MATHS

BOOKS - CHHAYA PUBLICATION MATHS (BENGALI ENGLISH)

MAXIMA AND MINIMA

Illustrative Examples

1. For what value of x will $(x-1)(3-x)$ have its maximum ?

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2. For what value of x will $x(12-2x)^2$ have its minimum ?

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3. Using calculus find the values of x for which the value of cos x is minimum and maximum , also find its minimum and maximum value.

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4. Show that the function $2x^3 + 3x^2 - 36x + 10$ has a maximum value at $x = -3$ and a minimum value at x=2 , also find the maximum and minimum values of the function.

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5. Examine for maxima and minima of the function

$f(x) = x^4 - 8x^3 + 22x^2 - 24x + 8.$

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6. Show that the function $f(x) = \frac{2}{3}x^3 - 6x^2 + 20x - 5$ has neither a maximum nor a minimum value.

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7. Prove that the function $\frac{\sin(x+\alpha)}{\sin(x+\beta)}$ has no critical point.

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8. Given $f(x) = x^3 - 12x^2 + 45x + 8$ Find the values of x for which f'(x)=0 . Hence , determine the maximum and minimum values of f(x).

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9. Show that , the maximum value of $2x + \frac{1}{2x}$ is less than its minimum value.

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10. Using calculus find the maximum and minimum values of $a \sin x + b \cos x$.

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11. For what values of x the function $y = 2 \sin x + \cos 2x (0 < x < 2\pi)$ attains the maximum and minimum values?

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12. Show that the function $\sin^3 x \cos x$ has a maximum value at $x = \frac{\pi}{3}$.

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13. Find the maximum value of $x^{\frac{1}{x}}$.

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14. Prove that , the minimum value of $9e^x + 25e^{-x}$ is 30.

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15. Using calculus prove that ,

if the sum of two positive quantities be given , their product is maximum , when they are equal ,

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16. if the product of two positive quantities be given, their sum is least , when they are equal.

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17. Given , $x + y = 3$, find the maximum and minimum values of $\frac{9}{x} + \frac{36}{y}$.

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18. Show that , the function $f(x) = |x - 1|$ is not differentiable at x=1 but it has a local minimum at x=1.

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19. Prove that ,the greatest rectangle inscribed in a given circle is a square.

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20. A particle is moving in a straight line and its distance x cm from a fixed point on the line at any time t seconds is given by ,

$$x = \frac{1}{12}t^4 - \frac{2}{3}t^3 + \frac{3}{2}t^2 + t + 15.$$

At what time is the velocity minimum ? Also find the minimum velocity.

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21. The space s described in time t by a paricle moving in a strainght line is given by , $s = t^5 - 40t^3 + 30t^2 + 80t - 250.$ Find the minimum value of its acceleration.

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22. A cylindrical tin can , closed at both ends of a given volume , has to be constructed . Prove that , the amount of tin required will be least , when the height of the can is equal to its diameter .

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23. Prove that , a conical tent of given capacity will require the least amount of canvas , when the height is $\sqrt{2}$ times the radius of the base .

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24. Find out at which points on the curve $y = f(x) = 3x^5 - 5x^3 + 5x - 7$, the rate of change of the function f(x) is minimal . What is the value of the rate of change ?

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25. A firm produces x tonnes of output at a total cost of Rs. R where

$$R = \frac{1}{10}x^3 - 5x^2 + 10x + 5.$$

At what level of output will the marginal cost and the average variable cost attain their respective minima ?

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26. A radio manufacturer finds that he can sell x radios per week at Rs. P each , where

$p = 2\left(100 - \frac{x}{4}\right)$. His cost of production of x radios per week is $\left(120x + \frac{x^2}{2}\right)$. Show that his profit us maximum when the production is 40 radios per week . Find also his maximum profit per week.

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27. If the sum of the lengths of hypotenuse and a side of a right - angled trianle is given , show that the area of the triangle is maximum when the angle between them is $\frac{\pi}{3}$

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28. Show that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere .

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29. Show that the semi - vertical angle of a cone of maximum volume and given slant height is $\tan^{-1}(\sqrt{2})$.

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30. Find the volume of the largest cylinder that can be inscribed in a sphere of radius r cm .

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31. Show that the volume of the greatest cylinder which can be inscribed in a cone of height h and semi - vertical angle α is $\frac{4}{27}\pi h^3 \tan^2 \alpha$. Also show that the height of the cylinder is $\frac{h}{3}$.

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32. A point on the hypotenuse of a right - angled triangle is at distance a and b from sides of the triangle . Show that the minimum length of the hypotenuse is $\left(a^{\frac{2}{3}}+b^{\frac{2}{3}}\right)^{\frac{3}{2}}$

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33. A wire of length 36 cm is cut into two pieces . One of the pieces will be bent into the shape of a square and the other into the shape of an equilateral triangle . Find the length of each

piece so that the sum of the areas of the square and triangle is minimum.

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34. A window in the form of a rectangle , is surmounted by a semicircular opening . The total perimeter of the window is 10 m Find the dimensions of the rectangular part of the window to admit maximum light through the whole opening .

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35. A tank with rectangular base and rectangular sides , open at the top is to be constructed so that its depth is 2 m and volume is $8m^3$. If building of tank costs Rs. 70 per square metre for the base and Rs. 45 per square metre for sides, what is the cost of least expensive tank ?

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36. Find the coordinates of a point on the parabola

$y = x^2 + 7x + 2$ which is closest to the straight

line

$y = 3x - 3.$

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37. The length of the hypotenuse of a right angled triangle is 3 ft Find the volume of the greatest cone that can be generated by revolving the triangle about a side.

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38. Let (h,k) be a fixed point where $h > 0, k > 0$. A straight line passing through this point cuts the positive directions of the coordinate axes at the points P and Q . Find the minimum area of the triangle OPQ , O being the origin.

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Exercise Mcq

1. If a differentiable function f(x) attains a local extremum at x=a , then -

A. $f'(a) = 0, f''(a) < 0$

B. $f'(a) < 0$

C. $f'(a) = 0, f''(a) \neq 0$

D. $f'(a) = 0, f''(a) > 0$

Answer: C

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2. If a function is not differentiable at x=c , then the function-

A. may attain a local maximum

B. may attain a local minimum

C. cannot attain an extremum

D. may attain both a maximum or a minimum

Answer: D

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3. In a given interval a function-

A. can have two consecutive maxima

B. can have two consecutive minima

C. possesses maximum and minimum values alternately

D. cannot have more than two extreme values.

Answer: C

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4. The function $f(x) = 4x - x^2 - 3$ has a maximum value at-

A. $x = 3$

B. $x = 2$

C. $x = -2$

D. none of these

Answer: B

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5. The minimum value of the functon $y = x^2 - 6x + 11$ is -

A. 2

B. -2

C. 3

D. -3

Answer: A

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6. Let $f(x) = x^3 - 9x^2 + 30x + 5$ be a differentiable function of x , then -

A. f(x) is minimum at x =3

B. minimum value of f(x) is 8

C. minimum value of f(x) is greater than its maximum value

D. f(x) possesses neither a maximum nor a minimum .

Answer: D

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7. If $0 \leq x \leq 2\pi$, the function f(x) = sinx is minimum at -

A. $x = \frac{3\pi}{2}$

B. $x = \pi$

C. $x = \frac{3\pi}{4}$

D. $x = 2\pi$

Answer: A

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8. The maximum value of the function $f(x) = 5 - x - x^2$ is -

A. $\frac{17}{4}$

B. $\frac{21}{4}$

C. -1

D. $\frac{19}{4}$

Answer: B

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9. Let x=c be a point in the domain of definition of a differentiable function , then f(x) will have a local maximum at x=c when -

A. $f'(c) = 0, f''(x) \neq 0$

B. $f'(c) = 0, f''(x) > 0$

C. $f'(c) = 0, f''(c) < 0$

D. $f'(c) = 0, f''(c) = 0$

Answer: C

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10. The minimum value of the function $f(x) = x^2 - x + 2$ is -

A. $\frac{1}{2}$

B. $-\frac{1}{2}$

C. $-\frac{7}{4}$

D. $\frac{7}{4}$

Answer: D

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11. If $-\pi \leq x \leq \pi$, then f(x) =cos x is maximum at

-

A. x=0

B. $x = \frac{\pi}{2}$

C. $x = \pi$

D. $x = -\pi$

Answer: A

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12. The critical points of the function $f(x) = \frac{2}{3}x^3 - \frac{3}{2}x^2 - 2x + 5$

A. $\frac{1}{2}, -2$

B. $-\frac{1}{2}, 2$

C. $\frac{1}{2}, 2$

D. $-\frac{1}{2}, -2$

Answer: B

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Exercise Very Short Answer Type Questions

1. State the conditions for maxima and minima of a function $y = f(x)$ at a point , where $\frac{d^2y}{dx^2} \neq 0$

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2. Using calculus find the values of x , for which , $4 + 2x - x^2$ is a maximum

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3. Using calculus find the values of x , for which ,

$\left(3x^2 - 5x + 4\right)$ is a minimum

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4. Using calculus find the values of x , for which ,

$x(12 - 2x)^2$ is a minimum

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5. Using calculus find the values of x , for which ,

sin x is a maximum

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6. Using calculus find :

the maximum value of $(1-x)(2+3x)$

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7. Using calculus find

the minimum value of $2x^2 - 4x + 10$

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8. Using calculus find

the maximum value of cos x

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9. Show that , the following functions have neither a maximum nor a minimum value :

$x^3 - 3x^2 + 9x - 5$

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10. Show that , the following functions have neither a maximum nor a minimum value :

$$\frac{ax+b}{cx+d}$$

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11. Show that , the following functions have neither a maximum nor a minimum value :

$$\frac{\cos(x+a)}{\cos(x+b)}$$

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12. Show that , the following functions have neither a maximum nor a minimum value :

$2x + \tan^{-1} x$

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13. Show that , the following functions have neither a maximum nor a minimum value :

e^x

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14. Show that , the following functions have neither a maximum nor a minimum value :

logx

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15. Show that , the maximum value of the function $x + \frac{1}{x}$ is less than its minimum value .

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16. Show that , $f(x) = x^2 + \frac{250}{x}$ has a minimum value at x=5.

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17. What do you mean by an extremum of a single - valued function State the condition for which f(x) possesses an extremum at x = c . Under what codition the extremum is a

(i) maximum , (ii) minimum ?

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18. Find the maximum value of the product of the two numbers , if their sum is 12.

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19. If $x > 0, y > 0$ and $xy = 25$, find the minimum value of x+y.

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20. If $x + y = 15$, show that $x^2 + y^2$ is least , when $x = y = \frac{15}{2}$.

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Exercise Short Answer Type Questions

1. Divide 24 into two parts such that their product is maximum.

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2. Show that , the function represented by the equation $y^2 = (x+1)\left(2x^2 - 7x + 7\right)$ has two critical points . Find these points.

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3. Find for what values of x the following functions are maximum and minimum Also find the corresponding maximum and minimum values :

$y = x^3 - 3x^2 + 5$

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4. Find for what values of x the following functions are maximum and minimum Also find

the corresponding maximum and minimum values :

$2x^3 - 21x^2 + 36x - 20$

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5. Find for what values of x the following functions are maximum and minimum Also find the corresponding maximum and minimum values :

$x^3 - 9x^2 + 24x - 12$

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6. Find for what values of x the following functions are maximum and minimum Also find the corresponding maximum and minimum values :

$$f(x) = \frac{x^2 - 7x + 6}{x - 10}$$

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7. Find for what values of x the following functions are maximum and minimum Also find the corresponding maximum and minimum

values :

$$y = 10 - x^2 - x^3 - \frac{1}{4}x^4$$

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8. Find for what values of x the following functions are maximum and minimum Also find the corresponding maximum and minimum values :

$$\frac{1}{2}x^4 - 2x^3 - 6x^2 + 16x + 1$$

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9. Given $f(x) = x^3 - 6x^2 + 9x - 8$. Find the values of x for which f'(x)=0 . Hence determine the maximum and minimum values of f(x) by the criterion involving signs of f'(x).

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10. Show that , the maximum value of the function $\frac{1}{3}x^3 - 2x^2 + 3x + 1$ is $\frac{4}{3}$ more than its minimum value.

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11. Find the maximum value of sinx +cosx and the value of x for which it is maximum.

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12. Show that , the function $\log\cos^2 x + \sec x$ is maximum at x=0 and minimum at $x = \frac{\pi}{3}$.

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13. Prove that , the minimum value of

(i) $4e^{2x} + 9e^{-2x}$ is 12 , (ii) $\frac{x}{\log x}$ is e.

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14. Show : $4^x - 8x \log_e 2$ is minimum at x=1

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15. Show : $x^2 \log \frac{1}{x}$ is maximum at $x = \frac{1}{\sqrt{e}}$

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16. Show : $\sin x(1 + \cos x)$ is maximum at $x = \frac{\pi}{3}$

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17. Show : $\frac{2\theta - \sin 2\theta}{\theta^2}(\theta > 0)$ is maximum at $\theta = \frac{\pi}{2}$

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18. If $\frac{x}{2} + \frac{y}{3} = 1$, find the minimum value of $x^2 + y^2$.

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19. If $2x + 3y = 4$, find the maximum or minimum value of xy.

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20. Show that , the difference between the maximum and minimum values of the function $x^3 - 27x + 108$ is 108.

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21. Find the critical points of each of the following functions :

$\sin 2x - x\left(-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}\right)$

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22. Find the critical points of each of the following functions :

$y = e^x \sin x$

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23. Find the critical points of each of the following functions :

$$f(x) = \frac{a^2}{x} + \frac{b^2}{a - x}$$

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24. The perimeter of a rectangle is 100 cm. If the area of the rectangle is maximum , find the lenghts of its sides.

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25. Show that , of all rectangles of given permeter , the square has the greatest area.

26. Prove that , of all rectangles of given area , the square has the least perimeter.

27. Find the dimensions of the rectangle of maximum perimeter that can be inscribed in a

circle of radius a.

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28. The perimeter of a triangle is 8 cm .If one of the sides is 3 cm , find the lengths of the other sides so that the area of the triangle may be a maximum.

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29. A printed page is to contain $96cm^2$ of printed area , a margin of 1.5 cm at the top and bottom

and a margin of 1 cm at the sides . What are the dimensions of the smallest page that would fulfil the requirements ?

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30. The height of a particle projected with velocity u at an angle α with the horizontal is u $\sin\alpha . t - \frac{1}{2}gt^2$ (g= constant) at any time t. Find the greatest height attained and the time of reaching it .

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31. What will be the radius of the base of a solid circular cylinder of volume 16π for which the total surface area will be smallest ?

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32. The function $f(x) = 4x^3 + ax^2 + bx + 2$ has an extremum at (2,-2) , find the values of a and b . Show that , the function possesses a minimum value at the extreme point.

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33. The cost c of manufacturing a certain article is given by the formula , $c = 5 + \frac{48}{x} + 3x^2$, where x is the number of articles manufactured . Find the minimum value of c .

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34. Determine when will the function $\sin 3x - 3\sin x$ will be greatest or least in the interval $0 < x < 2\pi$.

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35. Obtain the maximum and minimum values of the function $\cos^2 x + \cos x + 3$ in the interval $0 \leq x \leq \frac{\pi}{2}$.

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36. Find the maximum and minimum values of :

$3\cos x + 4\sin x$

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37. Find the maximum and minimum values of :

$$\cos^2 x - \cos x \left(0 \leq x \leq \frac{\pi}{2}\right)$$

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38. Find the maximum and minimum values of :

$$\sin x + \cos^2 x \left(\frac{\pi}{2} < x \leq \frac{3\pi}{2}\right)$$

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39. Find the maximum and minimum values of :

$$1 + 2\sin x + 3\cos^2 x \left(0 \leq x \leq \frac{\pi}{2}\right)$$

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40. Prove that function

$y = (x - a_1)^2 + (x - a_2)^2 + \ldots + (x - a_n)^2$ is minimum when $x = \frac{1}{n}(a_1 + a_2 + \ldots + a_n)$.

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41. If $A > 0, B > \ > 0$ and $A + B = \frac{\pi}{3}$, find the maximum value of tan A tan B .

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42. Prove that $f(x) = \cos x - 1 + \frac{x^2}{2!} - \frac{x^4}{4!}$ possesses a maximum value at x=0.

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Exercise Long Answer Type Questions

1. Show that ,the maximum value of $\left(\frac{1}{x}\right)^x$ is $e^{\frac{1}{e}}$

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2. If x+y=2 , that the maximum value of $z = \frac{4}{x} + \frac{36}{y}$ is less than its minimum value .

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3. Find the maximum and minimum values of $y = 6x^2 - 3x^4 - 1$ in the interval $y = -2 \leq x \leq 3$

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4. Investigate for what values of x, the fuction $\frac{x-1}{x^2+x-1}$ is a maximum or minimum .also show that the maximum value of the fuction is less than its minimum value

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5. Obtain the maximum and minimum values of the function $y = \frac{x}{(x-1)(x-4)}$

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6. If f (c) does not exist , can f(x) have an extremum at x=c ? Justify your answer taking $f(x) = |x|$ and $c = 0$.

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7. Determine a point on the parabola $x^2 = 8y$ which is nearest to the point (2,4).

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8. Find the abscissa of the point on the parabola $y^2 = 2px$, which is nearest to the point (a,0).

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9. Find the point on the line 2x+3y =6 which is closest to the origin.

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10. Using calculus find the length of the perpendicular from the point (2,-1) upon the line

$3x - 4y + 5 = 0$.

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11. Divide 5 into two parts such that the product of the square of one and the cube of the other may be the greatest possible.

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12. Given the sum of the perimeters of a square is equal to the diameter of the circle .

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13. A particle is moving in a straight line its distance from a fixed point on the line at the end of t second is x metres , where $x = t^4 - 10t^3 + 24t^2 + 36t - 10$ When is it moving most slowly ?

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14. An open tank of a given capacity has square base with vertical sides . Prove that the expense of lining the the tank with cement will be

minimum if the height of the tank is half the width.

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15. A particle moving in a straight line describes a distance x cm from a fixed point on the line at time at time t s where $x = t^5 - 40t^3 + 30t^2 + 180t + 240$. Find when the acceleration is minimum and find the minimum value of acceleration.

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16. A particle moves in a straight line such that its distance x from a fixed point on it at any time t is given by $x = \frac{1}{4}t^4 - 2t^3 + 4t^2 - 7$ Find the time when its velocity is maximum and the time when its acceleration is minimum .

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17. A cylindrical tin can open at the top , of a given capacity has to be constructed . Show that the amount of the tin required will be least if the height of the can is equal to its radius

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18. Show that the height of a cylinder of given total surface area and open at the top has maximum volumes is equal to the radius of its base .

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19. The length of the hypotenuse of a right angled triangle is a Find the lenghts of its other

sides , so that

the sum of the sides maximum ,

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20. The length of the hypotenuse of a right angled triangle is a Find the lenghts of its other sides , so that

the area of the triangle is maximum.

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21. A box with square top and bottom is to be made to contain 250 cm^3 Material for the top and bottom costs Rs. 2 per cm^2 and the material for the vertical sides costs Rs. 1 per cm^2 . What is the cost of the least expensive box that can be made?

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22. A firm produces x units of output per week at a total cost of Rs. $\left(\frac{1}{3}x^3 - x^2 + 5x + 3\right)$ .Find the output levels at which the marginal cost, the

average variable cost attain their respective minima.

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23. A radio manufacturer produces x sets per week at a total cost of Rs. $\left(x^2+78x+2500\right)$. . He is a monopolist and the demand function for his product is $x=\frac{600-p}{8}$, where the price is Rs. P per set . Show that the maximum net revenue (i.e., profit is obtained when 29 sets are produced per week What is the monopoly price ?

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24. Prove that the triangle of maximum area that can be incsribed in a given circle is an equilateral triangle.

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25. A figure consists of a semi - circle with a rectangle on its diameter . Given the perimeter of the figure , find the dimensions of the rectangle in order that the area may be maximum.

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26. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius a is $\frac{2a}{\sqrt{3}}$

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27. An open box with a square base is to be made out of a given quantity of card board of area c^2 sq . Unit . Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cubi unit .

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28. A wire of length 28 m is to be cut into two pieces . One of the pieces is to be made into a square and the other into a circle . What should be the lengths of the two pieces so the two pieces so that the combined area of the circle and the square is minimum.

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29. A closed right circular cylinder has volume $2156cm^3$. What will be the radius of its base so

that its total surface area is minimum ?

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30. Show that the height of the cone of maximum volume that can be inscribed in a sphere of radius 12 cm is 16 cm .

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31. Show that the surface area of a closed cuboid with square base and given volume is minimum when it is a cube .

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32. Manufacturer can sell x items at a price of Rs. $\left(5-\frac{x}{100}\right)$ each . The cost price is Rs. $\left(\frac{x}{5}+500\right)$. Find the number of items he should sell to earn maximum profit.

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33. Show that the maximum volume of the cylinder which can be inscribed in a sphere of radius $5\sqrt{3}$ cm is $500\pi cm^3$.

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34. An open box is to be made out of a piece of cardboard measuring $(24cm \times 24cm)$ by cutting off equal squares from the

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35. If the sum of the lengths of the hypotenuse and another side of a right angled triangle is given , show that the area of the triangle is

maximum , when the angle between these sides is 60°.

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36. The three sides of a trapezium are each of length 8 cm . Find the maximum area of the trapezium.

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37. A straight line passing through the point (a,b) [where $a > 0$ and $b > 0$] intersects positive

coordnate axes at the points p and Q respectively

. Show that the minimum value of $(OP + OQ)$ is

$\left(\sqrt{a} + \sqrt{b}\right)^2$.

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38. Find the coordinates of the point on the curve

$y = \frac{x}{1 + x^2}$ where the tangent to the curve has

the greatest slope.

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Sample Questions For Competitive Examination A

1. The function $f(x) = \sin x \cos^2 x$ has extremum at -

A. $x = \frac{\pi}{2}$

B. $x = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

C. $x = \cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$

D. $x = \cos^{-1}\left(-\sqrt{\frac{2}{3}}\right)$

Answer: A::C::D

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2. Let $f(x)\sqrt{\left(1-x^2\right)\left(1+2x^2\right)}$ defined on [-1,1] then -

A. the greatest value of f(x) is 1

B. the greatest value of f(x) is $\frac{3}{\sqrt{8}}$

C. the least value of f(x) is 0

D. the least value of f(x) is -1

Answer: B::C

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3. Let =cos x sin 2x , then-

A. min $f(x) > -\frac{7}{9}, x \in [-\pi, \pi]$

B. min $f(x) > > -\frac{9}{7}, x \in [-\pi, \pi]$

C. min $f(x) > -\frac{1}{8} x \in [-\pi, \pi]$

D. min $f(x) > -\frac{2}{9}, x \in [\pi, \pi]$

Answer: A,B

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4. If f(x) $= \tan^{-1} x - \frac{1}{2} \log x$. Then -

A. The greatest value of f(x) on $\left[\frac{1}{\sqrt{3}}, \sqrt{3}\right]$ is $\frac{\pi}{6} + \frac{1}{4}\log^3$

B. The least value of f(x) on $\left[\frac{1}{\sqrt{3}}, \sqrt{3}\right]$ is $\frac{\pi}{3} - \frac{1}{4}\log 3$

C. f(x) decreases on $(0, \infty)$

D. f(x) increases on $(-\infty, 0)$

Answer: A::B::C

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5. Let f(x)

$$= \begin{cases} x^3 + x^2 - 10x, & 1 \leq x < 0 \\ 1 + \cos x & \frac{\pi}{2} \leq x \leq \pi \\ 1 + \cos x, \frac{\pi}{2} \leq x \leq \pi & \end{cases}$$ then

f(x) has,

A. local maxima at $x = \frac{\pi}{2}$

B. local minima at $x = \frac{\pi}{2}$

C. absolute maxima at x =0

D. absolute maxima at $x = \frac{\pi}{2}$

Answer: A::C

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Sample Questions For Competitive Examination B

1. The maximum value of $7e|x \log x|$ for $0 < x \leq 1$ is-

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2. If f(x) $= \log_x \left(\frac{1}{a}\right) - \log_3 x^2 (x > 1)$ the max f(x) is -

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3. If f(x) $= \log = x^2 \log x$ on [1,e] , thenlog (greatest of f(x) - least of f(x)) is equal to -

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4. If the greatest value of $y = \frac{x}{\log x}$ on $\left[e, e^3\right]$ is u them $\frac{e^3}{u}$ is equal to -

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5. The least natural number a for which $x + ax^{-2} > 2$,AAx in $(0, \infty)$ is -

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Sample Questions For Competitive Examination C

1. Match the following column

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Sample Questions For Competitive Examination D

1. Let

$f(x) = (x-1)^m(2-x)^n, m, n \in \mathbb{N}$ and $m, n > 2$

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2. $\begin{cases} -x^2 + 4x + a, x \leq 3 \\ ax + b, 3 < x < 4 \\ -\frac{b}{4}x + 6, x \geq 4 \end{cases}$

If x=3 is the only point of minima in its neighbourhood and x=4 is neither a point of maxima nor a point of minima , then which of the following true ?

A. $a > 0, b < 0$

B. $a < 0, b < 0$

C. $a > 0, b \in \mathbb{R}$

D. none of these

Answer: A

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3. $\begin{cases} -x^2 + 4x + a, x \leq 3 \\ ax + b, 3 < x < 4 \\ -\frac{b}{4}x + 6, x \geq 4 \end{cases}$

If x=4 is the only point of maxima in its

neighbourhood but x=3 is neither a point of maxima nor a point of minima then which of the following is true ?

A. $a < 0, b > 0$

B. $a > 0, b < 0$

C. $a > 0, b > 0$

D. not possible

Answer: D

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4. $\begin{cases} -x^2+4x+a, x \leq 3 \\ ax+b, 3<x<4 \\ -\frac{b}{4}x+6, x \geq 4 \end{cases}$

If x=3 is a point of minima and x=4 is a point of mxima then which of the following is true?

A. $a<0, b>0$

B. $a>0, b<0$

C. $a>0, b>0$

D. $a<0, b<0$

Answer: C

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5. Amongst the several applications of maxima and minima one of the application find the largest term of a sequence . Let $\{a_n\}$ be a sequence . Consider f(x) obtained on replacing x by n,e.g let $a_n = \frac{n}{n+1}$. Consider f(x) $= \frac{x}{x+1}$ on $[1, \infty), f'(x) = \frac{1}{(x+1)^2} > 0$ For all x. Hence max $f(x) = \lim_{x \to \infty} f(x) = 1$

The largest term of $a_n = \frac{n^2}{n^3 + 200}$ is -

A. $\frac{29}{453}$

B. $\frac{49}{543}$

C. $\frac{43}{543}$

D. $\frac{41}{451}$

Answer: B

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6. Amongst the several applications of maxima and minima one of the application find the largest term of a sequence . Let $\{a_n\}$ be a sequence . Consider f(x) obtained on replacing x by n,e.g let $a_n = \frac{n}{n+1}$. Consider f(x) $= \frac{x}{x+1}$ on $[1, \infty), f'(x) = \frac{1}{(x+1)^2} > 0$ For all x. Hence max $f(x) = \lim_{x \to \infty} f(x) = 1$

The largest term of sequence $a_n = \frac{n}{(n^2 + 10)}$ is

-

A. $\frac{3}{19}$

B. $\frac{2}{13}$

C. 1

D. $\frac{1}{7}$

Answer: A

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7. Amongst the several applications of maxima and minima one of the application find the largest term of a sequence . Let $\{a_n\}$ be a sequence . Consider f(x) obtained on replacing x by n,e.g let $a_n = \frac{n}{n+1}$. Consider f(x) $= \frac{x}{x+1}$ on $[1, \infty), f'(x) = \frac{1}{(x+1)^2} > 0$ For all x. Hence max $f(x) = \lim_{x \to \infty} f(x) = 1$

If f(x) is the function required to find largset term in ques . (i) then -

A. f increases for all x

B. f decreases for all x

C. f has a maximum at $x = \sqrt[3]{400}$

D. f increases on [0,9]

Answer: C

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Sample Questions For Competitive Examination E

1. Let a ,b $\in \mathbb{R}$ be such that the function f given by $f(x) = \log|x| + bx^2 + ax, x \neq 0$

Statement - I : f has local maximum at x=-1 and x=2.

Statement - II : f" $(-1) < 0$ and also $f''(2) < 0$

A. Statement - I is true , Statement - II is true , Statement -II is a correct explanation for Statement - I

B. Statement - I is True , Statement - II is True , Statement -II is not a correct explanation for Statement - I

C. Statement - I is True , Statement - II is False.

D. Statement - I is False , Statement - II is False.

Answer: A

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2. Statement - I : If f(x) $= \sin x, then f'(0) = f'(2\pi)$

Statement - II : If f(x) = sin x , then f(0) $= f(2\pi)$.

A. Statement - I is true , Statement - II is true , Statement -II is a correct explanation for Statement - I

B. Statement - I is True , Statement - II is True , Statement -II is not a correct explanation for Statement - I

C. Statement - I is True , Satement - II is False.

D. Statement - I is true , Statement - II is true , Statement -II is a correct explanation for Statement - I

Answer: B

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