doubtnut
India's Number 1 Education App

MATHS

BOOKS - CHHAYA PUBLICATION MATHS (BENGALI ENGLISH)

MAPPING OR FUNCTION

Example

1. Let A = $\{0, 1, 2, 3, \}B = \{-3, -2, -1, 0, 1\}$ and $F: A \to B$ the mapping defined by $f(x) = x - 3$, for all $x \in A$. Show that f is one -one.

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2. Let $\mathbb{R}$ be the set of real number and $f: \mathbb{R} \to \mathbb{R}$, be given by $f(x) = 2x^2 - 1$. .Is this mapping one -one ?

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3. Show that the function $f: \mathbb{Z} \to \mathbb{Z}$ defined by $f(x) = 2x^2 - 3$ for all $x \in \mathbb{Z}$, is not one-one , here $\mathbb{Z}$ is the set of integers.

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4. Let $A = \{-2, 2, -3, 3\}$, $B = \{1, 4, 9, 16\}$ and $f: A \to B$ be given by $f(x) = x^2$, show that f is a many -one mapping.

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5. If $\mathbb{Z}$ be the set of integers, prove that the function $f: \mathbb{Z} \rightarrow \mathbb{Z}$ defined by $f(x) = |x|$, for all $x \in Z$ is a many -one function.

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6. Let $A = \{2, 3, 4, 5, 6\}, B = \{5, 8, 11, 14, 17\}$ and $f: A \rightarrow B$ be given by $y = f(x) = 3x - 1$ where $x \in A$ and $y \in B$. Show that ,f is an onto mapping.

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7. Let $\mathbb{N}$ be the set of natural numbers and D be the set of odd natural numbers. Then show that the mapping $f: \mathbb{N} \rightarrow D$, defined by $f(x) = 2x - 1$, for all $x \in \mathbb{N}$ is a surjection.

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8. Discuss the surjectivity of the following mapping: $f: \mathbb{Z} \to \mathbb{Z}$ defined by $f(x) = 2x - 1$, for all $x \in \mathbb{Z}$, where $\mathbb{Z}$ is the set of integers.

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9. Let $A = \{1, 2, 3\}, B = \{4, 5, 6\}$ and $f: A \to B$ be the mapping defined by, $f = \{(1, 4\}, (2, 5), (3, 6)\}$. Show that, f is a bijective mapping

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10. Let $\mathbb{Q}$ be the set of rational numbers and $f: \mathbb{Q} \to \mathbb{Q}$ be defined by ,

$f(x) = ax + b$

where $a, b, x \in \mathbb{Q}$ and $a \neq 0$. Prove that ,f is a bijection

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11. Discuss the bijectivity of the following mapping :$f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = ax^3 + b, x \in \mathbb{R}$ and $a \neq 0' \mathbb{R}$ being the set of real numbers

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12. function f and g are defined as follows: $f: \mathbb{R} - \{1\} \to \mathbb{R}$, where $f(x) = \dfrac{x^2 - 1}{x - 1}$ and $g: \mathbb{R} \to \mathbb{R} g(x) = x + 1, \mathbb{R}$ being the set of real numbers .Is $f = g$? Give reasons for your answer.

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13. Let $A = \left\{-1, -2, 0, 1\frac{5}{2}, 3\right\}$, $B = \{-6, -5, 0, 1, 4, 9\}$ and $f: A \to B$ be defined by $f(x) = 2x^2 - 3x - 5$. Find $f(A)$. Is $f(A) = B$?

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14. Prove that the function $f: \mathbb{R} \to \mathbb{R}$ defined by, $f(x) = \sin x$, for all $x \in \mathbb{R}$ is neither one -one nor onto.

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15. Let A be the set of triangles in a plane and $\mathbb{R}^+$ be the set of positive real numbers. Then show that, the function $f: A \to \mathbb{R}^+$ defined by ,$f(x) =$ area of triangle x, is many -one and onto.

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16. Let $\mathbb{R}$ be the set of real numbers and $A = R - \{3\}, B = R - \{1\}$. Show that , $f: A \rightarrow B$ defined by ,$f(x) = \frac{x-1}{x-3}$ is a one-one onto function.

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17. Let $\mathbb{C}$ and $\mathbb{R}$ be the sets of complex numbers and real numbers respectively . Show that, the mapping $f: \mathbb{C} \rightarrow \mathbb{R}$ defined by, $f(z) = |z|$, for all $z \in \mathbb{C}$ is niether injective nor surjective.

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18. Let $\mathbb{R}$ be the set real numbers and

$A = \{x \in \mathbb{R}: -1 \leq x \leq 1\} = B$

Examine whether the function f from A into B defined by

$f(x) = x|x|$ is surjective, injective or bijective.

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19. Show that , the function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by

$f(x) = x^3 + x$ is bijective, here $\mathbb{R}$ is the set of real numbers.

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20. Let $\mathbb{N}$ be the set of natural numbers: show that the mapping

$f\mathbb{N} \rightarrow \mathbb{N}$ given by,

$$f(x) = \begin{cases} \frac{(x)+1}{2} \text{when x is odd} \\ \frac{x}{2} \text{ when x is even} \end{cases}$$

is many -one onto.

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21. If $\mathbb{N}$ be the set natural numbers, then prove that, the mapping $f: \mathbb{N} \rightarrow \mathbb{N}$ defined by ,$f(n) = n - (-1)^n$ is a bijection.

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22. Let A be a finite set . If $f: A \rightarrow A$ is a one-one function, show that , f is a bijection.

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23. Let $A = \{a, b, c\}$. Write all one-one functions from A to A.

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24. Let S and T be two non- empty sets. Show that, $f: S \times T \to T \times S$ defined by , $f(a, b) = (b, a)$ for all $(a, b) \in S \times T$ is a bijection.

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25. Let NN be the set of natural number and $f: \mathbb{N} - \{1\} \to \mathbb{N}$ be defined by:

$f(n)$= the highest prime factror of n .

Show that f is a many -one into mapping

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26. Let the mapping $f: A \to B$ and $g: B \to C$ be defined by $f(x) = \frac{5}{x} - 1$ and $g(x) = 2 + x$ Find the product mapping (g o f).

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27. Let $A = \{x, y, z, t\}$ and the function $f: A \to A, g: A \to A$ be defined by,

$f(x) = z, f(y) = t, f(z) = y, f(t) = x$ and

$g(x) = y, g(y) = t, g(z) = x, g(t) = z$

Find $(gof)(t), (fog)(x), (fog)(y)$ and $(gof)(z)$.

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28. Let $\mathbb{R}$ be the set of real numbers . If the functions $f:\mathbb{R}\to\mathbb{R}$ and $g:\mathbb{R}\to\mathbb{R}$ be defined by , $f(x)=3x+2$ and $g(x)=x^2+1$, then find (g o f) and (f o g) .

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29. Let the function $f:\mathbb{R}\to\mathbb{R}$ and $g:\mathbb{R}\to\mathbb{R}$ be defined by, $f(x)=x^2-4x+3$ and $g(x)=3x-2$. Find formulas which define the composite functions

(i) f o f (ii) g o g (iii) f o g and (iv) g o f

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30. Let the functions f and g on the set of real numbers $\mathbb{R}$ be defined by, $f(x)=\cos x$ and $g(x)=x^3$. Prove that, (f o g) $\neq$

(g o f).

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31. Let the function f and g be defined by ,

$f = \{(a, b), (c, e), (d, a)\}$ and

$g = \{(b, c), (e, a), (a, c)\}$

Show that ,(g o f) and (f o g) are both defined. Also find (g o f) and (f o g) as sets of ordered pairs.

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32. The function f maps the set $A = \{a, b, c, d\}$ into itself, such that $f(a) = b, f(b) = d, f(c) = a, f(d) = c.$ Find the composition (f o f)

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33. Let $\mathbb{R}$ be the set of real numbers and $f: \mathbb{R} \rightarrow \mathbb{R}, g: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = 5|x| - x^2$ and $g(x) = 2x - 3$ Compute

(i) (g o f) (-2) (ii) (f o g) (-1)

(iii) (g o f)(5) (iv) (f o g)(5)

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34. Let $\mathbb{R}$ be the set of real numbers and $f: \mathbb{R} \rightarrow \mathbb{R}, g: \mathbb{R} \rightarrow \mathbb{R}$ be two functions such that, $(gof)(x) = 4x^2 + 4x + 1$ and $(fog)(x) = 2x^2 + 1$. Find $f(x)$ and $g(x)$.

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35. Let $\mathbb{R}$ be the set real numbers and $f: \mathbb{R} \rightarrow \mathbb{R}$ be given by $f(x) = ax + 2$, for all $x \in \mathbb{R}$. If $(fof) = I_{\mathbb{R}}$, find the value of a

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36. Let Q be the set of rational numbers and $f: \mathbb{Q} \rightarrow \mathbb{Q}$ be defined by , $f(x) = 3x - 2$, find $g: \mathbb{Q} \rightarrow \mathbb{Q}$, such that $(gof) = I_{\mathbb{Q}}$.

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37. Let $\mathbb{R}$ be the set of real numbers and $f: \mathbb{R} \rightarrow \mathbb{R}, g: \mathbb{R} \rightarrow \mathbb{R}, h: \mathbb{R} \rightarrow \mathbb{R}$ be defined by , $f(x) = \sin x, g(x) = 3x - 1, h(x) = x^2 - 4$.Find the formula which defines the product function h o (g o f) and hence compute [h o(g o f)] $\left(\frac{\pi}{2}\right)$

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38. Let $\mathbb{R}$ be the set of real numbers and $f:\mathbb{R}\to\mathbb{R}$ be defined by , $f(x)=2x+1.$ Find g: $\mathbb{R}\to\mathbb{R}$, such that $(gof)(x)=10x+10$

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39. Let $\mathbb{R}$ and$\mathbb{Q}$ be the sets of real numbers and rational numbers respectively. If $a\in\mathbb{Q}$ and f: $\mathbb{R}\to\mathbb{R}$ is defined by ,

$$f(x)=\begin{cases} x & \text{when} x\in\mathbb{Q} \\ a-x & \text{when } x\notin\mathbb{Q}\end{cases}$$

then show that ,$(fof)(x)=x$, for all $x\in\mathbb{R}$

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40. Let $\mathbb{Z}$ be that set of integers and $f: \mathbb{Z} \to \mathbb{Z}$ be defined by $f(x) = 2x$, for all $x \in \mathbb{Z}$ and g: $\mathbb{Z} \to \mathbb{Z}$ be defined by, (for all $x \in \mathbb{Z}$)

$$g(x) = \begin{cases} \frac{x}{2} & \text{when } \text{x is even} \\ 0 & \text{when x is odd} \end{cases}$$

Show that, $(gof) = I_{\mathbb{Z}}$, but $(fog) \neq I_{\mathbb{Z}}$.

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41. Let $f: \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} \frac{|x|}{x} & \text{when } x \neq 0 \\ 0 & \text{when } x = 0 \end{cases}$$

and the function $g: \mathbb{R} \to \mathbb{R}$ be defined by $g(x) = [x]$ where [x] is the greatest integer function. Prove that the functions (f o g) and (g o f) are same in [-1,0).

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42. Let $A = \{a, b, c, d, e\}$ and $f: A \to A$ be defined by $f(a) = d$, $f(b) = a$, $f(c) = d$, $f(d) = b$ and $f(e) = d$ find (i) $f^{-1}(b)$ (ii) $f^{-1}(e)$ (iii) $f^{-1}(d)$ and (iv) $f^{-1}\{a, b\}$.

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43. Let $\mathbb{Z}$ be the set of integers and $f: \mathbb{Z} \to \mathbb{Z}$ be defined by , $f(x) = x^2$. Find $f^{-1}(16)$ and $f^{-1}(-4)$

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44. Let $\mathbb{C}$ be the set of all complex numbers and $f: \mathbb{C} \to \mathbb{C}$ be defined by , $f(x) = x^2 + 2$. Find $f^{-1}(-1)$ and $f^{-1}(6)$

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45. Let $\mathbb{R}$ be the set of real numbers and $f:\mathbb{R}\to\mathbb{R}$ be defined by , $f(x)=2x^2-5x+6$ .Find $f^{-1}(5)$ and $f^{-1}(2)$

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46. Let $\mathbb{R}$ be the set of real numbers and $f:\mathbb{R}\to\mathbb{R}$ be given by $f(x)=x^2+2$. Find

(i)$f^{-1}\{11, 16\}$ (ii) $f^{-1}\{11\leq x\leq 27\}$

$(iii)f^{-1}\{0\leq x\leq 6\}$ (iv) $f^{-1}\{-2\leq x\leq 2\}$

(v) $f^{-1}\{-\infty< x\leq 4\}$

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47. Let $A=\{3, 6, 9, 12\}$ and $B=\{1, 2, 3, 4\}$. If $f:A\to B$ be defined by $f(x)=\frac{x}{3}$, find f and f^{-1} as sets of ordered pairs.

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48. Let $\mathbb{Q}$ be the set of rational numbers, If $f: \mathbb{Q} \to \mathbb{Q}$ is defined by $f(x) = ax + b$, where a, b, $x \in \mathbb{Q}$ and $a \neq 0$, then show that f is invertible and hence find f^{-1}

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49. Let $\mathbb{R}$ be the set of real numbers and $f: \mathbb{R} \to \mathbb{R}$ be defined by , $f(x) = x^3 + 1$, find $f^{-1}(x)$

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50. Let $A = \{x : -1 \leq x \leq 1\}$ and $f: A \to A$ be defined by $f(x) = \sin\frac{\pi x}{2}$. Show that f is a one- one onto mapping and

hence find a formula that defines f^{-1}

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51. Let $\mathbb{R}^{+}$ be the set of positive real numbers and $f: \mathbb{R} \rightarrow \mathbb{R}^{+}$ be defined by $f(x) = e^{x}$. Show that, f is bijective and hence find $f^{-1}(x)$

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Exercise 2 A

1. let $\mathbb{N}$ be the set of natural numbers and $f: \mathbb{N} \cup \{0\} \rightarrow \mathbb{N} \cup [0]$ be definedby :

$$f(n) = \begin{cases} n+1 & \text{when n is even} \\ n-1 & \text{when n is odd} \end{cases}$$

Show that ,f is a bijective mapping . Also that $f^{-1} = f$

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2. Let the function $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = x^2 (\mathbb{R}$ being the set of real numbers), then f is __

A. many - one and onto mapping

B. one-one and onto mapping

C. one-one and into mapping

D. many-one and into mapping

Answer: D

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3. Let the function $g: \mathbb{Q} - \{3\} \rightarrow \mathbb{Q}$ be defined by $g(x) = \frac{2x+3}{x-3}$ ($\mathbb{Q}$ being the set of rational numbers), then f is

___

A. surjective but not injective mapping

B. injective but not surjective

C. neither injective nor surjective

D. bijective mapping

Answer: B

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4. State which of the following statement is true?

A. If $y^2 = x$ then y amy be regarded as a function of x .

B. The function $f(x) = \frac{x^2}{x}$ and $\phi(x) = x$ are identical

C. A constant function is an onto function if its codomain contains only element.

D. Let $\mathbb{C}$ be the set of all complex number and the function $f: \mathbb{R} \to \mathbb{R}, g: \mathbb{C} \to \mathbb{C}$ be defined by , $f(x) = x^2$ and $g(x) = x^2$. State with reasons whether $f = g$ or not.

Answer: C

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5. State which of the following statement is false ?

A. If

$A=\{0,1,2,3\}, B=\{-3,-2,-1,0,1\}$ and $f: A \to B$

is the mapping defined by $f(x)=x-3$ for all $x \in A$,

then f is a one-one mapping

B. A constant mapping will be One-one when its domain

constans only one element.

C. Functions f and g are defined as follows: $f: \mathbb{R}-\{2\} \to \mathbb{R}$,

where $f(x)=\dfrac{x^2-4}{x-2}$ and $g: \mathbb{R} \to \mathbb{R}$, where

$g(x)=x+2$, then $f=g$

D. $f(x)=\sqrt{x^2+4x-1}$ then $f(-2)$ is not exist.

Answer: C

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6. The domain for which the functions $f(x) = 3x^2 - 2x$ and $g(x) = 3(3x - 2)$ are equal will be ___

A. $\left\{1, \frac{2}{3}\right\}$

B. $\{1, 3\}$

C. $\left\{\frac{2}{3}, 3\right\}$

D. $\left\{-\frac{2}{3}, 3\right\}$

Answer: C

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7. Let the mapping $f: [0. \infty) \to [0, 2]$ be defined by, $f(x) = \frac{2x}{x+1}$ then mapping f will be ____

A. injective but not surjective

B. neither injective nor surjective

C. onto but not one-one

D. one-one and into

Answer: A

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8. Let the mapping $f: \mathbb{N} \to \mathbb{N}$ defined by

$$f(x) = \begin{cases} x + 1, \text{ when} x \in \mathbb{N}, \text{ an odd} \\ x - 1, \text{ when} x \in \mathbb{N}, \text{ an even} \end{cases}$$

The mapping f will be ___

A. many-one and into

B. one-one and onto

C. many -one and onto

D. bijective mapping

Answer: B

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9. For any one-empaty set A, the identity mapping on A will be____

A. bijective

B. surjective but not injective

C. injective but not surjective

D. neither injective nor surjective

Answer: A

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10. Let A $=\{-1, 0, 1, 2, \} B=\{1, 1, 2, 3, -3\}$ and $f: A \rightarrow B$ be the mapping defined by ,$f(x)=2x-1$, for all $x \in A$ .Then f will be ___

A. one-one and into

B. one-one and onto

C. many- one and into

D. many-one and onto

Answer: A

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11. The mapping $f: \mathbb{Z} \rightarrow \mathbb{Z}$ defined by , $f(x)=3x-2$, for all $x \in \mathbb{Z}$, then f will be ___

A. onto but not one-one

B. one-one but not onto

C. many-one and into

D. many-one and onto

Answer: B

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12. The largest domain on which the function $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2$ is ___

A. $-\infty < x < 0 \text{ or } 0 < x < \infty$

B. $-\infty < x < 0 \text{ or } 0 \leq x < \infty$

C. $-\infty < x \leq 0 \text{ or } 0 \leq x < \infty$

D. $-\infty < x < 0$ or $0 < x < \infty$

Answer: c

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Exercise 2 A Very Short Answer Type Questions

1. Let $A = \{a, b\}$, write all one-one mappings from A to itself.

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2. Let $A = \{1, 2, 3\}$, write all one-one function from $A \rightarrow A$.

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3. Let $A = \{0, 1\}, B = \{2, 6\}$ and $f: A \to B$ be given by, $f(x) = 6 - 4x$ and $g: A \to B$ be given by, $g(x) = x^2 - 5x + 6$ State whether $f = g$ or not.

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4. Prove that the mapping $f: \mathbb{R} \to \mathbb{R}$ defined by ,$f(x) = x^2 + 1$ for all $x \in \mathbb{R}$ is neither one-one nor onto.

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5. Prove that the mapping $f: \mathbb{R} \to \mathbb{R}$ defined by ,$f(x) = x^2 + 1$ for all $x \in \mathbb{R}$ is neither one-one nor onto.

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6. Let $A = \{-1, 1, 2, -3\}, B = \{2, 8, 18, 32\}$ and $f: A \to B$ be defined by, $f(x) = 2x^2$, prove that, f is a many- one mapping of A into B`

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7. Prove that the function $f: \mathbb{R} \to \mathbb{R}$ defined by, $f(x) = \sin x$, for all $x \in \mathbb{R}$ is neither one -one nor onto.

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8. Show that the modulus function $f: \mathbb{R} \to \mathbb{R}$, given by $f(x) = |x|$ is neither one-one nor onto Where

$$|x| = \begin{cases} x \quad \text{when} \quad x \geq 0 \\ -x \quad \text{when} \quad x < 0 \end{cases}$$

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9. Show that, the mapping $f: \mathbb{N} \to \mathbb{N}$ defined by $f(x) = 3x$ is one-one but not onto

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10. Prove that, the function $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^3 + 3x$ is bijective .

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11. Let A be a finite set If $f: A \to A$ is an onto mapping , show that it is one-one aslo .

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12. Let A be the set of quadrilaterals in a plane and $\mathbb{R}^{+}$ be the set of positive real numbers. Prove that, the function $f: A \rightarrow \mathbb{R}^{+}$ defined by $f(x)$= area of quadrilateral x, is * many-one and onto.

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13. Let $A=\{-1,1,-2,2\}, B=\{3,4,5,6\}$ and $f: A \rightarrow B$ be the mapping defined by

$f=\{(1,6),(-1,4),(2,3),(-2,5)\}$.

Prove that, f is a bijective mapping

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14. Let D be the set of odd natural numbers . Then show that the mapping $f: \mathbb{N} \to D$, defined by, $f(x) = 2x - 3$ is onto but the mapping $g: \mathbb{Z} \to \mathbb{Z}$ defined by ,$g(x) = 2x - 3$ is not onto.

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15. Show that, the mapping $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = mx + n$, where $m, n, x \in \mathbb{R}$ and $m \neq 0$, is a bijection.

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16. Let $A = \mathbb{R} - \{2\}$ and $B = \mathbb{R} - \{1\}$. Show that, the function $f: A \to B$ defined by $f(x) = \dfrac{x-3}{x-2}$ is bijective .

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17. Let $\mathbb{C}$ be the set of complex numbers and $f: \mathbb{C} \to \mathbb{R}$ be defined by $f(z) = |z|$, for all $z \in \mathbb{C}$. Show that f is neither one-one nor onto.

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18. Show that the signum function $f: \mathbb{R} \to \mathbb{R}$, given by

$$f(x) = \begin{cases} 1 \text{ if } x > 0 \\ 0 \text{ if } x = 0 \\ -1 \text{ if } x < 0 \end{cases}$$

is neither one-one nor onto.

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19. Let $A = \{x \in \mathbb{R}: -1 \leq x \leq 1\} = B$. Show that, the mapping $f: A \to B$ defined by $f(x) = x|x|$ is bijective.

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20. Let $A = \{x \in \mathbb{R}: -1 \leq x \leq 1\} = B$. Prove that , the mapping from A to B defined by $f(x) = \sin \pi x$ is surjective.

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21. Prove that , the mapping $f: \mathbb{N} \rightarrow \mathbb{N}$ defined by,

$$f(x) = \begin{cases} x+1 & \text{when} \quad x \in \mathbb{N} \quad \text{is odd} \\ x-1 & \text{when} \quad x \in \mathbb{N} \quad \text{is even} \end{cases}$$

is one-one and onto.

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22. Prove that the greatest integer function $f: \mathbb{R} \rightarrow \mathbb{R}$, given by $f(x) = [x]$, is neither one-one nor onto, where [x] denotes the greatest integer less than or equal to x.

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Exercise 2 B

1. Let the function $f: \mathbb{R} \to \mathbb{R}$ be defined by , $f(x) = 3x - 2$ and $g(x) = 3x - 2(\mathbb{R}$ being the set of real numbers), then $(fog)(x) =$

A. $7x - 8$

B. $9x - 7$

C. $9x - 8$

D. $8x - 9$

Answer: c

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2. Two functions f and g are defined on the set of real numbers $\mathbb{R}$ by , $f(x) = \cos x$ and $g(x) = x^2$, then, $(fog)(x) =$

A. $\cos^2 x$

B. $\cos x^2$

C. $\sin^2 x$

D. $\sin x^2$

Answer: b

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3. If the function $f: \mathbb{R} \to \mathbb{R}$ and $g: \mathbb{R} \to \mathbb{R}$ are given by $f(x) = 3x + 2$ and $g(x) = 2x - 3(\mathbb{R}$ being the set of real numbers), state which of the following is the value of $(gof)(x)$?

A. $6x - 7$

B. $6x + 1$

C. 3x+5

D. $6x + 4$

Answer: b

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4. Let $\mathbb{R}$ be the set of real numbers and the mapping $f:\mathbb{R} \to \mathbb{R}$ and $g:\mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 5 - x^2$ and $g(x) = 3x - 4$, state which of the following is the value of $(fog)(-1)$?

A. 8

B. -44

C. 54

D. 16

Answer: b

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5. If $g(x) = x^2 + x - 2$ and $(gof)(x) = 2(2x^2 - 5x + 2)$, then $f(x) =$

A. $2x - 3$

B. $2x + 3$

C. $2x^2 - 3x + 1$

D. $2x^2 - 3x - 1$

Answer: a

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6. If $f(x) = \sin^2 x \text{ and } g(f(x)) = |\sin x|,$ then $g(x) =$

A. $\sqrt{x - 1}$

B. $\sqrt{x}$

C. $\sqrt{x + 1}$

D. $-\sqrt{x}$

Answer: b

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Exercise 2 B Very Short Answer Type Questions

1. What do you mean by composition of mapping ?

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2. Let $A = \{1, 2, 3, 4\}$ and the mapping $f: A \rightarrow A, g: A \rightarrow A$ be defined by

$f(1) = 3, f(2) = 4, f(3) = 2, f(4) = 1$

and $g(1) = 2, g(2) = 4, g(3) = 1, g(4) = 3$

Find $(i)(gof)(4), (ii)(fog)(1), (iii)(gof)(3), (iv)(fog)(2)$

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3. Let the function $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = x^2$ and $g(x) = x + 3$, evaluate

(f o g) (2) , (ii) (g o f) (3)

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4. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ and $g : \mathbb{R} \rightarrow \mathbb{R}$ be two mapping defined by $f(x) = 2x + 1$ and $g(x) = x^2 - 2$, find (g o f) and (f o g).

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5. Let the function $f : \mathbb{R} \rightarrow \mathbb{R}$ and $g : \mathbb{R}$ be defined by $f(x) = \sin x$ and $g(x) = x^2$. Show that, $(gof) \neq (fog)$.

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6. Let the functions $f : \mathbb{R} \rightarrow \mathbb{R}$ and $g : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = x + 1$ and $g(x) = x - 1$ Prove that , $(gof) = (fog) = I_{\mathbb{R}}$

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7. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by $f(x) = ax + b$, for all $x \in \mathbb{R}$. If $(fof) = I_{\mathbb{R}}$

Find the value of a and b.

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8. Let $f: \mathbb{Q} \rightarrow \mathbb{Q}$ be the function defined by ,

$f(x) = 2x + 5$, for all $x \in \mathbb{Q}$

Find the function $g: \mathbb{Q} \rightarrow \mathbb{Q}$ such that $(gof) = I_{\mathbb{Q}}$.

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9. Let the function $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by ,$f(x) = 4x - 3$.Find the function $g: \mathbb{R} \rightarrow \mathbb{R}$, such that $(gof)(x) = 8x - 1$

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10. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be the function defined by $f(x) = x + 1$. Find the function $g: \mathbb{R} \rightarrow \mathbb{R}$, such that $(gof)(x) = x^2 + 3x + 3$

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Exercise 2 B Short Answer Type Questions

1. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ be two mapping defined by $f(x) = x^2 + 3x + 1$ and $g(x) = 2x - 3$. Find formulas which define the composite mappings

(i) (f o f), (ii)(g o g), (iii) (g o f) , (iv) (f o g)

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2. Let the functions $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ be given by $f(x) = 3x - 2$ and $g(x) = 3|x| - x^2$. Find

(i) (g o f) (-1) , (ii) (f o g) (-2) , (iii) (g o f) (3), (iv) (f o g) (4)

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3. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ be two functions such that $(gof)(x) = \sin^2 x$ and $(fog)(x) = \sin(x^2)$. Find $f(x)$ and $g(x)$.

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4. let the functions $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = 3x + 5$ and $g(x) = x^2 - 3x + 2$. Find

$(i)(gof)(x^2 - 1)$, $(ii)(fog)(x + 2)$

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5. Let the functions f and g be defined by,

$f = \{(1, 2), (2, 3), (3, 4), (4, 1)\}$

and $g = \{(2, -1), (4, 2), (1, -2), (3, 4)\}$

Show that, (g o f) is defined but (f o g) is not defined . Also find (g o f) as set of ordered pairs.

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6. Let the functions f and g be defined by,

$f = \{(1, 2), (3, -2), (-1, 1)\}$

and $g = \{(2, 3), (-2, 1), (1, 3)\}$

Prove that , (g o f) and (f o g) are both defined . Also find (g o f) and (f o g) as sets of ordered pairs.

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7. Let the functions $f: \mathbb{R} \to \mathbb{R}, g: \mathbb{R} \to \mathbb{R}$ and $h: \mathbb{R} \to \mathbb{R}$ by given by,

$f(x) = \cos x, g(x) = 2x + 1 \text{ and } h(x) = x^3 - x - 6$

Find the value of the product function h o (g o f) and hence compute $[ho(go)]\left(\frac{\pi}{3}\right)$.

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8. If the function $f: \mathbb{R} \to \mathbb{R}$ be defined by,

$$f(x) = \begin{cases} x & \text{when } x \in \mathbb{Q} \\ 1 - x & \text{when } x \in \mathbb{Q} \end{cases}$$

then prove that , $(fof) = I_{\mathbb{R}}$.

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9. If $f:\mathbb{R}-\left\{\frac{7}{5}\right\}\to\mathbb{R}-\left\{\frac{3}{5}\right\}$ be defined as $f(x)=\frac{3x+4}{5x-7}$ and $g:\mathbb{R}-\left\{\frac{3}{5}\right\}\to\mathbb{R}-\left\{\frac{7}{5}\right\}$ be defined as $g(x)=\frac{7x+4}{5x-3}$ .Then find fog .

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Exercise 2 C

1. Let RR be the set of real numbers and the mapping $f:\mathbb{R}\to\mathbb{R}$ be defined by $f(x)=2x^2$, then $f^{-1}(32)=$

A. $\{4,\ -4\}$

B. $\{1,\ -1\}$

C. $\{2,\ -2\}$

D. $\{3,\ -3\}$

Answer: a

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2. The mapping $f: A \rightarrow B$ is invertible if is ___

A. injective but not surjective

B. surjective but not injective

C. bijective

D. none of these

Answer: c

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3. Let $A = \{a, b, c, d\}$ and $f: A \to A$ be defined by, $f(a) = d$, $f(b) = a$, $f(c) = b$ and $f(d) = c$. State which of the following is equal to $f^{-1}(b)$?

A. $\{a\}$

B. $\{b\}$

C. $\{c\}$

D. $\{d\}$

Answer: c

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4. Let ZZ be the set of integers and the mapping $f: \mathbb{Z} \to \mathbb{Z}$ be defined by, $f(x) = x^2$. State which of the following is equal to $f^{-1}(4)$?

A. 2

B. -2

C. -2i

D. 2i

Answer: d

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5. Let the function $f: A \rightarrow B$ have an inverse function $f^{-1}: B \rightarrow A$, then the nature of the function f is __

A. one-one and onto

B. one-one and into

C. many-one and onto

D. many-one and into

Answer: a

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Exercise 2 C Vary Short Answer Quations

1. If $f:\mathbb{R}-\left\{\frac{7}{5}\right\}\to\mathbb{R}-\left\{\frac{3}{5}\right\}$ be defined as $f(x)=\frac{3x+4}{5x-7}$ then find $f(-1)$.

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2. Let $A=\{-2,-1,0,1,2\}$ and $f:A\to A$ be defined by $f(-2)=1, f(-1)=-2, f(0)=1, f(1)=-1, f(2)=1.$

Find

(i) $f^{-1}(-1), (ii) f^{-1}(2)(ii) f^{-1}(1), (iv) f^{-1}\{-2, -1\}$

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3. Let the function $f: \mathbb{R} \to \mathbb{R}$ be defined by, $f(x) = x^2,$. Find

(i) $f^{-1}(25), (ii) f^{-1}(5), (iii) f^{-1}(-5)$

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4. Let $A = \{a, b, c\}$ and $B = \{p, q, r, \}$, defined three one-one and onto mappings from A to B and also find their inverse mappings

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5. Let $\mathbb{C}$ be the set of all complex numbers and $f: \mathbb{C} \rightarrow \mathbb{C}$ be given by, $f(x) = 3x^2 + 16$. Find

(i) $f^{-1}(1)$, $(ii) f^{-1}(-11)$, $(iii) f^{-1}(28)$

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6. Let the function $f: \mathbb{R} \rightarrow \mathbb{R}$ be given by , $f(x) = 3x^2 - 14x + 10$. Find

$(i) f^{-1}(4)$, $(ii) f^{-1}(-8)$

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7. Let the function $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = x^2 - 2$. Find

(i) $f^{-1}\{-1, 7\}$, $(ii) f^{-1}\{2 \leq x \leq 34\}$, (iii)

$f^{-1}\{-5 \leq x \leq 14\}$ (iv) $f^{-1}\{-6 \leq x \leq -2\}$,(v) $f^{-1}\{-\infty < x \leq 2\}$.

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8. Let the function $f: \mathbb{Q}$ be defined by $f(x) = 4x - 5$ for all $x \in \mathbb{Q}$. Show that f is invertible and hence find f^{-1}

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Exercise 2 C Short Answer Quations

1. Let $A = \{x \in \mathbb{R}: -1 \leq x \leq 1\}$ and functions f and g from A to A be defined by , $f(x) = x^2$ and $g(x) = x^5$. Show that g^{-1} exists but f^{-1} does not exist

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2. Let $A = \mathbb{R} - \{3\}$ and $B = \mathbb{R} - \{1\}$. Prove that the function $f: A \to B$ defined by , $f(x) = \frac{x-2}{x-3}$ is one-one and onto. Find a formula that defines f^{-1}

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3. let $A = \left\{x : -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}\right\}$ and $B = \{x : -1 \leq x \leq 1\}$. Show that the function $f: A \to B$ defined by, $f(x) = \sin x$ for all $x \in A$, is bijective . Hence, find a formula that defines f^{-1}

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4. let $A = \mathbb{R} - \left\{-\frac{1}{2}\right\}$ and $B = \mathbb{R} - \left\{\frac{1}{2}\right\}$. Prove that function $f: A \to B$ define by ,$f(x) = \frac{x+2}{2x+1}$ is invertible and hence find $f^{-1}(x)$

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5. Let the functions $f: \mathbb{Q} \to \mathbb{Q}$ and $g: \mathbb{Q} \to \mathbb{Q}$ be defined by, $f(x) = 3x$ and $g(x) = x + 3$. Assuming that f and g are both invertible , verify that , $(gof)^{-1} = \left(f^{-1} o g^{-1}\right)$.

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6. Let the function $f: \mathbb{R} \to \mathbb{R}$ be defined by, $f(x) = x^3 - 6$, for all $x \in \mathbb{R}$. Show that, f is bijective. Also find a formula that defines $f^{-1}(x)$.

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7. Let $A = \{0, 1, 2, 3\}$, $B = \{1, 4, 7, 10\}$,

$C = \{5, 11, 17, 23\}$ and $f: A \to B, G: B \to C$ be defined by

$f(x) = 3x + 1$ and $g(x) = 2x + 3$, verify that,

$(gof)^{-1} = (f^{-1})og^{-1}$

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8. Consider $f: \mathbb{R}_+ \to [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$.

Show that f is invertible with $f^{-1}(y) = \dfrac{\sqrt{y+6} - 1}{3}$

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Sample Questions

1. Let $f(x) = 2x - \sin x$ and $g(x) = 3\sqrt{x}$ then __

A. range of g o f is R

B. g o f is one- one

C. both f and g are one-one

D. both f and g are onto

Answer: a,b,c,d

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2. Let $f(x)\begin{cases}0 & \text{for} x=0 \\ x^2 \sin\frac{\pi}{x} & \text{for} \quad -1 < x < 1, (x \neq 0) \\ x|x| & \text{for} x \geq 1 \text{ or } \leq -1\end{cases}$ then_

A. $f(x)$ is an odd function

B. $f(x)$ is an even function

C. $f(x)$ is an either odd nor even

D. $f'(x)$ is an even function

Answer: a,d

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3. If $e^{x}+e^{f(x)}=e$,then for $f(x)$___

A. domain =$(-\infty, 1)$

B. range $(-\infty, 1)$

C. domain=$(-\infty, 0]$

D. range =$(-\infty, 1]$

Answer: a,b,c,d

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4. If the function f satisfies the reation $f(x+y)+f(x-y)=2f(x)f(y)Aax, y \in \mathbb{R}$ and $f(0) \neq 0$ then ____

A. $f(x)$ is an function

B. $f(x)$ is an odd function

C. If $f(2)=a$ then $f(-2)=a$

D. If $f(4)=b$ then $f(-4)=-b$

Answer: a,c

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5. If $f:\mathbb{R}^{+} \rightarrow \mathbb{R}^{+}$ is a polynomial function satisfying the functional equation $f\{f(x)\}=6x-f(x)$, then $f(17)$ is equal to ___

A. 17

B. -15

C. 34

D. -34

Answer: b,c

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Sample Questions Integer Anawer Type B

1. If $f(x) = (a^x) + \frac{a^{-x}}{2}$ and $f(x+y) + f(x-y) = Kf(x)f(y)$, then the value of K.

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2. Let $f(x) = x|x|$ and $g(x) = \sqrt{|x|}$ then the number of elements in the set $\{x \in \mathbb{R}\} f(x) = g(x)$ is equal to K. Find the value of K

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3. Let $g(x) = 1 + x - [x]$ [where [x] denote the gratest integer not exceeding x] and $f(x) = sgn. x$ [Where $f(x) = sgn. X = 1$ if $x > 0, f(x) = 0$ if $x = 0$ and $f(x) = -1$ if$x < 0]$ then for all x, $fog(x)$ is equal to λ .Find the value of λ

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4. Let f and g be two functions defined by $f(x) = \dfrac{x}{x+1}, g(x) = \dfrac{x}{1-x}$. If $(fog)^{-1}(x)$ is equal to kx,

then find the value of k.

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5. The value of $n \in \mathbb{Z}$ for which the function $f(x)=\dfrac{\sin n\pi}{\sin\left(\frac{x}{n}\right)}$ has 4π as its period is K, find the value of K.

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Sample Questions Matrix Match Type C

1. If $f:\mathbb{R}-\left\{\frac{7}{5}\right\}\to\mathbb{R}-\left\{\frac{3}{5}\right\}$ be defined as $f(x)=\dfrac{3x+4}{5x-7}$ then find $f(0)$.

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2. If $f : \mathbb{R} - \left\{ \frac{7}{5} \right\} \to \mathbb{R} - \left\{ \frac{3}{5} \right\}$ be defined as $f(x) = \frac{3x + 4}{5x - 7}$ then find $f(1)$.

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Sample Questions Comprehension Type C

1. Let $f(x) = \frac{1}{1 + x^2}$ and $g(x)$ is the inverse of $f(x)$,then find $g(x)$

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2. D$(f + g) =$

A. $\mathbb{R} - [-2, 0)$

B. $\mathbb{R} - [-1, 0)$

C. $\left[-2, \frac{1}{2}\right]$

D. none of these

Answer: b

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3. $R(f) =$

A. $\left[-\frac{1}{2}, \frac{1}{2}\right] - \{0\}$

B. $\left[-2, \frac{1}{2}\right]$

C. $[-2, 0]$

D. $[-1, 0]$

Answer: a

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4. $r(f) \cap R(g) =$

A. $\left[-2, \frac{1}{2}\right]$

B. $\left[-\frac{1}{2}, \frac{1}{2}\right] - \{0\}$

C. $[-1, 0]$

D. none of these

Answer: b

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5. Value of $F(3) =$

A. 1

B. -3

C. 5

D. 13

Answer: c

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6. Value of $F(4) =$

A. 1

B. -3

C. 5

D. 13

Answer: b

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Sample Questions Assertion Reason Type C

1. Value of $f(5) =$

A. 1

B. -3

C. 5

D. 13

Answer: d

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2. Let $f(x) = \sin + \cos x, \ g(x) = \dfrac{\sin x}{1 - \cos x}$

Statement-I: f is neither an odd function nor an even function

Statement -II: g is an odd function.

A. Statement -I is True , Statement -II is True , Statement II is a correct explanation for statement -I

B. Statement-I is True, Statement-II is True, Statement-II is not a correct explanation for Statement-I

C. Statement -I is True , Statement -II is False

D. Statement-I is False , Statement-II is True

Answer: b

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3. Let $A = (2, 3, 7, 9\}, \ f : A \rightarrow B$ is a function defined as $f(x) = x^2$. Then find the range of $f(x)$.

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