MATHS

BOOKS - CHHAYA PUBLICATION MATHS (BENGALI ENGLISH)

MISCELLANEOUS EXAMPLES

SET THEORY AND RELATION (OR FUCNTION)

1. For three sets A,B and C,if $A \cap C = B \cap C$ and $A \cup C = B \cup C$, then prove that $A = B$

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2. For any three sets A, B and Cn if $A \cap B = A \cap C$ and $A \cup GB = A \cap C$ then prove that B=C

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3. Two finite sets A and B consist of m and n elements respectively. The number of subsets in A exceeds that of B by 112. Find the values of m and n.

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4. A survey shows that 75% of the stubents of a school like Mathematics and 65% like Physics. If x% of the students like both Mathematric and Physice, find the maximum and minimum values of x.

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5. In a survey of 35 students of a class it was found that 17 students like mathematics and 10 like Mathematics but not Biology. Find the number of students who like (i) Biology, (ii) Biology but not Mathematics, it being given that each student takes at least one of the two subjects.

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6. An inquiry into 100 candidates who failed in English of H.S. Examination revealed the following data : failed in Aggregate-66, failed in Paper I-37, failed Paper II-59, failed in Aggregate and Paper I-17, failed in Aggregate and Paper II-43 and failed in both papers-13.

Find the number of candidates who failed in

(i) Aggregate or Paper II but not in Paper I

(ii) Aggregate but not in Paper I and Paper II.

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7. In a survey it was found that 76 men read magazine A, 30 read magazine B, 40 read magazine C and 6 men read all the thee magazines. If the total number of men who read magazines be 116, find how many men read exactly two of the three magazines.

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8. For two sets A and B the three elements of $A \times B$ are $(a, x), (b, y), (c, x)$, find $B \times A$

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9. If (a,b) and (b,c) are elements of $A \times A$, find the set A and other elements of $A \times A$

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10. If two-empty sets A and B have n elements in common, then show that $A \times B$ and $B \times A$ have n^2 elements in common.

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11. A relation R on the set of integers Z is defined as follows :

$(x, y) \in R \Rightarrow x \in Z, |x| < 4 \text{ and } y = |x|$

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12. Let {x} and [x] denote the fractional and integral parts of a real number x respectively. Solve $4\{x\} = x + [x]$.

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13. Find the natural number a for which $\sum_{k=1}^{n} f(a+k) = 16\left(2^n - 1\right)$ where the function f satisfies the relation $f(x+y) = f(x)f(y)$ for all natural numbers x,y and further $f(1) = 2$.

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14. If $f(x) = \cos\left[\pi^2\right]x + \cos\left[-\pi^2\right]x$ where [x] denotes the greatest integer function, then show that

$f(-\pi) = 0$

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15. If $f(x) = \cos\left[\pi^2\right]x + \cos\left[-\pi^2\right]x$ where [x] denotes the greatest integer function, then show that

$$f\left(\frac{\pi}{2}\right) = -1$$

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16. If $f(x) = \cos\left[\pi^2\right]x + \cos\left[-\pi^2\right]x$ where [x] denotes the greatest integer function, then show that

$$f\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

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17. Find the domain of definiion of each of the functions :

$$f(x) = \frac{1}{\log_{10}(1-x)} + \sqrt{x+2}$$

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18. Find the domain of definiion of each of the functions :

$g(x) = \sqrt{|x| - x}$

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19. Find the domain of definition of each of the functions :

$f(x) = \log\dfrac{3 + x}{3 - x}$

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20. If $f(x) = \max\left(x, \dfrac{1}{x}\right)$ for $x > 0$, where max (p,q) denotes the greater of p and q, find the value of $f(a)f\left(\dfrac{1}{a}\right)$, wheres $a > 0$

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21. If $f(x + y) = f(x) + f(y)$ for all real x and y, show that $f(x) = xf(1)$.

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22. Find the domain of definition of each of the functions :

$$f(x) = \frac{\sqrt{4 - x^2}}{\sin^{-1}(2 - x)}$$

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23. Find the domain of definition of each of the functions :

$$y = \frac{\sqrt{\cos x - \frac{1}{2}}}{\sqrt{6 + 35x - 6x^2}}$$

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24. Find the domain of definition of each of the functions :

$$f(x) = \sin\left[\log\frac{\sqrt{4 - x^2}}{1 - x}\right]$$

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25. Find the domain of definition of each of the functions :

$$\sqrt{\left(\log_3 x\right)^2 - 3\log_3 x - 4}$$

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26. If $f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2}$ find f(x).

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27. If $2f\left(\frac{1}{x}\right) + f(x) = 3x$, find $f\left(x - \frac{1}{x}\right)$

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28. $f(x) = \tan\left(x - \frac{\pi}{4}\right)$, find $f(x) \cdot f(- x)$

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29. A cubic f(x) satisfies the relation $f(x) + f\left(\frac{1}{x}\right) = f(x) \cdot f\left(\frac{1}{x}\right)$, show that $f(x) = 1 + x^3$ or $1 - x^3$. Further , if $f(2) = 9$, show that $f(4) = 65$

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TRIGONOMETRY

1. If $2\cos\theta = a + \frac{1}{a}$ and $2\cos\phi = b + \frac{1}{b}$, show that $2\cos(\theta - \phi) = \frac{a}{b} + \frac{b}{a}$

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2. If angles α and β satisfy the equation $a\cos\theta + b\sin\theta = c(a, b, c$ are constants), prove that-

(a) $\sin(\alpha + \beta) = \frac{2ab}{a^2 + b^2}$

(b) $\cos(\alpha + \beta) = \dfrac{a^2 - b^2}{a^2 + b^2}$

(c) $\cos(\alpha - \beta) = \dfrac{2c^2 - \left(a^2 + b^2\right)}{a^2 + b^2}$

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3. If $\sec\alpha$ and $\operatorname{cosec}\alpha$ are the roots of $x^2 - px + q = 0$ then show that $p^2 = q(q + 2)$

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4. If $\cot^2\theta = \cot(\theta - \alpha)\cot(\theta - \beta)$, prove that $\cot\alpha + \cot\beta = 2\cot 2\theta$

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5. If $\cos 2A \sin 2B = \cos 2C \sin 2D$, prove that $\tan(C + A)\tan(C - A)\tan(B + D) = \tan(D - B)$

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6. If $\sin(y+z-x)$, $\sin(z+x-y)$ and $\sin(x+y-z)$ are in A.P., show that tan x, tan y and tan z are also in A.P.

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7. If $\cos\beta\cos(\gamma-\alpha)=\cos(\alpha-\beta+\gamma)$, prove that $\cot\alpha+\cot y=2\cot\beta$

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8. If $\tan^2\alpha+2\tan\alpha\tan 2\beta=\tan^2\beta+2\tan\beta\tan 2\alpha$, show that $\tan\beta=\pm\tan\alpha$.

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9. If $\operatorname{cosec}2A+\operatorname{cosec}2B+\operatorname{cosec}2C=0$, prove that $\tan A+\tan B+\tan C+\cot A+\cot B+\cot C=0$

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10. Let $f(x - 1) = 5x - 3$.then find $f^{-1}(x)$

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11. If $\frac{\tan(\alpha - \beta + \gamma)}{\tan(\alpha + \beta - \gamma)} = \frac{\tan\beta}{\tan\gamma}$, show that either $\sin(\beta - \gamma) = 0$ or $\sin 2\alpha + \sin 2\beta + \sin 2\gamma = 0$

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12. If $0 < \alpha < \frac{\pi}{2}, \frac{\pi}{2} < \beta < \pi$ and $\cos\alpha = \frac{4}{5}, \cos\beta = -\frac{3}{5}$, find the value of $\cos\frac{\alpha - \beta}{2}$. Show also that $\beta = \frac{\pi}{2} + \alpha$.

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13. If α and β be two roots of the equation $a\cos\theta + b\sin\theta = c$, show that $\sin\alpha + \sin\beta = \frac{2bc}{a^2 + b^2}, \sin\alpha\sin\beta = \frac{c^2 - a^2}{a^2 + b^2}$ and $\tan(\alpha + \beta) = \frac{2ab}{a^2 - b^2}$

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14. If $-360^{\circ} < A < -270^{\circ}$, show that $\sin\frac{A}{2} = -\sqrt{\frac{1 - \cos A}{2}}$

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15. If $2\sin A = \sqrt{1 + \sin 2A} - \sqrt{1 - \sin 2A}$ then find the limits of values of A.

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16. Prove that :

$\tan 11^{\circ} 15' = \sqrt{4 + 2\sqrt{2}} - \sqrt{2} - 1$

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17. In triangle ABC , if C is a right prove that $\frac{\sin^2 A}{\sin^2 B} - \frac{\cos^2 A}{\cos^2 B} = \frac{a^4 - b^4}{a^2 b^2}$

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18. Using the identity $\tan A = \frac{\sin 2A}{1 + \cos 2A}$ find the value of $\tan 15^\circ$ and $\tan 75^\circ$ and hence solve that the equation $\sec^2\theta = 4\tan\theta$.

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19. Prove that the lengths of the chord of a circle which subtends an angle108° at the centre is equal to the sum of the lenths of the chords which subtend angles 36° and 60° at the centre of the same circle.

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20. In any triangle $ABC \text{if} \frac{b+c}{a} = \cot\frac{A}{2}$, prove that the triangle is right-angled.

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21. If the angles of a triangle be, A,B,C and $\cos\theta(\sin B + \sin C) = \sin A$, prove that $\tan^2\frac{\theta}{2} = \tan\frac{B}{2}\tan\frac{C}{2}$

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22. If the sides a,b,c of a triangle ABC are in A.P., prove that $\cos A\cot\frac{A}{2}, \cos B\cot\frac{B}{2}, \cos C\cot\frac{C}{2}$ are also in A.P.

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23. If a is the smallest side of a triangle and a,b,c are in A.P. prove that $\cos A = \dfrac{4c - 3b}{2c}$

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24. In a triangle ABC if $b + c = 3a$ then find the value of $\cot\left(\frac{B}{2}\right)\cot\left(\frac{C}{2}\right)$

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25. If the medium AD is perpendicular to the side AB in the triangle ABC, show that $\tan A + 2\tan B = 0$.

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26. Show that the perimeter of any triangle is equal to $2\cos\frac{A}{2}\cos\frac{B}{2}\sec\frac{A+B}{2}$

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27. If $\sin\theta + \cos\theta = a, \tan\theta + \cot\theta = b,$ show that $\frac{a^2 - 1}{2} = \frac{1}{b}$

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28. If the three side of a triangle be a, b, c and $\tan\theta = \frac{2\sqrt{ab}}{a - b}\sin\frac{C}{2}$, then show that $\sec\theta = \frac{c}{a - b}$.

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29. In the triangle ABC, if $\cos 3A + \cos 3B + \cos 3C = 1$, show that the triangle is obtuse-angled.

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30. Show that :

$\tan 20^\circ + \tan 70^\circ = 2\operatorname{cosec} 40^\circ$

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31. Show that :

$\sin 2\frac{\pi}{7} + \sin\frac{4\pi}{7} - \sin\frac{6\pi}{7} = 4\sin\frac{\pi}{7}\sin\frac{2\pi}{7}\sin\frac{3\pi}{7}$

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32. Show that :

$\tan 70^{\circ} = 2\tan 40^{\circ} + \tan 20^{\circ} + 4\tan 10^{\circ}$

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33. The length of hypotenuse and one side of a right -angle triangle are $[3 + 2(\sin\theta + \cos\theta)]$ and $[2(1 + \sin\theta) + \cos\theta)]$ respectively, show that the lengths of the other side of the triangle is $[2(1 + \cos\theta) + \sin\theta]$

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34. If $\cos A = \frac{3}{4}$, show that $32\sin\frac{A}{2}\sin\frac{5A}{2} = 11$

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35. If $\cos A = \tan B, \cos B = \tan C, \cos C = \tan A,$ show that $\sin A = \sin B = \sin C = 2\sin 18^{\circ}$

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36. If $\tan(\pi\cot\theta) = \cot(\pi\tan\theta),$ then show that $\tan\theta = \frac{1}{4}\left[2n + 1 \pm \sqrt{4n^2 + 4n - 15}\right]$, where $n > 1$ or $n < -2.$

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37. If $\tan(\pi\cos\theta) = \cot(\pi\sin\theta)$, show that $\cos\left(\theta - \frac{\pi}{4}\right) = \pm\frac{1}{2\sqrt{2}}$

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38. If $u_n = 2\cos n\theta$, then show that $u_{n+1} = u_1 u_n - u_{n-1}$, hence, show that $2\cos 5\theta = u_1^5 - 5u_1^3 + 5u_1$

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39. Show that the value of $\frac{\cot 3x}{\cot x}$ cannot lie between $\frac{1}{3}$ and 3.

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40. If $(\sin\theta + \cos\theta) = \sqrt{2}\cos\theta$, show that $\cot\theta = \left(\sqrt{2} + 1\right)$.

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41. Show that $3(\sin x - \cos x)^4 + 4\left(\sin^6 x + \cos^6 x\right) + 6(\sin x + \cos x)^2 = 13$

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42. Show that

$1 + \cos 56^\circ + \cos 58^\circ - \cos 66^\circ = 4\cos 28^\circ \cos 29^\circ \sin 33^\circ$

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43. If $\alpha = \frac{2\pi}{7}$, then prove that $\cos\alpha + \cos 2\alpha + \cos 4\alpha = -\frac{1}{2}$. Hence, deduce that $\sin\alpha + \sin 2\alpha + \sin 4\alpha = \frac{\sqrt{7}}{2}$

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44. In any triangle ABC if $\sin A \sin B \sin C + \cos A \cos B = 1$, then show that $a:b:c = 1:1:\sqrt{2}$.

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45. If $\sin\theta + \sin\phi = \sqrt{3}(\cos\phi - \cos\theta)$, then show that $\sin 3\theta + \sin 3\phi = 0$

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46. Prove that :

$\tan 70^\circ + \tan 10^\circ - \tan 50^\circ = \sqrt{3}$

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47. Prove that :

$4\sin 50^{\circ} - \sqrt{3}\tan 50^{\circ} = 1$

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48. If α, β, γ lengths of the altitudes of a triangle ABC, prove that $\alpha^{-2} + \beta^{-2} + \gamma^{-2} = \frac{1}{\Delta}(\cot A + \cot B + \cot C)$, where Δ is the area of the triangle.

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49. If $a\sin\theta + b\cos\theta = 0$,then find θ

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50. If $\cos^3\theta = k\cos(\alpha - 3\theta)$ and $\sin^3\theta = k\sin(\alpha - 3\theta)$, then show that $2k^2 - k\cos\alpha - 1 = 0$

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51. Show that the general solution of $\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x$ is $\frac{n\pi}{2} + \frac{\pi}{8}$, where n is any integer.

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52. If k is a positive integer, show that

$$2[\cos x + \cos 3x + \cos 5x + .. + \cos(2k - 1)x] = \frac{\sin 2kx}{\sin x}$$

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53. If $\alpha = \frac{\pi}{8n}$ then prove that $\sin^2\alpha + \sin^2 3\alpha + \sin^2 5\alpha + ..$ to 2n terms =n.

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54. If $\sin A + \tan A = a$ and $\cos A + \cot A = b$, then show that $(1 + a)^{-2} + (1 + b)^{-2} = (1 - ab)^{-2}$

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55. If $a^2(1 - \sin\theta) + b^2(1 + \sin\theta) = 2ab\cos\theta$ then show that $2\tan\theta = \frac{a}{b} - \frac{a}{a}$

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56. Show that

$$\frac{1 + \sin A}{\cos A} + \frac{\cos B}{1 - \sin B} = \frac{2(\sin A - \sin B)}{\sin(A - B) + \cos A - \cos B}$$

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57. If n is positive integer and $\cos\frac{\pi}{2n} + \sin\frac{\pi}{2n} = \frac{\sqrt{n}}{2}$, then prove that $4 \le n \le 8$

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58. Solve :

$\sec\theta + \text{cosec}\theta = 2\sqrt{2}$

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59. Solve :

$\tan^3\theta + \cot^3\theta = 8\text{cosec}^3 2\theta + 12$

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60. Solve :

$$\frac{\sin\frac{x}{2}\cos\frac{x}{2}}{1 - 8\sin^2\frac{x}{2}\cos^2\frac{x}{2}} + \frac{\cos x\left(4\cos^2 x - 3\right)}{2\sin 4x} = 0$$

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61. Solve :

$$\cos\theta\cos 2\theta\cos 3\theta = \frac{1}{4}(0 \le \theta \le \pi)$$

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62. If $\cos\beta$ is the geometric mean of $\sin\alpha$ and $\cos\alpha$, then show that

$$\cos 2\beta = -2\cos^2\left(\frac{\pi}{4} + \alpha\right)$$

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63. Prove that $\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\sin\frac{7\pi}{14} = \frac{1}{8}$

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64. The sides of a triangles are in A.P.and the greatest angle exceeds the least by 90°, prove that the side are proportional to $(\sqrt{7}+1), \sqrt{7}$ and $(\sqrt{7}-1)$

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65. Let $0 < x < \pi, 0 < y < \pi$ and $\cos x + \cos y - \cos(x+y) = \frac{3}{2}$. Prove that $x = y = \frac{\pi}{3}$

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66. Eliminate x and y from the following equations : $a\sin^2 + b\cos^2 x = c, b\sin^2 y + a\cos^2 y = d, a\tan x = b\tan y.$

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67. Find the root of equation $2x^2 + 3x + 11 = 0$.

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68. If $a\cos\theta + b\sin\theta = c$ then find the quadratic equation whose root are $\sin^2\alpha$ and $\sin^2\beta$

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69. If $\alpha = \frac{2\pi}{7}$ then prove that $\sin\alpha\sin2\sin4\alpha = -\frac{\sqrt{7}}{8}$

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70. If A,B,C are the angles of a triangle, show that that greatest value of $\sin2A + \sin2B + \sin2C \text{is} \frac{3\sqrt{3}}{2}$

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71. If A,B,C are the angles of a triangle, find the maximum values of :

$\sin A + \sin B + \sin C$

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72. If A,B,C are the angles of a triangle, find the maximum values of :

$\sin A \sin B \sin C$

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73. If A,B,C are the angles a triangle, prove that $\cos A + \cos B + \cos C \leq \frac{3}{2}$

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74. If A,B,C are the angles of a triangle, prove that the maximum value of

$\cos A \cos B \cos C \text{is} \frac{1}{8}$

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75. If $\theta + \phi = \alpha$, where $0 < \theta < \frac{\pi}{2}, 0 < \phi < \frac{\pi}{2}$ and α is constant, find the minimum values of

$\text{cosec}\theta + \text{cosec}\theta$

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76. If $\theta + \phi = \alpha$, where $0 < \theta < \frac{\pi}{2}, 0 < \phi < \frac{\pi}{2}$ and α is constant, find the minimum values of

$\sec\theta + \sec\phi$

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77. If $\theta + \phi = \alpha$, where $0 < \theta < \frac{\pi}{2}, 0 < \phi < \frac{\pi}{2}$ and α is constant, find the minimum values of

$\tan\theta + \tan\phi$

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78. Show that $\frac{1}{3} < \frac{\sec^2\theta - \tan\theta}{\sec^2\theta + \tan\theta} < 3.$

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79. Find the minimum values of:

$$\tan^2\frac{A}{2} + \tan^2\frac{B}{2} + \tan^2\frac{C}{2}$$

where A,B,C are the angles of a triangle.

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80. Find the minimum values of:

$$\cot^2 A + \cot^2 B + \cot^2 C$$

where A,B,C are the angles of a triangle.

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81. Find the minimum values of:

$4\sin A + 3\cos A$

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82. Find the minimum values of:

$\sin^2\frac{A}{2} + \sin^2\frac{B}{2} + \sin\frac{C}{2}$

where A,B,C are the angles of a triangle.

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83. If $A + B + C = \pi$ and $\tan\frac{B + C - A}{4}\tan\frac{C + A - B}{4} \cdot \tan\frac{A + B - C}{4} = 1$, then prove that $\cos A + \cos B + \cos C = -1$

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84. Prove that,

$$\begin{vmatrix} 6a & a & a \\ 12b & 2b & 2b \\ 9c & c & -2c \end{vmatrix} = 0$$

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85. If $\cos\alpha + \cos\beta = a\sin\alpha + \sin\beta = b$, then show that

$$\tan\frac{\alpha}{2} + \tan\frac{\beta}{2} = \frac{4b}{a^2 + b^2 + 2a}$$

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86. Determine the value of k for which $x - 1$ is a factor of $\left(x^2 + 1\right) + (x + k)$.

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87. Prove that :

$$\sec\alpha + \sec\left(120^\circ + \alpha\right)\sec\left(120^\circ - \alpha\right) = -3\sec 3\alpha$$

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88. Prove that :

$$\operatorname{cosec}\alpha + \operatorname{cosec}\left(\frac{2\pi}{3} + \alpha\right) + \operatorname{cosec}\left(\frac{\pi}{3} + \alpha\right) = 2\operatorname{cosec}3\alpha$$

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89. find the differtiation of $\tan\left(a^x\right)$

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90. If α, β, γ are positive acute angles, prove that $\sin\alpha + \sin\beta + \sin\gamma > \sin(\alpha + \beta + \gamma)$

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91. find the differtiation of $\sin\left(e^x\right)$

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92. If $\cos\alpha + \cos\beta = a$ and $\sin\alpha + \sin\beta = b$, then show that $\sin 2\alpha + \sin 2\beta = 2ab\left(1 - \frac{2}{a^2 + b^2}\right)$.

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93. Find $\sin\frac{A}{2}$ and $\cos\frac{A}{2}$ in terms of $\tan\frac{A}{4}$.

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94. If $\tan\theta(\cos\alpha + \sin\beta)$, then show that one value of $\tan\frac{\alpha}{2}\tan\left(\frac{\pi}{4} - \frac{\beta}{2}\right)$.

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95. If $\sin(\pi\cot\theta)=\cos(\pi\tan\theta)$ and n is an integer, then prove that the value of $\cot 2\theta$ is of the form $\frac{1}{4}+k$.

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96. Prove that the maximum value of $(\cos\alpha_1)(\cos\alpha_2)\ldots(\cos\alpha_r)$ uder the restrictions

$0 \le \alpha_r \le \frac{\pi}{2}(r=1,2,\ldots,n)$ and $(\cot\alpha_1)(\cot\alpha_2)\ldots(\cot\alpha_r)=1 \text{is} \frac{1}{2^{\frac{n}{2}}}$

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97. Show that

$\sin^2 12^\circ+\sin^2 21^\circ+\sin^2 39^\circ+\sin^2 48^\circ=1+\sin^2 9^\circ+\sin^2 18^\circ$

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98. If $\alpha+\beta=\frac{\pi}{2}$ and $\beta+\gamma=\alpha$ then prove that $\tan\alpha=\tan\beta+2\tan\gamma$.

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99. θ and θ_2 are two distinct values of $\theta\left[0 \le \theta_1 2\pi, 0 \le \theta_2 < 2\pi\right]$ satisfying the equation $\sin(\theta + \alpha) = \frac{1}{2}\sin 2\alpha$. Prove that, $\frac{\sin\theta_1 + \sin\theta_2}{\cos\theta_1 + \cos\theta_2} = \cot\alpha$. s

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100. Find the values of α and $\beta\left[0 < \alpha, \beta < \frac{\pi}{2}\right]$ satisfying the equation $\cos\alpha\cos\beta\cos(\alpha + \beta) = -\frac{1}{8}$

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101. Find all the values of α for which the equation $\sin^4 x + \cos^4 x + \sin 2x + \alpha = 0$ is valid.

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102. In $\sin\alpha + \sin\beta = \frac{1}{2}$ and $\cos\alpha + \cos\beta = \frac{5}{4}$, find the value of $\tan\alpha + \tan\beta$.

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103. A 12 cm long wire is bent to form a triangle with one of its angles as 60°. Find the sides of the triangle when its area is the largest.

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104. If $\frac{\cos\alpha + \cos\beta + \cos\gamma}{\cos(\alpha + \beta + \gamma)} = \frac{\sin\alpha + \sin\beta + \sin\gamma}{\sin(\alpha + \beta + \gamma)}$, then show that each side is euqal to $\cos(\alpha + \beta) + \cos(\beta + \gamma) + \cos(\gamma + \alpha)$.

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105. If $a\tan x + b\tan y + c\tan z = m$, then show that the minimum value of $\tan^2 x + \tan^2 y + \tan^2 z is \frac{m^2}{a^2 + b^2 + c^2}$.

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106. Prove that

$$\tan\left(\alpha+\frac{\pi}{3}\right)\tan\left(\alpha-\frac{\pi}{3}\right)+\tan\alpha\tan\left(\alpha+\frac{\pi}{3}\right)+\tan\left(\alpha-\frac{\pi}{3}\right)\tan\alpha+3=0$$

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107. If $\sin\alpha+\sin\gamma+\sin\alpha\sin\beta\sin\gamma=0$, show that $\cos^2\alpha(1+\sin\beta\sin\gamma)^2=\cos^4\beta(1+\sin\gamma\sin\alpha)^2=\cos^4\gamma(1+\sin\alpha\sin\beta)^2$.

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108. If $\cos\alpha+\cos\beta+\cos\gamma+\cos\alpha\cos\beta\cos\gamma=0$, show that $\sin\alpha(1+\cos\beta\cos\gamma)=\pm\sin\beta\sin\gamma$.

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109. Given the product p of sines of the angles of a triangle and the prodcut q of their cosines, find the cubic equation whos coefficients are functions of p and q.

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110. If $0 < x < \pi$ and $\sin x + \sin^2 x + \sin^3 x = 1$, find the minimu value of $\cot^2 x$.

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111. If $0 \leq \theta < \frac{\pi}{2}$ and $\cos\theta + \cos^2\theta + \cos^3\theta = 1$ find the minimum value of $\tan\theta$.

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112. If $p\sin(\alpha + \beta) = \cos(\alpha - \beta)$, then show that $\frac{1}{1 - p\sin 2\alpha} + \frac{1}{1 - p\sin 2\beta} = \frac{2}{1 - p^2}$.

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113. If $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ and $x\sin\theta - y\cos\theta = \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$, prove that $\frac{x^2}{a} + \frac{y^2}{b} = a + b$.

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114. If $\cot x = \left(a + a^3 + a^3\right)^{\frac{1}{2}}$, $\cot y = \left(1 + a + a^{-1}\right)^{\frac{1}{2}}$ and $\cot x = \left(a^{-1} + a^{-2} + a^{-3}\right)^{\frac{1}{2}}$ prove that $dx + y = z$.

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115. Show that :

$$\frac{1}{\cos\alpha - \cos 3\alpha} + \frac{1}{\cos\alpha - \cos 5\alpha} + \frac{1}{\cos\alpha - \cos 7\alpha} + \ldots + \frac{1}{\cos\alpha - \cos(2n+1)\alpha} = \frac{\operatorname{cosec}\alpha}{2}$$

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116. If $\cos\alpha = \frac{\sin y}{\sin x}, \cos\beta = \frac{\sin z}{\sin x}$ and $\cos(\alpha - \beta) = \sin y \sin z,$ prove that $\tan^2 x = \tan^2 + \tan^2 z.$

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117. Show that the value of $\frac{\tan x + 2\tan 2x}{\tan x}$ cannot lie within 1 and 5.

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118. If $\alpha + \beta = \frac{\pi}{3}(\alpha > 0, \beta > 0)$ find the maximum value of $\tan\alpha\tan\beta.$

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119. Solve $\sqrt{2\cos^2 x + 1} + \sqrt{2\sin^2 x + 1} = 2\sqrt{2}$

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120. If $\tan\alpha = x\tan\beta (x > 0)$, show that $\tan^2(\alpha - \beta) \leq \frac{(x - 1)^2}{4x}$

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121. Prove that $\cos\theta\left(\sin\theta \pm \sqrt{\sin^2\theta + \sin^2\alpha}\right)$ always lies between $\pm\sqrt{1 + \sin^2\alpha}$.

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122. If $\cos^2\theta = \frac{a^2 - 1}{3}$ and $\tan^2\frac{\theta}{2} = \tan^{\frac{2}{3}}\alpha$ prove that $\cos\left(\frac{2}{3}\right)\alpha + \sin^{\frac{2}{3}}\alpha = \left(\frac{2}{a}\right)^{\frac{2}{2}}$

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123. Prove that $\cos^8\frac{\pi}{8}+\cos^8\frac{3\pi}{8}+\cos^8\frac{5\pi}{8}+\cos^8\frac{7\pi}{8}=\frac{17}{16},$

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124. Prove that

$\cos^2 40^\circ \cos^2 80^\circ + \cos^2 80^\circ \cos^2 20^\circ + \cos^2 200^\circ \cos^2 40^\circ = \frac{17}{16}$

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125. If $x+a=a(2\cos\theta-\cos 2\theta)$ and $y=a(2\sin\theta-\sin 2\theta)$,show that $\left(x^2+y^2+2ax\right)^2=4a^2\left(x^2+y^2\right)$.

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126. Show that the value of $\tan\left(\theta+\frac{\pi}{6}\right)\cot\left(\theta-\frac{\pi}{6}\right)$ cannot lie between $\left(2+\sqrt{3}\right)^2$ and $\left(2-\sqrt{3}\right)^2$.

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127. In a triangle of base a, the ratio of the other two sides is $r(<1)$. Show that the altitude of the triangle is less than or equal to $\dfrac{ar}{1-r^2}$

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128. In an acute angled triangle ABC, prove that $\tan^2 a + \tan^2 B + \tan^2 C \geq 9$.

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129. Given $\sin\alpha + \cos\alpha + \tan\alpha + \cot\alpha + \sec\alpha = 7$, find the value of $\sin 2\alpha$.

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130. If $\tan\theta = \sqrt{\dfrac{a-b}{a+b}}\tan\dfrac{A}{2}$ and $\cos\varphi = \dfrac{b + a\cos A}{a + b\cos A}$, show that $\varphi = 2\theta$.

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131. Solve : $\sqrt{\tan\alpha x + \sin x} + \sqrt{\tan x - \sin x} = 2\cos\sqrt{\tan x}$

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132. Determine the smallest positive value of x (in degrees for which)

$\tan\left(x + 100^{\circ}\right) = \tan\left(x + 50^{\circ}\right)\tan x \cdot \tan\left(x - 50^{\circ}\right)$

Watch Video Solution

133. Find the value of $\sin\frac{\pi}{18}\sin\frac{5\pi}{18}\sin\frac{7\pi}{18}$

Watch Video Solution

134. If $\tan\alpha = \frac{x}{y}$, then show that

$x\operatorname{cosec}\frac{\alpha}{3} - y\sec\frac{\alpha}{3} = 3\sqrt{x^2 + y^2}\left(0 < \alpha < \frac{\pi}{2}\right)$

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135. In triangle ABC if $A = 60°$, show that $\cos^2 B + \cos^2 C + \cos B \cos C = \frac{3}{4}$.

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136. If

$\sin x + \sin(x + y)\sin(x + y + z) = 0$ and $\cos x + \cos(x + y) + \cos(x + y + x) = 0.$

Show that $y = z = 120°$.

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137. If $\tan^2 z = \tan(x + y)\tan(x - y)$, show that $\cot^2 y + \cot(z + x)\cot(z - x) = 0$

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138. If $\cos\alpha, \sin\alpha$ and $\cot\alpha$ are in G.P. show that $\tan^6\alpha - \tan^2\alpha = 1$

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139. Solve:

$$4\left[\sin^4\theta + \sin^4\left(\theta + \frac{\pi}{4}\right) + \sin^4\left(\theta - \frac{\pi}{4}\right)\right] = 5$$

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140. Solve:

$$\tan^2\theta + \sec 2\theta = 1$$

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141. Solve :

$$e^{\sin x + \sqrt{3}\cos x - 1} = 1(-2\pi < x < 2\pi)$$

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142. Solve :

$$3^{\sin 2x + 2\cos^2 x} + 3^{1 - \sin 2x + 2\sin^2 x} = 28$$

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143. If $x + y + z = n\pi (n = 0, \pm 1, \pm 2, ...)$ and $\frac{\cos(y+z)}{\cos x} + \frac{\cos(x+y)}{\cos z} = \frac{2\cos(z+x)}{\cos y}$, then show that $\tan x + \tan x = 2\tan y$.

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144. If $\frac{\sin(\theta - \alpha)}{\sin(\theta - \beta)} = \frac{x}{y}$ and $\frac{\cos(\theta - \alpha)}{\cos(\theta - \beta)} = \frac{a}{b}$, then show that $\cos(\alpha - \beta) = \frac{ax + by}{bx + ay}$.

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145. If $\sin^2\theta = \frac{\cos 2\alpha \cos 2\beta}{\cos^2(\alpha + \beta)}$ then, show that one value of $\tan^2\frac{\theta}{2}$ is $\tan\left(\frac{\pi}{4} - \alpha\right)\tan\left(\frac{\pi}{4} + \beta\right)$.

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146. Find all the values of m for which the equation $m\sin\left(\theta + \frac{\pi}{4}\right) = 9 + \sin 2\theta$ is valid.

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147. If $2n\alpha = \frac{\pi}{2}$, then show that

$\tan\alpha\tan 2\alpha\tan 3\alpha \ldots . \tan(2n - 2)\alpha\tan(2n - 1)\alpha = 1$

Watch Video Solution

148. If $(m+2)\sin\theta + (2m-1)\cos\theta = (2m+1)$, then show that $\tan\theta = \ = \frac{4}{3}$ or $\frac{2m}{m^2-1}$.

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ALGEBRA

1. Solve the equation $z^2 + |Z| = 0$ where z is a complex quantity.

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2. Find the real values of θ when $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ will be purely real

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3. Find the real values of θ when $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ will be purely imaginary.

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4. Find the conjugate, modulus and amplitude of the complex number $\frac{\sqrt{3}-i\sqrt{2}}{\sqrt{2}-i\sqrt{2}}$.

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5. Find the modulus -amplitude fomr of the complex number $\left(-2\sqrt{3}-2i\right)$.

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6. If $z=x+iy$, show that $\sqrt{2}|z| \geq |x|+|y|$.

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7. If the modulus of the complex number $a + ib(b \neq o)$ is 1, showthat the complex number can be represented as follows :

$a + ib = \dfrac{c + i}{c - i}$, where c is a real quantity.

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8. find the cube roots of i

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9. If z_1, z_2, z_3,are three complex numbers, prove that $z_1. Im\left(\bar{z}_2 z_3\right) + z_2. Im\left(\bar{z}_3 z_1\right) + z_3 Im\left(\bar{z}_1 z_2\right) = 0$

where $Im(W)$= imaginary part of W,where W is a complex number.

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10. If ω be an imaginary cube root or unity, prove that

$$\frac{1}{1+2\omega} - \frac{1}{1+\omega} + \frac{1}{2+\omega} = 0$$

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11. If ω be an imaginary cube root or unity, prove that

$$\left(x + y\omega + z\omega^2\right)^4 + \left(x\omega + y\omega^2 + z\right)^4 + \left(x\omega^2 + y + z\omega\right)^4 = 0$$

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12. If ω be an imaginary cube root or unity, prove that

$\left(1 - \omega + \omega^2\right)\left(1 - \omega^2 + \omega^4\right)\left(1 - \omega^4 + \omega^8\right)\ldots \text{to} 2nth$ factor $= 2^{2n}$

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13. Prove that $\left(\frac{-1+\sqrt{-3}}{2}\right)^{n}+\left(\frac{-1-\sqrt{-3}}{2}\right)^{n}$

=2, when n is positive integer multipel of 3, $= -1$ when n is positive integer but not a multiple of 3.

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14. If m, n and p are arbitrary integers, show that the equation $x^{3m}+x^{3n+1}+x^{3p+2}=0$ is satisfied by the roots of the equations $x^{2}+x+1=0$

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15. The sides of a right angled triangle are in A.P. ,' show that the sides of the triangle are proportional to the numbers $3, 4, 5$

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16. The sum of first p terms of an A.P. is zero. Show that the sum of a next q terms is $\left[-\dfrac{aq(p+q)}{p-1}\right]$, a being the first term of the A.P.

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17. If $a_1, a_2, \ldots a_k$ are in A.P. then the equation $\dfrac{S_m}{S_n} = \dfrac{m^2}{n^2}$ (where S_k is the sum of the first k terms of the A.P) is satisfied . Prove that $\dfrac{a_m}{a_n} = \dfrac{2m-1}{2n-1}$

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18. If $f(x) = \dfrac{1}{x^2}$, show that $f(x) - f(x+1) = \dfrac{2m-1}{x^2(x+1)^2}$, hence find the the sum of first n terms of the series $\dfrac{3}{1^2 \cdot 2^2} + \dfrac{5}{2^2 \cdot 3^2} + \dfrac{7}{3^3 \cdot 4^2} + \ldots.$

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19. Let t_p be the p th term of an A.P. if its n th tem is zero, show that $t_1 + t_3 + t_5 + \ldots +$ to n th term=0.

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20. Let a,b,c,d, be four quantities such that $a + c = 2b$ and $ab + cd + ad = 3bc(b \neq 0)$. Prove that a,b,c and d are in A.P.

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21. Does there exist a geometric progression containing 27, 8 and 12 as three of its terms? If it exists, how many such progressions are possible ?

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22. If $a = 2$ and $e = 18$, then find three numbers b,c,d betweem a and e such that (i) their sum is 25, (ii) 2,b,c are in A.P. (iii) c,d, 18 are in G.P.

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23. Prove that any sequence of numbers $a_1, a_2, ...a_n$ satisfying the condition,

$$\frac{1}{a_1a_2} + \frac{1}{a_1a_2} + \dots + \frac{1}{a_{n-1}a_n} = \frac{n-1}{a_1a_n}$$

for every $n \geq 3$, is in arithmetic progression.

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24. If $\frac{a-bx}{a+bx} = \frac{b-cx}{b+cx} = \frac{c-dx}{c+dx}$ show that a,b,c , d are in G.P.

Watch Video Solution

25. Prove that

$1 + (1+x) + \left(1+x+x^2\right) + \left(1+x+x^2+x^3\right) + \dots +$ to n terms

$$= \frac{n}{1-x} - \frac{x\left(1-x^n\right)}{(1-x)^2}$$

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26. Show that in A.P. the sum of two terms equidistant from the begining and end is constant. Hence. Prove that

$$\frac{1}{x_1x_n}+\frac{1}{x_2x_{n-1}}+\frac{1}{x_3x_{n-2}}+\ldots+\frac{1}{x_{n-1}x_2}+\frac{1}{x_nx_1}=\frac{2}{x_1+x_n}\left(\frac{1}{x_1}+\frac{1}{x_2}+\frac{1}{x_3}+\ldots+\right.$$

where $x_1, x_2, x_3, \ldots x_n$ are in A.P.

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27. If there be m quantities in a G.P. whose common ratio is r and S_m denote the sum of first m terms, prove that the sum of their products taken two and two together is $\frac{r}{r+1}\cdot S_m\cdot S_{m-1}$.

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28. If a,b,c are in G.P. are $\log_c a, \log_b c, \log_a b$ are in A.P. , prove that the common difference of the A.P. is $\frac{3}{2}$.

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29. Find the sum to n terms of the following series. $5 + 555 + 55555 + \ldots$

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30. If $x \in \mathbb{R}$, solve the inequation $\frac{5x + 8}{4 - x} < 2$

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31. The population of a twon at the beginning of a year is N and the figure becomes $a + bN$ at the end of the year. Show that the population of the town at the end of k years will be $\frac{a}{1 - b} + \left(N - \frac{a}{1 - b}\right)b^k$.

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32. Examine the validity of the following statement:

If a and b ar odd integers then ab is an odd integer.

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33. Show that the total number of permutations of n diferent things not more than r things at a time (repetition being allowed) is $\frac{n\left(n^r - 1\right)}{n - 1}$.

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34. A and B stand in a line with 10 other people. In how many of the permutations there are 3 people between A and B ?

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35. Find the number of permutations of the letters x, x, x, y, y, y, z, w taken 5 at a time.

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36. A bag contains 50 tickets $1, 2, 3, \ldots 50$ of which 5 are drawn at random and aranged in ascending order of their numbers $x_1 < x_2 < x_3 < x_4 < x_5$. Find the number of selections so that $x_3 = 30$

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37. In $n > 7$ prove that ${}^{n-1}C_3 + {}^{n-1}C_4 > {}^{n}C_3$

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38. If $\frac{{}^{n}C_{r-1}}{a} = \frac{{}^{n}C_{r}}{b} = \frac{{}^{n}C_{r}}{c}$ prove that $n = \frac{ab + 2ac + bc}{b^2 - ac}$ and $r = \frac{a(b + c)}{b^2 - ac}$.

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39. A,B and C have respectively 4,3 and 2 differrent books. In how many different ways can they interchange the books among themselves, without altering the total number initially possessed by each ?

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40. Find the modulus and ampltidue of the complex in number

$$z = \frac{-2 - i2\sqrt{3}}{\sqrt{3} - i}$$

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41. Find the amplitude of

$$z = 1 + i\tan\frac{3\pi}{5}$$

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42. Find the modulus and amplitude of

$$z = \sin\frac{6\pi}{5} + I\left(1 + \cos\frac{6\pi}{5}\right)$$

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43. There are m A.P. Whose common differences are 1,2,3,.. M respectively, the first term of each being unity. Show that the sum of their nth terms is $\frac{m}{2}(mn - m + n + 1)$.

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44. If $S_1, S_2, S_3, \ldots S_m$ be the sum of first n terms of n A.P. Whose first terms are $1, 2, 3, \ldots M$ aand common differnece are $1, 3, 5, \ldots(2m - 1)$ respectively, show that $S_1 + S_2 + S_3 + \ldots + S_m = \frac{mn}{2}(mn + 1)$.

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45. Show that the sum of the numbers in the n th bracket of the series

$(1) + (3 + 5) + (7 + 9 + 11) + \ldots + \text{is} n^3$.

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46. Show that the following two A.P. is have 14 common terms

$3, 7, 11, 15 \ldots, 407$ and $2, 9, 16, 23, \ldots, 709$,

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47. Show that there are 17 identical terms in the following two

$A. P.\ 2, 5, 8 \ldots$ To 60th term and $3, 5, 7, \ldots$ to 50 th term.

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48. Find the n th term and the sum of first n terms of thefollowing

sequence $2, 7, 20, 57, 166, \ldots$

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49. Solve $\left|x^2 + 4x + 3\right| + 2x + 5 = 0$

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50. For $a > 0,$ determine all the roots of the equations $x^2 - 2a|x - a| - 3a^2 = 0$

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51. How many five -digit numbers divisible by 3 can do formed using the digits $0, 1, 2, 3, 4$ and 5 when no digit is repeated ?

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52. How many five-digit telephone numbers with pairwise distinct digits can be formed ?

Watch Video Solution

53. Five speakers A,B,C,D and E will speark at a meeting. In how many ways can they take their turrns if (i) A speask immediately before B, and (ii) B does not speak befor A?

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54. If $z_1 = 3i$ and $z_2 = -1 - i$, where $i = \sqrt{-1}$ find the value of arg $\left(\frac{z_1}{z_2}\right)$

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55. Show that the number of solutions of equations $|x|^2 - 3|x| + 2 = 0$ is 4.

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56. Show that $\frac{1}{2}\left[(x-y)^4+(y-z)^4+(z-x)^4\right]$ is perfect square.

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57. In an arithmetic progression of n terms (n is even) , the two middle terms are p-q,p+q respectively. Prove that the sum of the squares of all the term of the progression is n $\left[p^2+\frac{n^2-1}{3}q^2\right]$.

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58. In an A.P. fo n terms (n is even), the two middle terms are $(\alpha-\beta),(\alpha+\beta)$ respectively. Show that the sum of the cubes of all the terms of the A.P. is $n\alpha\left[\alpha^2+\left(n^2-1\right)\beta^2\right]$

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59. Prove that the ineuations $\frac{2x+1}{7x-1} > 5$ and $\frac{x+7}{x-8} > 2$ have no solutions

.

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60. If the argument of $(z-a)\left(\bar{z}-b\right)$ is equal to that of $\frac{\left(\sqrt{3}+i\right)\left(1+\sqrt{3}i\right)}{1+i}$, where a,b are two real numbers and $\bar{z}$ is the complex conjugate of the Argand diagram. Find the values of a and b so that be locus becomes a circle having its centre at $\frac{1}{2}(3+i)$.

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61. Which term of the series $3+4+6+9+13+18+\ldots$ is 5053?

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62. z_1 and $z_2(\neq z_1)$ are two complex numbers such that $|z_1| = |z_2|$. Sho

that the real part of $\dfrac{z_1 + z_2}{z_1 - z_2}$ is zero.

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63. If $x = \cos\theta + I\sin\theta$ and $y = \cos\psi + I\sin\psi$, then show that

$\dfrac{x}{y} + \dfrac{y}{x} = 2\cos(\theta - \psi)$.

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64. If $\omega$is an imaginary cube root of unity, show that

$\left(x + \omega + \omega^2\right)\left(x - \omega^2 - \omega^4\right)\left(x + \omega^4 + \omega^8\right)\left(x - \omega^8 - \omega^{16}\right)\ldots$ to 2n factors

$= \left(x^2 - 1\right)^n$

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65. If $\omega$is an imaginary cube root of unity, show that

$$\frac{a + b\omega + c\omega^2}{b + c\omega + a\omega^2} + \frac{a + b\omega + c\omega^2}{c + av + b\omega^2} = -1$$

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66. If $\omega$is an imaginary cube root of unity, show that

$$\frac{\omega}{9}\left[(1 - \omega)\left(1 - \omega^2\right)\left(1 - \omega^4\right)\left(1 - \omega^8\right) + 9\left(\frac{c + a\omega + b\omega^2}{a\omega^2 + b + c\omega}\right)^2\right] = -1$$

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67. Find the value of $4 + 5\left[-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right]^{344} + 3\left[-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right]^{365}\left(i = \sqrt{-1}\right)$

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68. If $x = \omega^{3}\sqrt{y} + \omega + \omega^{23}\sqrt{z}$, then prove that

$\left(x^3 - y - z\right)^3 = 27x^3yz$

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69. Find the square root of $\dfrac{a^2}{b^2} + \dfrac{b^2}{a^2} + \dfrac{1}{2i}\left(\dfrac{a}{b} + \dfrac{b}{a}\right) + \dfrac{31}{16}$.

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70. z_1 and z_2 are two non-zero complex numbers such that $z_1^2 + z_1z_2 + z_2^2 = 0$. Prove that the ponts z_1, z_2 and the origin form an isosceles triangle in the complex plane.

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71. Observing that $1^3 = 1, 2^3 = 3 + 5, 3^3 = 7 + 9 + 11, 4^3 = 13 + 15 + 17 + 19$, find a general formula for the cube of natural number n.

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72. Obtain the sum of $\frac{1}{x+1}+\frac{2}{x^2+1}+\frac{4}{x^4+1}+\ldots+\frac{2^n}{x^{2^n}+1}$

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73. Let A and B be two comoplex numbers such that $\frac{A}{B}+\frac{B}{A}=1$. Prove that the origin and the two points represented by A and B form vertices of an equilaterla triangle.

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74. Show that the complex numbers z_1, z_2, z_3 satisfying $\frac{z_1-z_3}{z_2-z_3}=\frac{1-\sqrt{3}}{2}$ are the verticels of an equilaterla triangle.

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75. If z_1, z_2, z_3 are complex numbers such that $|z_1| = |z_2| = |Z_3| = \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = 1$, then find $|z_1 + z_2 + z_3|$.

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76. Show that $(666...n\text{times})^2$+(888.... N times) =(444.... 2n times).

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77. Let $z_1 = 10 + 6i$ and $z_2 = 4 + 6i$. If z is complex number such that the argument of $\frac{z - z_1}{z - z_2}$ is $\frac{\pi}{4}$ then prove that $|z - 7 - 9i| = 3\sqrt{2}$.

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78. z_1 and z_2 are two non-zero complex numbers, they form an equilateral triangle with the origin in the complex plane. Prove that $z_1^2 - z_1z_2 + z_2^2 = 0$

Watch Video Solution

79. If the complex numbers z_1, z_2, z_3 represents the vertices of an equilaterla triangle such that $|z_1| = |z_2| = |z_3|$, show that $z_1 + z_2 + z_3 = 0$

Watch Video Solution

80. If ω is an imaginary cube root of unity. Find the value of the expression $1(2 - \omega)\left(2 - \omega^2\right) + 2(3 - \omega) + \dots + (n - 1)(n - \omega)\left(n - \omega^2\right)$.

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81. Find the sum of all multiples 3 or 4 between 1 and 325.

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82. The vertices A,B,C of an isosceles right angled triangle with right angle at C are represented by the complex numbers $z_1 z_2$ and z_3 respectively. Show that $\left(z_1 - z_2\right)^2 = 2\left(z_1 - z_3\right)\left(z_3 - z_2\right)$.

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83. If $n \geq 0$ is integer, using induction method prove that,

$$\frac{1}{1+x} + \frac{2}{1+x^2} + \frac{4}{1+x^4} + \ldots + \frac{2^n}{1+x^{2n}} = \frac{1}{x-1} + \frac{2^{n+1}}{1-x^{2^{n+1}}}.$$

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84. Find the differential coefficient of:

x^{5x}

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85. If $x + y = a + b$ and $x^2 + y^2 = a^2 + b^2$, then by mathematical induction prove that $x^n + y^n = a^n + b^n$. For all $n \in \mathbb{N}$.

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86. By the principle of mathematical induction prove that, $\left(2^{2^n} + 1\right)$ has 7 in unit's place for all integer $n \geq 2$.

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87. Find the differential coefficient of:

$x^9 - 5x^2 + 3x$

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88. Find the solution set of inequation $\frac{x - 2}{x + 5} > 2$.

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89. Find the differential coefficient of:

$x^5 + 4x$

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90. Find the differential coefficient of:

x^{2x}

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91. Using mathematical induction prove that,

$\tan^{-1}\frac{x}{1 \cdot 2 + x^2} + \tan^{-1}\frac{x}{2 \cdot 3 + x^2} + \ldots + \tan^{-1}\frac{x}{n \cdot (n+1) + x^2} + \tan^{-1}x - \tan^{-1}\frac{1}{n +}$

.

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92. Using mathematical induction prove that,

$$\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} + \ldots + \tan^{-1}\frac{1}{n^2 + n + 1} = \tan^{-1}(n + 1) - \frac{\pi}{4}\text{for all } n \in \mathbb{N}.$$

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93. Using mathematical induction prove that,

$$\tan^{-1}\left(\frac{1}{2 \cdot 1^2}\right) + \tan^{-1}\left(\frac{1}{2 \cdot 2^2}\right) + \ldots + \tan^{-1}\left(\frac{1}{2 \cdot n^2}\right) = \tan^{-1}(2n + 1) + \frac{\pi}{4}, \text{ for all}$$

.

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94. If $\left(1 + x + x^2\right)^n = a_0 + a_1 + x + a_2x^2 + \ldots + a_{2n}x^{2n}$, show that $a_0 + a_3 + a_6 + \ldots = a_1 + a_4 + a_7 + \ldots. + = a_2 + a_5 + a_8 + \ldots 3^{n-1}.$

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95. If $(1+x)^n = C_0 + C_1 + x + C_2x^2 + \dots + C_nx^n$

Find the value of $C_1 + 2^2 \cdot C_2x + 3^2 \cdot C_3x^2 + \dots + n^2 \cdot C_nx^{n-1}$ and hence find the sum of $C_1 - 2^2 \cdot C_2 + 3^2 \cdot C_3 - \dots. + (-1)^{n-1} \cdot n^2 \cdot C_n$

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96. If $(1+x)^n = C_0 + C_1 + x + C_2x^2 + \dots + C_nx^n$

Show that $C_1 - \frac{1}{2} \cdot C_2 + \frac{1}{3} \cdot C_3 - \dots + (-1)^{n-1} \cdot \frac{1}{n}C_n = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}$

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97. If $(1+x)^n = C_0 + C_1 + x + C_2x^2 + \dots + C_nx^n$

Show that $1^2 \cdot C_1^2 + 2^2 \cdot C_2 \cdot + 3^2 \cdot C_3 + \dots + n^2 \cdot c_n^2 = n^2 \cdot \frac{(2n-2)!}{[(n-1)!]^2}$

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98. If $(1+x)^n = C_0 + C_1 + x + C_2x^2 + \ldots + C_nx^n$

Show that $C_0^2 + 2 \cdot C_1^2 + 3 \cdot C_2^2 + \ldots + (n+1) \cdot C_n^2 = \dfrac{(n+2)(2n-1)!}{n!(n-1)!}$

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99. If $(1+x)^n = C_0 + C_1 + x + C_2x^2 + \ldots + C_nx^n$

Show that $\dfrac{2^2 \cdot C_0}{1 \cdot 2} + \dfrac{2^3 \cdot C_1}{2 \cdot 3} + \ldots. + \dfrac{2^{n+2}C_n}{(n+1)(n+2)} = \dfrac{3^{n+2} - 2n - 5}{(n+1)(n+2)}$

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100. If $(1+x)^n = C_0 + C_1 + x + C_2x^2 + \ldots + C_nx^n$

For all values of a show that

$$\frac{C_0}{a} - \frac{C_1}{a+a} + \frac{C_2}{a+2} - \ldots + (-1)^n \frac{C_n}{a+n} = \frac{n!}{a(a+1)(a+2)\ldots(a+n)}$$

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101. If $(1+x)^n = C_0 + C_1 + x + C_2x^2 + \ldots + C_nx^n$

Show that $C_1^2 + 2 \cdot C_2^2 + 3 \cdot C_3^2 \ldots . + n \cdot C_n^2 = \dfrac{(2n-1)!}{[(n-1)!]^2}$

Watch Video Solution

102. If $(1+x)^n = C_0 + C_1 + x + C_2x^2 + \ldots + C_nx^n$

show that

$$\left(C_0\right)^2 + \left(\frac{1}{2} \cdot C_1\right)^2 + \left(\frac{1}{3} \cdot C_2\right)^2 + \left(\frac{1}{4} \cdot C_3\right)^2 + \ldots + \left(\frac{1}{n+1} \cdot C_n\right)^2 = \frac{1}{(n+1)^2} \cdot$$

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103. Show that the sum of the coefficient of first $(r+1)$ terms in the expansions of $(1-x)^{-n}$ is $\dfrac{(n+1)(n+2)\ldots(n+r)}{r!}$

Watch Video Solution

104. Show that $4^{m}\left[1+\frac{m}{2}+\frac{m(m+1)}{2\cdot 4}+\frac{m(m+1)(m+2)}{2\cdot 4\cdot 6}+...\right]$

$7^{m}\left[1+\frac{m}{7}+\frac{m(m-1)}{7\cdot 14}+\frac{m(m-1)(m+2)}{7\cdot 14\cdot 21}+...\right]$

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105. It is given that n is an odd integer greater than 3 but n is not multiple of 3. prove that $\left(x^{3}+x^{2}+x\right)$ is a factor of $(1+x)^{n}-x^{n}-1$.

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106. If $(1+x)^{n}=C_{0}+C_{1}+x+C_{2}x^{2}+...+C_{n}x^{n}$ show that,

$1^{2}\cdot C_{1}+2^{2}\cdot C_{2}+3^{2}\cdot C_{3}+...+n^{2}\cdot C_{n}=n(n+1)2^{n-2}$,

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107. If $(1 + x)^n = C_0 + C_1 + x + C_2x^2 + \ldots + C_nx^n$ show that,

$C_0 - 2^2 \cdot C_1 + 3^2 \cdot C_2 - \ldots + (-1)^n \cdot (n + 1)^2 \cdot C_n = 0(n > 2)$

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108. Using binomial theorem for a positive integral index, prove that $\left(2^{3n} - 7n - 1\right)$ is divisble by 49, for any positive integer n.

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109. Find the largest coefficient in the expansion of $(1 + x)^n$, given that the sum of coefficients of the terms in its expansion is 4096.

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110. Given that the 4th term in the expansion of $\left(2 + \frac{3}{8}x\right)^{10}$ has the maximum numerical value, find the range of x for which the statement will be true.

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111. If $x \in \mathbb{R}$ then find the solution set of the inquation $|3x - 7| > 4$.

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112. Solve the following quadratic equation: $x^2 - (5 - i)x + (18 + i) = 0$

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113. For every natural number n, show that $7^{2n} + 2^{3n-3} \cdot 3^{n-1}$ is always divisible by 25.

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114. If the coefficients of the $(r - 1)$ th and $(2r + 3)$ th terms in the equation of $(1 + x)^{15}$ are equal, find the value of r.

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115. If n is a positive integer then using the indentiy $(1 + x)^n = (1 + x)^3(1 + x)^{n-3}$, prove that

$${}^nC_r = {}^{n-2}C_r + 3 \cdot {}^{n-3}C_{r-1} \cdot {}^{n-3}C_{r-2} + {}^{n-3}C_{r-3}$$

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116. If $S_1, S_2, S_3, \dots S_n$ are the sums of infinite geometric series, whose first terms are $1, 2, 3, \dots n$ whose ratios are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots \frac{1}{n+1}$ respectively, then find the value of $S_1^2 + S_2^2 + S_3^2 + \dots + S_{2n-1}^2$.

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117. If $u_r = \frac{1}{r!}$ then show that

$$u_0 u_n + 8u_1 + u_{n-1} + 8^2 u_2 u_{n-2} + \dots + 8^n u_n u_0 = \frac{3^{2n}}{n!}$$

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118. a series is given by the following form :

$1 + \left(x + x^2\right) + \left(x^3 + x^4 + x^5\right) + \left(x^6 + x^7 + x^8 + x^9\right) + \dots +$ find the first term and sum of the terms in the n th bracket.

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119. If $x = \cos\theta + i\sin\theta$ and $1 + \sqrt{1 - a^2} = na$, show that,

$$\frac{a}{2n}(1 + nx)\left(1 + \frac{n}{x}\right) = 1 + a\cos\theta$$

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120. Show that $\left(a+\omega+\omega^2\right)\left(a+\omega^2+\omega^4\right)\left(a+\omega^4+\omega^8\right)$... upto 2n factosrs $=(a-1)^{2n}$.

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121. Let z and ω tow complex number. If $|z|=|\omega|$ and areg (z) +arg (ω) $=\pi$ then show that $z=-\bar{\omega}$.

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122. If z_1+z_2 are two complex number and $\left|\frac{\bar{z}_1-2\bar{z}_2}{2-z_1\bar{z}_2}\right|=1, \left|z_1\right|\neq$, then show that $\left|z_1\right|=2$.

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123. If $x + iy$ moves on the line $3x + 4y + 5 = 0$, prove that the minimum value of $|x + iy|$ is 1.

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124. If $\left|z^2\right| = 9|z|$, find the maximum and minimum values of $|z|$.

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125. Find the greatest value of $|z|$ when $\left|z - \frac{6}{z}\right| = 2$, z being a complex number.

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126. Find the least value of $\left|z + \frac{1}{z}\right|$ if $|z| \geq 3$ (z being a complex number)

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127. If the roots of the equation $ix^2 - (b^2 + c^2)x + (c^2 + d^2)^2 = 0$ be in the ratio $3:4$, show that $\frac{(b^2 + c^2)^2}{(c^2 + d^2)^2} = \frac{49i}{12}$

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128. If one root of the equation $ax^2 - 2ibx - ic = 0$ be square of the other, then prove that $i = \frac{c^2a - 6abc}{8b^3 - ca^2}$.

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129. Solve

$$\frac{7y^2 + 1}{y^2 - 1} - 4\left(\frac{y^2 - 1}{7y^2 + 1}\right) = -3$$

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130. Solve

$$\left(x - \frac{x}{x+1}\right)^2 + 2x\left(\frac{x}{x+1}\right) = 3$$

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131. If α and β are the roots of $ax^2 + bx + cid = 0$ find the equation whose rootsare α^{-3}, β^{-3}.

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132. Show that,

$$3^n - \frac{n}{1!}3^{n-1} + \frac{n(n-1)}{2!}3^{n-2} - \dots + (-1)^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_2$$

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133. Find the coefficient of x^{-5} in

$${}^nC_0 - {}^nC_1\left(\frac{2x-1}{x}\right) + {}^nC_2\left(\frac{2x-1}{x}\right)^2 - \dots + (-1)^n\left(\frac{2x-1}{x}\right)^n$$

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134. Find the degree of theexpansion,

$$\left\{x+\left(x^3-1\right)^{1/2}\right\}^5+\left\{x-\left(x^3-1\right)^{1/2}\right\}^5$$

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135. Find the term independent of x in the expansion,

$$\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}$$

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136. Find the remainder if number 2^{2003} is divided by 17.

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137. Find last three digits of the number 17^{256}.

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138. Find the sum of the series,

$\frac{1^2}{3} + \frac{1^2 + 2^2}{5} + \frac{1^2 + 2^2 + 3^2}{7} + \dots$ upto n th term.

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139. Find the value of $\sum_{r=1}^{n} \frac{r}{1 + r^2 + r^4}$

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140. If $a_1, a_2, a_3, \dots$ from a G.P. with common ratio r, find in terms of r and a_1 the sum of $a_1 a_2 + a_2 a_3 + \dots + a_n + a_{n+1}$

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141. Show that $\frac{111...1}{91\text{digits}}$ is a composite number.

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142. If $A = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$,prove by induction that, $A^n = \begin{pmatrix} 1+2n & -4n \\ n & 1-2n \end{pmatrix}$ where n is a positive integer.

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143. If A = $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, prove that, $A^2 = \begin{bmatrix} 2^2 & 0 & 0 \\ 0 & 3^2 & 0 \\ 0 & 0 & 5^2 \end{bmatrix}$, hence by induction method show that, $A^n = \begin{bmatrix} 2^n & 0 & 0 \\ 0 & 3^n & 0 \\ 0 & 0 & 5^n \end{bmatrix}$

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144. Solve by Cramer's rule : ax+by+cz=1,

cx+ay+bz=0, bx+cy+az=0, given that, A,B,C are the cofactors of the elemets

a,b,c in D where

$$D = a\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$$

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145. Without expanding prove that,

$$\begin{vmatrix} 1 & bc & b+c \\ 1 & ca & c+a \\ 1 & ab & a+b \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

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146. If D= $\begin{vmatrix} 1 & \sin a & 1 \\ -\sin a & 1 & \sin a \\ -1 & -\sin a & 1 \end{vmatrix}$, show that, $2 \le D \le 4$.

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147. Prove that,

$$\begin{vmatrix} -2a & a & a \\ -2b & b & b \\ c+a & b+c & -2c \end{vmatrix} = 0$$

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148. Prove that,

$$\begin{vmatrix} (b+c)^2 & c^2 & b^2 \\ c^2 & (c+a)^2 & a^2 \\ b^2 & a^2 & (a+b)^2 \end{vmatrix} = 2(bc+ca+ab)^3$$

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149. If $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix} \times \begin{vmatrix} \alpha - i\beta & \gamma - i\delta \\ -\gamma - i\delta & \alpha + i\beta \end{vmatrix} = \begin{vmatrix} A - iB & C - iD \\ -C - iD & A + iB \end{vmatrix}$, write down the values of A, B, C, $\left(i = \sqrt{-1}\right)$. Hence show that, the product of two sums, each of four squares, can be expressed as the sum of four squares.

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150. If $s_r = x^r + y^r + z^r$, prove by considering the square of the determinant $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix}$ that $\begin{vmatrix} s_0 & s_1 & s_2 \\ s_1 & s_2 & s_3 \\ s_2 & s_3 & s_4 \end{vmatrix} = (x - y)^2(y - z)^2(z - x)^2$

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151. Prove that for all values of θ,

$$\begin{vmatrix} \sin\theta & \cos\theta & \sin 2\theta \\ \sin\left(\theta + \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) & \sin\left(2\theta + \frac{4\pi}{3}\right) \\ \sin\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta - \frac{2\pi}{3}\right) & \sin\left(2\theta - \frac{4\pi}{3}\right) \end{vmatrix} = 0$$

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152. Prove that,

$$\begin{vmatrix} \sin^3 A & \cos A & \sin A \\ \sin^3 B & \cos B & \sin B \\ \sin^3 C & \cos C & \sin C \end{vmatrix}$$

=sin(A-B)sin(B-C)sin(C-A)sin(A+B+C).

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153. Find the inverse of the matrix $B = \begin{bmatrix} 4 & -2 \\ 0 & 5 \end{bmatrix}$. Hence, find a matrix A such that, $AB + \begin{bmatrix} -1 & 3 \\ -9 & 6 \end{bmatrix} = \begin{bmatrix} 3 & 16 \\ 7 & 8 \end{bmatrix}$.

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154. If $A = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $B = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, show that,

$A^2 + B^2 + C^2 = \frac{3}{2}I$.

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155. Prove that,

$$\begin{bmatrix} -bc & b^2 + bc & c^2 + bc \\ a^2 + ac & -ac & c^2 + ac \\ a^2 + ab & b^2 + ab & -ab \end{bmatrix} = (ab + bc + ca)^3$$

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156. Without expanding prove that

$$\begin{bmatrix} b^2 - ab & b - c & bc - ac \\ ab - a^2 & a - b & b^2 - ab \\ bc - ac & c - a & ab - a^2 \end{bmatrix} = 0$$

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157. If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}, B = \begin{bmatrix} x & 1 \\ y & -1 \end{bmatrix}$ and $(A + B)^2 = A^2 + B^2$, find x and y.

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158. Using matrices , solve that following system of equations:

8x+4y+3z=18, 2x+y+z=5, x+2y+x=5.

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159. Using elementary row operations, find the inverse of the matrix

$\begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$.

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160. Using elementary row operations, find the inverse of the matrix

$$\begin{bmatrix} 1 & 3 & 2 \\ -3 & -3 & -1 \\ 2 & 1 & 0 \end{bmatrix}.$$

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161. Find the inverse of the following matrix using elementary operations :

$$A=\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

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162. Using properties of determinants prove that,

$$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$$

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163. Using properties of determinants, prove that,

$$\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 1+3p+2q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix} = 1$$

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164. By using properties of determinants, prove that,

$$\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(x-4)^2$$

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165. Let $A = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}$. Express A as sum of two matrices such that one is symmetric and the other is skew-symmetric.

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COORDINATE GEOMETRY (TWO DIMENSIONAL COORDINATE GEOMETRY)

1. If O be the origin and if coordinates of any two points Q_1 and Q_2 be (x_1, y_1) and (x_2, y_2) respectively, prove that

$OQ_1 \cdot OQ_2 \cos \angle Q_1OQ_2 = x_1x_2 + y_1 + y_2.$

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2. The ends of a rod of length l move on two mutually perpendicular lines. Find the locus of the point on the rod which divides it in the ratio $1:2$.

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3. If the coordinates of the vertices A,B and C of ΔABC be (x_1, y_1), (x_2y_2) and (x_3, y_3) respectively, then show that the coordinates of the in-centre of the triangle are $\left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}\right)$ where $BC = aCA = b$ and $AB = c$

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4. Prove that if $P(x, y)$ be any point on the line segment joining $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ then

$x = x_1 + k(x_2 - x_1)$ and $y = y_1 + k(y_2 - y_1)$, where $\overline{P_1P}, \overline{P_1P_2} = k$

What conclusion can you draw about the position of P if

$0 < k < 1$

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5. Prove that if $P(x, y)$ be any point on the line segment joining $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ then

$x = x_1 + k(x_2 - x_1)$ and $y = y_1 + k(y_2 - y_1)$, where $\overline{P_1P}, \overline{P_1P_2} = k$

What conclusion can you draw about the position of P if

$k > 1$

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6. Prove that if $P(x, y)$ be any point on the line segment joining $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ then

$x = x_1 + k(x_2 - x_1)$ and $y = y_1 + k(y_2 - y_1)$, where $\overline{P_1P}, \overline{P_1P_2} = k$

What conclusion can you draw about the position of P if

$k = 0$

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7. Prove that if $P(x, y)$ be any point on the line segment joining $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ then

$x = x_1 + k(x_2 - x_1)$ and $y = y_1 + k(y_2 - y_1)$, where $\overline{P_1P}, \overline{P_1P_2} = k$

What conclusion can you draw about the position of P if

$k = 1$

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8. Prove that if $P(x, y)$ be any point on the line segment joining $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ then

$x = x_1 + k(x_2 - x_1)$ and $y = y_1 + k(y_2 - y_1)$, where $\overline{P_1P}, \overline{P_1P_2} = k$

What conclusion can you draw about the position of P if

$k = \frac{1}{2}$?

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9. (3,2) and (-3,2) are the vertices of an equilateral triangle which contains the origin within it, what are the coordinates of the third vertex ?

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10. Show that the points $A(a, b), B(a + \alpha + b, \ + \beta), C(a + \alpha + p, b + \beta + q)$ and $D(a + p, b + q)$ when joined in order, form a parallelogram. Find the cooditions for which the paralleogram is a(i)rectangle (ii) rhombus.

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11. Prove that it is impossible to have an equilateral triangle for which of the coordinates of the vertices are all rational numbers.

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12. A straight line moves in such a way that the algebraic sum of the perpendicular distance on it form the vertices of a given triangle is always zero. Show that the stright line always passes through the centroid of the triangle.

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13. The coordinates of two point opposite vertices oc a square are (3,4) and (1,-1), find the coordinates of the other two vertices.

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14. If L,M,N divide the sides BC,CA and AB of a triangle ABC in the same ratio, then show that the traiangle ABC and dLMN have the same centroid.

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15. Find the equation to the locus represented by $x = \frac{2 + t + 1}{3t - 2}, y = \frac{t - 1}{t + 1}$ wher t is variable parameter.

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16. Find the equation to the locus represented by the parametric equations $x = 2t^2 + t + 1, y = t^2 - t + 1$.

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17. If t is a variable parameter, then the equation to the locus defined by the equations $x = 2(\sec t + \tan t) - 1$ and $y = 2(\sec t - \tan t) - 2$ is-

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18. The four point $A(a, 0), B(b, 0), C(c, 0)$ and $D(d, 0)$ are such that a and b are the roots of the equation $p_1x^2 + 2q_1x + r_1 = 0$ and c and d are the roots of the equation $P_2x^2 + \; + 2q_2 + r_2 = 0$. Show that the sum of the equation $p_2x^2 + 2q_2x + r_r0$. Show that the sum of the ratios in which C and D divide AB is zero if $p_1r_2 + p_2r_1 = 2q_1q_2$.

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19. The straight lines $y = x = 2$ and $y = 6x + 3$ are parallel to two sides of the rhombus ABCD. If the vertex A lies on y-axis and the diagonals of the rhombus intersect at (1,2) find the coordinat of A.

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20. The eqution of the side BC,CA and AB of the triangle ABC are $u_1 = a_1x + b_1y + c_1 = 0.\ u_2 = a_2x + b_2y + c_2 = 0$ and $u_3 = a_3x + b_3y + c_3 = 0$ respectively. Prove that the eqution of the straight line through A and paralle to BC is.

$\left(a_3b_1 - a_1b_3\right)u_2 + \left(a_1b_2 - a_2b_1\right)u_3 = 0$

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21. A striaght line intersect x-axis at A(7,0) and y-axis at B(0,-5). The straight line PQ is perpendicular to AB and intersect the x and y-axes at P adn Q respectively. Find the eqution to the locus of the point of intersection of the line AQ and BP.

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22. The equation of two diameters of a circle of area 154 sq unit are $2x - 3y = 5$ and $3x - 4y = 7$. Find the equation of the circle.

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23. If the four points $\left(m_i, \frac{1}{m_i}\right)$ where $m_1 > 0(I, 1, 2, 3, 4)$ are concyclic, then prove that $m_1 m_2 m_3 m_4 = 1$.

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24. The ends A and B of a rod AB of length 8 unit slide along the lines $y = 2$ and $x = 4$ respectively. Find the equation to the locus of the mid point of the rod.

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25. A is a point on the circle $x^2 + y^2 = 36$. Find the locus of the point P which divides the ordinate $\overline{AN}$ internally in the ratio $2:1$.

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26. The coordinates of the points A and B are (1,3) and (-2,1) respectively and the point P lies on the line $x + 7y + c = 0$ and $a' + x + b' + c' = 0$. Hence find the condition that the diagonals are perpendicular to one another.

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27. Find the equation of the diagonals of the parallelogram formed by the lines $ax + by$

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28. Show that the lines $y = 0, \sqrt{3y} - xy = \sqrt{3}$ and $\sqrt{3y} + x - 10 = 0$ form a cyclic trapezium. Determine the centre and the radius of the circle and also the area of the trapezium.

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29. Prove analytically that the perpendicular bisector of the sides of a triangle ar concurrent.

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30. The coordinates of the vertex A of the triangle ABC are (-5,2) the equation of AC is $4x + 3y + 14 = 0$ and the alttitude of the point A meets BC at the point D(3,-2). If $\angle DAB = 45^\circ$, find the equastion and gradient of AB, the coordinates of B and C and the ratio $BD : DC$.

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31. The coordinates of the vertices of a triangle are $\left[at_1t_2, a\left(t_1 + t_2\right)\right]$, $\left[at_2, t_3, a\left(t_2 + t_3\right)\right]$ and $\left[at_3t_1, t_3 + t_1\right)\right]$. Find the coordinates of the ortocentre of the triangle.

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32. Prove that diagonals of the parallelogram formed by the four straight lines $\sqrt{3}x + y = 0$, $\sqrt{3}y + x = 0$, $\sqrt{3}x + y = 1$ and $\sqrt{3}y + x = 1$ are the right angles to one another.

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33. Find the locus of the foot of the perpendicular from the origin upon a stright line which always passes through the point (2,3).

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34. Show that for all values of a,b and c the line $(b - c)x + (c - a)y + a - b = 0$ always passes through the point (2,3).

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35. Show that the locus of the point of intersection of the stright lines $x\sin\theta - y(\cos\theta - 1) = a\sin\theta$ and $x\sin\theta - y(\cos\theta + 1) + a\sin\theta = 0$ is a circle, find the equation of the circle.

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36. Prove analytically that in any triangle the perpendiculars drawn from the vertices upon the opposite sides are concurrent.

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37. Prove analytically that the bisectors of the interior angles of a triangle are concurrent.

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38. If d_1, d_2, d_3 be the distances of a fixed point from the straight lines $x\cos\alpha + y\sin\alpha_1 = p_1 x\cos\alpha_2 + y\sin\alpha_2 = p_2$ and $x\cos\alpha_3 = P_3$ respectively, show that

$$\left(d_1 + p_1\right)\sin\left(\alpha_2 - \alpha_3\right) + \left(d_2 + p_2\right)\sin\left(\alpha_3 - \alpha_1\right) + \left(d_3 p_3\right)\sin\left(\alpha_1 - \alpha_2\right) = 0$$

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39. Show that the quadrilateral formed by the four lines $y = mx + a\sqrt{a + m^2}, x + a\sqrt{1 + m^2}, y = mx - a\sqrt{1 + m^2}$ and $y = mx - a\sqrt{1 + m^2}$ is a square of rhombus accroding as $mm = -$ or $mm \neq -1$

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40. Show that the lines $x\cos\alpha + y\alpha = p, x\cos\left(\alpha + \frac{2\pi}{3}\right) + y\sin\left(\alpha + \frac{2\pi}{3}\right) = p$ and $x\cos\left(\alpha - \frac{2\pi}{3}\right) + y\sin\left(\alpha -$ from an equilaterla triangle.

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41. $2x - y + 4 = 0$ is a diameter of a circle which circumscribes a rectangle ABCD. If the coordinates of A and B are (4,6) and (1,9) resepectively. Find the area of ABCD.

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42. ABC is right-angle triangle, right -angled at A. The coordinates of B and C are (6,4) and (14,10) respectively. The angle between the side AB and x-axis is 45°. Find the coordinates of A.

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43. The line through $A(a\cos\theta, 0)$ and perpendicular to the x-axis meets the lines $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ and $x\cos\theta + y\sin\theta = a$ at B and C respectively. Show that $AC : AB = a : b$.

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44. P and Q are two point on the line $x - y + 1 = 0$, if O is the origin and $\overline{OP} = \overline{OQ} = 5$ unit, find the area of ΔOPQ.

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45. O(0,0) and A(4,4) are two given points and B is any point. Find the equation to the locus of the point of intersection of the perpendicular bisector of $\overline{OB}$ and $\overline{AB}$.

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46. Show that the image of the point (h,k) with respect to the striaight line $x\cos\alpha + y\sin\alpha = p$ is the point $(2p\cos\alpha - h\cos2\alpha - k\sin2\alpha, 2p\sin\alpha - h\sin2\alpha - k\cos2\alpha)$.

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47. A point moves so that sum of the squares of its distances from the vertices of a triangle is always constant. Prove that the locus of the moving point is a circle whose centre is the centroid of the given triangle.

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48. Find the locus of mid points of chords of the cirlce. $x^2 + y^2 = a^2$ which subtend angle 90° at the point (c,o).

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49. A straight line is such that its segment between lines $5x - y - 4 = 0$ and $3x + 4y - 4 = 0$ is bisectedat the point (1,5) . Find its equation.

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50. A rod of length a unit slides on the x-axis and another rod of length b unit slides on the y-axis in such a way that the four extremities of the rod are concylic. Show that the locus of the centre of the circle is $4\left(x^2 - y^2\right) = a^2 - b^2$.

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51. A circle cuts intercepts of length 2a unit and 2b unit from the x-axis and y -axis respectively. Show that the locus of the centre of the circle is $x^2 - y^2 = a^2 - b^2$.

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52. Let $x^2 + y^2 - 4x - 2y - 11 = 0$ be a fixex circle.A pair of tangents to the circle from the point (4,5) with a pair of its radii form quadrilaterla. Find the area of the quadrilaterla.

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53. The circle $x^2 + y^2 - 4x - 4y + 4 = 0$ is inscribed in a triangle which has two of its sides along the coordinate axes. The locus of the circumcentre of the triangle is $x + y - xy + k\left(x^2 + y^2\right)^{1/2} = 0$. Find k.

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54. Find the equation of the circle passing through the point (2,0) and touching two given lines $3x - 4y = 11$ and $4x + 3y = 13$ and it's center lies on y axis.

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55. Tangents PQ and PR are drawn from the point (α, β) to the circle $x^2 + y^2 = a^2$. Show that the area of ΔPQR is $\dfrac{a\left(\alpha^2 + \beta^2 - a^2\right)^{3/2}}{\alpha^2 + \beta^2}$.

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56. Let $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ be a given circle. Find the of the foot of perpendicular drawn from the origin upon any chord of S, which subtends a right angle at the origin.

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57. The line $2x + 3y = 12$ meets the x-axis at A and the y-axis at B. the line through (5,5) perpendicular to AB meets the axes andtheline AB at C,D,E respectively. If O is the origin of coordinates, find the ara of the figure OCEB.

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58. Find the equation of the line passing through the point (2,3) and making intercept of length 2 units bewteen the lines $y + 2x = 3$ and $y + 2x = 5$.

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59. Let ABC be a triangle with AB=AC. If D is the mid point of BC, E, foot of the perpendicular drawn from D to AC and F, the mid point of DE: prove that AF is perpendicular to BE.

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60. Lines $L_1 = ax + by + c = 0$ and $L_2 \equiv lx + \mu + n = 0$ intersect at the point P and makes an angle θ with each other. Find the eqauation of a line L different from L_2 which passes through P and makes the same angle θ with L_1.

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61. Detemine all values of α for which the point $\left(\alpha, \alpha^2\right)$ lines inside the triangle formed by the lines $2x + 3y - 1 = 0$, $x + 2y - 3 = 0$ and $5x - 6y \equiv 1$

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62. A line through $A(-5-4)$ meets of lines $x+3y+2=0, 2x+y+4$ and $x-y-5=0$ at the point B,C and D respectively. If $\left(\frac{15}{AB}\right)^2+\left(\frac{10}{AC}\right)^2=\left(\frac{6}{AD}\right)^2$, find the equation of the line.

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63. Find the radius of the smallest circle which touches the straight line $3x-y=6$ at (1-,3) and also touches the line y=x. compute upto one place of decimal only.

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64. If a circle passes through the points of intersection of the coordinates axes with the lines $\lambda x-y+1=0$ and $x-2y+3=0$, then find the value of λ.

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65. Let a circle be given by $2x(x - a) + y(2y - b) = 0(a \neq 0, b \neq 0)$. Find the condition on a and b if two chords, each bisected by the x-axis, can be drawn to the circle from $\left(a, \frac{b}{2}\right)$.

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66. The equation of two side of a triangle are $y = m_1 x$ and $y = m_2 x$, where m_1 and m_2 are two roots of the equation $bx^2 + 2hx + a = 0$. If the orthocentre of the triangle be (a,b) , then show that the equation of the third side of the triangle is $(a + b)(ax + by) = ab(a + b - 2h)$.

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67. In a triangle ABC, coordinates of A are (1,2) and the equations of the medians through B and C are $x + y = 5$ and $x = 4$ respectively. Find the coordinates of B and C.

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68. One diagonal of a square is the portion of the line $\frac{x}{a} + \frac{y}{b} = 1$. Intercepte by the coordinate axe. Find the coordinates of the ends of its other diagonal.

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69. A circle of radius r passes through the origin and intersects the x-axis and y-axis at P and Q respectively. Show that the equation to the locus of the foot of the perpendicular drawn form the origin upon the line segment $\overline{PQ}$ is $\left(x^2 + y^2\right)^3 = 4r^2x^2y^2$.

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70. The coordinates of A,B,C are (2,1) (6,-2) and (8,9) respectively. Find the equation of the internal bisector of the triangle ABC that bisects the angle at A.

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71. Find the equations to the straight lines passing through the foot of the perpendicular from the point (α, β) upon the striaght line $lx + my + n = 0$ and bisecting the angles between the perpendicular and the given striaght line.

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72. By parallel transformation of coordinate axes to a properly chosen point (h,k), prove that the equation $12x^2 - 10xy + 2y^2 + 11x - 5y + 2 = 0$ can be reduced to one containing only terms of the second degree.

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73. Show thathte coefficients of x^2, xy and $y^2 \text{in} ax^2 + 2hxy + by^2$ are invariants under the translatio of axes.

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74. Prove that the area of the triangle with vertices (p, q), $\left(x_1, y_1\right)$ and $\left(x_2, y_2\right)$ where p, x_1, x_2 are in G.P. with common ratio r_1 and q, y_1, y_2 and in G.P with common ratio r_2 is $\frac{1}{2}pq\left(r_1 - 1\right)\left(r_2 - 1\right)\left(r_2 - r_1\right)$ sq unit.`

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75. The base of triangle passes through a fixed point (f,g) and its other side sare bisected at right angles by the lines $y^2 - 8xy - 9x^2 = 0$. Detrmine the locus of its vertex.

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76. The straight line $ax + by = 1$ intersect the circle $x^2 + y^2 = c^2$ at P and Q , if the chord $\bar{P}Q$ substends and angle 45° at the centre of the circle, show that $c^2\left(a^2 + b^2\right) = 2\left(2 - \sqrt{2}\right)$.

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77. The equations of two equal sides AB and AC of an isosceles triangle ABC are $x + y = 5$ and $7x - y = 3$ I respectively. Find the equations of the side BC if the area of the triangle ABC is 5 sq unit.

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78. One diagonal of a square is the portion of the straight line $7x + 5y = 35$ intercepted by the axes. Obtain the coordinates of the extremities of other diagonal.

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79. If the straight lines $a^2x + ay + k = 0, b^2x + by + k = 0$ and $c^2x + cy + k = 0(k \neq 0)$ are concurrent, show that at least two of the three constant a,b,c are equal.

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80. Show that the circle $x^2 + y^2 - 2y - 15 = 0$ lies completely within the circle $x^2 + y^2 - x - 30 = 0$

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81. Two sides of a rhombus ABCD are parallel to the lines $x - y = 5$ and $6x - y = 3$. If the diagonals of the rhombus intersects at the point (2,1), find the equation of the diagonals. Further, find the possible coordinates of the vertex A if it lies on the x-axis.

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82. Two sides of a rhombus lyin in the first quadrant are given by $3x - 4y = 0$ and $12x - 5y = 0$. If the length of the longer diagonal is 12 units, find the equationsof the other two sides of the rhombus.

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83. A line joining two points A (2,0) and B(3,1) is rotated about A in the anit-clockwise direction through an angle 15°. Find the equation of the line in the new position. If B goes to C in th new position, what are coordinates of C ?

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84. A (3,0) and B(6,0) are two fixed points and $P(x_1, y_1)$ is a variable point of the plane. AP and BP meet the y-axis at the points C and D respectively. AD intersects OP at Q. Prove that line CQ always passes through the point (2,0).

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85. Find the coordinates of the centroid of the triangle, formed by the lines $x + 2y - 5 = 0, y + 2x - 7 = 0$ and $x - y + 1 = 0$.

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86. The points (1,3) and (5,1) are two oppsite vertices of a rectangle. The other two vertices lie on the line $y = 2x + c$, find c and the coordinates of the remains vertices.

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87. Show that the lines $ax + by + c = 0, ax - by + c = 0, ax - by = cax + by - c = 0(a \neq b)$ enclose a rhombus whose area is $\frac{2c^2}{ab}$ sq unit.

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88. Let $A\left(ct_1, \frac{c}{t_1}\right), B\left(ct_2, \frac{c}{t_2}\right)$ and $C\left(ct_3, \frac{c}{t_3}\right)$ be the vertices of the triangle ABC. Show that the ortocentre of the triangle lies on $xy = c^2$.

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89. The circle A and B of a line segment of constant length c unit slides upon the fixex rectangular axes OX and OY. If P be a point on the plane such that OAPB is a rectangle, then show that locus of the foot of the perpendicular drawn from P to AB is $x^{2/3}+y^{2/3}=c^{2/3}$.

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90. A variable striaght line of slope 4 intresects the hyprbola $xy=1$ at two points. Find the locus of the point which divides the line segment between theses two points in the ration $1:2$.

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91. If $x\cos\alpha+y\sin\alpha=p$, where $p=\dfrac{-\sin^2\alpha}{\cos\alpha}$ be a straight, line, prove that the perpendicular p_1, p_2 and p_3 on this line drawn from the point $\left(m^2, 2m\right), (mm', m+m' \text{ and })\left((m')^2, 2m'\right)$ respectively, are in geometric progression. $\left(m>0, m>0, 0<\alpha<90^{\circ}\right)$.

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92. A tangent drawn from the point (4,0) to the circle $x^2 + y^2 = 8$ touches it at a point A in the first quadrant. Find the coordinates of another point Bon the circle such that $AB = 4$.

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93. $A(x_1y_1)$ and $B(x_2, y_2)$ are two given points on a circle. If the chord AB substends an angle θ at a point p on its circumferene, show that the equation of the circle is

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = \pm \cos\theta[(x - x_1)(y - y_2) - (x - x_2)(y - y_1)$$

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94. If $4a^2 - 5b^2 + 6a + 1 = 0$ find the equation of the circle for which the straight line $ax + by + 1 = 0$ is a tangent.

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95. Find the point of the circle $x^2 + y^2 - 6x + 4y = 0$ which is (i) nearest (ii) farthest to the line $2x - 3y + 14 = 0$.

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96. Find the locus of mid points of chords of the cirlce. $x^2 + y^2 = a^2$ which subtend angle 90° at the centre of the circle.

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97. Vertices $A(1, 1), B(4, -2)$ and $(5, 5)$ of a triangle are given , find the equation of the perpendicular dropped from C to the internal bisectorof the angle A.

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98. Prove that the locus of middle points of chords of constant length 2d unit of the hyperbola $xy = c^2$ is $\left(x^2 + y^2\right)\left(xy - c^2\right) = d^2xy$.

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99. Find the integer values of m for which x-coordinate of the point of intersection of the striaght lines $3x + 4y = 9$ and $y = mx + 1$ is also integer.

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100. A chord AB of the circle $x^2 + y^2 = a^2$ subtends a right angle at its centre. Show that the locus of the centroid of the angle PAB as P moves on the circle is another circle.

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101. The corodinates of the three vertices of a triangle are $(a, a\tan\alpha), (b, b\tan\beta)$ and $(c, c\tan\gamma)$. If the circumcentre and orthocentre of the triangle are at (0,0) and (h,k) respectively, prove that $\frac{h}{k} = \frac{\cos\alpha + \cos\beta + \cos\gamma}{\sin\alpha + \sin\beta + \sin\gamma}$.

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102. The triangle PQR is inscribed in the circle $x^2 + y^2 = 25$. If Q and R have coordinates (3,4) and (-4,3) respectively. Then find $\angle QPR$.

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103. Let L_1 be a striaght line passing through the origin and L_2 be the straight $x + y = 1$. If the intercepts made by the circle $x^2 + y^2 - x + 3y = 0$ on L_1 and L_2 are equal, find the equation of the line L_1.

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104. A rectangle PQRS has its side PQ paralle to the line $y = mx$ and vertices P,Q and S are on the straight lines $y = a, x = b$ and $x = -b$ respectively. Find the locus of the vertex R.

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105. If two distinct chords, drawn from the point (p,q) on the circle $x^2 + y^2 = px + qy$ (where $pq \neq 0$) are bisected by the x-axis then show that, $p^2 > 8q^2$

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106. A variable line L passing through the point B(2,5) intesects the lines $2x^2 - 5xy + 2y^2 = 0$ at P and Q. Find the locus of the point Ron L such that distancesBP,BR and BQ are in harmonic progression.

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107. For all values of a and, b show that the circle $(x - 2)(x - 2 + a) + (y + 3)(y + 3 + b) = 0$ bisects the circumference of the circle $(x - 2)^2 + (y + 3)^2 = 36$.

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108. A circle C_1 of diameter 6 units, is in the first quadrant and it touches the striaght lines $5x + 12y - 10 = 0$ and $5x - 12y - 40 = 0$. Another circle C_2 concentric with C_1 intercepts chords of length 8 units. Form the two given straight lines. Find the equation of the cirlce C_2.

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109. Find he point on the straight line $y = 2x + 11$ which is nearest to the circle $16\left(x^2 + y^2\right) + 32x - 8y - 50 = 0$.

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110. On the parabola $y^2 = 4ax, P$ is the point with parameter t,Q is the opposite xtremity of the focal chord through P and R is the point for which QR is paralle to PK where K is the point (2a,0). Show that R has parameter $\frac{t^2 - 1}{t}$.

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111. Through the vertex A of a parabola the chords AP and AQ are drawn at right angles. Show that the striaght line PQ intersets the axis at a fixed point.

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112. Show that the sum of the ordinate of end of any chord of a system of paralel chords of the parabola $y^2 = 4ax1$ is constant.

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113. An equilateral triangle is inscribed within the parabola $y^2 = 4ax$ with one vertex at the chords of the parabola. Find the lenghtof side of triangle .

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114. P,Q and R three points on the parabola $y^2 = 4ax$. If Pq passes through the focus and PR is perpendicular to the axis of the parabola, show that the locus of the mid point of QR is $y^2 = 2a(x + a)$.

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115. A line of length (a+b) unit moves in such a way that its ends are always on two fixed perpendicular striaght lines. Prove that the locus of a point on this line which divides it into of lenghts a unit and b unit is an ellipse.

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116. show that the length of the focal chord of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ which makes an angleθ with the major axis is $\frac{2ab^2}{a^2\sin^2\theta + b^2\cos^2\theta}$ unit.

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117. A rod of given length (a+b) unit moves so that its ends are always on coordinates. Prove that the locus of a point, which divides the line into two portions of lenghts a unit and b units, is an ellipse. State the situation when the locus will be a circle.

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118. If θ and ϕ be the eccentric angles of the extremities of a focal chord passing passing through the focus (ae,0) of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, shos that $\tan\frac{\theta}{2}\tan\frac{\phi}{2} = \frac{e - 1}{e + 1}$ (e is the eccentricity of the ellipse).

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119. θ and ϕ are the eccentric angles of two points on an ellipse whose length of major axis is 2a unit. If theline joinin theses points intersects the major axis at a distance c unit from the origin, then show that, $\tan\frac{\theta}{2}\tan\frac{\phi}{2} = \frac{c - a}{a + b}$.

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120. PQ and PR are two focal chords of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, if $2\alpha, 2\beta$ and 2γ are the eccentric angles of the point P,.Q and R respectively, prove that, $\cot\alpha\cot\beta = \tan\gamma\tan\beta$.

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121. Find the locus of middle points of chords of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ which subtend right angle at its center.

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122. Find the length of chord of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$, whose middle point is $\left(\frac{1}{2}, \frac{2}{5}\right)$.

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123. An ellipse has OB as a semi-minor axis, F and F' are its two foci and the angle FBF' is a right angle. Find the eccentricity of the ellipse.

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124. Let P b any point on the hyperbola $\frac{x^2}{a^2}$ - $\frac{y^2}{b^2} = 1$ whose ordinate is PN. AA' is its transverse axis. If the point Q divides AP in the ratio $a^2 : b^2$, then NQ is?

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COORDINATE GEOMETRY (THREE DIMENSIONAL COORDINATE GEOMETRY)

1. Determine the perpendicular distances of the points (-4,3,4) from the coordinates axes.

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2. C is a point on the line-semgment joining the points A (4,-2,6) and B(2,-3,4), if y-coordinates of C is 0, find its z-coordinate.

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3. The coordinates of the vetex A of the triangle ABC are (2,5,-3), if centroid of the triangle is at (-2,1,3), find the coordinates of the mid points on the side BC.

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4. Show that the points $(0, 7, 10), (-1, 6, 6)$ and $(-4, 9, 6)$ are the vertices of a right angled isosceles triangle.

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5. The cordinates of the mid points of the sides BC, CA and AB of the triangle ABC are $(5, 2, 8), (2, -2, -3)$ and $(2, -3, 4)$, find the coordinates of the centroid of the triangle.

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6. Find the ratios in which the yz- plane , zx-plane and xy-plane divide the line segment joining the points $(-2, 4, 7)$ and $(3, -5, 8)$

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7. The points A and B trisect the line -segment joining the points $P(2, 1 - 3)$ and $Q(5, -8, 3)$, find the coordinates of the points A and B.

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8. The coordinates of the vertices A,B and C of the triangle ABC are $(-1, 4, 2,)(3, -2, 0)$ and $(1, 2, 4)$ respectively find the length of the median through the vertex A.

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9. The point P is equidistant from the points $(-1, 4, 2), (2, 1, 2), (3, 2, 2)$ and $(0, 5, 6)$, find the coordinates of the point P.

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10. $P(-2, -3, 6)$ is a given point and O is the origin. Find the coordinates of seven other points in three dimensional space such that the distance of each point from the origin O is equal to OP.

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CALCULUS

1. Without using graph paper, draw the graph of the function $f(x) = \sin\frac{1}{x}$ and determine from the graph whether $\lim_{x \to 0} f(x)$ exsits or not.

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2. What is the domain and range of the function $f(x) = \frac{x^2 - 3x + 2}{x^2 + x - 6}$. Find the limit of f(x) as x approaches 2.

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3. Show that :

$$\lim_{x \to \pi} \frac{1 + \cos^3 x}{\tan^2 x} = \frac{3}{2}$$

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4. Evaluate :

$$\lim_{x \to \frac{\pi}{2}} \frac{\cos 3x + 3\cos x}{\left(\frac{\pi}{2} - x\right)^3}$$

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5. Show that :

$$\lim_{x \to a} \frac{\sin x - \sin a}{\sqrt{x} - \sqrt{a}} = 2\sqrt{a}\cos a$$

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6. Show that :

$$\lim_{x \to a} \frac{\sin x - \sin a}{\sqrt{x} - \sqrt{a}} = 2\sqrt{a}\cos a$$

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7. Show that :

$$\lim_{x \to 0} \frac{\sin\log(1 + x)}{\log(1 + \sin x)} = 1$$

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8. Show that :

$$\lim_{x \to 0} \frac{5x\cos x - 2\sin x}{3x + \tan x} = \frac{3}{4}$$

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9. Show that :

$$\lim_{x \to 0} \frac{10^x - 2^x - 5^x + 1}{x^2} = \log_e 2 . \log_e 5$$

Watch Video Solution

10. Show that :

$$\lim_{x \to 4} \frac{(\cos\alpha)^x - (\sin\alpha)^x - \cos 2\alpha}{x - 4} = \cos^4\alpha \log_e(\cos\alpha) - \sin^4\alpha \log_e(\sin\alpha)$$

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11. Show that :

$$\lim_{x \to 2} \frac{\sin\left(e^{x-2} - 1\right)}{\log(x - 1)} = 1$$

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12. Show that :

$$\lim_{x \to \pi} \frac{\sqrt{2 + \cos x} - 1}{(\pi - x)^2} = \frac{1}{4}$$

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13. Show that :

$$\lim_{x \to 0}\left[\frac{1}{x} - \frac{\log(1+x)}{x^2}\right] = \frac{1}{2}$$

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14. Show that :

$$\lim_{x \to \frac{\pi}{4}}\frac{\sqrt{2} - (\sin x + \cos x)}{(4x - \pi)^2} = \frac{1}{16\sqrt{2}}$$

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15. If $f(x) = \lim_{n \to \infty}\frac{x^n g(x) + h(x)}{x^n + 1}$ show that

f(x) =h(x), when $0 < x < 1$

$= \frac{1}{2}[h(x) + g(x)]$, when x=1

=g(x), when $x > 1$

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16. If $\lim_{x \to 0} \frac{\sin 2x + a\sin x}{x^3}$ is finite, find a and the limit.

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17. Evaluate the following limits:

$$\lim_{x \to 0} \frac{\sqrt{x^2 - 1} + \sqrt{x - 1}}{\sqrt{x^2 - 1}}$$

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18. Evaluate the following limits:

$$\lim_{x \to 2a} \frac{\sqrt{x - 2a} + \sqrt{x} - \sqrt{2a}}{\sqrt{x^2 - 4a^2}}$$

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19. Evaluate the following limits:

$$\lim_{x \to 0} \frac{\sin x^2 \left(1 - \cos x^2\right)}{x^6}$$

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20. Evaluate the following limits:

$$\lim_{x \to 0} \frac{xe^x - \log(1 + x)}{x^2}$$

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21. Evaluate the following limits:

$$\lim_{h \to 0} \frac{(a + h)^2 \sin(a + h) - a^2 \sin a}{h}$$

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22. Evaluate the following limits:

$$\lim_{n \to \infty}\left[\frac{n}{n^2}+\frac{n+1}{n^2}+\frac{n+2}{n^2}+\ldots\ldots\ldots+\frac{2n}{n^2}\right]$$

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23. Evaluate the following limits:

$$\lim_{x \to 3}\frac{3-\sqrt{6+x}}{3\sqrt{3}-\sqrt{6-x}}$$

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24. Evaluate the following limits:

$$\lim_{x \to 0}\frac{\sqrt{\cos x}-\sqrt[3]{\cos x}}{\sin^2 x}$$

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25. Evaluate the following limits:

$$\lim_{x \to 0}\left[coec^3x\cot x - 2\cot^3 x\operatorname{cosec} x + \frac{\cot^4 x}{\sec x}\right]$$

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26. Evaluate the following limits:

$$\lim_{x \to \frac{\pi}{2}}\left(x\tan x - \frac{\pi}{2}\sec x\right)$$

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27. Show that $\lim_{x \to 1}\frac{\sqrt{1 - \cos(x - 1)}}{x - 1}$ does not exist.

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28. Evaluate:

$$\lim_{x \to 2} \frac{2^x + 2^{3-x} - 6}{\sqrt{2^{-x}} - 2^{1-x}}$$

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29. Evaluate:

$$\lim_{x \to \frac{\pi}{2}} \left(\frac{2x\sec x}{\pi} - \tan x \right)$$

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30. Evaluate:

$$\lim_{x \to 0} (\cos x)^{\cot^2 x}$$

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31. Evaluate:

$$\lim_{x \to \frac{1}{2}} \frac{\cos^2 \pi x}{e^{2x} - 2ex}$$

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32. Evaluate:

$$\lim_{x \to \frac{\pi}{3}} \frac{\tan^3 x - 3\tan x}{\cos\left(x + \frac{\pi}{6}\right)}$$

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33. Evaluate:

$$\lim_{x \to a} \frac{ae^x - xe^a}{x - a}$$

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34. Evaluate:

$$\lim_{x \to 0} \frac{e^x - \log(e + ex)}{x}$$

Watch Video Solution

35. Evaluate:

$$\lim_{x \to \frac{\pi}{2}} (1 + \cos x)^{3\sec x}$$

Watch Video Solution

36. Evaluate:

$$\lim_{x \to \frac{1}{2}} \frac{\sin\left(\frac{\pi x}{2}\right) - \cos\left(\frac{\pi x}{2}\right)}{\frac{1}{2} - x}$$

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37. Evaluate:

$$\lim_{x \to 0}\left(\frac{x - 1 + \cos x}{x}\right)^{1/x}$$

Watch Video Solution

38. Evaluate:

$$\lim_{x \to 2}\frac{\sqrt{1 + \sqrt{2 + x}} - \sqrt{3}}{x - 2}$$

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39. Evaluate:

$$\lim_{x \to 0}(\sin x + \cos x)^{1/x}$$

Watch Video Solution

40. Evaluate:

$$\lim_{x \to 0} \left[\tan\left(x + \frac{\pi}{4}\right)\right]^{1/x}$$

Watch Video Solution

41. Evaluate:

$$\lim_{x \to 0} \frac{1}{x^3}[\sin(3x + a) - 3\sin(2x + a) + 3\sin(x + a) - \sin a]$$

Watch Video Solution

42. Evaluate:

$$\lim_{x \to 2} \frac{\sqrt{2x + 5} - 3\left(\sqrt{2x - 3}\right)}{\sqrt[3]{2x - 3}}$$

Watch Video Solution

43. Evaluate:

$$\lim_{x \to 0}(1 + \sin 2x)^{\operatorname{cosec} x}$$

Watch Video Solution

44. Evaluate:

$$\lim_{x \to 1}\frac{x^x - 1}{x \log x}$$

Watch Video Solution

45. Evaluate:

$$\lim_{x \to 0}\frac{\log\left(1 + x + x^2\right) + \log\left(1 - x + x^2\right)}{\sec x - \cos x}$$

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46. Prove that $\lim_{x \to 0} \frac{x\sqrt{2ax - x^2}}{\left(\sqrt{8ax - 4x^2} + \sqrt{8ax}\right)^3} = \frac{1}{128a}$

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47. Evaluate [without using L' Hospital's rule]:

$$\lim_{x \to 2} \frac{2 - \sqrt{2 + x}}{\sqrt[3]{2} - \sqrt[3]{4 - x}}$$

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48. Evaluate [without using L' Hospital's rule]:

$$\lim_{x \to \frac{\pi}{4}} \frac{4\sqrt{2} - (\cos x + \sin x)^5}{1 - \sin 2x}$$

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49. Find the differential coefficient of:

$\cos\left(ax^2 + bx + c\right)$

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50. Find the differential coefficient of:

$\tan^2(ax)$

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51. Find the differential coefficient of:

x^x

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52. Find from first principle the differential coefficient of:

$x\tan^{-1}x at x = 1$

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53. Examine the differentiability of $f(x) = |\sin x - \cos x|$ at $x = \frac{\pi}{4}$

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54. Prove that the function $f(x) = |x|$ is not differentiable there at x=0.

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55. Let $f(x) = 2x + 3$ for $x \leq 1$

$= ax^2 + bx$ for $x > 1$

If f(x) is everywhere differentiable, then prove that $f'(2) = -4$.

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56. If $f(2) = 4, g(2) = 9$ and $f'(2) = 2g'(2)$, then evaluate $\lim_{x \to 2} \frac{\sqrt{f(x)} - 2}{\sqrt{g(x)} - 3}$

.

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57. If $f(9) = 9, f'(9) = 4$, then evaluate $\lim_{x \to 9} \frac{\sqrt{f(x)} - 3}{\sqrt{x} - 3}$.

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58. Let $f\left(\frac{x + y}{2}\right) = \frac{1}{2}[f(x) + f(y)]$ for all real x and y. If f'(0) exists and equals (-1), find f'(2)`.

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59. If f(x) is differentiable at x=a, find the value of

$\lim_{x \to a} \frac{(x + a)f(x) - 2af(a)}{x - a}$

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60. The value of $\lim_{x \to 0} f(x)$ where $f(x) = \dfrac{x^3 + x^2}{2x^3 - 27x^2}$

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61. If $f(x + y) = f(x)f(y)$ for all x, y and $f(x) = 1 + xg(x)$, where $\lim_{x \to 0} g(x) = 1$. Show that $f'(x) = f(x)$.

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62. If $f(x + y + z) = f(x)f(y)f(z) \neq 0$, for all x, y, z and $f(2) = 4, f'(0) = 3$, find $f'(2)$.

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63. If $f(3) = 6$ and $f'(3) = 2$, find the value of $\lim_{x \to 3} \dfrac{xf(3) - 3f(x)}{x - 3}$.

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64. If $y = f\{f(x)\}, f(0) = 0$ and $f'(0) = 5$, find $\left[\frac{dy}{dx}\right]_{x=0}$

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65. Find the derivative of $|f(x)|$ with respect to x, hence, write down the derivative of $|\cos x|$.

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66. Prove that,

$$\lim_{n \to \infty} e^{\frac{2}{n}+1} = e$$

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67. Prove that,

$$\lim_{n \to \infty} \left[\frac{n}{n^2} + \frac{n+1}{n^2} + \frac{n+2}{n^2} + \ldots\ldots. + \frac{2n}{n^2} \right] = \frac{3}{2}$$

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68. Prove that,

$$\lim_{n \to \infty} \frac{3n^3 - 5n^2 + 4}{5 + 3n^2 - 4n^3} = -\frac{3}{4}$$

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69. Prove that,

$$\lim_{x \to \infty} \frac{4x^3 - 5x^2 + 6x + 9}{3x^4 + 4x^2 - 11} = 0$$

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70. Prove that,

$$\lim_{n \to \infty} \frac{1^2 + 2^2 + 3^2 + ... + (n+2)^2}{n^3} = \frac{1}{3}$$

Watch Video Solution

71. Prove that,

$$\lim_{x \to \infty} \left(\frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + ... + \frac{1}{4^x} \right) = \frac{1}{3}$$

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72. Prove that,

$$\lim_{x \to \infty} \frac{1 + 2 + 3 + ...\text{to}(4x+1)\text{terms}}{(x+1)^2} = 8$$

Watch Video Solution

73. Prove that,

$$\lim_{n \to \infty} \frac{3.5 + 5.7 + 7.9 + \ldots\ldots. + (2n+1)(2n+3)}{n^3} = \frac{4}{3}$$

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74. Prove that,

$$\lim_{x \to \infty} \sqrt{x}\left(\sqrt{x+3} - \sqrt{x}\right) = \frac{3}{2}$$

Watch Video Solution

75. Prove that,

$$\lim_{n \to \infty} \left(\sqrt{1 + n + n^2} - n\right) = \frac{1}{2}$$

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76. Evaluate :

$$\lim_{x \to \infty} \frac{pe^x + qe^{-x}}{re^x + se^{-x}} (r \neq 0)$$

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77. Evaluate :

$$\lim_{x \to -\infty} \frac{ae^{x} + be^{-x}}{ce^{x} + de^{-x}} (d \neq 0)$$

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78. Find from first principle the differential coefficients of the following functions :

$\cos(\log x)$

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79. Find from first principle the differential coefficients of the following functions :

$\log(\sin x)$

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80. Find from first principle the differential coefficients of the following functions :

$\sqrt{\cot x}$

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81. Find from first principle the differential coefficients of the following functions :

$e^{\sqrt{x}}$

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82. Find the differential coefficients of the following functions :

$\sin(\log x)$

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83. Find from first principle the differential coefficients of the following functions :

$\sin\left(x^2+1\right)$

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84. Find from first principle the differential coefficients of the following functions :

x^x

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WBHS ARCHIVE 2017 (UNIT-1)

1. If $B \subseteq A$, then the set B-A will be

A. B

B. A

C. ϕ

D. A'

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2. A relation R is defined from $A = \{1, 2, 4, 5\}$ to $B = \{1, 2, 3, 4\}$ in such a way that, $(x, y) \in R \Rightarrow x > y$ Express R as a set of ordered pairs.

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3. If $y = f(x) = \dfrac{px + q}{rx - p}$, then show that x=f(y).

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4. For any three sets A, B and C, prove that

$A - (B \cup C) = (A - B) \cap (A - C)$.

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WBHS ARCHIVE 2017 (UNIT-2)

1. Show that, $\frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ} = 4.$

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2. Find the value of $2ac\sin\left(\frac{A - B + C}{2}\right)$ for ΔABC.

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3. If $\frac{\sin^4\alpha}{a} + \frac{\cos^4\alpha}{b} = \frac{1}{a+b}$, then show that $\frac{\sin^8\alpha}{a^3} + \frac{\cos^8\alpha}{b^3} = \frac{1}{(a+b)^3}$

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4. If for a trianlge ABC, $\cot A + \cot B + \cot C = \sqrt{3}$, then show that the triangle is equilateral.

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5. Solve: $4\sin x \sin 2x \sin 4x = \sin 3x$.

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6. If $\tan\frac{\theta}{2} = \sqrt{\frac{1-e}{1+e}}\tan\frac{\phi}{2}$. Then prove that $\cos\phi = \frac{\cos\theta - e}{1 - e\cos\theta}$.

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WBHS ARCHIVE 2017 (UNIT-3)

1. If ω be the imaginary cube root of 1, then the value of $\left(3 + \omega + 3\omega^2\right)^4$ will be

A. 16

B. 16ω

C. $16\omega^2$

D. none of these

Answer: (B)

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2. If the difference between the roots of the quadratic equation $x^2 + px + 8 = 0$ be 2, then the value of p will be

A. ± 2

B. ± 4

C. ± 6

D. ± 8

Answer: (C)

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3. If $^{16}C_r = {}^{16}C_{2r+1}$, then the value of r will be

A. 6

B. 5

C. 4

D. 3

Answer: (B)

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4. If z be a complex number and $|z+5| \le 6$, then find the maximum and minimum values of $|z+2|$.

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5. If ${}^{n}P_{r} = \; = 504$ and ${}^{n}C_{r} = 84$, then find the value of n and r.

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6. Find the coefficients of

x^{-2} in the expansion of $\left(2x^{3} - \frac{1}{x^{2}}\right)^{6}$

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7. If P-th term of an arithmetic progression be Q and Q-th term be P, then show that (P+Q) -th term is 0.

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8. If $n \ne \mathbb{N}$, then prove by mathematical induction that $7^{2n} + 2^{3(n-1)}3.^{n-1}$ is always a multiple of 25.

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9. If $z = x + iy$ and $\frac{z - i}{z + 1}$ is purely imaginary, then show that the point z always lies on a circle.

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10. How many odd numbers of five digits can be formed with the digits 3, 6, 7, 2, 0 when no digit is repeated?

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11. Show that the mid-term in the expansion of $(1 + x)^{2n}$ is $\frac{1.3.5.......(2n - 1)}{n!}.2^n.x^n$ (where $n \neq \mathbb{N}$).

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12. If the ratio of the sum of 1st n terms of two arithmetic series is $(4n - 13):(3n + 10)$, then find the ratio of their ninth terms.

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13. Solve the following inequation : $\dfrac{|x + 2| + 2x}{x + 2} > 2.$

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14. Solve applying formula: $3x^2 - (2 - 2i)x + 10 - 4i = 0.$

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15. Out of 14 marbles 10 are red in colour and remaining 4 are of different colours. How many ways can you select 10 marbles out of these 14 marbles?

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16. Find the sum to n terms: $\frac{1}{2} + \frac{3}{2^2} + \frac{5}{2^3} + \ldots\ldots + \frac{2n - 1}{2^n}.$

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WBHS ARCHIVE 2017 (UNIT-4)

1. The equation of the directirix of the parabola $x^2 - 4x - 8y + 12 = 0$ is-

A. y=1

B. x=1

C. x=-1

D. y=-1

Answer: (A)

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2. The coordinates of B and C of the triangle ABC are (5, 2,8) and (2, -3, 4) respectively. If the centroid of the triangle is (3,-1,3), then the coordinates of A are

A. (2,-2,2)

B. (2,-2,-3)

C. (2,2,-3)

D. (-2,-2,-3)

Answer: (B)

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3. The perimeter of the triangle formed by the straight line $4x + 3y - k = 0$ with the coordinate axes is 24 unit , find the value of k.

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4. Find the coordinates of the point lies on the plane YOZ which is equidistant from the points $A(1, -1, 0)$, $B(2, 1, 2)$ and C(3,2,-1).

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5. Find the equations of the lines passing through the point (4,5) making equal angles with the lines $3x = 4y + 7$ and $5y = 12x + 6$.

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6. If the equation of the side BC of an equilateral triangle ABC is $x + y = 2$ and the coordinate of the vertex A is (2,3) then find the equation of the other two sides.

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7. A circle in the first quadrant touches both the axes and its centre lies on the straight line $lx + my + n = 0$. find the equation of that circle

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8. Prove that the locus of the mid-points of chords of length 2d unit of the hyperbola $xy = c^2$ is $\left(x^2 + y^2\right)\left(xy - c^2\right) = d^2xy$.

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9. The coordinates of end points of a focal chord of an ellipse are $\left(x_1, y_1\right)$ and $\left(x_2, x_2\right)$ Prove that $y_1y_2 + 4x_1x_2 = 0$.

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10. The ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ passes through the point of intersection of the lines 7x + 13 y - 87 = 0 and 5x - 8y + 7 = 0 and its length of latus rectum is $\frac{32\sqrt{2}}{5}$, find a and b .

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WBHS ARCHIVE 2017 (UNIT-5)

1. The value of $\lim_{x \to 0} \frac{e^x - e^{-x}}{x}$ is

A. 0

B. 1

C. 2

D. 3

Answer: (C)

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2. If $y = \cos^2\frac{x}{2}$, the value of $\frac{dy}{dx}$ is

A. $\cos x$

B. $\frac{1}{2}\cos x$

C. $-\frac{1}{2}\sin x$

D. $-\sin x$

Answer: (C)

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3. Evaluate : $\lim_{x \to 0} \frac{\cos 5x - \cos 7x}{\cos x - \cos 5x}$.

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4. If $(x+4)y = x$, then show that $x\frac{dy}{dx} + y(y-1) = 0$.

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5. Evaluate : $\lim_{x \to 0} \dfrac{\left(e^x - 1\right)\log(1 + x)}{\sin^2 x}$.

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6. Find from the first principle, the derivative of $f(x) = \sec 2x$ at $x = \dfrac{\pi}{8}$

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WBHS ARCHIVE 2017 (UNIT-6)

1. Prove by method of contradiction that $\sqrt{5}$ is an irrational number.

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2. Prove by Truth Table: $\sim(p \vee q) = \sim p \wedge \sim q$.

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WBHS ARCHIVE 2017 (UNIT-7)

1. If $P(A \cap B) = \frac{7}{13}$, then the value of $P\left(A^c \cup B^c\right)$ is

A. $\frac{4}{13}$

B. $\frac{6}{13}$

C. $\frac{8}{13}$

D. $\frac{12}{13}$

Answer: (B)

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2. If the variance of a distribution is 4 and coefficient of variation is 5%, then mean of the distribution is

A. 20

B. 40

C. 60

D. 80

Answer: (B)

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3. If a coin is tossed 3 times in succession, then find the prbabillity of obtaining tail at least once.

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4. The standard deviation of 32 numbers is 5. If the sum of the numbers is 80, then find the sum of the squares of the numbers.

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5. If 30 dates are named at random, find the probability that 5 of them will be Sundays.

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6. If $4^x = 8^y$ then calculate the value of $\frac{x}{y} - 1$

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WBJEE ARCHIVE 2017 (UNIT-2)

1. The equation $\sin x(\sin x + \cos x) = k$ has real solutions, where k is a real number, Then

A. $0 \le k \le \frac{1 + \sqrt{2}}{2}$

B. $2 - \sqrt{3} \le k \le 2 + \sqrt{3}$

C. $0 \le k \le 2 - \sqrt{3}$

D. $\frac{1 - \sqrt{2}}{2} \leq k \leq \frac{1 + \sqrt{2}}{2}$

Answer: D

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WBJEE ARCHIVE 2017 (UNIT-3)

1. In a GP series consisting of positive terms, each term is equal to the sum of next two terms. Then the common ratio of this GP series is

A. $\sqrt{5}$

B. $\frac{\sqrt{5} - 1}{2}$

C. $\frac{\sqrt{5}}{2}$

D. $\frac{\sqrt{5} + 1}{2}$

Answer: B

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2. If $\left(\log_5 x\right)\left(\log_x 3x\right)\left(\log_{3x} y\right) = \log_x x^3$, then y equals

A. 125

B. 25

C. $\frac{5}{3}$

D. 243

Answer: A

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3. The expression $\frac{(1 + i)^n}{(1 - i)^{n - 2}}$ equals

A. $-i^{n+1}$

B. i^{n+1}

C. $-2i^{n+1}$

D. 1

Answer: C

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4. Let $z = x + iy$, where x and y are real. The points (x,y) in the xy-plane of which $\frac{z+1}{z-1}$ is purely imaginary, lie on

A. a straight line

B. an ellipse

C. a hyperbola

D. a circle

Answer: D

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5. If p,q are odd integers, then the roots of the equation $2px^2 + (2p + q)x + q = 0$ are

A. rational

B. irrational

C. non-real

D. equal

Answer: A

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6. Out of 7 consonants and 4 vowels, words are formed each having 3 consonants and 2 vowels. The number of such words that can be formed is

A. 210

B. 25200

C. 2520

D. 302400

Answer: B

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7. The number of all numbers having 5 digits, with distinct digits is

A. 99999

B. $9 \times {}^{9}P_{4}$

C. ${}^{10}P_{5}$

D. ${}^{9}P_{4}$

Answer: B

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8. The greatest integer which divides $(p+1)(p+2)(p+3)......(p+q)$ for all $p \in \mathbb{N}$ and fixed $q \in \mathbb{N}$ is

A. $p!$

B. $q!$

C. p

D. q

Answer: A

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9. Let $\left(1+x+x^2\right)^9 = a_0 + a_1x + a_2x^2 + ... + a_{18}x^{18}$. Then

A. $a_0 + a_2 + ... + a_{18} = a_1 + a_3 + ... + a_{17}$

B. $a_0 + a_2 + ... + a_{18}$ is even

C. $a_0 + a_2 + ... + a_{18}$ is divisible by 9

D. $a_0 + a_2 + \ldots\ldots + a_{18}$ is divisible by 3 but not by 9

Answer: B

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10. The probability that a non-leap year selected at random will have 53 Sundays is

A. 0

B. $\frac{1}{7}$

C. $\frac{2}{7}$

D. $\frac{3}{7}$

Answer: B

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11. Let α and β be the roots of $x^2 + x + 1 = 0$. If n be positive integer, then $\alpha^n + \beta^n$ is

A. $2\cos\frac{2n\pi}{3}$

B. $2\sin\frac{2n\pi}{3}$

C. $2\cos\frac{2n\pi}{3}$

D. $2\sin\frac{n\pi}{3}$

Answer: A

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12. The complex number z satisfying the equation $|z - i| = |z + 1| = 1$ is

A. 0

B. 1+i

C. $-1 + i$

D. 1-i

Answer: A::C

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13. If a, $b \in \{1, 2, 3\}$ and the equation $ax^2 + bx + 1 = 0$ has real roots, then

A. $a \geq b$

B. $a \leq b$

C. number of possible ordered pairs (a,b) is 3

D. $a < b$

Answer: C::D

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WBJEE ARCHIVE 2017 (UNIT-4)

1. Transforming to parallel axes through a point (p,q) the equation $2x^2 + 3xy + 4y^2 + x + 18y + 25 = 0$ becomes $2x^2 + 3xy + 4y^2 = 1$. Then

A. $p = -2, q = 3$

B. $p = 2, q = -3$

C. $p = 2, q = -4$

D. $p = -4, q = 3$

Answer: B

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2. Let $A(2, -3)$ and $B(-2, 1)$ be two angular points of ΔABC. If the centroid of the triangle moves on the line $2x + 3y = 1$, then the locus of the angular point C is given by

A. $2x + 3y = 9$

B. $2x - 3y = 9$

C. $3x + 2y = 5$

D. $3x - 2y = 3$

Answer: A

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3. The point P(3,6) is first reflected on the line y=x and then the image point Q is again reflected on the line y=-x to get the image point Q'. Then the circumcentre of the $\Delta PQQ'$ is

A. (6,3)

B. (6,-3)

C. (3,-6)

D. (0,0)

Answer: D

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4. Let d_1 and d_2 be the lengths of the perpendiculars drawn from any point of the line $7x - 9y + 10 = 0$ upon the lines $3x + 4y = 5$ and $12x + 5y = 7$ respectively. Then

A. $d_1 > d_2$

B. $d_1 = d_2$

C. $d_1 < d_2$

D. $d_1 = 2d_2$

Answer: B

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5. The common chord of the circles $x^2 + y^2 - 4x - 4y = 0$ and $2x^2 + 2y^2 = 32$ subtends at the origin an angle equal to

A. $\frac{\pi}{3}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{6}$

D. $\frac{\pi}{2}$

Answer: D

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6. The locus of the mid-points of the chords of the circle $x^2 + y^2 + 2x - 2y - 2 = 0$ which make an angle of 90° at the centre is

A. $x^2 + y^2 + 2x - 2y - 2 = 0$

B. $x^2 + y^2 - 2x + 2y = 0$

C. $x^2 + y^2 + 2x - 2y = 0$

D. $x^2 + y^2 + 2x - 2y - 1 = 0$

Answer: C

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7. Let P be the foot of the perpendicular from focus S of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ on the line $bx - ay = 0$ and let C be the centre of the hyperbola. Then the area of the rectangle whose sides are equal to that of SP and CP is

A. 2ab

B. ab

C. $\frac{\left(a^2 + b^2\right)}{2}$

D. $\frac{a}{b}$

Answer: B

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8. B is extremity of the minor axis of an ellipse whose foci are S and S'. If $\angle SBS'$ is a right angle, then the eccentricity of the ellipse is

A. $\frac{1}{2}$

B. $\frac{1}{\sqrt{2}}$

C. $\frac{2}{3}$

D. $\frac{1}{3}$

Answer: B

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9. The axis of the parabola $x^2 + 2xy + y^2 - 5x + 5y - 5 = 0$ is

A. $x + y = 0$

B. $x + y - 1 = 0$

C. $x - y + 1 = 0$

D. $x - y = \frac{1}{\sqrt{2}}$

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10. The line segment joining the foci of the hyperbola $x^2 - y^2 + 1 = 0$ is one of the diameters of a circle. The equation o f the circle is

A. $x^2 + y^2 = 4$

B. $x^2 + y^2 = \sqrt{2}$

C. $x^2 + y^2 = 2$

D. $x^2 + y^2 = 2\sqrt{2}$

Answer: C

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11. If one of the diameters of the curve $x^2 + y^2 - 4x - 6y + 9 = 0$ is a chord of a circle with centre (1,1), the radius of this circle is

A. 3

B. 2

C. $\sqrt{2}$

D. 1

Answer: A

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12. Let $A(-1, 0)$ and $B(2, 0)$ be two points. A point M moves in the plane in such a way that $\angle MBA = 2\angle MAB$. Then the point M moves along

A. a straight line

B. a parabola

C. an ellipse

D. a hyperbola

Answer: D

Watch Video Solution

13. The fouce of the parabola $x^2 - 6x + 4y + 1 = 0$ is

A. (2,3)

B. (3,2)

C. (3,1)

D. (1,4)

Answer: C

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WBJEE ARCHIVE 2017 (UNIT-7)

1. Mean of n observations $x_1, x_2, \ldots\ldots, x_n$ is $\bar{x}$. If an observation x_q' then the new mean is

A. $\bar{x} - x_q + x_q'$

B. $\dfrac{(n-1)\bar{x} + x_q'}{n}$

C. $\dfrac{(n-1)\bar{x} - x_q'}{n}$

D. $\dfrac{\bar{n}x - x_q + x_q'}{n}$

Answer: D

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HS (XI) AND WBJEE 2018 (GROUP - A)

1. All possible subsets of set ϕ is

A. 0

B. 1

C. 2

D. None of these

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2. Value of $\omega^n + \omega^{2n}$, where $\omega = \dfrac{-1 + i\sqrt{3}}{2}$ and $n = 3k + 1$, is

A. 0

B. -1

C. 1

D. None of these

Answer: A::B

Watch Video Solution

3. If ${}^nC_p = {}^nC_q$, then

A. $n \neq p$ or $p + q = n$

B. $p = q$ or $p - q = n$

C. $n = p = q$ or $p + q \neq n$

D. $p = q$ or $p + q = n$

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4. Value of $\sin 36^\circ$ is

A. $\frac{1}{4}\sqrt{10 - 2\sqrt{5}}$

B. $\frac{1}{4}\sqrt{10 + 2\sqrt{5}}$

C. $\frac{1}{4}\sqrt{10 + \sqrt{5}}$

D. $\frac{1}{4}\sqrt{10 - \sqrt{5}}$

Watch Video Solution

5. The value of $\lim_{x \to 4}\left(\frac{e^x - e^4}{x - 4}\right)$ is

A. e^{-4}

B. e^4

C. 1

D. None of these

Answer: A::D

Watch Video Solution

6. Find the point of z-axis which is equidistant from the points $(1, 5, 7)$ and $(5, 1, -4)$

A. $\left(0, 0\frac{3}{2}\right)$

B. (0,0,5)

C. (0,5,0)

D. (4,2,3)

Answer: B::C

Watch Video Solution

7. The angle made by the straight line $x\cos\alpha + y\sin\alpha = p$ with the negative direction of x-axis is

A. $\frac{\pi}{2} + \alpha$

B. α

C. $-\alpha$

D. $\frac{\pi}{2} - \alpha$

Answer: A::B

Watch Video Solution

8. If $f(x) = |x|$, then f'(0) is

A. 0

B. 1

C. -1

D. None of these

Answer: D

Watch Video Solution

9. In single throw of two dice, the probability of obtaining a total of 8' is

A. $\frac{8}{36}$

B. $\frac{3}{36}$

C. $\frac{9}{36}$

D. $\frac{5}{36}$

Answer: A::C

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10. If $y = 2x + 5$ and variance of y is 16, then the standard deviation of x is

A. - 1

B. 4

C. 1

D. 2

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HS (XI) AND WBJEE 2018 (GROUP - B)

1. If $A \cap B' = \phi$, then show that $A = A \cap B$ and hence show that $A \subseteq B$.

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2. Find the domain and range of the real function $f(x) = \dfrac{1}{\left(1 - x^2\right)}$.

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3. Prove that $\cos^2 48^\circ - \sin^2 12^\circ = \frac{\left(\sqrt{5}+1\right)}{8}$.

Watch Video Solution

4. Show that, $\cot 2x \cot x - \cot 3x \cot 2x - \cot 3x \cot x = 1$

Watch Video Solution

5. Find the value of n, so that $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ may be the geometric mean between a and b.

Watch Video Solution

6. Find the value of r, if the coefficients of (2r+4) -th and (r-2) -th terms in the expansion of $(1+x)^{18}$ are equal.

Watch Video Solution

7. Find the principal amplitude of (-1-i).

Watch Video Solution

8. Find the number of squares in Chesboard.

Watch Video Solution

9. Find the focus of the parabola $y = x^2 + x + 1$.

Watch Video Solution

10. Find the equation of a circle with centre (h,k) and touching both the axes.

Watch Video Solution

11. Evaluate : $\lim_{x \to \frac{\pi}{6}} \frac{\sqrt{3}\sin x - \cos x}{\left(x - \frac{\pi}{6}\right)}$

Watch Video Solution

12. Prove that the derivative of and odd function is an even function.

Watch Video Solution

13. Find the variance of first n natural numbers.

Watch Video Solution

14. If $P(A) = \frac{2}{3}, P(B) = \frac{1}{2}, P(A \cap B) = \frac{1}{6}$, then find the value of $P(A \cap B')$ and $P(A \cup B)$.

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HS (XI) AND WBJEE 2018 (GROUP - C)

1. Let $A = \left\{x \in \mathbb{N} : x^2 - 5x + 6 = 0\right\}, B = \{x \in W : 0 \leq x < 2\}$ and $C = \{x \in \mathbb{N} : x < 3$

, then verify that $A \times (B \cup C) = (A \times B) \cup (A \times C)$

Watch Video Solution

2. Prove that in any $\Delta ABC, (b - c)\cot\frac{A}{2} + (c - a)\cot\frac{B}{2}\cot\frac{C}{2} = 0.$

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3. Solve : $\sec x - \tan x = \sqrt{3}.$

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4. If p-th, q-th and r-th terms of and AP as well as those of a GP are a, b, c respectively, then prove that $a^{b-c}.\, b^{c-a}.\, c^{a-b} = 1$

Watch Video Solution

5. Prove that $x^n - y^n$ is divisible by (x-y) for all $n \in \mathbb{N}$

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6. If $z = x + iy$ and $w = \frac{1 - iz}{z - i}$ such that $|w| = 1$, then show that z is purely real.

Watch Video Solution

7. Find the rank of the word 'MOTHER' in dictionary format.

Watch Video Solution

8. If the coefficients of 2nd, 3rd and 4th terms in the expansion of $(1 + x)^{2n}$ are in AP, Show that $2n^2 - 9n + 7 = 0$.

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9. (2a,0) and (0,a) are the extremities of the base of an isosceles triangle, and the equation of one of the equal sides x=2a. Find the equations of other two sides and the area of triangle.

Watch Video Solution

10. A variable straight line passes through the point of intersection of the straight lines $\frac{x}{a} + \frac{y}{b} = 1$ and $\frac{x}{b} + \frac{y}{a} = 1$ and intersects the axes at P and Q. Find the locus of midpoint of PQ.

Watch Video Solution

11. The abscissae of the two points A and B are the roots of the equation $x^2 + 2ax - b^2 = 0$ and their ordinates are the roots of the equation $x^2 + 2px - q^2 = 0$. Find the equation and the radius of the circle with AB as diameter.

Watch Video Solution

12. If $2f(x) + f(-x) = 1 + x$ find $f'(10)$ where f'(x) denote derivative of f(x).

Watch Video Solution

13. Evaluate : $\lim_{x \to \frac{\pi}{4}} \frac{4\sqrt{2} - (\cos x + \sin x)^5}{1 - \sin 2x}$

Watch Video Solution

14. Write the negation of each of the following statements:

p : For every number $x, x^2 > x$

q : For every real number x, either $x > 1$ or $x < 1$.

(b) "Mathematics is fun" check whether this sentence is a statement.

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15. Consider the statement:

p: If x is real number such that $x^3 + 4x = 0$, then x=0, prove that p is true statement, using

(a) method of contradication and

(b) method of contrapositive

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16. A bag contains 5 white and 4 black balls. If 3 balls are drawn at random, find the probability that at least two of them are white.

Watch Video Solution

17. The arithmetic mean and standard deviation of 7 observations are respectively 8 and 16. If five of the observations are 2, 4, 10, 12 and 14 then find the values of the remaining two.

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HS (XI) AND WBJEE 2018 (GROUP - D)

1. If $x = a(\cos\theta + \sin\theta\sin2\theta)$ and $y = a(\sin\theta + \cos\theta\sin2\theta)$, then show that $(x+y)^{\frac{2}{3}} + (x-y)^{\frac{2}{3}} = 2a^{\frac{2}{3}}$.

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2. Show that,

$$3\left[\sin^4\left(3\frac{\pi}{2} - \alpha\right) + \sin^4(3\pi + \alpha)\right] - 2\left[\sin^6\left(\frac{\pi}{2} + \alpha\right) + \sin^6(5\pi - \alpha)\right] = 1.$$

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3. Draw the graph of the solution set of the inequations $2x + y \geq 2, x - y \leq 1, x + 2y \leq 8, x \geq 0$ and $y \geq 0$, also shade the solution region. (Graph paper not necessary)

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4. Find the number of permutations and the number of combinations in the letters of the word 'EXPRESSION' taken four at a time.

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5. Find the sum of the integers between 90 and 890 which are perfect squares.

Watch Video Solution

6. If z_1 and z_2 be two non-zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then prove that $argz_1 - argz_2 = 0$.

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7. The directrix of a parabola is $x + y + 4 = 0$ and vertex is at $(-1, -1)$. Find the position of the focus and the equation of parabola.

Watch Video Solution

8. Prove that the major axis of an ellipse is greater than its minor aixs.

Watch Video Solution

9. Find the eccentricity of a hyperbola whose conjugate axis and latus rectum are equal.

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WBJEE 2018

1. The domain of definition of $f(x) = \sqrt{\frac{1 - |x|}{2 - |x|}}$ is

A. $(-\infty, -1) \cup (2, \infty)$

B. $[-1, 1] \cup (2, \infty) \cup (-\infty, -2)$

C. $(-\infty, 1) \cup (2, \infty)$

D. $[-1, 1) \cup (2, \infty)$

Answer: A::B::C

Watch Video Solution

2. Given that n numbers of AMs are inserted between two sets of numbers a, 2b and 2a, b where $a, b \in R$. Suppose further that the m th means between these sets of numbers are same, then the ratio a:b equals

A. $n - m + 1 : m$

B. $n - m + 1 : n$

C. $n : n - m + 1$

D. $m : n - m + 1$

Answer: A::B

Watch Video Solution

3. If $x + \log_{10}\left(1 + 2^x\right) = x\log_{10}5 + \log_{10}6$ then the value of x is

A. $\frac{1}{2}$

B. $\frac{1}{3}$

C. 1

D. 2

Answer: A

Watch Video Solution

4. If $Z_r = \sin\frac{2\pi r}{11} - i\cos\frac{2\pi r}{11}$ then $\sum_{r=0}^{10} Z_r =$

A. -1

B. 0

C. $-i$

D. i

Watch Video Solution

5. If z_1 and z_2 be two non-zero complex numbers such that $\frac{z_1}{z_2} + \frac{z_2}{z_1} = 1$, then the origin and the points represented by z_1 and z_2

A. lie on a straight line

B. form a right angled triangle

C. form and equilateral triangle

D. form an isosceles triangle

Answer: B

Watch Video Solution

6. If $b_1b_2 = 2(c_1 + c_2)$ and b_1, b_2, c_1, c_2 are all real numbers, then at least one of the equations $x^2 + b_1x + c_1 = 0$ and $x^2 + b_2x + c_2 = 0$ has

A. real roots

B. purely imaginary roots

C. roots of the form a+ib $(a, b \in R, ab \neq 0)$

D. rational roots

Answer: A

Watch Video Solution

7. The number of selection of n objects from 2n objects of which n are identical and the rest are different is

A. 2^n

B. 2^{n-1}

C. $2^n - 1$

D. $2^{n-1} + 1$

Answer: A::B::C

Watch Video Solution

8. If $(2 \le r \le n)$, then ${}^nC_r + 2.\ {}^nC_{r+1} + {}^nC_{r+2}$ is equal to

A. $2.\ {}^nC_{r+2}$

B. ${}^{n+1}C_{r+1}$

C. ${}^{n+2}C_{r+2}$

D. ${}^{n+1}C_r$

Answer: A::B::C

Watch Video Solution

9. The number $(101)^{100}$ - 1 is divisible by

A. 10^4

B. 10^6

C. 10^8

D. 10^{12}

Answer: A::B::C

Watch Video Solution

10. If n even positive integer, then the condition that the greatest term in the expansion of $(1 + x)^n$ may also have the greatest coefficient is

A. $\frac{n}{n+2} < x < \frac{n+2}{n}$

B. $\frac{n}{n+1} < x < \frac{n+1}{n}$

C. $\frac{n+1}{n+2} < x < \frac{n+2}{n+1}$

D. $\frac{n+2}{n+3} < x < \frac{n+3}{n+2}$

Answer: A

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11. If $\sin 6\theta + \sin 4\theta + \sin 2\theta = 0$, then general value of θ is

A. $\frac{n\pi}{4}, n\pi \pm \frac{\pi}{3}$

B. $\frac{n\pi}{4}, n\pi \pm \frac{\pi}{6}$

C. $\frac{n\pi}{4}, 2n\pi \pm \frac{\pi}{3}$

D. $\frac{n\pi}{4}, 2n\pi \pm \frac{\pi}{6}$

Answer: A::C::D

Watch Video Solution

12. Without changing the direction of the axes, the origin is transferred to the point (2,3). Then the equation $x^2 + y^2 - 4x - 6y + 9 = 0$ changes to

A. $x^2 + y^2 + 4 = 0$

B. $x^2 + y^2 = 4$

C. $x^2 + y^2 - 8x - 12y + 48 = 0$

D. $x^2 + y^2 = 9$

Answer: B::D

Watch Video Solution

13. The point Q is the image of the point P(1,5) about the line y=x and R is the image of the point Q about the line y=-x. The circumcentre is the ΔPQR is

A. (5,1)

B. (-5,1)

C. (1,-5)

D. (0,0)

Answer: A::C

Watch Video Solution

14. The angular points of a triangle are $A(-1, -7), B(5, 1)$ and $C(1, 4)$. The equation of the bisector of the angle $\angle ABC$ is

A. $x = 7y + 2$

B. $7y = x + 2$

C. $y = 7x + 2$

D. $7x = y + 2$

Answer: A::C

Watch Video Solution

15. If one of the diameters of the circle, given by the equation $x^2 + y^2 + 4x + 6y - 12 = 0$, is a chord of a circle S , whose centre is $(2, \; - 3)$, the radius of S is

A. $\sqrt{41}$ unit

B. $3\sqrt{5}$ unit

C. $5\sqrt{2}$ unit

D. $2\sqrt{5}$ unit

Answer: A::B::C::D

Watch Video Solution

16. A chord AB is drawn from the point A(0,3) on the circle $x^2 + 4x + (y - 3)^2 = 0$, and is extended to M such that AM=2AB. The locus of M is

A. $x^2 + y^2 - 8x - 6y + 9 = 0$

B. $x^2 + y^2 + 8x + 6y + 9 = 0$

C. $x^2 + y^2 + 8x - 6y + 9 = 0$

D. $x^2 + y^2 - 8x + 6y + 9 = 0$

Answer: B::C

Watch Video Solution

17. Let the eccentricity of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be reciprocal to that of the ellipse $x^2 + 9y^2 = 9$, then the ratio $a^2 : b^2$ equals

A. $8:1$

B. $1:8$

C. $9:1$

D. $1:9$

Answer: A::B

Watch Video Solution

18. Let A, B be two distinct points on the parabola $y^2 = 4x$. If the axis of the parabola touches a circle of radius r having AB as diameter, the slope of the line AB is

A. $-\frac{1}{r}$

B. $\frac{1}{r}$

C. $\frac{2}{r}$

D. $-\frac{2}{r}$

Answer: B

Watch Video Solution

19. Let $P\left(at^2, 2at\right), Q, R\left(ar^2, 2ar\right)$ be three points on a parabola $y^2 = 4ax$. If PQ is the focal chord and PK, QR are parallel where the coordinates of k is (2a,0), then the value of r is

A. $\dfrac{t}{1 - t^2}$

B. $\dfrac{1 - t^2}{t}$

C. $\dfrac{t^2 + 1}{t}$

D. $\dfrac{t^2 - 1}{t}$

Answer: A::B

Watch Video Solution

20. Let P be a point on the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line through P parallel to the y-axis meets the circle $x^2 + y^2 = 9$ at Q, where P,Q are on the same side of the x-axis. If R is a point on PQ such that $\frac{PR}{RQ} = \frac{1}{2}$, then the locus of R is

A. $\dfrac{x^2}{9} + \dfrac{9y^2}{49} = 1$

B. $\dfrac{x^2}{49} + \dfrac{y^2}{9} = 1$

C. $\dfrac{x^2}{9} + \dfrac{y^2}{49} = 1$

D. $\frac{9x^2}{49} + \frac{y^2}{9} = 1$

Answer: A::B::D

Watch Video Solution

21. Let z_1 and z_2 be complex numbers such that $z_1 \neq z_2$ and $|z_1| = |z_2|$. If $Re(z_1) > 0$ and $Im(z_2) < 0$, then $\frac{z_1 + z_2}{z_1 - z_2}$ is

A. one

B. real and positive

C. real and negative

D. purely imaginary

Answer: A::B

Watch Video Solution

22. From a collection of 20 consecutive natural numbers, four are selected such that they are not consecutive. The number of such selections is

A. 284×17

B. 285×17

C. 284×16

D. 285×16

Answer: A::B::D

Watch Video Solution

23. A line cuts the x-axis at A(5,0) and the y-axis at B(0,-3). A variable line PQ is drawn perpendicular to AB cutting the x-axis at P and the y-axis at Q. If AQ and BP meet at R, then the locus of R is

A. $x^2 + y^2 - 5x + 3y = 0$

B. $x^2 + y^2 + 5x + 3y = 0$

C. $x^2 + y^2 + 5x - 3y = 0$

D. $x^2 + y^2 - 5x - 3y = 0$

Answer: B::C

Watch Video Solution

24. Let A be the centre of the circle $x^2 + y^2 - 2x - 4y - 20 = 0$. Let B(1,7) and (4,-2) be two points on the circle such that tangents at B and D meet at C. The area of the quadrilateral ABCD is

A. 150 sq units

B. 50 sq units

C. 75 sq units

D. 70 sq units

Answer: A

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25. Consider the parabola $y^2 = 4x$. Let P and Q be points on the parabola wher $P(4, -4)$ and $Q(9, 6)$. Let R be a point on the area of the parabola between P and Q. Then the area of ΔPQR is largest when

A. $\angle PQR = 90°$

B. R(4,4)

C. $R\left(\frac{1}{4}, 1\right)$

D. $\left(1, \frac{1}{4}\right)$

Answer: A::D

Watch Video Solution

26. If the equation $x^2 - cx + d = 0$ has roots equal to the fourth powers of the roots of $x^2 + ax + b = 0$, where $a^2 > 4b$, then the roots of $x^2 - 4bx + 2b^2 - c = 0$ will be

A. both real

B. both negative

C. both positive

D. one positive and one negative

Answer: A::D

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27. On the occasion of Dipawali festival each student of a class sends greeting cards to others. If there are 20 students in the class, the number of cards sends by students is

A. ${}^{20}C_2$

B. ${}^{20}P_2$

C. $2 \times {}^{20}C_2$

D. $2 \times {}^{20}P_2$

Answer: A::B::C

Watch Video Solution

28. The area of the triangle formed by the intersectionf of a line parallal to x-axis and passing through P(h,k), with the lines y=x and x+y=2 is h^2. The locus of the point P is

A. x=y-1

B. x=-(y-1)

C. x=1+y

D. x=-(1+y)

Answer: A::C

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29. A hyperbola, having the transverse axis of length $2\sin\theta$ is confocal with the ellipse $3x^2 + 4y^2 = 12$. Its equation is

A. $x^2\sin^2\theta - y^2\cos^2\theta = 1$

B. $x^2 cosec^2\theta - y^2\sec^2\theta = 1$

C. $\left(x^2 + y^2\right)\sin^3\theta = 1 + y^2$

D. $x^2 cosec^2\theta = x^2 + y^2 + \sin^2\theta$

Answer: A::B::C

Watch Video Solution

30. Consider the function $y = \log_a\left(x + \sqrt{x^2 + 1}\right), a > 0, a \neq 1$. The inverse of the function

A. does not exist

B. is $x = \log_{\frac{1}{a}}\left(y + \sqrt{y^2 + 1}\right)$

C. is $x = \sinh$ (ylna)

D. is $x = \cosh\left(- y \text{ In } \frac{1}{a}\right)$

Answer: A

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HS (XI) AND WBJEE 2019 (HS (XI) 2019) (GROUP -A)

1. If $A = \{a, b, c\}$ and $B = \{1, 2\}$, then the number of relations from A to B is

A. 2^6

B. 2^5

C. 2^9

D. 2^8

Watch Video Solution

2. If $z^2 + z + 1 = 0$, then the value of

$$\left(z + \frac{1}{z}\right)^2 + \left(z^2 + \frac{1}{z^2}\right)^2 + \ldots\ldots. + \left(z^7 + \frac{1}{z^7}\right)^2$$ will be

A. 27

B. 45

C. 13

D. 7

Answer: A::B::C

Watch Video Solution

3. The sum of the coeficients in the expansion of $(2x - 3y)^{15}$ will be

A. 0

B. -1

C. 5^{15}

D. 1

Answer: A::B::C

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4. If $\cos\alpha = \frac{1}{\sqrt{5}}\left(0^\circ < \alpha < 90^\circ\right)$ and $\cos\beta = \frac{1}{\sqrt{10}}, \left(270^\circ < \beta < 360^\circ\right)$ then the value of $\sin(\alpha + \beta)$ is

A. $\frac{1}{\sqrt{2}}$

B. $\frac{1}{5\sqrt{2}}$

C. $-\frac{3}{5\sqrt{2}}$

D. $-\frac{1}{5\sqrt{2}}$

Answer: A::B

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5. The equation $\frac{x^2}{10 - \lambda} + \frac{y^2}{4 - \lambda} = 1$ represents an ellipse if

A. $\lambda < 4$

B. $\lambda > 4$

C. $\lambda < 4 < 10$

D. $\lambda > 10$

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6. The value of $\lim_{x \to \frac{\pi}{4}} \frac{\sin x - \cos x}{x - \frac{\pi}{4}}$ is

A. $\sqrt{2}$

B. 2

C. $\frac{1}{\sqrt{2}}$

D. $\frac{1}{2}$

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7. The ratio in which YOZ plane divides the line segment joining the points $(3, -2, -4)$ and $(2, 4, -3)$ is

A. $1:2$

B. $-4:3$

C. $-2:3$

D. $-3:2$

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8. If $f(x) = \frac{e^x}{g(x)}, g(0) = 6, g'(0) = 2$, then f'(0) is

A. 1

B. $\frac{2}{3}$

C. $\frac{1}{9}$

D. $\frac{2}{9}$

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9. If $P(A \cup B) = \frac{3}{4}, P(A \cap B) = \frac{1}{4}, P(A) = \frac{2}{3},$ so the value of $P\left(A^C \cap B\right)$ equals to

A. $\frac{3}{8}$

B. $\frac{5}{12}$

C. $\frac{7}{12}$

D. $\frac{1}{12}$

Answer: A::B

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10. If the values of variable X are $x_1, x_2, ... x_n$, then the variance of $ax_1, ax_2, ... ax_n$ (a is any non-zero real number) is

A. a var (X)

B. a^2 var (X)

C. a^3 var (X)

D. a^n var (X)

Answer: A::B

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HS (XI) AND WBJEE 2019 (HS (XI) 2019) (GROUP -B)

1. If $A = \{x : -1 < x \leq 5\}$ and $B = \{x : -3 \leq x < 4\}$, then find $A \cap B$.

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2. Find the range of the function $f(x) = \frac{1}{2 + \sin 3x}$

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3. Find the value of $\cos^2\frac{\pi}{8} + \cos^2\frac{3\pi}{8} + \cos^2\frac{5\pi}{8} + \cos^2\frac{7\pi}{8}$

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4. Show that the value of $\cos^2\theta + \cos^2(\theta + \alpha)$ - $2\cos\alpha\cos\theta\cos(\theta + \alpha)$ is independents of θ.

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5. If A be the arithmetic mean (A.M.) and p, q be two geometric means (G.M.) between two given numbers then prove that $\frac{p^2}{q} + \frac{q^2}{p} = 2A$.

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6. Find the value of $\sqrt{i} + \sqrt{-i}$.

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7. Prove that ${}^{2n}P_n = \{1.3.5......(2n-1)\}2^n$

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8. Find the coefficient of x^{-11} in the expansion of $\left(x^2 - \frac{1}{x^3}\right)^{12}$

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9. The coordinaters of one end of a diameter of a circle $x^2 + y^2 - 8x - 4y + 15 = 0$ is (2,1). Find the coordinates of the other end of the diameter.

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10. Find the eccentricity of the hyperbola whose latusrectum and transverse axis are of same length.

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11. If $f(x) = \frac{|x|}{x}$ then discuss with justification whether $\lim_{x \to 0} f(x)$ exists or not.

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12. Find the value of $\frac{d}{dx}\left(x\frac{e^x + e^{4x}}{e^x + e^{-2x}}\right)$.

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13. Two dice are thrown simultaneously. What is the probability that the sum of the points on the two dice is 5?

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14. Show the "difference between the arithmetic mean and median can never be greater then the standard deviation".

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HS (XI) AND WBJEE 2019 (HS (XI) 2019) (GROUP -C)

1. For any three sets A, B and C prove that $A - (B \cup C) = (A - B) \cap (A - C)$

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2. Show that the value of $\sin 18^{\circ}$ is $\dfrac{\sqrt{5} - 1}{4}$

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3. For a triangle, if $\frac{1}{a+c}+\frac{1}{b+c}=\frac{3}{a+b+c}$, then show that $\angle C=60^{\circ}$.

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4. Using the principle of mathematical induction, prove that $1^2+3^2+5^2+\ldots\ldots+(2n-1)^2, =\frac{n}{3}\left(4n^2-1\right)$, where $n\in N$.

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5. Show that if $n\geq 1$ is an integer, then $9^{n+1}-8n-9$ is divisible by 64.

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6. How many different numbers of 5 significant digits can be formed with the digits $0, 2, 5, 6, 7$ if repetition is (a) allowed, (b) not allowed?

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7. Solve $: z + |z| = 1 + 2i$, where $z = x + iy, x, y \in \mathbb{R}$.

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8. There numbers are in G.P. whose product and sum are respectively 216 and 21. Find the numbers.

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9. Find the distance from the point P(4,1) to the line 4x-y=0 measured along the line making and angle 135° with the positive direction of x-axis.

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10. Find the equations of the circle which touches y-aixs at (0,5) and whose centre lies on the line 2x+y=13.

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11. Find the locus of the foot of the perpendicular drawn from the origin to the straight line which passes through the fixed point (a,b).

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12. If $\lim_{x \to 3} \frac{px^2 - q}{x - 3} = 6$, then find the values of p and q.

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13. If $y = \sqrt{x} + \frac{1}{2\sqrt{x}}$ and $2x\frac{dy}{dx} + y = f(x)$, then find f(x).

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14. Prove that for any two statements p and q the statements $\sim(p \leftrightarrow \sim q)$ and $p \leftrightarrow q$ are equivalent.

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15. Using contrapositive method, prove that -"If x is an interger and x^2 is an odd number then x will be an odd number".

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16. A number is selected at random among the first 50 positvie integers. Find the probability that the selected number is divisible by 4 or 5.

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17. Calculate the value of : $(i)^{61}$

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HS (XI) AND WBJEE 2019 (HS (XI) 2019) (GROUP -D)

1. Find the general solution of the equation $\sec\theta + 1 = \left(2 + \sqrt{3}\right)\tan\theta$.

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2. Prove that $\cos\frac{\pi}{11}\cos\frac{2\pi}{11}\cos\frac{3\pi}{11}\cos\frac{4\pi}{11}\cos\frac{5\pi}{11} = \frac{1}{32}$

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3. Solve : $\frac{|x + 3| + 2x + 1}{x + 1} > 5, \forall x \in \mathbb{R}, x \neq -1.$

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4. One root of the quadratic equation $(2 + 3i)x^2 - bx + (3 - i) = 0$ is (2-i).

Find its other root and the value of b.

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5. If a_1, a_2, a_3 and a_4 be the coefficients of four consecutive terms in the expansion of $(1+x)^n$, then prove that $\frac{a_1}{a_1+a_2}, \frac{a_2}{a_2+a_3}$ and $\frac{a_3}{a_3+a_4}$ are in A.P.

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6. A committee of members is to be formed among 6 men and 4 women. Find the number of ways this can be done in such a way that each committee has at least one woman and two men.

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7. The focal chord of the parabola $y^2 = 4ax$ makes an angle θ with its aixs. Show that the length of the chord will be $4a\text{cosec}^2\theta$.

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8. Find the equation of hyperbola and length of its latus rectum, whose vertices are $(9, 2), (1, 2)$ and the ecentricity is $\frac{5}{4}$.

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WBJEE 2019

1. The three sides of a right-triangle are in GP (geometric progression). If the two actue angles be α and β, then $\tan\beta$ are

A. $\frac{\sqrt{5}+1}{2}$ and $\frac{\sqrt{5}-1}{2}$

B. $\sqrt{\frac{5+1}{2}}$ and $\sqrt{\frac{5-1}{2}}$

C. $\sqrt{5}$ and $\frac{1}{\sqrt{5}}$

D. $\frac{\sqrt{5}}{2}$ and $\frac{2}{\sqrt{5}}$

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WBJEE 2020

1. If $\log_2 6 + \frac{1}{2x} = \log_2\left(2^{\frac{1}{x}} + 8\right)$, that the values of x are

A. $\frac{1}{4}, \frac{1}{3}$

B. $\frac{1}{4}, \frac{1}{2}$

C. $-\frac{1}{4}, \frac{1}{2}$

D. $\frac{1}{3}, \frac{1}{-2}$

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WBJEE 2021

1. Let z be a complex number such that the principal value of argument, $arg z > 0$. Then $arg z - arg(-z)$ is

A. $\frac{\pi}{2}$

B. $\pm\pi$

C. π

D. -π

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WBJEE 2023

1. Let a, b, c be real numbers such that $a + b + c < 0$ and the quadratic equation $ax^2 + bx + c = 0$ has imaginary roots. Then

A. $a > 0, c > 0$

B. $a > 0, c < 0$

C. $a < 0, c > 0$

D. $a < 0, c < 0$

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WBJEE 2024

1. A candidate is required to answer 6 out of 12 questions which are divided into two parts A and B, each containing 6 questions and he/she is not permitted to attempt more that 4 questions from any part. In how many different ways can he/she make up his/her choice of 6 questions?

A. 850

B. 800

C. 750

D. 700

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WBJEE 2025

1. There are greetings cards, each of a different colour and 7 envelopes of same 7 colours as that of the cards. The number of ways in which the cards can be put in envelopes, so that exactly 4 of the cards go into envelopes of respective colour is,

A. ${}^{7}C_{3}$

B. $2.\ {}^{7}C_{3}$

C. $3!\,{}^{4}C_{4}$

D. $3!\,{}^{7}C_{3}\,{}^{4}C_{3}$

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WBJEE 2026

1. $7^{2n} + 16n - 1 (n \in \mathbb{N})$ is divisble by

A. 65

B. 63

C. 61

D. 64

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2. The number of irrational terms in the expansion of $\left(3^{\frac{1}{6}}+5^{\frac{1}{4}}\right)^{84}$ is

A. 73

B. 78

C. 75

D. 76

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3. Let P and T be the subsets of X-Y plane defined by

$$p = \left\{(x, y) : x > 0, y > 0 \text{ and } x^2 + y^2 = 1\right\}$$

$T = \left\{(x, y) : x > 0, y > 0 \text{ and } x^2 + y^8 < 1\right\}$ The $P \cap T$ is

A. the vaid set ϕ

B. P

C. T

D. $P - T^c$

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4. If $e^{\sin x} - e^{-\sin x} - 4 = 0$, then the number of real values of x is

A. 0

B. 1

C. 2

D. 3

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5. The angles of a triangle are in the ratio $2:3:7$ and the raidus of the circumscirbed circle is 10 cm. The length of the smallest side is

A. 2 cm

B. 5 cm

C. 7 cm

D. 10 cm

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6. A variable line passes through a fixed point (x_1, y_1) and meets the axes at A and B. If the rectangle OAPB be completed, the locus of P is, (O being the origin of the system of axes)

A. $\left(y-y_1\right)^2=4\left(x-x_1\right)$

B. $\frac{x_1}{x}+\frac{y_1}{y}=1$

C. $x^2+y^2=x_1^2+y_1^2$

D. $\frac{x^2}{2x_1^2}+\frac{y^2}{y_1^2}=1$

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7. A straight line through the point (3, -2) is inclined at an angle 60° to the line $\sqrt{3}x+y=1$. If it intersects the X-axis, then its equation will be

A. $y+x\sqrt{3}+2+3\sqrt{3}=0$

B. $y-x\sqrt{3}+2+3\sqrt{3}=0$

C. $y-x\sqrt{3}-2-2\sqrt{3}=0$

D. $y-x\sqrt{3}+2-3\sqrt{3}=0$

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8. A variable line passes through the fixed point (α, β). The locus of the foot of the perpendicular from the origin on the line is,

A. $x^2 + y^2 - \alpha x - \beta y = 0$

B. $x^2 - y^2 - 2\alpha x - 2\beta y = 0$

C. $\alpha x + \beta y \pm \sqrt{\left(\alpha^2 + \beta^2\right)} = 0$

D. $\dfrac{x^2}{\alpha^2} + \dfrac{y^2}{\beta^2} = 1$

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9. If the point of intersection of the lines $2ax + 4ay + c = 0$ and $7bx + 3by - d = 0$ lies in the 4th quadrant and is equidistant from the two axes, where a, b, c and d are non-zero numbers, then $ad : bc$ equals to

A. $2 : 3$

B. $2:1$

C. $1:1$

D. $3:2$

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10. A variable circle passes through the fixed point A(p,q) and touches x-axis. The locus of the other end of the diameter through A is

A. $(x - p)^2 = 4qy$

B. $(x - q)^2 = 4py$

C. $(y - p)^2 = 4qx$

D. $(y - q)^2 = 4px$

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11. If $P(0, 0,)Q(1, 0)$ and $R\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ are three given points, then the centre of the circle for which the lines PQ, QR and RP are the tangents is

A. $\left(\frac{1}{2}, \frac{1}{4}\right)$

B. $\left(\frac{1}{2}, \frac{\sqrt{3}}{4}\right)$

C. $\left(\frac{1}{2}, \frac{1}{2\sqrt{3}}\right)$

D. $\left(\frac{1}{2}, \frac{-1}{\sqrt{3}}\right)$

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12. For the hyperbola $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$, which of the following remains fixed when α varies?

A. directrix

B. vertices

C. foci

D. eccentricity

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13. S and T are the foci of an ellipse and B is the end point of the minor axis. If STB is equilateral triangle, the eccentricity of the ellipse is

A. $\frac{1}{4}$

B. $\frac{1}{3}$

C. $\frac{1}{2}$

D. $\frac{2}{3}$

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14. The equation of the directrices of the hyperbola $3x^2 - 3y^2 - 18x + 12y + 2 = 0$ is

A. $x = 3 \pm \sqrt{\frac{13}{6}}$

B. $x = 3 \pm \sqrt{\frac{6}{13}}$

C. $x = 6 \pm \sqrt{\frac{13}{3}}$

D. $x = 6 \pm \sqrt{\frac{3}{13}}$

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15. P is the extremity of the latusrectum of ellipse $3x^2 + 4y^2 = 48$ in the first quadrant. The eccentric angle of P is

A. $\frac{\pi}{8}$

B. $\frac{3\pi}{4}$

C. $\frac{\pi}{4}$

D. $\frac{\pi}{3}$

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16. For any non-zero complex number z, the minimum value of $|z| + |z - 1|$ is

A. 1

B. $\frac{1}{2}$

C. 0

D. $\frac{3}{2}$

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17. The polar coordinate of point P is $\left(2, \frac{\pi}{4}\right)$. The polar coordinate of the point Q, which is such that the line joining PQ is bisected perpendicularly by the initial line, is

A. $\left(2, \frac{\pi}{4}\right)$

B. $\left(2, \frac{\pi}{6}\right)$

C. $\left(-2, \frac{\pi}{4}\right)$

D. $\left(-2, \frac{\pi}{6}\right)$

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18. The length of conjugate axis of a hyperbola is greater than the length of transverse axis. Then the eccentricity e is,

A. $= \sqrt{2}$

B. $> \sqrt{2}$

C. $< \sqrt{2}$

D. $< \frac{1}{\sqrt{2}}$

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19. The value of $\lim_{x \to 0+} \frac{x}{p}\left[\frac{q}{x}\right]$ is

A. $\frac{[q]}{p}$

B. 0

C. 1

D. ∞

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20. Let x_1, x_2 be the roots of $x^2 - 3x + a = 0$ and x_3, x_4 be the roots of $x^2 - 12x + b = 0$. If $x_1 < x_2 < x_3 < x_4$ and x_1, x_2, x_3, x_4 are in GP then ab equals

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21. If $\theta \in \mathbb{R}$ and $\dfrac{1 - i\cos\theta}{1 + 2i\cos\theta}$ is real number, then θ will be (when I: set of integers)

A. $(2n + 1)\frac{\pi}{2}, n \in I$

B. $\dfrac{3n\pi}{2}, n \in I$

C. $n\pi, n \in I$

D. $2n\pi, n \in I$

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22. Straight lines $x - y = 7$ and $x + 4y = 2$ intersect at B. Points A and C are so chosen on these two lines such that AB=AC. The equation of line AC passing through (2,-7) is

A. $x - y - 9 = 0$

B. $23x + 7y + 3 = 0$

C. $2x - y - 11 = 0$

D. $7x - 6y - 56 = 0$

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23. Equation of a tangent to the hpyerbola $5x^2 - y^2 = 5$ and which passes through an external point (2,8) is

A. $3x - y + 2 = 0$

B. $3x + y - 14 = 0$

C. $23x - 3y - 22 = 0$

D. $3x - 23y + 178 = 0$

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1. A relation R is defined on the set of natural numbers $\mathbb{N}$ as follows :

$(x, y) \in R \Rightarrow y$ is divisible by x, for all x,$y \in \mathbb{N}$.

Show that, R is reflexive and transitive but not symmetric on $\mathbb{N}$.

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2. Let $\mathbb{N}$ be the set of all natural numbers and R be the relation on $\mathbb{N} \times \mathbb{N}$ defined by :

$(a, b)R(c, d) \Rightarrow ad(b + c) = bc(a + d)$

Check wheather R is an equivalance relation on $\mathbb{N} \times \mathbb{N}$.

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3. Let Z be the set of all integers and Z_0 be the set of all non-zero integers. Let a relation R on $\mathbb{Z} \times \mathbb{Z}_0$ be defined as follows :

$(a, b)R(c, d)r \Rightarrow ad = bc,$ for all

$(a, d), (c, d) \in \mathbb{Z} \times \mathbb{Z}_0$

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4. Find the number of equivalence relations on the set A={a,b,c} containing elements (b,c) and (c,b).

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5. Show that the relation R on the set A={1,2,3,4,5} given by R={(a,b):|a-b| is even} is an equivalence relation.

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6. Prove that the relation R on the set A={a in ZZ:1 |x-y| is a multiple of 4} is an equivalence relation.

Find also the elements of set A which are related to 2.

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7. Let P(A) be the power set of a non-empty set A. A relation R on P(A) is defined as follows :

$R = \{(X, Y) : X \subseteq Y\}$

Example (i) reflexivity, (ii)symmetry and (iii) transitivity of R on P(A).

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8. Let $\mathbb{R}$ be the set of real numbers and $A = \{x \in \mathbb{R} : -1 < x < 1\} = B$. Is the mapping $f : A \to B$defined by f(x) $= \dfrac{x}{1 + |x|}$ bijective ? Justify your answer.

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9. Let $\mathbb{R}$ be the set of all real numbers and $A = \{x \in \mathbb{R} : 0 < x < 1\}$. Is the mapping $f : A \to \mathbb{R}$defined by f(x) $= \dfrac{2x - 1}{1 - |2x - 1|}$ bijective ?

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10. If $\mathbb{R}$ is the set of real numbers, then functions $f: \mathbb{R} \rightarrow \mathbb{R}$and $g: \mathbb{R} \rightarrow \mathbb{R}$ are defined respectively by

f(x) $= \cos^2 x + \cos^2\left(\frac{2\pi}{3} + x\right) + \cos^2\left(\frac{2\pi}{3} - x\right)$ and g(x)=2 for all x in $\mathbb{R}$. Show that $(gof): \mathbb{R} \rightarrow \mathbb{R}$ is a constantfunction.

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11. For all $x \in \mathbb{Z}$, the function $f: \mathbb{Z} \rightarrow \mathbb{Z}$ is defined by f(x)=3x+4. Find function $g: \mathbb{Z} \rightarrow \mathbb{Z}$ such that (g o f) $= I_Z$.

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12. Determine the value of the constant $k(\neq 0)$ for which the function f(x)=1+kx is the inverse of itself.

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13. If f(x)=sin x , g(x) $= x^2$ and h(x)=logx, find the composite function [h o (g o f)](x).

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14. Find the domain of definitions of each of the following functions :

$f(x) = \cos^{-1}2x$

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15. Find the domain of definitions of each of the following functions :

$y = \sin^{-1}3x$

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16. Find the domain of definitions of each of the following functions :

$y = \sin^{-1}\left[\log_2\left(\frac{1}{2}x^2\right)\right]$

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17. Find the domain of definitions of each of the following functions :

$$f(x) = \frac{\sqrt{4 - x^2}}{\sin^{-1}(2 - x)}$$

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18. Find the domain of definitions of each of the following functions :

$$y = \sqrt{\sin^{-1}\left(\log_2 x\right)}$$

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19. Find the domain of definitions of each of the following functions :

$$f(x) = \sin^{-1}\left(\frac{2x - 3}{3}\right)$$

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20. Find the domain of definitions of each of the following functions :

$$\phi(x) = \cos^{-1}\frac{x-4}{3} + \log(5-x)$$

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21. Prove that

$$2\tan^{-1}\left[\tan\left(\frac{\alpha}{2}\right)\tan\left(\frac{\pi}{4}-\frac{\beta}{2}\right)\right] = \tan^{-1}\left(\frac{\sin\alpha\cos\beta}{\cos\alpha+\sin\beta}\right)$$

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22. Prove that

$$\tan^{-1}\sqrt{\frac{a(a+b+c)}{bc}} + \tan^{-1}\sqrt{\frac{b(a+b+c)}{ca}} + \tan^{-1}\sqrt{\frac{c(a+b+c)}{ab}} = \pi(a, b, c > 0)$$

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23. Prove that

$$\tan^{-1}\frac{ap-q}{aq+p} + \tan^{-1}\frac{b-a}{ab+1} + \tan^{-1}\frac{c-b}{bc+1} = \tan^{-1}\frac{p}{q} - \tan^{-1}\frac{1}{c}$$

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24. If $\frac{n\tan\theta}{\cos^2(\alpha-\theta)}=\frac{m\tan(\alpha-\theta)}{\cos^2\theta}$, then show that,

$$2\theta=\alpha-\tan^{-1}\left(\frac{n-m}{n+m}\tan\alpha\right)$$

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25. If $\phi=\tan^{-1}\frac{x\sqrt{3}}{2k-x}$ and $\theta=\tan^{-1}\frac{2x-k}{k\sqrt{3}}$, then show that one value of $(\phi-\theta)$ is 30°.

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26. If $\phi=\tan^{-1}\frac{x\sqrt{3}}{2k-x}$ and $\theta=\tan^{-1}\frac{2x-k}{k\sqrt{3}}$, then show that one value of $(\phi-\theta)$ is 30°.

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27. Prove that,

$$\tan^{-1}\frac{1}{2\times 1^2}+\tan^{-1}\frac{1}{2\times 2^2}+\tan^{-1}\frac{1}{2\times 3^2}+\ldots\infty=\frac{\pi}{4}$$

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28. If $\phi = \cot^{-1}\sqrt{\cos 2\theta} - \tan^{-1}\sqrt{\cos 2\theta}$, then show that, $\sin\phi = \tan^2\theta$.

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29. Prove that,

$$\sum_{n=1}^{\infty}\cot^{-1}\left(2n^2\right)=\frac{\pi}{4}$$

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30. Prove that

$$\sum_{r=1}^{n}\tan^{-1}\frac{1}{1+r+r^2}=\tan^{-1}(n+1)-\frac{x}{4}-\frac{\pi}{4},$$ hence deduce that,

$$\sum_{n=1}^{\infty} \tan^{-1}\frac{1}{1+n+n^2} = \frac{\pi}{4}$$

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31. Prove that

$$\sum_{r=1}^{n} \tan^{-1}\frac{2r}{r^4+r^2+2} = \tan^{-1}\left(n^2+n+1\right) - \frac{\pi}{4}$$

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32. Solve :

$\sin^{-1}x - \cos^{-1}x = \sin^{-1}(3x-2)$

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33. Solve :

$\cos^{-1}x - \sin^{-1}x = \cos^{-1}\left(x\sqrt{3}\right)$

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34. Solve that

$2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$

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35. Evaluate : $\cos^{-1}x + \cos^{-1}\left[\frac{x}{2} + \frac{\sqrt{3 - 3x^2}}{2}\right]\left(\frac{1}{2} \le x \le 1\right)$

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36. Prove that $\left(\sin^{-1}\frac{2ab}{a^2 + b^2} + \sin^{-1}\frac{2cd}{c^2 + d^2}\right)$ can be expressed in the form $\sin^{-1}\frac{2xy}{x^2 + y^2}$ where x and y are rational functions of a,b,c and d.

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37. Prove that :

$$\cos^{-1}\sqrt{\frac{2}{3}} - \cos^{-1}\frac{\sqrt{6}+1}{2\sqrt{3}} = \frac{\pi}{6}$$

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38. Prove that :

$$\cos^{-1}b - \sin^{-1}a = \cos^{-1}\left(b\sqrt{1-a^2} + a\sqrt{1-b^2}\right)$$

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39. Using principal values, express the following as a single angle :

$$2\tan^{-1}\frac{1}{\sqrt{3}} + 2\tan^{-1}\sqrt{3}$$

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40. If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$ and $x + y + z = \frac{3}{2}$, then show that, x=y=z.

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41. If $\theta = \tan^{-1}\left(2\tan^2\theta\right) - \frac{1}{2}\sin^{-1}\left(\frac{3\sin 2\theta}{5 + 4\cos 2\theta}\right)$, then find the general values of θ.

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42. If $\tan^{-1}\sqrt{\frac{a^2 - x^2}{a^2 + x^2}} + \tan^{-1}\sqrt{\frac{b^2 - y^2}{b^2 + y^2}} = \frac{\alpha}{2}$, then show that,

$$\frac{x^4}{a^4} - 2\frac{x^2y^2}{a^2b^2}\cos\alpha + \frac{y^4}{b^4} = \sin^2\alpha$$

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43. If $\tan^{-1}\left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)=\theta$, then prove that, $\sin 2\theta = x^2$.

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44. Prove that, $\tan^{-1}a+\tan^{-1}b+\tan^{-1}\left(\frac{1-a-b-ab}{1+a+b-ab}\right)=\frac{\pi}{4}$.

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45. Prove that, $\tan^{-1}\left(\frac{3\sin 2x}{5+3\cos 2x}\right)+\tan^{-1}\left(\frac{1}{4}\tan x\right)=x$.

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46. Prove that, $\tan^{-1}\frac{a}{b}-\tan^{-1}\left(\frac{a-b}{a+b}\right)=\frac{\pi}{4}$.

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47. Prove that , $\tan^{-1}\left(\frac{2\sin 2x}{1+2\cos 2x}\right)-\frac{1}{2}\sin^{-1}\left(\frac{3\sin 2x}{5+4\cos 2x}\right)=x.$

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48. If $\sin^{-1}\frac{x}{a}+\sin^{-1}\frac{y}{b}=\sin^{-1}\frac{c^2}{ab}$, prove that,

$b^2x^2+2xy\sqrt{a^2b^2-c^4}+a^2y^2=c^4$

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49. Prove that, $2\sin^{-1}\frac{2}{\sqrt{13}}+\frac{1}{2}\cos^{-1}\frac{7}{25}+\tan^{-1}\frac{63}{16}=\pi$

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50. If g(x) is the inverse of f(x) an d $f'(x)=\frac{1}{1+x^3}$, show that g'(x) $=1+[g(x)]^3$.

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51. If f(x+h) $= f(x) + hf'(x + \theta h)$, find θ, given $x = -a$, h=2a and f(x) $= \sqrt[3]{x}$.

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Limit, Continuity and Differentiability

1. Evaluate the following limits :

$$\lim_{x \to 0} \frac{\sin\log(1 + x)}{\log(1 + \sin x)}$$

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2. Evaluate the following limits :

$$\lim_{x \to 0} \frac{12^x - 2^{2x} - 3^x + 1}{x^2}$$

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3. Evaluate the following limits :

$$\lim_{x \to a} \frac{ae^{x} - xe^{a}}{x - a}$$

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4. Evaluate the following limits :

$$\lim_{x \to 0} \frac{e^{x} - \log(e + ex)}{x}$$

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5. Evaluate the following limits :

$$\lim_{x \to 0} \frac{e^{px} - e^{-qx}}{x}$$

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6. Evaluate the following limits :

$$\lim_{x \to 0} \frac{e^{-\frac{x}{2}} - 1}{\log(1 - 3x)}$$

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7. Evaluate the following limits :

$$\lim_{x \to \frac{1}{2}} \frac{e^{\log 2x} - 1}{e^{2x-1} - 1}$$

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8. Show that,

$$\lim_{x \to 0} (\cos x)^{\cot^2 x} = \frac{1}{\sqrt{e}}$$

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9. Show that,

$$\lim_{x \to \frac{\pi}{2}} (1 + \cos x)^{3\sec x} = e^3$$

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10. Show that,

$$\lim_{x \to 0} \left(\frac{x - 1 + \cos x}{x}\right)^{\frac{1}{x}} = \frac{1}{\sqrt{e}}$$

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11. Show that,

$$\lim_{x \to 0} (\sin x + \cos x)^{\frac{1}{x}} = e$$

Watch Video Solution

12. Show that,

$$\lim_{x \to 0}\left[\tan\left(x + \frac{\pi}{4}\right)\right]^{\frac{1}{x}} = e^2$$

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13. Show that,

$$\lim_{x \to 0}(1 + \sin 2x)^{\operatorname{cosec} x} = e^2$$

Watch Video Solution

14. Show that,

$$\lim_{x \to 1}\frac{x^x - 1}{x\log x} = 1$$

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15. Show that,

$$\lim_{x \to 0} \frac{\log\left(1 + x + x^2\right) + \log\left(1 - x + x^2\right)}{\sec x - \cos x} = 1$$

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16. Find the value of :

$$\lim_{x \to 0}(1 + 3x)^{\frac{1}{x}}$$

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17. Find the value of :

$$\lim_{x \to 0}(1 - 2x)^{\frac{2}{x}}$$

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18. Find the value of :

$$\lim_{x \to 0}(1 + kx)^{\frac{k}{x}}$$

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19. Find the value of :

$$\lim_{x \to 0}[1 + 3x]^{\frac{x+3}{x}}$$

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20. Prove that, $\lim_{x \to 0}\left(\frac{1 + 6x^2}{1 + 2x^2}\right)^{\frac{1}{x^2}} = e^4$

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21. Let f(x) $= \begin{cases} \frac{1 - \cos 4x}{x^2}, & \text{when} x < 0 \\ a, & \text{when} x = 0 \\ \frac{x}{\sqrt{16 + \sqrt{x}} - 4}, & \text{when} x > 0 \end{cases}$

Determine the value of a, if possible, so that f(x) is continuous at x=0.

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22. Given, f(x) $= \frac{\log\left(1 + x^2 \tan x\right)}{\sin x^3}$, when $x \neq 0$. Find the assigned value of f(0), if f(x) is to be continuous at x=0.

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23. The function f(x) $= \frac{1 - \sin x}{(\pi - 2x)^2}$ is underfined at $x = \frac{\pi}{2}$.Redefine the function f(x) so as to make it continuous at $x = \frac{\pi}{2}$.

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24. A function ϕx is defined as follows :

$$\phi x = \begin{cases} -2\sin x, & \text{when} -\pi \le x \le \frac{\pi}{2} \\ p\sin x + q, & \text{when} -\frac{\pi}{2} < x < \frac{\pi}{2} \\ \cos x, & \text{when} \frac{\pi}{2} \le x \le \pi \end{cases}$$

If θ(x) is continous in the interval $-\pi \le x \le \pi$, find the values of p and q.

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25. Prove that the function f(x)=|x+1|+|x-1| is not differetiable at $x = -1$ and at x=1.

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26. Find the from first principle the differential coefficients of the following functions :

$\tan^{-1}x$

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27. Find the from first principle the differential coefficients of the following functions :

$\sin^{-1}x$

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28. Find from the first principle the differential coefficients of the following functions :

$x\tan^{-1}x$ at x=1

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29. Examine the differentiability of the function f(x)=|sinx-cosx| at $x = \frac{\pi}{4}$.

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30. Examine the differentiability of the function f(x)=|cosx| at $x = \frac{\pi}{2}$.

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31. A function f(x) is defined as follows :

$$f(x) = \begin{cases} x^2 - 2x + 3, & \text{for} x < 1 \\ 2, & \text{for } x = 1 \\ 2x^2 - 5x + 5, & \text{for} x > 1 \end{cases}$$

Examine the continuity of f(x) at x=1.

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32. If f(x) $= \lim_{n \to \infty} \frac{x^n g(x) + h(x)}{x^n + 1}$, show that,

$$f(x) = \begin{cases} h(x), & \text{when} 0 < x < 1 \\ \frac{1}{2}[h(x) + g(x)], & \text{when} x = 1 \\ g(x), & \text{when } x > 1 \end{cases}$$

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Differentiation

1. If y=f{f(x)}, f(0)=0 and f'(0)=5, find $\left[\frac{dy}{dx}\right]_{x=0}$

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2. Find the derivative of |f(x)| with respect to x, hence, write down the derivartive of |cos x|.

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3. Find the differential coefficients of the following functions w.r.t. x:

$\log_3(\log_3 x)$

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4. Find the differential coefficients of the following functions w.r.t. x:

$$x^{\log x} + (\sin x)^x + 5x$$

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5. Find the differential coefficients of the following functions w.r.t. x:

$$\sin^{-1}\left(x^2\sqrt{1-x} - \sqrt{x}\sqrt{1-x^4}\right)$$

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6. Find the differential coefficients of the following functions w.r.t. x:

$$\log(1 + \sin 2x) + 2\log\sec\left(\frac{\pi}{4} - x\right)$$

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7. Find the differential coefficients of the following functions w.r.t. x:

$$\left(\sqrt{x}\right)^x + (x)^{\sqrt{x}}$$

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8. Find the differential coefficients of the following functions w.r.t. x:

$$\sin^{-1}\left[\frac{1}{13}\left(5x+12\sqrt{1-x^2}\right)\right]$$

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9. Find $\frac{dy}{dx}$, when

$$y=e^{x\sin x^3}+(\tan x)^x$$

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10. Find $\frac{dy}{dx}$, when

y=log|x|

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11. Find $\frac{dy}{dx}$, when

$$y = (x\log x)^{\log(\log x)}$$

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12. Find $\frac{dy}{dx}$, when

$$y = \tan^{-1}\frac{\sqrt{x} - 4\sqrt{x}}{1 + 4\sqrt{x^3}}$$

Watch Video Solution

13. Find $\frac{dy}{dx}$, when

$$y = \tan^{-1}\frac{x}{1 + \sqrt{1 - x^2}} + \sin\left(2\tan^{-1}\sqrt{\frac{1 - x}{1 + x}}\right)$$

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14. If $y = \log\left(\tan\frac{x}{2}\right) + \sin^{-1}(\cos x)$ show that,

$\frac{dy}{dx} = \operatorname{cos}ec - 1$

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15. If $x^{\log y} = \log x$, prove that, $\frac{x}{y} \cdot \frac{dy}{dx} = \frac{1 - \log x \log y}{(\log x)^2}$

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16. If $\sqrt{1 - x^4} + \sqrt{1 - y^4} = k\left(x^2 - y^2\right)$, prove that,

$\frac{dy}{dx} = \frac{x\sqrt{1 - y^4}}{y\sqrt{1 - x^4}}$

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17. If $\sqrt{1-x^{2n}}+\sqrt{1-y^{2n}}=a^n\left(x^n-y^n\right)$, prove that,

$$\frac{dy}{dx}=\left(\frac{x}{y}\right)^{n-1}\cdot\sqrt{\frac{1-y^{2n}}{1-x^{2n}}}$$

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18. If $x-\sqrt{a^2-y^2}=a\ \log\frac{a-\sqrt{a^2-y^2}}{y}$, show that,

$$\frac{dy}{dx}=\frac{y}{\sqrt{a^2-y^2}}$$

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19. If $xy=a\left[y+\sqrt{y^2-x^2}\right]$, prove that,

$$x^3\frac{dy}{dx}=y^2\left(y+\sqrt{y^2-x^2}\right)$$

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20. If $y = \frac{1}{3}\log\frac{x+1}{\sqrt{x^2 - x + 1}} + \frac{1}{\sqrt{3}}\tan^{-1}\frac{2x - 1}{\sqrt{3}}$, show that,

$$\frac{dy}{dx} = \frac{1}{x^3 + 1}$$

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21. If $x = \tan\frac{y}{2} + \log\tan\frac{y}{2} - 2\log\left(1 + \tan\frac{y}{2}\right)$, show that,

$$\frac{dy}{dx} = \frac{1}{2}\sin(1 + \sin y + \cos y)$$

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22. If $y = \frac{1}{3}.\frac{a^2 - b^2}{a^2 + b^2}.x\left[\frac{p}{p+1}.p\sqrt{x} + \frac{q}{q+1}.q\sqrt{x}\right]$, prove that,

$$\frac{dy}{dx} = \left(\frac{a+b}{a-b}\right)^{\frac{q+p}{q-p}} \text{ at } x = \left(\frac{a+b}{a-b}\right)^{\frac{2pq}{q-p}}$$

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23. If $3ax^2 = y^2\left(a - x^6\right), \dfrac{dy}{dx} = ?$.

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24. If $f(x) = \log_x(\log x),\ \ \text{then find}\ \ f'(x) at x = e$

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25. IF $y^2\left(1 - x^2\right) = x^2 + 1$, show that, $\left(1 - x^4\right)\left(\dfrac{dy}{dx}\right)^2 = y^4 - 1$.

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26. If $\cos y = \sqrt{\dfrac{\cos 3x}{\cos^3 x}}$, prove that, $\dfrac{dy}{dx} = \sqrt{3\sec x \sec 3x}$.

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27. Find the value of $\frac{dy}{dx}$ in the simplest form when

$$y = \frac{1}{4\sqrt{2}}\log\frac{1 + x\sqrt{2} + x^2}{1 - x\sqrt{2} + x^2} + \frac{1}{2\sqrt{2}}\tan^{-1}\frac{x\sqrt{2}}{1 - x^2}$$

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28. If $y = 2\frac{\sin^{-1}(x - 2)}{\sqrt{6}} - \sqrt{2 + 4x - x^2}$, show that, $\frac{dy}{dx}$ at x=2 is $\frac{2}{\sqrt{6}}$.

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29. If $f'(x) = \sin(\log x)$ and $y = f\left(\frac{2x + 3}{3 - 2x}\right)$, find $\frac{dy}{dx}$.

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30. If $y = \log\frac{a + b\tan\frac{x}{2}}{a - b\tan\frac{x}{2}}$ and $z = \frac{1}{a^2\cos^2\frac{x}{2} - b^2\tan\frac{x}{2}}$, then show that,

$$\frac{dy}{dz} = \frac{ab}{a^2 + b^2}\left(a^2\cot\frac{x}{2} - b^2\tan\frac{x}{2}\right).$$

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31. If $y = \sqrt{x + \sqrt{x + \sqrt{x + \dots\infty}}}$, show that, $\frac{dy}{dx} = \frac{1}{2y - 1}$.

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32. If $y = (\log x)^{(\log x)^{(\log x)^{\dots\infty}}}$ prove that,

$x\log x\frac{dy}{dx} = \frac{y^2}{1 - y\log(\log x)}$

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33. If $y = (\sin x)^{(\sin x)^{(\sin x)^{\dots\infty}}}$ show that,

$\frac{dy}{dx} = \frac{y^2\cot x}{1 - y\log(\sin x)}$

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34. If S_n be the sum of first n terms of a G.P. whose common ratio is r, then show that,

$$(r - 1)\frac{dS_n}{dr} = (n - 1)S_n - nS_{n-1}$$

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35. From the relation $\sin x \sin\left(\frac{\pi}{n} + x\right)\sin\left(\frac{2\pi}{n} + x\right)\ldots \sin\left(\frac{n-1}{n}\pi + x\right) = \frac{\sin x}{2^{n-1}}$ deduce that,

$$\cot x + \cot\left(\frac{\pi}{n} + x\right) + \cot\left(\frac{2\pi}{n} + x\right) + \ldots + \cot\left(\frac{n-1}{n}\pi + x\right) = n\cot nx.$$

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36. Find the differential coefficient of:

x^{nx}

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37. Find solution of

$$\frac{dy}{dx} = \frac{b}{a(b+2y)}.$$

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38. If $x = \sin t\sqrt{\cos 2t}$ and $y = \cos t\sqrt{\sin 2t}$, find $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$.

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39. If $y = e^{x^{e^{x^{\cdots\infty}}}}$,show that, $\frac{dy}{dx} = \frac{y^2 \log y}{x(1 - y\log x\log y)}$.

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40. If $y\sqrt{x^2+1} = \log\left(\sqrt{x^2+1} - x\right)$, show that,

$$\left(x^2+1\right)\frac{dy}{dx} + xy + 1 = 0$$

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41. If $x = \operatorname{cosec}\theta - \sin\theta$ and $y = \operatorname{cosec}^n\theta - \sin^n\theta$, prove that, $\left(x^2+4\right)\left(\frac{dy}{dx}\right)^2 = n^2\left(y^2+4\right)$.

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42. If $y = x^{y^x}$, prove that, $\frac{dy}{dx} = \frac{y\log y(1 + x\log x\log y)}{x\log x(1 - x\log y)}$.

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43. If $y(>0)$ is a differentiable function of x, find $\frac{d}{dx}\left(y^y\right)$.

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44. If f(x) $= \tan^{-1}\left(\frac{x}{1+20x^2}\right)$, show that,

$f'(x) = \frac{5}{1+25x^2} - \frac{4}{1+16x^2}$

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45. Find the differential coefficient of:

$\left(x^4 - 1\right)\left(x^4 + 1\right)$

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46. If $x^2 + y^2 = t - \frac{1}{t}$ and $x^4 + y^4 = t^2 + \frac{1}{t^2}$, show that,

$x^3y\frac{dy}{dx} = 1$

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47. Find the differential coefficient of:

$\frac{x^4 - 1}{x^2 + 1}$

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48. Find the differential coefficient of:

$$\frac{x^6 - 1}{x^2 - 1}$$

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49. If $y = \tan^{-1}\frac{4x}{1 + 5x^2} + \tan^{-1}\frac{2 + 3x}{3 - 2x}$, prove that,

$$\frac{dy}{dx} = \frac{5}{1 + 25x^2}$$

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50. Find the differential coefficient of:

$x^5 - 5x^2$

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51. Find the differential coefficient of:

n^x

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52. If $2y = x\left(1 + \frac{dy}{dx}\right)$, prove that, $\frac{d^2y}{dx^2}$=constant.

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53. If $y = x\log\left(\frac{1}{ax} + \frac{1}{a}\right)$, prove that, $x(x + 1)y_2 + xy_1 = y - 1$.

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54. If $p^2 = a^2\cos^2\theta + b^2\sin^2\theta$, show that, $p + \frac{d^2p}{d\theta^2} = \frac{a^2b^2}{p^3}$.

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55. If $(a + bx)e^{\frac{y}{x}} = x$, show that, $x^3\frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2$.

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56. If $(x - a)^2 + (y - b)^2 = r^2$, show that, $\frac{\left(1 + y_1^2\right)^{\frac{3}{2}}}{y_2} = - r.$

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57. If $y = px^n + qx^{-n}$, show that $x^2y_2 + xy_1 - n^2y = 0.$

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58. Find the differential coefficient of:

$x^6 + 6x^4 - 4x^2$

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59. IF $x^2 + xy + y^2 = a^2$, show that, $(x + 2y)^3\frac{d^2y}{dx^2} + 6a^2 = 0.$

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60. If $x = \sqrt{3}(3\sin\theta + \sin 3\theta), y = \sqrt{3}(3\sin\theta + \cos 3\theta)$, find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{3}$.

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61. IF $y = f\left(x^2\right)$ and $f'(x) = \sqrt{3x^2 + 1}$, find $\left[\frac{dy}{dx}\right]_{x=2}$.

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62. If $y = xe^{-\frac{1}{x}}$, prove that, $x^3y_2 - xy_1 + y = 0$.

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63. If $y = \frac{x\sin^{-1}x}{\sqrt{1 - x^2}} + \log\sqrt{1 - x^2}$, show that,

$$\left(1 - x^2\right)^2\frac{D^2y}{dx^2} - 3x\left(1 - x^2\right)\Bigg)\frac{dy}{dx} = 1$$

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64. If $x^3 + y^3 = 3ax^2$, show that, $\frac{d^2y}{dx^2} + \frac{2a^2x^2}{y^5} = 0.$

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65. If $x = \tan t$ and $y = \tan pt$, prove that,

$\left(1 + x^2\right)y_2 + 2(x - py)y_1 = 0$

Watch Video Solution

66. If $e^x + x = e^y$, Find, $\frac{d^2y}{dx^2}$.

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67. If $y = e^{ax}\cos bx$, Then find $\left[\frac{d^2y}{dx^2}\right]_{x=0}$

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68. Find the differential coefficient of:

$x^3 - 9x - 6$

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69. If $x\sqrt{1 - y^2} + y\sqrt{1 - x^2} = k$, find $\left[\frac{d^2y}{dx^2}\right]_{x=0}$

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70. Find the differential coefficient of:

$x^n - nx$

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71. If $\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{n}\right)^n$, prove that, $x^2y_2 + xy_1 + n^2y = 0$.

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72. If $x + y = e^{x-y}$, prove that, $\frac{d^2y}{dx^2} = \frac{4(x+y)}{(x+y+1)^3}$.

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73. If $u\sin t + v\cos t = 5$ and $u\cos t - v\sin t = 7$, find the value of uvu v, where u,v denote the first and second derivatives of u with respect to t and v,u have sililar meanings.

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74. If $y = x^n[a\cos(\log x) + b\sin(\log x)]$, show that,

$$x^2\frac{d^2y}{dx^2} + (1-2n)x\frac{dy}{dx} + \left(n^2+1\right)y = 0$$

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75. If $y = \frac{1}{3}\log\frac{x+1}{\sqrt{x^2 - x + 1}} + \frac{1}{3}\tan^{-1}\frac{2x-1}{\sqrt{3}}$, show that,

$$\frac{d^2y}{dx^2} = -\frac{3x^2}{\left(1 + x^3\right)^2}$$

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76. If $f(x + y + z) = f(x)f(y)f(z) \neq 0$for all x,y,z and f(2)=5, f'(0)=2, find f'(2).

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77. If f(x+y)=f(x)f(y) for all x,y and f(x)=1+xg(x), where $\lim_{x \to 0} g(x) = 1$, show that, f'(x)=f(x).

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78. If f(x) is differentiable at x=a, find the value of

$$\lim_{x \to a} \frac{(x + a)f(x) - 2af(a)}{x - a}.$$

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79. If $f(x)$ is differentiable and $f'(4) = 5,$ then the vlaue of

$$\lim_{x \to 2} \frac{f(4) - f\left(x^2\right)}{x - 2}$$ is equal to-

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80. If $h(x) = [f(x)]^2 + [g(x)]^2$ and f'(x)=g(x),

$f''(x) = -f(x)$, h(5)=10 find h(10).

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81. If f(x+y)=f(x)f(y) for all real x and y and f(5)=2, f'(0)=3 ,find f'(5).

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82. Let $f\left(\frac{x+y}{2}\right) = \frac{1}{2}[f(x) + f(y)]$ for all real x and y. If f'(0) exists and equals (-1), $f(0) = 1$, find f(2).

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83. If $y = \left(\sin^{-1}x\right)^2 + \left(\cos^{-1}x\right)^2$, then show that, $\left(1 - x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 4$

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84. If $y = t^2 + t^3$ and $x = t - t^4$, then find $\frac{d^2y}{dx^2}$.

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Integral Calculus

1. $\int \frac{e^{4x} - 2e^{3x} + 5e^{x} - 2}{e^{x} + 1} dx$

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2. $\int \frac{dx}{\left(e^{x} - 1\right)^{2}}$

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3. $\int \frac{x^{2} - 1}{x^{4} + 1} dx$

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4. $\int \frac{x^{2} + 1}{x^{4} + 1} dx$

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5. $\int \frac{\sqrt{x}dx}{1 + 4\sqrt{x^3}}$

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6. $\int \frac{\sqrt{1 + x^2}}{x^4} dx$

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7. $\int \sqrt{a^{\frac{1}{3}} + x^{\frac{1}{3}}} dx$

Watch Video Solution

8. $\int \frac{dx}{x\sqrt{x^4 - 1}}$

Watch Video Solution

9. $\int \frac{x^2+1}{x^4+x^2+1}dx$

Watch Video Solution

10. $\int \frac{(x^2-1)dx}{x^4+x^2+1}$

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11. $\int \frac{(x^2-1)dx}{x\sqrt{x^4+1}}$

Watch Video Solution

12. $\int \frac{dx}{x^2-4x+13}$

Watch Video Solution

13. $\int e^x\left(\frac{1 - x}{1 + x^2}\right)^2 dx$

Watch Video Solution

14. $\int \frac{1 - x^2}{1 + x^2} \cdot \frac{dx}{\sqrt{1 + x^4}}$

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15. Evaluate: $\int \frac{dx}{2\sin x + \sec x}$

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16. $\int \frac{dx}{\sin^4 x + \cos^4 x}$

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17. $\int \frac{\left(x^4 + 1\right)dx}{\left(1 - x^4\right)^{\frac{3}{2}}}$

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18. $\int \frac{dx}{1 + \sin x - \cos x}$

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19. $\int \frac{dx}{\cos x(5 + 3\cos x)}$

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20. $\int \frac{dx}{\sin x(a + b\cos x)}$

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21. $\int \frac{x^7 dx}{x^{12} - 1}$

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22. $\int \frac{dx}{\sqrt{x} + \sqrt{x - 2}}$

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23. $\int \frac{e^{2x}}{e^{2x} + 4} dx$

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24. $\int \frac{dx}{x^4 + 4}$

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25. $\int \frac{(\sin\theta - \cos\theta)d\theta}{(\sin\theta + \cos\theta)\sqrt{\sin^2\theta\cos^2\theta + \sin\theta\cos\theta}}$

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26. $\int \frac{\sqrt{\tan x} - \sqrt{\cot x}}{1 + 3\sin 2x}dx$

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27. $\int \sqrt{\cot x}dx$

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28. $\int x\sin^{-1}\sqrt{\frac{2a - x}{2a}}dx$

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29. $\int \frac{dx}{\sin x + \sec x}$

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30. $\int \frac{(x+1)dx}{x\left(1+xe^{x}\right)^{2}}$

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31. $\int \frac{dx}{\cos^{3} x\sqrt{\sin 2x}}$

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32. $\int \frac{\left(x^{2}-1\right)dx}{x\sqrt{x^{4}+3x^{2}+1}}$

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33. $\int \cos 2\theta \log\left(\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}\right)d\theta$

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34. $\int \frac{x3\sqrt{x^2} + 26\sqrt{x}}{x\left(1 + 3\sqrt{x}\right)}$

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35. $\int\left(\frac{\log\left(1 + 6\sqrt{x}\right)}{3\sqrt{x} + \sqrt{x}} + \frac{1}{3\sqrt{x} + 4\sqrt{x}}\right)dx$

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36. $\int \frac{dx}{1 + n\sqrt{x + 1}}$, n is a positive integer.

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37. $\int \frac{\sin x + \cos x}{\sin^4 x + \cos^4 x} dx$

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38. Evaluate: $\int \frac{dx}{\sqrt{\sin^3 x \sin(x + \alpha)}}$

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39. $\int \frac{dx}{\sqrt{2e^x - 1}} = 2\sec^{-1}\left(\sqrt{2}e^{\frac{x}{2}}\right) + c$

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40. $\int \frac{1 + \sqrt{x}}{1 + 4\sqrt{x}} dx = \frac{4}{5}x^{\frac{5}{4}} - x + \frac{8}{3}x^{\frac{3}{4}} - 4x^{\frac{1}{2}} + 8x^{\frac{1}{4}} - 8\log\left|x^{\frac{1}{4}} + 1\right| + c$

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41. $\int\left[\sqrt{\frac{a+x}{a-x}}-\sqrt{\frac{a-x}{a+x}}\right]dx=-2\sqrt{a^2-x^2}+c$

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42. $\int\frac{dx}{\left(a+\sqrt{x}\right)^{\frac{3}{2}}}=4\sqrt{1+\sqrt{x}}+\frac{4}{\sqrt{1+\sqrt{x}}}+c$

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43. $\int\frac{\sqrt{x}dx}{a+x}$

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44. $\int\frac{\sqrt{x^2+4x}}{x^2}dx$

Watch Video Solution

45. $\int \frac{\tan x dx}{1 - \sin x} = \frac{1}{2}\left[\frac{1}{1 - \sin x} + \log|\sec x - \tan x|\right] + c$

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46. $\int \frac{dx}{\sqrt{1 + \sqrt{x}}} = \frac{4}{3}\left(\sqrt{x} - 2\right)\sqrt{1 + \sqrt{x}} + c$

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47. $\int \frac{x^2 dx}{(x\sin x + \cos x)^2}$

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48. $\int \frac{1 + x^2}{1 - x^2} \cdot \frac{dx}{\sqrt{1 + x^4}} = \frac{1}{\sqrt{2}}\log\left|\frac{\sqrt{1 + x^4} + x\sqrt{2}}{1 - x^2}\right| + c$

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49. $\int \frac{1+x^2}{1-x^2} \cdot \frac{dx}{\sqrt{1+x^2+x^4}}$

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50. $\int \frac{\sqrt{\left(1-\sqrt{x}\right)}}{1+\sqrt{x}} \cdot \frac{dx}{x}$

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51. $\int \sqrt{1+\operatorname{cosec} x}\,dx$

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52. $\int \frac{\sqrt{x}dx}{\sqrt{a^3-x^3}}$

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53. Evaluate:

$$\int x^{\frac{13}{2}}\left(1+x^{5/2}\right)^{\frac{1}{2}}dx$$

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54. $\int \frac{x^{24}dx}{x^{10}+1}$

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55. $\int \frac{x^{2}dx}{x^{4}+1}$

Watch Video Solution

56. $\int \frac{xdx}{x^{4}-x^{2}+1}$

Watch Video Solution

57. $\int \frac{dx}{\left(1+x^4\right)^{\frac{1}{4}}}$

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58. $\int \frac{x^2 dx}{(x\cos x - \sin x)^2}$

Watch Video Solution

59. $\int \left[\frac{1-\cos x}{\cos x(1+\cos x)(2+\cos x)}\right]^{\frac{1}{2}} dx$

Watch Video Solution

60. $\int \frac{dx}{x^4+x^2+1}$

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61. If f(x) $= \int \frac{2\sin x - \sin 2x}{x^3} [x \neq 0]$, evaluate $\lim_{x \to 0} f'(x)$.

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62. If $I_n = \int \sin^n x dx$, show that,

$I_n = -\frac{1}{n}\sin^{n-1}x\cos x + \frac{n-1}{n}. I_{n-2}$. Hence, evaluate,

$\int \sin^6 x dx$.

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63. If $I_n = \int \cos^n x dx$, prove that,

$I_n = \frac{1}{n}\cos^{n-1}x\sin x + \frac{n-1}{n}. I_{n-2}$.Hence, evaluate,

$\int \cos^5 x dx$.

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64. Putting $b\tan^2 x = a\tan^2\theta$, prove that,

$$\int \frac{dx}{\left(a\cos^2 x + b\sin^2 x\right)^2} = \frac{(a+b)\theta}{2(ab)^{\frac{3}{2}}} - \frac{a-b}{4(ab)^{\frac{3}{2}}}.\sin 2\theta + c.$$

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65. If $\phi(x) = f(x) + xf'(x)$, then the value of $\int \phi(x)dx$ is -

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66. Using the definition of definite integral as the limit of a sum, evaluate :

$\int_a^b (2x+3)dx$

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67. Using the definition of definite integral as the limit of a sum, evaluate :

$\int_a^b 3k - 9dx$

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68. evaluate :

$\int_a^b \sqrt{x} dx$

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69. evaluate :

$\int_a^b \frac{dx}{\sqrt{x}}$

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70. Using the definition of definite integral as the limit of a sum, evaluate :

$\int_a^b 2^x dx$

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71. $\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9 + 16\sin 2x} dx$

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72. $\int_1^e \frac{(x+1)^3}{x^2} \log x dx$

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73. $\int_0^{\frac{\pi}{4}} \sec x \sqrt{\frac{1 - \sin x}{1 + \sin x}} dx$

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74. $\int_0^{\pi} \sec x \sqrt{\frac{1 - \sin x}{1 + \sin x}} dx$

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75. $\int_0^{\frac{\pi}{4}} \frac{\sec x dx}{1 + 2\sin^2 x}$

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76. $\int_0^1 \log\left[\sqrt{1 - x} + \sqrt{1 + x}\right]dx$

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77. $\int_0^{\frac{\pi}{2}} \log(\cot x)dx$

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78. $\int_0^3 \frac{2x^5 + x^4 - 2x^3 + 2x^2 + 1}{\left(x^2 + 1\right)\left(x^4 - 1\right)}dx$

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79. $\int_0^{\pi} \frac{\sqrt{2}x dx}{\sqrt{2} + \sin x}$

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80. $\int_0^{\frac{\pi}{2}} \left(\sqrt{\tan x} + \sqrt{\cot x}\right) dx$

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81. Evaluate :

$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} |\sin x| dx$

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82. Evaluate :

$\int_{-1}^{2} \left|1 - x^2\right| dx$

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83. Evaluate :

$$\int_{-2}^{2}(|x| + |x - 1|)dx$$

Watch Video Solution

84. Evaluate :

$$\int_{0}^{\pi}(\sin x + \cos x)dx$$

Watch Video Solution

85. Evaluate :

$$\int_{0}^{\pi}\sqrt{\frac{1}{2}(1 + \cos 2x)}dx$$

Watch Video Solution

86. Evaluate :

$$\int_{-1}^{\frac{3}{2}} |x\sin \pi x| dx$$

Watch Video Solution

87. Evaluate :

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{\cos x - \cos^3 x} dx$$

Watch Video Solution

88. Evaluate :

$$\int_{\left(\frac{1}{e}\right)}^{e} |\log x| dx$$

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89. Show that, $\int_0^{n\pi + v} |\sin x| dx = 2n + 1 - \cos v$, where n is a positive integer and $0 \le v \le \pi$.

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90. $\int_0^1 \frac{dx}{(x+2)(x+1)}$

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91. $\int_0^{\frac{\pi}{2}} \frac{\cos^{\frac{3}{2}} x dx}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}$

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92. $\int_0^{\pi} \frac{x dx}{1 + \cos\alpha \sin x}$

Watch Video Solution

93. $\int_0^1 \frac{dx}{1 - x + x^2}$

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94. $\int_0^{\frac{\pi}{2}} \frac{dx}{\left(a^2\cos^2 x + b^2\sin^2 x\right)^2} = \frac{\pi}{4}\cdot\frac{a^2+b^2}{a^3b^3}$

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95. $\int_0^1 \frac{\log(1+x)dx}{1+X^2} = \frac{\pi}{8}\log 2$

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96. $\int_{-\infty}^{\infty} \frac{1}{1+x^2}dx$

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97. If m and n are integers and $m \neq n$, then show that,

$$\int_0^{\pi} \sin mx \cos nx dx = \begin{cases} \frac{2m}{m^2-n^2} & \text{when}(m-n) \text{ is odd} \\ 0 & \text{when}(m-n) \text{is even} \end{cases}$$

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98. $\int_0^1 x\log x dx$

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99. $\int \frac{dx}{e^x - 1}$

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100. $\int_0^{\frac{\pi}{2}} \frac{\cos^2 x \sin x}{\sqrt{1 + \cos^2 x}} dx$

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101. $\int_0^5 \frac{x^3}{x^3 + (5 - x)^3} dx$

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102. $\int_0^{\infty} \frac{dx}{\left[x + \sqrt{1 + x^2}\right]^n}$

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103. $\int_0^{\frac{\pi}{6}} \frac{\sqrt{3\cos 2x - 1}}{\cos x} dx$

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104. $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1 + \cos x}$

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105. $\int_{-\pi}^{\pi} \frac{2x(1 + \sin x)}{1 + \cos^2 x} dx$

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106. $\int_0^{\pi} \frac{x \tan x dx}{\sec x + \tan x}$

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107. $\int((x-a)(b-x))dx$

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108. $\int_0^1 \tan^{-1}\left(1-x+x^2\right)dx$

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109. $\int_0^1 \tan^{-1}\frac{2x-1}{1+x-x^2}dx$

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110. $\int_0^{\frac{\pi}{2}} \frac{\cos x dx}{1+\sin^2 x}$

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111. $\int_0^{\pi} \frac{x dx}{1+\cos\alpha \sin x}[0<\alpha<\pi]$

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112. $\int_0^{\infty} \log\left(x+\frac{1}{x}\right)\frac{dx}{1+x^2}$

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113. $\int_0^2 \frac{(x-1)^2 \sin(x-1)dx}{(x-1)^2+\cos(x-1)}$

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114. $\int_0^{\frac{\pi}{2}} \frac{\sin x dx}{1 - \cos^2 + \cos^4 x}$

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115. Evaluate $\int_0^1 x\log(1 + x)dx$ and hence show that,

$$\frac{1}{1 \times 3} - \frac{1}{2 \times 4} + \frac{1}{3 \times 5} - \frac{1}{4 \times 6} + .. = \frac{1}{4}$$

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116. If $(1 + x)^n = C_0 + C_1x + C_2x^2 + \ldots + C_nx^n$, show that,

$$\frac{C_0}{1} + \frac{C_1}{2} + \frac{C_2}{3} + \ldots + \frac{C_n}{n + 1} = \frac{2^{n+1}}{n + 1}$$

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117. If $(1 + x)^n = C_0 + C_1x + C_2x^2 + \ldots + C_nx^n$, show that,

$$\frac{2^2.C_0}{1 \times 2} + \frac{2^3.C_1}{2 \times 3} + \ldots + \frac{2^{n+2}.C_n}{(n + 1)(n + 2)} = \frac{3^{n+2} - 2n - 5}{(n + 1)(n + 2)}$$

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118. Evaluate : $\int \frac{2x}{1+x^2}dx$

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119. Evaluate : $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}[f(x)+f(-x)][g(x)-g(-x)]dx$

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120. Given $\frac{\sin 2kx}{\sin x} = 2[\cos x + \cos 3x + \ldots + \cos(2k-1)x]$, where k is a positive integer, show that,

$\int_0^{\frac{\pi}{2}} \sin 2kx \cot x dx = \frac{\pi}{2}.$

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121. Prove that, $\int_0^{\pi} \log(1+\cos x)dx = -\pi\log 2$, given $\int_0^{\frac{\pi}{2}} \log((\sin x))dx = \frac{\pi}{2}\log\frac{1}{2}$.

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122. Evaluate : $\int_0^{\pi} \frac{x^2 \sin 2x \sin\left(\frac{\pi}{2}\cos x\right)}{2x-\pi}dx$

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123. If f(z) is an odd function, prove that, $\int_a^x f(z)dz$ is an even function.

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124. Prove that, $\int_0^{\frac{\pi}{2}} \cos^n x \cos nx\, dx = \frac{\pi}{2^{n+1}}$.

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125. Evaluate $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin 2x dx}{1 + \cos x}$

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126. Let f(x) be a function satisfying f'(x)=f(x) with f(0)=1 and g(x) be the function satisfying $f(x) + g(x) = x^2$.Prove that,

$\int_0^1 f(x)g(x)dx = \frac{1}{2}\left(2e - e^2 - 3\right)$

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127. Evaluate :

$$\lim_{n \to \infty} \frac{1}{n}\left[\sin\left(\frac{\pi}{2n}\right) + \sin\left(\frac{2\pi}{2n}\right) + \sin\left(\frac{3\pi}{2n}\right) + \dots + \sin\left(\frac{n\pi}{2n}\right)\right]$$

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128. Evaluate :

$$\lim_{n \to \infty} \frac{1}{n}\left[\tan\frac{\pi}{4n} + \tan\frac{2\pi}{4n} + \tan\frac{3\pi}{4n} + \ldots + \tan\frac{n\pi}{4n}\right]$$

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129. Evaluate :

$$\lim_{n \to \infty} \left[\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\left(1+\frac{3}{n}\right)\ldots\left(1+\frac{n}{n}\right)\right]^{\frac{1}{n}}$$

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130. Evaluate :

$$\lim_{n \to \infty} \left[\left(\left(1+\frac{1^2}{n^2}\right)\right)\left(1+\frac{2^2}{n^2}\right)\left(1+\frac{3^2}{n^2}\right)\ldots\left(1+\frac{n^2}{n^2}\right)\right]^{\frac{1}{n}}$$

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131. Evaluate :

$$\lim_{n \to \infty} \left[\frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2} + \ldots + \frac{1}{4n}\right]$$

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132. Evaluate :

$$\lim_{n \to \infty} \left[\frac{n!}{n^n}\right]^{\frac{1}{n}}$$

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133. Evaluate :

$$\lim_{n \to \infty} \left[\frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2} + \ldots + \frac{1}{3n}\right]$$

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134. Evaluate :

$$\lim_{n \to \infty} \frac{n}{(n!)^{\frac{1}{n}}}$$

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135. Evaluate :

$$\lim_{n \to \infty} \left[\frac{(2n)!}{n!n^n}\right]^{\frac{1}{n}}$$

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Differential Equation

1. Solve:

$$(1 + xy)ydx + (1 - xy)xdy = 0$$

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2. Solve:

$x^2(xdx + ydy) + 2y(xdy - ydx) = 0$

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3. Solve:

$x\frac{dy}{dx} + 2y = \sqrt{1 + x^2}$, given y=1, when x=1

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4. $\left(x + 2y^3\right)\frac{dy}{dx} = y$

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5. Solve:

$\left(xy^2 - e^{\frac{1}{x^3}}\right)dx - x^2ydy = 0$, given y=0, when x=1

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6. Solve:

$$\frac{dy}{dx} = \frac{1}{x\cot y + \sec y}$$

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7. Solve:

$$\frac{dy}{dx} - \frac{\tan y}{1 + x} = (1 + x)e^{x}\sec y$$

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8. Determine the equation of the curve passing through the origin, in the form y=f(x) which satisfies the differential equation $\frac{dy}{dx} = \sin(10x + 6y)$.

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9. Solve : $\frac{dy}{dx} = \frac{\cos\left(\log_{e^x}\right)}{\log_{e^y}}$

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10. If $\int_0^x f(t)dt = x + \int_x^1 tf(t)dt$, find the value of f(1).

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11. The tangent at any point P to a curve C intersects the coordinate axes at A and B. If P be the mid-point of the line segment AB and the curve passes through the point (1,1), find the equation of the curve C.

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12. The differential equation $t^2\frac{d^2y}{dt^2} + \alpha t.\ \frac{dy}{dt} + \beta y = 0$ is known as Euler's equation. Show that $y = t^r$ is a solution of Euler's equation if

$r^2 + (\alpha - 1)r + \beta = 0.$

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13. Solve : $x\cos x \frac{dy}{dx} + y(x\sin x + \cos x) = 1$

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14. $y\frac{dy}{dx} = xe^{x^2 + y^2}$

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15. $e^{x-y}dx + e^{y-x}dy = 0$

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16. Find the differential equation of the family of circles which touch the coordinate axes in the third quadrant.

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Application of Calculus

1. The adiabatic law for the expansion of a gas is given by the equation $pv^{1.4} = k$, where k is a constant. At a given time when p is 50 dynes per square cm and v is 20 c c, then v is decreasing at the rate of 2 c c, per second. Find the rate of change of p at that instant.

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2. The pressure p and the volume v of a gas are connected by the equation $pv^{1.4} = k$, where k is a constant. Prove that a decrease of $0.5\,\%$ in the volume of the gas corresponds to an increase of $0.7\,\%$ in the pressure.

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3. Prove that, the function $\dfrac{\sin\theta + 2\cos\theta}{3\sin\theta + 4\cos\theta}$ decreases for all real values of θ.

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4. Find the intervals of monotonicity of the function $y = 2x^2 - \log|x|[x \neq 0]$.

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5. Find the intervals in which the function

f(x) $= 3\cos^4 x + 10\cos^3 x + 6\cos^2 x - 3(0 \leq x \leq \pi)$

is monotonically increasing or decreasing.

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6. Prove that, $2\sin x + \tan x \geq 3x$,where $0 \leq x \leq \dfrac{\pi}{2}$.

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7. Prove that, f(x) = $\frac{1 - 2x - x^2}{1 + x - 2x^2}$ continually diminishes as x continually increases.

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8. If $ax + \frac{b}{x} \geq c$, for all positive values of x, then show that, $4ab \geq c^2$, where a,b,c are positive constants.

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9. For all positive values of x, if $ax^2 + \frac{b}{x} \geq c$(where $a \geq 0, b \geq 0$), prove that, $27ab^2 \geq 4c^3$.

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10. Show that, $\frac{\sin x}{x}$ decreases steadily and $\frac{\tan x}{x}$ increases monotonically in $0 < x < \frac{\pi}{2}$ and also $\frac{\tan x}{x} > \frac{\sin x}{x}$.

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11. For all x in $0 \leq x \leq \frac{\pi}{2}$, show that,

cos(sin x) > sin(cos x)

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12. The rate of decay of a radioactive substance at any time is proportional to its mass at that instant. If m_0 be the mass of the substance at time t=0, find the law of variation of its mass as a function of time t.

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13. Prove that the curve $x = 1 - 3t^2, y = t - 3t^3$ is symmetrical with respect to x-axis. If the tangent to the curve makes an angle $\psi$with the positive x-axis, show that, $\tan\psi \pm \sec\psi = 3t$.

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14. Tangents are drawn from the origin to the curve $y = \sin x$. Prove that their points of contact lie on $x^2y^2 = x^2 -^2$.

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15. Prove that all points on the curve $y^2 = 4a\left[x + a\sin\left(\frac{x}{a}\right)\right]$ at which the tangent is parallel to the x-axis lie on a parabola.

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16. The curve $y = ax^3 + bx^2 + cx + 5$ touches the x-axis at $P(-2, 0)$ and cuts the y-axis at a point Q where its gradient is 3. Find a,b,c.

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17. In the curve $x^m y^n = k^{m+n}(m, n, k > 0)$ prove that the portion of the tangent intercepted between the coordinate axes is divided at its point of contact in a constant ratio,

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18. Find the abscissa of the point on the curve $xy = (c + x)^2$, the normal at which cuts off numerically equal intercepts from the axes of coordinates.

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19. If p,q be the portions of the intercepts upon the coordinate axes by the tangent to the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ at the point $\left(x_1, y_1\right)$ thenprove that (p,q) lies on the circle $x^2 + y^2 = a^2$.

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20. If c=a+b, then show that the curves $x^{\frac{2}{3}} + y^{\frac{2}{3}} = c^{\frac{2}{3}}$ and $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ touch each other.

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21. Find the equation of the normal to the curve $x^3 + y^3 = 8xy$ at the point where it meets the curve $y^2 = 4x$, other than origin.

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22. Three normals are from the point (c,0) to the curve $y^2 = x$. Show that c must be greater than $\frac{1}{2}$.One normal is always the x-axis. Find c for which the other two normals are perpendicular to each other.

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23. If p_1 and p_2 be the lengths of the perpendiculars from the origin on the tangent and normal ot the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ at the point (x_1, y_1), than show that, $4p_1^2 + p_2^2 = a^2$.

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24. A spherical ice-ball is melting the radius decreasing at a constant rate of 0.1 cm per second. Find the amount of water formed in one second when the radius of the sphere is 7 cm.[Given $\pi = \frac{22}{7}$, sp.gr ice $= 0.9$].

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25. Show that the straight line $x\cos\alpha + y\sin\alpha = p$ is a tangent to the curve

$\frac{x^m}{a^m} + \frac{y^m}{b^m} = 1$, if

$(a\cos\alpha)^{\frac{m}{m-1}} + (b\sin\alpha)^{\frac{m}{m-1}} = p^{\frac{m}{m-1}}$

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26. If the line ax+by+c=0 is a normal to the curve xy=1 at the point (1,1), then prove that a and b are of opposite signs.

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27. If $0 < \alpha < \beta < \frac{\pi}{2}$, prove that, $\tan\alpha - \tan\beta < \alpha - \beta$.

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28. Find the equation of the normal to the curve

$y = (1 + x)^y + \sin^{-1}\left(\sin^2 x\right)$ at x=0.

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29. Prove that the the middle points of the normal chords of the parabola $y^2 = 4ax$ is on the curve $\frac{y^2}{2a} + \frac{4a^3}{y^2} = x - 2a$

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30. Prove that the normal at $\left(am^2, 2am\right)$ to the parabola $y^2 = 4ax$ meets the curve again at an angle $\tan^{-1}\left(\frac{1}{2}m\right)$.

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31. Show that the tangents at the end of any focal chord of the ellipse $x^2b^2 + y^2a^2 = a^2b^2$ intersect on the directrix.

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32. The tangent at the point θ on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, meets its auxiliary circle at two points whose join subtends a right angle at the centre, show that the eccentricity of the ellipse is given by,

$\frac{1}{e^2}=1+\sin^2\theta$

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33. Show that the tangent at any point on an ellipse and the tangent at the corresponding point on its auxiliary circle intersect on the major axis.

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34. $\int\frac{dx}{x\left(x^6+1\right)}$

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35. $\int \frac{dx}{a + be^x}$

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36. $\int \cos 2x \sin x dx$

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37. $P(\alpha, \alpha)$ is a point on the parabola $y^2 = 4ax$. Show that the normal chord of the parabola at P subtends a right angle at its focus.

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38. $\int \frac{dx}{e^x + e^{-x} + 2}$

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39. $\int \frac{1 - x^6}{1 - x} dx$

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40. The normal to the parabola $y^2 = 4ax$ at $P\left(am_1^2, 2am_1\right)$ intersects it again at $Q\left(am_2^2, 2am_2\right)$.If A be the vertex of the parabola then show that the area of the triangle

APQ is $\frac{2a^2}{m_1}\left(1 + m_1^2\right)\left(2 + m_1^2\right)$.

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41. If the normal at one end of lotus rectum of an ellipse passes through one end of minor axis then prove that,

$e^4 + e^2 - 1 = 0$

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42. If the normal at one end of lotus rectum of an ellipse passes through one end of minor axis then prove that,

$e^2 = \frac{\sqrt{5} - 1}{2}$, where e is the eccentricity of the ellipse.

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43. Find for what values of x the following functions are maximum and minimum :

$y = x^5 - 5x^4 + 5x^3 - 1$

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44. Find for what values of x the following functions are maximum and minimum :

$f(x) = x^2(x - 1)^3$

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45. Find for what values of x the following functions are maximum and minimum :

$5x^6 - 18x^5 + 15x^4 - 10.$

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46. Prove that the maximum value of $\frac{(2x-1)(x-8)}{(x-1)(x-4)}$ is less then its minimum value.

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47. Show that, f(x) $= \sin x - \frac{x^3}{3!} - \frac{x^5}{5!}$ is neither maximum nor minimum at x=0.

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48. If $f(x)=\dfrac{ax+b}{(x-1)(x-4)}$ has an extreme value at $(2,-1)$, find a and b and show that the extreme value is a maximum.

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49. $\int \dfrac{1-x^4}{1-x}dx$

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50. Prove that a conical tent of given capacity will required the least amount of canvas when the height is $\sqrt{2}$times the radius of the base.

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51. Prove that the function f(x) $= kx\left(x^2+a^2\right)^{-\frac{5}{2}}$has a maximum value at $x=\dfrac{a}{2}$.

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52. The total surface area of a right circular cone is given . Show that the volume of the cone is maximum when the semi-vertical angle is $\sin^{-1}\left(\frac{1}{3}\right)$.

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53. $\int \frac{dx}{\sin^4 x + \cos^4 x}$

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54. Show that the radius of the right circular cylinder of greatest curved surface which can be inscribed in given cone is half that of the cone.

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55. An open tank of volume 32 cu. Metre consists of a square base with vertical sides. Find the dimensions of the tank when the expense of lining it with lead is minimum.

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56. The cost of fuel of an engine varies as the square of its velocity and the cost of fuel is ₹ 48 per hour when the velocity is 16 km per hour. If other expenses be ₹ 300 per hour, then show that the most economical velocity for a journey of a given distance is 40 km per hour.

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57. A wire of length l is to be cut into to pieces, one being bent of form a square and the other to form a circle.How should the wire be cut if the sum of areas enclosed by the two pieces to be a minimum ?

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58. Find the point of the hyperbola $\frac{x^2}{24} - \frac{y^2}{18} = 1$ which is nearest to the line 3x+2y+1=0 and compute the distance between the point and the line.

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59. $\int \frac{(x+1)dx}{x\left(1+xe^x\right)}$

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60. $\int \frac{x^2 dx}{\left(x^3-1\right)\left(x^3+4\right)}$

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61. Evaluate $\int \frac{x^2-1}{x^4+x^2+1}$

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62. Evaluate $\int \frac{dx}{1 + \sin x + \cos x}$

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63. $\lim_{x \to 4} \frac{\sqrt{1 + \sqrt{2 + \sqrt{x}}} - \sqrt{3}}{x - 2}$

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64. $\lim_{x \to \infty} \left(\sqrt{x + \sqrt{x + \sqrt{x}}} - \sqrt{x}\right)$

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65. $\lim_{x \to \infty} \frac{\sqrt{1 + x^4} - \left(1 + x^2\right)}{x^2}$

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66. Divide 64 into two parts such that the sum of the cubes of the two parts is minimum.

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67. What normal to the curve $y = x^2$ forms the shortest chord ?

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68. Find the point on the curve $4x^2 + a^2y^2 = 4a^2, 4 < a^2 < 8$ that is farthest from the point $(0, -2)$.

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69. Determine the points of maxima and minima of the function

f(x) $= \frac{1}{8}\log x - bx + x^2, x > 0$, where $b \geq 0$ is a constant.

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70. Investigate for maxima and minima of the function

f(x) = $\int_1^x \left[2(t-1)(t-2)^3 + 3(t-1)^2(t-2)^2\right]dt.$

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71. A cubic function f(x) vanishes at $x = -2$ and has relative maximum/minimum at $x = -1$ and $x = \frac{1}{3}$. If $\int_{-1}^{1} f(x)dx = \frac{14}{3}$, find the cubic function f(x).

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72. $\lim_{x \to \infty} \left(\sqrt{x - \sqrt{x}} - \sqrt{x}\right)$

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73. Find the area bounded by the curves $y = \sin x$ and y=cos x between two consecutive points of their intersection.

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74. Find the area bounded by the parabola $y = x^2 - 6x + 10$ and the straight lines x=6 and y=2.

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75. Show that the common area between $y^2 = ax$ and $x^2 + y^2 = 4ax$ is $\left(3\sqrt{3} + \frac{4\pi}{3}\right)a^2$ square units.

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76. $\lim_{x \to 0} \frac{\sqrt{1 - x^2} - \sqrt{1 + x^2}}{x^2}$

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77. If the area enclosed by the parabola $x^2 = 72y$ and the line y=k be $64\sqrt{2}$ square units, then show that the given line touches the circle $x^2 + y^2 - 4y = 0$.

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78. $\lim x \to \infty \left(\sqrt{x^2 + 2x + 1} - x\right)$

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79. Find the area of the region bounded by the curves $2y^2 = x, 3y^2 = x + 1, y = 0$.

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80. If $sec\theta - \tan\theta = \frac{a+1}{a-1}$, then $\cos\theta$=

A. $\frac{a^2+1}{a^2-1}$

B. $\frac{a^2-1}{a^2+1}$

C. $\frac{2a^2}{a^2+1}$

D. $\frac{2a^2}{a^2-1}$

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81. Find the area bounded by the curve $y = x(x-1)^2$, the y-axis and the line y=2.

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82. If $\tan\theta = \frac{p}{q}$ then find the value of $\frac{p\sin\theta - q\cos\theta}{p\sin\theta + q\cos\theta}$

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83. Find the area bounded by the curve $y = 4x(x - 1)(x - 2)$ and the x-axis.

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84. Find the area in the first quadrant between $y^2 = 4x, y6(2) = 16x$ and the straight line x=9.

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85. Show that the function f(x) $= \sin^4 x + \cos^4 x$ is increasing in $\frac{\pi}{4} < x < \frac{3\pi}{8}$

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86. If f(x) $= \sin x - a\sin 2x - \frac{1}{3}\sin 3x + 2ax$ is an increasing function for all real values of x, show that, $a > 1$.

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87. Find the greatest and least value of $a\sin\theta + b\cos\theta$

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88. Find the cylinder of maximum volume which can be inscribed in a cone of height h and semivertical angle α.

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89. Find the equation of the straight line which is tangent at one point and normal at another point of the curve $x = 3t^2, y = 2t^3$.

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90. If the tangent at any point of the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ meets the coordinate axes in A and B, then show that the locus of mid-points of AB is a circle.

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91. If $\frac{2\sin\alpha}{1 + \cos\alpha + \sin\alpha} = x$ then find $\frac{1 - \cos\alpha + \sin\alpha}{1 + \sin\alpha}$

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92. If f(x)=$\int e^x(x - 1)(x - 2)dx$, then show that f(x) is a decreasing function $1 < x < 2$.

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93. Prove that, f(x) $= x + 2 + (x - 2)e^x$ is positive for all positive values of x.

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94. Show that the normal at the point $\left(3t, \frac{4}{t}\right)$ to the curve xy=12 cuts the curve again at the point whose parameter t_1 is given by $t_1 = -\frac{16}{9t^3}$

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95. The points (-2,-5), (2,-2), (8,a) are collinear, then find the value of a

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96. Show that of all isosceles triangles inscribed in a given circle, the equilateral triangle has the greatest area.

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97. If $x > a \geq e$, then show that, $a^x > x^a$.

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98. Find the values of x for which the function f(x) $= x^2(x - 2)^2$ will be an increasing function.

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99. The locus of $z = x + iy$ satisfying $\left|\frac{z - i}{z + i}\right| = 3$ then find the radious of the circle

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100. Determine the constant c such that the straight line joining the points (0,3) and $(5 - 2)$ is a tangent to the curve $y = \frac{c}{x + 1}$.

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101. Show that the minimum value of the length of a tangent to the ellipse $b^2x^2 + a^2y^2 = a^2b^2$ intercepted between the axes is (a+b).

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102. The parametric equations of a curve are given by $x - \sec^2 t, y = \cot t$.If the tangent at $P\left(t = \frac{\pi}{4}\right)$ meets the curve again at Q, then show that, $PQ = \frac{3\sqrt{5}}{2}$.

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103. If the tangent to the curve $x^3 + y^3 = a^3$ at the point $\left(x_1, y_1\right)$ intersects the curve again at the point $\left(x_2, y_2\right)$, then show that, $\frac{x_2}{x_1} + \frac{y_2}{y_1} + 1 = 0$.

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104. Find all the tangents to the curve $y = \cos(x + y), - 2\pi \leq x \leq 2\pi$ that are parallel to the line $x + 2y = 0$.

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105. The locus of $z = x + iy$ satisfying $\left|\frac{z - i}{z + i}\right| = 1$

A. $x = 0$

B. $y = 0$

C. $x = y$

D. $x + y = 0$

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106. find the area enclosed by the parabola $ay = 3\left(a^2 - x^2\right)$ and the x-axis.

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107. find the area of the region bounded by the parabola $y = x^2$, the line $y = x + 2$ and the x-axis.

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108. Show that the minimum distance from the origin to a point of the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ is $\left(\frac{a}{\sqrt{8}}\right)$.

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109. The normal at a point P on the hyperbola $b^2x^2 - a^2y^2 = a^2b^2$ of eccentricity e, intersects the coordinates axes at Q and R respectively. Prove that the locus of the mid-point of QR is a hyperbola of eccentricity $\frac{e}{\sqrt{e^2 - 1}}$.

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110. Find the abscissa of the point on the curve $x^3 = ay^2$, the normal at which cuts off equal intecepts from the coordinate axes.

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111. The parametric equation of a curve is given by, $x = a\left(\cos t + \log\tan\left(\frac{t}{2}\right)\right)$, y=a sin t. Prove that the portion of its tangent between the point of contact and the x-axis is of constant length.

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112. The locus of $z = x + iy$ satisfying $\left|\frac{z - i}{z + i}\right| = 2$

A. $3\left(x^2 + y^2\right) + 10y - 3 = 0$

B. $3\left(x^2 + y^2\right) + 10y + 3 = 0$

C. $3\left(x^2 + y^2\right) - 10y - 3 = 0$

D. $x^2 + y^2 - 5y + 3 = 0$

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113. The radius of a right circular cone is measured as 10 cm with a possible error of 0.02 cm and height as 16 cm with a possible error of 0.08 cm. Find the percentage error in computting the volume of the cone.

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114. The effeciency of a machine is given by, $E = \frac{\tan\theta}{\tan(\theta + \alpha)}$ where α is constant. Prove that, E is maximum at $\theta = \frac{\pi}{4} - \frac{\alpha}{2}$ and its maximum value is $\frac{1 - \sin\alpha}{1 + \sin\alpha}$.

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115. A normal is drwn at a point P(x,y) of a curve. It meets the x-axis at Q. If PQ is of constant length k, then show that the differential equation describing such curves is $y\frac{dy}{dx} = \pm\sqrt{k^2 - y^2}$. Find the equation of such a curve passing through (0,k).

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116. Find the area enclosed between the parabolas $y^2 = 4b(b - x)$ and $y^2 = 4a(x + a)$.

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117. Assuming the petrol burnt per hours in driving a motor boat varies as the cube of its velocity, show that the most economical speed when going against a current of c km per hour is $\frac{3c}{2}$ km per hour.

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118. The volume of a right prism is 16 cu.m. The base of the prism is in equilateral triangle. What must be the length of the side of the base for the least total surface area of the prism ?

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119. $\dfrac{1+\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}.....\infty}{1+\frac{1}{3!}+\frac{1}{5!}+\frac{1}{7!}+....\infty}$ equals

A. $\dfrac{e^2+1}{e^2-1}$

B. $\dfrac{e^2-1}{e^2+1}$

C. e^2+1

D. e^2-1

Answer: A

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120. Find the normal to the ellipse $4x^2 + 9y^2 = 36$ which is farthest from its centre.

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121. Find the dimensions of the rectangle of maximum area that can be inscribed in a semicircle of radius r.

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122. $1 + \frac{3^2}{2!} + \frac{3^4}{4!} + \frac{3^6}{6!} + \ldots\ldots\infty$ is equal to

A. $\frac{1}{2}\left(e^3 - e^{-3}\right)$

B. $\frac{1}{2}\left(e^3 + e^{-3}\right)$

C. e^3

D. e^{-3}

Answer: B

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123. A box is constructed from a rectangular metal sheet 21 cm by 16 cm cutting equal squares of sides x cm from the corners of the sheet and then turning up the projected portions. For what value of x the volume of the box will be maximum ?

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124. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius r is $\frac{2r}{\sqrt{3}}$.

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125. The total surface area of a right circular cone is given . Show that the volume of the cone is maximum when the semi-vertical angle is $\sin^{-1}\left(\frac{1}{3}\right)$.

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126. The area bounded by the parabola $y = x - x^2$ and the line y=mx equals $\frac{9}{2}$, find m.

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127. Prove that the radius of the right circular cylinder of greatest curved surface which can be inscribed in a given cone is half that of the cone.

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128. Let LL' be the latus rectum of the parabola $y^2 = 4ax$and P P' be a double ordinate between the vertex and the latus rectum. Prove that the area of the trapezium L L' P P' is maximum, when the distance of P'P from vertex is $\frac{a}{9}$.

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129. $1+\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+\ldots\ldots\infty$ is equal to

A. $\frac{1}{2}\left(e+\frac{1}{e}\right)$

B. $\frac{1}{2}\left(e-\frac{1}{e}\right)$

C. e

D. e^{-1}

Answer: A

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130. A point P is given on the circumference of a circle of radius r. The chord QR is parallel to the tangent line at P. Find the maximum area of

the triangle PQR.

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131. Let $f(x) = \sin^3 x + k\sin^2 x, \ -\frac{\pi}{2} < x < \frac{\pi}{2}$. Find the interval in which k should lie in order that f(x) has exactly one minimum and exactly one maximum.

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132. $1 + \frac{1}{3!} + \frac{1}{5!} + \frac{1}{7!} + \ldots\ldots\infty$ is equal to

A. $\frac{1}{2}\left(e + \frac{1}{e}\right)$

B. $\frac{1}{2}\left(e - \frac{1}{e}\right)$

C. e

D. e^{-1}

Answer: B

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133. Show that the semivertical angle of a cone of given slant height and maximum volume is $\tan^{-1}\sqrt{2}$.

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134. $\frac{2}{1!} + \frac{3}{2!} + \frac{4}{3!} + \ldots\ldots\infty$ is equal to

A. $e - 1$

B. $2e - 1$

C. $2e$

D. 1

Answer: B

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135. A curve y=f(x) passes through the point P(1,1). The normal to the curve at P is a(y-1)+(x-1)=0. If the slope of the tangent at any point on the curve is proportional to the ordinate of the point, find the equation of the curve.

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136. $\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+......\infty$ is equal to

A. e

B. $e-1$

C. $e+1$

D. 1

Answer: D

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137. $\dfrac{1+\dfrac{1}{2!}+\dfrac{2}{3!}+\dfrac{2^2}{4!}+\dfrac{2^3}{5!}+\ldots.\infty}{1+\dfrac{1}{2!}+\dfrac{1}{4!}+\dfrac{1}{6!}+\ldots\infty}$ equals

A. $\dfrac{e}{4}$

B. $8e$

C. $\dfrac{e}{2}$

D. $\dfrac{e\left(e^2+1\right)}{2\left(e^2-1\right)}$

Answer: C

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138. The sum of the surface of a sphere and a cube is given. Prove that the sum of their volumes is least when the diameter of the sphere is equal to the edge of the cube.

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139. Find the area bounded by the parabola $y^2 = 9x$ and the straight line $x - y + 2 = 0$

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140. In a certain culture, the number of bacteria at any instant increases at a rate proportional to the cube root of the number present at that instant. If the number becomes 8 times in 3 hours, when the number will be 64 times?

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141. Show that the length of the tangent to the curve $x^m y^n = a^{m+n}$ at any point of it, intercepted between the coordinate axes is divided internally by the point of contact in the ratio m:n.

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142. Area bounded by the parabola $2y = x^2$ and the straight line $x = y - 4$is

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143. Show that the locus of the midpoints of the chords of the circle $x^2 + y^2 = a^2$ which are tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $\left(x^2 + y^2\right)^2 = a^2x^2 - b^2y^2$.

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144. Find the equation of the curve whose length of the tangent at any point on ot, intercepted between the coordinate axes is bisected by the point of contact.

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145. Find those values of a for which the function

$$f(x) = \left(\frac{\sqrt{a+4}}{1-a} - 1\right)x^5 - 3x + \log 5$$

is decreasing for all real values of x.

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146. The function $y = a\log|x| + bx^2 + x$ has two extreme values for $x = -1$ and x=2. Find the values of a and b.

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Vector and Three Dimensional Coordinate Geometry

1. Find the angle between the vectors $\vec{a} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - \hat{k}$.

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2. Given $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ are two perpendicular vectors, find λ.

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3. Vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ and $|\vec{a}|| = 2, |\vec{b}\,| = 4$ and $|\vec{c}| = 6$, prove that, $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = -28$.

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4. Find the cosine of the angle made by the vector $\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right)$ with the posititve z-axis.

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5. For what value of a, the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $a\hat{i} - 6\hat{j} + 8\hat{k}$ are collinear ?

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6. The vector $\vec{a}$ is perpendicular to each of the vectors $\vec{b} = 4\hat{i} + 5\hat{j} - \hat{k}$, $\vec{c} = \hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{a}.\vec{d} = 21$, where $\vec{d} = 3\hat{i} + \hat{j} - \hat{k}$, find the vector $\vec{a}$.

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7. The vectors $\frac{29}{3}\hat{i} - 4\hat{j} + 5\hat{k}, 2\hat{i} + m\hat{j} + \hat{k}$ and $\vec{i} + 2\hat{j} + \hat{k}$ are coplanar, find m.

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8. The position vectors of the vertices A,B,C of the triangle ABC are $\left(\hat{i} + \hat{j} + \hat{k}\right)$, $\left(\hat{i} + 5\hat{j} - \hat{k}\right)$ and $\left(2\hat{i} + 3\hat{j} + 5\hat{k}\right)$ respectively. Find the greatest angle of the triangle.

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9. Show, by vector method, that the angle in a semicircle is a right angle.

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10. $\vec{a} = 3\hat{i} - 4\hat{j} - 4\hat{k}, \vec{b} = 3\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$ are three edges of a rectangular parallelopiped, prove that the volume of the parallelopiped is 49 cu unit.

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11. If the vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}, \vec{b} = \hat{i} + m\hat{j} - 3\hat{k}$ and $\vec{c} = 3\hat{i} - 4\hat{j} + 5\hat{k}$ are coplanar, then find m.

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12. Find the equation of the plane which contains the line of intersection of the planes $\vec{r}.\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) - 4 = 0$ and $\vec{r}.\left(2\hat{i} + \hat{j} - \hat{k}\right) + 5 = 0$ and which is perpendicular to the plane $\vec{r}.\left(5\hat{i}+3\hat{j}-6\hat{k}\right)+8=0$

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13. Find the cartesian and vector equations of the planes through the line of intersection of the planes $\vec{r}.\left(2\hat{i}+6\hat{j}\right)+12=0$ and $\vec{r}.\left(3\hat{i}-\hat{j}+4\hat{k}\right)=0$ which are at a unit distance from the origin.

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14. Find the equation of the plane passing through the line of intersection of the planes $\vec{r}.\left(\hat{i}+\hat{j}+\hat{k}\right)=1$ and $\vec{r}.\left(2\hat{i}+3\hat{j}-\hat{k}\right)+4=0$ and parallel to x-axis.

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15. Find the coordinates of the foot of the perpendicular drawn from the point A(1,2,1) on the line joining the points B(,1,4,6) and C(5,4,4,).

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16. Find the vector equation of the following plane in scalar product form :

$\vec{r} = \left(\hat{i} + \hat{j}\right) + \lambda\left(\hat{i} + 2\hat{j} - \hat{k}\right) + \mu\left(-\hat{i} + \hat{j} - 2\hat{k}\right)$

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17. Find the distance of the point $(-1, -5, -10)$ from point of intersection of the line $\vec{r} = \left(2\hat{i} - \hat{j} + 2\hat{k}\right) + \lambda\left(3\hat{i} + 4\hat{j} + 2\hat{k}\right)$ and the plane $\vec{r}.\left(\hat{i} - \hat{j} + \hat{k}\right) = 5.$

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18. Find the equation of the plane passing through the point (1,2,1) and perpendicular to the line joining the points (1,4,2) and (2,3,5). Also find the

coordinates of the foot of the perpendicular and the perpendicular distance of the point (4,0,3) from the above found plane.

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19. Find the equation of the plane passing through the points (3,4,1) and (0,1,0) and parallel to the line $\frac{x+3}{2} = \frac{y-3}{7} = \frac{z-2}{5}$.

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20. Find the equation of the plane passing through the points (3,1,1), $(1-2, 3)$ and parallel to y-axis.

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21. Find the vector equation of the line passing through the point $(2, -3, 1)$ and parallel to the planes $\vec{r}.\left(2\hat{i} - 3\hat{j} - \hat{k}\right) = 3$ and $\vec{r}.\left(\hat{i} + \hat{j} - \hat{k}\right) = 4.$

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22. Show that the line whose vector equation is $\vec{r} = \left(2\hat{i} - 2\hat{j} + 3\hat{k}\right) + \lambda\left(\hat{i} - \hat{i} + 4\hat{k}\right)$ is parallel to the plane whose vector equation is $\vec{r}.\left(\vec{i} + 5\hat{j} + \hat{k}\right) = 5$. Also, find the distance between them.

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23. Show that the line $\frac{x+3}{4} = \frac{y-5}{-1} = \frac{z+7}{2}$ lies in the plane $x - 2y - 3z - 8 = 0$.

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24. Find the angle between the line $\frac{x-2}{1} = \frac{y+1}{3} = \frac{z-4}{2}$ and the plane $3x - 2y + 4z = 6$.

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25. Find the vector and cartesion equations of a plane containing the two lines $\vec{r} = 2\hat{j} + \hat{j} - 3\hat{k} + \lambda\left(\hat{i} + 2\hat{j} + 5\hat{k}\right)$ and $\vec{r} = 3\hat{i} + 3\hat{j} + 2\hat{k} + \mu\left(3\hat{i} - 2\hat{j} + 5\hat{k}\right)$ .Also show that the line $\vec{r} = 2\hat{i} + 5\hat{j} + 2\hat{k} + p\left(3\hat{i} - 2\hat{j} + 5\hat{k}\right)$ lies in the plane.

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26. Find the vector equation of the plane that contains the lines $\vec{r} = \left(\hat{i} + \hat{j}\right) + \lambda\left(\hat{i} + 2\hat{j} - \hat{k}\right)$ and $\vec{r} = \left(\hat{i} + \hat{j}\right) + \mu\left(-\hat{i} + \hat{j} - 2\hat{k}\right)$.Also find the length of the perpendicular drawn from the point (2,1,4) to the plane thus obtained.

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27. The equation of a line are $\frac{4 - x}{2} = \frac{y + 3}{2} = \frac{z + 2}{1}$. Find the direction cosines of a line parallel to the above line.

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28. Find the equation of a line passing through the point $P(2, -1, 3)$ and perpendicular to the lines $\vec{r} = \left(\hat{i} + \hat{j} - \hat{k}\right) + \lambda\left(2\hat{i} - 2\hat{j} + \hat{k}\right)$ and $\vec{r} = \left(2\hat{i} - \hat{j} - 3\hat{k}\right) + \mu\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$.

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29. If the lines $x = a_1y + b_1, z = c_1y + d_1$ and $x = a_2y + b_2, z = c_2y + d_2$ are perpendicular, prove that, $1 + a_1a_2 + c_1c_2 = 0$.

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30. Computting the shortest between the following pair of lines determine whether they intersect or not:

$\vec{r} = -3\hat{i} + 6\hat{j} + \lambda\left(-4\hat{i} + 3\hat{i} + 2\hat{k}\right)$ and $\vec{r} = -2\hat{i} + 7\hat{k} + \mu\left(-4\hat{i} + \hat{j} + \hat{k}\right)$

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31. Find whether the lines $\vec{r} = \left(\hat{i} - \hat{j} - \hat{k}\right) + \lambda\left(2\hat{i} + \hat{j}\right)$ and $\vec{r} = \left(2\hat{i} - \hat{j}\right) + \mu\left(\hat{i} + \hat{j} - \hat{k}\right)$ intersect or not. If intersecting find their point of intersection.

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32. If the area bounded by the curves $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16a^2}{3}$ sq-unit then find the area bounded by $y^2 = 2x\text{and}x^2 = 2y$

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33. Find the equation of the line passing through the point $(1, 2, -4)$ and perpendicualr to two lines

$\frac{x - 8}{3} = \frac{y + 19}{-16} = \frac{z - 10}{7}$ and $\frac{x - 15}{3} = \frac{y + 29}{8} = \frac{z - 5}{-5}$.

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34. Find the coordinate of the foot of the perpendicular, the equation of the perpendicular and the perpendicular distance of the point P(3,2,1) from the plane $2x - y + z + 1 = 0$. Find also the image of the point in the plane.

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35. Find the equation of the plane passing through the point (1,1,1) and containing the line

$\vec{r} = \left(-3\hat{i} + \hat{j} + 5\hat{k}\right) + \lambda\left(3\hat{i} - \hat{j} - 5\hat{k}\right)$

Also show that the plane contains the line

$\vec{r} = \left(-\hat{i} + 2\hat{j} + 5\hat{k}\right) + \lambda\left(\hat{i} - 2\hat{j} - 5\hat{k}\right)$.

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36. Find the shortest distance between the following the following lines whose vector equations are

$\vec{r} = (1 - t)\hat{i} + (t - 2)\hat{j} + (3 - 2t)\hat{k}$ and $\vec{r} = (s - 1)\hat{i} + (2s - 1)\hat{j} + (2s + 1)\hat{k}$.

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37. Find the equation of a line passing throught the points A(0,6,-9) and B(-3,-63). If D is the foot of the perpendicular drawn from a point C(7,4,-1) on the line AB, then find the coordinates of D and the equation of line CD.

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Probability

1. In a school, there are 1000 students, out of which 430 are girls. It is known that out of 430, 10% of the girls study in class XII. What is the probability that a student chosen randomly studies in class XII, given that the chosen student is a girl ?

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2. On a multiple choice examination with three possible answers(out of which only one is correct) for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing ?

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3. There are two bags, bag I and bagII. Bag I contains 4 white 3 red balls while bag II contains 3 white and 7 red balls. One ball is drawn at random from one of the bags and it is found to be white. Find the probability that it was drawn from bag I.

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4. In a class, 5% of the boys and 10% of the girls have an IQ more than 150. In the class 60% of the students are boys and rest girls. If a student is selected at random and found to have an IQ of more that 150, find the probability that the student is a boy.

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5. A bag contains 4 red and 4 black balls. Another bag contains 2 red and 6 black balls. One of the bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag.

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6. Three cards are drawn seccessively with replacement from a well shuffled pack of 52 playing cards. If getting a card of spade is considered success, find the probability distribution of the number of successes.

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7. Calculate the value of : $(i)^{-22}$

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8. Two groups are competing for the position on the Board of Directors of a corporation. The probabilities that the first and the second groups will win are 0.6 and 0.4 respectively. Further, if the first group wins, the probability of introducing a new product is 0.7 and the corresponding probability in 0.3 if the second group wins. Find the probability that the new product was introduced by the second group,

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