lim_{x \to 0} \frac{\tan x^{\circ}}{x}$

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18. Evaluate : $\lim_{x \to 0} \frac{\sin^{-1} x}{2x}$

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19. $\lim_{x \to \frac{\pi}{2}} \frac{\cos x}{\frac{\pi}{2} - x} =$

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20. Evaluate : $\lim_{x \to \pi} \frac{\sin 3x}{\sin 2x}$

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21. Evaluate : $\lim_{x \to \pi} \frac{1 + \cos x}{(x - \pi)^2}$

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22. Evaluate : $\lim_{x \to 0} \frac{\sin x(1 - \cos x)}{x^3}$

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23. Evaluate : $\lim_{x \to 0} \frac{x^2 - \tan 2x}{\tan x}$

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24. Evaluate : $\lim_{x \to 0} \frac{\cos ecx - \cot x}{x}$

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25. Evaluate : $\lim_{x \to 0} \frac{\cos 5x - \cos 7x}{\cos x - \cos 5x}$

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26. Evaluate : $\lim_{x \to y} \frac{\sin^2 x - \sin^2 y}{x^2 - y^2}$

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27. Evaluate : $\lim_{x \to \frac{\pi}{2}} \left(\frac{\pi}{2} - x\right) \tan x$

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28. Evaluate : $\lim_{x \to 1} (1 - x) \tan \frac{\pi x}{2}$

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29. Evaluate : $\lim_{x \to a} \frac{x \sin a - a \sin x}{x - a}$

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34. Prove that, $\lim_{x \to 0} \frac{5^x - 4^x}{x} = \log_e \left(\frac{5}{4} \right)$

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35. Evaluate: $\lim_{x \to 0} \frac{\log(1+\alpha x)}{e^{2x}-1}$

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36. Evaluate : $\lim_{x \to 0} \frac{e^{ax}-1}{\sin bx}$

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37. Evaluate : $\lim_{x \to 0} \frac{\sin x}{\log_e (1+x)^{1/4}}$

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38. Evaluate : $\lim_{x \to 0} \frac{(e^x-1)\log(1+x)}{\sin^2 x}$

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39. Evaluate : $\lim_{x \to 0} \frac{3^x - 1}{(4+x)^{1/2} - 2}$

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40. Evaluate : $\lim_{n \to \infty} \frac{n^4 - n^2 + 6n + 1}{2n^4 - 5n + 4}$

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41. Evaluate : $\lim_{x \to \infty} \left(\sqrt{x^2 + ax} - x\right).$

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42. Prove that $\lim_{n \to \infty} \frac{1^3 + 2^3 + 3^3 + \ldots + n^3}{n^4} = \frac{1}{4}$

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43. Show that $\lim_{x \to 0} \frac{x \tan 2x - 2x \tan x}{(1 - \cos 2x)^2} = \frac{1}{2}$

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44. Examine the existence of the following limits :

$\lim_{x \to 2} \left(x + \sqrt{x - 2}\right)$

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45. Examine the existence of the following limits :

$\lim_{x \to 3} \frac{1}{2 + e^{\frac{1}{x-3}}} [2 < e < 3]$

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46. Prove that $\lim_{x \to 0} \frac{3x + |x|}{7x - 5|x|}$ does not exist .

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47. Does $\lim_{x \to a} f(x) = \lim_{x \to a} \phi(x)$ always imply $f(x) = \phi(x)$? Justify your answer .

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48. Examine the existence of the following of limits :

$$\lim_{x \to 0} \sin \frac{1}{x}$$

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49. Examine the existence of the following of limits : $\lim_{x \to 0} x \sin \frac{1}{x}$

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50. If $\lim_{x \to 0} (1 + x)^{\frac{1}{x}} = e$ prove that $\lim_{x \to 0} (1 + 3x)^{\frac{x+2}{x}} = e^6$.

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51. A function f (x) is defined as follwos :

$$f(x) \begin{array}{ll} = 2x + 1 & \text{when } x \leq 1 \\ = 3 - x & \text{when } x > 1 \end{array}$$

Examine whether $\lim_{x \to 1} f(x)$ exists or not.

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52. If $f(x) = \lim_{n \to \infty} \frac{1}{1 + x^{2n}}$ find f (x) for all real x .

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53. If α and β are the roots of the equation $ax^2 + bx + c = 0$, then show that $\lim_{x \to \alpha} \frac{1 - \cos\left(ax^2 + bx + c\right)}{(x - \alpha)^2} = \frac{a^2}{2}(\alpha - \beta)^2$

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Exercise 2 Multiple Choice Type Questions

1. $\lim_{x \to 0} \frac{\sin x}{\sqrt{x}}$ is -

A. 0

B. $-\frac{1}{2}$

C. 1

D. The limit does not exist

Answer: A

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2. $\lim_{x \to 0} \frac{\sin 3x}{\sin x}$ =

A. 2

B. 3

C. 1

D. 0

Answer: B

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3. $\lim_{x \to \frac{\pi}{2}} \frac{\cos x}{\frac{\pi}{2} - x} =$

A. 3

B. 2

C. 1

D. 0

Answer: C

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4. $\lim_{x \to 0} \frac{\sin^{-1} x}{x} =$

A. 4

B. 3

C. 2

D. 1

Answer: D

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5. $\lim_{x \to 0} \frac{e^{3x} - 1}{x} =$

A. $\frac{1}{2}$

B. $-\frac{3}{2}$

C. $\frac{3}{2}$

D. $-\frac{1}{2}$

Answer: C

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6. $\lim_{x \to 0} \frac{a^{2x} - 1}{x} =$

A. $\log_e a$

B. $2 \log_e a$

C. $-2 \log_e a$

D. $-\log_e a$

Answer: B

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7. $\lim_{x \to 0} \frac{\log(1 + 4x)}{x} =$

A. 4

B. -4

C. -3

D. 3

Answer: A

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8. $\lim_{x \to 1} \frac{x + x^2 + x^3 + \ldots . + x^n - n}{x - 1} =$

A. n

B. $\frac{n(n+1)}{2}$

C. $\frac{n(n-1)}{2}$

D. $\frac{(n+1)}{2}$

Answer: B

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9. $f(x) = \frac{\sqrt{x} + 3}{x + 1}$ then the value of $\lim_{x \to -3-0} f(x)$ is -

A. 0

B. $\frac{1}{2}$

C. $\frac{1}{2}$

D. 1

Answer: A

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10. If $\lim_{x \to 3} \frac{\alpha x^2 - \beta}{x - 3} = 12$, then -

A. $\alpha = 1, \beta = 2$

B. $\alpha = 2, \beta = 9$

C. $\alpha = 2, \beta = 18$

D. $\alpha = 2, \beta = 36$

Answer: C

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11. $\lim_{x \to -\frac{1}{2}} \frac{2x^2 - 3x + 1}{2x - 1} = ?$

A. 3

B. $\frac{3}{2}$

C. $-\frac{3}{2}$

D. -3

Answer: C

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12. $\lim_{x \to 1} \left[\cos\left(\frac{\pi}{2}\left(\sqrt{x^2 - 2x + 2}\right)\right)\right] = ?$

A. 0

B. 1

C. -1

D. $\frac{1}{2}$

Answer: A

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13. $\lim_{h \to 3} \sqrt[3]{2h^2 - 3h - 1} = ?$

A. 1

B. 2

C. 3

D. 0

Answer: B

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14. $\lim_{h \to 1} \left(2h^2 - 2h + 9\right)^{\frac{3}{2}} = ?$

A. 9

B. 3

C. 27

D. 81

Answer: C

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15. $\lim_{x \to 1} \frac{x-1}{\sqrt{x}-1} = ?$

A. 2

B. 1

C. 0

D. 3

Answer: A

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16. $\lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4} = ?$

A. 1

B. $\frac{1}{2}$

C. $\frac{1}{4}$

D. $\frac{1}{8}$

Answer: C

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17. $\lim_{x \to 2} \frac{x^5 - 32}{x - 2} = ?$

A. 10

B. 20

C. 40

D. 80

Answer: D

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18. $\lim_{x \to 3} \frac{x^3 - 27}{x^2 - 9} = ?$

A. $\frac{9}{2}$

B. $\frac{9}{4}$

C. $\frac{27}{2}$

D. 9

Answer: A

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19. $\lim_{h \to 0} \frac{3h^2 - 2h}{h} = ?$

A. 1

B. 3

C. 2

D. -2

Answer: D

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20. $\lim_{x \to 16} \frac{x^{\frac{1}{4}} - (16)^{\frac{1}{4}}}{x - 16} = ?$

A. $\frac{1}{32}$

B. $\frac{1}{16}$

C. $\frac{1}{8}$

D. $\frac{1}{4}$

Answer: A

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Exercise 2 Very Short Answer Type Questions

1. Evaluate the following limits: $\lim_{x \to 1} \left(\frac{\sqrt{2-x}-1}{1-x} \right)$

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2. Distinguish between $\lim_{x \to a} f(x)$ and $f(a)$

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3. Using $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$ deduce that , $\lim_{x \to 0} \frac{a^x - 1}{x} = \log_e a [a > 0]$.

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4. Starting from $\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$ deduce that , $\lim_{x \to 0} \frac{(1+x)^n - 1}{x} = n$.

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5. Using $\lim_{x \to a} \frac{e^x - 1}{x}$,show that , $\lim_{x \to 0} \frac{\log_e(1+x)}{x} = 1$

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6. Prove each of the following limits by graphical methods :

$$\lim_{x \to -2} 4 = 4$$

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7. Prove each of the following limits by intuitive and graphical methods :

$$\lim_{x \to 3} 2x = 6$$

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8. Prove each of the following limits by intuitive and graphical methods :

$$\lim_{x \to 0} \frac{2x}{x} = 2$$

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9. Prove each of the following limits by intuitive and graphical methods :

$$\lim_{x \to 0} |x| = 0$$

Watch Video Solution

10. Prove that :

$$\lim_{x \to 3} \frac{x^2 - 9}{x - 3} = 6$$

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11. If $f(x) = \sqrt{x - 3}, \quad \text{does} \quad \lim_{x \to 3} f(x)$ exist ? Give reasons .

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12. A function $\phi(x)$ is defined as follows :

$$\phi(x) \begin{array}{ll} = 2 & \text{when} x \neq 1 \\ = 0 & \text{when} x = 1 \end{array}$$

Does $\lim_{x \to 1} \phi(x)$ exist ? If so , find its value .

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13. If $f(x) = \frac{|x|}{x}$, show that , $\lim_{x \to 0} f(x)$ does not exist .

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14. Evaluate the following limits :

$$\lim_{x \to 2} \frac{x\left(x^2 - 4\right)}{x^2 - 5x + 6}$$

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15. Evaluate the following limits :

$$\lim_{h \to 1} \frac{h^2 + 2h - 3}{h - 1}$$

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16. Evaluate the following limits :

$$\lim_{h \to -\frac{1}{2}} \frac{6x^2 - 7x - 5}{2x^2 - x - 1}$$

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17. Evaluate the following limits :

$$\lim_{u \to -3} \left[\frac{1}{u+3} + \frac{6}{u^2 - 9} \right]$$

Watch Video Solution

18. Evaluate the following limits :

$$\lim_{x \to 1} \frac{x^3 - 1}{x^2 - 1}$$

Watch Video Solution

19. Evaluate the following limits :

$$\lim_{x \to 1} \frac{x^2 + 5x - 6}{x^2 - 3x + 2}$$

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20. Evaluate the following limits :

$$\lim_{x \to \frac{a}{2}} \frac{8x^2 - 10ax + 3a^2}{4x^2 + 4ax - 3a^2}$$

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21. Evaluate the following limits :

$$\lim_{x \to \frac{1}{2}} \frac{8x^3 - 1}{6x^2 - 5x + 1}$$

Watch Video Solution

22. Evaluate the following limits :

$$\lim_{x \to 0} \frac{\sqrt{1 + x} - \sqrt{1 - x}}{x}$$

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23. Evaluate the following limits :

$$\lim_{x \to 0} \frac{1 - \sqrt{1 - x^2}}{x^2}$$

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24. Evaluate the following limits :

$$\lim_{h \to 1} \frac{h - 1}{\sqrt{h} - 1}$$

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25. Evaluate the following limits :

$$\lim_{x \to 2} \frac{\sqrt{x} - \sqrt{2}}{x^2 - 4}$$

Watch Video Solution

26. Evaluate the following limits :

$$\lim_{x \to 0} \frac{x^2}{a - \sqrt{a^2 - x^2}}$$

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27. Evaluate the following limits :

$$\lim_{x \to 0} \frac{\sqrt{a + x} - \sqrt{a}}{x}$$

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28. Evaluate the following limits :

$$\lim_{x \to 3} \frac{x - 3}{\sqrt{x - 2} - \sqrt{4 - x}}$$

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29. Evaluate the following limits :

$$\lim_{x \to a} \frac{\sqrt{x - b} - \sqrt{a - b}}{x^2 - a^2}$$

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30. Evaluate the following limits :

$$\lim_{x \to 1} \frac{x^2 - 1}{\sqrt{3x + 1} - \sqrt{5x - 1}}$$

Watch Video Solution

31. Evaluate the following limits :

$$\lim_{x \to 2} \frac{\sqrt{1 + 2x} - \sqrt{1 + x^2}}{2 - x}$$

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32. $\lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4} = ?$

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33. Evaluate the following limits :

$$\lim_{x \to 0} \frac{\sqrt{1+x+x^2}-1}{x}$$

Watch Video Solution

34. Evaluate the following limits :

$$\lim_{x \to 0} \frac{\sqrt{1+2x^2}-\sqrt{1-2x^2}}{x^2}$$

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35. Show : $\lim_{x \to 2} \frac{x^9-512}{x^4-16} = 72$

Watch Video Solution

36. Show : $\lim_{x \to 2} \frac{(1+x)^n-3^n}{x-2} = n.3^{n-1}$

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37. show : $\lim_{x \to 3} \frac{x^{-6} - 3^{-6}}{x^{-4} - 3^{-4}} = \frac{1}{6}$

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38. show : $\lim_{h \to 0} \frac{\sqrt[3]{h+1} - 1}{h} = \frac{1}{3}$

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39. show : $\lim_{x \to a} \frac{\sqrt[3]{x} - \sqrt[3]{a}}{x - a} = \frac{1}{3} a^{-\frac{2}{3}}$

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40. show : $\lim_{x \to a} \frac{x^{\frac{3}{8}} - a^{\frac{3}{8}}}{x^{\frac{5}{3}} - a^{\frac{5}{3}}} = \frac{9}{40} a^{-\frac{31}{24}}$

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41. show : $\lim_{x \to 0} \frac{1}{x}\left[(1+x)^8 - 1\right] = 8$

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42. show : $\lim_{x \to 0} \frac{(1+x)^9 - 1}{(1+x)^6 - 1} = \frac{3}{2}$

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43. If $f(x) = ax^2 + bx + c$, show that ,

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = 2ax + b$$

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44. If $f(x) = \frac{1}{x}$, prove that , $\lim_{h \to 0} \frac{f(2+h) - f(2)}{h} = -\frac{1}{4}$

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45. Evaluate the following limits :

$$\lim_{x \to 0} \frac{\sin 2x}{2x^2 + x}$$

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46. Evaluate the following limits :

$$\lim_{x \to \frac{\pi}{2}} \frac{\cos x}{\pi - 2x}$$

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47. Evaluate the following limits :

$$\lim_{x \to 0} \frac{\sin 3x}{\sin 2x}$$

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48. Evaluate the following limits :

$$\lim_{x \to \pi} \frac{\sin x}{\pi - x}$$

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49. Evaluate the following limits :

$$\lim_{x \to \frac{\pi}{2}} \frac{2x - \pi}{\cos x}$$

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50. Evaluate the following limits :

$$\lim_{x \to 0} \frac{\sin x^{\circ}}{x}$$

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51. Evaluate :

$$\lim_{x \to 0} \frac{e^{4x} - 1}{x}$$

Watch Video Solution

52. Evaluate :

$$\lim_{x \to 0} \frac{e^x - e^{-x}}{x}$$

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53. Evaluate :

$$\lim_{x \to 0} \frac{e^{13x} - e^{7x}}{x}$$

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54. Evaluate :

$$\lim_{x \to 0} \frac{e^{\alpha x} + e^{\beta x} - 2}{x}$$

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55. Evaluate :

$$\lim_{x \to 0} \frac{e^{\sin x} - 1}{x}$$

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56. Evaluate :

$$\lim_{h \to 0} \frac{e^{\tan h} - 1}{h}$$

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Exercise 2 Short Answer Type Questions

1. The function f (x) is defined as follows :

$$f(x) \begin{array}{ll} = 2 - 3x & \text{when } x < 0 \\ = 3x - 2 & \text{when } x \leq 0 \end{array}$$

Evaluate $\lim_{x \to 0+} f(x)$ and $\lim_{x \to 0-} f(x)$ and hence state whether $\lim_{x \to 0} f(x)$ exists or not .

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2. If f (x)=[x] , where [x] denotes greatest integer less than or equal to x ,show that , $\lim_{x \to 2} f(x)$does not exist .

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3. Evaluate $\lim_{x \to 2} \frac{x^2 - 3x + 2}{x^3 - 4x + 3}$

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4. Evaluate : $\lim_{x \to -2} \frac{x^2 - 7x + 6}{x^4 + 5x - 5}$

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5. Evaluate : $\lim_{x \to 0} \frac{\sqrt{1 + x^2} - \sqrt{1 + x}}{\sqrt{1 + x^3} - \sqrt{1 + x}}$

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6. Evaluate $\lim_{x \to a} \frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} [a \neq 0]$

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7. If $G(x) = . - \sqrt{25 - x^2}$, find the value of $\lim_{x \to 1} \frac{G(x) - G(1)}{x - 1}$

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8. Evaluate :

$$\lim_{x \to \infty} \frac{6 - 5x^2}{8x - 15x^2}$$

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9. Evaluate :

$$\lim_{x \to \infty} \frac{2x - 5}{3x + 2}$$

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10. Evaluate :

$$\lim_{n \to \infty} \frac{3n^2 - 4n + 6}{n^2 + 6n - 7}$$

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11. Evaluate :

$$\lim_{n \to \infty} \frac{1 + \sqrt{n}}{1 - \sqrt{n}}$$

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12. Evaluate :

$$\lim_{x \to \infty} \frac{4x^3 - 3x^2 + 6x - 2}{3 + 5x^2 + 5x^3}$$

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13. Evaluate :

$$\lim_{x \to \infty} e^{\frac{1}{2x} + 2}$$

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14. Evaluate :

$$\lim_{n \to \infty} \frac{2n^2 + 3n - 9}{3n^3 + 2n + 7}$$

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15. Evaluate :

$$\lim_{x \to \infty} \frac{x^4 + 3x^3 - 2x^2 + 5}{x^3 - 3x^2 + 2x - 1}$$

Watch Video Solution

16. Evaluate :

$$\lim_{n \to \infty} \frac{\sqrt{1 + n^2} - \sqrt{1 + n}}{\sqrt{1 + n^3} - \sqrt{1 + n}}$$

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17. Evaluate the following limits :

$$\lim_{x \to 0} \frac{\sqrt{\cos x} - \sqrt[3]{\cos x}}{\sin^2 x}$$

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18. Evaluate the following limits :

$$\lim_{x \to 0} \frac{1 - \cos x}{x^2}$$

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19. Evaluate the following limits :

$$\lim_{h \to 0} \frac{1 - \cosh}{h \quad \sin \quad h}$$

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20. Evaluate the following limits :

$$\lim_{x \to 0} \frac{1 - \cos 4x}{x^2}$$

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21. Evaluate the following limits :

$$\lim_{h \to 0} \frac{\cos ah - \cos bh}{h^2}$$

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22. Evaluate the following limits :

$$\lim_{x \to 0} \frac{\tan^{-1} x}{x}$$

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23. Evaluate the following limits: $\lim_{x \to 0} \left[\tan\left(\frac{\pi}{4} + x\right)\right]^{\frac{1}{x}}$

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24. Evaluate the following limits :

$$\lim_{x \to 0} \frac{x \tan x}{1 - \cos x}$$

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25. Evaluate the following limits :

$$\lim_{h \to 0} \frac{\sin(x + h) - \sin x}{h}$$

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26. Evaluate the following limits :

$$\lim_{h \to 0} \frac{\tan(x + h) - \tan x}{h}$$

Watch Video Solution

27. Evaluate the following limits :

$$\lim_{x \to 0} \frac{\sin(x^2 + 4x)}{x^3 - 5x^2 + 2x}$$

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28. $\lim_{x \to 0} \frac{\sin x(1 - \cos x)}{x^3}$

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29. Evaluate the following limits :

$\lim_{x \to 0} \tan 5x \quad \text{cosec} 4x$

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30. Evaluate the following limits :

$\lim_{x \to 0} \frac{2 \sin x - \sin 2x}{x^3}$

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31. Evaluate the following limits :

$$\lim_{x \to a} \frac{1 - \cos(x - a)}{(x - a)^2}$$

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32. Evaluate the following limits :

$$\lim_{x \to 0} \frac{\cos x - \sec x}{x^2}$$

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33. Evaluate the following limits :

$$\lim_{x \to 0} \frac{1 - \cos 4x}{1 - \cos 5x}$$

Watch Video Solution

34. Evaluate the following limits :

$$\lim_{x \to \pi} \frac{1 + \cos x}{\pi - x}$$

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35. Prove :

$$\lim_{x \to 0} \frac{\sin \alpha x^{\circ}}{\sin \beta x^{\circ}} = \frac{\alpha}{\beta}$$

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36. Prove :

$$\lim_{x \to 0} \left(\frac{1}{\sin x} - \frac{1}{\tan x} \right) = 0$$

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37. Prove :

$$\lim_{x \to 0} \frac{\tan x - \sin x}{x^3} = \frac{1}{2}$$

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38. Prove :

$$\lim_{x \to 0} \frac{\tan^{-1} x - x}{\sin^{-1} x} = 1$$

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39. Prove :

$$\lim_{\theta \to \frac{\pi}{4}} \frac{1 - \tan\theta}{1 - \cot\theta} = -1$$

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40. Prove :

$$\lim_{h \to 0} \frac{\sec(x + h) - \sec x}{h} = \sec x \tan x$$

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41. Prove :

$$\lim_{x \to 0} \frac{\cos 9x - \cos 7x}{\cos 7x - \cos 5x} = \frac{4}{3}$$

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42. Prove :

$$\lim_{x \to 0} \frac{\sin 6x + \sin 8x}{\sin 4x + \sin 6x} = \frac{7}{5}$$

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43. Prove :

$$\lim_{\theta \to \frac{\pi}{2}} (\sec \theta - \tan \theta) = 0$$

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44. Prove :

$$\lim_{x \to y} \frac{\cos^2 x - \cos^2 y}{x^2 - y^2} = -\frac{\sin 2y}{2y}$$

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45. Prove :

$$\lim_{x \to a} \frac{\sin x - \sin a}{x - a} = \cos a$$

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46. Prove :

$$\lim_{x \to a} \frac{\tan x - \tan a}{x - a} = \sec^2 a$$

Watch Video Solution

47. Prove :

$$\lim_{x \to 0} \frac{\tan x - \sin x}{1 - \cos x} = 0$$

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48. Prove :

$$\lim_{x \to 1} = \frac{\cos\left(\frac{\pi x}{2}\right)}{1 - x} = \frac{\pi}{2}$$

Watch Video Solution

49. Prove :

$$\lim_{\theta \to \frac{\pi}{4}} \frac{2 - \operatorname{cosec}^2\theta}{1 - \cot\theta} = 2$$

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50. Prove :

$$\lim_{x \to 0} \frac{\tan 2x - x}{3x - \sin x} = \frac{1}{2}$$

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51. Evaluate : $\lim_{x \to 0} \frac{e^{(x+h)^2} - e^{x^2}}{h}$

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52. Evaluate : $\lim_{x \to 0} \frac{e^{2x} - 1}{\sin 3x}$

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53. Evaluate : $\lim_{x \to 0} \frac{4^x - 1}{\sin 2x}$

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54. Evaluate : $\lim_{h \to 0} \frac{e^{2h} - 1}{e^{3h} - 1}$

Watch Video Solution

55. Evaluate : $\lim_{x \to 0} \frac{a^{\alpha x} - b^{\beta x}}{x}$

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56. Evaluate : $\lim_{x \to 0} \frac{\log(1 + 3x)}{x}$

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57. Evaluate : $\lim_{x \to 0} \frac{2^x - 1}{\sqrt{1+x} - 1}$

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58. Evaluate : $\lim_{x \to 0} \frac{e^x + e^{-x} - 2}{x^2}$

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59. Prove : $\lim_{x \to 0} \frac{\log(1+2x)}{\sin 3x} = \frac{2}{3}$

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60. Prove : $\lim_{x \to 1} \frac{\log x}{x - 1} = 1$

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61. Prove : $\lim_{x \to e} \frac{\log x - 1}{x - e} = \frac{1}{e}$

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62. Prove : $\lim_{x \to 0} \frac{p^x - q^x}{\tan x} = \log \frac{q}{q}$

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63. Prove : $\lim_{x \to 0} \frac{\sin \log(1+x)}{x} = 1$

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64. Prove : $\lim_{x \to 0} \frac{\sin \log(1+x)}{e^x - 1} = 1$

Watch Video Solution

65. Prove : $\lim_{x \to 0} \frac{\log \cos x}{\sin^2 x} = -\frac{1}{2}$

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66. Prove : $\lim_{h \to 0} \frac{\log(x+h) - \log x}{h} = \frac{1}{x}$

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67. Prove : $\lim_{x \to 0} \frac{\log(1+\sin x)}{x} = 1$

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68. Prove : $\lim_{x \to 0} \frac{\sqrt{1+x} - 1}{\log(1+x)} = \frac{1}{2}$

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69. Evaluate :

$$\lim_{x \to 0} \frac{\sin x}{\log_e (1+x)^{\frac{1}{2}}}$$

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70. Evaluate :

$$\lim_{x \to 0} \frac{e^{x^2} - 1}{\sin^2 x}$$

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71. Evaluate :

$$\lim_{x \to 0} \frac{(e^x - 1)\log(1 + x)}{\sin x}$$

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72. Evaluate :

$$\lim_{x \to 4} \frac{x^{7/2} - 4^{7/2}}{\log_e(x - 3)}$$

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73. Evaluate :

$$\lim_{x \to 0} \frac{\log(1 + \alpha x)}{\sin \beta x}$$

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74. Evaluate :

$$\lim_{x \to 0} \frac{e^{3x} - 1}{\log(1 + 5x)}$$

Watch Video Solution

75. Evaluate :

$$\lim_{x \to 0} \frac{2^{3x} - 1}{3^{2x} - 1}$$

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76. Find the limit (iF exists) : $\lim_{x \to 0} \frac{|x|}{x}$

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77. Find the limit *when it exists) : $\lim_{x \to 0} \frac{\sin|x|}{x}$

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78. The value of $\left[\lim_{x \to 0}\left(\frac{\sin x}{x}\right)^{\frac{\sin x}{x-\sin x}} + \lim_{x \to 1} x^{\frac{1}{1-x}}\right]$ is

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79. Find the limit (if exists) : $\lim_{x \to 0} \frac{\sin\{x\}}{\{x\}}$, (where {x} denotes the frectional part of x)

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80. Find the value of $\lim_{x \to 3} \frac{\sqrt{16x^2+112}-16}{3x^2-15x+18}$

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Exercise 2 Long Answer Type Questions

1. Draw the graph of the function $y = \frac{x^2}{x}$ and from the graph evaluate $\lim_{x \to 0} \frac{x^2}{x}$

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2. Let $f(x) \begin{array}{ll} = 1 & ,\text{for} \quad x > 0 \\ = x^2 & ,\text{for} \quad x \leq 0 \end{array}$

,discuss whether $\lim_{x \to 0} f(x)$ exists or not .

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3. Prove : $\lim_{x \to 4} \frac{3 - \sqrt{5 + x}}{1 - \sqrt{5 - x}} = -\frac{1}{3}$

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4. Prove : $\lim_{x \to 1} \frac{x^2 - \sqrt{x}}{\sqrt{x} - 1} = 3$

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5. Prove : $\lim_{x \to 0} \frac{1 - \cos x \sqrt{\cos 2x}}{x^2} = \frac{3}{2}$

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6. Prove : $\lim_{n \to \infty} \left(\sqrt{n+1} - \sqrt{n}\right) = 0$

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7. Prove : $\lim_{x \to \infty} \sqrt{x}\left[\sqrt{x+2} - \sqrt{x}\right] = 1$

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8. Prove : $\lim_{x \to \infty} \left[\sqrt{1 + x + x^2} - x\right] = \frac{1}{2}$

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9. Prove : $\lim_{n \to \infty} \left[\sqrt[3]{n+1} - \sqrt[3]{n}\right] = 0$

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10. Find the values of the following limts :

$$\lim_{n \to \infty} \frac{1+2+3+\ldots+n}{n^2}$$

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11. Find the values of the following limts :

$$\lim_{n \to \infty} \frac{1^2+2^2+3^2+\ldots+n^2}{n^3}$$

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12. Find the values of the following limts :

$$\lim_{n \to \infty} \left(1+\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\ldots. \text{ to n terms}\right)$$

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13. Find the values of the following limts :

$$\lim_{n\to\infty}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\ldots.+\frac{1}{3^n}\right)$$

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14. Find the values of the following limts :

$$\lim_{n\to\infty}\frac{1+2+3+\ldots.\text{to}(3n+2)\text{terms}}{(n+1)^2}$$

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15. Find the values of the follwing limts :

$$\lim_{n\to\infty}\frac{1.2+2.3+3.4+\ldots.n(n+1)}{n^3}$$

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16. prove the $\lim_{n\to\infty}\frac{a.\,e^x+b.\,e^{-x}}{c.\,e^x+d.\,e^{-x}}=\frac{a}{c}[2<e<3, c\neq 0].$

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17. Show that $\lim_{n \to \infty} \frac{a.\, e^x + b.\, e^{-x}}{c.\, e^x + d.\, e^{-x}} = \frac{b}{d} [2 < e < 3, d \neq 0].$

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18. Do the following limits exist ?

$$\lim_{x \to \pi} \frac{1}{\pi - x}$$

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19. Do the following limits exist ?

$$\lim_{x \to 0} \mathrm{X} \cos \frac{1}{x}$$

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20. Does $\lim_{x \to 4} \sqrt{4 - x}$ exist ?

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21. Does $\lim_{x \to 4} \sqrt{4 - x}$ exist ?

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22. Prove :

$$\lim_{x \to 0} x\cos\frac{1}{x} = 0$$

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23. Prove :

$$\lim_{x \to 0} x^2 \sin\frac{1}{x} = 0$$

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24. Prove :

$$\lim_{x \to 0} \left(x^2 \sin^{\frac{1}{x}}\right) = 0$$

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25. Prove :

$$\lim_{x \to \infty} \frac{\sin x}{x} = 0$$

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26. Prove :

$$\lim_{x \to \infty} \frac{\cos x}{x} = 0$$

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27. Putting $z = x - \frac{\pi}{4}$ show that $\lim_{x \to \frac{\pi}{4}} \frac{1 - \tan x}{1 - \sqrt{2} \sin x} = 2.$

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28. Putting $x = \theta - \frac{\pi}{4}$ prove that $\lim_{\theta \to \frac{\pi}{4}} \frac{\sin \theta - \cos \theta}{\theta - \frac{\pi}{4}} = \sqrt{2}.$

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29. A function f (x) is defined as follwos :

f(x) $\begin{matrix} = 1 & , & \text{when} & x \neq 0 \\ = 0 & , & \text{when} & x = 0 \end{matrix}$

Does $\lim_{x \to 0} f(x)$ exist ?

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30. Prove :

$$\lim_{x \to 1} \frac{4^x - 4}{x - 1} = 8 \log_e 2$$

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31. Prove :

$$\lim_{x \to 2} \frac{\log(2x - 3)}{2(x - 2)} = 1$$

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32. If $\lim_{x \to 0} (1+x)^{1/x} = e$, show that

$\lim_{x \to 0} (1+2x)^{1/x} = e^2$

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33. If $\lim_{x \to 0} (1+x)^{1/x} = e$, show that

$\lim_{x \to 0} (1+3x)^{3/x} = e^9$

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34. If $\lim_{x \to 0} (1+x)^{1/x} = e$, show that

$\lim_{x \to 1} x^{\frac{1}{1-x}} = e^{-1}$

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35. If $\lim_{x \to 0} (1+x)^{1/x} = e$, show that

$\lim_{x \to 0} (1+4x)^{1/x} = e^4$

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36. If $f(x) = \lim_{n \to \infty} \frac{1}{1 + n \sin^2 \pi x}$, find f (x) for all real values of x .

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37. Prove :

$$\lim_{x \to \infty} \frac{\sin x}{x} = 0$$

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38. Evaluate the following limits :

$$\lim_{x \to 0} \frac{e^{px} - e^{-qx}}{x}$$

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Sample Questions For Competitive Exams Multiple Correct Answers Type

1. $\lim_{n \to \infty} \frac{-3n + (-1)^n}{4n - (-1)^n}$ is equal to

A. $-\frac{3}{4}$

B. 0 when n is even

C. $-\frac{3}{4}$, when n is odd

D. $\frac{3}{4}$

Answer: A::C

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2. Let $f(x) = \begin{matrix} 1 + \frac{2x}{a} & ,0 \leq x < 1 \\ ax & ,1 \leq x < 2 \end{matrix}$ if $\lim_{x \to 1} f(x)$ is defined then value of 'a' will be -

A. 1

B. -1

C. 2

D. -2

Answer: B::C

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3. If $K^2 = \lim_{x \to 0} \left(\frac{x}{\sqrt{1+x} - \sqrt{1-x}} \right)$, then the value of K will be -

A. 1

B. 2

C. -2

D. -1

Answer: A::D

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4. If $\lim_{x \to 0} \frac{(1+x)^8 - 1}{(x+1)^2 - 1} = p^2$, then the value of P will be -

A. 3

B. 2

C. -2

D. -3

Answer: B::C

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5. If $\lim_{x \to \infty} \frac{(1+x)^{10} + (x+2)^{10} + ... + (x+100)^{10}}{x^{10} + 10^{10}} = K^2$, then the value of K will be -

A. 10

B. -10

C. 1

D. -1

Answer: A::B

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Sample Questions For Competitive Exams Integer Answer Type

1. If $\lim_{x \to 2} \frac{x-2}{\sqrt{x}-\sqrt{2}} = K\sqrt{2}$ then K is equal to -

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2. $\lim_{x \to \infty} \left[\frac{x}{x-2} - \frac{4}{x^2-2x}\right]$ is -

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3. If $\lim_{x \to 0} \frac{1-\cos 4x}{1-\cos 6x} = \frac{4}{K}$ then K is equal to -

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4. If $\lim_{x \to 0} \frac{\tan x - \sin x}{x^3} = \frac{M}{N}$, then value of N (when M =3) is equal to -

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5. $\lim_{x \to 0} \frac{\tan(x^2 - 1)}{2(x - 1)}$ is -

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Sample Questions For Competitive Exams Comprehension Type

1.

$\lim_{x \to 0} f(x) = 2$ where $f(x) = \frac{a \sin x - b \sin 3x + c \sin 5x}{x}$ and $a : b : c = 3 : 2$

, then -

then value of a is

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2.

$\lim_{x \to 0} f(x) = 2$ where $f(x) = \frac{a \sin x - b \sin 3x + c \sin 5x}{x}$ and $a : b : c = 3 : 2$

, then -

the value of b is

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3.

$\lim_{x \to 0} f(x) = 2$ where $f(x) = \frac{a \sin x - b \sin 3x + c \sin 5x}{x}$ and $a : b : c = 3 : 2$

, then -

the value of $2a - b + c$ is

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4. If α and β be the roots of $ax^2 + bx + c = 0$ and $L = \lim_{x \to \beta} \frac{1 - \cos\left(ax^2 + bx + c\right)}{(x - \beta)^2}$, then -

L = ?

A. $\frac{a^2(\beta + \alpha)^2}{2}$

B. $\frac{b^2(\beta - \alpha)^2}{2}$

C. $\frac{a^2(\beta-\alpha)^2}{2}$

D. $\frac{b^2(\beta+\alpha)^2}{2}$

Answer: C

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5. If α and β be the roots of $ax^2+bx+c=0$ and $L=\lim_{x\to\beta}\frac{1-\cos\left(ax^2+bx+c\right)}{(x-\beta)^2}$, then -

if $L=0$, then -

A. $\alpha=\beta$

B. $\alpha>\beta$

C. $\beta>\alpha$

D. none of these

Answer: A

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Sample Questions For Competitive Exams Assertion Reason Type

1. Statement -I :

$$\lim_{x \to \infty}\left(\frac{1^2}{x^3}+\frac{2^2}{x^3}+\frac{3^2}{x^3}+\ldots\ldots+\frac{x^2}{x^3}\right)=\lim_{x \to \infty}\frac{1^2}{x^3}+\lim_{x \to \infty}\frac{2^2}{x^3}+\ldots.+\lim_{x \to}$$

Statement - II :

$$\lim_{x \to a}\{f_1(x)+f_2(x)+\ldots.+f_n(x)\}=\lim_{x \to a}f_1(x)+\lim_{x \to a}f_2(x)+\ldots+\lim_{x \to a}f_n$$

A. Statement -I is true , Statement -II is true and Statement-II is a correct explanation for Statement - I .

B. Statement -I is true , Statement -II is true but Statement -II is is not a correct explantion of Statement -I .

C. Statement -I is true , Statement - II is false .

D. Statement - I is false , Statement -II is true .

Answer: D

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2.

$$\lim_{x \to 0} f(x) = 2 \quad \text{where} f(x) = \frac{a \sin x - b \sin 3x + c \sin 5x}{x} \quad \text{and} \quad a : b : c = 3 : 2$$

, then -

the value of $2a - c$ is

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